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Informed Manipulation

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Informed Manipulation [1]

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Abstract

In asymmetric information models of financial markets prices (imperfectly) reveal private information held by traders. Informed insiders thus have an incentive not only to trade less aggressively but also to trade in the "wrong" direction so as to "confuse" the market and increase the noise in the trading process. They thus manipulate the information content of the market prices for private profit. The result holds when the value of the insider's information does not decay immediately and when the market is uncertain about the presence of an informed insider. We prove the result for a Glosten-Milgrom type Bid-Ask model as well as for a Kyle type of market order model where the insider faces price uncertainty.

1 Introduction

In asymmetric information models of financial markets, prices (imperfectly) reveal private information held by traders. An informed trader is thus hurt precisely because his trading reveals (partially) the private information which allowed him to trade profitably in the first place. Kyle (1985) in fact shows that informed insiders strategically choose to trade less aggressively in a situation where their trading affects the equilibrium price compared to the competitive situation where it does not. In other words, the absolute value of the informed traders position on an asset is likely to be smaller in a strategic context, where the insider is informationally large in the market.

However, in the linear equilibrium of Kyle's model, the informed trader's trading strategy was monotonic in his information in the sense that he bought the asset when the asset was undervalued given his information, (i.e., when the value of the asset given his information was more than the (expected) price in the market for buying the asset) and sold the asset when the asset was overvalued (when the value of the asset given his information was less than the (expected) price in the market for selling the asset). To quote from Kyle (1985), the unique linear equilibrium of the model, "rules out a situation in which the insider can make unbounded profits by first destabilizing prices with unprofitable trades made at the nth auction, then recouping the losses and much more with profitable trades at future auctions." In fact, Kyle shows when the time between trades is short enough, the insider's dynamic strategy is essentially the same as the his strategy in the static case when he only gets to trade once, before his private information is revealed.

In this paper we model a different kind of strategic trading by informed insiders. In a dynamic setting, not only might informed traders want to hide their trades by trading less aggressively, but they might also find it in their interest to confuse the other market participants by trading in the "wrong" direction for short term losses but long term profits. For example, an insider who knows that the prospects of a certain asset are not good might actually start buying the asset to drive its price up and then sell it without its price falling too fast; and vice versa.

We look at a Kyle-type of model of repeated trading by an informed trader in a market with noise traders and competitive price-setters or market makers. However, our model differs from Kyle's model in a number of ways.

First, unlike Kyle's model we assume that the market does not know that an informed insider exists in the market for sure, in addition to the fact that they do not know the nature of his information (if he exists). Second, we assume that noise trader demand is bounded (or, equivalently, there is a upper bound on the order size which can be submitted in any period)[1].

We show that when the number of periods of trading (before all private information is revealed) is large enough then there exists no equilibrium in which any type of the informed trader always trades in the direction of his information, i.e., buys when he has good news and sells when he has bad news. Since we also prove the existence of an equilibrium, this shows that *every* equilibrium involves *every* type of the informed trader "manipulating" the market by sometimes trading in the wrong direction.

The kind of manipulation we consider is pure trade-based manipulation to use the classification scheme proposed by Allen and Gale (1992). The asymmetric information in the market is of course crucial for any kind of trading to take place. But apart from that, the informed trader does not announce any kind of false information (information-based manipulation; see for example Benabou and Laroque (1992)); neither does the informed trader manipulate based on actions that change the actual or perceived value of the asset (action-based manipulation).

The boundedness in the noise trades means that the insider faces a high degree of adverse selection pressure when he trades on his information. The same boundedness and the uncertainty the market faces about his existence then allows him to profitably manipulate so as to reduce the responsiveness of prices to trades and reduce the adverse selection pressure. In effect, by manipulating, the insider creates his own liquidity or noise.

One conclusion of the model is that the lower the anonymity of the trading process from the perspective of the insider the greater is the adverse selection pressure that he faces. Therefore, the greater is his incentive to manipulate the market. This suggests

[1] In fact we assume that noise trades have a finite (and therefore bounded) support. However, as will be clear later, boundedness is crucial and the finiteness assumption is made for technical reasons).

an interesting link between this paper and the recent literature on disclosure laws and manipulation (see, for example, John and Narayanan (1997), Fishman and Hagerty (1995): see Review of the Literature) , where disclosure laws effectively make the market non-anonymous from the insider's perspective.

Moreover, the incentive to manipulate by one type of informed trader depends on the strategies of other types of informed trader. In particular, manipulation by one type makes price less informative and less responsive to trades, and reduces the pressure on all types of informed traders to trade against their information and manipulate. Nevertheless, every type manipulates, given sufficiently many periods of trading.

