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Li, Fei and Tian, Can
University of Pennsylvania

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Fei Li
University of Pennsylvania

Can Tian*
University of Pennsylvania

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Abstract

In this note, we consider the impact of job rotation in a directed search model in which firm sizes are endogenously determined, and match quality is initially unknown. A large firm benefits from the opportunity of rotating workers so as to partially overcome mismatch loss. As a result, in the unique symmetric subgame perfect equilibrium, large firms have higher labor productivity and lower separation rate. In contrast to the standard directed search model with multi-vacancy firms, this model can generate a positive correlation between firm size and wage without introducing any exogenous productivity shock or imposing non-concave production function assumption.

Keywords: Directed Search, Job Rotation, Firm Size and Wage, Firm Size and Labor Productivity

JEL Classification Codes: L11; J31; J64

*Department of Economics, University of Pennsylvania, 3718 Locust Walk, Philadelphia, PA 19104; Email: tiancan@sas.upenn.edu

1 Introduction

Job rotation practice is commonly observed in large firms. In the literature, it is well known that the job rotation policy mainly results from learning of pair-wise match quality between workers and jobs. However, little work has been done to address the impact of job rotation on the labor market. One reason is that the study of job rotation requires a framework to simultaneously consider internal labor market of a firm and external labor market. Yet, in the job search model, labor economists' favorite work horse, a firm is treated as a single job vacancy, and therefore it is impossible to distinguish internal and external labor market. Recently, many job search papers, including Hawkins (2011), Kaas and Kircher (2010), Lester (2010) and Tan (2011), have shed light on endogenous determination of firm size, which have the potential to study the interaction between a firm's internal and external labor market.

In this note, we employ a directed search model with multi-vacancy firms to examine the role of job rotation in labor market. In particular, we assume that a firm can choose its size by determining the number of job vacancies. A large firm can hire more workers, which requires higher fixed cost. All workers are ex ante identical, but they may be good at different jobs, which is initially unknown. The match quality between a worker and a job is uncertain when the worker is hired, but can be learned afterwards. Large firms can freely reallocate workers over jobs, and therefore partially overcome the loss of mismatch.

Our main result highlights the impact of job rotation in labor market. In the unique symmetric subgame perfect equilibrium, we obtain a positive correlation between firm size, labor productivity and wage, which is consistent with empirical findings, when the mismatching risk is severe enough. Without the opportunity of job rotation, however, the correlation between firm size, labor productivity and wage is negative for all parameters, which is the result of a standard directed search model.

Our note is related to the literatures in two ways. First, Meyer (1994) and Ortega (2011) point out the learning role of job rotation in firms. They provide justification of job rotation, but both of them narrow the study within the boundary of a single firm. As a step further, we apply their insight in a competitive labor market model to study the effect of within-firm job rotation on external labor market. Papageorgiou (2011) is the only note that studies the impact of job rotation on labor market but with a different focus. He pays more attention to the interaction between tenure effect and job reallocation within a firm, while, in contrast, we focus on how internal labor market in presence of job rotation affects the job allocation on the external labor market. In his model, firm sizes are exogenously given rather than endogenously determined as in ours. In addition, he utilizes a Pissarides-Mortenson model and introduces heterogenous firms, so the pricing mechanism in his paper is Nash bargaining instead of wage posting, and the search is random rather than directed.

Second, the directed search model we employed is along the lines of Montgomery(1991), Peters (1991), Burdett, Shi and Wright (2001), and their later extension by Lester (2010) to multi-vacancy case. Kaas and Kircher (2011) also study a directed search model with multi-vacancy firms. However, none of the above can generate a relationship between firm size, wage and labor productivity that is in line with observations without introducing exogenously dispersed random productivity. Shi (2002) introduces a frictional product market to overcome this problem. In his paper, large firms have more incentive to attract workers since they have bigger share in product market and are anxious to produce enough product. Tan (2011) allows local convexity in production function to generate a positive size-wage differential. Yet, in our model, the production function is concave.

