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# The maximum economic yield management of an age-structured salmon population 

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#### Abstract

This chapter develops a sustainable economic yield harvesting model for the wild Atlantic salmon (Salmo salar) where the population comprises different age classes. It is shown that the weight-fecundity relationship of the spawning population, comprising two age classes, is crucial for the maximum sustainable yield fishing composition. In a next step the optimal selective fishing is replaced by an optimal non-selective fishing pattern, and the discrepancy between these two schemes is analyzed.


Key words: Salmon fishery, age classes, maximum economic yield

## X.1. Introduction

For many years, the North Atlantic salmon (Salmo salar) has been one of the most important fish species in Norway because of its social, cultural, and economic importance. It was traditionally harvested for food, but is today most important to recreational anglers (NOU 1999). The abundance has been declining during the last few decades, especially since the 1990s. There are a combination of various factors behind this development, such as sea temperature, diseases, and human activity, both in the spawning streams and through the strong growth of salmon sea farming (NASCO, 2004). As the wild stock began to decrease during the 1980s, the Norwegian government imposed gear restrictions to limit the marine harvest. Drift net fishing was banned in 1989, and the fishing season of bend net fishing, taking place in the fjords and close to the spawning rivers, has been restricted several times. At the same time, the sport fishing season in the spawning rivers has been subject to various restrictions (NOU, 1999). However, despite all these measures taken to secure and rebuild the stock, the abundance of wild salmon seems to be at only half the level experienced in the

1960s and 1970s. Today, farmed salmon is regarded as the main threat to the viability of the wild salmon population because of the spread of diseases through sea lice infection, escapees, and environmental pollution (Hindar et al., 2006).

Wild salmon fishing has been analyzed in many papers from an economic perspective (see, e.g., Routledge, 2001; Laukkanen, 2001; Olaussen and Skonhoft, 2008). These are all studies based on a biomass approach where 'a fish is a fish', while Kulmala et al. (2008) studied numerically an age-structured dynamic salmon model. The age model formulated in this chapter is much simpler than that of Kulmala et al. as we aim to say something analytically about the basic driving forces behind a harvest composition that maximizes the economic yield (MEY). For this reason, only biological equilibrium is considered. The analysis has similarities with Reed (1980) and Getz and Haight (1988), but we study a different biological system in which all the spawning fish, i.e., salmon, die after spawning. This contrasts with Reed's model, where a fixed fraction of the spawning fish (e.g., cod) survives, and enters an older year class after spawning. While our analysis is directly related to Atlantic salmon, we will find that it fits various Pacific salmon species, such as pink and chum salmon, which also die after spawning (see, e.g., Groot and Margolis 1991).

This chapter is organized as follows. In section two the population model is formulated, and where we consider two spawning, and hence two harvestable, age classes. In section three, we find the maximum sustainable economic yield fishing policy. The economic benefit of our selective harvesting scheme is next in section four compared to a uniform fishing pattern. The theoretical reasoning is numerically illustrated in section five while section six finally summarizes and concludes this chapter.

## X.2. Population model

Atlantic salmon is an anadromous species that has a complex life cycle with several distinct phases. Freshwater habitat is essential in the early development stages, as this is where it spends the first one to four years from spawning to juvenile rearing before undergoing smoltification and seaward migration. Then, it stays for one to three years in the ocean for feeding and growing and, when mature, returns to its natal or 'parent' river to spawn. After spawning, most salmon die, as less than $10 \%$ of the female salmon spawn twice (Mills 1989). The Atlantic salmon is subject to fishing when it migrates back to its parent river. In Norway,
most sea fishing takes place in fjords and inlets with wedge-shaped seine and bend nets. This fishing is commercial or semi commercial. In the rivers, salmon are caught by recreational anglers with rods and hand lines. The recreational fishery is by far the most important from an economic point of view (NOU, 1999).