Manipulative trading benefits the informed trader: his profits are higher than the benchmark where he is replaced by many short-lived informed traders, who buy when they have good news and sell when they have bad news. Since market makers earn zero expected profits, the gains of the informed trader are transformed into losses for the noise traders.

In the next section we discuss and relate this paper to the literature on manipulation. In section 2 we introduce the simple bid-ask model in the spirit of Glosten and Milgrom (1985) and prove that when information is sufficiently long-lived in every equilibrium every type of the informed trader will manipulate. The section concludes with a simple three-period example which demonstrates our result. In section 3 we introduce the Kyle-like market order model where we prove the identical result that the insider will manipulate in every equilibrium, if his information is long-lived. The difference between the two models is that the insider faces price uncertainty (or execution risk) in the market order model and does not in the bid ask model. Section 4 concludes.

1.1 Review of the Literature

The seminal paper on strategic trading by an informed insider with long-lived private information is by Kyle (1985). Kyle demonstrates the existence of a unique linear equilibrium where the insider trades in the direction of his information. In fact, Kyle shows that when the time between trades is short enough, the dynamic strategy of the informed trader is very like the trading strategy of a short-lived informed trader who can trade only once. Our model differs from Kyle's model in two important ways. We assume that the market faces uncertainty about whether the insider exists. Further, we also suppose that noise trading is bounded. We show that then the dynamic trading strategy of the insider is very different from the strategy of the informed trader who trades only once: the dynamic strategy involves manipulation.

A number of other authors have considered the definition and possibility of manipulation. Jarrow (1992) formulates sufficient conditions for manipulation to be unprofitable. These sufficient conditions are properties of the "reduced-form" price function[2] which implicitly incorporates the beliefs of the market about the equilibrium strategy used by the large trader. In contrast to Jarrow, we set up a model of strategic trading where the dependence of prices on the market's beliefs about the insider's strategy is made explicit. Further, while Jarrow looks at manipulation by an uninformed trader, we look at informed manipulation in this paper.

Allen and Gale (1992) also consider an explicit model of strategic trading in which some equilibria involve manipulation. However, manipulation by the informed trader is not profitable in their model. Manipulation by an uninformed trader is profitable, in the presence of certain restrictions on the strategy of the informed trader.

Allen and Gorton (1992) consider a model of uninformed manipulation in which an asymmetry in buys and sells by noise traders trades creates the possibility of manipulation. In every equilibrium of their model, the manipulator makes zero profits. Kumar and Seppi (1992) show how an uninformed trader makes profits when an information event takes place. The manipulator, by a sequence of trades in the spot and futures markets, is able to profit knowing that informed traders are about to trade based on the private information they receive. This example requires a "cash-settled" futures contract along with the underlying asset. Chakraborty and Yılmaz (1999) show that both the informed and uninformed insider can manipulate in an economy where there are a large number of rational traders, called followers, who have better information than the market about and noise traders and as a result seek to follow the insider's trades.

Fishman and Hagerty (1995) provide a one-period equilibrium model of profitable

[2]A reduced form price function is one in which the response of the market to the large trader's trades has already been incorporated, so that it is only a function of the large trader's trades.

manipulation when an uninformed insider successfully misleads the market into thinking he is informed. The profitable opportunity arises due to the mandatory disclosure of the insider's trades. John and Narayanan (1997) and Huddart, Hughes and Levine (1999) also look at the effect of mandatory disclosure laws on the insider's incentive to manipulate. The present paper is complementary to this literature because, under the assumption of bounded noise trades, if trading happens for long enough, the existence of the insider and the information that he has will be disclosed to the market in finite time.

Kyle (1984) considers a model of market manipulation in futures markets where a large trader can undetectably acquire a large position and manipulate by implementing a profitable squeeze. Benabou and Laroque (1992) consider a reputational model of manipulation in which an insider sometimes tells the truth to build a reputation and sometimes lies and profitably manipulates. They assume that he is a price-taker and can trade without being detected so that his trades do not affect prices and only his announcements do. Therefore, they do not consider a pure trade based model of manipulation in which no announcements are made or actions (other than trading) taken. Bagnoli and Lipman (1996) develop a model of action-based manipulation where the manipulator pools with someone who can take an action that alters the value of the firm. In their model, the manipulator takes a position, announces a takeover bid and unwinds his position. Vila (1989) presents another model of action-based manipulation where the manipulator pools with someone who is buying stock prior to a takeover bid in which the value of the firm will be increased.

2 The Bid Ask Model

2.1 Notation and Definitions

2.1.1 The Asset

We consider a market for one asset. The long-term return or the fundamental value of the asset, v, is not known to all the participants in the market. In particular, we assume $v \in V = \{0, 1\}$, with the prior probability that $v = 1$ equal to $\overline{v} \in (0, 1)$.