The rest of this note is organized as follows. We first set up the model and characterize the unique symmetric subgame perfect equilibrium. Next, we derive implications of our model and discuss the result and compare them to empirical evidences.

2 The Model

2.1 Setup

There are N workers and M firms on the market, both of which are ex ante identical. Denote $\lambda = M/N$ as the ratio of firms to workers. Note that λ is not the labor market tightness since the number of vacancy is endogenous in this model. Following the literature, we first consider individual decision problem given N, M as finite numbers, then we fix λ and take N, M to infinity to approximate the equilibrium in a large labor market.

A match of a worker-job pair is good with probability $\rho \in (0, 1]$, and bad with a complementary probability. If the match is good, we say the quality is 1 meaning the worker-job match can produce 1 unit revenue; otherwise, 0. The match quality is initially unknown, and learned later. We assume the match quality is independent across jobs and workers, even within a multi-job firm.

The game has three stages: job posting stage (I), job searching stage (II) and production stage (III). At Stage I, the job posting stage, each firm decides how may vacancies to post, k, and at what wage level, w, where w is potentially a function of k. For simplicity, we assume that they can create $k \in \{1, 2\}$ vacancies with cost $C(k)$, thus the market tightness is $\theta \in [\lambda, 2\lambda]$. Without loss of generality, we assume convex cost function with $C(1) = 0$, $C(2) = C$, $0 < C < \rho$ and let $c = C/\rho \in (0, 1)$. We assume that wage, $w \in [0, 1]$, does not depend on any further information such as the realized number of applicants and revealed match quality.

At Stage II, the job searching stage, each worker observes (k, w_k) of every firm and applies for the firms that offer the highest expected payoff. We assume that workers can only apply for a firm instead instead of a specific position in that firm. If the number of workers that apply for a particular firm exceeds the number of vacancies posted, the firm randomly hire just enough workers; otherwise the firm hires all applicants. Then the firm assign job positions randomly to employees. Hence, a worker's expected payoff from applying for a firm is determined jointly by both the posted wage and the probability of getting a job.

At Stage III, the production stage, a firm with k jobs and h employees, $1 \leq h \leq k$, learns match qualities of all $P_h^k = k!/\left(k-h\right)!$ possible worker-job pairs, which have $2^{P_h^k}$ possible realizations. A large firm with $k = 2$ has the freedom to assign jobs to employee(s) to derive the highest revenue, which creates a potential benefit margin compared to a small firm ($k = 1$). For example, if a firm posts 2 jobs, A and B, and hires 2 workers I and II, it can observe the match qualities of pairs $\{(I, A), (I, B), (II, A), (II, B)\}$, with the value of, say $\{1, 0, 0, 1\}$. In this specific case, clearly the firm shall let I do job A and II goes to B to earn 2 as the total revenue, provided that the firm pays $2w_2$ to workers. The job reallocation benefit can be fully described as follows. From the point of view of an employee hired by a two-job firm, his match quality state is $s \in \{AB, \bar{A}\bar{B}, A\bar{B}, \bar{A}B\}$, where AB means his match quality is 1 with both job A and B, and $\bar{A}\bar{B}$ means 0 with each, and the both $A\bar{B}, \bar{A}B$ can be interpreted as the similar way. For a firm with $\left(k, h\right), h \geq 1$, the optimized payoff matrix is given as following tables.

INSERT TABLES HERE

When two workers' state are $\left(\bar{A}B, A\bar{B}\right)$, the firm can match between I and job B and II and job A. Hence, the probability to overcome one or two mismatch can generate extra revenue for a large firm. We define $F\left(\rho; k, h\right)$ the expected revenue function of a firm with (k, h) before match qualities are observed. For a small firm,

$$F(\rho; 1, 1) = \rho.$$

Similarly, for a large firm,

$$\begin{aligned} F\left(\rho; 2, 1\right) &= 1 - \left(1-\rho\right)^2 > \rho, \\ F\left(\rho; 2, 2\right) &= -2\rho^4 + 4\rho^3 - 4\rho^2 + 4\rho > 2\rho, \ \forall \rho \in (0, 1). \end{aligned}$$

