Why ethnic fractionalization? Polarization, ethnic conflict and growth

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Abstract

This paper is an attempt to clarify the relationship between fractionalization, polarization and conflict. The literature on the measurement of ethnic diversity has taken as given that the proper measure for heterogeneity can be calculated by using the fractionalization index. This index is widely used in industrial economics and, for empirical purposes, the ethnolinguistic fragmentation is ready available for regression exercises. Nevertheless the adequacy of a synthetic index of heterogeneity depends on the intrinsic characteristics of the heterogeneous dimension to be measured. In the case of ethnic diversity there is a very strong conflictive dimension. For this reason we argue that the measure of heterogeneity should be one of the class of polarization measures. In fact the intuition of the relationship between conflict and fractionalization do not hold for more than two groups. In contrast with the usual problem of polarization indices, which are of difficult empirical implementation without making some arbitrary choice of parameters, we show that the RQ index, proposed by Reynal-Querol (2002), is the only discrete polarization measure that satisfies the basic properties of polarization. Additionally we present a derivation of the RQ index from a simple rent seeking model. In the empirical section we show that while ethnic polarization has a positive effect on civil wars and, indirectly on growth, this effect is not present when we use ethnic fractionalization.

Keywords: ethnic heterogeneity, civil wars, economic development.
JEL classification: O11, Z12, O55

1 Introduction

The increasing incidence of ethnic conflicts and its much publicized consequences have attracted the interest of many researchers in the social sciences. Many studies have addressed directly the issue of ethnic diversity and its effects on social conflicts and civil wars (Reynal-Querol 2002a, Sambanis 2001 or Fearon and Laitin 2000). Political scientists have stressed the importance of institutions in the attenuation, or intensification, of social conflict in ethnically divided societies. Recently economist have connected ethnic diversity with important economic phenomena like investment (Mauro 1995), growth (Easterly and Levine 1997) or the quality of government (La Porta et al. 1999). The number of papers dealing with the effects of ethnic diversity on issues of economic interest is growing at a fast rate, mostly with an empirical content[1].

In this respect is common place in recent work to include as a regressor in empirical growth estimations an index of ethnic diversity. There are several mechanisms that explain the need to include such an indicator. First of all, and most important, an ethnically diverse society has a higher probability of ethnic conflicts, which may lead in the worst of cases to a civil war. If not the political instability caused by ethnic conflicts has a negative impact on investment and, indirectly, on growth. Secondly ethnic diversity may generate a high level of corruption which, in turn, could also deter investment. Finally it has been argued that in heterogeneous societies the diffusion of technological innovations is more difficult, specially when there is ethnic conflict among groups in a country. Business as usual is not possible in a society with a high level of ethnic conflict since this situation affects all levels of economic activities. Trade may be restricted to individuals of the same ethnic group; public infrastructures can have an ethnic bias; government expenditure may favor some ethnic groups to the detriment of others, etc. The common element to all these mechanisms is the existence of a conflict which, through social and political channels, spreads to the economy. Therefore the effect of ethnic diversity on growth depend on the incidence of the distribution of ethnic groups on the political process and the fight for economic resources.

In clear contrast with these mechanisms the empirical studies have used as an indicator of the probability of ethnic conflicts the indices of fractionalization. However there are no theoretical reasons that justify the use of this kind of measures. In fact as a summary measure, the index of fractionalization has important shortcomings generating Rankins of diversity that are widely counterintuitive as we will show. In this paper we argue that different indices of ethnic diversity can give rise to very different Rankins of ethnic "conflictiveness". We show that, opposite to the frac-

[1] For instance Bluedorn (2001), Vigdor (2002) or Caselli and Coleman (2002).

tionalization index, there are other kind of indicators that can be derived naturally from theoretical models of conflict or rent seeking. In particular we propose an index of polarization that has a clear theoretical support and performs very well empirically.

This paper is organized as follows. Section 2 describes the theoretical characteristics of the index of fractionalization and compares it with the index of polarization. Section 3 shows how the index of polarization can be derived from a model of ethnic conflict. Section 4 presents the empirical results obtained by the application of the index of fractionalization and the index of polarization to data on ethnic diversity. It is shown that for very high levels of fractionalization the readings of the index of polarization can be very low. In fact, for high levels of diversity the correlation between fractionalization and polarization is negative. Section 5 compares the empirical performance of the polarization index proposed in this paper vis a vis the fractionalization index in the explanation of economic growth and civil wars. Section 6 summarizes the conclusions.

2 Measuring ethnic diversity

Several authors have stressed the importance of ethnic diversity in the explanation of growth, investment, the efficiency of government or civil wars. Easterly and Levine (1997) argue that the very high ethnic fragmentation of countries in Africa explains an important part of their poor economic performance. The effect of ethnic diversity on growth goes through an indirect channel: the choice of poor public policies which, at the end, has an influence on long-run growth. In particular ethnic diversity transforms economic policy in a rent seeking mechanism. Additionally ethnic heterogeneity generate also problems in the design of structural policies related with infrastructures or education. Mauro (1995), using a similar reasoning, shows that ethnic fractionalization affects investment by increasing corruption and political instability. Since corruption has a negative effect on investment and the latter is shown to be a robust determinant of growth then ethnic diversity is also an important factor in the explanation of growth. La Porta et al. (1999) point out that ethnic diversity leads to corruption and low efficiency governments that expropriate the ethnic losers.

All these papers use the index of ethnolinguistic fragmentation (ELF) as the indicator of ethnic diversity. The raw data come from the Atlas Narodov Mira (1964) compiled in the former Soviet Union which refer to the situation in 1960. The criteria for group formation was based on the historical linguistic origin. The measure ELF was calculated by Taylor and Hudson (1974) which summarizes the data of the Atlas using the Herfindahl index. In particular the index takes the form

$$ELF_j{=}1\text{-}\sum_{i=1}^{I_j}\left(\frac{n_{ij}}{N_j}\right)^2{=}1\text{-}\sum_{i=1}^{I_j}s_{ij}^2 \qquad\qquad i=1,...,I_j \qquad (1)$$

where n_{ij} is the number of people that belong to ethnolinguistic group i in country j, N_j is the size of the population of the country j and I_j is the total number of ethnic groups in country j. The broad popularity of the ELF index is based on its simple interpretation as the probability that two randomly selected individuals from a given country will not belong to the same ethnolinguistic group. The fact that it could be used without the need to start it from scratch also helped its popularity.

Recently many authors have proposed alternative measures of ethnic diversity to overcome the shortcoming of ELF. There are at least four issues that have been discussed: the multidimensional nature of ethnicity, the source of raw data, the relevant classification of ethnic groups and the measure that summarizes the data.

Social scientists have recognized for long time the multidimensional nature of ethnicity. The ethnic identity of a group includes its language, race, color and/or religion. The use of ELF was prompted by the ready-made nature of the index. However many researchers have turned recently to the issue of the measure of other dimensions of ethnicity as well as the separate identification of ethnic and linguistic differences. Montalvo and Reynal-Querol (2002) and Alesina et al. (2002) discuss the construction and the sources for indices of linguistic, religious and ethnic diversity.

The issue of data sources is particularly relevant for the classification criteria. In general there could be two kinds of problems in the classification: either grouping problems- the aggregation in a single group of distinct ethnic groups which sometimes are even highly antagonistic- and the inclusion problem or enumeration of many ethnic groups that are irrelevant from the point of view of their political relevance[2]. Posner (2000) points out how the Atlas includes in the same group the Nyamwezi and the Sukuma despite the fact that these groups are easily distinguishable from each other and they are political competitors in Tanzania's ethnic arena. In other situations groups are aggregated into larger groups even though in the original source they appear as disjoint sets. However the proposal of including only the politically relevant groups is quite problematic mainly because what you want to capture is not only actual conflict but also potential conflict. In addition there is a clear endogeneity issue: the ethnic distribution of the society could affect the representation of the heterogeneous ethnic groups and, therefore, their political representation. Therefore a dictatorial political system may only represent one group when there are many others that, precisely because

[2] See Posner (2000) for a lengthy discussion of these two questions.

they are not represented, can increase the likelihood of a social conflict[3].

Finally there is also the issue of how to construct an index which is appropriate to measure the relevant aspects of ethnic diversity. This is the basic issue discussed in this paper. Imagine that there are two countries, A and B, with three ethnic groups each. In country A the distribution of the groups is (0,49, 0,49, 0,01) while in the second country, B, is (0,33, 0,33, 0,34). Which country will have a higher probability of social conflicts and, therefore, less growth?. Using the index of fractionalization the answer is B. However, Montalvo and Reynal-Querol (2000) and Reynal-Querol (2002a) have argued that the answer is A. They use the index of polarization RQ which takes the form

$$RQ_i = 1 - \sum_{i=1}^{N} \left(\frac{1/2 - \pi_i}{1/2} \right)^2 \pi_i$$

In this paper we argue that any index of ethnic heterogeneity should be theoretically oriented. It should accommodate the interpretations and mechanism that different authors have proposed in the explanation of the effect of ethnic diversity on, for instance, growth. We will show that the index of fractionalization is at odds with the basic explanations and, therefore, cannot capture the relevant dimensions of ethnic divisions. The main reason why the index has been widely use is it simple interpretation as the probability of being matched with an individual of a different ethnic group. However in the context of conflict and rent seeking models this measure is not a relevant indicator of the intensity of the conflict while the RQ indicator can be easily justified. The objective of the section is to show that the RQ index is the only measure among the family of discrete polarization measures, that satisfies the usual properties of polarization.

2.1 The index of fractionalization

The index of fractionalization has, at least, two theoretical justification in completely different contexts. In industrial organization the literature on the relationship between market structure and profitability has used the Herfindahl index to measure the level of market power in oligopolistic markets[4]. However the derivation of the index in this context starts with a non-cooperative game where oligopolistic firms play Cournot strategies. Therefore the index can summarize the market

[3] Montalvo and Reynal-Querol (2000) and Reynal-Querol (2002a) discuss the impact on the index of religious fragmentation of the aggregation of religious groups.

[4] This index has been also used in antitrust cases.

power in games that work through the market. Its ability to measure the structure of power in political or military processes as they appear represented by rent seeking or conflict models is very limited and none when compare with the proper measures (polarization). But, as we argued before, the conflictive nature of ethnic divisions is precisely the basic argument put forward in the explanation of the relationship between ethnic diversity, investment and growth.

The second theoretical foundation for the index of fractionalization comes from the theory of inequality measurement. We are going to pursue this line in detail since it is not a well-known relationship and it connects nicely with our discussion on the foundations of the polarization index.

One important family of inequality measures is the one derived from the concept of entropy. In this respect the measure has an information theory base. The general form of this kind of measures is

$$ET_{\beta} = \frac{1}{1+\beta}\left(\sum_{i=1}^{N}\frac{1}{N}h\left(\frac{1}{N}\right) - \sum_{i=1}^{N}s_i h(s_i)\right) \tag{2}$$

where N is the total population, s_i is the share of income of group i and β discriminates across different families of indicators. The function $h(p)$ measures the information convey by the argument. If an event is quite likely, p is near 1, the information is not very interesting while if an event is very unlikely its informational content is very high. Therefore $h' < 0$. Let's assume that h takes the form $h(s) = -\frac{1}{\beta}s^{\beta}$ which includes as a particular case, for $\beta = 0$, the $h(s) = -\log s$. For each value of β we have a particular inequality measure. For instance for $\beta = 0$ the function is

$$ET_0 = \sum_{i=1}^{N} s_i \left(\log s_i - \log(\frac{1}{N})\right)$$

which is precisely Theil's inequality index. For $\beta = 1$ the index can be written as

$$ET_1 = 0.5\left(\sum_{i=1}^{N} s_i^2 - \frac{1}{N}\right) = 0.5\left(H - \frac{1}{N}\right)$$

where H is simply Herfindahl's index. For a given population ET_1 and H are cardinally equivalent. The important issue with respect to this measures is the effect of changes in the size of the groups on the index. In general for the family exposed above we have that a small transfer of Δs from rich man 1 (R1) to poor man 1 (P1) implies

$$\Delta ET_\beta = (h(s_{R1}) - h(s_{P1}))\Delta s = -\frac{1}{\beta}\left(s_{R1}^{\beta} - s_{P1}^{\beta}\right)\Delta s$$

Depending on β the distance $h(s_R) - h(s_P)$ will take a particular form. For instance when $\beta = 0$ the distance between R1 and P1 is the same as the distance between R2 and P2 if the ratios s_{R1}/s_{P1} and s_{R2}/s_{P2} are the same. In the case of Herfindahl's index the distance is even simpler since it is an absolute distance measure: the distance between a group with a 1% share and a group with a 2% is the same as the distance between a group with the 9% and a group with a 10%. For this reason Herfindahl's index fulfills the strong principle of transfer. However, and opposite to all other measures of inequality, it is not independent of proportional increases in income and population but it decreases with the size of the population. In addition H moves between 0 and 1 but the minimum value is always greater than 0.

