Cost structures and the movement of the innovation locus: a derived network approach

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Abstract

We consider the problem faced by a manager commissioning an innovative product requiring multi-stage sequenced innovation, when innovating agents have different costs and information transfer is expensive. We specify their optimisation problem and present a polynomial time solution method. We use the method to consider how cost networks influence centre choice switching by solving a series of stochastically generated networks and running logistic regressions on switching frequencies. The effect of expected innovation costs and its standard deviation are shown to be distribution dependent. Expected transfer costs are shown to have an unambiguous dampening effect on the amount of centre switching. Network size sensitivity is considered. Transfer costs are found to be far more influential on switching than innovation costs in a symmetric model. Cost trends that leave average costs unchanged are shown typically to have a significant non-zero effect on switching. A cost structure is introduced to model dichotomous expertise and to link innovation and transfer costs, and agent switches shown to be highly sensitive to an underlying learning cost measure. We then consider the set of sequences constrained to reach each possible final stage agent to reflect managerial specificity. Distributional parameters are found to have a dampened effect on within series changes, and their effect on cross series diversity is demonstrated to be opposite to that on within series changes.

1 Introduction

Information exchange links between innovators have often been found to change over time (Owen-Smith and Powell, 2004; Powell *et al.*,

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2005; Schilling and Phelps, 2007). Commercial or non-commercial partners may enter or leave formal or informal arrangements, and preliminary investigations may become research, development, or commercialisation.

Candidate explanations for the fluctuations include variations in link determinants, such as social proximity (Sorenson, 2003; Cantner and Graf, 2006), resource complementarity (Frenken, 2000), or past ties (Beckman *et al.*, 2004). Von Hippel (1994) proposes that when the cost of acquiring necessary information at an innovation site is high, the innovation locus may shift to an alternative site that already has the information, and that the cost of locus transfer may lead to a further refinement in the process. A common feature of many explanations is that they present a network of information exchange links emerging from an antecedent social, geographical, or information network.

This paper follows Von Hippel (1994) in examining the movement of the problem solving locus under costly knowledge acquisition and transfer. We consider the managerial problem of selecting agents responsible for innovation at each stage of a multistage, sequential process, while minimising total costs incurred. Agents' innovation costs are stage specific, and transfer costs are incurred when different agents are selected for successive innovation stages. The problem faced is equivalent to the selection of a path through a network where nodes represent agent costs and edges represent transfer costs.

For any such cost network, we present a dynamic programming algorithm that offers a polynomial time solution to the managerial problem, in contrast to the exponential time exhaustive search solution. The solution describes the locus of innovation and the movement of information between the loci. The algorithm is used to examine the effect on agent switching of specific connections within innovation and transfer cost networks generated stochastically.

We start by examining the effect of parameter variation in independent identically distributed innovation costs, holding transfer costs constant. Increased expected innovation costs are found to increase the number of agent switches for a lognormal distribution but not a uniform distribution when we also allow for standard deviation, while the cross product of standard deviation and expectations has a distribution dependent effect. We rationalise the observations with reference to distribution shapes and a condition for agent switching.

We then examine agent switching consequent on transfer cost distribution parameter variation, with innovation costs generated by an unchanging uniform distribution. We find greater expected transfer costs tend to decrease the number of agent changes, as the savings from locus transfer are less likely to exceed the costs. We find parameter standard deviation and the cross product are both significant too, and argue for distribution dependent ambiguity in their effects.

We then look at whether our results are sensitive to the network size. We depart from our base assumptions of five agents and thirteen innovation steps to examine many agents or many steps or both. For both the innovation and transfer generated networks, the effect of expected value did not change in its high significance or sign on moving to larger networks, and had relatively modest variation in its size. Standard deviation showed much greater adjustment for the innovation network, gaining significance and a tendency to reduce agent switching. For the transfer network, standard deviation became a much less positive influence on switching. The cross product of expectations and standard deviation similarly had a less positive effect in large networks, but sign and significance were unaltered.

We next examine the effect of parameter variation when the network is generated simultaneously by innovation and transfer cost variation. Transfer costs are found to be much more important than innovation costs for determining agent changes, when the two cost types are compared over the same parameter ranges. The observation is rationalised by noting that transition costs have positive means whereas the difference between two successive innovation costs is symmetric around zero, so we would expect transition costs to be more influential on changes.

