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Evaluating and Investing in Equity Mutual Funds

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Evaluating and Investing in Equity Mutual Funds

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Abstract

Our framework for evaluating and investing in mutual funds combines observed returns on funds and passive assets with prior beliefs that distinguish pricing-model inaccuracy from managerial skill. A fund's "alpha" is defined using passive benchmarks. We show that returns on non-benchmark passive assets help estimate that alpha more precisely for most funds. The resulting estimates generally vary less than standard estimates across alternative benchmark specifications. Optimal portfolios constructed from a large universe of equity funds can include actively managed funds even when managerial skill is precluded. The fund universe offers no close substitutes for the Fama-French and momentum benchmarks.

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1. Introduction

Evaluating mutual fund performance combines data and judgment. The data typically consist of returns on the funds and one or more benchmarks, and the judgment typically involves specifying the benchmarks and their role in defining performance. Investing in mutual funds also combines data and judgment. Rather than accept the standard performance measures at face value, investors can allow their decisions to reflect doubts about either the adequacy of the benchmarks or the stock-picking ability of fund managers.

In this study, we develop and implement a framework in which views about the adequacy of the benchmarks—their ability to price other passive assets—can be incorporated formally into both performance evaluation and the investment decision. The framework also allows separate beliefs about potential managerial skill to enter the investment decision. To accomplish these tasks, we introduce passive "non-benchmark" assets that are not used in previous approaches. These assets provide information that can be used to estimate a fund's performance more precisely. They also allow beliefs that distinguish benchmark inadequacy from managerial skill, and they help account for common variation in returns across funds that is not captured by the benchmarks.

A mutual fund's performance is often measured by alpha, the intercept in a regression of the fund's return on one or more passive benchmark returns.[1] The choice of benchmarks is often guided by a pricing model, as in Jensen's (1969) pioneering use of the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) to investigate mutual fund alphas relative to a single market-index benchmark. Other studies, beginning with Lehmann and Modest (1987), examine fund alphas with respect to a set of multiple benchmarks viewed as the relevant factors for pricing in a multifactor model, such as the Arbitrage Pricing Theory of Ross (1976).

Alpha is typically computed by ordinary-least-squares (OLS) estimation of the regression

$$r_{A,t} = \alpha_A + \beta_A' r_{B,t} + \epsilon_{A,t}, \tag{1}$$

where $r_{A,t}$ is the fund's return in month t, $r_{B,t}$ is a $k \times 1$ vector containing the benchmark returns, and α_A denotes the fund's alpha. This standard approach ignores information about alpha provided by returns on other non-benchmark passive assets. Even though such assets play no role in the *definition* of alpha in (1), they can play a useful role in its *estimation*.

[1] Throughout our discussion, "returns" are rates of return in excess of a riskless interest rate, or they are payoffs on zero-investment spread positions.

To take one example, assume that the benchmark assets do indeed price other passive assets. Consider the regression of a non-benchmark return $r_{n,t}$ on the benchmark returns,

$$r_{n,t} = \alpha_n + \beta_n' r_{B,t} + \epsilon_{n,t}, \tag{2}$$

where the correlation between $\epsilon_{A,t}$ and $\epsilon_{n,t}$ is positive. If the benchmarks price other passive assets, then $\alpha_n = 0$. Now suppose that over the same sample period used to obtain the OLS estimate of α_A, the OLS estimate of α_n is less than its true value of zero. Given this outcome and the positive correlation between $\epsilon_{A,t}$ and $\epsilon_{n,t}$, the OLS estimate of α_A is expected to be less than its true value as well, and this information can be used in estimating α_A.

Pricing models often motivate the choice of benchmarks, but non-benchmark assets can provide additional information about a mutual fund's alpha even with no assumption about the benchmarks' pricing ability. The explosive growth of the mutual fund industry in recent years presents investors with many funds that have relatively short histories. Consider a fund whose available return history is shorter than the histories of $r_{n,t}$ and $r_{B,t}$. Suppose that the OLS estimate of α_n computed for the sample period of the fund's available history is less than the OLS estimate of α_n computed for a longer sample period. Since the latter estimate is more precise, the first estimate is more likely to be less than the true (unknown) value of α_n. Given the positive correlation between $\epsilon_{A,t}$ and $\epsilon_{n,t}$, the same can be said of the OLS estimate of α_A relative to its true value, and this information can be used in estimating α_A. The additional information comes not through a pricing model, as in the first example, but through the longer histories of the passive asset returns.

In the two examples described above, α_n is assumed to be either zero or completely unknown. One may well prefer an intermediate version in which the benchmarks are believed to be relevant for pricing other passive assets, but not without error. In such a case, which we handle in a Bayesian framework, non-benchmark assets can play a role that combines aspects of both examples. Additional information about α_A is provided by the extent to which the short-history estimate of α_n differs from zero as well as from its long-history estimate.

Our study does not recommend a particular set of benchmarks for defining alpha. Recent academic studies compute mutual fund alphas with respect to a single market benchmark (e.g., Malkiel (1995)) as well as sets of multiple benchmarks (e.g., Carhart (1997) and Elton, Gruber, and Blake (1996)). We compute alphas in both single-benchmark and multiple-benchmark settings. Alphas defined with respect to a single market benchmark may be of interest whether or not one believes in the CAPM. We offer just two of many examples of their use in practice: Morningstar, the leading provider of mutual fund information, reports alphas computed with respect to one of several broad market indexes; Capital Resource

Advisors, one of the largest providers of performance information to institutional clients, reports alphas computed with respect to the S&P 500 Index. Our approach allows one to estimate alpha under various assumptions about whether the benchmarks that define alpha price other passive investments.

We investigate the performance of a large sample of equity mutual funds and find that the information about a fund's alpha provided by non-benchmark returns can be substantial, especially for certain types of funds. Suppose, for example, that one has no confidence in the CAPM's pricing ability but nevertheless wishes to report a small-company growth fund's traditional alpha defined with respect to a single market benchmark. The absolute difference between the OLS estimate and an alternative estimate that incorporates information in non-benchmark returns has a median value across such funds of 8.3% per annum. If instead one has complete confidence in the CAPM's pricing ability, then the median absolute difference in estimates is 7.2%. In both cases the alternative estimate is more precise, and its variance is only about one-third that of the OLS estimate for the median small-company growth fund.

A number of studies observe that OLS estimates of mutual fund alphas are sensitive to the specification of the benchmarks that define those alphas.[2] When the estimation of a fund's alpha incorporates non-benchmark assets, the definition of alpha typically becomes less important and, in some cases, even irrelevant. We estimate alphas defined with respect to the CAPM and with respect to the three Fama and French (1993) benchmark factors, which include size and value factors in addition to the market factor. When estimated using OLS, the median difference in alphas between the two models is 2.3% per annum for all funds and 8.1% for small-company growth funds. When the estimation incorporates non-benchmark assets but does not rely on the benchmarks to price them, those values fall to 1.2% and 2.0%. If the benchmarks are instead assumed to price the non-benchmarks exactly, the estimates of a fund's alpha are identical under the two models, even though the definitions of the alphas differ. In general, if alphas are defined with respect to different benchmarks but estimated using the same set of passive assets (benchmark and non-benchmark), then the estimates are identical if in each case the benchmarks are assumed to price the non-benchmark assets exactly. Loosely speaking, if you believe that *some* pricing model holds exactly and want a fund's alpha with respect to it, you need not identify the model.

As in numerous previous studies, we find that estimated alphas for the majority of equity mutual funds are negative.[3] For each investment objective and each age group, we find a

[2]An early example is the study by Lehmann and Modest (1987).

[3]Grinblatt and Titman (1995) review the literature on mutual fund performance.

posterior probability near 100% that the average of the funds' CAPM alphas is negative when the non-benchmark assets are excluded. Alphas for most funds remain negative when defined with respect to multiple benchmarks as well as when the information in the non-benchmark assets is used the estimation.

Non-benchmark assets permit a framework for defining a measure of managerial skill when a pricing model is viewed as possibly flawed but not useless. A common interpretation of alpha is that it represents the skill of the fund's manager in selecting mispriced securities. That interpretation is subject to a number of potential complications, including a concern that the benchmarks used to define alpha might not price all passive investments.[4] This concern can be addressed by defining skill with respect to an expanded set of passive assets, thus weakening the link to a pricing model. Preserving some role for a pricing model can still be useful to an investor, in that a somewhat inaccurate pricing model can be of some help in identifying optimal portfolios. We allow an investor to have prior beliefs about a skill measure defined as the intercept in a regression of the fund's return on an expanded set of passive assets that includes both non-benchmark assets as well as the benchmarks relevant to a particular pricing model. At the same time, we allow the investor to have prior beliefs about the potential mispricing of the non-benchmark assets with respect to the benchmarks. In other words, an investor can have prior beliefs that distinguish managerial skill from pricing-model inaccuracy.

Performance evaluation is a topic of long-standing interest in the academic literature, but few if any studies have pursued its obvious practical motivation: constructing a portfolio of mutual funds. We compute portfolios having the maximum Sharpe ratio constructed from an investment universe of over 500 no-load equity funds. Optimal portfolios are obtained by combining the information in historical returns with an investor's prior beliefs, accounting for parameter uncertainty. We entertain priors representing a range of beliefs about both managerial skill as well as the accuracy of each of three pricing models: the CAPM, the three-factor Fama-French model, and the four-factor model of Carhart (1997). The last model supplements the three Fama-French benchmarks with a "momentum" factor, the current month's difference in returns between the previous year's best- and worst-performing stocks. Returns on passive benchmark and non-benchmark assets are used in the modeling and estimation, but only the mutual funds are assumed to be eligible for investment. The compositions of the optimal portfolios are influenced substantially by prior beliefs about both managerial skill and pricing models. We also find that a "hot-hand" portfolio of the

[4] Other well-known complications include the possibility that the manager uses information to change the value of β_A through time as conditional expected returns on the benchmarks fluctuate.

previous year's best-performing mutual funds does not enter the optimal portfolio under any set of prior beliefs considered, even if the investor has complete confidence in the four-factor model, which contains the momentum factor. Real estate funds, interestingly, tend to exhibit positive sensitivities to that factor as well as to the three Fama-French factors, so those funds occupy much of the optimal portfolio for such an investor.

We find that when the passive benchmarks used to define α_A are not available for investment, there need not exist close substitutes for them in the universe of mutual funds. For an investor who believes completely in the accuracy of the Fama-French model and precludes managerial skill, the perceived maximum Sharpe ratio is only 66 percent of what could be achieved by direct investment in that model's benchmarks. For a believer in the Carhart four-factor model, the corresponding value is 54 percent. Interestingly, actively managed funds can be better substitutes for the benchmarks than existing passive funds, so active funds can be selected even by investors who admit no possibility of managerial skill.

Our investment problem is related to the recent study by Baks, Metrick, and Wachter (2000), who also consider investment in mutual funds under prior beliefs about manager skill. Their investment universe includes mutual funds as well as the passive benchmarks used to define α_A, and they do not include beliefs about possible pricing-model errors. In their setting, an investor buys an actively managed fund only if it appears to have a positive α_A. Those authors do not construct optimal portfolios, but they conclude that unless one is extremely skeptical about the possibility of managerial skill, some actively managed funds would be selected by an investor who wants a high overall Sharpe ratio. Our investment universe differs from theirs in that we do not assume the benchmark returns can be earned costlessly. Rather than attempt to construct after-cost versions of those returns, we simply confine investors to the universe of mutual funds.

Section 2 discusses the econometric issues and reports results from computing alternative estimates of alpha for 2,609 equity mutual funds. Section 3 presents the results of the investment problem, and Section 4 briefly reviews our conclusions.

2. Estimating alpha

This section begins with some basic concepts underlying the use of non-benchmark assets. We then describe the construction of our data, explain our methodology, and report results based on a large sample of equity mutual funds. In this section, dealing with estimation, the prior beliefs about α_A are "diffuse," or completely non-informative. In the spirit of much of the previous literature, the fund's track record determines its estimated alpha without any adjustment for what one might think to be reasonable magnitudes for that parameter. In the next section, dealing with an investment problem, we consider the effects of informative prior beliefs about managerial skill.

2.1. The role of non-benchmark assets

Let $r_{N,t}$ denote the $m \times 1$ vector of returns in month t on m non-benchmark passive assets, so the multivariate version of the regression in (2) is written as

$$r_{N,t} = \alpha_N + B_N r_{B,t} + \epsilon_{N,t}, \tag{3}$$

where the variance-covariance matrix of $\epsilon_{N,t}$ is denoted by Σ. Let σ_ϵ^2 denote the variance of the disturbance $\epsilon_{A,t}$ in (1). Also define the regression of the fund's return on all $p\,(= m + k)$ passive assets,

$$r_{A,t} = \delta_A + c'_{AN} r_{N,t} + c'_{AB} r_{B,t} + u_{A,t}, \tag{4}$$

where the variance of $u_{A,t}$ is denoted by σ_u^2. All regression disturbances are assumed to be normally distributed, independently and identically across t. A key to understanding the role of non-benchmark assets is given by the equality

$$\alpha_A = \delta_A + c'_{AN} \alpha_N, \tag{5}$$

which follows by taking expectations in (1) and (4) and applying the relation

$$\beta_A = B'_N c_{AN} + c_{AB}, \tag{6}$$

which is easily verified using standard regression theory.

Intuition for how non-benchmark assets can help to estimate α_A in (1) is developed most easily if the second-moment parameters β_A, c_{AN}, and c_{AB} are viewed as known. Assume the fund's history contains S observations, and define estimators of the intercepts in (1), (3), and (4) as

$$\bar{\alpha}_A = (1/S) \sum_{t=1}^{S} (r_{A,t} - \beta'_A r_{B,t}), \tag{7}$$

$$\bar{\alpha}_N = (1/S) \sum_{t=1}^{S} (r_{N,t} - B_N r_{B,t}), \tag{8}$$

and

$$\bar{\delta}_A = (1/S)\sum_{t=1}^{S}(r_{A,t} - c'_{AN}r_{N,t} - c'_{AB}r_{B,t}). \tag{9}$$

Note using (6) that $\bar{\alpha}_A$ is also equal to the result from substituting $\bar{\delta}_A$ and $\bar{\alpha}_N$ into the right-hand side of (5):

$$\begin{aligned}\bar{\delta}_A + c'_{AN}\bar{\alpha}_N &= (1/S)\sum_{t=1}^{S}(r_{A,t} - (B'_N c_{AN} + c_{AB})' r_{B,t}) \\ &= \bar{\alpha}_A. \end{aligned} \tag{10}$$

Suppose first that the benchmarks have no assumed pricing ability. Then α_N is a vector of unknown parameters, but it can be estimated more precisely than in (8) if the available history of $r_{N,t}$ and $r_{B,t}$ is longer than the S observations in the fund's history. Substituting $\bar{\alpha}_N$ and $\bar{\delta}_A$ into the right-hand side of (5) gives $\bar{\alpha}_A$ as an estimator of α_A. Substituting the more precise estimator of α_N (along with $\bar{\delta}_A$) produces a more precise estimator of α_A, since $\bar{\delta}_A$ is uncorrelated with either estimator of α_N. Suppose instead that the benchmarks are assumed to price the non-benchmark assets exactly, so $\alpha_N = 0$ and thus $\alpha_A = \delta_A$. Then both $\bar{\alpha}_A$ and $\bar{\delta}_A$ are unbiased estimators of α_A, but the sampling variance of $\bar{\delta}_A$, σ_u^2/S, is less than or equal to the sampling variance of $\bar{\alpha}_A$, σ_ϵ^2/S. In this case, the non-benchmark asset returns explain additional variance of the fund's return and thereby provide a more precise estimator of its alpha.

The basic idea is that a more precise estimator of α_A is obtained by evaluating the right-hand side of (5) at $\bar{\delta}_A$ and a more precise estimator of α_N than $\bar{\alpha}_N$. That more precise estimator of α_N can be obtained by using a sample period longer than that available for the fund, as in the case where the benchmarks are not assumed to have any pricing ability, or by simply setting $\alpha_N = 0$, as in the case where the benchmarks are assumed to price the non-benchmark assets perfectly. When $\epsilon_{A,t}$ is correlated with the elements of $\epsilon_{N,t}$ (i.e. when $c_{AN} \neq 0$), then the difference between $\bar{\alpha}_N$ and a more precise estimator of α_N supplies information about the likely difference between $\bar{\alpha}_A$ and α_A. When the more precise estimator of α_N relies on a longer history, the additional information about α_A is provided in essentially the same way that sample means of long-history assets provide information about expected returns on short-history assets, as in Stambaugh (1997).