If we do not take into account the distances between groups and only consider the size of the groups we get the index of fractionalization as

$$FRAG_j = 1 - \sum_{i=1}^{I_j}\left(\frac{n_{ij}}{N_j}\right)^2 = 1 - \mathrm{H}_j \qquad i = 1, ..., I_j \tag{3}$$

2.2 Polarization measures

An alternative measure of ethnic diversity can be constructed using the concept of polarization. This conceptualization is particularly suited to capture the generation of social tensions, revolutions, civil wars or social unrest in general. Esteban and Ray (1994) (ER94) is concerned with the conceptualization and measurement of polarization. What do they mean by polarization? Suppose that a population of individuals may be grouped according to some vector of characteristics into "clusters", such that each cluster is very similar in terms of the attributes of its members, but different clusters have members with very "dissimilar" attributes. In that case the society is polarized even though the measurement of inequality could be low. In fact ER94 emphasize the difference between their measure of polarization and the usual measures of inequality.

Following ER94, "every society can be thought of as an amalgamation of groups, where two individuals drawn from the same group are "similar", and from different groups, are "different" relative to some given set of attributes or characteristics. The polarization of a distribution of individual attributes must exhibit the following features:

Feature1 : There must be a high degree of homogeneity within each group.

Feature 2: There must be a high degree of heterogeneity across groups.

Feature 3: There must be a small number of significantly sized groups. In particular, groups of insignificant size (e.g., isolated individuals) carry little weight[5].

Therefore the measure of polarization should be such that intra-group homogeneity and inter-group heterogeneity increases it. The identification felt by an individual with respect to its group increases with the number of people in the group. This fact implies an identification function $I(p)$ such that $I' > 0$[6]. On the other hand the alienation felt by an individual from others depend on how far away are the latter from the individual. This concept deals with the fact that inter-group heterogeneity increases polarization. An individual y feels an alienation $a(\delta(y, y'))$ with respect to an individual y', where $\delta(.,.)$ stands for the distance $|y - y'|$. The polarization measure mixes both concepts as follows

$$P(\pi, \mathbf{y}) = \sum_{i=1}^{N} \sum_{j=1}^{N} \pi_i \pi_j T(I(\pi_i), a(\delta(y_i, y_j)))$$

where $T(I, a)$ is taken to be an increasing function of a. Furthermore $T(I, 0) = 0$. ER94 narrow down the class of allowable measures by imposing three axioms. Only one measure, P^*, of the family of P satisfies this three axioms. It has the form

$$P^*(\pi, \mathbf{y}) = \mathbf{k} \sum_{i=1}^{N} \sum_{j=1}^{N} \pi_i^{1+\alpha} \pi_j |y_i - y_j|$$

for some constants $k > 0$ and $\alpha \in (0, \alpha^*)$ where $\alpha^* \simeq 1.6$. Notice that when $\alpha = 0$ this polarization measure is precisely the Gini coefficient. Therefore it is the fact that the share of each group is raised to the $1+\alpha$ power that exceeds one what makes the polarization measure significantly different from inequality measures. This is the reason why we are going to emphasize the relevance of α in the following discussions.

2.3 From income polarization to ethnic polarization

The ER94 index of polarization was though initially as a measure of income or wealth polarization. As such it has problems for its empirical implementation since its value depends critically on the

[5] Notice that these three features apply naturally to all ethnic dimensions.

[6] The identification function could also include other characteristics. In our case it may depend on the ethnic characteristic analyzed. For instance some ethnic characteristics generate a higher sense of identity than others.

number of groups. However in terms of income or wealth it is not clear which are the levels that distinguish different groups with a common identity. Where does the middle class start? How "rich" is rich? This strong difficulty together with the uncertainty over the right parameter for α has reduce the empirical applicability of the polarization index.

In the case of ethnic diversity the identity of the groups is less controversial, even though data availability may generate some problems[7]. However in this case the "distance between antagonists" is less clear than the application to income or wealth[8]. Instead of assuming an absolute distance for the antagonists function we take $\delta(.,.)$ as generated by a discrete metric. This implies that

$$\begin{aligned}\delta(y,y') &= 0 \qquad if\ y = y' \\ &= 1 \qquad if\ y \neq y'\end{aligned}$$

This discrete metric implies that the distance between two individual of the same ethnic group is 0 while the distance from an individual of any other group is equal to 1[9]. Therefore ethnic polarization could be measured by DP defined as the index of discrete polarization[10]:

$$DP(\alpha,k) = k\sum_{i=1}^{N}\sum_{j=1}^{N}\pi_i^{1+\alpha}\pi_j 1(i \neq j)$$

where $1(.)$ is an indicator function[11]. Notice that in this case, opposite to the general polarization index, the parameter $\alpha \in [0,\infty)$.The use of a discrete metric is reasonable since the identity of the groups is clear and it is difficult, and probably quite arbitrary, to try to measure the antagonism

[7] We already mentioned the so called "grouping problem" with respect to the Atlas data.

[8] Caselli and Coleman (2002) build a theory where "group distance" is also important. However in their empirical application they include the usual fractionalization index, based only on relative group size. We agree with these authors that more research should be done in order to capture "distances" across ethnic groups.

[9] Notice that potentially we could determine, without defining explicitly the exact "distance" between two groups, if different groups are sufficiently "different" with respect to their preferences and discretize the distances.

[10] This is a loose definition since, as we will show, this index only captures the basic characteristics of a polarization measure if $\alpha = 1$.

[11] By analogy we will call discrete Gini index the discrete polarization family with $\alpha = \mathbf{0}$. We are going to distinguish this discrete Gini index ($DP(\mathbf{0},k)$) from the general discrete polarization ($DP(\alpha > \mathbf{0},k)$) even though the earlier is a particular case of the DP family. As we argued before the fact that $\alpha > \mathbf{0}$ is the basic attraction of polarization measures versus inequality indices.

across ethnic groups by numbers in R^{+}[12]. The same is true for the index of fractionalization.

There is an additional problem, which is going to be finally an advantage, when embedding a discrete metric into the polarization measure P^*. It is known that the discrete metric and the Euclidean metric are not equivalent in R. For this reason the apparently minor change of the metric implies that the discrete polarization measure does not fulfill anymore the axioms that give support to its original construction. Fortunately there is a value of α that recover the usual properties of the polarization measure, as we will discuss in the next section. At this point we should already notice that this indicator, used by Montalvo and Reynal-Querol (2000, 2002) and Reynal-Querol (2002a) to measure religious heterogeneity[13] (RQ) may help to solve the situation found in different empirical studies[14] where the probability of civil wars, or poor economic performance, does not have a monotonic relationship with ELF. When we put together polarization and fractionalization we find out that to high levels of fractionalization it corresponds low levels of polarization.

2.3.1 The Case of two groups: N=2

In this section we show that for the case of two groups, indices of fractionalization, discrete polarization and the discrete Gini index are the same. Notice, that for the case of two groups the only difference between the case of discrete case and the general case, controlling for distances, will only be a scale problem. Therefore the result that FRAG, GINI and DP are the same, will also apply to the measurement of income fragmentation, Gini inequality and polarization when we take into account the distances. This has serious implications in the empirical analysis, given that many times the literature refer to the case of bipolarization, that is, they previously divide the society among two groups and then they compute the index of polarization. The results show, that the index of bipolarization is the same as fractionalization and the Gini index[15].

Proposition 1: For N=2 and $\alpha = 1$ the polarization measures, $DP(1,k)$, provides the same

[12] As far as we know there are no data available on the measure of the "distance" between ethnic groups even though Posner (2000) considers that the depth of the ethnic division should be an information included in the indices of ethnic diversity. In particular Posner (2000) argues that "by not capturing the depth of the ethnic cleavages, indices of ethnic fractionalization leave out a potentially important part of the explanation". Humphreys (2001) makes a similar argument.

[13] From now on we will call it the RQ index.

[14] Collier (2001) or Humphreys (2001).

[15] However, as we will show in next section, only a particular case of the DP family of polarization indices will have this property when N>2. Nevertheless some authors argue that the fragmentation index has this property for N=2 and implicitly assume it has the same property for N>2.

ranking order as the fragmentation measures, FRAG. That is $DP(1,k) = \frac{k}{2}FRAG$. Additionally if $\alpha = 1$ and $k = 2$, both measures are the same. That is $DP(1,2) = FRAG$.

Proof:

$DP(\alpha,k) = k\sum_{i=1}^{2}\pi_i^{1+\alpha}(1-\pi_i) = k[\pi_1^{1+\alpha}\pi_2 + \pi_2^{1+\alpha}\pi_1] = k[\pi_1^{1+\alpha}(1-\pi_1) + (1-\pi_1)^{1+\alpha}\pi_1] =$

$DP(1,k) = k[\pi_1^2(1-\pi_1) + (1-\pi_1)^2\pi_1] = k[\pi_1(1-\pi_1)[\pi_1 + (1-\pi_1)]] = k[\pi_1 - \pi_1^2]$

$FRAG = 1 - \sum_{i=1}^{2}\pi_i^2 = 1 - \pi_1^2 - (1-\pi_1)^2 = 2[\pi_1 - \pi_1^2]$

Therefore, $DP(1,k) = \frac{k}{2}FRAG$, and $DP(1,2) = FRAG$ for N=2.■

Proposition 2: For N=2 the discrete Gini index , $DP(0,k)$, is k times the fragmentation index. That is, $DP(0,k) = k * FRAG$. Moreover the discrete Gini index normalized by $k = 1$ is exactly the fragmentation index, $DP(0,1) = FRAG$.

Proof:

Discrete Gini index can be written as $DP(0,k) = k\sum_{i=1}^{2}\pi_i(1-\pi_i) = k(\pi_1(1-\pi_1) + \pi_2(1-\pi_2)) =$

$k(\pi_1 - \pi_1^2 + \pi_2 - \pi_2^2) = k(1 - \sum_{i=1}^{2}\pi_i^2) = k * FRAG$

Therefore the discrete Gini index, $DP(0,k) = k * FRAG$, and Gini $DP(0,1) = FRAG$.■

Proposition 3: For N=2, the discrete Gini index is two times the polarization index, when $\alpha = 1$, except when the k used for calculating the discrete Gini, k, is $k = (1/2)k'$, where k' is the k used to calculate polarization. That is, $DP(0,k) = DP(1,k')$, the discrete polarization index, where $k = (1/2)k'$.

Proof:

We know that $DP(1,k') = k'[\pi_1 - \pi_1^2]$

and the discrete Gini index is $DP(0,k) = k\sum_{i=1}^{2}\pi_i(1-\pi_i) = k(\pi_1(1-\pi_1) + \pi_1(1-\pi_{1})) =$ $k(\pi_1(1-\pi_1)) * 2 =$

$k[\pi_1 - \pi_1^2] * 2 = 2 * DP(1,k) = DP(1,k')$

Therefore $DP(0,k) = 2DP(1,k) = DP(1,k')$,the discrete polarization measure, where $k = (1/2)k'$■

Corollary:: $DP(0,1) = DP(1,2)$ and $DP(0,2) = DP(1,4)$

Proof:

This corollary is a direct application of proposition 3.

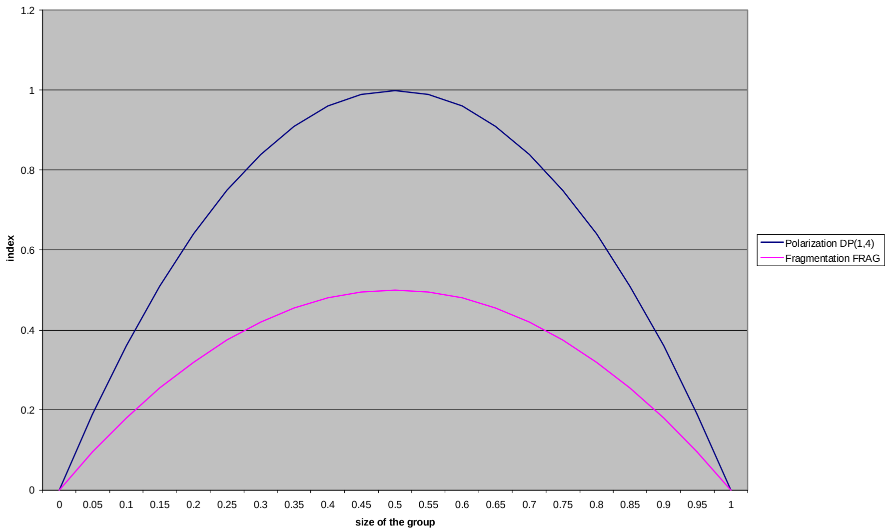

Figure 1: polarization versus fragmentation for N=2

Proposition 4: For N=2 the discrete Gini index, the fragmentation index and the discrete polarization measure, with a particular normalization, are the same. Without the normalization they provide the same ranking order.

Proof:

It follows from Proposition 1 and 2 that $DP(0,1) = DP(1,2) = FRAG$. ■

Figure 1 shows the non-normalized discrete Gini index (the fractionalization index) and the index DP(1,4) for the case of two groups. As we can see the only difference among them is their normalization. All of them present a maximum at the configuration (π_1, π_2) =(0.5,0.5).

2.3.2 The Case of N≥2

Proposition 5: For any number of groups the index of discrete polarization DP(1, 4) is equal to the RQ index.