We then look at parameter influence on agent changes in the presence of a linear trend in either the innovation or transfer costs. We find that a trend in innovation costs that averages to one nevertheless tends to produce an overall increase in the number of changes. Trends in transfer costs averaging to one tend to reduce the amount of agent switching. We reason that the non-neutrality is a result of the relative distributional concentrations of innovation and transfer costs.

We next examine two network representations of a more dichotomous cost structure. The first assumes that some agents have expertise at some steps in the innovation sequence, and other agents have to pay to acquire the knowledge in order to innovate. We measure this expense by a learning cost parameter multiplier to a Bernoulli variable added to innovation costs. Over a given parameter range, expected learning costs tend to increase agent changes to a much greater extent than the expected innovation costs. The relation is explained by the greater dispersion of the learning costs compared with the innovation costs.

The second dichotomous representation adds the additional assumption that when an agent has to spend money to acquire expertise for innovation, any transfers to or from them will also be more costly. The idea is that there will be greater difficulties in communicating innovation requirements and information about past developments. We find that learning cost again leads to a very large increase in switching. The increase is noted to decompose into parts due to changes in innovation and transfer cost distributions.

We then examine agent switching when innovation sequences are constrained to be at a certain agent in their last stage, where the final agent is successively taken to be each possible one. This might arise if a commissioning agent is the final user or marketer of a new technology, and there is some freedom from cost competition driving out inefficient producers. We consider responses to cost parameter changes within series using aggregated changes, and responses in the diversity across series using counts of distinct agents. We find the effects of expectation and standard deviation changes on within variation tend to be lower than on a single optimal series, while parameter effects on cross variation tend to act in the opposite direction to the effect on within variation. The difference is explained by observing that when series are constrained to pass through a particular agent, factors that tend to reduce within variation also tend to keep series locked into that agent. Thus they reduce common agent usage with series constrained to pass through different agents, and so increase cross diversity.

The rest of the paper has the following structure. Section 2 presents the model and solution method we use. Section 3 presents particular network structures and their estimation results. Section 4 concludes.

2 The model and its solution

2.1 Specification

In this section we present our model of the movement of the problem solving locus. It is characterised by cost minimisation on a network of innovation loci with heterogeneous transfer and innovation expenses.

There are n agents who can initiate an innovation process while minimising total costs. Innovation is a sequential process consisting of s stages. Any initiating agent can be chosen to be responsible for a particular stage of innovation. The cost of innovation varies by the agent and the stage. Selecting the agent for the successive stages is costly with a transfer cost dependent on the stage and on the agent used in the current and next stage. Both innovation and transfer costs are stochastic. The sequence ends at the initiating agent. For most of the analysis, we study the sequence of the initiating agent with the lowest cost sequence. Later, we look at all of the sequences together.

We can represent the innovation process as a sequence $(p_1, p_2, \ldots, p_s)$ where each p_i is chosen from $1, \ldots, n$ indexing the n possible agents who can be selected at each stage. The optimal process is the sequence minimising the total cost

$$I_{p_1,1} + T_{p_1,p_2,1} + I_{p_2,2} + \cdots + T_{p_{s-1},p_s,s-1} + I_{p_s,s}$$

where $I_{p_t,t}$ is the cost of innovation at stage t of the agent with index p_t, and $T_{p_{t-1},p_t,t-1}$ is the cost of transferring at the end of stage t-1 from agent indexed p_{t-1} to agent indexed p_t.

2.2 Solution

Our base analysis of the dynamics proceeds by generating random samples of innovation and transfer costs under various distributional assumptions and then determining the lowest cost sequence of agents for each sample. Next we measure the number of agent changes in the sequence, before logistically regressing the number on the distribution parameters. We then consider modifying factors.