Much of the intuition developed when β_A, c_{AN}, and c_{AB} are known extends to the actual setting in which those parameters must be estimated. Equation (5) also holds when all quantities are replaced by OLS estimators based on the sample of S observations. That is,

$$\hat{\alpha}_A = \hat{\delta}_A + \hat{c}'_{AN}\hat{\alpha}_N, \tag{11}$$

where $\hat{\alpha}_A$, $\hat{\alpha}_N$, and $\hat{\delta}_A$ are the OLS estimates of the intercepts in (1), (3), and (4), respectively, and $\hat{c}_{AN}$ is the OLS estimate in (4). As before, the information in non-benchmark assets is incorporated by replacing $\hat{\alpha}_N$ with a more precise estimator based either on a longer history or some degree of belief in a pricing model. When all parameters are unknown, substituting a more precise estimator of α_N can in some cases produce an estimator of α_A that is less precise than the usual estimate of the fund's alpha, $\hat{\alpha}_A$. For example, if one assumes that $\alpha_N = 0$ and substitutes that value into (11) in place of $\hat{\alpha}_N$, the resulting alternative estimator of α_A is simply $\hat{\delta}_A$. The mean of $\hat{\delta}_A$ is α_A, but the variance of $\hat{\delta}_A$ can exceed that of $\hat{\alpha}_A$. Since c_{AN} must be estimated and $\hat{\delta}_A$ and the elements of $\hat{c}_{AN}$ are correlated, replacing $\hat{\alpha}_N$ with a lower-variance quantity need not lower the variance of $\hat{\alpha}_A$. Such an outcome is most likely to occur as the number of non-benchmark assets increases without a sufficient increase in the R-squared in (4). In essence, the degrees-of-freedom effect can outweigh the additional explanatory power. We use between five and seven non-benchmark assets, depending on the number of benchmarks, and we find that the information provided by those assets produces a more precise estimate of α_A for most funds in our sample. In the Bayesian framework explained below, we also apply a moderate degree of shrinkage to the slope coefficients in (4) to increase their precision and thereby enhance the information provided by the non-benchmark assets. A potential direction for future research is the use of higher frequency data to increase the precision of the slope coefficients.

Suppose two researchers agree on an overall set of p passive assets to include when *estimating* α_A, but they disagree about the subset of those passive assets to designate as benchmarks for *defining* α_A. Their chosen benchmark subsets might not even have any members in common. Moreover, suppose each researcher believes his benchmarks price the remaining passive assets perfectly. Then those researchers' estimates of α_A will be identical, even though their definitions of α_A are not. That is, the definition of α_A is irrelevant to its estimation if, for whatever benchmarks might be designated for defining α_A, the remaining non-benchmark assets would be assumed to be priced exactly by those benchmarks. Perhaps ironically, if the benchmarks are not assumed to have perfect pricing ability, their designation becomes relevant not only for defining α_A but also for estimating it.

To understand the above statements, consider first the maximum-likelihood estimator (MLE) of α_A under the restriction that $\alpha_N = 0$. That estimator is given by $\hat{\delta}_A$, the OLS estimator of the intercept in (4), which does not depend on which of the p assets are designated as the benchmarks. Note that the disturbances $\epsilon_{N,t}$ and $u_{A,t}$ are by construction uncorrelated and, given the normality assumption, independent. The likelihood function can therefore be expressed as a product of two factors, one for each regression. The restriction on α_N does not affect the MLE of δ_A, which is $\hat{\delta}_A$, since α_N appears in the other factor. Substituting δ_A along with the restricted MLE of α_N (the zero vector) into the functional relation in (5) gives $\hat{\delta}_A$ as the MLE of α_A as well. It can also be verified that $\hat{\delta}_A$ arises as the restricted estimator in a seemingly-unrelated-regression model, or SURM.[5] That is, let regressions (1) and (3) jointly constitute a SURM, and consider the estimation of the model subject to the restriction $\alpha_N = 0$. The restricted coefficient estimator requires the unknown joint covariance matrix of $(\epsilon_{A,t}\ \epsilon'_{N,t})$. If that matrix is replaced by the sample covariance matrix of the residuals from the first-pass unrestricted OLS estimation, the resulting "feasible" restricted SURM estimator of α_A is again simply $\hat{\delta}_A$. With no restriction on α_N, then of course both the MLE and SURM estimator of α_A is simply the usual estimator $\hat{\alpha}_A$. When shrinkage is applied to the slope coefficients in (4), as in the Bayesian setting described below, the same type of result obtains. That is, the assumption $\alpha_N = 0$ implies that the posterior mean of α_A is equal to the posterior mean of δ_A, which doesn't depend on the designation of the benchmarks.

2.2. Data

Monthly returns on the benchmark and non-benchmark passive assets are constructed for the $35\frac{1}{2}$-year period from July 1963 through December 1998. The sample period for any given fund, typically much shorter, is a subset of that overall period. We specify up to three benchmark series, consisting of the three factors constructed by Fama and French (1993), updated through December 1998.[6] The first of these, MKT, is the excess return on a broad market index. The other two factors, SMB and HML, are payoffs on long-short spreads constructed by sorting stocks according to market capitalization and book-to-market ratio. We estimate "Fama-French" alphas, defined with respect to all three benchmarks, as well as "CAPM" alphas, defined with respect to just MKT.

When estimating CAPM alphas, SMB and HML become two of the non-benchmark series. Five additional non-benchmark series are used in the estimation of both CAPM and Fama-French alphas. The first of these, denoted as CMS, is the payoff on a characteristic-matched spread in which the long position contains stocks with low HML betas (in a multiple regression including MKT and SMB) and the short position contains stocks with high HML betas. The long and short positions are matched in terms of market capitalization and book-

[5]Zellner (1962) introduces methods for estimating seemingly unrelated regressions. For a textbook treatment, including estimation under linear restrictions, see Theil (1971).

[6]We are grateful to Ken French for supplying these data.

to-market ratio, and the overall spread position is formed from a set of triple-sorted equity portfolios constructed as in Pástor and Stambaugh (2000), who closely follow the procedures of Daniel and Titman (1997) and Davis, Fama, and French (2000). At the end of June of each year s, all NYSE, AMEX, and NASDAQ stocks in the intersection of the CRSP and Compustat files are sorted and assigned to three size categories and, independently, to three book-to-market categories. The nine groups formed by the intersection can be denoted by two letters, designating increasing values of size (S, M, B) and book-to-market (L, M, H). We then construct beta spreads within the four extreme groups of size and book-to-market: SL, SH, BL, and BH. The stocks within each group are sorted by their HML betas and assigned to one of three value-weighted portfolios.[7] A spread within each group is constructed each month (from July of year s through June of year $s+1$) by going long \$1 of the low-beta portfolio and short \$1 of the high-beta portfolio, and the value of CMS in month t is the equally weighted average of the four spread payoffs in month t.

The second non-benchmark series, denoted as MOM, is the "momentum" factor constructed by Carhart (1997). At the end of each month $t-1$, all stocks in the CRSP file with return histories back to at least month $t-12$ are ranked by their cumulative returns over months $t-12$ through $t-2$. The value of MOM in month t is the payoff on a spread consisting of a \$1 long position in an equally weighted portfolio of the top 30% of the stocks in that ranking and a corresponding \$1 short position in the bottom 30%. This factor is included as a fourth benchmark in some of the analysis in the next section, dealing with investment, but this section confines the estimation of alphas to those based on the CAPM and the three-factor Fama-French model.

The remaining three non-benchmark assets, whose returns are denoted as IP1, IP2, and IP3, are portfolios constructed from a universe of 20 value-weighted industry portfolios formed using the same classification scheme as Moskowitz and Grinblatt (1999). The three portfolios mimic the first three principal components of the disturbances in multiple regressions of the 20 industry returns on the other passive returns: MKT, SMB, HML, CMS, and MOM. The vector of weights for IP1 is proportional to the eigenvector for the largest eigenvalue of the sample covariance matrix of the residuals in those regressions, and the other two portfolios are similarly formed using eigenvectors for the second and third eigenvalues.

[7] Using up to 60 months of data through December of year $s-1$, the "pre-formation" HML betas are computed in a regression of the stock's excess returns on "fixed-weight" versions of the FF factors, which hold the weights on the constituent stocks constant at their June-end values of year s. Using the fixed-weight factors, as suggested by Daniel and Titman (1997), increases the dispersion in the "post-formation" betas of the resulting portfolios.

The specification of non-benchmark assets must be somewhat arbitrary, but our selection of the five described above is based on several considerations. Recall that non-benchmark assets supply information about α_A, the fund's alpha, when they explain additional variance of the fund's returns, i.e. when $c_{AN} \neq 0$. Also, except when the benchmarks are assumed to price the non-benchmark assets perfectly, the latter assets also provide information about α_A when they are mispriced by the benchmarks, i.e. when $\alpha_N \neq 0$. Our inclusion of the three industry portfolios is motivated primarily by the first consideration, explaining variance. Although we don't dismiss the possibility of their being mispriced, those portfolios are constructed to capture the most important sources of industry-related variation that is not accounted for by the other passive assets. On the other hand, our inclusion of CMS and MOM is motivated chiefly by the second consideration, mispricing. Evidence in other studies indicates that those spread positions may not be priced completely by the three benchmark factors, MKT, SMB, and HML. For example, Daniel and Titman (1997) conclude that, during the post-1963 period, characteristic-matched spreads in HML beta produce nonzero alphas with respect to the Fama-French three-factor model.[8] Fama and French (1996) report a large three-factor alpha for the momentum strategy of Jegadeesh and Titman (1993). Of course, to be useful in estimating α_A, CMS and MOM also have to explain additional variance of the fund's returns, and we find that to be the case for many funds.

Parsimony is a consideration limiting our number of non-benchmark assets to five. As discussed earlier, the degrees-of-freedom effect in finite samples argues against indiscriminately specifying a large number of non-benchmark assets. One might instead include a larger number of the characteristic-matched spreads, say one for each size/book-to-market subgroup, or include all 20 industry portfolios instead of constructing the smaller set of three. We tried such alternatives and found that they quite often produce estimates of α_A similar to those obtained using the smaller set of five, but the precision of the estimates based on the larger set is lower. The OLS estimators of δ_A and c_{AN} are undefined, or essentially infinitely imprecise, when the total number of passive assets exceeds the length of the fund's history. The shrinkage estimator (explained below) can still be computed in that case, but it often yields a less precise inference than when fewer non-benchmark assets are used. It is likely that future research could refine the selection of non-benchmark assets and further increase the precision of estimated alphas. For example, a different set of non-benchmark assets could be specified for each fund, so that the assets have a high correlation with the specific fund at hand. A larger number of non-benchmark assets could be used for a fund with a longer history, since the degrees-of-freedom problem is then less severe. Our specification of

[8] Davis, Fama, and French (2000) find that a hypothesis of zero mispricing for such spreads cannot be rejected within the longer 1929–97 period.

non-benchmark assets, motivated chiefly by simplicity, probably understates the potential gains from using non-benchmark assets to help estimate fund performance.

The mutual-fund data come from the 1998 CRSP Survivor Bias Free Mutual Fund Database.[9] Our sample contains 2,609 domestic equity mutual funds.[10] We exclude multiple share classes for the same fund as well as funds with only a year or less of available returns. Funds are assigned to one of seven broad investment objectives, using information that the CRSP database provides about classifications by Wiesenberger, ICDI, and Strategic Insight. Table 1 lists the number of funds in each objective, further classifying funds within each objective by the number of months in the fund's available return history. For each fund we compute the monthly return in excess of that on a one-month Treasury bill.

2.3. The Bayesian framework

We compute the posterior mean of α_A by computing posterior means for the parameters of the regressions in (3) and (4). Recall from an earlier discussion that the likelihood function for each fund can be expressed as a product of two factors, one for each regression. We assume that the disturbances in (4) are uncorrelated across funds, which implies that the likelihood functions across funds are independent. Non-benchmark assets thus play yet another role, in that they account for covariance in fund returns that is not captured fully by the benchmarks. We also specify prior beliefs in which the parameters of the regression in (3) are independent across funds as well as independent of the parameters of (4). Given the latter property, the posterior distribution of α_N is independent of δ_A and c_{AN}, so the posterior mean of α_A is obtained simply by evaluating the right-hand side of (5) at the posterior means of α_N, δ_A and c_{AN}. The independence of the prior and the likelihood across funds allows us to conduct the analysis fund by fund. We also examine the posterior standard deviations of α_A, which can be computed using the posterior first and second moments of α_N, δ_A and c_{AN}. Derivations of the posterior moments are provided in the Appendix. The specification of prior beliefs is discussed below.

First consider the parameters of the regression in (3). The prior distribution for Σ, the covariance matrix of $\epsilon_{N,t}$, is specified as inverted Wishart,

$$\Sigma^{-1} \sim W(H^{-1}, \nu). \tag{12}$$

[9]CRSP, Center for Research in Security Prices, Graduate School of Business, The University of Chicago 1999, crsp.com. Used with permission. All rights reserved.

[10]We are grateful to Thomas Knox and the authors of Baks, Metrick, and Wachter (2000) for providing us with a number of corrections to the CRSP Mutual Fund Database.

We set the degrees of freedom $\nu = m + 3$, so that the prior contains very little information about Σ. From the properties of the inverted Wishart distribution (e.g., Anderson (1984)), the prior expectation of Σ equals $H/(\nu - m - 1)$. We specify $H = s^2(\nu - m - 1)I_m$, so that $\mathrm{E}(\Sigma) = s^2 I_m$. Following an "empirical Bayes" approach, the value of s^2 is set equal to the average of the diagonal elements of the sample estimate of Σ obtained using OLS. Conditional on Σ, the prior for α_N is specified as a normal distribution,

$$\alpha_N | \Sigma \sim N(0, \sigma^2_{\alpha_N}(\frac{1}{s^2}\Sigma)). \tag{13}$$

Pástor and Stambaugh (1999) introduce the same type of prior for a single element of α_N, and Pástor (2000) and Pástor and Stambaugh (2000) apply the multivariate version in (13) to portfolio-choice problems. Having the conditional prior covariance matrix of α_N be proportional to Σ is motivated by the recognition that there exist portfolios of the passive assets with high Sharpe ratios if the elements of α_N are large when the elements of Σ are small. Such combinations receive lower prior probabilities under (13) than when each element of α_N has standard deviation σ_{α_N} but is distributed independently of Σ. The prior distribution for B_N is diffuse and independent of α_N and Σ.

Our earlier discussion focuses on the cases in which the benchmarks' ability to price the non-benchmark assets is assumed to be either perfect or nonexistent. That is, either α_N is set to the zero vector or the prior beliefs about α_N are diffuse. These two cases represent the opposite extremes on a continuum characterized by σ_{α_N}, the marginal prior standard deviation of each element in α_N. Specifying $\sigma_{\alpha_N} = 0$ is equivalent to setting $\alpha_N = 0$, corresponding to perfect confidence in the benchmarks' pricing ability. A diffuse prior for α_N corresponds to $\sigma_{\alpha_N} = \infty$. With a nonzero finite value of σ_{α_N}, prior beliefs are centered on the pricing restriction, but some degree of mispricing is entertained. We refer to σ_{α_N} as "mispricing uncertainty."

Next consider the parameters of the regression in (4). The prior for σ^2_u, the variance of $u_{A,t}$, is specified as inverted gamma, or

$$\sigma^2_u \sim \frac{\nu_0 s_0^2}{\chi^2_{\nu_0}}, \tag{14}$$

where $\chi^2_{\nu_0}$ denotes a chi-square variate with ν_0 degrees of freedom. Define $c_A = (c'_{AN}\ c'_{AB})'$. Conditional on σ^2_u, the priors for δ_A and c_A are specified as normal distributions, independent of each other:

$$\delta_A | \sigma^2_u \sim N(\delta_0,\ \left(\frac{\sigma^2_u}{\mathrm{E}(\sigma^2_u)}\right) \sigma^2_\delta), \tag{15}$$

and

$$c_A|\sigma_u^2 \sim N(c_0,\ \left(\frac{\sigma_u^2}{\mathrm{E}(\sigma_u^2)}\right)\Phi_c). \tag{16}$$

The marginal prior variance of δ_A is σ_δ^2, and the marginal prior covariance matrix of c_A is Φ_c. In this section, focused on estimation, we set $\sigma_\delta^2 = \infty$, which implies that the prior for α_A is diffuse and that δ_0 is irrelevant. In the next section, dealing with investment, we set σ_δ^2 to finite values and specify δ_0 to reflect the fund's costs. The conditional prior variance of δ_A is positively related to σ_u^2 for a reason similar to that given for the corresponding assumption in (13). If the variation in the fund's return is explained well by that of the benchmarks, so that σ_u^2 is low, then it is less likely that the fund's manager can achieve a large value for δ_A.

Values for s_0, ν_0, c_0, and Φ_c in (14) through (16) are specified using an empirical-Bayes procedure. The basic idea is that a given fund is viewed as a draw from a cross-section of funds with the same investment objective, so the prior uncertainty about a parameter for the fund is governed by the cross-sectional dispersion of that parameter. The empirical-Bayes procedure uses the data to infer those properties of the cross-section. The prior mean and covariance matrix of c_A, denoted by c_0 and Φ_c, are set equal to the corresponding sample cross-sectional moments of $\hat{c}_A$, the OLS estimate of c_A, for all funds with at least 60 months of data and the same investment objective as the fund at hand. (Recall that the investment objectives are displayed in Table 1.) Setting Φ_c equal to the sample covariance matrix of $\hat{c}_A$, without adjusting for the sampling variation in those estimates, overstates the dispersion across funds in the true values of c_A. In that sense, our use of this empirical-Bayes procedure is conservative, in that it applies an intentionally modest degree of shrinkage toward the cross-sectional mean of $\hat{c}_A$ when computing the posterior moments of c_A for a given fund. With a diffuse prior on c_A, or no shrinkage, the estimate (posterior mean) of c_A is simply the OLS value $\hat{c}_A$. The degree of shrinkage applied here, albeit conservative, gives a more precise estimate of c_A, especially for a short-history fund, and thereby allows the non-benchmark assets to reveal more of their information about the fund's alpha.