2.1.2 Traders, Types & Information Sets

There is one dynamic trader in the market who trades repeatedly before his private information is revealed. The private information or type of the informed trader is denoted by $\theta \in \Theta = \{I_1, I_0, N\}$. When $\theta = I_1$ the dynamic trader is informed and knows that the value of the asset $v = 1$. When $\theta = I_0$, the dynamic trader knows that the value of the $v = 0$. When $\theta = N$, the dynamic trader is a noise trader and his trading is driven by exogenous (e.g., liquidity) motives. The existence of this last type of trader is meant to capture the notion that the market faces an uncertainty regarding the existence of an informed dynamic trader who trades on the basis of his information[3].

Denote by $\mu(\theta|v)$ the probability that the dynamic trader is of type θ conditional on the value of the asset being v. We suppose that

$$\begin{aligned} \mu(\theta &= I_1|v = 1) = \mu(\theta = I_0|v = 0) \equiv \mu_I \in (0, 1) \\ \mu(\theta &= I_1|v = 0) = \mu(\theta = I_0|v = 1) = 0 \\ \mu(\theta &= N|v = 1) = \mu(\theta = N|v = 0) \equiv 1 - \mu_I \end{aligned}$$

2.1.3 The Trading Game and the Market Structure

We consider a model of sequential trading where market makers post bid and ask prices every period. The dynamic trader submits a demand in each period which the market

[3] In the bid ask model that we develop here, with only one trader, the existence of the type $\theta = N$ is also necessary to have profitable trade.

makers observe and post a new bid and ask price for the next period. Trading occurs for T periods before all private information is publicly revealed. Let $t = 1, 2...$ index the periods of trading.

We suppose that the market makers are competitive so that the prices they post in any period is equal to the expected value of the asset conditional on the observed history of trades up to and including that period. Notice that with probability μ_I *all* the trades have been submitted by the informed dynamic trader (whose trades are correlated with the fundamental value of the asset) and with probability $1 - \mu_I$, *all* the trades have been submitted by a noise trader (whose trades are uncorrelated with the fundamental value of the asset).

We assume that in each period the dynamic trader can submit a demand equal to a buy order of size 1, or a sell order of size 1 or can choose not to trade. Let $E = \{b, n, s\}$ denote the set of possible trade events which can occur in any period, with e its generic element. Thus $e = b$ denotes a buy order, $e = s$ a sell order and $e = n$ a no trade. The restriction to unit-sized buy and sell orders is done purely for simplicity and all results generalize to the case where a finite number of possible trade sizes can be submitted.

Let E^t denote the $t-$fold Cartesian product of E. This the set of possible $t-$period histories of trades observed by the market makers. Let e^t denote the generic element of E^t and let $e_{t'}(e^t)$ the $t^{'}-$th element of e^t, $t^{'} \leq t$. Denote by $E^0 = \{e^0\}$ the null history. Let $\overline{E} = \cup_{t=0}^{\infty} E^t$.

Let $H = E^{\infty}$ denote the set of infinitely long histories of trades with h being its generic element. Denote by $e^t(h)$ the first $t-$elements of h and by $e_t(h)$ the $t-$th element of h.

2.1.4 Strategies, Prices and Equilibrium

A (behavior) strategy for the dynamic trader is a function $\sigma : \Theta \times \overline{E} \to \Delta(E)$. Let $\sigma(\theta, e^t)(e)$ be the probability that the strategy σ assigns to action e after history e^t given his type θ. Let Σ be the set of strategies for the dynamic trader. Note that the definition of strategies above applies to the infinitely long version of the game (when $T = \infty$) and, when suitably truncated, applies to any finite version of the game also.

We suppose that when the dynamic trader is not informed, $\theta = N$, he buys, sells and does not trade with probability bounded away from zero for all histories:

$$(N) \quad \text{There exists } \varepsilon > 0 \text{ s.t. } \min_{e \in E} \sigma(N, e^t)(e) > \varepsilon \text{ for all } e^t \in \overline{E}$$

Therefore the distribution of the noise traders trades could depend on the history of trades.

The market makers choose a price function $p : \overline{E} \times \Sigma \to \mathbb{R}$. Note that like strategies, the price function has also been defined to apply to finite as well as infinite versions of the game.

We suppose that the market makers are "competitive" and set the prices equal to the expected value of the asset conditional on their information. Their information in any period is the history of trades up to and including that period. Thus,

$$(MM) \quad p(e^t; \widehat{\sigma}) = E[v|e^t; \widehat{\sigma}] = \Pr[v = 1|e^t; \widehat{\sigma}] \text{ for all } e^t \in \overline{E}, \text{ and for all } \widehat{\sigma}$$

The price in any period will also depend on the beliefs $\widehat{\sigma}$ of the market maker about the strategy of the dynamic trader. This dependence has been made explicit above, but will be dropped for notational convenience when it causes no confusion. Note that the conditional expectation in (MM) is well defined as long as σ satisfies (N).