and and define the rate of marginal gain for a large firm as $g(\rho) = \frac{F(\rho;2,2) - F(\rho;2,1)}{\rho} = -2\rho^3 + 4\rho^2 - 3\rho + 2$, where $g(\rho) \in (1, 2)$ for $\rho \in (0, 1)$, and it is strictly decreasing in ρ. We highlight two features here. First, observe that $F\left(\rho; 2, 2\right) - 2F\left(\rho; 2, 1\right) = -(1-\rho)^2 < 0$, meaning the marginal labor productivity in a large firm's is decreasing in the number of employees. Second, we model the learning of match quality and the practice of job rotation in a reduced form. In general, one can assume infinitely many substages in the production stage where the firm can reallocate workers over jobs in each substage. In the first substage, given the job allocations at Stage II, the firm learns the quality of each match. In the following substage, the firm is given the choices of reallocating workers over jobs and firing workers. This setup can generate similar continuation payoff as our reduced form game.

2.2 Analysis

The solution concept we adopt is symmetric subgame perfect equilibrium (SSPE), in which each firm chooses to be a large firm with the same probability and posts the same wage, and each worker applies for a large firm with same probability. We will solve the game backwards. Given any history, which will be defined later, in Stage II, the job searching stage, a firm reallocates workers over jobs optimally in Stage III if possible. Therefore we start from the job searching stage and characterize the symmetric Nash equilibrium in this subgame for any given history in which firms play symmetric strategies. Then, we will characterize each firm's strategy given the strategies of workers.

Stage II: Job Searching Stage. The history of job posting in Stage I can be summarize by a vector $H = (w^1, ..w^M; k^1, ..k^M)$ listing wages and sizes of all M firms. Let $\mathcal{H}$ be the set of all possible H's. In principle, workers strategy is defined as $\gamma : \mathcal{H} \to [0,1]^M$. Given a history H, a worker chooses a vector γ such that (1) γ^j is the probability that he applies for firm $j \in \{1, 2, ..M\}$ and (2) $\sum_{j=1}^{M} \gamma^j = 1$.

Consider the problem of worker i who is deciding whether and for which firm to apply. When the rest $N-1$ workers play identical strategies γ and firm j posts k^j positions and wage w^j, for $j \in \{1, 2, ..M\}$, this worker chooses strategy $\hat{\gamma}$ to maximize his expected utility

$$\max_{\hat{\gamma}} E_{\hat{\gamma}} \left[\Omega\left(\gamma\right) w\right] = \sum_{j=1}^{M} \hat{\gamma}^j \Omega_{k^j}(\gamma^j) w^j,$$

where

$$\begin{aligned}
\begin{bmatrix} \Omega_1(\gamma^j) \\ \Omega_2(\gamma^j) \end{bmatrix} &= \begin{bmatrix} \sum_{n=0}^{N-1} \left[\frac{(n-1)!}{n!(N-1-n)!}\right] \left(\gamma^j\right)^n \left(1-\gamma^j\right)^{N-1-n} \frac{1}{n+1} \\ \left(1-\gamma^j\right)^{N-1} + \sum_{n=1}^{N-1} \left[\frac{(n-1)!}{n!(N-1-n)!}\right] \left(\gamma^j\right)^n \left(1-\gamma^j\right)^{N-1-n} \frac{2}{n+1} \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{N\gamma^j} \left[1-\left(1-\gamma^j\right)^N\right] \\ \frac{2}{N\gamma^j} [1-(1-\gamma^j)^N] - (1-\gamma^j)^{N-1} \end{bmatrix}.
\end{aligned}$$

$\Omega_{k^j}(\gamma^j)$ stands for the probability that this worker is hired if he applies for firm j which posts k^j positions. Given any history H, we have $\gamma^*(H) = (\gamma^{*j}(H))$ as the symmetric Nash equilibrium in this subgame. Define the market utility level as $U^*(H) = \max_j\{\Omega_{k^j}(\gamma^{*j})w^j\}$. Apparently, for any $\gamma^{*j}(H)$ to be positive, applying for firm j must deliver the market utility to an arbitrary worker.