The inverted gamma prior density for σ_u^2 implies (e.g., Zellner (1971, p. 372)),

$$\mathrm{E}(\sigma_u^2) = \frac{\nu_0 s_0^2}{\nu_0 - 2}, \tag{17}$$

and

$$\nu_0 = 4 + \frac{2(\mathrm{E}(\sigma_u^2))^2}{\mathrm{Var}(\sigma_u^2)}. \tag{18}$$

We substitute the cross-sectional mean and variance of $\hat{\sigma}_u^2$ for $\mathrm{E}(\sigma_u^2)$ and $\mathrm{Var}(\sigma_u^2)$ in (17) and (18). The value of ν_0 is set to the next largest integer of the resulting value on the right-hand side of (18), and then that value of ν_0 implies the value of s_0^2 using (17). Here again, using the cross-sectional variance of $\hat{\sigma}_u^2$ without adjusting for sampling error produces a conservative amount of shrinkage toward the cross-sectional mean of $\hat{\sigma}_u^2$ for funds with the same objective.

Our framework assumes that funds' sensitivities to passive assets are constant over time. One way of relaxing this assumption is to model these coefficients as linear functions of state variables, as for example in Ferson and Schadt (1996). In such a modification, passive asset returns scaled by the state variables can be viewed as returns on additional passive assets (dynamic passive strategies), and the approach developed here could be extended to such a setting. Another approach to dealing with temporal variation in parameters could employ data on fund holdings. Daniel, Grinblatt, Titman, and Wermers (1997) and Wermers (2000), for example, use such data in characteristic-based studies of fund performance.

2.4. An alternative approach using GMM

The information in non-benchmark assets can also be incorporated in an estimate of the fund's alpha by using the generalized method of moments (GMM) of Hansen (1982). Let γ denote the vector of parameters corresponding to the elements of δ_A, c_{AN}, c_{AB}, α_N, and B_N. The estimate of γ can be obtained by minimizing $g(\gamma)'Wg(\gamma)$, where $g(\gamma)$ denotes the vector of $(1+m+k)+m(1+k)$ moment conditions

$$g(\gamma) \equiv \begin{pmatrix} \frac{1}{S}\sum_{t\in F}\left(r_{A,t}-\delta_A-c'_{AN}r_{N,t}-c'_{AB}r_{B,t}\right)\begin{pmatrix}1\\ r_{N,t}\\ r_{B,t}\end{pmatrix} \\ \text{vec}\left\{\frac{1}{T}\sum_{t=1}^{T}\left(r_{N,t}-\alpha_N-B_N r_{B,t}\right)\begin{pmatrix}1\\ r_{B,t}\end{pmatrix}'\right\}\end{pmatrix}, \tag{19}$$

and F denotes the subset of the periods $\{1,\ldots,T\}$ representing the fund's return history of length S. The first set of moment conditions in (19) corresponds to the regression in (4), and the second set corresponds to the regression in (3). The weighting matrix W is block diagonal, since the disturbance in (3) is uncorrelated with that in (4). GMM estimates of the fund's alpha can be constructed under two cases, corresponding to $\sigma_{\alpha_N}=\infty$ and $\sigma_{\alpha_N}=0$ in the Bayesian framework. With no restriction on α_N, the above moment conditions serve to exactly identify γ. Using the GMM estimate $\check{\gamma}$, the fund's estimated alpha can be computed as $\check{\alpha}_A=\check{\delta}_A+\check{c}'_{AN}\check{\alpha}_N$. With the pricing restriction $\alpha_N=0$, the second set of moment conditions can be dropped and the fund's alpha is estimated simply as $\check{\alpha}_A=\check{\delta}_A$.

The above GMM approach incorporates the information in non-benchmark assets in essentially the same manner as the Bayesian framework, but it permits a more general stochastic setting. The disturbances in (3) and (4) are not assumed to obey a specific distribution, and the weighting matrix W can be specified to allow non-i.i.d. behavior of those disturbances. Moreover, it is straightforward to modify the above moment conditions to allow the sensitivities (c_A, B_N) to be linear functions of state variables, as in Ferson and Schadt (1996), whereas incorporating such an extension in the likelihood-based Bayesian setting is more complicated. On the other hand, the Bayesian framework also offers some advantages. First, it permits flexible non-extreme beliefs about pricing and skill, i.e., values of σ_{α_N} and σ_δ other than zero or infinity. Second, it provides finite-sample inference about the funds' alphas. Third, it increases the precision of the estimates of c_A, particularly for short-history funds, by shrinking the elements of c_A to their cross-sectional averages. Finally, as demonstrated in the next section, the Bayesian framework permits the data to be analyzed in the context of mutual fund investment and accounts for parameter uncertainty in that decision problem.

2.5. Results

Table 2 reports medians, within various fund classifications, of CAPM alphas (Panel A) and Fama-French alphas (Panel B). The posterior mean of α_A, denoted as $\tilde{\alpha}_A$, is computed for σ_{α_N} equal to zero, two percent (annualized), and infinity. Recall that the usual OLS estimator, denoted as $\hat{\alpha}_A$, makes no use of non-benchmark assets. Also reported are median absolute differences between $\hat{\alpha}_A$ and $\tilde{\alpha}_A$. Not surprisingly, non-benchmark assets play a greater role in the estimation of CAPM alphas, since two of the non-benchmark assets in that case, SMB and HML, are already included as benchmarks when estimating Fama-French alphas. Across all funds, the median value of $|\hat{\alpha}_A - \tilde{\alpha}_A|$ is two percent per annum for CAPM alphas but about one percent or less, depending on σ_{α_N}, for Fama-French alphas. Note also that $|\hat{\alpha}_A - \tilde{\alpha}_A|$ is typically smaller for the funds with longer histories. With a longer history, $\hat{\alpha}_A$ becomes more precise, so the additional information in non-benchmark returns has a smaller impact.

The manner by which non-benchmark assets provide information is illustrated most dramatically in the case of CAPM alphas for small-company growth funds. For such funds, incorporating the information in non-benchmark assets typically makes a difference of between 7.2% and 8.3% per annum when estimating the CAPM alpha, depending on σ_{α_N}. Nearly half of those 413 funds have track records of three years or less (see Table 1), and the bulk of their track records fall toward the end of the overall period. In recent years, small-firm indexes have underperformed their CAPM predictions, which is relevant when $\sigma_{\alpha_N} = 0$, and they have also underperformed their long-run historical averages, which is relevant when $\sigma_{\alpha_N} = \infty$. (Both statements are relevant when $\sigma_{\alpha_N} = 2\%$.) Incorporating that information is accomplished in either case largely by including the size factor SMB as a non-benchmark asset.

An important issue in performance evaluation is whether the mutual fund industry adds value beyond standard passive benchmarks. We address this issue by computing posterior probabilities that average fund alphas within various fund classifications are negative. These probabilities are computed based on 100,000 draws of the average alpha from its posterior distribution. The probabilities are reported in Table 3, together with posterior means of average CAPM alphas (Panel A) and Fama-French alphas (Panel B). Some differences between the average alphas in Table 3 and the median alphas in Table 2 reflect skewness in the cross-sectional distribution of fund alphas. For example, the average of the OLS estimates of the CAPM alphas across all funds is -3.83%, compared to their median of -2.13%.

With few exceptions, Table 3 supports the inference that average fund alphas are negative. For example, for each investment objective and each age group, the average of the OLS estimates of the CAPM alphas is negative with 100% probability. The averages of the OLS estimates of the Fama-French alphas are mostly negative, although they are reliably positive for funds with histories longer than 10 years. When the non-benchmark assets are included, the average performance across all funds remains significantly negative, although the performance of long-history funds and aggressive growth funds improves with skeptical prior beliefs about pricing ($\sigma_{\alpha_N} = \infty$). The importance of beliefs about pricing can be illustrated by the average Fama-French alpha for small-cap growth funds. When the non-benchmark assets are not used, there is a 50% probability that the average alpha for those funds is negative. When the non-benchmark assets are included, the probability that the average alpha is negative rises to 100% when those assets are believed to be exactly priced by the benchmarks, but it drops to 9% when no pricing relation is used.

The alpha estimates in Tables 2 and 3 are generally higher for the funds with longer histories. These patterns could reflect survival effects, although similar age-related patterns emerge in the subsample of funds that existed at the end of 1998 (about 75% of our funds), and the median CAPM $\hat{\alpha}_A$'s for the shorter-lived funds are actually lower in that subsample than in the overall sample that includes non-surviving funds. Nevertheless, at least part of the age patterns could reflect survival effects, in that funds with poor track records are less likely to be long lived.

To investigate whether including the non-benchmark assets leads to a more precise inference about a fund's alpha, we examine the ratio of two posterior variances. The numerator of the ratio is the posterior variance of α_A under our model in which non-benchmark assets are used and the prior variance for the elements of α_N is as given in the column heading. Recall that the posterior mean of α_A in that case is denoted as $\tilde{\alpha}_A$. The denominator of the ratio is the posterior variance of α_A when the non-benchmark assets are not used and diffuse priors are assigned to all parameters. The posterior mean of α_A in that case is the OLS estimate $\hat{\alpha}_A$. For ease of discussion, we commit a slight abuse of notation and refer to the posterior variances in the numerator and denominator as the "variances" of $\tilde{\alpha}_A$ and $\hat{\alpha}_A$. These variances reflect the precision of inferences about α_A in the sense generally associated with standard errors in a frequentist setting. In fact, the denominator of the ratio equals the squared standard error computed in the usual regression model.

For most funds, a more precise inference about alpha is obtained by including non-benchmark assets. Table 4 reports the median ratio of the variance of $\tilde{\alpha}_A$ to the variance of $\hat{\alpha}_A$. Also reported is the fraction of those ratios that are less than one. For the CAPM alpha estimates, the median variance ratio across all funds is approximately 0.7, and the ratio is less than one for roughly 90% of the funds. For Fama-French alphas, the ratio has a median of about 0.85 and is less than one for roughly 80% of the funds. In general, the median ratio is higher for the funds with longer histories. Note from Table 1 that funds with track records of at least 20 years represent only about 5% or our sample (139 out of 2609). For those funds, the OLS estimates of Fama-French alphas are typically about as precise as the estimates that incorporate the non-benchmark assets: the median variance ratios are 1.00 or just slightly less, and the ratios cluster fairly tightly around that value. At the other extreme lies the variance ratio associated with estimating CAPM alphas for small-company growth funds. That variance ratio has a median between 0.33 and 0.39, depending on σ_{α_N}, and the ratio is less than one for all such funds in our sample. Thus, for small-company growth funds in particular, not only is the CAPM $\tilde{\alpha}_A$ substantially higher than the CAPM $\hat{\alpha}_A$, it is also substantially more precise.

Recall that estimates of α_A are identical across different specifications of the benchmarks when one assumes the non-benchmark assets are priced exactly under each specification. In Table 2, note that the median values of $\tilde{\alpha}_A$ are indeed the same in Panels A and B when $\sigma_{\alpha_N} = 0$ (which sets $\alpha_N = 0$). Table 5 compares estimates of CAPM and Fama-French alphas when $\sigma_{\alpha_N} = 2\%$ (Panel A) and $\sigma_{\alpha_N} = \infty$ (Panel B). As expected, the median absolute differences between models are typically larger in the second case, but those differences are still substantially less than the median absolute differences between OLS estimates (Panel

C). In other words, even when the non-benchmark assets are not believed to be priced whatsoever by either model's benchmarks, their presence in the estimation still makes the definition of α_A substantially less important than when they are not used at all. Across all funds, the median absolute difference between estimated CAPM and Fama-French alphas is 0.42% (per annum) under $\sigma_{\alpha_N} = 2\%$ and 1.24% under $\sigma_{\alpha_N} = \infty$, as compared to 2.28% for the OLS estimates. For small-company growth funds, the median difference is 0.69% under $\sigma_{\alpha_N} = 2\%$ and 2.03% under $\sigma_{\alpha_N} = \infty$, as compared to 8.07% for the OLS estimates.

Table 6 compares alphas defined for a given set of benchmarks and estimated with or without the pricing restriction imposed on the non-benchmark assets. That is, for α_A defined with respect to a given pricing model, we compare estimates under $\sigma_{\alpha_N} = 0$ to estimates under $\sigma_{\alpha_N} = \infty$. Across all funds, the median difference is 0.94% for CAPM alphas and 0.68% for Fama-French alphas. Interestingly, the median differences display little if any relation to the length of the fund's history. Most of the median differences in the two-way sort (by objective and age) are 2% or less, with the exception of sector funds. For the funds with the longest histories, the effect of imposing the pricing restriction on the non-benchmark assets is often as large as the effect of not including the non-benchmark assets at all (shown earlier in Table 2). The latter effect is more important for history lengths of ten years or less when estimating CAPM alphas and five years or less when estimating Fama-French alphas. Note from Table 1 that about 85% of the equity funds in our sample have history lengths less than 10 years, and about 60% have histories of five years or less.

3. Investing with priors about skill and pricing

Prior beliefs about pricing models can be useful to someone investing in mutual funds. A pricing model implies that a combination of the model's benchmark assets provides the highest Sharpe ratio within a passive universe. That implication is useful to an investor seeking a high Sharpe ratio, even if the investor has less than complete confidence in the model's pricing accuracy and cannot invest directly in the benchmarks. Prior beliefs about managerial skill are also important in the investment decision. One investor might believe completely in a model's accuracy in pricing passive assets but believe active managers may well possess stock-picking skill. Another investor might be skeptical about the ability of fund managers to pick stocks as well as the ability of academics to build accurate pricing models.

This section applies the econometric framework described in the previous section to an investment setting that allows an investor to combine information in the data with prior beliefs about both pricing and skill. Non-benchmark assets allow us to distinguish between pricing and skill, and they supply additional information about funds' expected returns in essentially the same manner as in the estimation problem of the previous section. In addition, non-benchmark assets help account for common variation in funds, making the investment problem feasible using a large universe of funds. We begin the section with a discussion of skill and prior beliefs, and then we present and discuss portfolios constructed under a range of priors about pricing models and skill.

3.1. Prior beliefs

In both commercial and academic settings, much interest attaches to alphas defined with respect to small sets of benchmarks identified by pricing models. Estimating such alphas is the subject of the previous section. Alpha is often interpreted as skill displayed by the fund's manager in selecting mispriced securities, but a nonzero alpha need not reflect skill if some passive assets can also have nonzero alphas. For example, a manager who starts a new fund investing in non-benchmark passive assets whose alphas have historically been positive can have a positive alpha in the absence of any skill. To address such concerns, one could expand the set of benchmarks to include more passive assets, even to the point of including all assets available to the manager. Indeed, as observed by Grinblatt and Titman (1989, p.412), "... the unconditional mean-variance efficient portfolio of assets that are considered tradable by the evaluated investor provides correct inferences about the investor's performance ... links between performance measures and particular equilibrium models are not necessary." Chen and Knez (1996) adopt a similar approach in a conditional setting, in that they evaluate funds with respect to a set of passive benchmarks selected without regard to a pricing model: "...we argue that for application purposes, one does not need to rely on asset pricing models to define an admissible performance measure" (p. 515).

In practice, the number of passive assets must be limited in some fashion. Our empirical design includes p passive assets, consisting of k benchmarks and m non-benchmark assets, and the benchmarks are associated with popular asset pricing models. Suppose one admits the possibility that the benchmarks do not price the non-benchmark assets exactly, that is $\alpha_N \neq 0$. Then δ_A, the intercept in (4), is a better measure of skill, in that it is defined with respect to the more inclusive set of passive assets. Of course, that measure might still be nonzero for passive assets omitted from the set of p. The point is simply that inadequacy of δ_A as a skill measure implies inadequacy of α_A, whereas δ_A can be adequate when α_A is not.

The skill measure δ_A is defined with respect to the overall set of p assets, but the investor nevertheless finds it useful to partition that set into k benchmark and m non-benchmark assets. Even though the investor is unwilling to assume that the k benchmarks price the non-benchmark assets exactly, he might nevertheless believe that the benchmarks possess some pricing ability. That pricing ability, albeit imperfect, helps the investor identify portfolios with high Sharpe ratios, as will be illustrated below. The investor's prior beliefs about pricing are represented as before, with a prior for α_N characterized by σ_{α_N}.

