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Population Aging and Legal Retirement Age*

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Abstract

This paper analyzes the effects of population aging on the preferred legal retirement age. What is revealed is the crucial role that the indirect 'macro' effects resulting from a change in the legal retirement age play in the optimal decision. Two Social Security systems are studied. Under a defined contribution scheme aging lowers the preferred

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legal retirement age. However, under a defined pension scheme the retirement age is delayed. This result shows the relevance of correctly choosing the parameter affected by the dependency ratio in the design of the Social Security programme.

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1. Introduction

Reforms of Social Security systems are now one of the main issues on the economic policy agenda of most industrialized countries. It is widely considered that, unless serious changes take place, the aging of the population implying a rise in the number of retirees relative to that of workers will threaten the viability of Pay-As-You-Go public pension systems in the long-run. However, it may be said that this is not a financial problem but a political one (see Cremer and Pestieau 2000). The threat of aging can be neutralized by raising taxes, cutting benefits or/and delaying the age of retirement. Indeed, this latter reform is one of the policy conclusions of *Maintaining Prosperity in an Ageing Society*, OECD (1998) "...a direct way to encourage people to work longer would be to raise the pensionable age". But delaying the retirement age may not be very popular among people. According to recent surveys, most of workers tend to manifest that they are happy with the current retirement age (see Cremer and Pestieau 2003) which suggests that reforms on the legal retirement age have currently become a very delicate matter for governments.

In this paper, we aim at providing an analysis of the preferred legal retirement age and how this is affected by the population aging. Using a life-cycle model we calculate the optimal legal retirement age and examine its relationship with the population growth rate via the dependency ratio. We contribute two main findings to the existing literature on Social Security and retirement. First, we highlight the indirect macro effects resulting from altering the legal retirement age, that is, the effects on the aggregate constraint of the adjustment made in the ratio of workers and retirees when the legal retirement age is lowered/delayed.

This crucial role is stressed when we compare our results with those obtained by Sheshinski (1978). In Sheshinski's model, the individual ignores the impact of her decision on the Social Security budget constraint, as she only optimizes her own retirement age. So, under a defined pension scheme

a reduction in the birth rate leads to a lower *individual* optimal retirement age. But when we analyze the preferred *legal* retirement age and these effects are taken into account, we find the opposite result. The optimal response to a population aging will be to postpone the retirement age.

Secondly, we also seek to underline the importance of correctly choosing the parameter affected by the dependency ratio in order to make the retirement age reform easier. Depending on the policy reaction, the aging of the population may affect the contribution rate or the pension benefits. That is, the demographic effects of a decrease in the population growth rate may lead, via changes in the dependency ratio, to higher contribution rates or to lower pension benefits. Consequently, we distinguish two Social Security programmes. First, a defined contribution scheme where the contribution rate is constant and pension benefits are affected by the dependency ratio, and secondly, a defined pension scheme where population aging leads to higher contribution rates and thus the pension is the constant parameter.

The term 'legal retirement age' usually refers to the age at which benefits are available. However, since there are also strong incentives to stop working after this standard entitlement age, we consider legal retirement in this model as the age at which workers are obliged to leave the labor force, that is, as a mandatory retirement. Indeed, in some countries there are direct restrictions on work beyond the standard age (in Portugal and Spain entitlement to pension benefits beyond the standard age is conditional on complete withdrawal from work), or, frequently, individuals have to leave their current jobs to receive their pensions (see Blondal and Scarpetta 1998, or Gruber and Wise 1999). So, we can observe that the average retirement age in some OECD countries, such as the United Kingdom, Portugal and Ireland (see Blondal and Scarpetta 1998) is very close to this standard retirement age.¹