In what follows, a specific salmon population (with its native river) is considered in terms of a number of individuals at time $t$ structured into recruits $N_{0, t}(y r<1)$, three young age classes, $N_{1, t}(1 \leq y r<2), N_{2, t}(2 \leq y r<3)$ and $N_{3, t}(3 \leq y r<4)$, and two adult, spawning classes $N_{4, t}(4 \leq y r<5)$, one sea winter (1SW) and $N_{5, t}(5 \leq y r \leq 6)$, two sea winter (2SW).

Recruitment is endogenous and density dependent, and the 2SW has higher fertility than the 1SW. Natural mortality is fixed and density independent and, as an approximation, it is assumed that the whole spawning population dies after spawning. It is further assumed that the proportion between the two adult age classes is fixed. This proportion may be influenced by a number of factors, such as the type of river ('small' salmon river vs. 'large' salmon river) and environmental factors (NOU, 1999). As fishing takes place when the fish returns back to its native river (see also above), only the adult spawning classes $N_{4, t}$ and $N_{5, t}$ are subject to fishing.

With $B_{t}$ as the size of the spawning population, adjusted for different fertility among the two spawning classes (see below), the stock recruitment relationship is given by:

$$
\begin{equation*}
N_{0, t}=R\left(B_{t}\right) . \tag{1}
\end{equation*}
$$

$R\left(B_{t}\right)$ may be a one-peaked value function (i.e., of the Ricker type) or it may be increasing and concave (i.e., of the Beverton-Holt type). In both cases, zero stock means zero recruitment, $R(0)=0$. The number of young is next defined as:
(2) $N_{a+1, t+1}=s_{a} N_{a, t}$,
where $a=0,1,2$, and with $s_{a}$ as the age-specific natural survival rate, assumed to be density independent and fixed over time.

As indicated, only the spawning classes are subject to fishing mortality (marine as well as river fishing). With $0<\sigma<1$ as the fixed proportion of the adult stock that returns to spawn in the first year, the number of spawning fish of this part of the adult population (1SW) is:

$$
\begin{equation*}
N_{4, t+1}=s_{3} N_{3, t} \sigma\left(1-f_{4, t}\right), \tag{3}
\end{equation*}
$$

where $f_{4, t}$ yields the fishing mortality. Accordingly, $H_{4, t}=s_{3} N_{3, t} \sigma f_{4, t}$ is the number of harvested 1SW fish in year $t$. As indicated, the parameter $\sigma$ may depend on various factors, but is considered as fixed and exogenous. The rest of this cohort $\tilde{N}_{4, t+1}=s_{3} N_{3, t}(1-\sigma)$ stays one year more in the ocean. When subject to natural mortality, as well as subsequent fishing mortality, on migration back to spawning in the home river, the size of the next year's spawning population (2SW) becomes:

$$
\begin{equation*}
N_{5, t+1}=s_{4} \tilde{N}_{4, t}\left(1-f_{5, t}\right) . \tag{4}
\end{equation*}
$$

Hence, $H_{5, t}=s_{4} \tilde{N}_{4, t} f_{5, t}$ is the number of harvested 2SW salmon year $t$. With $\gamma_{4}$ and $\gamma_{5}$ as the fecundity parameters of the 1SW and 2SW stocks, respectively, and where 2SW is more productive, $\gamma_{5}>\gamma_{4}$, the spawning population in year $t$ may be written as $B_{t}=\gamma_{4} N_{4, t}+\gamma_{5} N_{5, t}$, or:

$$
\begin{equation*}
B_{t}=\gamma_{4} s_{3} N_{3, t-1} \sigma\left(1-f_{4, t-1}\right)+\gamma_{5} s_{3} N_{3, t-2}(1-\sigma) s_{4}\left(1-f_{5, t-1}\right) . \tag{5}
\end{equation*}
$$

Equation (2) implies $N_{3, t+3}=s_{0} s_{1} s_{2} N_{0, t}$, or:
(6) $\quad N_{3, t+3}=s R\left(B_{t}\right)$,
when also using equation (2) and where $s=s_{0} s_{1} s_{2}$ comprises the previous years' survival rates. For given fishing mortalities, equations (6) and (5) yield a system of two difference equations of degree five for the two variables $N_{3, t}$ and $B_{t}$.