We assume that an investor selecting a portfolio of mutual funds generally has informative prior beliefs about a fund manager's ability to achieve a nonzero δ_A. Prior beliefs about δ_A, given by (15), are characterized by the prior mean and standard deviation, δ_0 and σ_δ. Recall that in the estimation problem addressed in the previous section, the prior beliefs about δ_A are diffuse ($\sigma_\delta = \infty$) and thus δ_0 is irrelevant. In the investment problem addressed here, σ_δ can be finite, and even zero, so the prior mean δ_0 must be specified as well.

If a fund manager possesses no skill, then δ_A should simply reflect costs, since the returns on the p passive assets used to define δ_A have no costs deducted. To represent a prior belief that precludes skill, we set $\sigma_\delta = 0$ and specify

$$\delta_0 = -\frac{1}{12}(expense \ + \ 0.01 \times turnover), \tag{20}$$

where *expense* is the fund's average annual expense ratio and *turnover* is the fund's average annual reported turnover. Multiplying the latter quantity by 0.01 is equivalent to assuming a round-trip cost per transaction of one percent, approximately the 95 basis points estimated by Carhart (1997) for the average fund in his sample. Carhart obtains that estimate as the average slope coefficient in monthly cross-sectional regressions of fund return on "modified" turnover (MTURN), which is reported turnover plus one-half the rate of change in total net assets (TNA) adjusted for investment returns and mergers. Reported turnover is the minimum of the fund's purchases and sales divided by its average TNA. MTURN, which essentially includes transactions arising from contributions and withdrawals, is the appropriate measure for estimating transactions costs in Carhart's regression. For example, a fund that sells nothing in a year but experiences contributions doubling its size will have a value of zero for *turnover* but a value of 0.50 for MTURN. The resulting purchases incur costs impacting the fund's return that year, and the year-by-year values of MTURN can better explain that component of return variation and thereby provide an estimate of transactions costs. In forecasting future transactions, however, it seems more reasonable to abstract from growth or shrinkage of the fund and instead view a fund with either no sales or no purchases as likely to be a low-turnover fund. Thus, we define *turnover* as the average of the reported turnover values.

When one admits some possibility of skill, the link between turnover and prior expected performance becomes less clear. If the manager does possess skill, then high turnover is likely to be accompanied by positive performance. On the other hand, if the manager possesses no skill, then high turnover can only hurt expected performance. If the investor is uncertain about whether the manager has skill, that is if $\sigma_\delta > 0$, then the relation between expected turnover and expected performance is ambiguous. A similar ambiguity arises with expense ratios. We follow an empirical Bayes approach in specifying how prior expected performance depends on *expense* and *turnover* when $\sigma_\delta > 0$.[11] Specifically, we estimate a cross-sectional regression of estimated δ_A on $\frac{1}{12}expense$ and $\frac{1}{12}turnover$, where the estimate of δ_A is the posterior mean obtained with $\sigma_\delta = \infty$. Across a number of alternative methods for including funds (e.g., minimum history length) and estimating the coefficients (OLS or weighted least squares), we find that the coefficient on $\frac{1}{12}expense$ is consistently about -1 and is at least twice its standard error. In contrast, the coefficient on $\frac{1}{12}turnover$ fluctuates within an interval roughly between -0.005 and 0.005 and is generally less than its standard error.[12] Guided by this result, we specify

$$\delta_0 = -\frac{1}{12}expense \tag{21}$$

as the prior mean of δ_A when $\sigma_\delta > 0$.

3.2. The investment problem

Under various prior beliefs about skill and pricing, we construct portfolios with the highest Sharpe ratio, defined as expected excess return divided by the standard deviation of return. The investment universe consists of 505 funds selected from the 2,609 equity mutual funds analyzed in the previous section. The 505 funds are those that (i) charge no load fee, (ii) exist at the end of 1998, (iii) have at least 36 months of return history under the most recent manager, and (iv) have data on expense ratios and turnover rates. We exclude funds that charge load fees simply because it is not clear how to treat the payment of such fees within the single-period setting implicit in maximizing the Sharpe ratio. The p passive assets used to define δ_A are included in the econometric specification, but since returns on those assets

[11] An alternative approach, proposed by Baks, Metrick, and Wachter (2000), is to specify a prior for performance that is truncated below at a point that reflects expenses as well as an estimate of transactions costs.

[12] Wermers (2000) finds that turnover does not exhibit a significant relation to net performance after adjusting for risk and asset characteristics.

do not include any implementation costs, only the mutual funds are assumed to be eligible for investment. In addition, short positions in funds are precluded.

The stochastic setting is as defined in Section 2. Let R denote the sample data, consisting of returns on passive assets and funds through month T, and let r_{T+1} denote the vector of returns on the funds in month $T+1$. In solving the investment problem, Sharpe ratios are computed using moments of the predictive distribution of the funds' returns,

$$p(r_{T+1}|R) = \int_{\theta} p(r_{T+1}|R,\theta)\, p(\theta|R)\, d\theta, \tag{22}$$

where $p(\theta|R)$ is the posterior distribution of the parameter vector, θ.[13] The first two moments of this predictive distribution are derived in the Appendix. The fund's history is used only back to the month beginning the most recent manager's tenure, whereas the return histories of the p passive assets begin in July 1963.

A meaningful investment universe can include only those funds that exist at the end of the sample period, December 1998, but this selection criterion raises the issue of survival bias. In particular, under prior beliefs that admit the possibility of skill ($\sigma_\delta > 0$), one might be concerned that the posterior mean of a manager's skill measure δ_A is overstated by a failure to condition on the fund's having survived. Baks, Metrick, and Wachter (2000) make the interesting observation that, if a fund's survival depends only on *realized* return histories, then the posterior distribution of the parameters for the surviving funds is unaffected by conditioning their survival. In other words, a sufficient assumption for this result is that once one conditions on all the return histories, the probability of a fund's surviving is unaffected by further conditioning on the model's unobserved parameters. The Bayesian posterior for the parameters conditions on the return histories in any event, and with this assumption those return histories subsume the information in knowing the fund survived. Like Baks, Metrick, and Wachter, we find the notion that survival depends only on realized returns to be plausible, and thus we proceed under that assumption.

3.3. Portfolio selections

Table 7 reports weights in the optimal portfolio for investors with various beliefs about managerial skill and mispricing of passive assets under the CAPM. (The weights in each column of Panel A add to 100 percent.) For convenience, we refer throughout to a portfolio having

[13]Early applications of Bayesian methods to portfolio choice, using diffuse prior beliefs, include Zellner and Chetty (1965), Klein and Bawa (1976), and Brown (1979). Recent examples, using informative priors, include Pástor (2000) and Pástor and Stambaugh (2000).

the highest Sharpe ratio within a given universe as "optimal." Mispricing uncertainty, σ_{α_N}, is assigned values of zero, one percent, and two percent (per annum), while skill uncertainty, σ_δ, is assigned values of zero, one percent, three percent, and infinity. Tables 8 and 9 report corresponding results for two other pricing models, the Fama-French three-factor model (Table 8) and the Carhart four-factor model (Table 9). Table 10 reports optimal weights for $\sigma_{\alpha_N} = \infty$, in which case the investor makes no use of the pricing models.

We stated earlier that a pricing model, even if not believed completely, helps identify the portfolios with high Sharpe ratios. This point can be illustrated in Table 7, for example, by examining the optimal portfolio's correlation with the market index, MKT. Reported in Panel B of Tables 7 through 10 is the optimal portfolio's predictive correlation with the portfolio having the highest Sharpe ratio among those that combine the factors of each pricing model. The latter portfolio is optimal for an investor who believes completely in the given pricing model and can invest only in the p passive assets. The investor here, in contrast, can invest only in mutual funds and not in the p passive assets. An investor who believes completely in the CAPM and in no possibility of fund-manager skill ($\sigma_{\alpha_N} = \sigma_\delta = 0$) selects a combination of market index funds that is virtually perfectly correlated with MKT. A value of $\sigma_{\alpha_N} = 1\%$ means that, before examining the data, the investor assigns about a five percent probability to the prospect that the expected return on a given non-benchmark passive asset violates its CAPM prediction by more than 200 basis points per annum in either direction. With that degree of mispricing uncertainty but the same belief about skill, the optimal portfolio is still essentially composed of market index funds and has a correlation with MKT that rounds to 1.00. With twice as much mispricing uncertainty ($\sigma_{\alpha_N} = 2\%$), the correlation with MKT is 0.89, which is still considerably higher than the value of 0.74 obtained in Table 10 when no pricing model is used.

The CAPM continues to influence portfolio choice when the investor admits the possibility of managerial skill. A value of $\sigma_\delta = 1\%$ means that, before examining a given fund's track record, the investor assigns about a 2.5% probability to the prospect that the fund's manager generates a positive skill measure gross of expenses of at least 200 basis points per year. (Of course, given the symmetry of our prior, the investor assigns the same probability to a negative skill measure of that magnitude, but the left tail is presumably less interesting with short sales precluded.) With that amount of skill uncertainty, the CAPM can still help the investor construct the portfolio with the highest Sharpe ratio, even with some uncertainty about the CAPM's ability to price passive assets. When $\sigma_\delta = 1\%$, the optimal portfolio has a correlation of 0.92 with MKT when $\sigma_{\alpha_N} = 2\%$ (Table 7), as compared to a correlation of only 0.76 when the CAPM is not used (Table 10). With three times as much skill uncertainty

($\sigma_\delta = 3\%$), the optimal portfolio's correlation with MKT is 0.93 when $\sigma_{\alpha_N} = 2\%$ and 0.87 when the model is not used. That is, even with a substantial degree of willingness to accept the possibility of managerial skill and only modest confidence in the CAPM, the investor's portfolio selection is still influenced by the pricing model.

Portfolio choice is influenced by beliefs in the other pricing models in essentially similar ways as noted above for the CAPM. For both the Fama-French and four-factor models, however, perfect confidence in the model does not result in an optimal portfolio of funds that mimics as closely the optimal combination of the benchmarks from the model. Table 8 reports optimal portfolios for investors with varying degrees of confidence in the three-factor Fama-French model. After seeing the data, an investor who has complete prior confidence in that model and admits no possibility of manager skill ($\sigma_{\alpha_N} = \sigma_\delta = 0$) believes that the optimal portfolio constructed from our universe of 505 no-load mutual funds has a correlation of only 0.75 with the optimal combination of the Fama-French benchmarks (Panel B). Moreover, as reported in Panel C, that investor judges the highest Sharpe ratio obtainable within the fund universe to be only 0.66 times that of the highest Sharpe ratio obtainable by combining the benchmarks.[14] Under the four-factor model, as reported in Table 9, the correlation between the optimal fund portfolio and the optimal combination of the four benchmarks is 0.61, and the Sharpe ratio of the first portfolio is only slightly more than half that of the second. Evidently, close substitutes for the Fama-French and Carhart benchmarks cannot be constructed with long positions in funds from our no-load universe.

The main reason for the lack of benchmark substitutes is our precluding short sales of mutual funds. When the short-sale constraint is removed, the Sharpe ratio of the optimal fund portfolio increases to 0.99 times the Sharpe ratio of the efficient benchmark combination under the Fama-French model and to 0.94 times the maximum under the Carhart model. Since only a relatively small subset of funds can be shorted in practice, precluding short sales in our fund universe seems reasonable. We also redid the analysis with an expanded investment universe of 919 funds that includes funds with load fees. The improvement from including the load funds is surprisingly small, despite the fact that we ignore their load fees. Under the four-factor model, the Sharpe ratio rises only to 0.55 times the maximum, as compared to multiple of 0.54 in the original no-load setting. Under the Fama-French model, the Sharpe ratio rises so little that it rounds, as before, to only 0.66 times the maximum.

[14] If δ_0 were set to zero for each fund, the correlation between the two portfolios would equal the Sharpe ratio of the fund portfolio divided by the Sharpe ratio of the benchmark portfolio. In that case, the second portfolio would have the highest possible Sharpe ratio for the overall universe of funds and passive assets with investment weights unconstrained (i.e., short sales permitted), and an exact relation between correlations and Sharpe ratios applies (e.g., Proposition 1 of Kandel and Stambaugh (1987)).

We conclude that the universe of all equity mutual funds, including the load funds, provides no close substitutes for the Fama-French and momentum benchmarks.

3.4. Portfolio comparisons

Differences in prior beliefs about pricing models and the potential for managerial skill can lead to economically significant differences between optimal portfolios of mutual funds. Table 11 and Panels D and E of Table 10 present comparisons of portfolios constructed under various specifications. The certainty-equivalent differences are computed for an investor who maximizes the mean-variance objective,

$$C_p = E_p - \frac{1}{2}A\sigma_p^2, \tag{23}$$

where E_P and σ_p^2 denote the mean and variance of the excess return on the investor's overall portfolio (including unrestricted riskless borrowing and lending). Risk-aversion, A, is set to 2.75, which is the level at which an investor would allocate 100% to MKT if the investment universe contained just that single risky position in addition to the riskless asset. In comparing portfolios obtained under different specifications, one portfolio is designated as optimal and the other as suboptimal, where the suboptimal portfolio is optimal under the alternative specification. We compare the certainty equivalent for the optimal portfolio, C_o, to the certainty equivalent for a suboptimal portfolio, C_s. Both certainty equivalents are computed using the predictive moments obtained under the prior beliefs associated with the optimal portfolio.

Panel A of Table 11 compares portfolios formed with the same σ_{α_N} and σ_δ but under different pricing models. The difference between any two models ranges between 1 and 61 basis points per month, depending on the prior uncertainty about mispricing and skill.[15] In general, sample averages receive more weight in estimating expected returns when one's prior beliefs about pricing and skill become less informative. As mispricing uncertainty increases, the portfolios formed with beliefs centered on different pricing models become more alike: the certainty-equivalent difference drops and the correlation increases. An increase in skill uncertainty also tends to make the cross-model difference less important, although not monotonically. The largest certainty-equivalent differences, which can exceed 50 basis points

[15] The reported certainty-equivalent difference is actually the average of two differences, one for each of the two pricing models designated as producing the optimal portfolio. The correlation reported in Panel A is similarly the average of two values, one for the predictive distribution associated with each model. Averaging in this fashion treats the pricing models symmetrically, although generally the two values being averaged are close to each other.

per month, tend to occur between the CAPM and the four-factor model when σ_{α_N} and σ_δ are small. The smallest differences occur between the Fama-French and four-factor models when σ_δ is large. When σ_{α_N} and σ_δ are both one percent or less, however, the certainty-equivalent difference between those two models is at least 19 basis points per month.

In Panel B of Table 11, the optimal portfolio under a given set of beliefs about skill and mispricing is compared to the portfolio selected by an investor who rules out any ability of academics to build models and any skill of portfolio managers to pick stocks. The portfolio of this "completely skeptical" investor, for whom $\sigma_{\alpha_N} = \infty$ and $\sigma_\delta = 0$, is designated as the suboptimal portfolio in computing the pairwise comparisons described previously. (Its weights are given in the first column of Table 10.) Suppose one forces that portfolio to be held by an investor who has a modest degree of confidence in the CAPM, say $\sigma_{\alpha_N} = 2\%$, and who admits some possibility of managerial skill, say $\sigma_\delta = 1\%$. Then that investor suffers a certainty-equivalent loss of 29 basis points per month, or about 3.5% per year. With beliefs centered around the Fama-French model but again with $\sigma_{\alpha_N} = 2\%$ and $\sigma_\delta = 1\%$, the certainty-equivalent loss falls to 15 basis points per month. When skill uncertainty is one percent or less, complete belief in the four-factor model produces a portfolio quite close to that obtained with no use of the model at all, with a certainty-equivalent difference of 6 basis points or less and a correlation of at least 0.97. As an investor's willingness to accept the prospect of managerial skill increases, so does the certainty-equivalent loss if forced to hold the portfolio of the completely skeptical investor. With $\sigma_\delta = 3\%$, for example, the loss is between 31 and 89 basis points per month with modest confidence ($\sigma_{\alpha_N} = 2\%$) in one of the three pricing models. With no use of a pricing model, the loss is 23 basis points, as reported in Panel D of Table 10.

Even with no belief in a pricing model and no preconceived limit on the magnitude of likely managerial skill, that is when both σ_{α_N} and σ_δ are infinitely large, the investor is generally ill-advised in using a fund's historical average return as the input for its expected return. If the fund's history is shorter than those of the passive assets, then the histories of the passive assets provide additional information about the fund's expected return. Under the above prior beliefs, the certainty-equivalent loss of holding the portfolio constructed using sample-averages instead of holding the portfolio constructed using that additional information about expected returns is 187 basis points per month, or more than 22 percent annually (Panel D of Table 10). The predictive covariance matrix obtained when $\sigma_{\alpha_N} = \infty$ and $\sigma_\delta = \infty$ is used to construct both portfolios. As prior beliefs about pricing or skill become informative, the loss incurred by holding the portfolio based on sample averages becomes even greater, as is apparent in Panel C of Table 11.