Many studies have analyzed the relationship between retirement and Social Security. Earlier literature has mainly focussed on the effect of the introduction of a pension system on the individual retirement decision (see among others Sheshinski 1978, Crawford and Lilien 1981, Kahn 1988, or Fabel 1994). There is also more recent literature dealing with the retirement decision in a political economy environment (see Casamatta et al. 2005 or Conde-Ruiz and Galasso 2003 and 2004). Related to our study, Conde-Ruiz et al. (2003) provide a long-term perspective on individual retirement behavior. They develop an overlapping generations model where they examine how incentives to retire early are affected by the pension system. They suggest that non-

actuarially fair pension systems may induce rational agents to retire early. In a similar framework, Conde-Ruiz et al. (2005) also find that aging may lead to earlier retirement. Unlike the present analysis, both these studies focus on the individual decision and only consider the defined contribution scheme. Crettez and Le Maitre (2002) also examine how the population growth rate affects the optimal retirement age, but in a different context. In an overlapping generations model without explicit Social Security variables, neither an explicit contribution rate nor explicit pension benefits, they analyze intergenerational optimal resource sharing when a social planner can choose the retirement age in addition to consumption and investment. They find that the optimal retirement age depends on the elasticity of substitution of old agents' labour for young agents' labour and on the chosen social welfare function.

To our knowledge our study is the first attempt to provide a theoretical analysis of the effect of the population growth rate on the preferred legal retirement age, and how this effect may be completely different depending on the type of Pay-As-You-Go Social Security system in place, that is, if it is a defined contribution system or a defined pension system.

The paper proceeds as follows. In Section 2 the model is developed. In Section 3 the optimal legal retirement age is obtained and the relationship between this optimal legal retirement age and the population growth rate is examined. In Section 4 we allow the interest rate to exceed the population growth rate in order to obtain some additional insight and we use a numerical example to illustrate the results. Section 5 contains the conclusions reached.

2. The model

We consider a small open economy in which individuals live exactly T years, of which the first R years represent working life. The population consists of identical agents, and can thus be represented by a single agent.

We assume that the individual has a stationary and temporally independent utility function, which is separable and strictly increasing in consumption and leisure. It is written as

$$U(c_i^t, l^t) = u(c_i^t) + v(l^t) \quad (1)$$

where c_i^t is the consumption at period t in scheme i . The utility of consumption is twice differentiable with $u' > 0$ and $u'' < 0$ for all c_i^t . Let l^t be

the leisure at period t , the utility of leisure being $v(l^t) = 0$, while working, and $v(l^t) = v$, while retired.² This utility function is similar to Crawford and Lilien (1981) and Sheshinski (1978). In addition, we assume that the coefficient of relative risk aversion, $\rho_r(c)$, is non-increasing and less than one.

The agent plans consumption, savings and retirement to maximize the discounted value of utility subject to her lifetime budget constraint. She is assumed to earn a constant stream of wage per unit of time, normalized to unity, for the sake of simplicity, until she retires. Later, from a Social Security programme $\langle \tau, p \rangle$, she receives a constant stream of pension benefits per unit of time, p , τ being the Social Security contribution rate. The amount of hours spent working cannot be varied as it is institutionally set. Therefore, the only way utility coming from leisure can be modified is by changing the legal retirement age.

We consider a perfectly competitive capital market with free lending and borrowing at a fixed interest rate, which is equal to the subjective discount rate ($\delta = r$). This assumption together with the separability and concavity of the utility function implies constant consumption in all periods, that is, $c_i^t = c_i$ for any t . We also assume a constant birth rate, n , equal to the interest rate, $n = r$.

Thus, the indirect utility function of an individual over her life-cycle can be reduced to

$$U(c_i; R) \equiv \frac{(1 - e^{-rT})}{r} u(c_i) + \frac{(e^{-rR} - e^{-rT})}{r} v \quad (2)$$

where

$$c_i = \frac{1}{(1 - e^{-rT})} ((1 - e^{-rR}) + \phi_i) \quad (3)$$

is the constant consumption per unit of time, ϕ_i being the discounted present value of net benefits from Social Security under each scheme.