Proof:

$$RQ = 1 - \sum_{i=1}^{n} \left(\frac{0.5-\pi_i}{0.5}\right)^2 \pi_i = \sum_{i=1}^{n} \pi_i - \sum_{i=1}^{n} \left(\frac{0.5-\pi_i}{0.5}\right)^2 \pi_i = \sum_{i=1}^{n} [\pi_i - \left(\frac{0.5-\pi_i}{0.5}\right)^2 \pi_i] =$$
$$\sum_{i=1}^{n} [\pi_i - \left(\frac{0.5}{0.5} - \frac{\pi_i}{0.5}\right)^2 \pi_i] = \sum_{i=1}^{n} [\pi_i - (1-2\pi_i)^2 \pi_i] = \sum_{i=1}^{n} \pi_i [1 - (1-2\pi_i)^2] =$$
$$\sum_{i=1}^{n} \pi_i [1 - \left(1 + 4\pi_i^2 - 4\pi_i\right)] = \sum_{i=1}^{n} \pi_i [1 - 1 - 4\pi_i^2 + 4\pi_i] = \sum_{i=1}^{n} \pi_i 4[-\pi_i^2 + \pi_i] =$$

$\sum_{i=1}^{n} \pi_i^2 4[1-\pi_i] = 4\sum_{i=1}^{n}\sum_{j\neq i} \pi_i^2 \pi_j d = DP(1,4)$ where $d=1$ if $j \neq i$, and $d=0$ if $i=j$

Notice that when $\alpha = 1$, the only k that normalize DP between 0 and 1 is $k=4$. Therefore the justification for $k=4$ is that in that case $DP(1,4) \in [0,1]$

■.

Proposition 6: For any number of groups, the discrete Gini index, DP(0,k)=k*FRAG and DP(0,1)=FRAG

Proof:

$DP(0,k) = k\sum_{i=1}^{n} \pi_i(1-\pi_i) = k(\pi_1(1-\pi_1) + \pi_2(1-\pi_2) + ... + \pi_n(1-\pi_n)) =$

$k(\pi_1 - \pi_1^2 + \pi_2 - \pi_2^2 + ... + \pi_n - \pi_n^2) = k(\sum_{i=1}^{n} \pi_i - \sum_{i=1}^{n} \pi_i^2) = k(1 - \sum_{i=1}^{n} \pi_i^2) = k * FRAG$

Therefore $DP(0,k) = k * FRAG$ and $DP(0,1) = FRAG$■

3 Properties of the discrete polarization index

In the last section we have pointed out the relationship among different measures of diversity that have been used in the literature. However not all those indices have the properties of a polarization measure as described in ER94. In fact there is only one discrete polarization measure that satisfy all the properties. This section shows that such a measure is the RQ index.

Before showing the application of the properties of polarization to the RQ index we are going to discuss some properties of the discrete polarization family of measures. Observe that the DP index can be written as

$DP(\alpha,k) = k\sum_{i=1}^{n} \pi_i^{1+\alpha}(1-\pi_i) = \sum_{i=1}^{n} \pi_i(1 - 1 + k\pi_i^{\alpha} - k\pi_i^{1+\alpha}) =$

$\sum_{i=1}^{n} \frac{1}{N} - \sum_{i=1}^{n} \pi_i k(\frac{1}{k} - \pi_i^{\alpha} + \pi_i^{1+\alpha}) = \sum_{i=1}^{n}[\frac{1}{N} - \pi_i k(\frac{1}{k} - \pi_i^{\alpha} + \pi_i^{1+\alpha})] = \sum_{i=1}^{n} f(\pi_i).$

The behavior of the index critically depends on the properties of the f function. By differentiation one can compute f' and f''.

$f' = [-(2+\alpha)\pi_i^{\alpha+1} + (1+\alpha)\pi_i^{\alpha} - \frac{1}{k}]k$

$f'' = k[-(2+\alpha)(\alpha+1)\pi_i^{\alpha} + \alpha(1+\alpha)\pi_i^{\alpha-1}]$

The f function is convex for $\pi < \frac{\alpha}{2+\alpha}$ and concave for $\pi > \frac{\alpha}{2+\alpha}$. Notice that if $\alpha = 1$, then it is convex for $\pi < 1/3$ and concave for $\pi > 1/3$ (see figure 2).

Understanding the shape of the function when $\alpha = 1$ it is crucial to understand the properties of the polarization index RQ and DP(1,k). The intuition is simple. If we transfer population from one group to another the effect on the polarization (conflict) level is different depending on the size

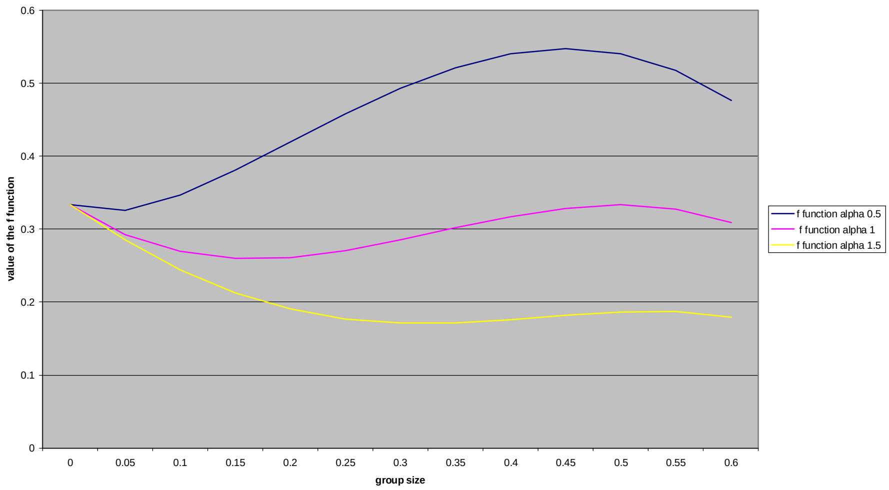

Figure 2: the f function

of the groups. Imagine a population composed by three groups distributed in the following way (0.5, 0.25, 0.25). If we transfer population from one small group to the other, polarization increases. We are in the convex part of the function. Therefore, $f(0.5) + 2f(0.25) < f(0.5) + f(0.3) + f(0.2)$ since by convexity $f(0.25) < \frac{f(0.3)+f(0.2)}{2}$. However if the distribution is (0.45, 0.45, 0.1), and we transfer population from one big group to the other, polarization decreases. This is because we are in the concave region. Therefore, $f(0.1) + 2f(0.45) > f(0.1) + f(0.4) + f(0.5)$ since by concavity $f(0.45) > \frac{f(0.4)+f(0.5)}{2}$. What is the intuition behind this result? In the first case, even that transfer implies that the distribution is more unequal in the new situation: one of the small groups is larger, respect to the big group, which means that we are closer to polarization. In the second case, the transfer implies that one of the large opponents became smaller, and therefore the new situation is less polarized. Notice that the results implies that this index does not satisfies the properties of the Lorenz curve about concavity. In a Lorenz curve this effect of moving people between small or big groups is the same. It is important to notice another difference with the Lorenz curve dominance, which is that our measure is global and the Lorenz curve is not. While in the Lorenz criteria establishes the impact on inequality of a local transfer independently of the shape of the rest of the distribution, in our case the effect on polarization of the transfer population from a group to another can not be established without knowing the entire distribution.

Now imagine that $\alpha = 0.5 < 1$. Then it is convex for $\pi < 1/5$ and concave for $\pi > 1/5$ (see figure 2). As above, imagine a population composed by three groups distributed in the following way (0.5, 0.25, 0.25). If we transfer population from one small group to the other, now the index decreases instead of increasing. We are in the concave part of the function. Now, $f(0.5)+2f(0.25) > f(0.5) + f(0.3) + f(0.2)$ since by concavity $f(0.25) > \frac{f(0.3)+f(0.2)}{2}$. If the distribution is (0.4, 0.4, 0.2), and we transfer population from one big group to the other, the index also decrease because we are also in the concave part of the function. Now it is not intuitive. In the first case, even that transfer implies that the distribution is more unequal in the new situation: one of the small groups is larger, respect to the big group, which means that we are closer to polarization, but the index does not indicate this, the index decreases. Which means that the index would not capture polarization in the entire distribution.

Now imagine that $\alpha = 1.5 > 1$. Then it is convex for $\pi < 3/7$ and concave for $\pi > 3/7$ (see figure 2). As above, imagine a population composed by three groups distributed in the following way (0.5, 0.25, 0.25). If we transfer population from one small group to the other, the index increases. We are in the convex part of the function. However if the distribution is (0.4, 0.4, 0.2), and we transfer population from one big group to the other, the index increase instead of decrease. This is because we are in the convex part of the function. Now, $f(0.2) + 2f(0.4) < f(0.2) + f(0.42) + f(0.38)$ since by convexity $f(0.4) < \frac{f(0.42)+f(0.38)}{2}$. Now it is also not intuitive. In the second case, the transfer implies that one of the big opponents became smaller, and therefore the new situation is less polarized. However, the index would indicate the opposite, because the index increases in the new situation instead of decreases, which means that the index would not capture polarization in the entire distribution.

3.1 The RQ index as a polarization measure

We have already shown that the discrete polarization measure can be written as:

$$DP(\alpha, k) = k \sum_{i=1}^{n} \sum_{j \neq i} \pi_i^{1+\alpha} \pi_j \qquad \text{where } \alpha \in [0, \infty)$$

and the RQ index is a particular case of the general expression

$$RQ = 1 - \sum_{i=1}^{n} \left(\frac{0.5 - \pi_i}{0.5}\right)^2 \pi_i = 4 \sum_{i=1}^{n} \sum_{j \neq i} \pi_i^2 \pi_j = DP(1, 4)$$

Therefore, for each possible α, we have a different DP measure. What is the admissible set of values of coefficient α if the DP measure has to satisfy the basic properties of polarization?

The polarization measure proposed by Esteban and Ray (1994) is obtained by imposing some "reasonable" axioms to the general class of polarization measures that we describe in section 2. The basic idea of the axioms is to conceptualize an index closely related to the concept of social tensions. We concentrate in the case where the location of the groups is measured by a discrete distance. Therefore, since distances are equal among all groups, the polarization measures only depend of the sizes of the groups. Does the DP measure constructed in this way satisfy the basic properties of a polarization measure?[16]

Property 1:

If there are three groups of sizes, p, q, and r, and $p > q$ and $q \geq r$, then if we merge the two smallest groups into a new group, $\widetilde{q}$, the new distribution is more polarized than the original one. That is, $POL(p, q, r) < POL(p, \widetilde{q})$ where $\widetilde{q} = q + r$[17].

Theorem 1: $DP(\alpha, k)$ satisfies property 1 if and only if $\alpha \geq 1$. (Proof in the appendix)

Property 1b: Suppose that there are two groups with size π_1 and π_2. Take any one group and split it into $m \geq 2$ groups in such a way that $\pi_1 = \widetilde{\pi}_1 \geq \widetilde{\pi}_i \; \forall_{i=2,..n+1}.$, where $\widetilde{\pi}$ is the new vector of population sizes, and clearly $\sum_{i=2}^{n+1} \widetilde{\pi}_i = \pi_2$. Then polarization under $\widetilde{\pi}$ is smaller than under π.

Theorem 2: The $DP(\alpha, k)$ measure satisfy property 1b if and only if $\alpha \geq 1$.

(Proof in the appendix).

Another property of polarization measures is that they attain their maximum at a bipolar symmetric distribution. In the case of the family of discrete polarization measures we can show the following lemma.

Lemma 1: The $DP(\alpha, k)$ index attains its maximum at a bipolar symmetric distribution if and only if $\alpha \geq 1$. (Proof in the appendix)

Property 2*: Assume that there are three groups of sizes p,q,p. Then if we shift mass from the q group equally to the other two groups, polarization increases. That is, $POL(p, q, p) < POL(p + x, q - 2x, p + x)$[18].*

[16] Notice that, in our case, we do not use the term "axiom" since we are not interested in describing and narrowing down a general class of discrete polarization measures. We only want to check if the DP measure proposed in this paper satisfies those properties.

[17] This property corresponds to axioms 1, 2 and 4 in ER94. Notice that the difference between axioms 1 and 2 is the original distance between groups which is the same for DP measures. Axiom 4 allows the size of one of the groups to be very small.

[18] This property corresponds to axiom 3 in ER94.