We now define an equilibrium for this model. An equilibrium is a strategy σ for the informed trader and a price function p where the price after any history is the expected value of the asset given the history and given that the market makers' believe that the dynamic trader is trading according to σ; and σ is sequentially rational given p. Formally,

Definition 1 *An equilibrium consists of a pair (σ, p) such that*

1. *the price after any history $e^t \in \overline{E}$ is given by $p(e^t; \sigma)$ and satisfies (MM).*
2. *the strategies $\sigma(I_1)$ and $\sigma(I_0)$ are sequentially rational given p.*

Notice that this definition would correspond to that of a perfect Bayesian equilibrium if we endogenized the behavior of the market makers and assumed that they were Bertrand competing.

2.1.5 Strategies and Probability Measures

A strategy for any type θ of the dynamic trader $\sigma(\theta, .)$ will induce a probability distribution over the set of finite and infinite histories. Since we will frequently talk about the asymptotic properties of prices and beliefs, we will fix the underlying probability space as $\{H, \mathcal{C}, Q(\sigma(\theta))\}$, where H is the set of infinite histories, $\mathcal{C}$ is a sigma-algebra on H generated, in the usual fashion, from the collection of cylinders (see Billingsley (1995), for example) and $Q(\sigma(\theta))$ is the unique extension of the measure generated on the collection of all cylinders to $\mathcal{C}$. Notice that $\mathcal{C}$ does not depend on $\sigma(\theta, .)$.

2.1.6 Manipulative Strategies

We now define our notion of manipulation. An informed trader, in selecting his trade in any period, has to balance the short term profit from the trade with the long term effect his trade has on the beliefs of the market and hence on future profits. Recall that in Kyle (1985), the unique linear equilibrium of the model, "rules out a situation in which the insider can make unbounded profits by first destabilizing prices with unprofitable trades made at the nth auction, then recouping the losses and much more with profitable trades at future auctions" (pp 1323, emphasis added). This is precisely our notion of manipulation. We say that a strategy is manipulative if it involves the informed trader undertaking a trade in any period which gives him strictly negative short term profit. If such a strategy is used in equilibrium, then it must be to manipulate the beliefs of the market regarding his private information, which will enable him to recoup the short term losses and more in the future.

Definition 2 *A strategy σ is non-manipulative if for all $\theta \neq N$, for all $e^t \in \overline{E}$*

$$\begin{array}{ll} (i) & \sigma(\theta, e^t)(b) > 0 \Rightarrow E[v|\theta] \geq p(e^t b) \\ (ii) & \sigma(\theta, e^t)(s) > 0 \Rightarrow E[v|\theta] \leq p(e^t s) \end{array}$$

Otherwise it is manipulative.

We should note that the conditions a non-manipulative strategy must satisfy according to above definition are very weak. For example, not buying with probability one even if the price is lower than its expected value is not considered manipulative. Similarly not trading even if it is profitable to trade is non-manipulative. In other words, it is very easy for a strategy to qualify to be a non-manipulative one. In return a manipulative strategy is very restricted. Considering the fact that our main result will state that every equilibrium involves manipulation, working with this definition of manipulation makes our results only stronger in terms of characterization of equilibrium behavior.

2.2 The Main Result

We show that in the model described above when the private information of the dynamic agent is sufficiently long lived, i.e., T is sufficiently large, then every equilibrium involves manipulative trading by all types of the informed trader. We first discuss the intuition behind the result and then state and prove our results formally.

Suppose we have an equilibrium in non-manipulative strategies. The informed trader of type I_1, is not supposed to sell with positive probability in any period in this candidate non-manipulative equilibrium as the price at which he can sell is always less than the expected value of the asset conditional on his information. But suppose he deviates and sells in the first period. This results in a loss in the first period equal to $-(1-B_1)$ where B_1 is the first period bid price in this candidate non-manipulative equilibrium..

However, given the sell in the first period, the market makers put zero weight on the type $\theta = I_1$, henceforth, as this type was not supposed to sell in the first period. Since the type $\theta = N$ will sell in the first period with positive probability and the type $\theta = I_0$ may sell with positive probability in the first period, the beliefs of the market makers will attach positive probability to these two types only. Since the expected value of the asset conditional on either of these two types is at most equal to the prior value $\overline{v}$, and since all histories can be generated by the type $\theta = N$ with positive probability, all prices (in particular, all ask prices) will be less than or equal to $\overline{v}$, after every history. Therefore after the first period deviation, the informed trader who knows $v = 1$ will be able to buy at a price at most equal to $\overline{v}$, for ever after.