As already indicated, we are concerned only with equilibrium fishing, or sustainable harvesting, in this paper. The population equilibrium for fixed fishing mortalities is defined for $N_{3, t}=N_{3}$ and $B_{t}=B$ for all $t$ such that:

$$
\begin{equation*}
B=\left[\gamma_{4} s_{3} \sigma\left(1-f_{4}\right)+\gamma_{5} s_{3}(1-\sigma) s_{4}\left(1-f_{5}\right)\right] N_{3}, \tag{5’}
\end{equation*}
$$

and
(6') $\quad N_{3}=s R(B)$.
In what follows, $\left(5^{\prime}\right)$ is referred to as the spawning constraint, whereas ( $6^{\prime}$ ) represents the recruitment constraint. An internal equilibrium ( $N_{3}>0$ and $B>0$ ) holds only if either $f_{4}$ or $f_{5}$, or both, are below one; that is, to exclude depletion, both mature classes cannot be totally fished down. Notice that this is a necessary but not sufficient condition. Figure X. 1 illustrates
the internal, unique equilibrium when the recruitment function is of the Beverton-Holt type, i.e., $R(0)=0, \partial R / \partial B_{t}=R^{\prime}>0$ and $R^{\prime \prime}<0$ (see also numerical section).

Figure X. 1 about here

## X.3. The maximum sustainable economic yield harvesting program

We start to analyze the optimal sustainable harvesting program under the assumption of perfect selectivity. With $w_{5}>w_{4}$ as the fixed weights (kg per fish) of the young and old mature population, respectively, and $p_{5} \geq p_{4}$ as the fishing values (NOK per kg), and where the 2SW is at least as valuable as the 1SW (Olaussen and Liu 2011),
$\pi=p_{4} w_{4} H_{4}+p_{5} w_{5} H_{5}=\left[p_{4} w_{4} s_{3} \sigma f_{4}+p_{5} w_{5} s_{3}(1-\sigma) s_{4} f_{5}\right] N_{3}$ then describes the yearly revenue in our salmon fishery. Therefore, the maximum sustainable economic yield problem is defined by finding fishing mortalities that maximize $\pi$ subject to the spawning constraint ( $5^{\prime}$ ) and the recruitment constraint ( $6^{\prime}$ ).

The Lagrangian of this problem may be written as
$L=\left[p_{4} w_{4} s_{3} \sigma f_{4}+p_{5} w_{5} s_{3}(1-\sigma) s_{4} f_{5}\right] N_{3}-\lambda\left[N_{3}-s R(B)\right]$
$-\mu\left\{B-\left[\gamma_{4} s_{3} \sigma\left(1-f_{4}\right)+\gamma_{5} s_{3}(1-\sigma) s_{4}\left(1-f_{5}\right)\right] N_{3}\right\}$, where $\lambda>0$ and $\mu>0$ (both in NOK per
fish) are the shadow prices of the recruitment and spawning constraints, respectively.
Following the Kuhn-Tucker theorem, the first-order necessary conditions (assuming $N_{3}>0$ and $B>0$ ) are:

$$
\begin{align*}
& \partial L / \partial f_{4}=N_{3}\left(p_{4} w_{4}-\mu \gamma_{4}\right) \frac{\geq}{<} 0 ; 0 \leq f_{4} \leq 1,  \tag{7}\\
& \partial L / \partial f_{5}=N_{3}\left(p_{5} w_{5}-\mu \gamma_{5}\right) \frac{\geq}{<} 0 ; 0 \leq f_{5} \leq 1,  \tag{8}\\
& \partial L / \partial N_{3}= \tag{9}
\end{align*}
$$