In order to analyze the demographic effects, let us consider a steady-state situation in which population is growing at a constant rate, n . As seen in Sheshinski's (1978) model, the steady-state age density function is equal to

$$f(t) = \frac{n}{1 - e^{-nT}} e^{-nt}. \quad (4)$$

The Social Security Programme is financed through the Pay-As-You-Go scheme where pensions of retirees are paid by working people through taxes. So, since the government budget is instantaneously balanced, we can obtain the following relationship between the contribution rate, pension benefits and the legal retirement age

$$(1 - e^{-nR})\tau = (e^{-nR} - e^{-nT})p. \quad (5)$$

3. Retirement age and population aging

Let us now study how the optimal legal retirement age of the individual is affected by a decrease in the birth rate and how these effects are different depending on the Social Security system in place. We focus only on the effect of a birth rate reduction, since its importance will be larger when the baby-boom generation of the last century enters retirement in the third decade of this century (see Weizsäcker 1991). It should be stressed that we study a steady-state economy with different birth rates, that is, we are not concerned with the transition period from a steady state with a determined population growth rate to another steady state with a different one.³

In the constant pension benefits system, the contribution rate is residually determined to balance the budget.⁴ This implies that under this scheme the benefits formula does not depend on the retirement age. On the other hand, in the constant contribution scheme, the pension is the parameter residually obtained through the budget constraint. Therefore, from (5) we obtain respectively

$$\tau(R, n, p) = \frac{(e^{-nR} - e^{-nT})}{(1 - e^{-nR})}p \quad (6)$$

and

$$p(R, n, \tau) = \frac{(1 - e^{-nR})}{(e^{-nR} - e^{-nT})}\tau. \quad (7)$$

Consequently, the discounted present value of net benefits from Social Security under each system is respectively

$$\phi_p = p \left((e^{-rR} - e^{-rT}) - (1 - e^{-rR}) \frac{(e^{-nR} - e^{-nT})}{(1 - e^{-nR})} \right) \quad (8)$$

and

$$\phi_\tau = \tau \left((e^{-rR} - e^{-rT}) \frac{(1 - e^{-nR})}{(e^{-nR} - e^{-nT})} - (1 - e^{-rR}) \right). \quad (9)$$

Notice that an increase in the legal retirement age reduces the absolute value of ϕ_p but it augments the absolute value of ϕ_τ .⁵

It can easily be verified that the utility function (2) is single-peaked for both pension schemes. So, we can obtain R_p^* and R_τ^* , the preferred legal retirement age with constant pension and with constant contribution, respectively. Analyzing the sign of $\partial R_p^*/\partial n$ and $\partial R_\tau^*/\partial n$, the following proposition can be stated.

Proposition 1 *i) Under the constant pension system, the lower the birth rate, the higher the preferred legal retirement age, that is, $\partial R_p^*/\partial n < 0$.*

ii) Under the constant contribution system, the lower the birth rate, the lower the preferred legal retirement age, that is, $\partial R_\tau^/\partial n > 0$.*

Proof: See appendix.

The first point of the proposition tells us that in a defined pension scheme a population aging would imply a delay in the optimal legal retirement age. This result can be explained by the income and the substitution effect.

The income effect tends to delay retirement. A lower population growth rate implies a higher dependency ratio. This increases the contribution rate which reduces individuals' lifetime income. The loss of income implies a lower demand for leisure which means a delay in the legal retirement age.

In order to explain the substitution effect, let us compare the result of the first point of this proposition with that obtained by Sheshinski (1978) in a similar setting to our constant pension scheme, in the sense that a decrease in the birth rate affects the contribution rate but the replacement ratio stays constant.