In contrast, if he follows his candidate equilibrium strategy then the price he trades at will converge to something greater than $\overline{v}$, almost surely. If the number of periods are long enough, then the deviation will yield him more profits. Therefore there will be no equilibrium where the strategies of both types of the informed trader are non-manipulative, for T large enough. In fact, below we show a stronger result, that for T large enough, there is no equilibrium where the strategy of one type of the informed trader is non-manipulative, regardless of the strategy of the other type.

We also prove below that an equilibrium always exists. Therefore we show, that for T large enough, every equilibrium involves manipulation by both types of the informed trader.

Below we first show that if one type of the informed trader uses a non-manipulative strategy, then the market makers learn his type in the long run, with probability one. Note that strategies in general, and non-manipulative strategies in particular, have all been defined for the case where $T = \infty$.

2.2.1 Convergence of Posteriors for Non-manipulative Strategies

We will look at a strategy σ and look at the induced probability measures $\{Q(\sigma(I_1), Q(\sigma(I_0), Q(\sigma(N))\}$ on $\mathcal{C}$. We will aim at proving convergence of the posteriors, $\Pr[\theta = I_1 | e^t, \sigma]$ of the market makers on Θ *to the truth with probability one,* given that $\sigma(I_1)$ is non-manipulative. Notice that convergence itself is an immediate consequence of the martingale convergence theorem.

For any $\mu_I < 1$, it is immediately seen that if $\sigma(I_1)$ is non-manipulative then $\sigma(I_1, e^t)(s) = 0$ for all e^t. To prove convergence, we will need to impose an additional restriction on σ, in particular on $\sigma(I_0)$. For any history, e^t and any $e \in E$, let $\pi_v(e, \sigma(I_v)|e^t, \sigma)$, $v = 0, 1$, denote the (expected) continuation payoff including that period of the informed trader of type I_v, from choosing action e after history e^t, and playing according to $\sigma(I_v)$ thereafter[4]. Recall that the way we have defined the payoffs, π_v could be infinity in the version of the trading game where $T = \infty$, because we are not discounting payoffs. The following is the additional restriction on σ that we will impose:

(R) For all $v \in \{0, 1\}$, for all e^t, if $\sigma(I_v, e^t)(e) > 0$

[4]The conditioning on σ is made explicit because the market makers beliefs and hence the prices and so the payoffs depend on σ.

$$\text{then either } e \in \arg\max_{e'} \pi_v(e', \sigma(I_v)|e^t, \sigma) \text{ or } \pi_v(e, \sigma(I_v)|e^t, \sigma) = \infty$$

Note that (R) is always satisfied by any equilibrium strategy of Γ_T for any T.

Lemma 1 *If $\sigma(I_1)$ is non-manipulative and $\sigma(I_0)$ satisfies (R) and σ_N satisfies (N), then the market makers learn the realization of $\theta = I_1$ with probability 1. Analogously, if $\sigma(I_0)$ is non-manipulative and $\sigma(I_1)$ satisfies (R) and σ_N satisfies (N), then the market makers learn the realization of $\theta = I_0$ with probability 1.*

Proof. Suppose first that $\theta = I_1$ so that $v = 1$. Suppose that $\sigma(I_1)$ is non-manipulative and $\sigma(I_0)$ satisfies (R) and σ_N satisfies (N). To prove that market makers will learn the realization of θ_D with probability 1, it suffices to find a set $A \subset H$ such that:

1. A is measurable: $A \in \mathcal{C}$
2. A has $Q(\sigma(I_1))-$probability $1 : Q(\sigma(I_1))(A) = 1$
3. A has $Q(\sigma(I_0))-$probability 0 and $Q(\sigma(N))-$probability $0 : Q(\sigma(I_0))(A) = Q(\sigma(N))(A) = 0$

Let

$$A = \{h \in H | e_t(h) \neq s \text{ for all } t\}$$

Since $A = \cap_n A_n$ is the countable intersection of the nested cylinders $A_n = \{h \in H | e_t(h) \neq s \ \forall \ t \leq n\}$, we must have $A \in \mathcal{C}$. Further,

$$Q(\sigma(I_1))(A) = 1 - Q(\sigma(I_1))(A_1^c) - \sum_{i=1}^{\infty} Q(\sigma(I_1))(A_{i+1}^c / A_i^c) = 1$$

