Optimal Budget Deficit Rules

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Abstract

This paper discusses the problem of the optimal determination of budget deficit limits in cases where the fiscal authority wishes to keep the budget deficit close to a reference value. It is assumed that the fiscal authority minimizes the expected discounted value of squared deviations from the reference value. Lump-sum and proportional intervention costs are considered. This paper is also an example of integration between stochastic process optimal control methods and the continuous time stochastic models. In fact, the characteristics of the stochastic process that rules the path of the budget deficit are taken from a previously developed continuous time stochastic model (Amador, 1999). Finally, simulation methods are used in order to conduct a comparative dynamics analysis. The paper concludes that, in the case of proportional intervention costs, the optimal ceiling depends positively on the cost parameter and on the variance of the budget deficit. On the contrary, the optimal ceiling depends negatively on the average budget deficit. These results remain valid in the case where there are both lump-sum and proportional intervention costs. Finally, in a stationary equilibrium context, we conclude that economies with higher tax rates and lower public expenditure should set higher budget deficit ceilings. The same is true for economies with a higher variance in technology and public expenditure shocks.

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1 Introduction

The analysis of the behaviour of budget deficits has been a key issue in macroeconomic theory. In fact, the literature on the sustainability of the public debt has always deserved special attention from the part of policy makers and fiscal authorities. It has been argued that there is a large potential for unsustainable budget deficit behaviour. This may be due to changes in the structural parameters of the economy, such as tax rates, public expenditure or the variance of shocks. Such concerns are stronger in the context of a monetary union such as the one that is underway in the European Union. In these cases, where there is a common, independent and low inflation oriented central bank, budget deficit financing is more difficult. The fact is that individual countries cannot finance the deficit through the issuing of new money. In addition, individual countries cannot reduce the real value of public debt through surprise inflation shocks. These restrictions to the financing of budget deficits add to another type of risk. In monetary unions countries do not hold any other major macroeconomic instrument other than the fiscal policy. As a result, they may be tempted to use it in a non-sustainable way, namely to promote short-run growth. If individual countries arrive at a situation where they can no longer finance the budget deficits, then they will have to be bailed-out or abandon the monetary union. In the first case, the credibility of the monetary union is put in danger. In fact, if any country is bailed-out by a central authority, a moral hazard problem emerges. In this situation other countries lack the incentives to keep their deficits under control. In the second case, if a country exits the monetary union, the common bond market is seriously affected. In this situation investors tend to run from public bonds, which leads to budgeting problems in the other countries. Thus, it is vital to keep budget deficits under control in monetary unions.

This is the reason why the Maastricht Treaty has included two public finance indicators in the set of nominal criteria that worked as pre-conditions for the entry in the European Monetary Union. Namely, the budget deficit should be lower than 3% of GDP and the public debt should be lower or heading closely towards 60% of GDP. The risks that have been discussed above are also the reason why the Stability and Growth Pact was signed. The Pact states that the upper barrier of 3% of GDP is to be kept and countries should make an effort to run a balanced budget. This would allow some scope for adjustments when needed. In addition, financial penalties were set to punish countries that repeatedly violate these constraints.

Nevertheless, there is no clear theoretical explanation for the choice of 3% of GDP as the optimal value for the budget deficit constraint. Furthermore, it can be questioned if this barrier is equally optimal for all countries or if it should depend on individual structural parameters and intervention costs. There are several papers which discuss this problem. A short and necessarily incomplete list includes Buiter, Corsetti and Roubini (1993), Alesina and Bayoumi (1996), Corsetti and Roubini (1996), Obstfeld and Peri (1998) and Perotti, Strauch and von Hagen (1998). Our paper examines the optimal setting of budget deficit constraints in an economy where the fiscal authority wishes to keep the deficit close to a reference value. Different types of intervention costs are considered and smooth pasting techniques are used to derive the solution.

This paper can also be taken as an example of integration between stochastic process optimal control methods and the continuous time stochastic models. The smooth pasting techniques have been widely used in models of optimal currency bands and speculative attacks. The key references in this area are Krugman (1991) and Krugman and Miller (1992). However, these models take the path of the exchange rate as a pre-defined stochastic process. In some cases it is said that the stochastic process is the reduced form solution of a non-specified model. In the same way, there is a class of continuous time stochastic models, developed by Turnovsky (1995), which generates stochastic processes as closed form solutions. This type of model is closely related with the optimal portfolio literature, but is focused instead on macroeconomic performance. Thus, there is scope for integration between the two literatures. In our paper, the characteristics of the stochastic process that rules the path of the budget deficit are taken from a previously developed continuous time stochastic model (see Amador, 1999). Due to this, it is possible to establish some relationships between the structural parameters of the economy and the optimal budget deficit rules.

The paper concludes that, in the case of proportional intervention costs, the optimal ceiling is positively affected by the cost parameter and by the variance of the deficit. On the contrary, the optimal ceiling is negatively affected by the average budget deficit. It is shown that these results remain valid in the case where there are both lump-sum and proportional intervention costs. Finally, in a stationary equilibrium context, it is shown that economies with higher tax rates and lower public expenditure should set higher budget deficit ceilings. The same is true for economies facing a higher variance in technology and public expenditure shocks.

The paper is divided into five sections. In the second section the problem of the fiscal authority is presented. Then, in the third section, different types of intervention costs are considered and the expressions for the optimal budget deficit rules are obtained. Next, in the fourth section we simulate the path of the budget deficit and a comparative dynamics analysis is presented. Finally, in the fifth section the main results are gathered in a brief conclusion.

2 The Problem Facing the Fiscal Authority

This section presents the problem of a fiscal authority whose concern is to keep the budget deficit near an established reference value. It will be assumed that the fiscal authority is concerned, not with the nominal budget deficit, but with the budget deficit-average output ratio. This variable is defined as $\frac{dB}{Y^D}$ and it measures the relative weight of the budget deficit in the economy[1].

It is also assumed that the fiscal authority aims to keep the budget deficit-average output ratio as close as possible to zero. That means that the objective of the fiscal authority is to run balanced budgets. This assumption is not restrictive. In fact, the model would be developed in the same way if we chose any other reference value. In such cases, the budget deficit could simply be measured in terms of deviations from the new reference value and the calculations would be identical to those presented below. In addition, it is assumed that the fiscal authority is increasingly worse-off the larger the deviations are from the reference value.