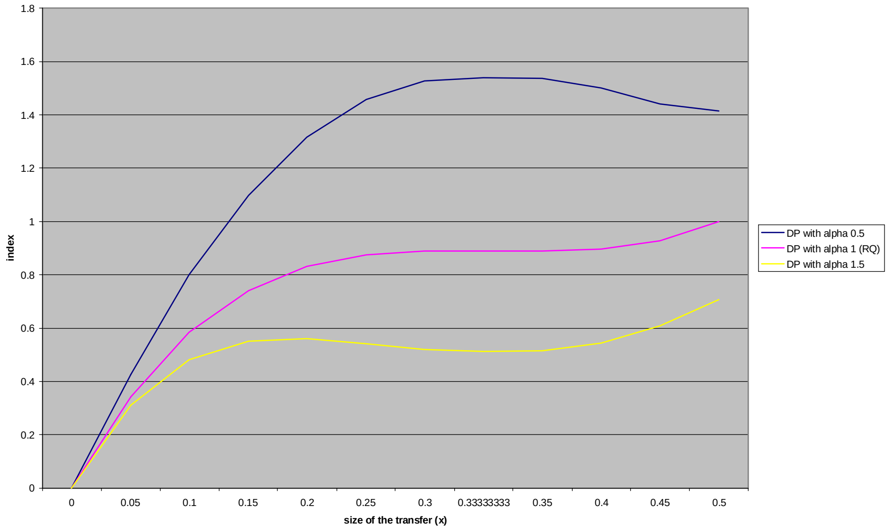

Figure 3: DP indices as a function of the transfer

Theorem 3: The only $DP(\alpha, k)$ measure that satisfy property 2 for any distribution is the one such that $\alpha = 1$. Notice that this is precisely the RQ polarization measure. (Proof in the appendix)

Figure 3 gives some intuition of the source of the result in theorem 3.

Corollary: The only family of DP measures that satisfies properties 1 and 2 is the one with $\alpha = 1, DP(1, k)$. Notice that when $\alpha = 1$, the only k that normalize DP between 0 and 1 is $k = 4$ and this is the RQ index.

Figure 4, to 7 show the shape of DP in the unit simplex for four values of the parameter, $\alpha = 0, \alpha = 1, \alpha = 0.5$ and $\alpha = 1.5$ respectively. The X and Y axises measure the size of two of the three groups. The Z axis represents the value of the index. In the case of $\alpha = 0$ (Figure 4) we can see that the index (which is the fragmentation index) increases monotonically up to the point (1/3,1/3,1/3) which implies equal size groups. Figure 5 shows that in the case of the RQ index, or DP(1,4), the maximum is reach when there are two groups of equal size and the index increases monotonically up to that point. Figure 6 presents the discrete polarization index for $\alpha = 0.5$. In this case (1/3,1/3,1/3) represents a global maximum while two groups of equal size represent a local maximum. This index, therefore, do not delivers a monotonic measure of heterogeneity. Finally, for $\alpha = 1.5$ the discrete polarization measure has the global maximum at two groups of equal size. However, it has a local minimum at (1/3,1/3,1/3) and, therefore, lacks also the monotonicity represented by property 2. From these figures we can see that only when $\alpha = 1$ the DP index is monotonic and shows a maximum at the bipolar situation (two groups with size). Therefore when distances among groups are discrete the only discrete polarization index that satisfies the properties of polarization (one and two) is the one such that $\alpha = 1$. Notice that this is precisely what we have called the RQ polarization index.

4 A theoretical justification: conflict and polarization

In the last section we have justified the use of the RQ index as a measure that satisfies the basic properties of polarization. However that is only a part of the theoretical foundations that we were seeking to justify the RQ index. In this section we show that the RQ index can be derived from a simple model of rent seeking[19].

Let's assume that the society is composed by N individuals distributed in M groups. Let π_i be the proportion of individuals in group i, $\pi_i = n_i/N$ .Society choose an outcome over the M possible issues. We identify issue i as the outcome most preferred by group i. We think of each outcome as a pure public good for the group members. Define u_{ij} as the utility derived by a member of group i if issue j is chosen by society. As we want to describe a pure contest case then $u_{ii} > u_{ij} = 0$ for all i, j with $i \neq j$.

Because of the rent seeking nature of the model we assume that agents can try to alter the outcome by spending resources in favor of their preferred outcome. Therefore there will be M possible outcomes depending on the resources spend by each of the M groups. Let's define x_i as the effort or the resources expended by an individual of group i. The total resources devoted to lobbying are $R = \sum_{i=1}^{M} \pi_i x_i$. Following this interpretation R can be thought as a measure of the intensity of social conflict. The cost of resources, or effort, x for each individual is $c(x)$. We are going to assume that the cost function, or effort disutility, is quadratic, $c(x) = (1/2)x^2$.

The basic element of any rent-seeking model is the contest success function, which defines the probability of success. We are going to use the traditional ratio form for the contest success function and define p_j as the probability that issue j is chosen, which depends on the resources spent by each group in favor of each outcome $j = 1,...M$, provided that $R > 0$.

[19] Esteban and Ray (1999) notice also the relationship between polarization and conflict although they did not proposed any particular indicator. The model presented in this section is a very simple rent seeking model which can also be interpreted as a particular case of Esteban and Ray (1999).

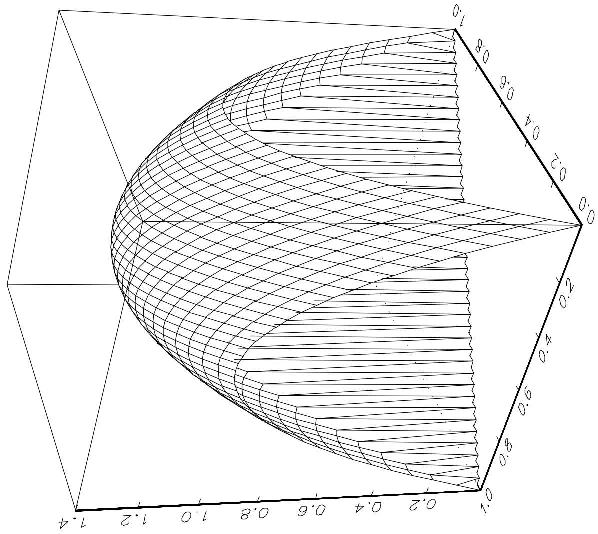

Figure 4: Index of fractionalization or discrete polarization with $\alpha = 0$.

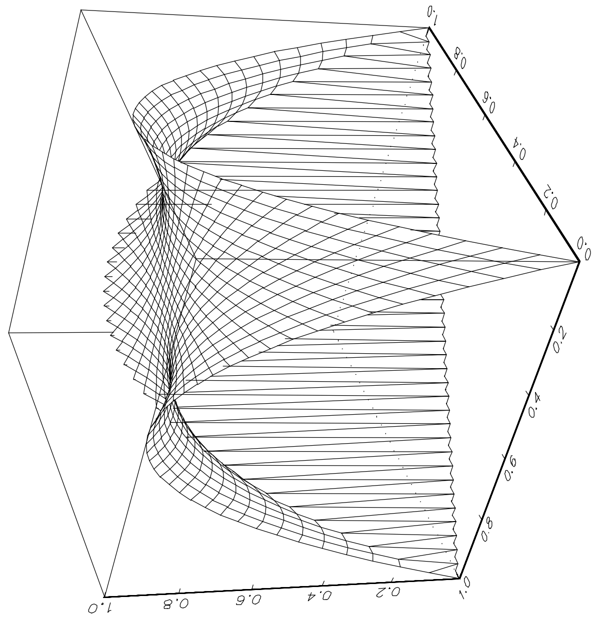

Figure 5: RQ index or discrete polarization with $\alpha = 1$

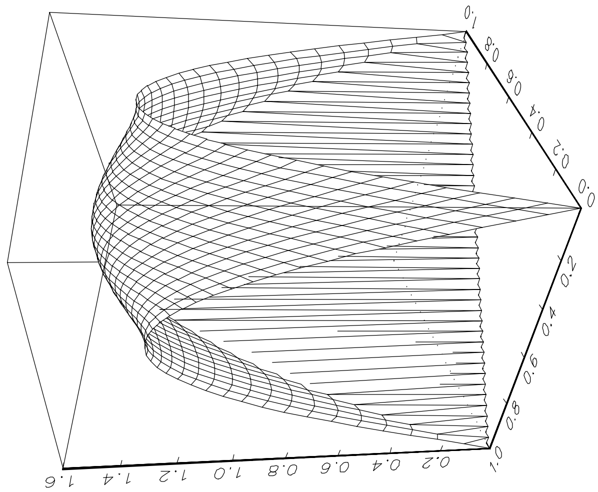

Figure 6: Discrete polarization for $\alpha = 0.5$

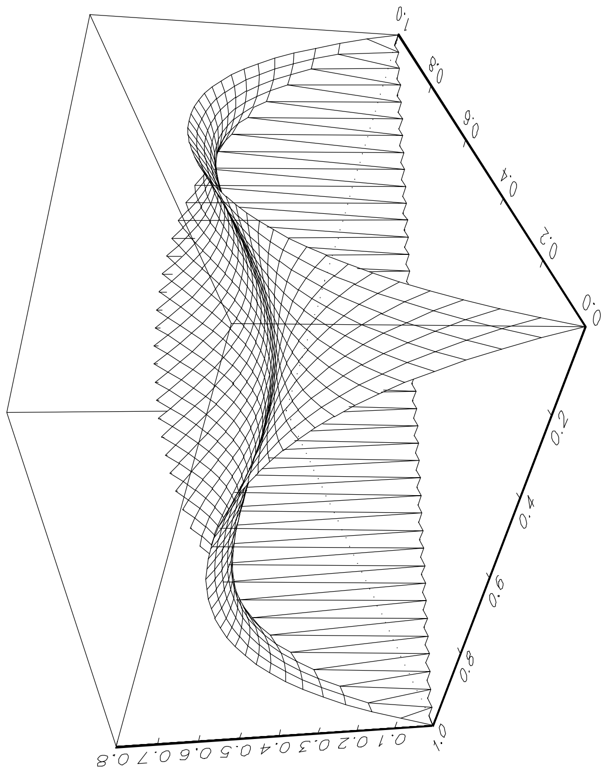

Figure 7: Discrete polarization for $\alpha = 1.5$.

$p_j = \frac{\pi_j x_j}{\sum_{j=1}^{M} \pi_j x_j} = \frac{\pi_j x_j}{R}$

In particular this function satisfies the property that an equiproportionate change in effort of all players would leave the winning probability of every player unchanged[20]. Then each member of group i has to decide the amount of resources she wants to expend in order to maximize the expected utility function taken into account that she doesn't care about non-preferred outcomes and the contest success function is of the ratio form.

$Eu_i = \sum_{j=1}^{M} p_j u_{ij} - c(x_i) = \sum_{j=1}^{M} p_j u_{ij} - (1/2)x_i^2 = p_i u_{ii} - (1/2)x_i^2$

subject to $p_j = \pi_j x_j / R$. As we assume a pure contest case and, $u_{ij} = 0$ for all $j \neq i$, and at least one group expend positive resources, $x_j > 0$, for some $j \neq i$, the first order conditions that solve the problem are

$\pi_i^2(u_{ii} - u_{ii}p_i) = \pi_i x_i R$

Adding all the first-order conditions we obtain the following expression:

$\sum_{i=1}^{M} \pi_i^2(u_{ii} - u_{ii}p_i) = R^2$

In the pure contest case the individuals only have a positive utility from their most preferred issue. Say that the utility $u_{ii} = k$

Therefore

$R^2 = \sum_{i=1}^{G} \pi_i^2(k - kp_i)$

Proposition 7: *If there are only two groups the normalize (squared) total cost can be written as* $R^2 = 1 - \sum_{i=1}^{G} \left(\frac{0.5-\pi_i}{0.5}\right)^2 \pi_i$, *which is the RQ index of polarization.*

Proof:

It is easy to show that if G=2 then the resources spend by each individual of any group are the same, $x_1 = x_2$, therefore $p_j = \pi_j$.

Therefore

$$R^2 = \sum_{i=1}^{2} \pi_i^2(k - k\pi_i) = \sum_{i=1}^{2} \pi_i(k\pi_i - k\pi_i^2) = \sum_{i=1}^{2} \pi_i(1 - 1 + k\pi_i - k\pi_i^2) = \sum_{i=1}^{2} \pi_i(1 - (1 - k\pi_i + k\pi_i^2)) = \sum_{i=1}^{2} \pi_i(1 - k(\tfrac{1}{k} - \pi_i + \pi_i^2)) = \sum_{i=1}^{2} \pi_i - \sum_{i=1}^{2} k(\tfrac{1}{k} - \pi_i + \pi_i^2)\pi_i = 1 - \sum_{i=1}^{2} k(\tfrac{1}{k} - \pi_i + \pi_i^2)\pi_i$$

As R is a measure of the total resources spent, or effort, for lobbying purposes, then it can be interpreted as an index of (potential) conflict. Notice that for $k = 4$ this index is normalized between 0 and 1, and can be rewritten as

[20] In general the ratio form of the contest success function takes the form $p_1/p_2 = (x_1/x_2)^z$ where z defines if there are diminishing returns ($z \leq 1$) to competitive efforts (x) or there are increasing returns ($z > 1$). In our case we set $z = 1$.