3.5. Who should buy actively managed funds?

One might presume that actively managed funds should be purchased only by those investors who admit some possibility that active fund managers possess stock-picking skill. For investors presented with our universe of 505 no-load funds, that need not be the answer. An investor who believes completely in the CAPM and admits no possibility of managerial skill does indeed invest only in market-index funds (Table 7). As the investor's beliefs depart from complete confidence in the CAPM, however, actively managed funds enter the optimal portfolio even if the investor still adheres to a belief that managerial skill is impossible. If one can invest directly and costlessly in the p passive assets used to define the skill measure δ_A, then indeed long positions in funds arise only when positive δ_A's are thought possible. Otherwise, one simply combines the passive assets to obtain the highest Sharpe ratio. Baks, Metrick, and Wachter (2000) essentially pose their active management question in that context. If instead the p passive assets are not available for investment, as in our setup, perfect substitutes for them need not exist in the mutual fund universe, let alone in its passively managed subset. As a result, some actively managed funds can become attractive even to investors who admit no chance of managerial skill.

A striking example of the above possibility occurs in the first column of Table 8. The investor in that case believes completely in the Fama-French model and in no chance of managerial skill. Nevertheless, the bulk of that investor's optimal portfolio is allocated to actively managed value funds and real-estate specialty funds: Legg Mason Total Return, Mutual Discovery, First American Investment Real Estate Securities and DFA AEW Real Estate Securities. Table 12 reports posterior means and "t statistics" (posterior mean divided by posterior standard deviation) of the intercept and slopes in (4) for all funds that receive at least a ten percent allocation in any of the porfolios in Tables 7 through 10. The selection of the above funds has nothing to do with their having superior historical performance. In fact, three of the four funds listed above have negative $\hat{\delta}_A$'s. With $\sigma_{\alpha_N} = \sigma_\delta = 0$, the expected returns on these funds, gross of costs, are assumed to conform exactly to the Fama-French model. The presence of these funds in the optimal portfolio is instead driven by their risk characteristics. Note, for example, that all four funds have relatively large positive (and "significant") slopes on HML.

3.6. Hot hands?

To the universe of 505 no-load funds, we also add a portfolio of funds with recent high returns, motivated by much previous research indicating short-run persistence in fund performance.[16] At the end of each year, starting with December 1962, we sort all existing equity funds by their total returns over the previous twelve months (including only funds with returns reported for those months) and form the equally weighted "hot-hand" portfolio of the top ten percent. As Carhart (1997) observes, this portfolio has a positive sensitivity to the momentum factor MOM, which is confirmed by the results in Table 12. The hot-hand portfolio appears in the last row of both panels, and the posterior mean of its coefficient on MOM is 0.19 (with a t statistic of 13.5). This portfolio does not enter any of the optimal portfolios reported in Tables 7 through 10. The same result occurs if the hot-hand portfolio contains only no-load funds, as constructed by Hendricks, Patel, and Zeckhauser (1993).

As Carhart (1997) points out, the hot-hand portfolio is a kind of momentum play. Even a strong belief in momentum, which in our setting amounts to a strong belief in Carhart's four-factor model, does not result in an allocation to the hot-hand strategy. As we discover, one reason for this outcome is the existence of other funds that apparently offer even stronger momentum plays, at least in the sense that they have higher coefficients on MOM. The first column of Table 9 displays the portfolio selected by an investor who rules out skill and has complete confidence in the four-factor model. Note that the bulk of this portfolio is invested in real estate funds. The regression results in Table 12 reveal that the posterior means of the MOM coefficients for many of these funds are higher than that for the hot-hand portfolio. Perhaps as importantly, the coefficients on SMB, HML, and MKT for these funds are also positive and relatively large.[17] The highest-Sharpe-ratio portfolio of the benchmarks in the four-factor model contains those three factors and MOM in positive amounts. In our sample, real estate funds offer exposures to all four factors, and that feature makes them attractive to investors who believe in that model. When prior beliefs admit the possibility of skill, funds enter the optimal portfolio due to their average realized returns as well as their risk characteristics. This doesn't help the hot-hand portfolio, since the posterior mean of its δ_A is only 1 basis point.[18]

[16]See, for example, Grinblatt and Titman (1992), Hendricks, Patel, and Zeckhauser (1993), and Brown and Goetzmann (1995)

[17]This is consistent with the evidence in Sanders (1997), who reports significantly positive SMB, HML, and MKT betas for real estate investment trust indices between 1978 and 1996.

[18]The hot-hand porfolio has a positive alpha with respect to the Fama-French benchmarks and receives a substantial positive allocation when the investment universe contains only those three benchmarks and the hot-hand portfolio. See Knox (1999) for a treatment of this case in a Bayesian portfolio-choice setting.

4. Conclusions

This study develops and applies a framework in which beliefs about pricing models and managerial skill play roles in both performance evaluation and investment decisions. Non-benchmark passive assets provide additional information about mutual funds' performance measures and expected returns, and they allow us to specify prior beliefs that distinguish mispricing from skill. In addition, non-benchmark assets help account for common variation in fund returns, making the investment problem feasible with a large universe of funds.

A mutual fund's performance measure, alpha, is defined relative to a set of passive benchmarks. The typically reported OLS estimate of alpha ignores information provided by returns on non-benchmark assets. The non-benchmark assets help estimate alpha if they are priced to some extent by the benchmarks or if their return histories are longer than the fund's.

Using a sample of 2,069 U.S. equity mutual funds, we demonstrate that the returns on non-benchmark assets contain substantial information about fund performance. For most funds, the estimates of alpha that incorporate this information are more precise than the standard estimates. In the case of small-company growth funds, for example, the non-benchmark assets allow alphas defined with respect to the market benchmark to be estimated with only one third of the variance associated with the usual OLS estimates of those alphas.

Compared to the usual estimates, the estimates of alpha that incorporate the information in the non-benchmark assets tend to exhibit less variation across different specifications of the benchmarks. At the extreme, if one believes that some subset of the passive assets used in the estimation prices the other passive assets exactly, then the estimate of alpha is the same regardless of which subset is designated as the benchmarks that define alpha. In other words, if one believes dogmatically in a pricing model, it does not matter which model that is when estimating alpha. For most funds, we find that including information in non-benchmark assets is more important than specifying the degree to which the non-benchmark assets are priced by the benchmarks. We also find that, across different beliefs about pricing, most funds have underperformed the CAPM and Fama-French benchmarks.

An important practical motivation for mutual-fund performance evaluation is to help an investor decide in which funds to invest. This study constructs portfolios with maximum Sharpe ratios from a universe of 505 no-load equity mutual funds. We find that the optimal portfolios are substantially affected by prior beliefs about pricing and skill as well as by including the information in non-benchmark assets. A pricing model is useful to an investor seeking a high Sharpe ratio, even if the investor has less than complete confidence in the model's pricing accuracy and cannot invest directly in the benchmarks. With investment in the benchmarks precluded, even investors who believe completely in a pricing model and rule out the possibility of manager skill can include active funds in their portfolios. The fund universe offers no close substitutes for the Fama-French and momentum benchmarks, and active funds can be better substitutes for the benchmarks than passive funds. We also find that the "hot-hand" portfolio of the previous year's best-performing funds does not appear in the portfolio of funds with the highest Sharpe ratio, even when momentum is believed to be priced. An investor who holds that belief and is skeptical about managerial skill instead invests heavily in real estate funds, which have higher exposures to the momentum factor and the Fama-French factors.

Maximizing the Sharpe ratio is only one of many investment objectives. With a multiperiod investment objective, for example, beliefs about pricing and skill could exhibit different effects. A multiperiod setting could also allow a meaningful consideration of the funds that charge load fees. Such extensions offer challenges for future research.

Appendix

This Appendix derives the posterior moments of α_A used in Section 2 as well as the predictive moments of the fund returns used in Section 3. The prior for the parameters of the regression in (4) is independent of the prior for the parameters of the regression in (3). In addition, both of those priors are assumed to be independent of the (diffuse) prior for E_B and V_{BB}, the mean and covariance matrix of the normal distribution for $r_{B,t}$. The independence of the priors and the independence of $u_{A,t}$ and $\epsilon_{N,t}$ imply that the posterior distribution for the parameters of the regression in (3) and the predictive distribution for $r_{N,T+1}$ and $r_{B,T+1}$ depend only on the sample of passive asset returns, not the fund returns. We first provide the moments of those distributions, relying on the derivations in Pástor and Stambaugh (2000), henceforth referred to as PS. Those moments are then combined with the posterior moments of δ_A and c_A to obtain the the posterior moments of α_A and the predictive moments of the fund returns.

A.1. Posterior distribution for the parameters of (3)

Define $Y = (r_{N,1}, \ldots, r_{N,T})'$, $X = (r_{B,1}, \ldots, r_{B,T})'$, and $Z = (\iota_T \; X)$, where ι_T denotes a T-vector of ones. Also define the $(k+1) \times m$ matrix $G = (\alpha_N \; B_N)'$, and let $g = \mathrm{vec}\,(G)$. For the T observations $t = 1, \ldots, T$, the regression model in (3) can be written as

$$Y = ZG + U, \quad \mathrm{vec}\,(U) \sim N(0, \Sigma \otimes I_T), \tag{A.1}$$

where $U = (\epsilon_{N,t}, \ldots, \epsilon_{N,T})'$. Define the statistics $\hat{G} = (Z'Z)^{-1}Z'Y$, $\hat{g} = \mathrm{vec}\,(\hat{G})$, $\hat{\Sigma} = (Y - Z\hat{G})'(Y - Z\hat{G})/T$, $\hat{E}_B = X'\iota_T/T$, and $\hat{V}_{BB} = (X - \iota_T\hat{E}_B')'(X - \iota_T\hat{E}_B')/T$. Let θ_P denote the parameters of the joint distribution of the passive asset returns (G, Σ, E_B, and V_{BB}), and define the $T \times p$ sample matrix of passive returns, $R_P = (X \; Y)$. The likelihood function for the passive returns can be factored as

$$p(R_P|\theta_P) = p(Y|\theta_P, X)\; p(X|\theta_P), \tag{A.2}$$

where

$$p(Y|\theta_P, X) \;\propto\; |\Sigma|^{-\frac{T}{2}} \exp\left\{-\frac{1}{2}\mathrm{tr}\,(Y - ZG)'(Y - ZG)\Sigma^{-1}\right\} \tag{A.3}$$

$$p(X|\theta_P) \;\propto\; |V_{BB}|^{-\frac{T}{2}} \exp\left\{-\frac{1}{2}\mathrm{tr}\,(X - \iota_T E_B')'(X - \iota_T E_B')V_{BB}^{-1}\right\}. \tag{A.4}$$

The joint prior distribution of all passive-return parameters is

$$p(\theta_P) = p(\alpha_N|\Sigma)\; p(\Sigma)\; p(B_N)\; p(E_B)\; p(V_{BB}), \tag{A.5}$$

where

$$
\begin{aligned}
p(\alpha_N|\Sigma) &\propto |\Sigma|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\alpha_N'(\frac{\sigma^2_{\alpha_N}}{s^2}\Sigma)^{-1}\alpha_N\right\} && \text{(A.6)}\\
p(\Sigma) &\propto |\Sigma|^{-\frac{\nu+m+1}{2}} \exp\left\{-\frac{1}{2}\text{tr}\, H\Sigma^{-1}\right\} && \text{(A.7)}\\
p(B_N) &\propto 1 && \text{(A.8)}\\
p(E_B) &\propto 1 && \text{(A.9)}\\
p(V_{BB}) &\propto |V_{BB}|^{-\frac{k+1}{2}}. && \text{(A.10)}
\end{aligned}
$$

The priors of B_N, E_B, and V_{BB} are diffuse. The prior of Σ is inverted Wishart with a small number of degrees of freedom, so that it is essentially noninformative. The prior on α_N given Σ is normal and centered at the pricing restriction. Let D denote a $(k+1)\times(k+1)$ matrix whose $(1,1)$ element is $\frac{s^2}{\sigma^2_{\alpha_N}}$ and all other elements are zero. Also let $F = D + Z'Z$ and $Q = Z'(I_T - ZF^{-1}Z')Z$. Applying the analysis in PS, the likelihood and the prior are combined to obtain the following moments of the posterior distribution:

$$
\begin{aligned}
\tilde{g} &= \text{E}(g|R_P) = (I_m \otimes F^{-1}Z'Z)\hat{g} && \text{(A.11)}\\
\tilde{\Sigma} &= \text{E}(\Sigma|R_P) = \frac{1}{T+\nu-m-k-1}(H + T\hat{\Sigma} + \hat{G}'Q\hat{G}) && \text{(A.12)}\\
\text{Var}(g|R_P) &= \tilde{\Sigma}\otimes F^{-1} && \text{(A.13)}\\
\tilde{E}_B &= \text{E}(E_B|R_P) = \hat{E}_B && \text{(A.14)}\\
\tilde{V}_{BB} &= \text{E}(V_{BB}|R_P) = \frac{T}{T-k-2}\hat{V}_{BB} && \text{(A.15)}\\
\text{Var}(E_B|R_P) &= \frac{1}{T-k-2}\hat{V}_{BB}. && \text{(A.16)}
\end{aligned}
$$

Posterior means are denoted using tildes for the remainder of the Appendix.

A.2. Predictive moments of the passive returns

These predictive moments are derived in PS. Define $r_{P,T+1} = (r'_{N,T+1}\ r'_{B,T+1})'$. Its predictive mean is

$$
E^*_P = \text{E}(r_{P,T+1}|R_P) = \begin{pmatrix} \tilde{\alpha}_N + \tilde{B}_N\tilde{E}_B \\ \tilde{E}_B \end{pmatrix}, \tag{A.17}
$$

where $\tilde{\alpha}_N$ and $\tilde{B}_N$ are obtained from Eq. (A.11) using $\tilde{g} = \text{vec}\,((\tilde{\alpha}_N\ \tilde{B}_N)')$.

Partition the predictive covariance matrix as

$$
V^*_P = \text{Var}(r_{P,T+1}|R_P) = \begin{bmatrix} V^*_{NN} & V^*_{NB} \\ V^*_{BN} & V^*_{BB} \end{bmatrix}. \tag{A.18}
$$

Denote the i-th row of B_N as b_i', the i-th column of G as g_i, and the (i,j) element of Σ as $\sigma_{i,j}$. The first submatrix, V_{NN}^*, can be represented in terms of its (i,j) element:

$$(V_{NN}^*)_{(i,j)} = \tilde{b}_i' V_{BB}^* \tilde{b}_j + \text{tr}\left[V_{BB}^* \text{Cov}(b_i, b_j'|R_P)\right] + \tilde{\sigma}_{i,j} + [1 \ \ \tilde{E}_B'] \text{Cov}(g_i, g_j'|R_P)[1 \ \ \tilde{E}_B']'. \quad \text{(A.19)}$$

Note that $\text{Cov}(b_i, b_j'|R_P)$ and $\text{Cov}(g_i, g_j'|R_P)$ are submatrices of $\text{Var}(g|R_P)$ in (A.13). The remaining submatrices in (A.18) can be shown to be equal to

$$\begin{aligned} V_{BB}^* &= \tilde{V}_{BB} + \text{Var}(E_B|R_P) \\ V_{NB}^* = V_{BN}^{*\prime} &= \tilde{B}_N \tilde{V}_{BB} + \tilde{B}_N \text{Var}(E_B|R_P). \end{aligned}$$

A.3. Posterior distribution for the parameters of (4)

Let r_A $(S \times 1)$ contain S observations of $r_{A,t}$, the return on a given fund A. We assume $S \leq T$ and that the (consecutive) months in which $r_{A,t}$ is observed form a subset of those in which $r_{N,t}$ and $r_{B,t}$ are observed. The assumption that the disturbances in (4) are independent across funds, coupled with the assumption that the priors of that regression's parameters are independent across funds, implies that the posterior distribution for a given fund's parameters of the regression in (4) does not depend on the observed returns of the other funds (conditional on the passive return sample R_P). Let θ_A denote the set of parameters δ_A, c_A, and σ_u^2. Our various modeling assumptions give

$$\begin{aligned} p(\theta_A, \theta_P|R_P, r_A) &\propto p(\theta_A, \theta_P) p(R_P, r_A|\theta_A, \theta_P) \\ &= p(\theta_A) p(\theta_P) p(r_A|R_P, \theta_A, \theta_P) p(R_P|\theta_A, \theta_P) \\ &= p(\theta_A) p(r_A|R_P, \theta_A) p(\theta_P) p(R_P|\theta_P) \\ &\propto p(\theta_A|R_P, r_A) p(\theta_P|R_P). \end{aligned} \quad \text{(A.20)}$$