Therefore, the objective function of the fiscal authority is the expected discounted value of a payoff function, which depends on $\frac{dB}{Y^D}$, that is:

$$F\left(\frac{dB}{Y^D}\right) = E\int_0^\infty f\left(\frac{dB}{Y^D}\right)e^{-\beta t}dt \tag{1}$$

where $f\left(\frac{dB}{Y^D}\right)$ stands for the payoff function and β is an exogenous intertemporal discount rate. Moreover, the fact of having a fiscal authority which increasingly dislikes larger budget deficits is included by assuming a quadratic loss function. That is:

$$f\left(\frac{dB}{Y^D}\right) = \left(\frac{dB}{Y^D}\right)^2 \tag{2}$$

At this point it is necessary to present the expression for the expected present value of this particular quadratic payoff function. This expression is presented by Dixit (1993) and we recall the main steps of its derivation in the Appendix. As a matter of fact, the expected present value of the quadratic payoff function is given by an expression of the type:

[1]Note that $\frac{dB}{Y^D}$ is different from the definition of structural budget deficit. In our model, the actual budget deficit dB is measured as a proportion of the average output. The total output is given by a stochastic production function $dY = \alpha K dt + \alpha K dy$, where α is a technological parameter, K is the capital stock in the economy and dy is a technology shock with zero mean and variance σ_y^2. In this context, we simply consider that $Y^D = \alpha K$.

$$F\left(\frac{dB}{Y^D}\right) = \frac{\sigma^2}{\beta^2} + \frac{2\mu^2}{\beta^3} + \frac{2\mu\left(\frac{dB}{Y^D}\right)}{\beta^2} + \frac{\left(\frac{dB}{Y^D}\right)^2}{\beta} + \\ Ae^{-a\left(\frac{dB}{Y^D}\right)} + Be^{b\left(\frac{dB}{Y^D}\right)} \tag{3}$$

where A and B are two constants to be determined afterwards, and a and b are the roots of the Fundamental Quadratic of Brownian motion defined as:

$$\psi(\lambda) \equiv \beta - \mu\lambda - \frac{1}{2}\sigma^2\lambda^2 \tag{4}$$

where, $\sigma^2$and μ are respectively the variance and the growth rate of the budget deficit-average output ratio. Additionally, through simple calculus, it is possible to determine the roots of the fundamental quadratic defined by equation 4. Note that since $\sigma^2 > 0$ and $\psi(0) = \beta > 0$, there are certainly two roots with opposite signs. These are:

$$a = -\frac{1}{\sigma^2}\left(\mu + \sqrt{\mu^2 + 2\sigma^2\beta}\right) < 0 \tag{2.5}$$

$$b = -\frac{1}{\sigma^2}\left(\mu - \sqrt{\mu^2 + 2\sigma^2\beta}\right) > 0 \tag{2.6}$$

Finally, it is necessary to guarantee that the expected present value $F\left(\frac{dB}{Y^D}\right)$ is finite. This issue is discussed in the Appendix, where we also explain the meaning of the fundamental quadratic.

The next section presents the conditions that define the optimum limits for the budget deficit. These conditions depend on the form of the payoff function, on the characteristics of the stochastic process that regulates the budget deficit and on the type of intervention costs faced by the fiscal authority when trying to regulate it.

3 Optimal Limits for the Budget Deficit

In the previous section we presented the expression for the expected discounted welfare loss of budget deficits, given the assumed quadratic payoff function of the fiscal authority. Nevertheless, it is shown in the Appendix A that, in the case of an uncontrolled budget deficit, the last two terms of equation 3 are not necessary to define $F\left(\frac{dB}{Y^D}\right)$. In such a case we would have $A = B = 0$. However, in the presence of a fiscal authority which aims at controlling the budget deficit, these constants are not zero and depend on the intervention costs faced by the fiscal authority. In the next subsection we describe these costs.

3.1 Intervention Costs

The intervention costs faced by the fiscal authority when it affects the normal path of the budget deficit are assumed to be given by:

$$C = \overline{C} - c.d\left(\frac{dB}{Y^D}\right) \tag{6}$$

where $\overline{C}$ and c are cost parameters.

The cost function above contains both lump-sum and proportional intervention costs. At this point, we briefly discuss the rationale for considering these two types of intervention costs. Firstly, we consider the lump-sum component $\overline{C}$. The existence of such a cost, which is independent of the size of the intervention, can be interpreted as a loss of credibility faced by the fiscal authority when it needs to intervene on the budget deficit. The second component $c.d\left(\frac{dB}{Y^D}\right)$ states that the costs faced by the fiscal authority are proportional to the size of the interventions. In fact, a unitary negative deviation in the budget deficit-average output ratio implies an intervention cost of c. This is a realistic assumption because the welfare of the fiscal authority is certainly linked to macroeconomic performance. In fact, to implement downward corrections in the budget deficit is a contractionary policy, which reduces growth and employment. Unfortunately, recessions are themselves a cause of bad budget deficit performance. This is due to the effect of automatic stabilisers. In this sense, the stronger the downward intervention in the budget deficit, the larger the negative effect on the economy and the larger the costs faced by the fiscal authority.

3.2 Value Matching and Smooth Pasting Conditions

After having introduced the basic assumptions of the problem, it is now possible to determine the optimal limits for the budget deficit. Essentially, what we do next is the optimal control of the Brownian motion that describes the path of the budget deficit. We proceed in two steps. First, we assume exogenous limits for the budget deficit. In this case, the main point is to understand what the fiscal authority does when the limits are reached. This is the role played by the value matching conditions. A second and more interesting step is to consider that the limits are endogenous and optimally set by the fiscal authority. In this case it is necessary to know, not only how to intervene when the limits are reached, but also where they should be placed. In this second step the smooth pasting conditions are used.