$p_{4} w_{4} s_{3} \sigma f_{4}+p_{5} w_{5} s_{3}(1-\sigma) s_{4} f_{5}-\lambda+\mu\left[\gamma_{4} s_{3} \sigma\left(1-f_{4}\right)+\gamma_{5} s_{3}(1-\sigma) s_{4}\left(1-f_{5}\right)\right]=0$,
and

$$
\begin{equation*}
\partial L / \partial B=\lambda s R^{\prime}(B)-\mu=0 . \tag{10}
\end{equation*}
$$

Control condition (7) indicates that the fishing mortality of the ISW population should take place at the point where the marginal biomass value gain is equal, below or above its marginal
biomass harvest loss, determined by the fecundity parameter and evaluated by the spawning constraint shadow price. Condition (8) is analogous for the 2SW. The stock condition (9) says that the harvestable population should be managed so that the recruitment constraint shadow price $\lambda$ is equal to the total marginal harvest value gain plus the total marginal spawning biomass value gain, evaluated at its shadow price. Finally, stock condition (10) indicates that the recruitment growth, evaluated at its shadow price, should be equal to the spawning constraint shadow price $\mu$.

From the control conditions (7) and (8), it is observed that only the weight value-fecundity ratio $p_{i} w_{i} / \gamma_{i}(i=4,5)$ determines the fishing mortality and the fishing composition and, hence, no other factors play a direct role. This outcome differs from the seminal Reed (1980) paper, who found that weight together with natural mortality ('biological discounted’ value) directly determined the fishing composition. As already indicated, the reason for this discrepancy is the different biological characteristics of the fish stocks. While the mature fish die after spawning in our salmon model, the spawning fish survive and enter older age classes in the Reed model.

Weight and fertility are related and larger and older fish in most instances, if not always, indicate higher fertility (e.g., Getz and Haight, 1989). According to McGinnity et al. (2003), this relationship for wild salmon is described such that fertility is an increasing, strictly concave function of weight (and age) and hence the weight-fertility ratio increases with weight (see also numerical section below). With $w_{5} / \gamma_{5}>w_{4} / \gamma_{4}$ together with $p_{5} \geq p_{4}$ and hence a higher marginal value gain-loss ratio for fishing the 2SW spawning fish, the maximum sustainable yield harvesting policy, given by conditions (7) and (8), indicates a higher fishing mortality for the 2SW than the 1SW. This is stated as the following proposition:

Proposition 1. With a higher weight-fecundity ratio for the old adult subpopulation, the maximum yield harvesting policy is governed by a higher fishing mortality of the old subpopulation.

There are three possible cases, all corner solutions, that may represent this optimal policy: i) $f_{5}^{*}=1$ and $0<f_{4}^{*}<1$, ii) $f_{5}^{*}=1$ and $f_{4}^{*}=0$, and iii) $0<f_{5}^{*}<1$ and $f_{4}^{*}=0$ (superscript ${ }^{* *}$, indicates optimal values). The spawning constraint (5')
$N_{3}=B /\left[\gamma_{4} s_{3} \sigma\left(1-f_{4}\right)+\gamma_{5} s_{3}(1-\sigma) s_{4}\left(1-f_{5}\right)\right]$ will be steeper in case i) than in case ii), which again will be steeper than that in case iii). Therefore, when taking the recruitment constraint $\left(6^{\prime}\right)$ into account (again, see Figure X.1), we find that the size of the spawning population $B^{*}$ as well as the harvestable stock $N_{3}^{*}$ will be highest with harvest option iii) and lowest if case i) represents the optimal policy. If $p_{5} w_{5} / \gamma_{5}$ is substantially higher than $p_{4} w_{4} / \gamma_{4}$, we may intuitively suspect that it is beneficial for the manager to invest in the salmon population by leaving the young mature population unexploited and harvesting the whole old adult population. Hence, case ii), where $f_{5}^{*}=1$ and $f_{4}^{*}=0$, should maximize the sustainable yield. On the other hand, with a 'small' weight value-fertility ratio difference, either case i), with harvesting of both mature populations, or case iii), with harvesting of the old mature population only, should possibly represent the optimal solution.