Stage I: Wage Posting Stage. Now take one step back and consider a firm's problem. A firm's strategy is to choose $(\phi, w_1, w_2) \in (0,1) \times (0,1) \times (0,1)$, which consists of a probability ϕ to become a small firm, and a size contingent wage menu (w_1, w_2). Since we work in a backward order, a firm expects the forms of γ^* and U^*. When all the other firms choose ϕ and post wage menu (w_1, w_2), if firm j posts a single vacancy, it chooses $\hat{w}_1^j$ to maximize the expected profit,

$$(\pi_1^*)^j = \max_{(\hat{w}_1)^j} E_\phi \left[[1 - (1 - \gamma^{*j}\left(\hat{H}_1\right))^N] \left(\rho - \hat{w}_1^j\right) \right],$$

where $\hat{H}_1^j = \hat{H}_1^j(\hat{w}_1; \phi, w_1, w_2) = (1, \hat{w}_1; k^{-j}(\phi), w^{-j})$ represents an arbitrary distribution of sizes and wages induced by (ϕ, w_1, w_2) of other firms provided that firm j posts a single vacancy and sets wage to be $\hat{w}_1$. The maximization is subject to the following constraint:

$$\Omega_1\left(\gamma^{*j}\left(\hat{H}_1^j\right)\right)\hat{w}_1^j = U^*\left(\hat{H}_1^j\right),$$

otherwise firm j would anticipate zero applicants. The firm j's expected profit is the product of the probability that at least one applicant arrives, $1-(1-\gamma^{*j})^N$, and the expected surplus, $\rho - \hat{w}_1^j$. Similarly, if firm j posts two vacancies, a similar problem must be solved:

$$(\pi_2^*)^j = \max_{\hat{w}_2^j} E_\phi \left\{ \begin{array}{l} \left[1 - N\gamma^{*j}\left(\hat{H}_2^j\right)\left(1-\gamma^{*j}\left(\hat{H}_2^j\right)\right)^{N-1} - \left(1-\gamma^{*j}\left(\hat{H}_2^j\right)\right)^N\right]\left(F(\rho;2,2) - 2\hat{w}_2^j\right) \\ + \left[N\gamma^{*j}\left(\hat{H}_2^j\right)\left(1-\gamma^{*j}\left(\hat{H}_2^j\right)\right)^{N-1}\right]\left(F(\rho;2,1) - \hat{w}_2^j\right) - C \end{array} \right\}$$

subject to

$$\Omega_2\left(\gamma^{*j}\left(\hat{H}_2^j\right)\right)\hat{w}_2^j = U^*\left(\hat{H}_2^j\right),$$

with $\hat{H}_2^j == \hat{H}_2^j(\hat{w}_2; \phi, w_1, w_2) = (2, \hat{w}_2; k^{-j}, w^{-j})$. In equilibrium, we have $\left(\hat{w}_1^j, \hat{w}_2^j\right) = (w_1, w_2) = (w_1^*, w_2^*)$, $\forall j$. Meanwhile, we have $\gamma^{*j} = \gamma_1^*$ if $k^j = 1$, and $\gamma^{*j} = \gamma_2^*$ if $k^j = 2$. Define $\pi^* = \max_\phi[\phi\pi_1^* + (1-\phi)\pi_2^*]$, and ϕ^* as the probability that a firm chooses to post one vacancy. Again, for ϕ^* to be positive, π_1^* must equal π^*.

Equilibrium Characterization. The equilibrium is characterized in the following proposition.