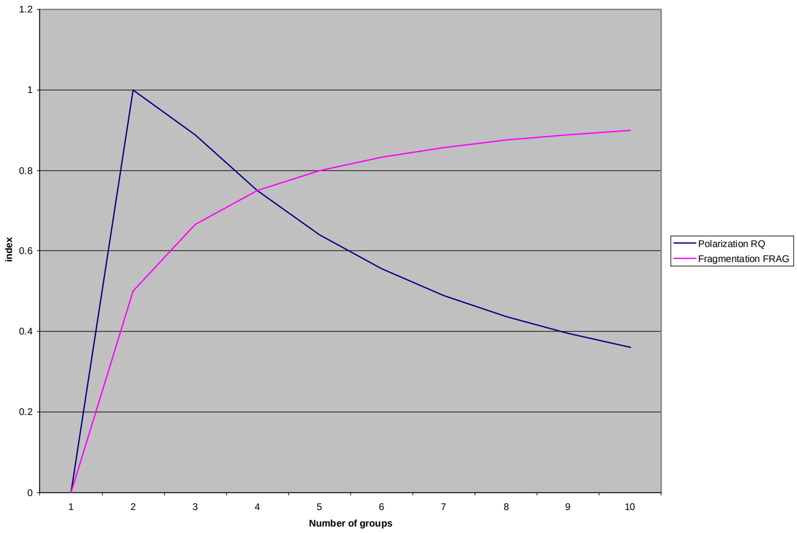

Figure 8: Polarization versus fragmentation for N groups of equal size.

$R^2 = 1 - \sum_{i=1}^{2} 4(\frac{1}{4} - \pi_i + \pi_i^2)\pi_i = 1 - \sum_{i=1}^{2} (\frac{0.5-\pi_i}{0.5})^2 \pi_i$
which is precisely the RQ index.■

Proposition 8: If there are *G groups of equal size*[21], $n_1 = ... = n_G$*, the normalize (squared) total cost can be written as* $R^2 = 1 - \sum_{i=1}^{G} \left(\frac{0.5-\pi_i}{0.5}\right)^2 \pi_i$

Proof: Immediate from proposition 1.

Figure 8 shows the graph of the fractionalization index and the polarization index in function of the number of groups when all of them have the same size. As we discussed in the previous section while the polarization index has a maximum at two groups the fractionalization index grows with the number of groups.

5 The empirical performance: fractionalization versus polarization

In this section we compare the empirical performance of measures of fractionalization and indicators of polarization in the explanation of economic growth. Political instability has been a ubiquous

[21] Notice that in the case of two groups this condition was not needed.

explanatory variable in empirical specifications of economic growth since the very beginning of the new empirical literature on growth. Barro (1991), and many authors after him, have included coups, revolutions and assassinations as explanatory variables and show that they have a negative and significant effect on growth. More recently some papers (Mauro 1995, Easterly and Levine 1997 or Barro 1997) have shown the negative effect of ethnolinguistic fractionalization on investment and growth. However we argued in the theoretical part of this article that the index of polarization is the proper indicator to capture the extent of social conflicts. Therefore ethnic polarization should have an indirect and negative effect on investment and growth. But then, is it polarization or fractionalization what matters in the explanation of growth in heterogeneous societies?

Before we answer this question we should address another matter. Are polarization and fractionalization indices very different? In principle, as we showed in section 2, polarization and fractionalization should have a high correlation when the number of groups is two but they maybe very different if the number of groups is greater than two. In this section we consider two dimensions of social heterogeneity that have been the focus of recent economic research: ethnic and religious diversity. The data come from Montalvo and Reynal-Querol (2000, 2002)[22]. Figure 9 presents the relationship between ethnic polarization and fractionalization. . Figure 9 shows that for low levels of fractionalization the relationship between ethnic fragmentation[23] and ethnic polarization is positive and close to linear. However for the medium range the relationship is zero and for high levels of fractionalization the relationship with polarization is negative[24]. Figure 10 presents the scatterplot of religious fragmentation versus religious polarization. It shows a similar pattern: for low levels of religious fractionalization the relationship with polarization is positive and close to linear. However for intermediate and high levels of religious fractionalization the relationship is zero. Therefore the correlation is low when there is high religious heterogeneity, which is the interesting case. This fact is important when studying issues of development given that most of the African countries present a high level of polarization. We should notice that this lack of correlation for intermediate and high levels of fractionalization is particularly important since this is the situation that we expect is more negative for conflict, investment or growth.

[22] The data on ethnic diversity uses the ethnolinguistic criterion of the WCE (World Christian Encyclopedia). The data on religious diversity comes from "L'Etat des Religions dans le monde".

[23] Ethnic fragmentation has a very high correlation with the traditional ethnolinguistic fragmentation indicator ELF.

[24] The correlation between ethnic fragmentation and ethnic polarization is 0.62. We know that the figure looks very similar for any source of data on ethnic diversity (figures upon request). As we argued before the difference between fragmentation and polarization is basically theoretical.

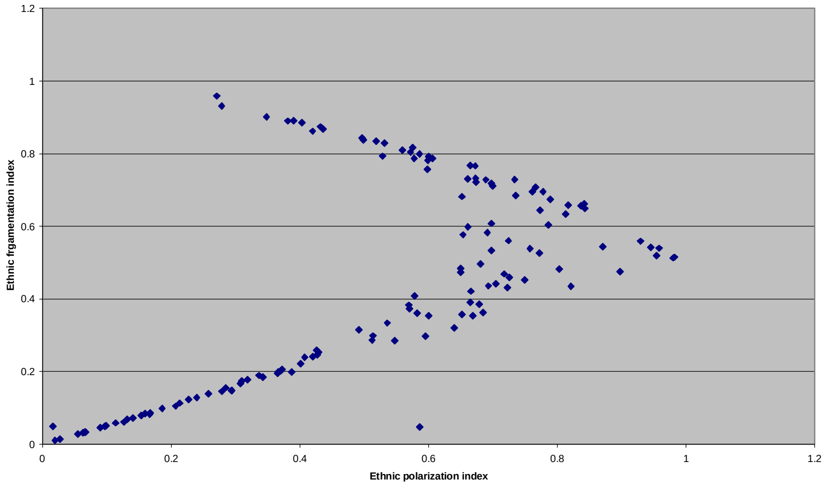

Figure 9: Relationship between ethnic fragmentation and polarization

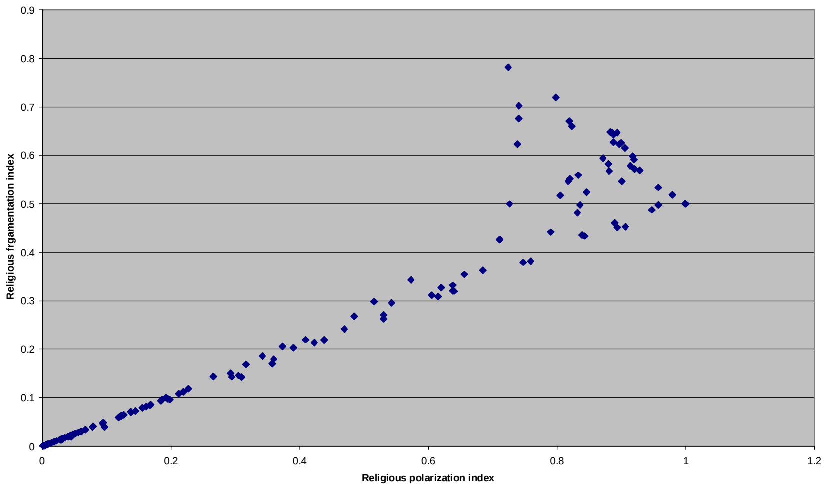

Figure 10: Relationship between religious fragmentation and polarization

Table 1 shows the ranking of the most ethnically polarized countries versus the most fragmented. We emphasize in italics the countries were there have been a civil war during the sample period. The source for civil wars is Doyle and Sambanis (2000). As far as we know this is the largest and most rigorous dataset on civil war available. Doyle and Sambanis (2000) define civil war as an armed conflict with the following characteristics: "(a) it caused more than one thousand deaths; (b) it challenged the sovereignty of an internationally recognized state; (c) it occurred within the recognized boundary of that state; (d) is involves the state as a principal combatant; (e) it included rebels with the ability to mount organized armed opposition to the state; and (f) the parties were concerned with the prospects of living together in the same political unit after the end of the war"[25].

As we can see in table 1 nine out of the ten most ethnically polarized countries have suffered a civil war during the sample period. In the case of ethnic fragmentation only four out of the ten most fragmented countries have suffered a civil war.

Table 1

Ranking of ethnic heterogeneity

Ethnic Polarization		Ethnic Fragmentation	
1	*Eritrea*	1	Tanzania
2	*Guatemala*	2	*Uganda*
3	Niger	3	*India*
4	*Nigeria*	4	Ivory Coast
5	*Sierra Leona*	5	Benin
6	*Bosnia*	6	*Mali*
7	*Liberia*	7	*Philippines*
8	*Jordan*	8	Gabon
9	*Kenya*	9	Guinea-Bissau
10	*Mozambique*	10	Cameroon

Table 2 presents the ranking of countries by religious polarization and fragmentation. We can see that, as in table 1 , civil wars are concentrated in the countries with the highest level of religious polarization. Seven out of ten of the countries with the highest level of religious polarization suffered a civil war during the sample period. However only four our of the ten countries with the highest level of religious fragmentation suffered a civil war.

[25] This definition is practically identical to Singer and Small (1982, 1994) and Licklider (1993, 1995).

Table 2

Ranking of religious heterogeneity

Religious Polarization		Religious Fragmentation	
1	*Eritrea*	1	Korea
2	Botswana	2	Suriname
3	*Dominican Republic*	3	Hong Kong
4	Madagascar	4	Malaysia
5	*Zimbabwe*	5	China
6	*Nigeria*	6	*Indonesia*
7	*Burundi*	7	Tanzania
8	Fiji	8	Cameroon
9	*Sierra Leone*	9	Ivory Coast
10	*Bosnia*	10	*Liberia*

Tables 1 and 2 are used only as an informal discussion on the issue of the relationship between polarization, fragmentation and civil wars. Table 3 presents the estimation of a logit model for the incidence of civil wars as a function of polarization and fragmentation measures of ethnic and religious diversity. The sample includes 138 countries over the period 1960-95. We divide the sample in periods of five years. The endogenous variable is the incidence of a civil war during each five years period. The data come from Doyle and Sambanis (2000) and, as in their case, we consider two alternative variables: civil wars and ethnic civil wars (not including ideological civil wars). The basic specification is taken from Doyle and Sambanis (2000) and Collier and Hoeffler (1998). Therefore the explanatory variables include the log of the population at the beginning of the period[26] (LPOP), the log of real GDP per capita in the initial year of each period (LGDPC), the level of democracy (DEMP3)[27] and regional dummies as well as the indices of fragmentation[28] and polarization[29]. Whenever any religious index (either polarization or fractionalization) is used as a

[26] This is necessary given that the traditional definition of civil war includes a minimum number of deaths as one of the defining criteria.

[27] We construct the democracy dummy using the Polity III dataset which is the largest and longest sample source of information on democracy. Notice that Gastil's index of democracy does not contain data for periods before 1970. Nevertheless the correlation between Gastil's index and our DEMP3 variable for the period in which they overlap is more than 0.9. To determine the democratic status of a country we choose level 4 as the limit. Therefore any country with a level less than 4 is considered an autocracy.

[28] Obviously Collier and Hoeffler (1998) and Doyle and Sambanis (2000) use only indices of fragmentation.

[29] Collier and Hoeffler (1998) also use primary exports as explanatory variable. However there are many missing

regressor we also include a set of dummy variables for the major religious group in order to avoid the confusion of the effect of the index with the characteristics of the major religion[30].

Table 3 shows the results for the incidence of any civil war. As in Doyle and Sambanis (2000) and Collier and Hoeffler (1998) initial GDP per capita has a negative effect while the size of population has a positive effect. The level of democracy has no effect on civil wars[31]. Specification (1) shows that ethnic polarization has a positive and significant effect on civil wars while ethnic fragmentation has no effect. Given this in (2) we eliminate ethnic fragmentation without having any effect on the basic results. Specification (3) compares the effect of religious polarization versus religious fragmentation. It is interesting to notice that religious polarization has a positive and very significant effect on the incidence of a civil war while religious fragmentation has a negative effect. We should notice at this point that, as we argued before, if the number of groups is larger than two the existence of many small groups increases fractionalization but reduces polarization and the probability of conflict. Another interesting fact about specification (3) is that it implies a larger pseudo-R^2 than when using the ethnic indices. Finally column (4) includes all the ethnic and religious indicators. It turns out that religious polarization and ethnic polarization continue having a positive and significant effect on the incidence of a civil war while ethnic fragmentation has no effect and religious fragmentation has even a negative effect.

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values and the size of the sample is reduced drastically. In addition they turn out to be not significantly different from 0.

[30] In any case, for most of the regressions these religious dummy variables are not significantly different from 0.

[31] Reynal-Querol (2002c) shows that democracy is not a totally effective vaccine against civil wars. It is the interaction between democracy and checks and balances what matters to avoid civil wars. Sambanis(2001) Hegre et al.(2001) and Reynal-Querol (2002a,c) find evidence that mid-level democracies are more prone to civil wars that full democracies or full autocracies. The interpretation may be that for starting a civil war some level of freedom is needed to let people organize.