In case i) with $f_{5}^{*}=1$ and $0<f_{4}^{*}<1$, the spawning constraint shadow price is determined through condition (7) as $\mu^{*}=p_{4} w_{4} / \gamma_{4}$. Combination with Eq (9) yields $\lambda^{*}=s_{3}\left[p_{4} w_{4} \sigma+p_{5} w_{5}(1-\sigma) s_{4}\right]$. When inserting into Eq (10), the optimal spawning biomass is governed by $R^{\prime}\left(B^{*}\right)=\mu^{*} / s \lambda^{*}=1 / s s_{3}\left[\gamma_{4} \sigma+\left(p_{5} w_{5} \gamma_{4} / p_{4} w_{4}\right)(1-\sigma) s_{4}\right] . N^{*}$ then follows from Eq. (6'), and we find the fishing mortality $f_{4}^{*}$ next through Eq ( $5^{\prime}$ ), $B^{*}=s_{3} \gamma_{4} \sigma\left(1-f_{4}\right) N_{3}^{*}$. If case ii) with $f_{5}{ }^{*}=1$ and $f_{4}{ }^{*}=0$ represents the optimal policy, the spawning constraint ( $5^{\prime}$ ) reads $B=\gamma_{4} s_{3} \sigma N_{3}$. Therefore, in this case, Eq (5) together with the recruitment constraint (6') $N_{3}=s R(B)$ alone determines $N_{3}^{*}$ and $B^{*}$. In case iii), where $0<f_{5}^{*}<1$ and $f_{4}^{*}=0$, the spawning constraint shadow price is determined through condition (8) as $\mu^{*}=p_{5} w_{5} / \gamma_{5}$, whereas Eq (9) determines the recruitment constraint shadow price as $\lambda^{*}=s_{3} p_{5} w_{5}\left[\gamma_{4} \sigma / \gamma_{5}+(1-\sigma) s_{4}\right]$. Therefore, just as in case $\left.i\right)$, the size of the spawning biomass is found through $\operatorname{Eq}(10)$, now as $R^{\prime}\left(B^{*}\right)=1 / s s_{3}\left[\gamma_{4} \sigma+\gamma_{5}(1-\sigma) s_{4}\right]$ while $N^{*}$ next follows from Eq. (6’). The optimal fishing mortality is again determined by the spawning constraint ( $5^{\prime}$ ), in this case iii) as $B^{*}=s_{3}\left[\gamma_{4} \sigma+\gamma_{5}(1-\sigma) s_{4}\left(1-f_{5}\right)\right] N_{3}^{*}$.

As indicated, changes in fishing prices may shift the optimal harvest policy from targeting only 2SW to targeting both stocks, and the vice versa. While price shifts have no effects
within case ii) and case iii), we find $\partial R^{\prime}\left(B^{*}\right) / \partial p_{4}>0$ within case i) as $R^{\prime}\left(B^{*}\right)=1 / s_{3}\left[\gamma_{4} \sigma+\left(p_{5} w_{5} \gamma_{4} / p_{4} w_{4}\right)(1-\sigma) s_{4}\right]$ then describes the optimal spawning stock. Therefore, $\partial \mathrm{B}^{*} / \partial p_{4}<0$ and also $\partial N_{3}^{*} / \partial p_{4}<0$ hold. The effects of $p_{5}$ are of the opposite. Because the spawning constraint ( $5^{\prime}$ ) writes $B=s_{3} \gamma_{4} \sigma\left(1-f_{4}\right) N_{3}$ in this case i), a higher $p_{4}$ is satisfied with a higher $f_{4}^{*}$. Not surprisingly, with $p_{4}=p_{5}$ we find zero stock and harvesting effects of price changes. There are then simply no price trade-offs present. These price effects found here are generally different from the standard biomass (lumped parameter) fishery model (e.g., Clark, 1990) where changing harvest value, in absence of stock dependent harvesting costs (and other possible stock values), have no effects when determining the maximum sustainable yield policy.