Proposition 1. *There exists a unique SSPE in this game.* $\exists\underline{\lambda}$ *and* $\bar{\lambda}$ *such that the equilibrium strategy profile* $(\gamma_1^*, \gamma_2^*, \phi^*, w_1^*, w_2^*)$ *satisfies one of following three condition:*

1. *if* $\lambda \leq \underline{\lambda}$, *then* $\phi^* = 0$ *and* $\gamma_1^* = 0$;

2. if $\lambda \geq \bar{\lambda}$*, then* $\phi^* = 1$ *and* $\gamma_2^* = 0$*;*

3. if $\lambda \in (\underline{\lambda}, \bar{\lambda})$*, then* $(\phi^*, \gamma_1^*, \gamma_2^*) \in (0,1) \times (0,1) \times (0,1)$*, and in this equilibrium, the wage and market tightness* θ *in small and large firm market are given by*

$$\begin{aligned} w_1 &= \frac{\rho e^{-q_1}}{1-e^{-q_1}}, w_2 = \frac{q_2 e^{-q_2}[F(\rho;2,1) + (F(\rho;2,2) - F(\rho;2,1))q_2]}{2(1-e^{-q_2}) - q_2 e^{-q_2}}; \\ \theta_1 &= 1/q_1,\ \theta_2 = 2/q_2 \end{aligned}$$

where $q_1 = \gamma_1^* N$ *and* $q_2 = \gamma_2^* N$ *are the queue lengths at a small or a large firm.*[1]

In the first two equilibria, there is no heterogeneity in firm size. The intuition behind these two equilibria are simple. When λ is too small, there are so few firms in the market relative to workers such that it is easy to hire two workers with a low wage. In equilibrium, no firm chooses to become a small one. Similarly, when λ is too large, there are too many firms, and it is hard and costly to fill both vacancies as a large firm. In equilibrium, no firm wants to be a large one. In next subsection, we focus on the last case, in which q_k is the expected number of applicants at a firm with k vacancies, and characterize the impact of job rotation on labor market variables.

2.3 Implications and Discussions

In this subsection, we look at some implications of the unique SSPE. The model gives predictions on relationships between firm size and productivity, wage, profit, as well as separation rate, which are roughly in line with empirical findings.

In our model, the job rotation rate is trivially increasing in firm size. We can generalize our model one step further and allow firms to post $1, 2, .., K$ vacancies. Now that a larger firm can overcome the mis-match loss even more via reassignment of jobs, a higher rotation rate shall appear. This is consistent with empirical findings by Papageorgiou (2011). We will see how this higher job rotation benefit in larger firms affect the labor market.

Size and Labor Productivity. The average labor productivity of a small firm is ρ, that of a large firm is a convex combination of $F(\rho;2,2)/2$ and $F(\rho;2,1)$, which is greater than ρ since $F(\rho;2,2) > 2\rho$ and $F(\rho;2,1) > \rho$ for any $\rho \in (0,1)$. As stated before, marginal labor productivity of a large firm is decreasing in size measured as number of employers, and therefore the production function of a large firm is concave.

[1] It is worth noting that the existence of heterogenous firm sizes is due to coordination failure friction rather than job rotation. In standard directed search model with multi-vacancy firms, one can also obtain a unique SSPE in which both large and small firms exist.

Size and Wage Differential. In standard directed search models, if all firms were to offer the same wage, then firms with more vacancies would attract more job seekers as the probability of filling a vacancy is higher. Hence, small firms must increase wages to compete in the labor market. In equilibrium, workers are indifferent to whichever firm to apply for, and large firms are associated with low wages. However, this contradicts the observations on labor market[2]. In our model, large firms have the opportunity to reallocate workers over jobs and partially overcome the mismatch between workers and jobs. This job rotation effect within a large firm results in higher expected productivity, and therefore higher wage in equilibrium. We claim that, when such effect is strong enough to offset the coordination failure, wage premium of large firms arises.

Result 1. *Large firms offer lower wages than small firms if there is no mismatch, $\rho = 1$. For any $c \in (0, 1)$, there exist a $\bar{\rho}(c)$ such that $w_2 > w_1$ when $\rho \in (0, \bar{\rho}(c))$.*

We provide a numerical illustration of this result due to difficult derivation of an analytical proof. In Figure 1, w_2/w_1 is the wage premium of a large firm. When $\rho = 1$, we replicate the result of standard directed search model with multi-vacancy firms, simply because there is no risk of mismatch. In this case, large firms offer lower wage for any positive c. When ρ is small, it is possible to obtain the wage premium of large firms. The intuition is as follows. Smaller ρ implies a higher probability of mismatch and, consequently, a greater job rotation benefit and a higher wage premium; thus the wage premium is decreasing in ρ. To avoid the inconvenience caused by the absolute scale of the entry cost, we normalize C as a fraction of ρ, looking at the dimension of c instead of C in the comparative statics with respect to ρ. When ρ goes to zero, the entry cost C goes to zero pro rata.