In the aforementioned paper, Sheshinski analyzes the optimal retirement decision and how it changes with reference to some parameters. He finds that higher population growth rates lead to increases in the equilibrium retirement age. In that case, the income effect lowers the retirement age. The higher population growth rate reduces the dependency ratio which decreases the contribution rate. Therefore, the increase in the lifetime income leads to a higher demand for leisure. On the contrary, the substitution effect generates

incentives to delay the retirement age. A lower contribution rate implies an increase in the net wage. Hence, the price of leisure rises and individuals relocate their demand from leisure to consumption. This means that the substitution effect causes individuals to postpone their retirement age. Since the total effect increases the equilibrium retirement age, it can be concluded that the substitution effect outweighs the income effect.

However, the sign of the substitution effect and, consequently, the positive relationship between the population growth rate and the optimal retirement age in Sheshinski's model heavily relies on the assumption that individuals ignore the impact of their decision on the aggregate constraint. In other words, individuals consider that their retirement decision only affects the length of their working period and they ignore their influence on the dependency ratio and, therefore, on the contribution rate. In Sheshinski's model, this is a plausible assumption since individuals optimize their own retirement age under competitive conditions. But since in our model we analyze the optimal legal retirement age, the macro effects on the dependency ratio of changes in the legal retirement age cannot be ignored.

Returning to our model, under a constant pension scheme the changes in the legal retirement age affect individuals' lifetime income in two ways: fixing the length of the working period and determining the contribution rate via the dependency ratio. For instance, a delay in the legal retirement age not only increases the working period but also reduces the contribution rate by decreasing the dependency ratio.

Furthermore, the larger the contribution rate, the bigger the indirect effect on the net wage of a change in the legal retirement age will be. The reason is the higher relative weight of the contribution rate on the individual's net wage. Consequently, the increase in the contribution rate caused by the population aging augments the importance of these indirect effects. In other words, population aging increases the relative price of leisure so that individuals relocate their demand from leisure to consumption. Hence, the substitution effect also means that individuals delay their retirement age. So, under a defined pension scheme both the substitution and the income effects are in the same direction, leading to a higher legal retirement age.

The opposite results between Sheshinski's model and our setting highlight the crucial role of the indirect macro effects of changes in the legal retirement age. And these play such an important role, that, when they are taken into

account, a decrease in the population growth rate will imply, under a defined pension scheme, an unambiguous delay in the optimal legal retirement age.

The second point of the proposition states that under a defined contribution scheme population aging would lower the optimal legal retirement age. This result can again be explained by the income and the substitution effect.

The income effect is similar to the previous one. The higher dependency ratio caused by the change in the ageing structure of the population decreases pension benefits. So, the reduction in individuals' lifetime income comes now from these lower pension benefits. The loss of income again implies a lower demand for leisure which means a delay in the legal retirement age.

However, under a defined contribution scheme the substitution effect is in the opposite direction. Under this scheme changes in the legal retirement age also affect individuals' lifetime income in two ways. First, fixing, as in the previous case, the length of the working period, and secondly, determining the pension benefits via the dependency ratio.

The lesser the pension benefits, the smaller the indirect effect on the lifetime income of a change in the legal retirement age. The reason is the lower relative weight of the pension benefits on the individuals' lifetime income. Consequently, the reduction in the pension benefits caused by the population aging decreases the importance of these indirect effects.

In other words, population aging reduces the relative price of leisure so that individuals relocate their demand from consumption to leisure. Hence, the substitution effect leads to a lowering of the retirement age. So, under a defined pension scheme, since the substitution effect outweighs the income effect, a decrease of the birth rate will lower the optimal legal retirement age.

The economic intuition behind this proposition is the following. When the birth rate decreases, the rate of return of the PAYG pension system also decreases.⁶ As a consequence, individuals may have incentives to limit the size of the system and to rely more heavily on private savings. This can be achieved by delaying the legal retirement age under the constant pension system, which would reduce the contribution rate, and lowering it under the constant contribution system, which would reduce the pension benefits.