## X.4. Non-selective fishing pattern

It may also be of interest to find the maximum economic yield when our optimal selective fishing pattern is replaced by an optimal fishing pattern with similar, or uniform, fishing mortalities. This scheme may hence indicate a non-selective fishing situation where 'a fish is a fish' as considered in biomass models used in the traditional bioeconomic analysis (e.g., Clark 1990). Our optimizing problem is then described by the goal of maximizing $\pi=\left[p_{4} w_{4} s_{3} \sigma f_{4}+p_{5} w_{5} s_{3}(1-\sigma) s_{4} f_{5}\right] N_{3}$ subject to the spawning and recruitment constraints, Eqs. (5') and (6'), respectively, and $f_{4}=f_{5}=f$.

The Lagrangian of this problem may be written as

$$
L=\left[p_{4} w_{4} s_{3} \sigma+p_{5} w_{5} s_{3}(1-\sigma) s_{4}\right] f N_{3}-\lambda\left[N_{3}-s R(B)\right]-\mu\left\{B-\left[\gamma_{4} s_{3} \sigma+\gamma_{5} s_{3}(1-\sigma) s_{4}\right](1-f) N_{3}\right\}
$$

when inserting for the uniform fishing pattern. The first order necessary conditions (again with $N_{3}>0$ and $B>0$ ) reads:

$$
\begin{equation*}
\partial L / \partial f=N_{3}\left\{\left[p_{4} w_{4} \sigma+p_{5} w_{5}(1-\sigma) s_{4}\right)\right]-\mu\left[\left(\gamma_{4} \sigma+\gamma_{5}(1-\sigma) s_{4}\right]\right\}=0 ; 0<f<1, \tag{11}
\end{equation*}
$$

and

$$
\begin{equation*}
\partial L / \partial N_{3}=\left[p_{4} w_{4} s_{3} \sigma+p_{5} w_{5} s_{3}(1-\sigma) s_{4}\right] f-\lambda+\mu\left[\gamma_{4} s_{3} \sigma+\gamma_{5} s_{3}(1-\sigma) s_{4}\right](1-f)=0, \tag{12}
\end{equation*}
$$ together with $\mathrm{Eq}(10)$. The control condition (11) must hold as equation as stock depletion never can be beneficial under this economic yield scenario with zero discount rent.

With $N_{3}>0$, this equation may also be written as
$\left[p_{4} w_{4} \sigma / s_{4}+p_{5} w_{5}(1-\sigma)\right]=\mu^{*}\left[\gamma_{4} \sigma / s_{4}+\gamma_{5}(1-\sigma)\right]$. Therefore, the optimal uniform fishing
pattern may be characterized by a situation where the 'biological discounted' marginal harvesting value (marginal gain) equalizes the 'biological discounted’ fertility (marginal loss), evaluated by the spawning constraint shadow value. This equation also determines the optimal spawning constrain shadow price, $\mu^{*}$.

The uniform harvesting pattern can never be more economic beneficial than the selective harvesting scheme as one more constraint is included in the uniform maximization problem. When combining Eqs (11) and (12) we find $\lambda=\mu s_{3}\left[\gamma_{4} \sigma+\gamma_{5}(1-\sigma) s_{4}\right]$ after some small rearrangements. Inserted into condition (10), the size of the spawning biomass is next described as $R^{\prime}\left(B^{*}\right)=1 / s_{3}\left[\gamma_{4} \sigma+\gamma_{5}(1-\sigma) s_{4}\right]$. Therefore, we find exactly the same optimal spawning population as in the above selective harvesting case iii) and the marginal harvesting value (marginal gain) has no influence on the optimal uniform fishing pattern. This is stated as:

Proposition 2. The marginal harvesting value (marginal gain) has no influence on the optimal uniform fishing pattern.