Next, since $\sigma(N)$ satisfies (N), there exists $\varepsilon > 0$, such that

$$Q(\sigma(I))(A) \leq \lim_{t \to \infty} (1 - \varepsilon)^t = 0.$$

It remains to show that $Q(\sigma(I_0))(A) = 0$. Since $\sigma(I_0)$ satisfies (R), to generate an infinite history with no sell orders, it must be true that along any finite history of no sell orders, not selling yields either a continuation payoff of infinity or it maximizes the continuation payoff. But not selling for ever gives non-positive payoff in contradiction to the first possibility and, since $\mu_I < 1$ and σ_N satisfies (N), selling just once gives a strictly positive payoff after each such history, in contradiction to the second possibility. Therefore, for any finite history of no sells there exists a period where $\sigma(I_0)(s) = 1$, so that $Q(\sigma(I_0))(A) = 0$. The proof for the case where $\theta = I_0$ and $v = 0$ is analogous. □

2.2.2 Existence of Manipulative Equilibrium

We prove our main result in the context of the bid-ask model in two steps. First we show that an equilibrium exists for all T. Then we show that for T large enough there does not exist an equilibrium where either type of the informed trader trades non-manipulatively. Therefore, for T large enough every equilibrium involves manipulation by every type of the informed trader.

Proposition 2 *For all $\mu_I < 1$, $T < \infty$ an equilibrium exists.*

Proof. Since for finite T we have a game of incomplete information with compact action spaces, the proof of existence follows from Kakutani's Fixed Point Theorem by standard arguments.□

Proposition 3 *Fix $\mu_I < 1$. Suppose $\sigma(N)$ satisfies (N). There exists $\overline{T}(\mu_I)$ such that if $T > \overline{T}(\mu_I)$ there does not exist an equilibrium where either $\sigma(I_1)$ or $\sigma(I_0)$ is non-manipulative.*

Proof. In an equilibrium where $\sigma(I_1)$ is non-manipulative, if the informed trader is trading and $\theta = I_1$ then there cannot be any sell orders (unless the price equals 1, which is impossible as the posterior beliefs on noise trading is always non-zero, for any finite history). But, since every history is generated with positive probability if $\theta = N$, if the informed trader deviates and sells once, then all prices will be bounded above by $\overline{v}$, after every history. Therefore, given the convergence result of the lemma above, this deviation will be more profitable than following the candidate strategy, if T is large enough.□

Notice that in equilibrium the market makers know that the type I_1 sells with positive probability and the type I_0 buys with positive probability. Therefore, along a history of all buy orders, the ask prices in equilibrium will all be lower than the corresponding ask prices when the market makers believe that the informed trader behaves myopically, i.e., when they believe that I_1 always buys and I_0 always sells. Therefore, for type I_1, the profits from always buying, when market makers have the equilibrium beliefs, are higher than the profits from always buying, when the market makers believe that the informed trader is behaving myopically. But the profits from this strategy of always buying with equilibrium beliefs for the market makers must be weakly lower than type I_1's equilibrium profits. Therefore, the informed trader earns higher profits from his dynamic manipulative strategy, *when the market makers know that he is manipulating*, over the benchmark where he trades myopically, *and the market makers set prices knowing that he is trading myopically.* In other words, a long-lived informed trader, trading strategically and manipulating, will earn higher equilibrium profits than the total equilibrium profits of many short-lived informed traders. Since market makers earn zero expected profits, this translates into lower expected profits for the noise traders.

2.2.3 A Three Period Example

If a dynamic trader convinces the market makers that he has no valuable information, i.e., he is a noise trader, then the prices converge to the prior, $\overline{v}$. Therefore, for a low prior value $\overline{v}$, the informed trader who knows $v = 1$, $\theta = I_1$, has a greater incentive to manipulate in comparison to the informed trader who knows $v = 0$, $\theta = I_0$. On the other hand for low probabilities of having a dynamic informed trader, μ_I, the prices are not very sensitive to trading since it is more likely to have a noise trader. Hence, both types of the informed trader have a small incentive to manipulate. Consequently, in order to have a short horizon example we will choose $\mu_I = 0.90$, a relatively large possibility. Similarly, in order to have a simple example with only one type of the informed trader manipulating we have to choose a very high or a very low $\overline{v}$. Hence, we choose $\overline{v} = 0.10$, so that only the informed trader who knows $v = 1$, $\theta = I_1$, manipulates. To simplify things further we assume that no trade is not an option, and a noise trader buys and sells with equal probabilities for all histories. Under these assumptions, having a three periods is sufficient to have manipulation.[5] As we pointed out earlier, type I_0 does not manipulate and therefore sells in every period with probability 1. On the other hand, type I_1 does not manipulate in the last two periods since he does not have time to recoup his losses. Therefore, he manipulates only in the first period by playing a mixed strategy of buying with probability 0.9889 and selling with probability 0.0111. Then the equilibrium prices becomes: $p(b) = 0.9563$, $p(s) = 0.0018$, $p(bb) = 0.9776$, $p(sb) = 0.3752$, $p(bbb) = 0.9999$, $p(sbb) = 0.5604$. Therefore, the expected profits of type I_1 is 0.0662.