Table 3

Pool Logit: five years periods between 1960-1995

Dependent variable: Civil War Incidence

Variable	(1)	(2)	(3)	(4)
C	0.38	0.51	1.36	0.86
	(0.23)	(0.31)	(0.72)	(0.43)
Log initial GDP per capita	-0.98	-0.94	-1.04	-1.05
	(5.80)	(5.68)	(5.20)	(5.16)
Log population	0.45	0.40	0.47	0.49
	(4.98)	(4.91)	(5.06)	(4.92)
Democracy P3	0.41	0.38	0.39	0.34
	(1.63)	(1.53)	(1.44)	(1.27)
Ethnic Polarization	2.18	1.78		1.59
	(3.53)	(3.24)		(2.29)
Ethnic Fragmentation	-0.82			-0.17
	(1.38)			(0.26)
Religious Polarization			6.01	5.38
			(3.38)	(2.94)
Religious Fragmentation			-9.50	-9.02
			(3.19)	(2.96)
Religious dummies	no	no	yes	yes
N	753	753	746	739
Pseudo-R^2	0.16	0.16	0.18	0.19

*Absolute t-statistics are between parenthesis.

Table 4 present the same specification but using as dependent variable the incidence of an ethnic civil war. The results of table 3 are basically confirmed by these estimations. Initial GDP has a negative effect while initial population has a negative effect and democracy has no effect. If we only include ethnic polarization and fragmentation, as in (1), both are significantly different from 0 but only ethnic polarization has a positive effect.

Table 4

Pool Logit: five years periods between 1960-1995

Dependent variable: Ethnic Civil War Incidence

Variable	(1)	(2)	(3)	(4)
C	-0.85	0.98	-0.70	-0.45
	(0.46)	(0.48)	(0.38)	(0.20)
Log initial GDP per capita	-1.04	-1.08	-1.11	-1.08
	(5.59)	(5.03)	(4.98)	(4.87)
Log population	0.59	0.53	0.62	0.57
	(5.60)	(4.99)	(5.44)	(5.43)
Democracy P3	0.45	0.43	0.36	0.35
	(1.58)	(1.37)	(1.13)	(1.12)
Ethnic Polarization	3.35		2.74	2.31
	(4.77)		(3.51)	(3.35)
Ethnic Fragmentation	-1.79		-0.87	
	(2.68)		(1.17)	
Religious Polarization		8.23	7.38	7.77
		(4.19)	(3.71)	(3.93)
Religious Fragmentation		-12.3	-11.4	-12.1
		(3.81)	(3.49)	(3.73)
Religious dummies	no	yes	yes	yes
N	753	707	700	700
Pseudo-R^2	0.22	0.24	0.26	0.26

*Absolute t-statistics are between parenthesis.

Specification (2) shows also that religious polarization has a positive effect while religious fragmentation has a negative effect. When we include together all the indicators then ethnic fragmentation becomes insignificant, polarization (religious and ethnic) has a positive effect on the probability of a civil war and religious fragmentation has a negative effect. From tables 3 and 4 we can conclude that polarization (ethnic and religious) has a positive impact on the incidence of civil wars while fractionalization has no effect, or even a negative effect. As it is well known the negative effect of political instability on growth we argue that polarization will have, through its incidence on civil wars, a negative effect on growth. There are many empirical articles that have documented the negative effect of political instability and civil wars on growth[32]. Nevertheless, as we are using a new and extend dataset on civil war, we should also show that with this data there is an indirect negative effect of polarization (ethnic and religious) on growth. For this purpose table 5 presents the estimation of the basic specification of Barro (1991) including polarization and fragmentation measures[33]. We also include all the political instability proxies used by that author and estimate a SURE system (growth and civil war regressions[34]) for the sample period 1960-1990 divided in five years periods. Columns (1) and (2) show the results for the ethnic measures. Column (1) confirms the negative and significant effect of civil wars on growth[35] while column (2) confirms the positive effect of ethnic polarization on civil wars and the insignificant effect of ethnic fragmentation. With respect to religious diversity, columns (3) and (4), the results show once again that it is religious polarization what has a positive effect on civil wars. Also in this system the effect of civil wars on growth is negative. Notice that we have argued that ethnic/religious polarization has a negative effect on growth, through its impact on the probability of civil wars, but ethnic/religious fragmentation has no effect through this channel[36].

6 Conclusions

The recent literature on the impact of ethnic diversity on growth has taken as given, without much discussion, that the proper measure for heterogeneity should be calculated by using the fractionalization index. The popularity of this index rests basically on its traditional use in the industrial economics literature. In addition, for empirical purposes, the ready-made nature of the ethnolin-

[32] Murdoch and Sandler (2002) find a negative effect of civil wars on growth using the Mankiw, Romer and Weil (1992) specification.

[33] Montalvo and Reynal-Querol (2002) report similar findings (the negative effect of religious polarization on growth while religious fragmentation has no significant effect) using the specification of Mankiw, Romer and Weil (1992).

[34] Notice that this estimation is simply a confirmatory exercise. From an econometric point of view we could improve this specification since we know that the linear probability model is not the best choice for dummy endogenous variables. Montalvo and Reynal-Querol (2000) show that the indirect effect of ethnic/religious polarization can be traced in two additional channels: the negative effect of ethnic/religious polarization on investment and its positive effect on public expenditure. Therefore their SURE system includes a growth equation and equations for investment, public expenditure and civil wars.

[35] In fact when civil wars are included all the other political instability variables (assassinations, coups, etc) turn out to have a non-significant parameter.

[36] However ethnic/religious fragmentation may have a direct effect on growth justified by a loss of communication argument. However fragmentation is not significant for civil wars and social conflicts.

Table 5: Ethnic Heterogeneity, civil wars and growth

Variable	Growth	Civil War	Growth	Civil war
C	0.68	0.58	0.68	0.38
	(7.35)	(2.76)	(7.38)	(1.52)
Average ratio Investment/GDP	0.51		0.51	
	(4.66)		(4.66)	
Log initial GDP per capita	-0.07	-0.12	-0.07	-0.09
	(-5.83)	(-5.58)	(-5.84)	(-3.40)
Secondary School	0.00		0.00	
	(0.78)		(0.78)	
Primary School	0.00		0.00	
	(0.62)		(0.62)	
Government spending	-0.51		-0.51	
	(-4.46)		(-4.46)	
Average revolutions per year	-0.00		-0.00	
	(-0.56)		(-0.56)	
Assassination per milion pop year	-0.00		-0.00	
	(-1.28)		(-1.27)	
Average coups d'etat per year	-0.01		-0.01	
	(-0.83)		(-0.83)	
Pish	-0.01		-0.01	
	(-0.46)		(-0.47)	
Ppdev	-0.02		-0.02	
	(-0.73)		(-0.73)	
Civil War	-0.06		-0.07	
	(-2.92)		(-3.05)	
Log population		0.05		0.04
		(4.04)		(3.29)
Democracy P3		0.07		0.06
		(2.07)		(1.75)
Ethnic Polarization		0.22		
		(2.51)		
Ethnic Fragmentation		-0.08		
		(-0.91)		
Religious Polarization				0.53
				(2.53)
Religious Fragmentation				-0.73
				(-2.17)
Regional dummies	Yes	Yes	Yes	Yes
N	448	448	448	448
R-squared	0.3007	0.1223	0.3007	0.1490

guistic fragmentation index, ELF, has favored its widespread use since it does not implied any effort of finding new data and constructing an index. We argue that the adequacy of a synthetic index of heterogeneity depends on the intrinsic characteristics of the heterogeneous dimension to be measured and the phenomenon under study. In the case of ethnic diversity there is a very strong conflictive dimension, besides the communication problems emphasized by the literature. For this reason we argue that the measure of heterogeneity should be one of the class of discrete polarization measures. In contrast with the usual problem of polarization indices, which are of difficult empirical implementation without making some arbitrary choice of parameters, we show that the only discrete polarization family that satisfies the basic properties of polarization is the one with a coefficient equal to 1. In addition the only scale parameter that normalizes this index between 0 and 1 takes the value 4. These parameters define a unique measure of discrete polarization which coincides with the indicator proposed by Reynal-Querol (2002a) (RQ) for the measurement of ethnic and religious heterogeneity. Additionally we show that the RQ index can be derived from a simple rent seeking model.

In the empirical section we show the correlation between ethnic fragmentation and ethnic polarization is positive and very high (close to linear) for low levels of fragmentation. However for intermediate and high levels of ethnic fractionalization the correlation is zero or even negative. The same results hold for the relationship between religious fragmentation and religious polarization. Using the most commonly used specification for the incidence of civil wars and growth we find that while polarization, either ethnic or religious, has a large impact on civil wars incidence and, indirectly, on growth, fractionalization has no impact on armed conflicts. Therefore, through this indirect channel ethnic/religious polarization has an important role in the explanation of long run growth.

Finally the RQ index can be used to measure polarization in a context different from the ethnic/religious conflict. In fact any situation that generates rent seeking activities, either in markets or in political institutions, can be suitable for this measure of heterogeneity. Therefore, the RQ index could be an alternative to traditional Herfindahl's index: depending on the theoretical context two firms can lead to a more competitive behavior than many firms.

APPENDIX:

Proof of Theorem 1:

Proof of sufficiency:

The general discrete polarization index can be written as

$$DP(\alpha,k) = k\sum_{i=1}^{n}\sum_{j\neq i}\pi_i^{1+\alpha}\pi_j = k\sum_{i=1}^{n}\pi_i^{1+\alpha}(1-\pi_i) = \sum_{i=1}^{n}k[\pi_i(\pi_i^{\alpha}-\pi_i^{1+\alpha})] =$$
$$\sum_{i=1}^{n}\pi_i(k\pi_i^{\alpha}-k\pi_i^{1+\alpha}) = \sum_{i=1}^{n}\pi_i(1-1+k\pi_i^{\alpha}-k\pi_i^{1+\alpha}) = \sum_{i=1}^{n}\pi_i(1-\tfrac{k}{k}+k\pi_i^{\alpha}-k\pi_i^{1+\alpha}) =$$
$$\sum_{i=1}^{n}\pi_i - \sum_{i=1}^{n}\pi_i k(\tfrac{1}{k}-\pi_i^{\alpha}+\pi_i^{1+\alpha}) = 1-\sum_{i=1}^{n}\pi_i k(\tfrac{1}{k}-\pi_i^{\alpha}+\pi_i^{1+\alpha}) \qquad (1)$$

For the three point distribution (p,q,r) the discrete polarization measure is

$$DP(\alpha,k)^{(p,q,r)} = 1-pk(\tfrac{1}{k}-p^{\alpha}+p^{1+\alpha})-qk(\tfrac{1}{k}-q^{\alpha}+q^{1+\alpha})-rk(\tfrac{1}{k}-r^{\alpha}+r^{1+\alpha})$$

For the alternative distribution $(p,\widetilde{q})$ the DP index is

$$DP(\alpha,k)^{(p,\widetilde{q})} = 1-pk(\tfrac{1}{k}-p^{\alpha}+p^{1+\alpha})-\widetilde{q}k(\tfrac{1}{k}-\widetilde{q}^{\alpha}+\widetilde{q}^{1+\alpha})$$

where $q+r=\widetilde{q}$

Therefore

$$DP(\alpha,k)^{(p,\widetilde{q})}-DP(\alpha,k)^{(p,q,r)} = qk(\tfrac{1}{k}-q^{\alpha}+q^{1+\alpha})+rk(\tfrac{1}{k}-r^{\alpha}+r^{1+\alpha})-\widetilde{q}k(\tfrac{1}{k}-\widetilde{q}^{\alpha}+\widetilde{q}^{1+\alpha}) =$$
$$qk(\tfrac{1}{k}-q^{\alpha}+q^{1+\alpha})+rk(\tfrac{1}{k}-r^{\alpha}+r^{1+\alpha})-(q+r)k(\tfrac{1}{k}-\widetilde{q}^{\alpha}+\widetilde{q}^{1+\alpha}) =$$
$$qk[(\tfrac{1}{k}-q^{\alpha}+q^{1+\alpha})-(\tfrac{1}{k}-\widetilde{q}^{\alpha}+\widetilde{q}^{1+\alpha})]+$$
$$+rk[(\tfrac{1}{k}-r^{\alpha}+r^{1+\alpha})-(\tfrac{1}{k}-\widetilde{q}^{\alpha}+\widetilde{q}^{1+\alpha})] =$$

Let's define $h(\pi) = (\frac{1}{k}-\pi^{\alpha}+\pi^{1+\alpha})$. The first derivative of this function is

$$h'(\pi) = -\alpha\pi^{\alpha-1}+(1+\alpha)\pi^{\alpha}$$

Notice that $h'(\pi^*)=0$ when $\pi^*=\frac{\alpha}{1+\alpha}$. Evaluating at the first derivative we obtain that $h(\pi)$ is a strictly increasing for all $\pi>\pi^*$ and a strictly decreasing function for all $\pi<\pi^*$.

We can write the difference in DP when we merge two small groups in function of $h(.)$ as

$$DP(\alpha,k)^{(p,\widetilde{q})}-DP(\alpha,k)^{(p,q,r)} = qk(h(q)-h(\widetilde{q}))+rk(h(r)-h(\widetilde{q}))$$

We want to show that if $\alpha\geq 1$ then $h(q)>h(\widetilde{q})$ and $h(r)>h(\widetilde{q})$ for all $q,r<\frac{1}{2}$ and, therefore, $DP(\alpha,k)^{(p,\widetilde{q})}-DP(\alpha,k)^{(p,q,r)}$ is positive for any distribution of p, q and r.