It is empirically difficult to find fiscal authorities that deal with excessive budget surpluses. In fact, budget surpluses are rare events so we will assume that there are only ceilings to the budget deficit. This assumption automatically sets the constant A equal to zero. In fact, if there is no floor and we take $\frac{dB}{Y^D}$ as being highly negative, it is reasonable to expect that it will not reach the ceiling during the period of analysis. Therefore, $F(x)$ would be the expected discounted value of the uncontrolled stochastic process. However, taking equation 3 and making $\frac{dB}{Y^D} \to -\infty$ we would have $F\left(\frac{dB}{Y^D}\right) \to +\infty$. This lack of consistency can only be solved by setting $A = 0$.

3.2.1 Value Matching Conditions

The fact of having considered a lump-sum component in the cost function, makes it optimum for the fiscal authority to reset the budget deficit to a lower value when the ceiling is reached. In fact, it is not optimum to continuously intervene near the ceiling because the lump-sum cost must be faced in each intervention. Nevertheless, it may not be optimum to reset the budget deficit to zero. In fact, there is a second component in the intervention cost that depends on the size of the intervention. Thus, if we define the ceiling as u and the value of the budget deficit after resetting as r, the following condition must be verified:

$$F(u) - F(r) = \overline{C} + c(u - r) \tag{7}$$

which, using equation 3, is equivalent to:

$$\frac{2\mu(u-r)}{\beta^2} + \frac{u^2 - r^2}{\beta} + B\left(e^{bu} - e^{br}\right) = \overline{C} + c(u - r) \tag{8}$$

3.2.2 Smooth Pasting Conditions

In addition, $F\left(\frac{dB}{Y^D}\right)$ is positively affected by the constant B, which depends on parameters u and r. Therefore, these parameters must be chosen so that $B_u = B_r = 0$. Then, differentiating equation 8 in order to r and u and substituting the conditions above, we obtain:

$$\frac{2\mu}{\beta^2} + \frac{2r}{\beta} + bBe^{br} = c \tag{9}$$

$$\frac{2\mu}{\beta^2} + \frac{2u}{\beta} + bBe^{bu} = c \tag{10}$$

These are the smooth pasting conditions and can be generically written as: $F'(r) = F'(u) = c$.

3.3 Optimal Limit Analysis

In the previous subsections we have presented the value matching and the smooth pasting conditions. As noted above, the optimum barriers must verify these conditions, which is equivalent to saying that it is necessary to solve the system formed by equations 8, 9 and 10, where u, r and B are the unknowns. However, this is a three-equation non-linear system, which is almost always impossible to solve explicitly. Therefore, in the next section, we proceed with the numerical solution of the system. Meanwhile, we examine the special case where there are just proportional intervention costs.

In fact, this special case allows us to obtain a closed form solution for the optimal budget deficit ceiling. The same is not true for the case where there are just lump-sum intervention costs. As pointed-out, when there are no lump-sum costs it is optimal to continuously intervene at the ceiling. In this case the value matching condition is just:

$$F^{'}(u) = c \tag{11}$$

that is:

$$\frac{2\mu}{\beta^2} + \frac{2u}{\beta} + bBe^{bu} = c \tag{12}$$

As expected, the limit is placed such that the increase in welfare from intervening, measured by $F^{'}(u)$, equals the marginal cost of intervention c. As discussed, it is also true that $B_u = 0$. Thus, using this condition and differentiating equation 12 in order to u, we obtain:

$$\frac{2}{\beta} + b^2 Be^{bu} = 0 \tag{13}$$

which is equivalent to $F^{''}(u) = 0$. This is analogous to the smooth pasting condition and it is known in the literature as the super contact condition. For more details see Dumas (1991). Next, putting together equations 12 and 13 it is possible to write:

$$u = \frac{\beta}{2}\left(\frac{2}{b\beta} + c - \frac{2\mu}{\beta^2}\right) \tag{14}$$

which, using equation 2.6, can be rewritten as:

$$u = -\frac{\sigma^2}{\mu - \sqrt{(\mu^2 + 2\sigma^2\beta)}} + \frac{\beta}{2}c - \frac{\mu}{\beta} \tag{15}$$

At this point it is important to examine equation 15, and see how the optimal ceiling depends on the intervention cost parameter (c), on the trend of the budget deficit (μ) and on the variance of the deficit (σ^2). Recall that we are just considering the existence of proportional intervention costs. As observed, the consideration of other types of intervention costs does not allow for the determination of the optimal limit as a closed form expression such as 15.

The effect on the optimal ceiling of an increase in the marginal intervention cost is given by:

$$\frac{\partial u}{\partial c} = \frac{\beta}{2} > 0 \tag{16}$$

which is always positive. An increase in parameter c means that the fiscal authority faces higher costs when it intervenes and these costs are higher the larger the intervention. Thus, taking the other things constant, if it is costlier to reach the ceiling, then it is optimal to reach it less often or to reach it later in time. This means that u should be set at a higher value.

The effect of an increase in the growth rate of the budget deficit μ is given by:

$$\frac{\partial u}{\partial \mu} = \frac{1 - \mu \left(\mu^2 + 2\sigma^2\beta\right)^{-\frac{1}{2}}}{\frac{1}{\sigma^2}\left(\mu - \sqrt{(\mu^2 + 2\sigma^2\beta)}\right)^2} - \frac{1}{\beta} \tag{17}$$

which, does not have a definite sign. However, considering the range of reasonable values for the parameters, this derivative is clearly negative. In fact, when the budget deficit grows faster, the marginal decrease in the welfare of the fiscal authority, given by $\partial F/\partial t$, is also higher. Therefore, given the marginal intervention cost, it is optimal to intervene sooner, which is equivalent to saying that the optimal ceiling should be set at a lower level.

Finally, we examine the effect of an increase in the variance of the budget deficit σ^2. This effect is given by the derivative:

$$\frac{\partial u}{\partial \sigma^2} = -\frac{\mu - \sqrt{(\mu^2 + 2\sigma^2\beta)} + \sigma^2\beta \left(\mu^2 + 2\sigma^2\beta\right)^{-\frac{1}{2}}}{\left(\mu - \sqrt{(\mu^2 + 2\sigma^2\beta)}\right)^2} \tag{18}$$

which is positive, for the range of acceptable values for the parameters. Therefore, it is optimal to set the ceiling at a higher level. There are two effects leading to this result. Firstly, an increase in parameter σ^2 means that, for each u, the average size of interventions is higher. As we have said, this translates into higher intervention costs. Secondly, the benefit of intervening when a negative shock occurs is smaller. In fact, a higher σ^2 means that a strong increase in the budget deficit tends to be reverted in the next period. Therefore, the optimal u is higher.