In order to calculate the benefits of manipulation we look at the prices when we restrict the strategies and the beliefs of the market makers to non-manipulative actions by the insider. Then the ask prices (along histories of all buys) become: $p(b) = 0.9567$, $p(bb) = 0.9778$, $p(bbb) = 0.9999$. Therefore, the expected profits of type I_1 is 0.0656. Hence the manipulation results in approximately 1% higher profits.

3 The Market Order Model

3.1 Notation and Definitions

3.1.1 The Asset

We consider, as before, a market for one asset. The long-term return or the fundamental value of the asset, v, is not known to all the participants in the market. In particular, we assume, as before, $v \in V = \{0, 1\}$, with the prior probability that $v = 1$ equal to $\overline{v} \in (0, 1)$.

[5] In fact, even under the assumption of $\overline{v} = 0.10$ we can get both sides manipulating if we choose a longer horizon.

3.1.2 Traders, Types & Information Sets

As before, there is one dynamic trader in the market who trades repeatedly before his private information is revealed. As before, the private information or type of the informed trader is denoted by $\theta \in \Theta = \{I_1, I_0, N\}$ with μ_I being the probability that he is informed and $1 - \mu_I$ being the probability that he is noise.

3.1.3 The Trading Game and the Market Structure

We consider a market order model of sequential trading with discrete trade sizes where market makers observe the aggregate market order in each period and post a price to execute the order. Let $t = 1, 2...$ index the periods of trading. The dynamic trader submits a order, x_t, in period t, and noise traders submit a order y_t. The market makers observe the aggregate order $e_t = x_t + y_t$. Trading occurs for T periods before all private information is publicly revealed.

We suppose that the market makers are competitive so that the prices they post in any period is equal to the expected value of the asset conditional on the observed history of market orders up to and including that period.

We assume that in each period the dynamic trader can submit an order x_t which belongs to the finite set $X \equiv \{-x_n, ... - x_1, 0, x_1, ..., x_n\}$ where $0 < x_1 < ... < x_n$. We assume that the noise trader demand, y_t, lies in the set $Y \equiv \{-y_m, ..., -y_1, 0, y_1, ..., y_m\}$ where $0 < y_1 < ... < y_m$. Let $E = X \oplus Y = \{-x_n - y_m, ..., 0, ..., x_n + y_m\}$ denote the set of possible market orders which can occur in any period, with e its generic element.

Let E^t denote the $t-$fold Cartesian product of E. This the set of possible $t-$period histories of orders observed by the market makers. Let e^t denote the generic element of E^t and let $e_{t'}(e^t)$ the $t^{'}-$th element of e^t, $t^{'} \leq t$. Denote by $E^0 = \{e^0\}$ the null history. Let $\overline{E} = \cup_{t=0}^{\infty} E^t$.

Let $H = E^\infty$ denote the set of infinitely long histories of orders with h being its generic element. Denote by $e^t(h)$ the first $t-$elements of h and by $e_t(h)$ the $t-$th element of h.

3.1.4 Strategies, Prices and Equilibrium

A (behavior) strategy for the dynamic trader is a function $\sigma : \Theta \times \overline{E} \to \Delta(X)$[6]. Let $\sigma(\theta, e^t)(x)$ be the probability that the strategy σ assigns to action x after history e^t conditional on θ. Let Σ be the set of strategies for the dynamic trader. Note that the definition of strategies above applies to the infinitely long version of the game (when $T = \infty$) and applies to any finite version of the game also.

We suppose that when the dynamic trader is not informed, $\theta = N$, he buys, sells and does not trade with probability bounded away from zero for all histories:

$$(N) \quad \text{There exists } \varepsilon > 0 \text{ s.t. } \min_{x \in X} \sigma(N, e^t)(x) > \varepsilon \text{ for all } e^t \in \overline{E}$$

Therefore the distribution of the noise traders trades could depend on the history of trades.

Let $g : \overline{E} \to \Delta(Y)$ describe the probability distribution of the noise in the market order, y_t, conditional on the history. We assume that the distribution of y_t is iid over time so that $g(y|e^t)$ does not depend on e^t. Further, $g(.)$ has full support on Y.

A strategy for the dynamic trader $\sigma : \Theta \times \overline{E} \to \Delta(X)$ and $g(.)$ define a function $\zeta : \Theta \times \overline{E} \to \Delta(E)$ which generates a probability distribution over E for each θ and e^t.

The market makers choose a price function $p : \overline{E} \times \Sigma \to \mathbb{R}$. Note that like strategies, the price function has also been defined to apply to finite as well as infinite versions of the game.