In principle we should analyze two possible cases: when the merge results in a group that is smaller than the original largest group ($\widetilde{q}\leq p$) and when the merge of the smallest groups is large than the originally largest group ($\widetilde{q}>p$).

CASE 1: $q+r=\widetilde{q}\leq p$**.**

In this case $q+r=\widetilde{q}\leq\frac{1}{2}$. and $r\leq q<\frac{1}{2}$

Since $\widetilde{q}$ is smaller than p, then $\widetilde{q} \leq \frac{1}{2}$, Therefore we need that $h(\pi_i) > h(\widetilde{q})$ for all $\pi_i \leqslant \widetilde{q} \leq \frac{1}{2}$.

Therefore if $h(q, r) > h(\widetilde{q})$ for all $q, r \leq \frac{1}{2}$, then $h(\pi)$ has to be a decreasing function for all $\pi \leq \frac{1}{2}$. This is only possible if $\pi^*(\alpha) \geq 1/2$.But since $\pi^* = \frac{\alpha}{1+\alpha} \geq \frac{1}{2}$, the latter is satisfied if and only if $\alpha \geq 1$.

Therefore for h being strictly decreasing for all $q, r \leq 1/2$, implies that the DP index has to satisfy property 1 if $\alpha \geq 1$.

CASE 2: $q + r = \widetilde{q} > p$

In this case the minimum value for p is, $p = \frac{1}{3} + \varepsilon$, and the maximum value for $\widetilde{q} = \frac{2}{3} - \varepsilon$. Notice that now q and r can not be any value between $(0, \frac{2}{3})$, otherwise would violate the assumption that $q, r < p$. Therefore, the maximum value for q and r is, $q = \frac{1}{3}, r = \frac{1}{3} - \varepsilon$. This is problematic because we don't need that h be decreasing for $\pi \leq \frac{2}{3}$.

Now for each value of $\widetilde{q}$,which means a value for p, there is a possible maximum value for q, which in the limit is p. Therefore what we need to show is that $h(\max q) > h(p) \geq h(\widetilde{q})$,

We have to show therefore, that $h(\max q) > h(p) \geq h(\widetilde{q})$ in all the rage of $\widetilde{q} \in [\frac{1}{2}, \frac{2}{3}]$.This means that we have to analyze the range of possibilities when $\frac{1}{3} < q < \frac{1}{2}$ when $\frac{1}{2} \leq \widetilde{q} < \frac{2}{3}$

Notice that as $\widetilde{q}$ decrease, then p increases, and then the range of possible q also increases, and therefore in the limit the maximum $q = p$, increases. Therefore,

If the following inequality $h(\frac{1}{3} + \varepsilon) \geq h(\frac{2}{3} - \varepsilon)$ is satisfied for all ε, means that when $\widetilde{q} > p$, then property 1 is satisfied.

So we look which families of DP measures satisfy this inequality:

$h(\frac{1}{3} + \varepsilon) \geq h(\frac{2}{3} - \varepsilon)$

$1 - (\frac{1}{3} + \varepsilon)^{\alpha} + (\frac{1}{3} + \varepsilon)^{1+\alpha} \geq 1 - (\frac{2}{3} - \varepsilon)^{\alpha} + (\frac{2}{3} - \varepsilon)^{1+\alpha}$

$-(\frac{1}{3} + \varepsilon)^{\alpha} + (\frac{1}{3} + \varepsilon)^{1+\alpha} \geq -(\frac{2}{3} - \varepsilon)^{\alpha} + (\frac{2}{3} - \varepsilon)^{1+\alpha}$

$(\frac{1}{3} + \varepsilon)^{\alpha}[\frac{1}{3} + \varepsilon - 1] \geq (\frac{2}{3} - \varepsilon)^{\alpha}[\frac{2}{3} - \varepsilon - 1]$

$(\frac{1}{3} + \varepsilon)^{\alpha}[-\frac{2}{3} + \varepsilon] \geq (\frac{2}{3} - \varepsilon)^{\alpha}[-\frac{1}{3} - \varepsilon]$

$$\left[\frac{\frac{1}{3}+\varepsilon}{\frac{2}{3}-\varepsilon}\right]^{\alpha} \geq \left[\frac{\frac{1}{3}+\varepsilon}{\frac{2}{3}-\varepsilon}\right]$$

Therefore in order this inequality be satisfied for all values of ε we need that $\alpha \geq 1$. It would also be true for r, given that $r \leq q \leq \frac{1}{2}$, and we have shown that h is decreasing function of $\pi \leq \frac{1}{2}$.

Therefore, $DP(\alpha, k)^{(p\widetilde{q})} \geq DP(\alpha, k)^{(p,q,r)}$ if $\alpha \geq 1$

Proof of necessity:

By contradiction. We can show that if $\alpha < 1$, then there always exist a distribution of p, q, r such that the polarization after merging the two smallest groups is smaller than the original, that is to say $DP(\alpha,k)^{(p\widetilde{q})} < DP(\alpha,k)^{(p,q,r)}$.

Consider the case such that $q = r$. Therefore $\widetilde{q} = 2q$.

Let's now compute,

$DP(\alpha,k)^{(p\widetilde{q})} - DP(\alpha,k)^{(p,q,r)} =$

$2q[k(\frac{1}{k} - q^{\alpha} + q^{1+\alpha})] - k2q[(\frac{1}{k} - (2q)^{\alpha} + (2q)^{1+\alpha})] =$

$2kq[\frac{1}{k} - q^{\alpha} + q^{1+\alpha} - \frac{1}{k} + (2q)^{\alpha} - (2q)^{1+\alpha}] =$

$2kq[-q^{\alpha} + q^{1+\alpha} + (2q)^{\alpha} - (2q)^{1+\alpha}] =$

We want to show that for $\alpha < 1$, there always exist a set of $q \in [q^*, \frac{1}{3})$, such that $[-q^{\alpha} + q^{1+\alpha} + (2q)^{\alpha} - (2q)^{1+\alpha}] < 0$

$q^{\alpha}(q-1) + (2q)^{\alpha}(1-2q) < 0$

$(2q)^{\alpha}(1-2q) < -q^{\alpha}(q-1)$

$(2q)^{\alpha}(1-2q) < q^{\alpha}(1-q)$

$(\frac{2q}{q})^{\alpha} < \frac{(1-q)}{(1-2q)}$

$2^{\alpha} < \frac{(1-q)}{(1-2q)}$

Notice that if $q = r$ then $q < \frac{1}{3}$. If $q = \frac{1}{3}$, then $\frac{(1-q)}{(1-2q)}$ evaluated at $\frac{1}{3}$ is 2.

Moreover, for $\alpha < 1$, $2^{\alpha} < 2$ .Therefore, there always exist a set of $q' \in [q^*, \frac{1}{3})$, such that $2^{\alpha} < \frac{(1-q')}{(1-2q')} < 2$.

Therefore, for any $\alpha < 1$, there exist a set of $q' \in [q^*, \frac{1}{3})$, such that $DP(\alpha,k)^{(p\widetilde{q})} < DP(\alpha,k)^{(p,q,r)}$ ■

Proof of Theorem 2:

Proof of sufficiency:

The general discrete polarization index can be written as

$$DP(\alpha,k)^{(N=n)} = 1 - \sum_{i=1}^{n} \pi_i k(\frac{1}{k} - \pi_i^{\alpha} + \pi_i^{1+\alpha}) \qquad (1)$$

For the two point distribution (N=2) the discrete polarization measure is

$DP(\alpha,k)^{(N=2)} = 1 - \sum_{i=1}^{2} \pi_i k(\frac{1}{k} - \pi_i^{\alpha} + \pi_i^{1+\alpha}) =$

$1 - \pi_1 k(\frac{1}{k} - \pi_1^{\alpha} + \pi_1^{1+\alpha}) - \pi_2 k(\frac{1}{k} - \pi_2^{\alpha} + \pi_2^{1+\alpha})$

For the alternative N point distribution $N = 1 + n$ the DP index is

$DP(\alpha,k)^{(N=n+1)} = 1 - \sum_{i=1}^{n+1} \widetilde{\pi}_i k(\frac{1}{k} - \widetilde{\pi}_i^{\alpha} + \widetilde{\pi}_i^{1+\alpha}) =$

$1 - \widetilde{\pi}_1 k(\frac{1}{k} - \widetilde{\pi}_1^{\alpha} + \widetilde{\pi}_1^{1+\alpha}) - \sum_{i=2}^{n+1} \widetilde{\pi}_i k(\frac{1}{k} - \widetilde{\pi}_i^{\alpha} + \widetilde{\pi}_i^{1+\alpha})$ where $\widetilde{\pi}_1 = \pi_1$ and $\sum_{i=2}^{n+1} \widetilde{\pi}_i = \pi_2$

Therefore

$DP(\alpha,k)^{(N=2)} - DP(\alpha,k)^{(N=n+1)} = \sum_{i=2}^{n+1} \widetilde{\pi}_i k(\frac{1}{k} - \widetilde{\pi}_i^{\alpha} + \widetilde{\pi}_i^{1+\alpha}) - \pi_2 k(\frac{1}{k} - \pi_2^{\alpha} + \pi_2^{1+\alpha}) =$

$\sum_{i=2}^{n+1} \widetilde{\pi}_i k(\frac{1}{k} - \widetilde{\pi}_i^{\alpha} + \widetilde{\pi}_i^{1+\alpha}) - \sum_{i=2}^{n+1} \widetilde{\pi}_i k(\frac{1}{k} - \pi_2^{\alpha} + \pi_2^{1+\alpha}) =$

$\sum_{i=2}^{n+1} \widetilde{\pi}_i [k(\frac{1}{k} - \widetilde{\pi}_i^{\alpha} + \widetilde{\pi}_i^{1+\alpha}) - k(\frac{1}{k} - \pi_2^{\alpha} + \pi_2^{1+\alpha})] =$

$\widetilde{\pi}_1 [k(\frac{1}{k} - \widetilde{\pi}_1^{\alpha} + \widetilde{\pi}_1^{1+\alpha}) - k(\frac{1}{k} - \pi_2^{\alpha} + \pi_2^{1+\alpha})] +$

$\widetilde{\pi}_2 [k(\frac{1}{k} - \widetilde{\pi}_2^{\alpha} + \widetilde{\pi}_2^{1+\alpha}) - k(\frac{1}{k} - \pi_2^{\alpha} + \pi_2^{1+\alpha})] + ...+$

$\widetilde{\pi}_{n+1} [k(\frac{1}{k} - \widetilde{\pi}_{n+1}^{\alpha} + \widetilde{\pi}_{n+1}^{1+\alpha}) - k(\frac{1}{k} - \pi_2^{\alpha} + \pi_2^{1+\alpha})]$

Let's define $h(\pi) = (\frac{1}{k} - \pi^{\alpha} + \pi^{1+\alpha})$.The first derivative of this function is

$h'(\pi) = -\alpha\pi^{\alpha-1} + (1+\alpha)\pi^{\alpha}$

Notice that $h'(\pi^*) = 0$ when $\pi^* = \frac{\alpha}{1+\alpha}$. Evaluating at the first derivative we obtain that $h(\pi)$ is a strictly increasing for all the $\pi > \pi^*$, and a strictly decreasing function for all the $\pi < \pi^*$.

We can write the difference in DP in function of $h(.)$ as

$DP(\alpha,k)^{N=2} - DP(\alpha,k)^{N=n+1} = \sum_{i=2}^{n+1} \widetilde{\pi}_i [h(\widetilde{\pi}_i) - h(\pi_2)]$

We want to show that if $\alpha \geq 1$ then $h(\widetilde{\pi}_i) > h(\pi_2)$ for all $\widetilde{\pi}_i < \frac{1}{2}$ and, therefore, $DP(\alpha,k)^{N=2} - DP(\alpha,k)^{N=n+1}$ is positive for any distribution..

In principle we should analyze two possible cases: when we split the small group ($\pi_2 \leq \pi_1$) and when we split the largest group ($\pi_2 > \pi_1$).

CASE 1: If $\pi_2 \leq \pi_1$.

In this case $\pi_2 \leq \frac{1}{2}$. and $\widetilde{\pi}_i < \frac{1}{2}$

Since π_2 is smaller than π_2, then $\pi_2 \leq \frac{1}{2}$, Therefore we need that $h(\widetilde{\pi}_i) > h(\pi_2)$ for all $\widetilde{\pi}_i < \pi_2 \leq \frac{1}{2}$.

Therefore if $h(\widetilde{\pi}_i) > h(\pi_2)$ for all $\widetilde{\pi}_i \leq \frac{1}{2}$, then $h(\pi)$ has to be a decreasing function for all $\pi \leq \frac{1}{2}$. This is only possible if $\pi^*(\alpha) \geq 1/2$.But since $\pi^* = \frac{\alpha}{1+\alpha} \geq \frac{1}{2}$, the latter is satisfied if and only if $\alpha \geq 1$.