In the next section we develop a comparative dynamics analysis in order to examine the more interesting case where there are both proportional and lump-sum intervention costs.

4 Comparative Dynamics

In this section we present different macroeconomic scenarios, illustrating economies with different characteristics. Each of these different scenarios is associated with a different calibration for the model. This allows for the numerical solution of the non-linear system of equations that defines the optimal limit in the case of mixed proportional and lump-sum intervention costs. This section is divided into three subsections. First, we compare two economies with different intervention costs. Then, we make use of a previously developed continuous time stochastic model (Amador, 1999) to examine the effects of public expenditure and taxation on the optimal budget deficit rules. Finally, the same thing is carried out for changes in the variance of technology and public expenditure shocks. As mentioned in the beginning, this paper is an example of integration between the stochastic process optimal control methods and the continuous time stochastic models. At this point this sentence is clearer. In fact, the continuous time stochastic model presents the effects of public expenditure, taxation and uncertainty on the behaviour of the deficit. Then, taking the trend and the variance of the deficit, and making use of the methods described above, the optimal deficit rules are determined.

4.1 The Effects of Changing Intervention Costs

This subsection takes equations 8, 9 and 10, and examines the effects of different intervention costs on the optimal budget deficit limits. To this purpose we introduce Economy 1 and Economy 2 in the two tables, below.

Each table contains the values of the exogenous and endogenous parameters in the model. The parameters in the first row, namely α (average rate of return on capital), β (intertemporal discount rate), g (proportion of public expenditure on output), τ (tax rate on output), σ_y^2 (variance of technology shocks) and σ_z^2 (variance of expenditure shocks) are exogenous[2].They en-

[2]The covariance between the two types of shocks is assumed to be zero. This is not a restrictive assumption. In fact, it is possible to examine the effects of such an exogenous parameter on the path of the budget deficit. See Amador (1999). We have also assumed the same intertemporal discount rate β for the representative consumer and the fiscal authority.

Parameter	α	β	g	τ	σ_y^2	σ_z^2
Value	0.15	0.05	0.25	0.35	0.01	0.01
Parameter	C	c	μ	σ^2	u	r
Value	0.0001	0.9	0.094	0.0017	0.0508	0.0124

Table 1: Economy 1

Parameter	α	β	g	τ	σ_y^2	σ_z^2
Value	0.15	0.05	0.25	0.35	0.01	0.01
Parameter	C	c	μ	σ^2	u	r
Value	0.0002	2.5	0.094	0.0017	0.0833	0.0349