We start by assuming five agents participating in an innovation process with thirteen steps. Costs are positive and are either drawn from a lognormal or uniform distribution with specified mean and standard deviation. For the lognormal distribution, cost samples are generated for means and standard deviations taking evenly spaced values. For the uniform distribution, the mean and lower bound are similarly generated. The cost of transfer from an agent to itself is fixed at zero, and the other transfer costs are generally of the same magnitude as the innovation cost. The relative rather than the absolute values of the costs are of interest for determining the dynamics, and reflect the possibility of a high expense of technology transfer compared with innovation. The assumptions relating to the numbers of agents and steps, and the cost distributions are varied in subsequent simulations.

An exhaustive check on the lowest cost sequence from all n possible choices of agent at each of the s stages is $O(n^s)$, which becomes prohibitively time-consuming for large n and s. We therefore propose a dynamic programming algorithm for use here. We know the costs for optimal series up to the first stage, conditional on the final agent at the first stage - they are just the innovation costs of each vertex. We can then calculate the costs for the optimal series up to the second stage conditional on the final agent at that stage, by comparing for each possible first stage precursor the sum of the costs of being at that first stage and the costs of transferring from that agent to the second stage agent. We repeat the process for later stages, and finally we compare total costs for sequences at each possible stage s end point to find the lowest cost one. The comparisons from the second stage onwards are $O(n^2)$, the final comparison is $O(n)$, and the whole procedure is $O(n^2 s)$.

Once we have the optimal sequences we calculate the number of agent changes within each of them. These numbers together with the cost distribution parameters form the determined and determinant variables in a logistic regression. We implement the data generation, solution algorithm, and MLE estimation in R code forthcoming on our website.

3 Network structures and their estimation

3.1 Innovation costs

In this section we examine the influence of innovation cost distribution parameters on the number of change points for costs generated by lognormal and uniform innovation cost distributions. Lognormal cost samples are generated for innovation cost means and standard deviations taking values from one to ten by increments of 0.2. Uniform distribution cost samples are generated for innovation cost means that are similarly defined, and lower bounds running from 0.2 to the mean less 0.2 at intervals of 0.2, so there are $46^2 = 2,116$ datapoints in each regression. Transfer costs are held constant at five.

Table 1 shows regressions of the number of agent changes on expected innovation costs and their standard deviation, where the cost variation is generated in innovation alone. In column one, a lognormal distribution of costs is used. Rises in expected innovation costs increase the number of agent changes. The effect is highly significant. In column two, the standard deviation also increases the number of changes as a distinct effect to expectation changes, with a highly significant coefficient. The operation of standard deviation on changes seems to operate more directly through the cross product with expectations, as shown in column three. Standard deviation loses its significance to a highly significant cross product effect. Column four introduces a uniform cost distribution, again finding expected costs increase the number of changes. In column five, expected costs have minimal effect when compared with standard deviation's positive effect. Expectations lose any significance in the regression. Column six finds expectations, standard deviation, and their cross product are all highly significant influences.

The influence of expected innovation costs and their standard deviation may be explained by reference to the selection algorithm and the cost distribution function. The criteria for a series showing no vertex change (staying at vertex p) at stage t rather than one showing a change from vertex q to vertex r is

$$C_{p,t-1} + I_{p,t} < C_{q,t-1} + T_{q,r,t-1} + I_{r,t}$$

where $C_{p,t}$ is the cumulative cost to stage t ending at agent p.

Table 1: Logistic regression of number of changes on innovation cost distribution parameters

	Lognormal	Lognormal	Lognormal	Uniform	Uniform	Uniform
Expected costs	0.239 ***	0.242 ***	0.126 ***	0.302 ***	-0.0131	0.253 ***
	0.00832	0.00837	0.0207	0.0156	0.0205	0.0413
St. dev. costs		0.117 ***	-0.0119		0.853 ***	1.88 ***
		0.00785	0.0227		0.0293	0.132
Expectation * st. dev			0.0193 ***			-0.121 ***
			0.0032			0.0152
Constant	-3.54 ***	-4.24 ***	-3.46 ***	-4.47 ***	-4.34 ***	-6.51 ***
	0.0596	0.0779	0.145	0.126	0.13	0.327
Pseudo R^2	0.3	0.38	0.39	0.18	0.63	0.66

* denotes ten percent significance, ** denotes five percent significance, and *** denotes one percent significance. Asymptotic standard errors are reported below the coefficients.