It is worthwhile noting that if we compare these optimal legal retirement ages with the preferred individual retirement age in the absence of a Social Security system, it can be deduced that the population aging would place

this individual retirement age between the two legal ones. The reasoning is the following. The optimal retirement age for an individual in the absence of Social Security is equal to the preferred legal one in the case in which $r = n$ since in that case net benefits from Social Security would be zero. The constant consumption (3) would be the same with or without a pension programme and so would the optimal retirement age.⁷ But, as we have seen above, population aging would lead to a delay in the optimal legal retirement age under a defined pension scheme, placing it beyond the individual optimal one in the absence of Social Security or to a lowering of the legal retirement age if the system has a defined contribution rate, in which case the legal retirement age would be below the individual optimal one in the absence of a pension programme.

4. Extension: Allowing the interest rate to exceed the population growth rate

For simplicity, all the previous analysis has been made under the assumption that the interest rate must be equal to the population growth rate. Since the most empirically relevant case is when $r > n$, in this extension we relax the aforementioned assumption.

When the interest rate exceeds the population growth rate, the analytical treatment becomes intractable. The former relationships between the birth rate and optimal legal retirement ages under each scheme cannot be mathematically proved. Instead, we proceed to calculate the optimal values of the legal retirement age for some alternative levels of n . These calculations are presented in the following subsection.

Even so, an interesting result arises in the case in which the real interest rate is allowed to be higher than the population growth rate. This holds for any strictly concave utility function.⁸

Proposition 2 *Let $r > n$. Let the indirect utility function $U(R; n)$ be strictly concave on R . Let (τ, p) be such that $\tau = \tau(n, p, R_p^*)$, then $R_p^* > R_\tau^*$.*

Proof: See appendix.

This proposition states that if the interest rate is larger than the population growth rate, and if the contribution rate were the same, irrespective of the scheme, then, the higher optimal legal retirement age would be reached under the defined pension scheme. The reasoning is the following. When

the interest rate exceeds the birth rate the discounted present values of net benefits from Social Security, (8) and (9), are negative.⁹ In that case, the smaller the relative weight of the pension system in the lifetime income, the better individuals will be. As mentioned above, this reduction is achieved by delaying the legal retirement age in a defined pension scheme, since this implies a lower contribution rate, and lowering the legal retirement age in a defined contribution scheme, since this results in lower pension benefits. These reductions in the Social Security forced savings would be compensated by voluntary savings and individuals would improve their utility levels.

Again, if we compare these optimal legal retirement ages with the preferred individual retirement age in the absence of a Social Security system, it can be deduced that this latter would be placed between the two legal ones. The same previous reasoning can be applied.

4.1 Numerical Example

In this numerical example we allow the interest rate to be higher than the population growth rate and calculate the optimal legal retirement age under each scheme for alternative values of n . We use the square root function, $u(c) = \sqrt{c}$. The results are summarized in table 1:

Table 1

In the first two columns, it can be seen how under a defined pension scheme the optimal legal retirement age is delayed almost 10 years due to the decrease in the birth rate from 0.0135 to 0.0115. However, under a defined contribution scheme, the same population aging leads to a lowering of the optimal legal retirement age of more than a year.¹⁰ This difference is caused by the opposite substitution effects embedded in each scheme.

These contrary substitution effects imply that under the defined pension scheme the income and the substitution effects are in the same direction, but under the defined contribution case these effects are opposite. This explains the great impact of the aging of the population on the optimal legal retirement age when pension benefits are constant, in contrast with the lower impact when the contribution rate is the constant parameter. Indeed, when the interest rate exceeds the population growth rate it cannot be assured that the relationship between n and R_r^* is always positive.¹¹

It has to be underlined that for $n = 0.0135$ individuals face the same contribution rate and the same pension benefits under both schemes for $R = 40$, so, the numerical example also shows the result stated in the last proposition. The optimal legal retirement age in the case of a constant contribution rate, $R_\tau^* = 40$, is smaller than the optimal retirement age in the case of constant pension benefits, $R_p^* = 46.86$.