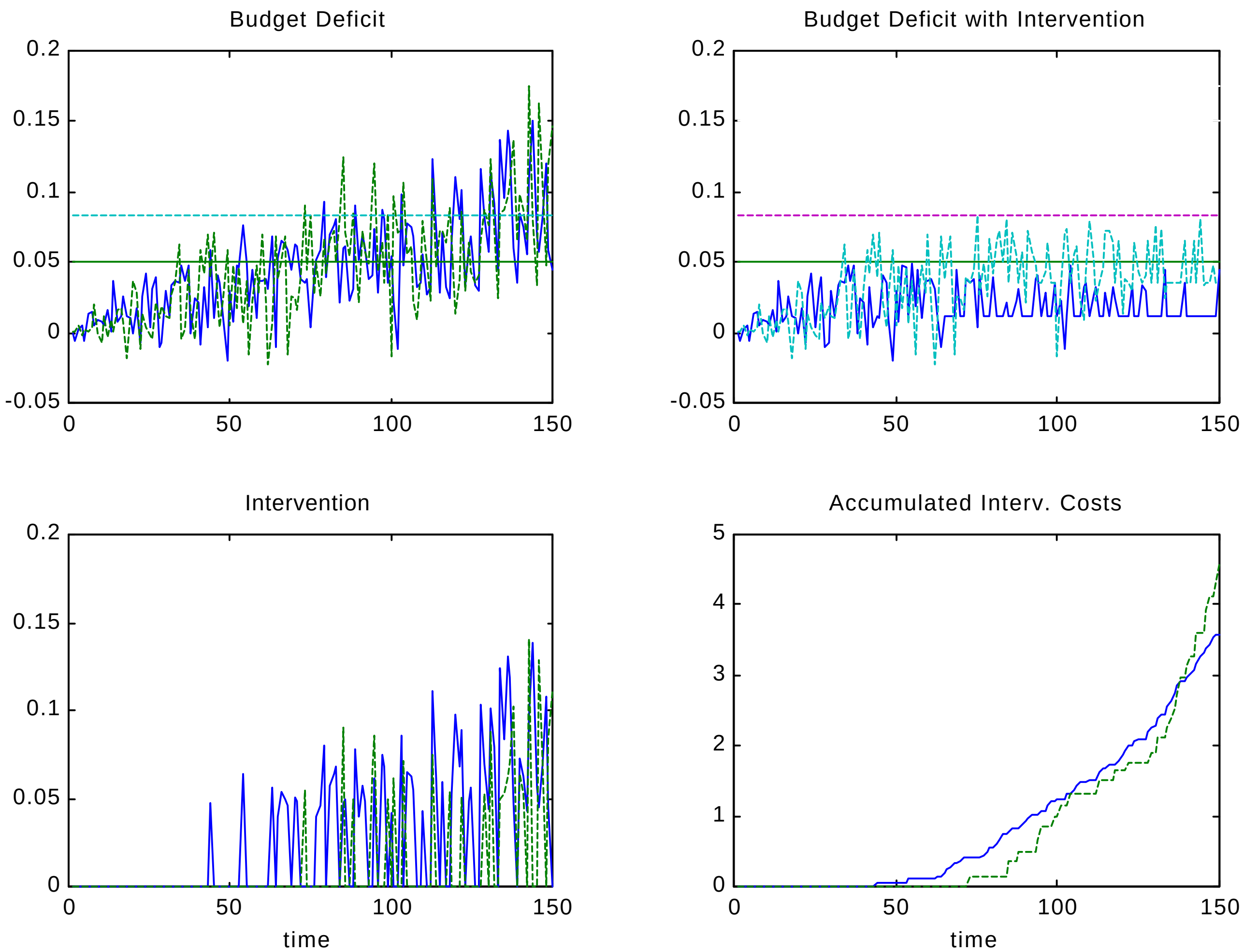
Table 2: Economy 2

dogenously determine μ (trend of the budget deficit-average output ratio) and σ^2 (variance of the budget deficit-average output ratio). These relationships are obtained from the previously mentioned continuous time stochastic model. Then, the parameters μ and σ^2, together with the intervention cost parameters C (lump-sum cost) and c (proportional cost), are used to obtain the numerical solution of the non-linear system of equations that defines the optimal budget deficit limit. As a result, parameters u (ceiling for the budget deficit) and r (budget deficit after resetting) are endogenously determined. The value for parameter B, which is also an unknown in the three-equation non-linear system, is not presented in the table because it is of no interest io our analysis. Note that the values chosen for the exogenous parameters do not describe any particular economy. Nevertheless, they provide realistic results for the endogenous variables[3]. Our objective is to compare different scenarios and not to simulate any particular economy.

The two economies described in tables 4.1 and 4.2 are identical in all aspects except for the costs faced by the fiscal authority when trying to affect the budget deficit. In fact, Economy 2 presents larger values for parameters C and c than does Economy 1. Due to this, the optimal values for parameters u and r are different. In fact, Economy 2 presents a higher optimal ceiling and a higher resetting value.

[3]There is no clear way of measuring the lump-sum and proportional costs faced by the fiscal authority when it intervenes to control the budget deficit. Part of these costs are clearly subjective. The same is true for the form of the payoff function of the fiscal authority. Therefore, we have chosen to calibrate parameters C and c in such a way that the endogenous variables present realistic values in terms of magnitude.

Figure 4.1: Economy 1 (-)   Economy 2 (--)



Nevertheless, the absolute difference between the ceiling and the resetting value is greater in Economy 2 than in Economy 1. This means that the adjustment in the budget deficit when the ceiling is reached is greater in Economy 2. The intuition for these results is clear. As was seen, the smooth pasting conditions state that, at the optimum, there is equality between marginal costs and marginal benefits of intervention. Therefore, an increase in the marginal intervention cost c must be matched by an increase in marginal benefits, which can only come about through a higher ceiling u. Next, we explain the reason why the resetting adjustment is greater in Economy 2. The value matching condition holds the answer to this question. It has been demonstrated that the total benefit of the intervention, measured in terms of the gain in the payoff function of the fiscal authority, should equal the total costs of intervention. Therefore, for reasonable values of the parameters, an increase in the lump-sum costs leads to an increase in the optimal difference between u and r, as a way of increasing the total gains of intervention. In fact, the higher the lump-sum cost C, the larger the resetting adjustment that is made. This is a way of reducing the frequency of adjustments, which means saving on the intervention costs. Figure 4.1 plots the results that we have been discussing. Finally, we discuss the path of the accumulated intervention costs curve. As a matter of fact, the accumulated intervention costs rise later in Economy 2 but build up faster through time. On the one hand, Economy 2 presents a higher ceiling which makes it normal that interventions take longer to occur. On the other hand, when interventions begin to occur, costs rise faster because parameters C and c are higher in Economy 2.

4.2 The Effects of Public Expenditure and Taxation

In this subsection we turn to the case of two economies that are different in terms of public expenditure and taxation parameters. In this context, Economy 3 presents a lower tax rate and a higher public expenditure-average output ratio than does Economy 4. Tables 4.3 and 4.4 describe these two scenarios.

As explained in Amador (1999), the stationary equilibrium budget deficit is smaller in the economy that follows a less rigorous fiscal policy. As a matter of fact, the stationary equilibrium budget deficit must be smaller in order to allow for a slower rate of public debt accumulation. Only a relatively lower stock of public debt can compensate the effects of a less rigorous fiscal policy and ensure the sustainability of the budget deficit. In addition, it can also be seen that Economy 3 endogenously generates a lower variance for the budget deficit. Next, we turn to the consequences of these two scenarios

Parameter	α	β	g	τ	σ_y^2	σ_z^2
Value	0.15	0.05	0.25	0.3	0.05	0.05
Parameter	C	c	μ	σ^2	u	r
Value	0.0001	0.9	0.045	0.0015	0.0541	0.0238

Table 3: Economy 3

Parameter	α	β	g	τ	σ_y^2	σ_z^2
Value	0.15	0.05	0.2	0.4	0.05	0.05
Parameter	C	c	μ	σ^2	u	r
Value	0.0001	0.9	0.158	0.0323	0.1447	0.0986

Table 4: Economy 4

in terms of the optimal budget deficit limits. An examination of Tables 4.3 and 4.4 reveals that Economy 4 presents a higher optimal ceiling and resetting value. It is important to understand this result. Some of the effects at work were discussed above in subsection 3.3 when the case of proportional intervention costs was considered. Firstly, it was remarked that the higher the budget deficit growth rate, the lower the optimal ceiling. Secondly, we concluded that the higher the variance of the budget deficit, the higher the optimal ceiling. In this section, the existence of lump-sum intervention costs adds some extra effects to the analysis. In the first place, the existence of lump-sum intervention costs reinforces the case for a higher optimal ceiling when the variance of the deficit is greater. In fact, the costs of responding to shocks, which tend to be reversed in the next period, are higher. Therefore, costs and benefits of intervention only equalize when the optimal ceiling is set at a higher value. In the second place, the lump-sum costs are by themselves a motive for higher ceilings. The lump-sum costs are something to be paid every time the budget deficit exceeds the ceiling. For each budget deficit growth rate, such events become more frequent as time goes by. Therefore, it is optimal to wait until the benefits of intervention are higher, which is equivalent to saying that the limit is set at a higher value. In our case, despite facing a higher μ, Economy 4 presents a higher limit because its σ^2 is also higher.