Different fish prices and differences in fish weigh among 1SW and 2SW have therefore no influence on the optimal fishing pattern. Moreover, if prices, fish weights and fertility are such that case iii) represents the optimal harvesting policy under the assumption of selective harvesting, the size of the spawning biomass and degree of exploitation will be similar under uniform harvesting.

## X.5. Numerical illustration

The above theoretical reasoning will now be illustrated numerically. Hansen et al. (1996) estimated a salmon recruitment function for a small river in Norway (the Imsa River) based on the Shepherd recruitment function, which includes three parameters. In our model, we choose a simpler approach and use the Beverton-Holt function (cf. Figure 2). This function may be specified as $R(B)=r \frac{B}{1+B / K}$, with $r>0$ as the intrinsic growth rate, or maximum number of recruits per (fertility adjusted) spawning salmon, and $K>0$ as the stock level for which density-dependent mortality equals density-independent mortality. The size of $r K$ yields the maximum number of recruits and scales the system ('size of the river'), which is assumed to be 40,000 (number of recruits). The value of $r$ indicates the 'quality' of the river,
and we choose $r=400$ (number of recruits per spawning salmon). Then, we find $K=100$. The fertility - weight relationship is based on McGinnity et al. (2003) given as $\gamma=4.83 w^{0.87}$ (weight $w$ is here measured gramme). In the river Imsa in Norway, the same functional form is estimated as $\gamma=5.10 w^{0.86}$ (personal communication senior researcher Ola Diserud Norwegian Institute of Natural Research, Trondheim).When normalizing the fertility parameter of the SW1 to one, $\gamma_{4}=1$, and using the McGinnity et al functional form, we find $\gamma_{5}=2.4$ under the assumption of fishing weights $w_{4}=2.0$ and $w_{5}=5.5(\mathrm{~kg} / \mathrm{salmon})$. These weights fit a 'typical’ medium-sized Norwegian salmon river (NOU, 1999). The survival parameters are based on NOU (1999), whereas the fishing prices are related to recreational fishery, which, as indicated, is far more important economically than the marine fishery. The assumption here is that the fishing permit price in a reasonably good river is about 200 NOK per day (see also Olaussen and Liu 2011). Based on average catch success, this permit price may translate into fishing prices in the range of $100-400(\mathrm{NOK} / \mathrm{kg})$, or even higher. We assume the same price for old and young and use $p_{4}=p_{5}=150(\mathrm{NOK} / \mathrm{kg})$. We then have $p_{5} w_{5} / \gamma_{5}=343.8>p_{4} w_{4} / \gamma_{4}=300$ (NOK/fish). Table X. 1 summarizes the baseline parameter values.

Table X. 1 about here

Because the weight-fertility value ratio is highest for the 2SW population, the economic yield maximizing fishing mortality will be highest for the old adult population (Proposition 1).

Table X. 2 (first row) demonstrates where case i) with $f_{5}^{*}=1$ and $f_{4}^{*}=0.31$ yields the optimal fishing mortality. Reducing the gain-loss ratio of the young mature subpopulation by lowering $p_{4}$ while keeping all other parameters at their baseline values, and hence the discrepancy between $p_{5} w_{5} / \gamma_{5}$ and $p_{4} w_{4} / \gamma_{4}$ increases, leads to the optimal fishing policy described by case ii), with no harvesting of young fish (row two). Row three indicates what happens when the natural survival rate of the young $s$ is reduced while all other parameters are kept at their baseline values. Such a reduction may be the result of infection through transmission of lice from farmed salmon. Indeed, as indicated in the introductory section, this is considered to be one of the most important threats to the wild Atlantic salmon (see, e.g., Verspoor et al. 2003). A $40 \%$ reduction yields quite dramatic effects. The spawning biomass
declines significantly and the profit is reduced by more than $50 \%$. Again, case i) with harvesting of the entire old adult population represents the optimal fishing policy.