5. Conclusions

This paper analyzes the effects of population aging on the preferred legal retirement age. The opposite results obtained in the two pension schemes highlight the importance of which parameter has to be affected by the dependency ratio in order to involve a lower degree of compulsion in the reform of the legal retirement age. If the dependency ratio modifies the contribution rate, a population aging would generate an income and a substitution effect that would lead to a delay in the preferred legal retirement age, which might make retirement age reform easier.

The importance of the indirect macro effects of changing the legal retirement age is also stressed by comparing our result in the defined pension case, higher preferred legal retirement age related to lower birth rate, with Sheshinski's (1978) result, lower preferred individual retirement age related to lower birth rate.

Summing up, our results suggest that if governments want to delay the legal retirement age, in order to make the future reforms easier, it would be better to transfer the effects of the aging of the population onto the contribution rate, instead of pension benefits.

Appendix

Proof Proposition 1

i) We have to prove that $\partial R_p^*/\partial n$ is negative. It can easily be checked that $\partial^2 U/\partial R^2$ is negative evaluated at $R = R_p^*$. Therefore, the sign of $\partial R_p^*/\partial n$ coincides with the sign of $\partial(\partial U/\partial R)/\partial n$.

$$\frac{\partial\left(\frac{\partial U}{\partial R}\right)}{\partial n} = (1 - e^{-rT}) \left(u''(c_i) \frac{\partial c_i}{\partial R} \frac{\partial c_i}{\partial n} + u'(c_i) \frac{\partial(\partial c_i/\partial R)}{\partial n} \right) \quad (10)$$

The constant consumption c_p is as follows:

$$c_p = \left(1 - \frac{(e^{-nR} - e^{-nT})}{(1 - e^{-nR})} p \right) \frac{(1 - e^{-rR})}{(1 - e^{-rT})} + p \frac{(e^{-rR} - e^{-rT})}{(1 - e^{-rT})}. \quad (11)$$

First, $\partial c_p/\partial R$ and $\partial c_p/\partial n$ are strictly positive: an increase in R augments the working period and reduces the contribution rate, both effects imply higher consumption; an increase in n reduces the dependency ratio and so the contribution rate, this effect also implies higher consumption.

Secondly, $\partial(\partial c_p/\partial R)/\partial n$ can be reduced to:

$$\frac{\partial(\partial c_p/\partial R)}{\partial n} = \frac{1}{(1 - e^{-rT})} \left(\frac{e^{-nR}(1 - e^{-nT})}{(1 - e^{-nR})^2} (1 - e^{-nR} - nR) \right). \quad (12)$$

It is easy to check that:

$$1 - e^{-nR} - nR < 0 \quad \forall nR \in (0, \infty) \quad (13)$$

and therefore $\partial(\partial c_p/\partial R)/\partial n < 0$. Q.E.D.

ii) We have to prove that $\partial R_\tau^*/\partial n > 0$. It is easy to check that (2) is strictly concave, therefore the sign of $\partial R_\tau^*/\partial n$ also coincides with the sign of (10).

The constant consumption c_τ is:

$$c_\tau = \frac{(1 - e^{-rR})}{(1 - e^{-rT})} (1 - \tau) + \tau \frac{(1 - e^{-nR})}{(e^{-nR} - e^{-nT})} \frac{(e^{-rR} - e^{-rT})}{(1 - e^{-rT})}. \quad (14)$$

Again $\partial c_\tau/\partial R$ and $\partial c_\tau/\partial n$ are strictly positive: an increase in R augments the working period and raises pension benefits, both effects imply

higher consumption; an increase in n reduces the dependency ratio and thus augments pension benefits, this effect also implies higher consumption.