INSERT FIGURE 1 HERE

For standard directed search models to generate positive correlation between firm size and wage, exogenous productivity difference is required. In particular, Kaas and Kircher (2011) and Lester (2010) assume that firms randomly draw their productivity levels from a pre-determined distribution before they enter the labor market, and high productivity firms decide to be large and low productivity firms choose otherwise. If the ex ante distribution of productivity is dispersed enough, this technology difference can overcome the frictional effect of coordination failure, and can generate reasonable size-wage differential. In their models, large firm size and wage premium are the consequence of high productivity. Our model suggests a somewhat reversed direction of such relationship: even with ex ante homogeneity assumed, large firms may emerge, taking

[2] For example, Brown and Medoff (1989), Oi and Idson (1999) point out that there exists a positive size-wage differential in labor market.

advantage of the opportunity of job rotation, which in turn induces high productivity and wage premium.

Size and Separation Rate. For tractability, we introduce job rotation together with separation in a combined and induced manner. Nevertheless, it is possible to explicitly model separation decision by assuming indefinitely many substages after initial worker-job match. When a firm gradually learns its workers' match quality with all positions, it has the chance to fire incapable employees. Due to the job rotation advantage, large firms have lower separation rate than small firms in our model. This prediction is also supported by recent empirical work. Papageorgiou (2011) finds that workers in larger firms are less likely to separate even conditional on the worker's wage by analyzing Survey of Income and Program Participation data.

3 Conclusion

We modified a standard directed search model to explain the size-wage differential observed in labor market, highlighting the effect of job rotation practice. However, in contrast to the standard directed search model with multi-vacancy firms, our modified model can generate a positive correlation between firm size and wage without introducing any exogenous productivity shock or imposing non-concave production function assumption. We assume ex ante homogeneous firms and workers, and initially unknown match quality that determines labor productivity. Firm sizes are endogenously determined. Paying extra cost, a large firm benefits from the opportunity of rotating workers so as to partially overcome mismatch loss. As a result, in the unique symmetric subgame perfect equilibrium, large firms have higher labor productivity and, when explicitly modeled, lower separation rate.

Appendix

3.1 Proof

Proof of Proposition 1. The proof is essentially same as the proof of existence theorem in Montgomery (1991), Burdett, Shi and Wright (2001), and Lester (2011). We starts with the equilibrium of the last case. Fix λ, let N, $M \to \infty$. Then workers' utility from applying small firm and large firm are given by

$$U_1 = \frac{1 - e^{-q_1}}{q_1} w_1, U_2 = (\frac{2}{q_2}(1 - e^{-q_2}) - e^{-q_2}) w_2$$

In equilibrium $U_1 = U_2 = U^*$.

Firms' problem become

$$\max_{q_1} \rho(1-e^{-q_1}) - q_1 U^*$$
$$\max_{q_2} F(\rho;2,2)(1-e^{-q_2}-q_2e^{-q_2}) + F(\rho;2,1)q_2e^{-q_2} - q_2U^* - C$$

by plugging $w_1 = \frac{U^*}{\frac{1-e^{-q_1}}{q_1}}$ and $w_2 = \frac{U^*}{(\frac{2}{q_2}(1-e^{-q_2})-e^{-q_2})}$ into firms' decision problems. They yield FOCs

$$w_1 = \frac{q_1\rho e^{-q_1}}{1-e^{-q_1}} \tag{1}$$
$$w_2 = \frac{q_2e^{-q_2}[F(\rho;2,1)+(F(\rho;2,2)-F(\rho;2,1))q_2]}{2(1-e^{-q_2})-q_2e^{-q_2}} \tag{2}$$