The last row in Table X. 2 finally illustrates the optimal non-selective and uniform fishing pattern under the baseline parameter values scenario and where the marginal fishing value (marginal gain) plays no role (Proposition 2).We find the fishing mortality to be 0.70 and the fish abundance reduces somewhat to the baseline selective scheme (first row). As expected the profit is lower, but the reduction is quite insignificant. The changes in the size of the harvestable population $N_{3}^{*}$ and spawning population $B^{*}$ are also quite modest. However, when case ii) with $f_{5}^{*}=1$ and $f_{4}^{*}=0$ represents the optimal scheme due to increased weightfecundity discrepancy (row two), the differences in the spawning stock becomes more profound as the uniform fishing mortality still be 0.70 in the uniform case as changes in fishing prices play no role here.

Table X. 2 about here

## X.6. Concluding remarks

In this chapter, we have from a theoretical point of view, studied the maximum sustainable yield management of an age-structured wild Atlantic salmon (Salmo salar) population with two spawning and harvestable classes. Under the assumption of perfect fishing selectivity, the basic finding is that the weight-fecundity ratio discrepancy between the harvestable classes determines the optimal fishing mortality and the fishing composition, and no other factors play a direct role. This is stated as Proposition 1. Our analysis and findings are based on the Atlantic salmon, but the results will also apply to, e.g., the various Pacific salmon stocks, which also die after spawning.

The model is also studied fishing under imperfect selectivity and similar fishing mortalities in the harvest. We find here that the marginal fishing value (gain) has no influence on the optimal fishing pattern. This is stated as Proposition 2. Only the marginal loss (fertility) counts, together with survival and composition of the 1SW and 2SW stock counts. The uniform fishing pattern yields lower profit that under the perfect selectivity pattern. In the numerical illustration this loss is quite small. However, as also demonstrated, the differences in the size of the spawning stock and harvestable stock size may be quite significant.

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| PARAMETER | DESCRIPTION | VALUE |
| :--- | :--- | :--- |
| S | Natural survival rate <br> young | 0.05 |
| $s_{3}$ | Natural survival rate <br> young adult | 0.5 |
| $s_{4}$ | Natural survival rate old <br> adult | 0.5 |
| $r$ | Intrinsic growth rate <br> recruitment function | 400 (\# of recruits/ <br> fertility adjusted <br> spawner) |
| $K$ | Scaling parameter <br> recruitment function | 100 (\# of spawners) |
| $\sigma$ | Migration parameter | 0.5 |
| $w_{4}$ | Weight young adult | 2.0 (kg/fish) |
| $w_{5}$ | Weight old adult | 5.5 (kg/fish) |
| $\gamma_{4}$ | Fecundity parameter |  |
| young adult | 1.0 |  |
| $\gamma_{5}$ | Fecundity parameter old <br> adult | 2.4 |
| $p_{5}$ | Fish price young adult | 150 (NOK/ kg) |

Table X.1. Biological and economic baseline parameter values

|  | $f_{4}^{*}$ | $f_{5}^{*}$ | $N_{3}(\#)$ | $B(\#)$ | $H_{4}^{*}(\#)$ | $H_{5}^{*}(\#)$ | $\pi^{*}(1000$ <br> NOK) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Baseline values | 0.31 | 1.00 | 1,420 | 245 | 110 | 178 | 179 |
| 200\% reduction <br> price young adult <br> $\left(p_{4}=50\right)$ | 0.00 | 1.00 | 1,600 | 400 | 0 | 200 | 165 |
| $40 \%$ reduction <br> natural survival <br> rate young <br> $(s=0.03)$ | 0.11 | 1.00 | 751 | 167 | 21 | 94 | 84 |
| Uniform fishing. <br> Baseline values | 0.70 | 0.70 | 1397 | 232 | 244 | 122 | 173 |

Table X.2: Maximum economic yield


Figure X.1: Internal equilibrium for fixed fishing motrtalities $0 \leq f_{4} \leq 1$ and $0 \leq f_{5} \leq 1$ (but not $f_{4}=f_{5}=1$ ). Beverton-Holt type recruitment function