The second factor in (A.20) is the posterior derived previously. The first factor, the posterior for θ_A, combines the priors given in (14) through (16) with the likelihood,

$$p(r_A|R_P, \theta_A) \propto \frac{1}{\sigma_u^T} \exp\left\{-\frac{1}{2\sigma_u^2}(r_A - Z_A\phi_A)'(r_A - Z_A\phi_A)\right\}, \quad \text{(A.21)}$$

where $R_{P,A}$ denotes the S rows of R_P corresponding to the months in which $r_{A,t}$ is observed, $Z_A = (\iota_S \ R_{P,A})$, and $\phi_A = (\delta_A \ c_A')'$. The prior densities corresponding to (14) through (16) are given by

$$p(\sigma_u) \propto \frac{1}{\sigma_u^{\nu_0+1}} \exp\left\{-\frac{\nu_0 s_0^2}{2\sigma_u^2}\right\} \quad \text{(A.22)}$$

and

$$p(\phi_A|\sigma_u) \propto \frac{1}{\sigma_u^{p+1}} \exp\left\{-\frac{1}{2\sigma_u^2}(\phi_A - \phi_0)'\Lambda_0(\phi_A - \phi_0)\right\}, \tag{A.23}$$

where $\phi_0 = (\delta_0 \; c_0')'$ and

$$\Lambda_0 = \left(\frac{\nu_0 s_0^2}{\nu_0 - 2}\right)\left[\begin{array}{cc} \sigma_\delta^2 & 0 \\ 0 & \Phi_c \end{array}\right]^{-1}. \tag{A.24}$$

The product of (A.21), (A.22), and (A.23) gives, after simplifying,[19]

$$\begin{aligned} p(\phi_A, \sigma_u|R_P, r_A) \;\; \propto \;\; & \frac{1}{\sigma_u^{p+1}} \exp\left\{-\frac{1}{2\sigma_u^2}(\phi_A - \tilde{\phi}_A)'(\Lambda_0 + Z_A'Z_A)(\phi_A - \tilde{\phi}_A)\right\} \\ & \times \frac{1}{\sigma_u^{T+\nu_0+1}} \exp\left\{-\frac{h_A}{2\sigma_u^2}\right\}, \end{aligned} \tag{A.25}$$

where

$$\tilde{\phi}_A = (\Lambda_0 + Z_A'Z_A)^{-1}(\Lambda_0\phi_0 + Z_A'r_A) \tag{A.26}$$

$$h_A = \nu_0 s_0^2 + r_A'r_A + \phi_0'\Lambda_0\phi_0 - \tilde{\phi}_A'(\Lambda_0 + Z_A'Z_A)\tilde{\phi}_A. \tag{A.27}$$

It follows from (A.25) that

$$\phi_A|R_P, r_A, \sigma_u \sim N(\tilde{\phi}_A, \; \sigma_u^2(\Lambda_0 + Z_A'Z_A)^{-1}) \tag{A.28}$$

$$\sigma_u^2|R_P, r_A \sim \frac{h_A}{\chi^2_{T+\nu_0}}, \tag{A.29}$$

and hence

$$\tilde{\sigma}_u^2 = E(\sigma_u^2|R_P, r_A) = \frac{h_A}{T + \nu_0 - 2} \tag{A.30}$$

$$\text{Var}(\phi_A|R_P, r_A) = \tilde{\sigma}_u^2(\Lambda_0 + Z_A'Z_A)^{-1}, \tag{A.31}$$

where the last equation follows from variance decomposition.

A.4. Posterior moments of a fund's alpha

Let $\tilde{\alpha}_N$ and V_{α_N} denote the posterior mean and covariance matrix of α_N, given by the appropriate submatrices of the moments in (A.11) and (A.13), and let V_{ϕ_A} denote the posterior covariance matrix of ϕ_A given in (A.31). From the previous discussion recall that the posteriors of ϕ_A and α_N are independent. Thus, from equation (5), the posterior mean of the fund's alpha is given by

$$\tilde{\alpha}_A = \tilde{\delta}_A + \tilde{c}_{AN}'\tilde{\alpha}_N, \tag{A.32}$$

[19]See Zellner (1971, pp. 75-76) for a similar derivation.

where $\tilde{\delta}_A$ and $\tilde{c}_{AN}$ are subvectors of the posterior mean of ϕ_A given in (A.26).

To obtain the posterior variance of α_A, rewrite (5) as

$$\alpha_A = \phi_A' d, \tag{A.33}$$

where $d = (1\ \alpha_N'\ 0)'$, and the posterior mean and covariance matrix of d are given by

$$\tilde{d} = \begin{bmatrix} 1 \\ \tilde{\alpha}_N \\ 0 \end{bmatrix}, \qquad V_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & V_{\alpha_N} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{A.34}$$

Let $\mathcal{D}$ denote the data, R_P and r_A, and note that by the variance-decomposition rule,

$$\mathrm{Var}(\alpha_A|\mathcal{D}) = \mathrm{E}[\mathrm{Var}(\phi_A' d|\mathcal{D}, d)|\mathcal{D}] + \mathrm{Var}[\mathrm{E}(\phi_A' d|\mathcal{D}, d)|\mathcal{D}]. \tag{A.35}$$

Using the independence of ϕ_A and d, the first term in (A.35) can expressed as

$$\begin{aligned} \mathrm{E}[\mathrm{Var}(\phi_A' d|\mathcal{D}, d)|\mathcal{D}] &= \mathrm{E}[d' V_{\phi_A} d|\mathcal{D}] \\ &= \mathrm{E}[\mathrm{tr}\,(V_{\phi_A} d d')|\mathcal{D}] \\ &= \mathrm{tr}\,[V_{\phi_A} \mathrm{E}(d d'|\mathcal{D})] \\ &= \mathrm{tr}\,(V_{\phi_A} V_d) + \tilde{d}' V_{\phi_A} \tilde{d}, \end{aligned} \tag{A.36}$$

and the second term can be expressed as

$$\begin{aligned} \mathrm{Var}[\mathrm{E}(\phi_A' d|\mathcal{D}, d)|\mathcal{D}] &= \mathrm{Var}[\tilde{\phi}_A d|\mathcal{D}] \\ &= \tilde{\phi}_A' V_d \tilde{\phi}_A. \end{aligned} \tag{A.37}$$

A.5. Predictive moments of fund returns

The derivation of the predictive moments of fund returns closely parallels the derivation in PS of the predictive moments of the non-benchmark returns, $r_{N,T+1}$. Let R denote all of the sample returns data on funds and passive assets through period T. Since c_A and E_P (the mean of $r_{P,t}$) are independent in the prior, the predictive mean of $r_{A,T+1}$ is

$$\mathrm{E}(r_{A,T+1}|R) = \mathrm{E}(\delta_A + c_A' E_P|R) = \tilde{\delta}_A + \tilde{c}_A' \tilde{E}_P. \tag{A.38}$$

The predictive variance of $r_{A,T+1}$ can be written as

$$\mathrm{Var}(r_{A,T+1}|R) = \mathrm{E}(\mathrm{Var}(r_{A,T+1}|R, \phi_A)|R) + \mathrm{Var}(\mathrm{E}(r_{A,T+1}|R, \phi_A)|R). \tag{A.39}$$

To evaluate both terms on the right-hand side, first note that the regression in (4) implies

$$r_{A,T+1} = \delta_A + c_A' r_{P,T+1} + u_{T+1} \tag{A.40}$$
$$= [1\ r_{P,T+1}']\phi_A + u_{T+1}. \tag{A.41}$$

To compute the first term on the right-hand side of (A.39), observe using (A.40) that

$$\text{Var}(r_{A,T+1}|R,\phi_A) = c_A' V_P^* c_A + \tilde{\sigma}_u^2, \tag{A.42}$$

and taking expectations gives

$$\text{E}(\text{Var}(r_{A,T+1}|R,\phi_A)|R) = \tilde{c}_A' V_P^* \tilde{c}_A + \text{tr}\,[V_P^* \text{Cov}(c_A, c_A'|R)] + \tilde{\sigma}_u^2. \tag{A.43}$$

To compute the second term on the right-hand side of (A.39), observe using (A.41) that

$$\text{E}(r_{A,T+1}|R,\phi_A) = [1\ \tilde{E}_P']\phi_A, \tag{A.44}$$

so

$$\text{Var}(\text{E}(r_{A,T+1}|R,\phi_A)|R) = [1\ \tilde{E}_P']\text{Cov}(\phi_A,\phi_A'|R)[1\ \tilde{E}_P']'. \tag{A.45}$$

Note that the posterior covariance matrix $\text{Cov}(\phi_A,\phi_A'|R)$ and its submatrix $\text{Cov}(c_A,c_A'|R)$ are given in (A.31).

Computing the predictive covariance of $r_{A,T+1}$ with the return on another fund J, $r_{J,T+1}$, is simplified by the independence across funds of (i) the disturbances in (A.40) and (ii) the posteriors for the coefficient vectors ϕ_A and ϕ_J. Applying the same approach as used above for the predictive variance gives

$$\text{Cov}(r_{A,T+1}, r_{J,T+1}|R) = \tilde{c}_A' V_P^* \tilde{c}_J. \tag{A.46}$$

Computing the predictive covariance of $r_{A,T+1}$ with the vector of returns on the passive assets, $r_{P,T+1}$, is simplified by the independence of the posterior for ϕ_A from that of E_P and V_P. Let θ denote the union of θ_P and θ_A. Using the law of iterated expectations and the variance decomposition rule gives

$$\begin{aligned}\text{Cov}(r_{A,T+1}, r_{P,T+1}|R) &= \text{E}(\text{Cov}(r_{A,T+1}, r_{P,T+1}|R,\theta)|R) + \text{Cov}(\text{E}(r_{A,T+1}|R,\theta), \text{E}(r_{P,T+1}|R,\theta)|R) \\ &= \text{E}(V_P c_A|R) + \text{Cov}(\delta_A + c_A' E_P, E_P|R) \\ &= \tilde{V}_P \tilde{c}_A + \text{Cov}(E_P, E_P'|R)\tilde{c}_A \\ &= V_P^* \tilde{c}_A. \end{aligned} \tag{A.47}$$

Table 1

Number of Equity Mutual Funds Classified by History Length and Investment Objective

The sample contains domestic equity mutual funds in the CRSP database with continuous return histories longer than one year. Multiple share classes for the same fund are excluded. Funds are assigned to one of seven broad investment objectives using information that the CRSP database provides about classifications by Wiesenberger, ICDI, and Strategic Insight.

	Length of fund's return history (months)						
Investment objective	13–23	24–35	36–59	60–119	120–239	≥ 240	All
Small-company growth	128	60	95	109	21	0	413
Other aggressive growth	40	30	41	32	10	0	153
Growth	213	130	226	251	92	60	972
Income	36	35	38	47	14	4	174
Growth and income	154	80	119	153	36	34	576
Maximum capital gains	9	10	17	16	13	41	106
Sector funds	61	37	45	68	4	0	215
All funds	641	382	581	676	190	139	2609

Table 2
Estimates of Alpha for Equity Mutual Funds

Each value in the table is the median across the set of designated funds, expressed in percent per annum. Fund performance, denoted by α_A, is defined as the intercept in the regression of the fund's excess return, $r_{A,t}$, on either the market benchmark index, MKT_t, (Panel A) or that benchmark index plus the size and value benchmark indexes, SMB_t and HML_t (Panel B). The OLS estimate of α_A, denoted by $\hat{\alpha}_A$, is based on the fund's available history and the corresponding history of the benchmarks. The posterior mean of α_A, denoted by $\tilde{\alpha}_A$, is based on the fund's available history as well as the returns from January 1963 through December 1998 on the benchmarks and additional non-benchmark indexes. The quantity σ_{α_N}, expressed in percent per annum, denotes the prior standard deviation of the intercept α_N in a regression of a non-benchmark return on the benchmark indexes. The prior for α_A is diffuse.

History length or		$\tilde{\alpha}_A$ for σ_{α_N} of			$\lvert\hat{\alpha}_A - \tilde{\alpha}_A\rvert$ for σ_{α_N} of		
investment objective	$\hat{\alpha}_A$	0	2%	∞	0	2%	∞
A. $r_{A,t} = \alpha_A + \beta_A \text{MKT}_t + \epsilon_{A,t}$							
13–23 months	-4.81	-2.07	-1.87	-1.34	3.27	3.29	3.33
24–35 months	-2.85	-1.64	-1.53	-1.17	2.53	2.47	2.72
36–59 months	-2.87	-1.61	-1.35	-1.13	2.44	2.24	2.10
60–119 months	-1.49	-0.97	-0.91	-0.56	1.35	1.29	1.42
120–239 months	-0.84	-0.09	-0.08	0.03	1.29	1.04	0.96
240 months and greater	-0.53	-0.17	-0.26	-0.14	0.70	0.53	0.17
Small-company growth	-8.45	-1.59	-0.97	-0.05	7.20	7.66	8.30
Other aggressive growth	-5.41	-0.97	-0.74	-1.06	4.80	4.65	4.58
Growth	-2.17	-0.97	-1.01	-1.17	1.64	1.48	1.52
Income	-0.39	-1.84	-1.40	-0.45	1.27	1.07	0.83
Growth and income	-0.51	-0.97	-0.87	-0.59	0.93	0.89	1.02
Maximum capital gains	-2.29	-1.47	-1.53	-1.95	2.16	1.75	1.34
Sector funds	-1.06	-3.96	-2.70	0.09	4.95	3.48	2.95
All funds	-2.13	-1.25	-1.07	-0.74	2.05	1.87	1.90
B. $r_{A,t} = \alpha_A + b_{A,1}\text{MKT}_t + b_{A,2}\text{SMB}_t + b_{A,3}\text{HML}_t + \eta_{A,t}$							
13–23 months	-1.68	-2.07	-1.96	-1.43	1.66	1.55	1.59
24–35 months	-1.63	-1.64	-1.52	-1.38	1.40	1.25	1.01
36–59 months	-1.29	-1.61	-1.46	-1.14	1.05	0.95	0.78
60–119 months	-0.92	-0.97	-0.94	-0.66	0.76	0.57	0.39
120–239 months	0.07	-0.09	-0.06	0.08	0.64	0.42	0.24
240 months and greater	0.12	-0.17	-0.13	0.17	0.76	0.50	0.05
Small-company growth	-0.41	-1.59	-1.16	-0.08	1.45	1.15	0.92
Other aggressive growth	-0.37	-0.97	-0.45	0.08	1.76	1.34	0.96
Growth	-0.88	-0.97	-0.86	-0.72	0.90	0.78	0.59
Income	-2.03	-1.84	-1.90	-1.74	0.74	0.61	0.47
Growth and income	-1.19	-0.97	-1.00	-1.11	0.79	0.68	0.44
Maximum capital gains	-0.28	-1.47	-1.32	-0.34	1.40	1.03	0.45
Sector funds	-1.84	-3.96	-3.51	-2.48	3.18	2.44	1.35
All funds	-1.07	-1.25	-1.14	-0.86	1.09	0.91	0.65

Table 3
Average Equity-Fund Alphas

The table reports the posterior mean of $\bar{\alpha}_A$, the average α_A across the set of designated funds, expressed in percent per annum. Also reported is the posterior probability (expressed in percent) that $\bar{\alpha}_A$ is negative. Fund performance, denoted by α_A, is defined as the intercept in the regression of the fund's excess return, $r_{A,t}$, on either the market benchmark index, MKT_t, (Panel A) or that benchmark index plus the size and value benchmark indexes, SMB_t and HML_t (Panel B). The OLS estimate of α_A, denoted by $\hat{\alpha}_A$, is based on the fund's available history and the corresponding history of the benchmarks. The posterior mean of α_A, denoted by $\tilde{\alpha}_A$, is based on the fund's available history as well as the returns from January 1963 through December 1998 on the benchmarks and additional non-benchmark indexes. The quantity σ_{α_N}, expressed in percent per annum, denotes the prior standard deviation of the intercept α_N in a regression of a non-benchmark return on the benchmark indexes. The prior for α_A is diffuse.