Plugging (1) and (2) into workers' utility and firms' profit yields

$$U_1(q_1) = \rho e^{-q_1}, U_2(q_2) = e^{-q_2}[F(\rho;2,1)+(F(\rho;2,2)-F(\rho;2,1))q_2]$$

and

$$\begin{aligned}\pi_1(q_1) &= (1-e^{-q_1})\rho - q_1\rho e^{-q_1}\\ \pi_2(q_2) &= F(\rho;2,2)(1-e^{-q_2}-q_2e^{-q_2}) + F(\rho;2,1)q_2e^{-q_2}\\ &\quad -q_2e^{-q_2}[F(\rho;2,1)+(F(\rho;2,2)-F(\rho;2,1))q_2] - C\end{aligned}$$

In equilibrium, firms are indifferent between posting one and two vacancies; thus

$$(1-e^{-q_1}(1+q_1))\rho = F(\rho;2,2) - e^{-q_2}\{F(\rho;2,2)(1+q_2) - F(\rho;2,1)q_2 \tag{3}$$
$$+F(\rho;2,1)q_2 + (F(\rho;2,2)-F(\rho;2,1))q_2^2\} - C \tag{4}$$

and workers indifferent condition yields

$$q_1 = q_2 - \ln[\frac{F(\rho;2,1)}{\rho} + (\frac{F(\rho;2,2)-F(\rho;2,1)}{\rho})q_2] \tag{5}$$

The equilibrium is pined down by finding a (q_1, q_2) satisfying (3) and (5).

Combining (3) and (5) yields

$$\begin{aligned}& e^{-q_2}[\frac{F(\rho;2,2)-F(\rho;2,1)}{\rho} + (F(\rho;2,1)/\rho\\ & +q_2(\frac{F(\rho;2,2)-F(\rho;2,1)}{\rho}))\ln(\frac{F(\rho;2,1)}{\rho} + (\frac{F(\rho;2,2)-F(\rho;2,1)}{\rho})q_2)]\\ =\ & \frac{F(\rho;2,2)-\rho-C}{\rho}\end{aligned}$$

The right hand side of above equation is a positive number; the left hand side is strictly decreasing in q_2, equals to $\frac{F(\rho;2,2)-F(\rho;2,1)}{\rho}$ at $q_2 = 0$, and converges to 0 as $q_2 \to \infty$; thus there is a unique solution.

Then define σ as the probability a worker will visit a one vacancy firm. It must hold that

$$q_1 = \frac{N\sigma}{M\phi} = \frac{\sigma}{\lambda\phi}, q_2 = \frac{N(1-\sigma)}{M(1-\phi)} = \frac{(1-\sigma)}{\lambda(1-\phi)}$$

Hence, the equilibrium q_1^*, q_2^* will uniquely give a $\phi^* = \frac{q_1(\lambda q_2 - 1)}{q_2 - q_1}, \sigma^* = \frac{q_2 - 1/\lambda}{q_2 - q_1}$. In any interior solution, $q_2^* \geq q_1^*$ due to equation (5). When $q_1^* < \frac{1}{\lambda} < q_2^*$, $0 < \phi^*, \sigma^* < 1$, and therefore $0 < \gamma_1^*, \gamma_2^* < 1$. Following the similar argument of Lester (2011), when $\frac{1}{\lambda} \leq q_1^*$, one can prove $\phi^* = 1$, when $\frac{1}{\lambda} \geq q_2^*$, $\phi^* = 0$. *Q.E.D.*

3.2 Tables and Figures

Table 1: $(k, h) = (2, 2)$

		Employee I				
		Prob(s_I)	ρ^2	$(1-\rho)^2$	$\rho(1-\rho)$	$\rho(1-\rho)$
	Prob(s_{II})	$s_{II} \backslash s_I$	AB	$\bar{\text{A}}\bar{\text{B}}$	A$\bar{\text{B}}$	$\bar{\text{A}}$B
	ρ^2	AB	2	1	2	2
Employee II	$(1-\rho)^2$	$\bar{\text{A}}\bar{\text{B}}$	1	0	1	1
	$\rho(1-\rho)$	A$\bar{\text{B}}$	2	1	1	2
	$\rho(1-\rho)$	$\bar{\text{A}}$B	2	1	2	1