These results are robust to different calibrations and state that, in the stationary equilibrium, higher tax rates or lower public expenditure correspond to higher optimal limits. Figure 4.2 illustrates this result.

Parameter	α	β	g	τ	σ_y^2	σ_z^2
Value	0.15	0.05	0.25	0.35	0.02	0.02
Parameter	C	c	μ	σ^2	u	r
Value	0.0001	0.9	0.092	0.0033	0.0595	0.0212

Table 5: Economy 5

Parameter	α	β	g	τ	σ_y^2	σ_z^2
Value	0.15	0.05	0.25	0.35	0.001	0.001
Parameter	C	c	μ	σ^2	u	r
Value	0.0001	0.9	0.095	0.0002	0.0429	0.0044

Table 6: Economy 6

4.3 The Effects of Technology and Public Expenditure Shocks

The analysis of differences in the variance of technology and public expenditure shocks is similar to that presented in the previous subsection. Thus, Tables 4.5 and 4.6 describe two different scenarios. In this case Economy 5 faces a variance of technology and public expenditure shocks that is higher than that of Economy 6. In addition, the covariance between the two types of shocks is assumed to be zero. The differences between these two economies endogenously generate a lower variance and a slightly higher trend for the budget deficit in Economy 6. As a consequence, this economy presents a lower optimal ceiling and a correspondingly lower resetting value. In conclusion it can be said that, the lower the volatility faced by the economies, the lower their optimal ceiling and resetting value. Figure 4.3 illustrates these results.

5 Concluding Remarks

In this section we briefly sum up the main results of the paper and discuss its limitations. As stated in the introduction, the paper uses the stochastic optimal control techniques to examine what it is that is behind the determination of optimal limits for the budget deficit. Furthermore, we make use of a previously developed continuous time stochastic model in order to establish the links between the structural parameters of the economy and these optimal limits. In this context, some interesting results are obtained. Firstly, in the case of proportional intervention costs, we conclude that the higher the cost parameter, the higher is the optimal value for the budget deficit ceiling. The same is true for greater variances of the budget deficit. On the contrary, the higher the average budget deficit, the lower the optimal limit. Secondly, in a stationary equilibrium context with both proportional and lump-sum intervention costs, we conclude that economies with higher tax rates and lower public expenditure should set higher budget deficit limits than less rigorous ones. Finally, the higher the volatility affecting the economy in the stationary equilibrium, the higher the optimal budget deficit ceiling will be.

There are two types of limitations in our analysis. Firstly, it is very difficult to know what the magnitude of the intervention cost faced by the fiscal authority is. Additional research work can be done to link these costs to the structural parameters in the economy. Secondly, our analysis presupposes the knowledge of the stochastic process that rules the path of the budget deficit. To this purpose, we used a previously derived continuous time stochastic model. However, this model only describes the path of the deficit in the stationary equilibrium. Therefore, part of our results apply only to the stationary equilibrium context and not to cases where the deficit is following an unsustainable path.

Figure 4.2: Economy 3 (-)   Economy 4 (--)