History length or investment objective	Posterior mean of $\bar{\alpha}_A$			Prob($\bar{\alpha}_A < 0$)		
		$\tilde{\alpha}_A$ for σ_{α_N} of			$\tilde{\alpha}_A$ for σ_{α_N} of	
	$\hat{\alpha}_A$	0	∞	$\hat{\alpha}_A$	0	∞
A. $r_{A,t} = \alpha_A + \beta_A \mathrm{MKT}_t + \epsilon_{A,t}$						
13–23 months	-6.31	-1.67	-1.19	100	100	98
24–35 months	-4.77	-1.67	-1.29	100	100	98
36–59 months	-4.12	-1.78	-1.39	100	100	100
60–119 months	-2.13	-0.99	-0.66	100	100	94
120–239 months	-1.07	-0.08	0.11	100	72	42
240 months and greater	-0.50	-0.27	-0.26	100	99	70
Small-company growth	-9.29	-1.28	0.01	100	100	50
Other aggressive growth	-6.79	-0.99	-1.39	100	98	89
Growth	-3.06	-1.00	-1.27	100	100	100
Income	-0.84	-1.56	-0.36	100	100	80
Growth and income	-1.02	-1.25	-1.01	100	100	100
Maximum capital gains	-4.74	-2.85	-3.01	100	100	100
Sector funds	-4.14	-2.45	-0.54	100	100	78
All funds	-3.83	-1.33	-0.97	100	100	98
B. $r_{A,t} = \alpha_A + b_{A,1}\mathrm{MKT}_t + b_{A,2}\mathrm{SMB}_t + b_{A,3}\mathrm{HML}_t + \eta_{A,t}$						
13–23 months	-1.54	-1.67	-1.06	100	100	100
24–35 months	-1.22	-1.67	-1.02	100	100	100
36–59 months	-1.32	-1.78	-1.10	100	100	100
60–119 months	-0.66	-0.99	-0.44	100	100	99
120–239 months	0.38	-0.08	0.46	0	72	1
240 months and greater	0.15	-0.27	0.21	10	99	17
Small-company growth	-0.00	-1.28	0.50	50	100	9
Other aggressive growth	0.27	-0.99	0.29	30	98	30
Growth	-0.78	-1.00	-0.61	100	100	99
Income	-2.04	-1.56	-1.64	100	100	100
Growth and income	-1.76	-1.25	-1.43	100	100	100
Maximum capital gains	-1.67	-2.85	-1.64	100	100	100
Sector funds	-1.43	-2.45	-1.25	99	100	99
All funds	-0.99	-1.33	-0.72	100	100	100

Table 4
Relative Precision of Estimates of Alpha for Equity Mutual Funds

The table reports statistics for the ratio of variances, $\text{var}(\tilde{\alpha}_A)/\text{var}(\hat{\alpha}_A)$. Fund performance, denoted by α_A, is defined as the intercept in the regression of the fund's excess return, $r_{A,t}$, on either the market benchmark index, MKT_t, (Panel A) or that benchmark index plus the size and value benchmark indexes, SMB_t and HML_t (Panel B). The OLS estimate of α_A, denoted by $\hat{\alpha}_A$, is based on the fund's available history and the corresponding history of the benchmarks. The posterior mean of α_A, denoted by $\tilde{\alpha}_A$, is based on the fund's available history as well as the returns from January 1963 through December 1998 on the benchmarks and additional non-benchmark indexes. The quantity σ_{α_N}, expressed in percent per annum, denotes the prior standard deviation of the intercept α_N in a regression of a non-benchmark return on the benchmark indexes. The prior for α_A is diffuse.

History length or	$\sigma_{\alpha_N} = 0$		$\sigma_{\alpha_N} = 2\%$		$\sigma_{\alpha_N} = \infty$	
investment objective	median	$\% < 1$	median	$\% < 1$	median	$\% < 1$
			A. $r_{A,t} = \alpha_A + \beta_A \text{MKT}_t + \epsilon_{A,t}$			
13–23 months	0.62	83	0.62	83	0.63	82
24–35 months	0.63	88	0.62	88	0.63	89
36–59 months	0.65	90	0.65	92	0.68	91
60–119 months	0.70	93	0.69	97	0.73	96
120–239 months	0.76	93	0.77	97	0.84	98
240 months and greater	0.84	88	0.87	92	0.98	74
Small-company growth	0.33	100	0.34	100	0.39	100
Other aggressive growth	0.54	90	0.55	90	0.59	90
Growth	0.76	87	0.76	89	0.80	87
Income	0.75	84	0.75	89	0.77	89
Growth and income	0.77	83	0.77	85	0.80	85
Maximum capital gains	0.70	92	0.73	95	0.85	88
Sector funds	0.62	96	0.63	96	0.66	95
All funds	0.68	89	0.68	91	0.72	89
			B. $r_{A,t} = \alpha_A + b_{A,1}\text{MKT}_t + b_{A,2}\text{SMB}_t + b_{A,3}\text{HML}_t + \eta_{A,t}$			
13–23 months	0.70	81	0.69	81	0.69	81
24–35 months	0.79	76	0.78	79	0.78	77
36–59 months	0.85	80	0.84	83	0.83	82
60–119 months	0.89	80	0.88	84	0.89	83
120–239 months	0.95	75	0.93	88	0.94	91
240 months and greater	1.00	53	0.98	71	0.99	83
Small-company growth	0.80	84	0.79	86	0.80	86
Other aggressive growth	0.81	86	0.79	88	0.78	88
Growth	0.88	78	0.86	82	0.88	82
Income	0.88	79	0.87	83	0.86	82
Growth and income	0.91	67	0.89	72	0.90	73
Maximum capital gains	0.96	66	0.94	75	0.96	83
Sector funds	0.74	95	0.74	95	0.76	95
All funds	0.86	78	0.84	82	0.85	82

Table 5

Comparison of Estimated CAPM and Fama-French Alphas under Alternative Roles for Non-Benchmark Assets

Each entry in Panels A and B is the median across funds of the absolute difference between posterior means of a fund's CAPM alpha and its Fama-French alpha, in percent per annum, under alternative prior beliefs about the degree to which each model prices the non-benchmark assets. Each entry in Panel C is the median absolute difference of the OLS alpha estimates. Fund performance, denoted by α_A, is defined as the intercept in the regression of the fund's excess return, $r_{A,t}$, on either the market benchmark index, MKT_t, (CAPM) or that benchmark index plus the size and value benchmark indexes, SMB_t and HML_t (Fama-French). The OLS estimate of α_A, denoted by $\hat{\alpha}_A$, is based on the fund's available history and the corresponding history of the benchmarks. The posterior mean of α_A, denoted by $\tilde{\alpha}_A$, is based on the fund's available history as well as the returns from January 1963 through December 1998 on the benchmark and additional non-benchmark indexes. The quantity σ_{α_N} denotes the prior standard deviation of the intercept α_N in a regression of a non-benchmark return on the benchmark returns, and the table compares $\tilde{\alpha}_A$ under a given non-zero σ_{α_N} but different specifications of the set of benchmark indexes. (The posterior means of the CAPM and Fama-French alphas are identical under $\sigma_{\alpha_N} = 0$.) The prior for α_A is diffuse.

	Length of fund's return history (months)						
Investment objective	13–23	24–35	36–59	60–119	120–239	≥ 240	All
A. Mispricing uncertainty for the non-benchmark assets (σ_{α_N}) equal to 2% per annum							
Small-company growth	0.58	1.01	0.74	0.67	0.70	na	0.69
Other aggressive growth	0.69	0.43	0.74	0.60	0.69	na	0.67
Growth	0.43	0.38	0.41	0.41	0.40	0.35	0.40
Income	0.48	0.38	0.47	0.49	0.57	0.41	0.47
Growth and income	0.18	0.20	0.26	0.19	0.33	0.23	0.20
Maximum capital gains	0.64	0.35	0.43	0.66	0.46	0.37	0.45
Sector funds	0.96	0.85	1.36	0.93	1.11	na	0.98
All funds	0.42	0.41	0.47	0.43	0.40	0.32	0.42
B. No reliance on the model to price the non-benchmark assets ($\sigma_{\alpha_N} = \infty$)							
Small-company growth	1.69	2.72	2.11	1.89	2.02	na	2.03
Other aggressive growth	2.10	1.29	2.38	1.75	2.20	na	1.89
Growth	1.25	1.11	1.18	1.13	1.13	1.02	1.14
Income	1.31	1.07	1.29	1.27	1.52	1.17	1.27
Growth and income	0.53	0.63	0.74	0.54	0.89	0.61	0.59
Maximum capital gains	2.00	1.16	1.64	2.15	1.30	1.12	1.34
Sector funds	2.43	2.41	3.40	2.37	2.91	na	2.47
All funds	1.18	1.18	1.38	1.26	1.20	0.99	1.24
C. No use of non-benchmark assets (OLS estimates)							
Small-company growth	11.21	11.33	7.98	4.72	3.96	na	8.07
Other aggressive growth	8.97	6.79	6.43	4.47	3.52	na	6.35
Growth	3.60	1.97	1.91	1.28	1.33	1.13	1.81
Income	1.80	1.43	1.30	1.44	1.62	0.93	1.44
Growth and income	1.79	1.73	1.54	1.01	0.85	0.68	1.25
Maximum capital gains	5.45	5.67	3.04	3.61	2.30	1.46	2.60
Sector funds	5.08	5.08	1.51	2.90	3.60	na	3.29
All funds	3.88	2.93	2.47	1.67	1.53	1.07	2.28

Table 6

Comparison of Estimated Alphas With and Without the Pricing-Model Restriction Applied to the Non-benchmark Assets

Each entry in the table is the median across funds of the absolute difference between posterior means of a fund's alpha, in percent per annum, under alternative prior beliefs about whether the pricing-model restriction applies to the non-benchmark assets. Fund performance, denoted by α_A, is defined as the intercept in the regression of the fund's excess return, $r_{A,t}$, on either the market benchmark index, MKT_t, (Panel A) or that benchmark index plus the size and value benchmark indexes, SMB_t and HML_t (Panel B). The posterior mean of α_A, denoted by $\tilde{\alpha}_A$, is based on the fund's available history as well as the returns from January 1963 through December 1998 on the benchmarks and additional non-benchmark indexes. The quantity σ_{α_N} denotes the prior standard deviation of the intercept α_N in a regression of a non-benchmark return on the benchmark indexes, and the table compares $\tilde{\alpha}_A$ under $\sigma_{\alpha_N} = 0$ versus $\sigma_{\alpha_N} = \infty$. The prior for α_A is diffuse.

	Length of fund's return history (months)						
Investment objective	13–23	24–35	36–59	60–119	120–239	≥ 240	All
A. $r_{A,t} = \alpha_A + \beta_A \text{MKT}_t + \epsilon_{A,t}$							
Small-company growth	1.69	1.40	1.46	1.57	1.08	na	1.55
Other aggressive growth	0.72	0.67	0.88	0.82	0.92	na	0.77
Growth	0.78	0.75	0.85	0.84	0.91	0.83	0.83
Income	1.32	1.00	1.30	1.23	1.56	1.19	1.28
Growth and income	0.48	0.56	0.69	0.52	0.64	0.48	0.54
Maximum capital gains	0.85	0.93	1.36	1.35	1.45	0.95	1.07
Sector funds	3.31	2.94	6.22	3.61	4.78	na	4.31
All funds	0.90	0.89	1.01	1.00	0.99	0.80	0.94
B. $r_{A,t} = \alpha_A + b_{A,1}\text{MKT}_t + b_{A,2}\text{SMB}_t + b_{A,3}\text{HML}_t + \eta_{A,t}$							
Small-company growth	1.58	2.05	1.39	1.62	1.76	na	1.62
Other aggressive growth	1.88	1.20	1.90	1.78	1.19	na	1.63
Growth	0.56	0.63	0.55	0.57	0.65	0.67	0.59
Income	0.33	0.40	0.51	0.41	0.84	0.67	0.40
Growth and income	0.34	0.37	0.41	0.36	0.46	0.36	0.37
Maximum capital gains	1.49	1.33	1.91	1.41	0.89	1.30	1.29
Sector funds	1.25	1.30	2.85	2.16	2.01	na	1.93
All funds	0.66	0.66	0.68	0.66	0.73	0.74	0.68

Table 7

Portfolios with the Highest Sharpe Ratio Under Priors for CAPM Mispricing and Skill of Fund Managers

The investment universe consists of 505 no-load equity mutual funds with at least three years of return history through December 1998. The benchmark factors are MKT, the excess return on the value-weighted stock market, SMB, the difference between returns on small and large stocks, HML, the difference between returns on high and low book-to-market stocks, and MOM, the difference between returns on stocks with high and low returns over the previous year (excluding the most recent month). The correlations and Sharpe ratios in Panels B and C are computed with respect to the same predictive distribution used to obtain the optimal fund portfolio in the same column. The weights in the combination of a given set of benchmarks with the highest Sharpe ratio are computed using the marginal predictive distribution of those benchmarks under diffuse priors (which is equivalent to using sample moments of the those benchmarks).

Mispricing uncertainty (σ_{α_N}) in percent per year:	0	0	0	0	1	1	1	1	2	2	2	2
Skill uncertainty (σ_δ) in percent per year:	0	1	3	∞	0	1	3	∞	0	1	3	∞
A. Portfolio Weights (×100)												
Ameristock Mutual Fund	-	-	22	-	-	-	22	-	-	-	20	-
BT Institutional:Equity 500 Index Fund	23	41	-	-	16	23	-	-	-	-	-	-
California Investment S&P 500 Index Fund	53	-	-	-	44	-	-	-	8	-	-	-
Cohen & Steers Realty Shares	-	-	-	-	-	-	-	-	3	5	-	-
Century Shares Trust	-	-	-	-	-	-	-	-	11	-	-	-
DFA AEW Real Estate Securities Portfolio	-	-	-	-	-	-	-	-	18	4	-	-
Elfun Trusts	-	-	-	-	-	5	-	-	-	-	-	-
First American Investment:Real Est Sec/Y	-	-	-	-	-	-	-	-	11	5	-	-
First Funds:Growth and Income Portfolio/I	-	6	-	-	-	5	-	-	-	-	-	-
Gabelli Asset Fund	-	-	-	-	-	-	-	-	-	-	5	-
Galaxy Funds II:Utility Index Fund	-	-	-	-	8	-	-	-	32	18	-	-
IDS Utilities Income Fund/Y	-	-	-	-	-	-	-	-	-	2	-	-
Legg Mason Eq Tr:Value Fund/Navigator	-	-	23	71	-	-	17	67	-	-	4	59
MassMutual Instl Funds:Small Cap Value Eqty/S	-	-	-	-	-	-	-	-	5	-	-	-
Oakmark Fund	-	-	1	-	-	-	4	-	-	-	8	-
Robertson Stephens Inv Tr:Information Age/A	-	-	-	15	-	-	-	11	-	-	-	5
T. Rowe Price Dividend Growth Fund	-	-	-	-	-	-	-	-	-	4	-	-
T. Rowe Price Equity Income Fund	-	30	-	-	-	59	-	-	-	57	-	-
UAM Fds Tr:Heitman Real Estate Portfolio/Inst	-	-	-	-	-	-	-	-	-	3	-	-
Vanguard Index Tr:Extended Market Port/Inv	24	-	-	-	32	-	-	-	12	-	-	-
Vanguard PrimeCap Fund	-	23	4	-	-	9	-	-	-	-	-	-
Weitz Series Fund:Hickory Portfolio	-	-	-	-	-	-	3	2	-	-	6	8
Weitz Series Fund:Value Portfolio	-	-	51	14	-	-	54	20	-	1	57	28
B. Correlation (×100) with the portfolio having the highest Sharpe ratio among all portfolios that combine the benchmark factors shown												
MKT	100	99	95	93	100	98	94	94	89	92	93	94
MKT, SMB, HML	48	49	51	34	49	56	53	37	64	68	55	43
MKT, SMB, HML, MOM	32	31	31	21	35	35	33	23	53	49	34	26
C. Sharpe ratio of the portfolio in Panel A divided by the highest Sharpe ratio for a portfolio that combines the benchmark factors shown (×100)												
MKT	96	104	152	234	96	104	153	230	107	114	156	224
MKT, SMB, HML	197	214	314	482	141	154	226	339	102	109	148	213
MKT, SMB, HML, MOM	292	318	466	716	144	157	230	346	81	86	118	169

Table 8

Portfolios with the Highest Sharpe Ratio Under Priors for Fama-French-Model Mispricing and Skill of Fund Managers

The investment universe consists of 505 no-load equity mutual funds with at least three years of return history through December 1998. The benchmark factors are MKT, the excess return on the value-weighted stock market, SMB, the difference between returns on small and large stocks, HML, the difference between returns on high and low book-to-market stocks, and MOM, the difference between returns on stocks with high and low returns over the previous year (excluding the most recent month). The correlations and Sharpe ratios in Panels B and C are computed with respect to the same predictive distribution used to obtain the optimal fund portfolio in the same column. The weights in the combination of a given set of benchmarks with the highest Sharpe ratio are computed using the marginal predictive distribution of those benchmarks under diffuse priors (which is equivalent to using sample moments of the those benchmarks).