Therefore for h being strictly decreasing for all $\widetilde{\pi}_i \leq 1/2$, implies that the DP index has to satisfy property 1 if $\alpha \geq 1$.

CASE 2: $\pi_2 > \pi_1$

In that case the maximum value that $\widetilde{\pi}_i$ can take in the limit would be π_1, that is $\max \widetilde{\pi}_i = \pi_1 - \varepsilon$.

The value for $\pi_2 = (1-\pi_1)$. Notice that now $\widetilde{\pi}_i$ can not be any value between $(0, 1-\pi_1)$, otherwise would violate the assumption that $\widetilde{\pi}_i < \pi_1$. Therefore, the maximum value for $\widetilde{\pi}_i$ is $\max \widetilde{\pi}_i = \pi_1 - \varepsilon$,. This is problematic because we don't need that h be decreasing for $\pi \leq \pi_2$.

Now for each value of π_2,which means a value for π_1, there is a possible maximum value for $\widetilde{\pi}_i$, which in the limit is π_1. Therefore what we need to show is that $h(\max \widetilde{\pi}_i) > h(\pi_1) \geq h(\pi_2)$,

We have to show therefore, that $h(\max \pi_1) > h(\pi_1) \geq h(\pi_2)$ in all the range of $\pi_2 \in [\frac{1}{2}, 1 - \pi_1]$.This means that we have to analyze the range of possibilities when $\pi_1 < \widehat{\pi}_i < \frac{1}{2}$ when $\frac{1}{2} \leq \pi_2 < 1 - \pi_1$

Notice that as π_2 decreases, then π_1 increases, and then the range of possible $\widetilde{\pi}_i$ also increases, and therefore in the $\max \widetilde{\pi}_i$ (that in the limit $=\pi_1$) also increases. Therefore,

If the following inequality $h(\pi_1 + \varepsilon) \geq h(1 - \pi_1 - \varepsilon)$. is satisfied for all ε, means that when $\pi_2 > \pi_1$, then property 1 is satisfied.

So we look which families of $DP(\alpha, k)$ measures satisfies this inequality:

$h(\pi_1 + \varepsilon) \geq h(1 - \pi_1 - \varepsilon)$

$1 - (\pi_1 + \varepsilon)^\alpha + (\pi_1 + \varepsilon)^{1+\alpha} \geq 1 - (1 - \pi_1 - \varepsilon)^\alpha + (1 - \pi_1 - \varepsilon)^{1+\alpha}$

$-(\pi_1 + \varepsilon)^\alpha + (\pi_1 + \varepsilon)^{1+\alpha} \geq -(1 - \pi_1 - \varepsilon)^\alpha + (1 - \pi_1 - \varepsilon)^{1+\alpha}$

$(\pi_1 + \varepsilon)^\alpha[\pi_1 + \varepsilon - 1] \geq (1 - \pi_1 - \varepsilon)^\alpha[1 - \pi_1 - \varepsilon - 1]$

$(\pi_1 + \varepsilon)^\alpha[\pi_1 + \varepsilon - 1] \geq (1 - \pi_1 - \varepsilon)^\alpha[-\pi_1 - \varepsilon]$

$\left[\frac{\pi_1+\varepsilon}{1-\pi_1-\varepsilon}\right]^\alpha \geq \left[\frac{\pi_1+\varepsilon}{1-\pi_1-\varepsilon}\right]$

Therefore in order this inequality be satisfied for all values of ε we need that $\alpha \geq 1$. Moreover it would also be true for all $\widetilde{\pi}_i \leq \max \widetilde{\pi}_i$ given that we have shown that h is a decreasing function of π.

Therefore, $DP(\alpha, k)^{N=2} \geq DP(\alpha, k)^{N=n+1}$ if $\alpha \geq 1$

Proof of necessity:

By contradiction. We can show that if $\alpha < 1$, then there always exist a distribution of π such that the polarization before spliting one group is smaller than the new distribution, that is to say $DP(\alpha, k)^{(N=2)} < DP(\alpha, k)^{(N=N+1)}$.

Consider the case such that the distribution among two groups is composed by $\pi_1$and π_2. Then the distribution of $N + 1$ groups is composed by $\pi_1$and $\widetilde{\pi}_2 = \widetilde{\pi}_3 = \widetilde{\pi}_4 = ... = \widetilde{\pi}_{N+1} = \pi$, such that $\sum_{i=2}^{N+1} \widetilde{\pi}_i = N\pi = \pi_2$.

Let's now compute,

$DP(\alpha, k)^{(2)} - DP(\alpha, k)^{(N+1)} =$

$N\pi[k(\frac{1}{k} - \pi^\alpha + \pi^{1+\alpha})] - k\pi_2(\frac{1}{k} - \pi_2^\alpha + \pi_2^{1+\alpha}) =$

$N\pi[k(\frac{1}{k} - \pi^{\alpha} + \pi^{1+\alpha})] - k(N\pi)(\frac{1}{k} - (N\pi)^{\alpha} + (N\pi)^{1+\alpha}) =$

$N\pi k[\frac{1}{k} - \pi^{\alpha} + \pi^{1+\alpha} - \frac{1}{k} + (N\pi)^{\alpha} - (N\pi)^{1+\alpha}] =$

$N\pi k[-\pi^{\alpha} + \pi^{1+\alpha} + (N\pi)^{\alpha} - (N\pi)^{1+\alpha}] =$

we want to show that for $\alpha < 1$, there always exist a set of $\pi \in [\pi^{**}, \frac{1}{N})$, such that $N\pi k[-\pi^{\alpha} + \pi^{1+\alpha} + (N\pi)^{\alpha} - (N\pi)^{1+\alpha}] < 0$

that $-\pi^{\alpha} + \pi^{1+\alpha} + (N\pi)^{\alpha} - (N\pi)^{1+\alpha} < 0$

$\pi^{\alpha}(\pi - 1) + (N\pi)^{\alpha}(1 - N\pi) < 0$

$(N\pi)^{\alpha}(1 - N\pi) < -\pi^{\alpha}(\pi - 1)$

$(N\pi)^{\alpha}(1 - N\pi) < \pi^{\alpha}(1 - \pi)$

$(\frac{N\pi}{\pi})^{\alpha} < \frac{(1-\pi)}{(1-2\pi)}$

$N^{\alpha} < \frac{(1-\pi)}{(1-N\pi)}$

Notice that if $\widetilde{\pi}_2 = \widetilde{\pi}_3 = \widetilde{\pi}_4 = ... = \widetilde{\pi}_{N+1} = \pi$, then $\pi < \frac{1}{N+1}$. If $\pi = \frac{1}{N+1}$, then $\frac{(1-\pi)}{(1-N\pi)}$ evaluated at $\frac{1}{N+1}$ is N.

Moreover, for $\alpha < 1$, $N^{\alpha} < N$ .Therefore, there always exist a set of $\pi' \in [\pi^{**}, \frac{1}{N+1})$, such that $N^{\alpha} < \frac{(1-\pi')}{(1-N\pi')} < N$.

Therefore, for any $\alpha < 1$, there always exist a set of $\pi' \in [\pi^{**}, \frac{1}{N+1})$, such that $DP(\alpha, k)^{(N=2)} < DP(\alpha, k)^{(N=N+1)}$■

Proof of Lemma 1:

Step 1: Suppose there are N groups of any size. Take the biggest one and separate it from the others. Then merge all the other groups into one group. By property 1b the DP measure increases if and only if $\alpha \geq 1$. That is, in the new distribution the index is larger than in the original one if and only if $\alpha \geq 1$. This means that, given any distribution of N groups, we can always find another distribution on two groups where the DP index is larger if and only if $\alpha \geq 1$. This does not mean that the new distribution is more polarize as explain above, but that the index is larger.

Step 2: Suppose now that we only have two groups of π and $(1-\pi)$ sizes. The polarization index $DP = k\sum_{i=1}^{2} \pi_i^{1+\alpha}(1 - \pi_i) = k[\pi_1^{1+\alpha}(1 - \pi_1) + (1 - \pi_1)^{1+\alpha}\pi_1]$

It is easy to verify that for any α this expression is maximized at $\pi_1 = \pi_2 = 0.5$. which means that $\pi_1 = \pi_2 = 0.5$ is a local maximum for any $\alpha < 1$. However it is a global maximum only if $\alpha \geq 1$.■

Proof of Theorem 3:

Any three points discrete distribution can be written in the form $(x, 1-2x, x)$ such that $x \in [0, \frac{1}{2}]$. Our purpose is to show under what conditions DP is an increasing function of x, the shifted mass from the q group to any other group,

$DP(x, 1-2x, x) < DP(\widetilde{x}, 1-2\widetilde{x}, \widetilde{x})$ for all $x < \widetilde{x}$.

Therefore the comparison of $DP(p, q, p)$ and $DP(p+x, q-2x, p+x)$. would be the same as the comparison of

$DP(x', 1-2x', x')$ and $DP(\widetilde{x}, 1-2\widetilde{x}, \widetilde{x})$ where $x' = p$ and $\widetilde{x} = p + x$

We can compute DP in this case as

$DP(\alpha, k) = k[(2x^{1+\alpha}(1-x) + (1-2x)^{1+\alpha}2x)] = k[2x^{1+\alpha} - 2x^{2+\alpha} + (1-2x)^{1+\alpha}2x]$

The first derivative of DP is

$\frac{\partial DP}{\partial x}(\alpha, k) = k[2(1+\alpha)x^{\alpha} - 2(2+\alpha)x^{1+\alpha} + (1+\alpha)(1-2x)^{\alpha}(-2)2x + (1-2x)^{1+\alpha}2] =$

$2k\{x^{\alpha}[(1+\alpha) - (2+\alpha)x] + (1-2x)^{\alpha}[-2(1+\alpha)x + (1-2x)]\} =$

$2k\{x^{\alpha}[1+\alpha-2x-x\alpha] + (1-2x)^{\alpha}[(1-2x) - 2x - 2x\alpha]\}$

Therefor $\frac{\partial DP}{\partial x}$ evaluated at $\alpha = 1$ is always positive given that

$\frac{\partial DP(1,k)}{\partial x} = 2k[1-3x]^2 > 0\ \forall x$.Therefore if $\alpha = 1$ then $\frac{\partial DP(1,k)}{\partial x} > 0$ for any distribution.

In addition the partial derivative, $\frac{\partial DP(\alpha,k)}{\partial x}$, evaluated at $x = \frac{1}{3}$ is always equal to 0

$2k\{(\frac{1}{3})^{\alpha}[1+\alpha-2\frac{1}{3}-\frac{1}{3}\alpha] + (1-2\frac{1}{3})^{\alpha}[(1-2\frac{1}{3}) - 2\frac{1}{3} - 2\frac{1}{3}\alpha]\} =$

$= k\{(\frac{1}{3})^{\alpha}[\frac{1}{3}+\frac{2}{3}\alpha] + (\frac{1}{3})^{\alpha}[-\frac{1}{3}-\frac{2}{3}\alpha]\} = 0$ for all values of α

The second derivative is

$\frac{\partial^2 DP(\alpha,k)}{\partial x \partial x} = 2k\{\alpha x^{\alpha-1}[(1+\alpha) - 2x - x\alpha] + x^{\alpha}[-2-\alpha]+$

$+\alpha(1-2x)^{\alpha-1}(-2)[1-4x-2x\alpha] + (1-2x)^{\alpha}[-4-2\alpha]\} =$

$2k\{\alpha x^{\alpha-1}[1+\alpha-2x-x\alpha] - x^{\alpha}[2+\alpha]-$

$-2\alpha(1-2x)^{\alpha-1}[1-4x-2x\alpha] - (1-2x)^{\alpha}[4+2\alpha]\}$

Evaluating the second derivative at $x = 1/3$ we obtain

$\frac{\partial^2 DP(\alpha,k)}{\partial x \partial x} = (\frac{1}{3})^{\alpha}[3(2\alpha^2 - 2)]$

This means that for $\alpha = 1$, then $\frac{\partial^2 DP(1,k)}{\partial x \partial x} = 0$, which implies that $x = \frac{1}{3}$ is an inflection point. However if $\alpha < 1$, then $\frac{\partial^2 DP(\alpha,k)}{\partial x \partial x} < 0$, which means that $x = \frac{1}{3}$ is a maximum and if $\alpha < 1$, then $\frac{\partial^2 DP(\alpha,k)}{\partial x \partial x} > 0$, which means that $x = \frac{1}{3}$ is a minimum. Therefore if $x = \frac{1}{3}$ is a maximum, this means that for any ball around x=1/3 then $DP(\alpha < 1, k)^{x=\frac{1}{3}+\varepsilon} < DP(\alpha < 1, k)^{x=\frac{1}{3}}$ which violates property 2. On the other side for $\alpha > 1$ $x = \frac{1}{3}$ is a minimum which implies that $DP(\alpha > 1, k)^{x=\frac{1}{3}-\varepsilon}$ $> DP(\alpha > 1, k)^{x=\frac{1}{3}}$which also violates property 2.Therefore the only DP measure that satisfy property 3 for any distribution has a parameter $\alpha = 1$.■

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