Table 2: $(k, h) = (2, 1)$

Prob(s)	ρ^2	$(1-\rho)^2$	$\rho(1-\rho)$	$\rho(1-\rho)$
Employee's s	AB	$\bar{\text{A}}\bar{\text{B}}$	A$\bar{\text{B}}$	$\bar{\text{A}}$B
Payoff	1	0	1	1

Table 3: $(k, h) = (1, 1)$

Prob(s)	ρ	$1-\rho$
Employee's s	A	$\bar{\text{A}}$
Payoff	1	0

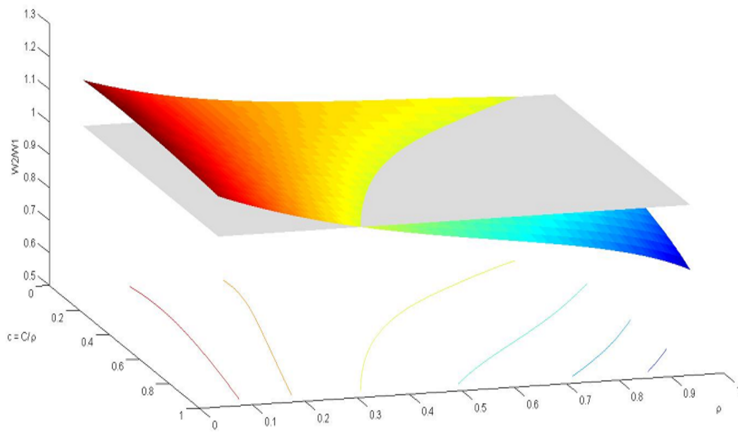

Figure 1. Wage ratio as a function of c and ρ.

References

[1] Brown C. and J. Medoff (1989): "The Employer Size-Wage Effect, " *Journal of Political Economy*, Vol. 97, pp. 1027-1059.

[2] Burdett K., S. Shi, and R. Wright (2001): "Pricing and Matching with Frictions," *Journal of Political Economy*, Vol. 109, pp. 1060-1085

[3] Hawkins W. (2011): "Competitive Search, Efficiency, and Multi-worker Firms," *mimeo* University of Rochester.

[4] Kaas L., and P. Kircher (2011): "Efficient Firm Dynamics in a Frictional Labor Market," *mimeo* LSE.

[5] Lester B. (2010): "Directed Search with Multi-vacancy Firms," *Journal of Economic Theory*, Vol. 149, pp. 2108-2132.

[6] Meyer M. (1994): "The Dynamics of Learning with Team Production: Implications for Task Assignment," *The Quarterly Journal of Economics*, Vol. 109, pp. 1157-1184.

[7] Montgomery J. (1991): "Equilibrium Wage Dispersion and Interindustry Wage Differentials," *The Quarterly Journal of Economics* , Vol. 106, pp. 163-179

[8] Oi W., and T. Idson (1999): "Firm Size and Wages," in O.C. Ashenfelter, D. Card, eds., *Handbook of Labor economics*, Vol.3, Amesterdam; New York: Elsevier.

[9] Ortega J. (2001): "Job Rotation as a Learning Mechanism," *Management Science*, Vol. 47, pp. 1361-1370.

[10] Papageorgiou T. (2011) "Large Firms and Internal Labor Market," *mimeo* Penn State University.

[11] Peters M. (1991): "Ex Ante Price Offers in Matching Games Non-Steady States," *Econometrica*, Vol. 59, pp. 1425-1454.

[12] Shi S. (2002): "Product Market and the Size-wage Differential," *International Economic Review*, Vol. 43, pp. 21-45.

[13] Tan S. (2011): "Directed Search and Firm Size," *International Economic Review*, forthcoming.