Thus

$$(C_{p,t-1} - C_{q,t-1}) + (I_{p,t} - I_{r,t}) < T_{q,r,t-1} \tag{1}$$

Under the lognormal distribution the distribution around the mean depends on the mean and variance. Changing the mean can vary the distribution of the cumulative difference (the first bracketed term), and the size of the incremental difference in costs (the second bracketed term). This alters the probability of the transfer function being sufficiently large to make movement inefficient, and so the probability of a switch. If we have a translation-invariant innovation cost distribution such as the uniform distribution satisfying $Prob(I_{p,t,m+c} > a) = Prob(I_{p,t,m} + c > a)$ where $I_{p,t,m}$ is $I_{p,t}$ with its dependence on the mean m made explicit and c is any constant, then $Prob(I_{p,t,m} - I_{p,t,m} > k)$ for any constant k is $Prob(I_{p,t,m} + c - (I_{p,t,m} + c) > k) = Prob(I_{p,t,m+c} - I_{p,t,m+c} > k)$, so that altering the mean does not alter the distribution function of the incremental difference. Writing the cumulative function C in terms of its components and then differencing shows that the mean adjustment also leaves the cumulative difference unchanged. For general cost distributions the effect is potentially negative or positive, while the standard deviation does alter the distribution around the mean and hence the difference's distribution.

3.2 Transfer costs

This section regresses agent changes on transfer cost distribution parameters. Transfer costs are generated from lognormal and uniform distributions with parameters varying as in section 3.1. Innovation costs are generated from a uniform distribution on (1,9); if we held innovation costs constant, no changes would occur for any transfer costs.

Table 2 shows the effect of transfer cost determinants on agent shifts when transfer costs are stochastic and innovation costs are stochastic with fixed parameters. Column one finds expected transfer costs are negatively associated with change frequency, with a highly significant coefficient. Column two finds that standard deviation has a distinct positive effect, while column three shows the cross product has a significant positive effect. Columns four to six repeat the regressions with a uniform distribution and find similar

Table 2: Logistic regression of number of changes on transfer cost distribution parameters

	Lognormal	Lognormal	Lognormal	Uniform	Uniform	Uniform
Expected costs	-0.357 ***	-0.374 ***	-0.544 ***	-0.296 ***	-0.499 ***	-0.53 ***
	0.00601	0.00622	0.0162	0.0104	0.0157	0.0197
St. dev. costs		0.182 ***	0.0504 ***		0.56 ***	0.389 ***
		0.00574	0.0125		0.0275	0.0711
Expectation * st. dev			0.0282 ***			0.0228 ***
			0.0024			0.00867
Constant	1.17 ***	0.222 ***	0.984 ***	0.0469	0.164 **	0.369 ***
	0.0321	0.0433	0.0783	0.0643	0.0655	0.101
Pseudo R^2	0.53	0.66	0.67	0.31	0.49	0.49

* denotes ten percent significance, ** denotes five percent significance, and *** denotes one percent significance. Asymptotic standard errors are reported below the coefficients.

results differing in the magnitude but not the direction or significance of the coefficients.

Inequality 1 again is helpful for explaining the observed results. Conditional on the costs to stage $n-1$, an increase in the transfer costs unambiguously decreases the likelihood of a change. The effect of transfer costs to stage $n-1$ is potentially ambiguous, which may offset the contemporaneous effect. The standard deviation of costs may also have an ambiguous effect, dependent on the innovation and transfer cost distributions.

3.3 Size sensitivity

This section regresses agent changes on innovation and transfer cost distribution parameters, when the number of agents or number of steps or both is much larger than the base case. The number of agents is varied up to 50 and the number of steps up to 100. We repeat the innovation and transfer estimations of Tables one and two for the full specification including cross products. Costs are generated from the lognormal distribution.