Mispricing uncertainty (σ_{α_N}) in percent per year:	0	0	0	0	1	1	1	1	2	2	2	2
Skill uncertainty (σ_δ) in percent per year:	0	1	3	∞	0	1	3	∞	0	1	3	∞
A. Portfolio Weights ($\times$100)												
Ameristock Mutual Fund	-	-	10	-	-	-	9	-	-	-	6	-
CGM Realty Fund	-	3	-	-	-	4	-	-	-	4	-	-
Cohen & Steers Realty Shares	-	-	-	-	-	-	-	-	6	5	-	-
Columbia Real Estate Equity Fund	-	-	-	-	-	-	-	-	3	3	-	-
DFA AEW Real Estate Securities Portfolio	13	1	-	-	17	4	-	-	21	8	-	-
DFA Invest Grp:US Large Cap Value Port	2	-	-	-	-	-	-	-	-	-	-	-
First American Investment:Real Est Sec/Y	19	13	-	-	20	15	-	-	21	15	-	-
Galaxy Funds II:Utility Index Fund	8	5	-	-	13	11	-	-	22	18	-	-
Legg Mason Eq Tr:Total Return Fund/Navigator	40	10	-	-	34	6	-	-	21	-	-	-
Legg Mason Eq Tr:Value Fund/Navigator	-	-	-	44	-	-	-	43	-	-	-	41
Mutual Discovery Fund/Z	18	39	37	26	15	35	34	24	6	25	28	18
Oakmark Fund	-	-	2	-	-	-	3	-	-	-	4	-
T. Rowe Price Equity Income Fund	-	29	7	-	-	25	8	-	-	17	12	-
UAM Fds Tr:Heitman Real Estate Portfolio/Inst	-	-	-	-	-	-	-	-	-	3	-	-
Weitz Series Fund:Hickory Portfolio	-	-	-	1	-	-	-	3	-	-	-	6
Weitz Series Fund:Value Portfolio	-	-	45	29	-	-	47	31	-	-	50	35
B. Correlation ($\times$100) with the portfolio having the highest Sharpe ratio among all portfolios that combine the benchmark factors shown												
MKT	88	89	89	94	87	89	89	94	83	86	90	94
MKT, SMB, HML	75	74	66	55	75	75	65	55	73	74	64	54
MKT, SMB, HML, MOM	52	47	38	32	54	50	38	32	57	55	37	32
C. Sharpe ratio of the portfolio in Panel A divided by the highest Sharpe ratio for a portfolio that combines the benchmark factors shown ($\times$100)												
MKT	137	147	180	234	139	147	179	232	146	151	176	229
MKT, SMB, HML	66	70	86	112	66	70	85	111	70	72	84	109
MKT, SMB, HML, MOM	99	106	130	169	89	94	114	149	75	78	91	118

Table 9

Portfolios with the Highest Sharpe Ratio Under Priors for Four-Factor-Model Mispricing and Skill of Fund Managers

The investment universe consists of 505 no-load equity mutual funds with at least three years of return history through December 1998. The benchmark factors are MKT, the excess return on the value-weighted stock market, SMB, the difference between returns on small and large stocks, HML, the difference between returns on high and low book-to-market stocks, and MOM, the difference between returns on stocks with high and low returns over the previous year (excluding the most recent month). The correlations and Sharpe ratios in Panels B and C are computed with respect to the same predictive distribution used to obtain the optimal fund portfolio in the same column. The weights in the combination of a given set of benchmarks with the highest Sharpe ratio are computed using the marginal predictive distribution of those benchmarks under diffuse priors (which is equivalent to using sample moments of the those benchmarks).

Mispricing uncertainty (σ_{α_N}) in percent per year:	0	0	0	0	1	1	1	1	2	2	2	2
Skill uncertainty (σ_δ) in percent per year:	0	1	3	∞	0	1	3	∞	0	1	3	∞
A. Portfolio Weights ($\times$100)												
Alpine US Real Estate Equity Fund/Y	-	1	-	-	-	1	-	-	-	1	-	-
CGM Realty Fund	1	10	6	-	-	9	6	-	-	8	5	-
Cohen & Steers Realty Shares	14	14	9	-	14	14	9	-	14	14	10	-
Columbia Real Estate Equity Fund	11	12	6	-	10	12	6	-	9	11	7	-
DFA AEW Real Estate Securities Portfolio	28	19	-	-	27	19	-	-	26	18	-	-
First American Investment:Real Est Sec/Y	20	16	4	-	19	16	5	-	18	15	5	-
Gabelli Asset Fund	-	-	8	-	-	-	6	-	-	-	1	-
Galaxy Funds II:Utility Index Fund	14	11	-	-	17	13	-	-	21	18	-	-
Legg Mason Eq Tr:Value Fund/Navigator	-	-	-	48	-	-	-	46	-	-	-	44
Lindner/Ryback Small Cap Fund/Investor	-	-	-	2	-	-	-	2	-	-	-	1
Morgan Stanley Dean Witter Ist:US Real Est/A	-	-	2	-	-	-	2	-	-	-	3	-
Mutual Discovery Fund/Z	-	-	4	7	-	-	3	6	-	-	2	5
T. Rowe Price Dividend Growth Fund	-	-	14	-	-	-	15	-	-	-	16	-
UAM Fds Tr:Heitman Real Estate Portfolio/Inst	13	16	9	-	12	16	9	-	11	15	9	-
Weitz Series Fund:Hickory Portfolio	-	-	-	10	-	-	-	11	-	-	1	12
Weitz Series Fund:Value Portfolio	-	-	38	34	-	-	39	35	-	-	40	37
B. Correlation ($\times$100) with the portfolio having the highest sample Sharpe ratio among all portfolios that combine the benchmark factors shown												
MKT	75	75	89	94	75	75	89	94	75	75	89	94
MKT, SMB, HML	67	68	66	51	67	68	65	51	67	68	65	51
MKT, SMB, HML, MOM	61	62	50	30	61	62	50	30	61	62	50	30
C. Sharpe ratio of the portfolio in Panel A divided by the highest Sharpe ratio for a portfolio that combines the benchmark factors shown ($\times$100)												
MKT	175	178	177	222	176	179	178	222	179	181	179	221
MKT, SMB, HML	84	85	85	106	84	85	85	106	85	86	86	106
MKT, SMB, HML, MOM	54	55	55	69	54	55	55	69	55	56	55	68

Table 10

Portfolios with the Highest Sharpe Ratio Under Priors for Skill of Fund Managers and No Use of a Pricing Model ($\sigma_{\alpha_N} = \infty$)

The investment universe consists of 505 no-load equity mutual funds with at least three years of return history through December 1998. The benchmark factors are MKT, the excess return on the value-weighted stock market, SMB, the difference between returns on small and large stocks, HML, the difference between returns on high and low book-to-market stocks, and MOM, the difference between returns on stocks with high and low returns over the previous year (excluding the most recent month). The correlations, Sharpe ratios, and certainty-equivalent differences in Panels B through E are computed with respect to the same predictive distribution used to obtain the optimal fund portfolio in the same column. The weights in the combination of a given set of benchmarks with the highest Sharpe ratio are computed using the marginal predictive distribution of those benchmarks under diffuse priors (which is equivalent to using sample moments of the those benchmarks). The certainty-equivalent difference is computed with relative risk aversion equal to 2.75.

Skill uncertainty (σ_δ) in percent per year:	0	1	2	3	∞
A. Portfolio Weights (×100)					
CGM Realty Fund	-	2	2	1	-
Cohen & Steers Realty Shares	13	14	13	11	-
Cappiello-Rushmore Trust:Utility Income Fund	-	1	-	-	-
Columbia Real Estate Equity Fund	7	9	9	7	-
DFA AEW Real Estate Securities Portfolio	20	14	2	-	-
First American Investment:Real Est Sec/Y	14	12	11	6	-
Galaxy Funds II:Utility Index Fund	37	32	21	8	-
IDS Utilities Income Fund/Y	-	5	5	-	-
Legg Mason Eq Tr:Value Fund/Navigator	-	-	-	-	32
Morgan Stanley Dean Witter Ist:US Real Est/A	-	-	2	5	3
Oakmark Fund	-	-	-	-	2
T. Rowe Price Dividend Growth Fund	-	-	-	6	-
UAM Fds Tr:Heitman Real Estate Portfolio/Inst	9	12	11	8	-
Weitz Partners Value Fund	-	-	-	-	2
Weitz Series Fund:Hickory Portfolio	-	-	-	6	18
Weitz Series Fund:Value Portfolio	-	-	24	42	43
B. Correlation (×100) with the portfolio having the highest Sharpe ratio among all portfolios that combine the benchmark factors shown					
MKT	74	76	82	87	94
MKT, SMB, HML	66	66	66	64	51
MKT, SMB, HML, MOM	59	60	57	50	31
C. Sharpe ratio of the portfolio in Panel A divided by the highest Sharpe ratio for a portfolio that combines the benchmark factors shown (×100)					
MKT	190	191	184	185	221
MKT, SMB, HML	91	92	88	89	106
MKT, SMB, HML, MOM	59	59	57	57	69
D. Comparison to the portfolio that is optimal under $\sigma_\delta = 0$					
Certainty-equivalent difference (basis pts. per mo.)	0	1	6	23	133
Correlation (×100)	100	100	97	89	71
E. Comparison to the portfolio that is optimal when expected returns equal sample means					
Certainty-equivalent difference (basis pts. per mo.)	477	440	367	310	187
Correlation (×100)	68	69	75	80	91

Table 11

Comparisons of Portfolios of No-Load Funds Formed Under Various Prior Beliefs About Manager Skill and Pricing Models

All portfolios being compared are formed from an investment universe of 505 no-load equity mutual funds with at least three years of return history through December 1998. The pricing models considered are the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965), the three-factor Fama-French (1993) model, and the four-factor model of Carhart (1997), which adds a momentum factor to the Fama-French model. All of the reported correlations and certainty-equivalent differences are computed using the predictive distribution formed under the prior mispricing uncertainty (σ_{α_N}) and skill uncertainty (σ_δ) in the column heading. The certainty-equivalent difference is computed with relative risk aversion equal to 2.75.

Mispricing uncertainty (σ_{α_N}) in percent per year:	0	0	0	0	1	1	1	1	2	2	2	2
Skill uncertainty (σ_δ) in percent per year:	0	1	3	∞	0	1	3	∞	0	1	3	∞
A. Comparison of the portfolios formed with the same σ_{α_N} and σ_δ under different pricing models												
Certainty-equivalent difference (basis points per month)												
CAPM versus Fama-French	26	25	19	28	23	19	14	21	9	8	8	10
CAPM versus four-factor	59	61	34	18	50	51	28	13	24	27	19	5
Fama-French versus four-factor	24	29	19	3	19	23	17	2	10	13	13	1
Correlation (×100)												
CAPM versus Fama-French	87	89	93	94	87	91	95	96	97	96	97	98
CAPM versus four-factor	73	71	91	96	76	75	92	97	94	89	93	99
Fama-French versus four-factor	89	84	94	99	92	88	94	99	97	94	94	100
B. Comparison of the optimal portfolio to the portfolio that is optimal under $\sigma_{\alpha_N} = \infty$ and $\sigma_\delta = 0$												
Certainty-equivalent difference (basis points per month)												
CAPM	71	74	138	299	53	57	121	275	26	29	89	232
Fama-French	28	35	94	227	22	27	85	217	11	15	67	196
Four-factor	4	6	35	153	3	5	34	151	2	3	31	147
Correlation (×100)												
CAPM	73	71	70	60	77	74	71	62	95	90	73	65
Fama-French	87	82	73	69	91	86	73	69	96	93	73	70
Four-factor	98	97	87	69	98	97	87	69	99	98	88	69
C. Comparison of the optimal portfolio to the portfolio that is optimal when expected returns equal sample means												
Certainty-equivalent difference (basis points per month)												
CAPM	393	356	259	172	395	358	262	171	404	367	270	171
Fama-French	414	380	281	171	417	383	282	172	426	390	285	174
Four-factor	465	428	302	185	466	429	303	185	468	431	305	185
Correlation (×100)												
CAPM	95	95	94	94	94	94	93	95	83	87	89	95
Fama-French	81	82	84	93	80	82	84	93	77	79	84	93
Four-factor	67	67	82	94	67	68	82	94	68	68	81	93

Table 12

Coefficients in Regressions of Fund Returns on the Passive Asset Returns

The table reports posterior means and "t statistics" (posterior mean divided by posterior standard) of the intercept (δ_A) and slope coefficients in a regression of the fund's return on the returns of the eight passive assets. The passive assets are CMS, a spread between stocks with high and low HML betas but with both legs matched in terms of market capitalization (size) and book-to-market ratios, IP1–IP3, three portfolios formed by applying principal-component analysis to a set of 20 industry portfolios, MOM, the difference between returns on stocks with high and low returns over the previous year (excluding the most recent month), SMB, the difference between returns on small and large stocks, HML, the difference between returns on high and low book-to-market stocks, and MKT, the excess return on the value-weighted stock market.

	δ	CMS	IP1	IP2	IP3	MOM	SMB	HML	MKT
A. Posterior Mean (×100)									
Ameristock Mutual Fund	0.39	0	-0	-7	6	-6	-34	8	90
BT Institutional:Equity 500 Index Fund	0.09	-1	0	1	1	-2	-25	-2	96
CGM Realty Fund	-0.14	4	-14	-21	3	19	58	63	109
California Investment S&P 500 Index Fund	0.07	-1	0	1	1	-2	-24	-2	96
Century Shares Trust	-0.45	-10	2	-28	6	9	8	43	132
Cohen & Steers Realty Shares	-0.22	-13	-9	-41	17	28	69	53	102
Columbia Real Estate Equity Fund	-0.26	16	-10	-22	8	22	46	61	94
DFA AEW Real Estate Securities Portfolio	-0.59	7	-9	-24	9	23	57	64	99
First American Investment:Real Est Sec/Y	-0.29	21	-12	-25	5	16	47	71	106
Galaxy Funds II:Utility Index Fund	-0.57	38	-9	-48	-9	13	13	63	140
Legg Mason Eq Tr:Total Return Fund/Navigator	-0.16	11	1	3	8	-6	10	67	91
Legg Mason Eq Tr:Value Fund/Navigator	0.84	-11	4	23	-2	-5	-24	-14	82
Mutual Discovery Fund/Z	0.34	-18	6	7	2	-10	39	57	64
Robertson Stephens Inv Tr:Information Age/A	1.13	55	-6	48	-21	8	32	-133	75
T. Rowe Price Dividend Growth Fund	0.21	7	-1	-1	3	4	-3	20	78
T. Rowe Price Equity Income Fund	0.16	17	1	3	3	-6	0	36	70
UAM Fds Tr:Heitman Real Estate Portfolio/Inst	-0.32	-17	-10	-40	15	31	70	52	107
Vanguard Index Tr:Extended Market Port/Inv	-0.08	-2	-1	2	-3	6	54	-3	103
Vanguard PrimeCap Fund	0.44	-13	5	30	-10	-3	9	-32	68
Weitz Series Fund:Hickory Portfolio	0.56	-11	7	-19	-8	0	61	18	108
Weitz Series Fund:Value Portfolio	0.41	-0	2	-7	-9	-4	19	18	72
Hot-Hand Portfolio	0.01	-7	0	9	-4	19	48	-11	89
B. "t-statistic" (posterior mean divided by posterior standard deviation)									
Ameristock Mutual Fund	2.0	0.0	-0.1	-1.2	1.8	-1.6	-5.5	0.8	11.2
BT Institutional:Equity 500 Index Fund	1.9	-0.3	0.5	0.6	1.2	-1.5	-12.0	-0.9	38.9
CGM Realty Fund	-0.7	0.2	-2.9	-1.2	0.3	2.3	4.0	2.9	4.8
California Investment S&P 500 Index Fund	1.4	-0.5	0.5	0.6	0.9	-1.7	-11.6	-0.6	38.1
Century Shares Trust	-1.6	-0.7	0.7	-3.1	1.0	1.7	0.9	3.6	10.7
Cohen & Steers Realty Shares	-0.9	-0.6	-2.2	-2.7	2.1	3.7	5.2	2.8	5.5
Columbia Real Estate Equity Fund	-0.8	0.7	-2.3	-1.3	1.0	2.8	3.4	3.0	4.5
DFA AEW Real Estate Securities Portfolio	-2.1	0.4	-2.7	-2.0	1.3	3.7	5.3	4.1	6.5
First American Investment:Real Est Sec/Y	-1.2	1.0	-3.3	-1.7	0.6	2.5	4.2	4.0	5.9
Galaxy Funds II:Utility Index Fund	-2.2	2.6	-3.2	-4.8	-1.7	2.7	1.5	5.0	11.3
Legg Mason Eq Tr:Total Return Fund/Navigator	-0.2	1.0	0.6	0.4	1.8	-1.2	1.2	5.8	9.0
Legg Mason Eq Tr:Value Fund/Navigator	2.8	-0.8	1.5	2.5	-0.3	-0.7	-2.4	-1.1	6.6
Mutual Discovery Fund/Z	1.4	-1.8	3.1	1.0	0.5	-1.9	5.2	5.7	7.0
Robertson Stephens Inv Tr:Information Age/A	1.6	1.4	-0.9	1.6	-1.5	0.6	1.5	-3.9	2.1
T. Rowe Price Dividend Growth Fund	2.0	1.4	-1.6	-0.4	1.5	1.4	-0.7	3.9	17.0
T. Rowe Price Equity Income Fund	2.1	3.7	1.9	1.3	1.7	-2.9	0.1	9.0	18.5
UAM Fds Tr:Heitman Real Estate Portfolio/Inst	-1.1	-0.8	-2.5	-2.6	1.8	4.0	5.3	2.8	5.8
Vanguard Index Tr:Extended Market Port/Inv	-0.8	-0.5	-1.5	0.9	-2.6	3.3	21.4	-0.9	33.4
Vanguard PrimeCap Fund	2.8	-1.5	3.6	6.8	-3.4	-0.8	1.6	-4.7	10.3
Weitz Series Fund:Hickory Portfolio	1.4	-0.5	1.9	-1.4	-1.0	0.0	3.9	0.9	5.9
Weitz Series Fund:Value Portfolio	2.4	-0.0	1.3	-1.3	-2.6	-1.0	3.0	2.3	9.6
Hot-Hand Portfolio	0.2	-2.2	0.4	4.6	-3.4	13.5	22.4	-4.1	30.6

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