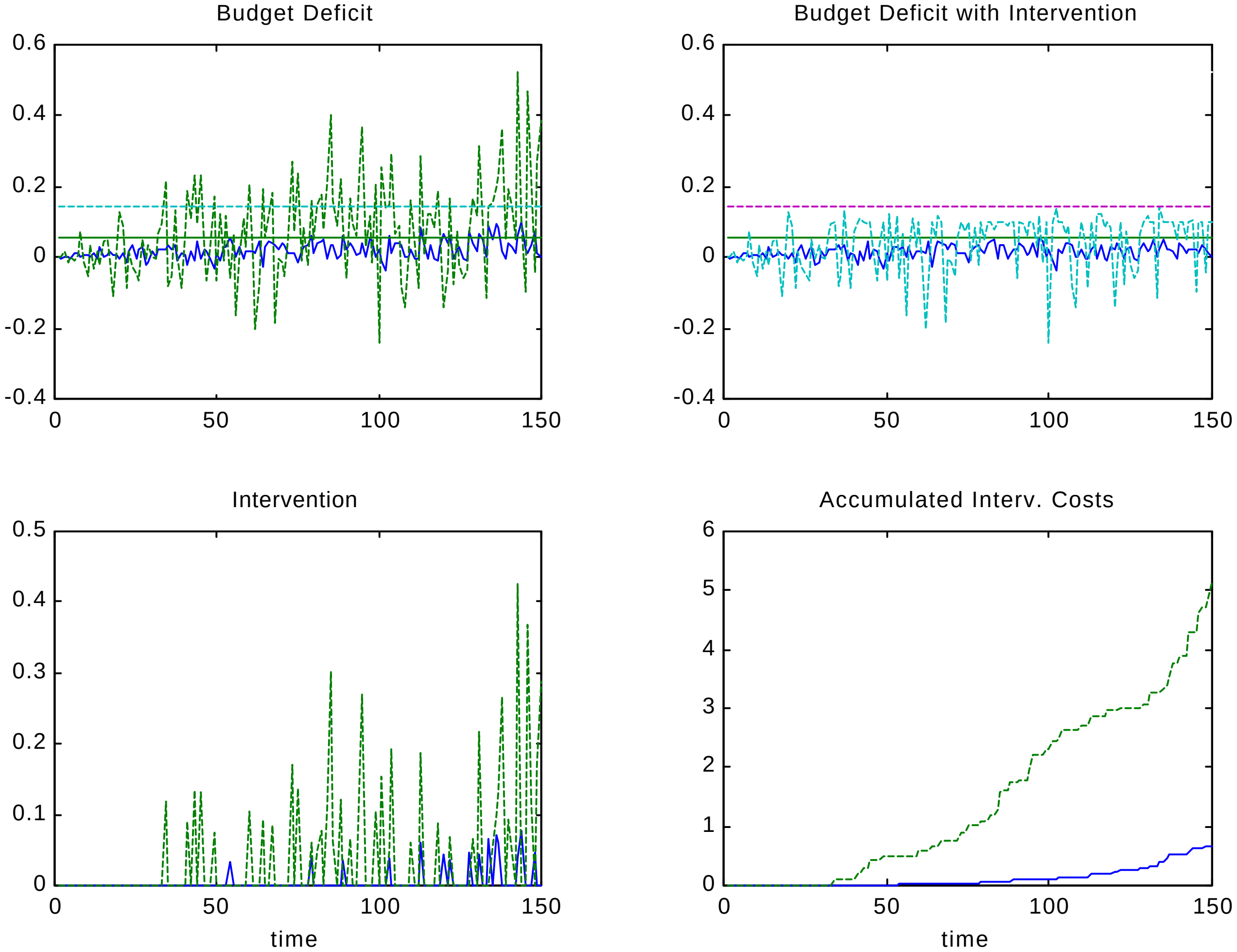
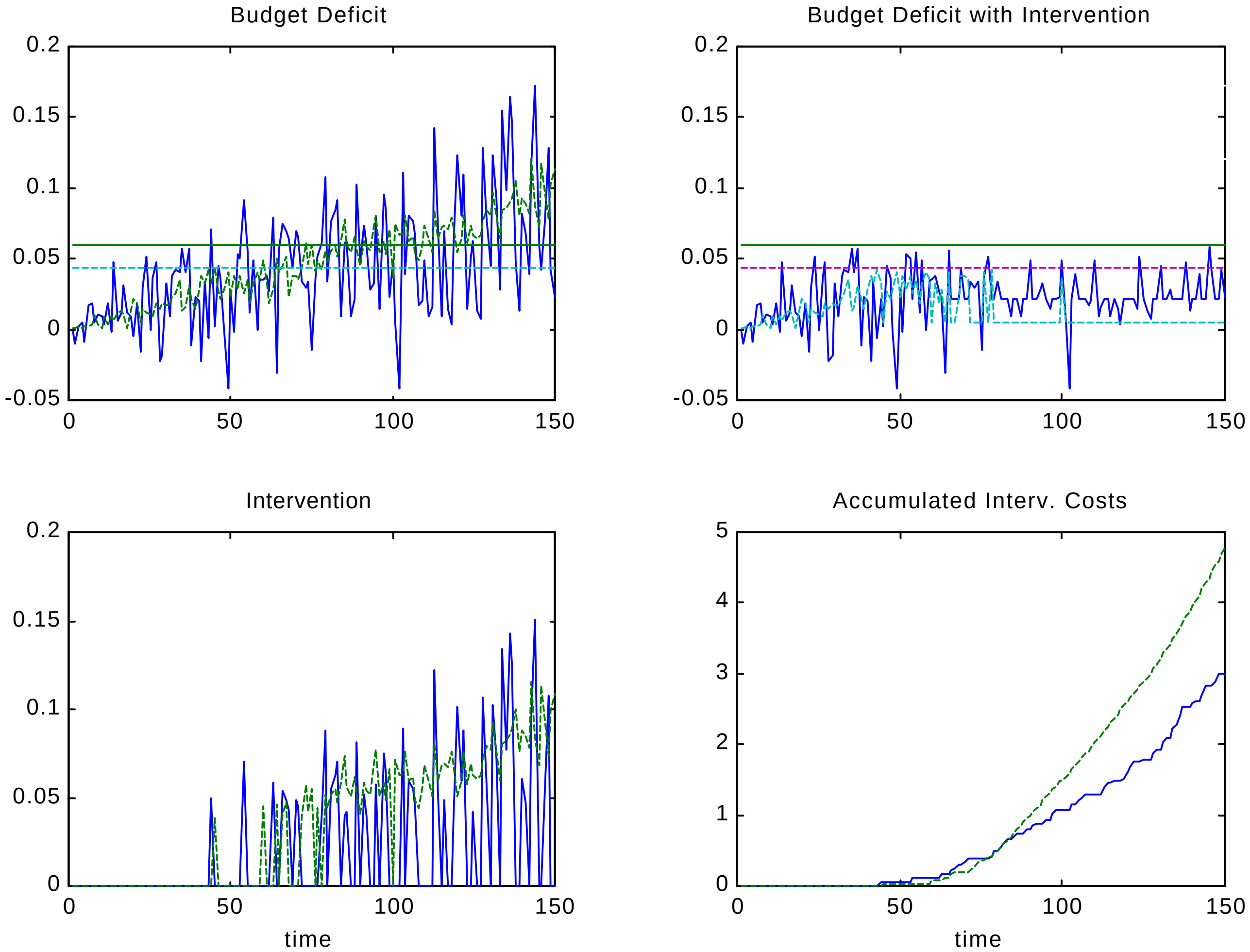


Figure 4.3: Economy 5 (-)   Economy 6 (--)



A Appendix

This appendix presents the main steps of the method used by Dixit (1993) to derive the solution of the differential equation that represents the expected present value of a quadratic payoff function. In this context, we also try to explain the meaning of the fundamental quadratic equation 4.

We start by observing that the expected present value function $F(x)$ can be decomposed as:

$$F(x) = f(x)dt + e^{-\beta dt}E[F(x + dx)] \tag{19}$$

Note that in this appendix we define x as a Brownian motion stochastic process with mean μ and variance σ^2. In order to simplify the notation, we will not use the random variable $\frac{dB}{Y^D}$, which has a specific meaning in the model. Simplifying equation 19 and ignoring the terms of order higher than dt, which go to zero when $dt \to 0$, we have:

$$F(x) = f(x)dt + (1 - \beta dt)\left(F(x) + \{E[F(x + dx)] - F(x)\}\right) \tag{20}$$

$$= f(x)dt + F(x) - \beta F(x)dt + \{E[F(x + dx)] - F(x)\} \tag{21}$$

Thus:

$$\beta F(x)\, dt = f(x)dt + \{E[F(x + dx)] - F(x)\} \tag{22}$$

$$= f(x)dt + E[dF] \tag{23}$$

Making use of Itô's Lemma, we rewrite:

$$E[dF(x)] = \mu F^{'}(x)dt + \frac{1}{2}\sigma^2 F^{''}(x)dt \tag{24}$$

Substituting equation 24 in 23 and dividing by dt, we obtain:

$$\frac{1}{2}\sigma^2 F^{''}(x) + \mu F^{'}(x) - \beta F(x) + f(x) = 0 \tag{25}$$