In Table 3, we examine the sensitivity of the previous results to the number of agents and steps. Columns one to four examine the effect of innovation distribution parameters on agent changes, where the data generation is by changes in innovation parameters and not transfer parameters. Column one replicates the estimation in column three of Table one with few (five) agents and few (thirteen) steps, with some small variations due to a different random seed. Column two has few agents and many (one hundred) steps, and the change effects of expected costs and standard deviation fall. They have the same level of significance, and their explanatory power increases sharply. Column three has many (fifty) agents and few steps. The change effects here increase for expected costs, while standard deviation and its product with expectations have a negative and less positive effect, respectively. Explanatory power increases a moderate amount. The many agents and many steps case is handled in column four. The overall effect of expected innovation costs is slightly more positive than for the few agents, few steps case. Standard deviation decreases the number of changes through its significant direct action, in contrast to the few agents, few steps case, while the product's effect is less positive.

Table 3: Logistic regression of number of changes on innovation (columns one to four) and transfer (columns five to eight) cost distribution parameters with generation by variation in single parameter sets

	Innovation				Transfer			
	Few, few	Few, many	Many, few	Many, many	Few, few	Few, many	Many, few	Many, many
Expected costs	0.117 ***	0.0821 ***	0.211 ***	0.139 ***	-0.542 ***	-0.48 ***	-0.505 ***	-0.475 ***
	0.0209	0.00652	0.0202	0.00593	0.0161	0.00531	0.0148	0.00501
St. dev. costs	0.0066	0.00193	-0.0536 **	-0.0517 ***	0.0468 ***	0.0562 ***	0.218 ***	0.21 ***
	0.0225	0.00689	0.024	0.00672	0.0125	0.00427	0.0154	0.00536
Expectation * st. dev	0.0189 ***	0.0168 ***	0.0157 ***	0.0152 ***	0.0267 ***	0.0218 ***	0.0133 ***	0.0114 ***
	0.0032	0.00101	0.00327	0.00096	0.0024	0.0008	0.00245	0.00084
Constant	-3.51 ***	-3.01 ***	-3.62 ***	-2.83 ***	1.06 ***	0.892 ***	1.54 ***	1.58 ***
	0.146	0.0441	0.147	0.041	0.0783	0.0266	0.0847	0.0294
Pseudo R^2	0.38	0.71	0.51	0.79	0.67	0.88	0.79	0.94

* denotes ten percent significance, ** denotes five percent significance, and *** denotes one percent significance. Asymptotic standard errors are reported below the coefficients. "Few, few" means few agents and few steps, and so on.

Columns five to eight regress agent changes on transfer distribution parameters, where the generation is by variation in those parameters alone. Column five replicates the estimation in Table two, column three for few agents and few steps. Column six looks at few agents and many steps. The effects of transfer distribution parameters are broadly similar to the base case, with a slightly less negative expectations effect and a slightly greater standard deviation effect. Explanatory power increases from its already high base level to a very high level as measured by R^2. In column seven, the many agents and few steps regression finds an expected transfer cost effect that is similar to the few agents, few steps case, and standard deviation and product effect that are over fifty percent lower. Signs and significance are retained, and overall explanatory power rises moderately. Column eight's many agents, many steps regression has a fall in the magnitude of the expected value effect, and large declines in the standard deviation and product effects. Again, signs and significance are retained. Almost all the variation in the data is explained by the determinant variables.

3.4 Innovation and transfer cost interaction

This section regresses agent changes on cost distribution parameters when the data is generated by parameter variation in both innovation and transfer costs. Costs are generated from a lognormal distribution. Owing to the large number of parameter combinations, the cost distribution parameters are spaced at 1 rather than 0.2, meaning there are 10,000 datapoints.

In Table 4, we generate data by varying both innovation and transfer cost distribution parameters simultaneously. Column one regresses changes in the data on innovation distribution determinants alone. Expected innovation costs have a less positive effect on change numbers than when the data is generated by innovation parameter variation alone, as estimated in Table 3.1, column three. Increased standard deviation reduces the number of changes, and the effect is significant. The product's effect is roughly the same. The regression's explanatory power is low. Column two shows the results of regression on transfer distribution determinants alone. Compared with the results in Table 3.2, column three where the data is generated by transfer parameter variation alone, there is a moderate

Table 4: Logistic regression of number of changes on innovation and transfer cost distribution parameters with generation by variation in both parameter sets