$\partial(\partial c_\tau/\partial R)/\partial n$ can be reduced to:

$$\frac{\partial(\partial c_\tau/\partial R)}{\partial n} = \frac{1}{(1 - e^{-rT})} \left(\frac{e^{-nR}(1 - e^{-nT})}{(e^{-nR} - e^{-nT})^2} e^{-nT} (e^{n(T-R)} - (n(T-R) + 1)) \right). \quad (15)$$

It is easy to check that:

$$e^{n(T-R)} - (n(T-R) + 1) > 0 \quad \forall n(T-R) \in (0, \infty). \quad (16)$$

Therefore, since $\partial(\partial c/\partial R)/\partial n > 0$, the next step is to prove that the positive part of (10) is larger than the negative part.

Taking common factor and rearranging terms, (10) will be

$$\frac{\partial \left(\frac{\partial U}{\partial R} \right)}{\partial n} = (1 - e^{-rT}) \frac{u'(c)}{c} \left(c \frac{\partial(\partial c/\partial R)}{\partial n} - \rho_r(c) \frac{\partial c}{\partial R} \frac{\partial c}{\partial n} \right). \quad (17)$$

Since $\rho_r(c) < 1$, (10) is positive if

$$\frac{c \frac{\partial(\partial c/\partial R)}{\partial n}}{\frac{\partial c}{\partial R} \frac{\partial c}{\partial n}} \geq 1. \quad (18)$$

Since we are evaluating at $n = r$, after some simplifications, the *LHS* of (18) can be reduced to the following expression:

$$LHS = \frac{(e^{nR} - 1)(e^{nT} - 1)(e^{nT} - e^{nR}(1 + n(T-R)))}{(e^{nT} - e^{nR})n(R(e^{nT} - 1) - T(e^{nR} - 1))}. \quad (19)$$

It can be shown numerically by means of a mathematical programme that $LHS > 1$ for any $n \in (0, 1)$ and for any $R \in (0, T)$. Indeed, the mathematical programme runs results lower than one for values of R very close to 0. So, we should impose a lower bound on R , although almost no restrictive, for instance $R \in (T/100, T)$. Q.E.D.

Proof Proposition 2

We have to prove that with strictly concave indirect utility functions, if $\tau(n, p, R_p^*) = \tau$ then $R_p^* > R_\tau^*$.

Disregarding the scheme, the first order condition of the maximization problem would be:

$$\frac{\partial U_i}{\partial R} = \frac{(1 - e^{-rT})}{r} u'(c_i) \left(\frac{\partial c_i}{\partial R} \right) - e^{-rR} v = 0. \quad (20)$$

Given that for $R = R_p^*$, c_p will be equal to c_τ , we have to prove that at that point

$$\frac{\partial c_p}{\partial R} > \frac{\partial c_\tau}{\partial R} \quad (21)$$

If this inequality holds, the single-peakness property of the utility function implies that $R_p^* > R_\tau^*$. We calculate $\partial c_p / \partial R$ and $\partial c_\tau / \partial R$, in order to compare them:

$$\frac{\partial c_p}{\partial R} = \frac{r e^{-rR}}{(1 - e^{-rT})} (1 - \tau(n, p, R_p^*)) + \frac{p n e^{-nR} (1 - e^{-rR})}{(1 - e^{-rT})(1 - e^{-nR})} \quad (22)$$

$$+ \frac{p n e^{-nR} (1 - e^{-rR})(e^{-nR} - e^{-nT})}{(1 - e^{-rT})(1 - e^{-nR})^2} - \frac{p r e^{-rR}}{(1 - e^{-rT})} \quad (23)$$

$$\frac{\partial c_\tau}{\partial R} = \frac{r e^{-rR}}{(1 - e^{-rT})} (1 - \tau) + \frac{\tau n e^{-nR} (e^{-rR} - e^{-rT})}{(1 - e^{-rT})(e^{-nR} - e^{-nT})} \quad (24)$$

$$+ \frac{\tau n e^{-nR} (1 - e^{-nR})(e^{-rR} - e^{-rT})}{(1 - e^{-rT})(e^{-nR} - e^{-nT})^2} - \frac{p(n, \tau, R_\tau^*) r e^{-rR}}{(1 - e^{-rT})} \quad (25)$$