The next step is to obtain the solution of the non-homogeneous linear differential equation above. The solution consists of the sum of the complementary function and the particular integral. As is known, the complementary function is the general solution of the homogeneous part of equation 25:

$$\frac{1}{2}\sigma^2 F^{''}(x) + \mu F^{'}(x) - \beta F(x) = 0 \tag{26}$$

The general solution of equation 26 is just the linear combination of two independent solutions. If we try a solution of the type $\exp(\lambda x)$ we obtain:

$$\exp(\lambda x)\left[\frac{1}{2}\sigma^2\lambda^2 + \mu\lambda - \beta\right] = 0 \tag{27}$$

which can only be true if:

$$\frac{1}{2}\sigma^2\lambda^2 + \mu\lambda - \beta = 0 \tag{28}$$

The interesting point is that this equation generates as solutions the roots of the fundamental quadratic that was introduced above. Therefore, we obtain the values for a and b that were defined in equations 2.5 and 2.6. As it was also said, these roots are different and have opposite signs. Consequently, they are independent and the general solution of equation 26 is simply:

$$Ae^{-ax} + Be^{bx} \tag{29}$$

where A and B are constants to be determined.

At this point it is important to explain the meaning of the fundamental quadratic equation 4. The expected present value of the exponential payoff function can be rewritten as:

$$F(x) = \int_0^\infty E\left[\exp(\lambda x)\, e^{-\beta t} dt\right] \tag{30}$$

$$= \exp(\lambda x) \int_0^\infty \exp\left[-\left(\beta - \lambda\mu - \frac{1}{2}\lambda^2\sigma^2\right) t\right] dt \tag{31}$$

$$= \frac{\exp(\lambda x)}{\beta - \lambda\mu - \frac{1}{2}\lambda^2\sigma^2} \tag{32}$$

Therefore, for the integral to converge, the denominator must be positive. This means that the fundamental quadratic determines the values for which the problem is valid.

Taking equation 25 again, we now turn to the determination of the particular integral. Given our flow payoff $f(x) = x^2$, the right guess for the particular integral is:

$$F(x) = \sum_{m=0}^{2} a_m x^m \tag{33}$$

Substituting this expression in 25 we obtain:

$$\sigma^2 a_2 + \mu(a_1 + 2a_2 x) - \beta\left(a_0 + a_1 x + a_2 x^2\right) + x^2 = 0 \tag{34}$$

Next, equating the coefficient of each power of x to zero, it is possible to determine the expressions for a_0, a_1 and a_2. Therefore, we obtain the following three conditions:

$$\sigma^2 a_2 + \mu a_1 - \beta a_0 = 0 \tag{35}$$

$$2\mu a_2 - \beta a_1 = 0 \tag{36}$$

$$-\beta a_2 + 1 = 0 \tag{37}$$

that is:

$$a_0 = \frac{\sigma^2}{\beta^2} + \frac{2\mu^2}{\beta^3} \tag{38}$$

$$a_1 = \frac{2\mu}{\beta^2} \tag{39}$$

$$a_2 = \frac{1}{\beta} \tag{40}$$

Substituting these parameters back into equation 33, the particular integral becomes:

$$F\left(x\right) = \frac{\sigma^2}{\beta^2} + \frac{2\mu^2}{\beta^3} + \frac{2\mu}{\beta^2}x + \frac{1}{\beta}x^2 \tag{41}$$

Finally, taking the sum of the general solution and the particular integral, we obtain the solution of the of the non-homogeneous linear differential equation 25. That is:

$$F\left(x\right) = \frac{\sigma^2}{\beta^2} + \frac{2\mu^2}{\beta^3} + \frac{2\mu}{\beta^2}x + \frac{1}{\beta}x^2 + Ae^{-ax} + Be^{bx} \tag{42}$$

which is similar to equation 3 presented in the text of the paper.

There is an alternative method for obtaining the expected present value of the quadratic payoff function. We will present this second method briefly because it provides an additional important result.

Taking again the flow payoff of the type $\exp\left(\lambda x\right)$ we can expand and rewrite:

$$F(x) = E\left[\int_0^\infty \sum_{n=0}^{\infty} \frac{1}{n!}\left(\lambda x_t\right)^n e^{-\beta t}dt \ \ |x_o = x\right] \tag{43}$$

$$= \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} E\left[\int_0^\infty x_t^n e^{-\beta t}dt \ \ |x_o = x\right] \tag{44}$$

In a similar way, equation 32 can also be expanded in powers of λ, in the neighbourhood of $\lambda = 0$. First, note that:

$$\exp\left(\lambda x\right) = \sum_{n=0}^{\infty} \frac{1}{n!}\left(\lambda x\right)^n \tag{45}$$

$$\left[\beta - \lambda\mu - \frac{1}{2}\lambda^2\sigma^2\right]^{-1} = \frac{1}{\beta}\sum_{n=0}^{\infty} \beta^{-n}\left(\lambda\mu + \frac{1}{2}\lambda^2\sigma^2\right)^n \tag{46}$$

Then, using the Binomial Theorem, it is possible to rewrite equation 32 as:

$$F(x) = \left(\sum_{n=0}^{\infty} \frac{1}{n!} (\lambda x)^n \right) \left(\frac{1}{\beta} \sum_{n=0}^{\infty} \beta^{-n} \sum_{j=0}^{n} \binom{n}{j} (\lambda n)^j \left(\frac{1}{2} \lambda^2 \sigma^2 \right)^{n-j} \right) \tag{47}$$

At this point, equating the powers $n = 2$ in 44 and 47 we obtain again:

$$F(x) = \frac{\sigma^2}{\beta^2} + \frac{2\mu^2}{\beta^3} + \frac{2\mu}{\beta^2} x + \frac{1}{\beta} x^2 \tag{48}$$

which means that the solution of the differential equation 25 is simply the particular integral. Therefore, the constants A and B are zero for the case of the uncontrolled Brownian motion with a quadratic payoff function.

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