	Innovation	Transfer	Both
Expected costs (innovation)	0.0499 ***		0.0597 ***
	0.00493		0.00546
St. dev. costs (innovation)	-0.0434 ***		-0.0548 ***
	0.00517		0.00568
Expectation * st. dev (innovation)	0.0144 ***		0.0183 ***
	0.00079		0.00088
Expected costs (transfer)		-0.42 ***	-0.438 ***
		0.00652	0.00668
St. dev. costs (transfer)		0.0926 ***	0.0993 ***
		0.00492	0.00505
Expectation * st. dev (transfer)		0.0127 ***	0.0128 ***
		0.00096	0.00098
Constant	-1.34 ***	0.35 ***	-0.222 ***
	0.0319	0.0312	0.0468
Pseudo R^2	0.1	0.54	0.66

* denotes ten percent significance, ** denotes five percent significance, and *** denotes one percent significance. Asymptotic standard errors are reported below the coefficients.

reduction in the negative effect of expected costs, a near doubling of the effect of standard deviation, and a rough halving of the product's effect. The explanatory power of the transfer regression is moderate, with an R^2 of 0.54. In column three, we regress changes on both the innovation and transfer cost parameters. There is some modest difference between the innovation cost distribution coefficients of columns one and three, and the transfer cost distribution coefficients are very similar. The explanatory power is quite high.

We can see that transfer costs will often be more important than innovation costs as influences on change points by examining the change criteria 1. The left hand side consists of the sum of two zero mean variables, by symmetry. The dispersion may be altered by the expected innovation cost and its standard deviation. By contrast, the right hand side variable is a positive mean random variable. Altering the expected transfer cost alters the mean as well as possibly the dispersion. For distributions where the mean shifting effect is more important than the dispersion effect, expected transfer costs will typically have a greater explanatory power than expected innovation costs.

3.5 Trends

This section looks at cost parameter influence on agent changes, when costs are subject to a linear trend. Incremental innovation and transfer costs are generated as for sections 3.1 and 3.2, and the trend is applied as a multiplier to them that equals one when at the midpoint of the step sequence. Thus, the expected cost over the mean trended sequence is the same as the expected cost over the mean untrended series. Costs are generated from the lognormal distribution. Cost and trend series are each split into 19 parts for generating the data, giving 6,859 datapoints.

Table 5: Logistic regression of number of changes on innovation and transfer cost distribution parameters and trends with generation by variation in single parameter sets

	Innovation	Transfer
Expected costs	0.139 ***	-0.461 ***
	0.0114	0.00801
St. dev. costs	0.0301 **	0.0618 ***
	0.0122	0.00644
Expectation * st. dev	0.015 ***	0.0194 ***
	0.00171	0.00121
Trend	1.24 ***	-1.2 ***
	0.124	0.0863
Constant	-3.7 ***	0.849 ***
	0.0807	0.0402
Pseudo R^2	0.39	0.66

* denotes ten percent significance, ** denotes five percent significance, and *** denotes one percent significance. Asymptotic standard errors are reported below the coefficients.

Table 5 contains regressions of agent switches on cost distribution determinants including a trend in estimation and generation. Column one looks at innovation cost determinants, and finds that the innovation cost trend has a large positive coefficient. We might think that the trend might have a positive effect over part of the series since it would increase expected costs at some point, so that it should share the same sign as expectations. On the other hand, it would be tend to have a negative effect over the rest of the series, so it is not immediately transparent to us why the overall effect should be positive. Column two examines transfer cost determinants. The transfer cost trend has a large negative effect, in common with the expected costs. Once again, it is not a priori clear to us why averaging to zero of the trend effect does not occur.

We may reason for a non-zero average effect by reference to cost distribution shapes, taking an extreme example for illustration. If the sum of expected cumulative costs and current innovation costs is slightly greater than the transfer costs and has low variance, and the transfer costs also have low variance, then by the inequality 1 there should be many agent changes. If we then introduce a transfer cost trend, then some of the transfer costs will be larger than the sum, and so no changes will occur for them. The other transfer costs will fall, so changes will still occur, and the overall effect is a reduction in the number of changes.