Given that for $R = R_p^*$, $\tau(n, p, R_p^*) = \tau$ and $p(n, \tau, R_p^*) = p$, we only have to compare the second and the third addends. In the defined pension scheme

$$\tau(R, n, p) = \frac{(e^{-nR} - e^{-nT})}{(1 - e^{-nR})} p \quad (26)$$

We know that $\tau = \tau(n, p, R_p^*)$. So, substituting (26) in (24) and (25), that is, the constant contribution τ for its value, and comparing the two derivatives, we obtain that (21) holds if:

$$\frac{(e^{-nR} - e^{-nT})}{(1 - e^{-nR})} > \frac{(e^{-rR} - e^{-rT})}{(1 - e^{-rR})}. \quad (27)$$

The inequality (27) will be true for $r > n$ since the dependency ratio is decreasing with respect to the population growth rate. Q.E.D.

Notes

¹If there is a possibility of early access to pension benefits with some adjustment to the value of retirement benefits, the average retirement age is usually found between this age at which pensions can be accessed and the standard retirement age. See Blondal and Scarpetta (1998) or Samwick (1998).

²In a further analysis, we shall extend the model to the more general case where the utility of leisure may not be independent of age. That is, the utility of leisure while retired will be $v(l^t) = v(t)$ with $v'(t) \geq 0$.

³Our analysis should be interpreted from a long-run perspective.

⁴Since $0 < \tau(n) < 1$, then $0 < p < \frac{(1-e^{-nR})}{(e^{-nR}-e^{-nT})}$.

⁵Obviously, *this is so when* the interest rate *is* different to the birth rate. If $r = n$, the Social Security is actuarially fair *as* the present value of Social Security benefits is equal to the present value of contributions. In other words, net benefits from the pension system are equal to zero, and therefore, changes in the legal retirement age are neutral.

⁶Notice the different effect of changes in n or r on the optimal decision. While a variation in the interest rate would affect the discount present value of the entire lifetime income, a change in the population growth rate would only affect either the working period or the retirement period depending on the defined parameter of the Social Security.

⁷The actuarially fair system in the case of $r = n$ would be completely equivalent to private savings. So, in our context of certain lifetimes and perfect capital markets individuals would only have to replace private savings with public savings if the contribution rate is not set at their desired level. See Crawford and Lilien (1981) for an extensive analysis of the effect of a pension system on the retirement decision when the three commonly maintained assumptions - perfect capital markets, actuarial fairness and certain lifetimes - are relaxed.

⁸When *the* interest rate is higher than *the* birth rate, $r > n$, the single-peakedness property of the indirect utility function $U(c, R)$ *cannot* be guaranteed for any value of *the* interest and birth *rates*.

⁹This means that the pension system creates intergenerational redistribution.

¹⁰These optimal retirement ages have been calculated for $T = 60$ and $v = 0.008513$. The constant parameters are $p = 0.908$ in the defined pension scheme and $\tau = 0.3$ in the defined contribution scheme.

¹¹For determined parameters we have observed how reductions in the birth rate lead to higher optimal legal retirement ages under the defined contribution scheme. For instance, calculated for $T = 60$, $\tau = 0.3$ and $v = 0.0206$, we have obtained $R_r^* = 40.00$ with $r = 0.06$ and $n = 0.02$, and $R_r^* = 40.19$ with $r = 0.06$ and $n = 0.019$.

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$r = 0.015$		$r = 0.015$	
n	R_p^*	n	R_τ^*
0.0135	46.86	0.0135	40.00
0.0130	49.01	0.0130	39.65
0.0125	51.31	0.0125	39.31
0.0120	53.78	0.0120	39.00
0.0115	56.46	0.0115	38.70

Table 1. Optimal legal retirement ages under each scheme