3.6 Dichotomous costs

In this section, we perform our regressions introducing a more dichotomous cost structure which links the distributional parameters in innovation and transfer costs. The first representation is that some agents at some steps have specific expertise that is necessary for innovation, whereas others lack the skills entirely. We model the assumption as a Bernoulli variable addition to all innovation costs, where the Bernoulli variable has probability 0.5 and is multiplied by a non-stochastic learning cost variable. The second representation makes the same assumption as the first, and additionally that any agent who has a non-zero learning cost will also have additional costs from receiving or transferring the innovation locus to another agent at the same stage. The rationale is that the same lack of innovation skills will lead to difficulties in communicating the innovation requirements and the specification of past innovation. The mathematical model is that for any agent who has non-zero learning cost we add the same amount to the transfer cost inwards or outwards from the agent.

We generate the first model with lognormal stochasticity in the innovation costs and fixed transfer costs, as in section 3.1. The innovation cost mean, its standard deviation, and the learning cost parameters are split at 0.5 intervals over (1,10) for the simulation, giving 6,859 datapoints. For the second model, we generate data using variation in the innovation and transfer cost distribution parameters, as well as in the learning cost. For generation purposes, the parameters are spaced at unit intervals over (1,10), giving 3,125 datapoints.

Table 6 shows regression results for the two models. Column one regresses agent changes on innovation cost determinants including learning cost, where these generate the data under the first model. The learning cost variable tends to increase the number of agent changes, and is highly significant. Expected costs have a much smaller positive effect compared with the results in Table 1, column three. The learning cost variable adds both to the expected innovation cost and its standard deviation. We would therefore expect it to have the same broad effect as these variables. Their different distributional assumptions present an explanation of the different coefficients and significances of the effects.

Columns two to four regress agent changes on cost parameters under the second model. Column two regresses agent changes on innovation parameters alone. Learning cost increases the number of agent changes, and is highly significant. Expected cost loses its significance compared to the no-learning case in Table 4 column one, while standard deviation and the product have significant negative and positive effects respectively, representing no qualitative and small quantitative changes from Table 4. Explanatory power is very low. Column three regresses agent changes on transfer cost distribution parameters alone. Learning cost increases agent changes, and is highly significant, while the other coefficients are comparable to Table 4, column two where there is no learning cost. A little over half the data's variation is explained by the model. In column four, both innovation and transfer cost parameters are determinants, and the estimated coefficients are very similar to those of the separate models. Explanatory power is barely greater than the transfer cost alone model.

Under the second model, learning cost increases both expected innovation cost and its product (tending to increase the number

Table 6: Logistic regression of number of changes on innovation and transfer cost distribution parameters and learning costs with generation by variation in single (column one) and both parameter sets (columns two to four)

	VH1	VH2 innovation	VH2 transfer	VH2 both
Expected costs (innovation)	0.0127 *	0.0106		0.0119
	0.00716	0.0176		0.0188
St. dev. costs (innovation)	-0.0118	-0.0413 **		-0.0475 **
	0.00726	0.0177		0.0189
Expectation * st. dev (innovation)	0.0118 ***	0.0265 ***		0.0305 ***
	0.00114	0.00529		0.00566
Expected costs (transfer)			-0.649 ***	-0.653 ***
			0.0205	0.0206
St. dev. costs (transfer)			0.0993 ***	0.1 ***
			0.0186	0.0186
Expectation * st. dev (transfer)			0.0431 ***	0.0433 ***
			0.00596	0.00598
Learning cost	0.176 ***	0.139 ***	0.158 ***	0.159 ***
	0.0032	0.00749	0.00801	0.00804
Constant	-2.59 ***	-0.915 ***	0.376 ***	0.211 **
	0.0495	0.063	0.066	0.0904
Pseudo R^2	0.42	0.06	0.56	0.58

* denotes ten percent significance, ** denotes five percent significance, and *** denotes one percent significance. Asymptotic standard errors are reported below the coefficients.

of change points), and expected transfer costs (tending to decrease change points) and its product (tending to increase change points). Here the net effect is positive.

3.7 Distinct fixed final agents

Previous sections have studied the single series of agents with the lowest overall costs. In this section we take the last agent in the series to be each possible agent in turn, so that five different series are generated. We can then look at variation within series to measure the number of agent changes, and variation across series to measure the divergence of paths.

We generate data as in sections 3.1 and 3.2, arising from innovation and transfer parameter variation respectively. We then run our dynamic programming algorithm to identify the lowest cost sequences conditional on the end agents, and do not perform the final selection step. For the within variation, the number of agent changes is summed across all series over all periods and divided by the number of possible changes to give the dependent variable in the logistic regression. The measure of the cross variation is the sum across the first to penultimate step of the number of distinct agents in each step, divided by the maximum number possible of differences. This is the corresponding logistic dependent variable.

Table 7 shows regressions of agent changes on cost distribution parameters, when the final step agents are constrained to be each of the possible ones in turn. Column one shows the regression of within series variation on the innovation cost distribution parameters when these are used to generate the variation in the data. Expected innovation costs have a much smaller positive effect than when the optimal series alone is examined in Table 3.1, column three, while the product has a little smaller effect. They both remain highly significant. Explanatory power is moderately high.

Column two looks at within series variation for data generated by and estimated for transfer cost parameter variation. The effects of expectations and standard deviations are slightly reduced compared with the non constrained case in Table 3.2, column three. Signs and significance are unchanged, and explanatory power is high.

Column three regresses cross series diversity on innovation cost distribution determinants. Expectations and the product both tend

Table 7: Logistic regression of the aggregate number of changes (columns one and two) or the aggregate number of distinct cross series agents (columns three and four) on innovation and transfer cost distribution parameters with generation by variation in single parameter sets

	Innovation within	Transfer within	Innovation cross	Transfer cross
Expected costs	0.0664 ***	-0.463 ***	-0.0313 ***	0.111 ***
	0.00781	0.00667	0.00393	0.00439
St. dev. costs	0.00444	0.0613 ***	0.00382	-0.00196
	0.00815	0.00548	0.00378	0.00491
Expectation * st. dev	0.0151 ***	0.0196 ***	-0.00864 ***	-0.00509 ***
	0.00121	0.00102	0.00067	0.00074
Constant	-2.73 ***	0.913 ***	-15.9 ***	-16.9 ***
	0.0519	0.0339	0.0225	0.0294
Pseudo R^2	0.47	0.73	0.36	0.47

* denotes ten percent significance, ** denotes five percent significance, and *** denotes one percent significance. Asymptotic standard errors are reported below the coefficients.

to reduce diversity, and are highly significant. The standard deviation is not significant. Explanatory power is moderate. Column four regresses diversity on transfer cost determinants. Expected costs increase diversity, while the product reduces it. Again they are highly significant, and the standard deviation is not significant.

The significant variables explaining the cross series diversity have the opposite effect on within series agent changes. Factors that tend to lock innovation managers into a given innovation agent reduce within variability. If the various innovation series are constrained to pass through different agents at some stage, as here, the same factors will tend to make those agents persist over the series increasing cross series diversity.

4 Conclusion

This paper examines the movement of the innovation locus when a cost minimising manager selects agents to innovate at each step of a multistage process. We present a dynamic programming algorithm able to solve the manager's problem in manageable time, and use it to analyse the effects of cost network structure on innovation locus selection. We show that the effect on agent switching of expected innovation costs and their standard deviation depends on the cost distribution shape. Increasing expected transfer costs tends to decrease switching, with a potentially ambiguous higher moment effect. The sensitivity to network size is demonstrated, with a general tendency to reduce the predicted parameter-induced increases in switching in large networks. Transfer cost effects are shown to dominate innovation cost effects in influencing switching. Trends are considered, and a neutral linear trend averaging to one is shown to have a non-zero effect on agent changes under many distributional assumptions. Two representations of cost structures with dichotomous expertise are presented, and the effects of learning costs for non-experts are shown to be much larger than the effects of normal innovation expected costs. When series are constrained so that innovation series are generated with each ending at a different agent, we find that parameter sensitivity of average within series variation is reduced compared with the single series case. We find that when distributional parameters tend to increase within series changes, they tend to decrease cross series diversity and vice versa.

This paper has generated a number of theoretical predictions without empirical investigation. Some of the predictions such as the offsetting between within variation and cross diversity lend themselves to immediate testing, whether on survey data or more aggregated data. We leave such testing to future work.

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