We suppose that the market makers are "competitive" and set the prices equal to the expected value of the asset conditional on their information. Their information in any period is the history of trades up to and including that period. Thus,

$$(MM) \quad p(e^t; \widehat{\sigma}) = E[v|e^t; \widehat{\sigma}] = \Pr[v = 1|e^t; \widehat{\sigma}] \text{ for all } e^t \in \overline{E}, \text{ and for all } \widehat{\sigma}$$

The price in any period will also depend on the beliefs $\widehat{\sigma}$ of the market maker about the strategy of the dynamic trader. This dependence has been made explicit above, but

[6]Strategies have been defined to be conditioned on the type of the dynamic trader and also on the public history of observed market orders. They could in principle depend on the history of the dynamic trader's own past trades, but in equilibrium will not, so that our restricition above is innocuous.

will be dropped for notational convenience when it causes no confusion. Note that the conditional expectation in (MM) is well defined as long as σ satisfies (N).

As before, an equilibrium is a strategy σ for the informed trader and a price function p where the price after any history is the expected value of the asset given the history and given that the market makers' believe that the dynamic trader is trading according to σ; and σ is sequentially rational given p. Formally,

Definition 3 *An equilibrium consists of a pair* (σ, p) *such that*

1. *the price after any history* $e^t \in \overline{E}$ *is given by* $p(e^t; \sigma)$ *and satisfies* (MM).
2. *the strategies* $\sigma(I_1)$ *and* $\sigma(I_0)$ *are sequentially rational given* p.

3.1.5 Strategies and Probability Measures

A strategy for any type θ of the dynamic trader $\sigma(\theta, .)$ will induce a probability distribution over the set of finite and infinite histories of market orders, through the induced distribution $\zeta(\theta, .)$. Since we will frequently talk about the asymptotic properties of prices and beliefs, we will fix the underlying probability space as $\{H, \mathcal{C}, Q(\sigma(\theta))\}$, where H is the set of infinite histories, $\mathcal{C}$ is a sigma-algebra on H generated, in the usual fashion, from the collection of cylinders and $Q(\sigma(\theta))$ is the unique extension of the measure generated on the collection of all cylinders to $\mathcal{C}$. Notice that $\mathcal{C}$ does not depend on $\sigma(\theta, .)$.

3.1.6 Manipulative Strategies

The notion of manipulation in this trading game is identical to the one we considered in the previous section. We say that a strategy is manipulative if it involves the informed trader undertaking a trade in any period which gives him strictly negative short term profit. If such a strategy is used in equilibrium, then it must be to manipulate the beliefs of the market regarding his private information, which will enable him to recoup the short term losses and more in the future. The only difference from the corresponding definition in the previous section is that now the price at which his trade will be executed is uncertain from the informed trader's perspective. Therefore, we define profits from any trade in terms of expected prices conditional on that trade.

Definition 4 *A strategy σ is non-manipulative if for all $\theta \neq N$, for all $e^t \in \overline{E}$*

$$\begin{array}{ll} (i) & \sigma(\theta, e^t)(x) > 0, x > 0 \Rightarrow E[v|\theta] \geq E[p(e^t e)|e^t, x] \\ (ii) & \sigma(\theta, e^t)(x) > 0, x < 0 \Rightarrow E[v|\theta] \leq E[p(e^t e)|e^t, x] \end{array}$$

Otherwise it is manipulative.

3.2 The Main Result

We show that in the model described above when the private information of the dynamic agent is sufficiently long lived, i.e., T is sufficiently large, then every equilibrium involves manipulative trading by all types of the informed trader. We first discuss the intuition behind the result and then state and prove our results formally.

Suppose we have an equilibrium in non-manipulative strategies. The informed trader of type I_1, who knows that the fundamental value of the asset $v = 1$, is not supposed to sell with positive probability in any period in this candidate non-manipulative equilibrium as the price at which he can sell is always less than 1, the expected value of the asset conditional on his information. But suppose he deviates and goes on selling in every period. Since the noise trader demand is bounded below by $-y_m$, the market makers will observe a market order $y < y_m$ with arbitrarily large probability, if he sells long enough. Since all histories can be generated by the type $\theta = N$, the market makers will then put zero weight on cannot be consistent with the type $\theta = I_1$ trading in the market, and the price will be less than or equal to $\overline{v}$, with probability 1, in every period to follow, no matter what history occurs. The trader can thus buy any amount at a price less than or equal to $\overline{v}$ in every period.

In contrast, if he follows his candidate equilibrium strategy then the price he trades at will converge to something greater than $\overline{v}$, almost surely. If the number of periods are long enough, then the deviation will yield him more profits. Therefore there will be no equilibrium where the strategies of both types of the informed trader are non-manipulative, for T large enough.

We prove our main result in the context of the market-order model in two steps, as in the bid-ask model. First we show that an equilibrium exists for all T. Then we show that for T large enough there does not exist an equilibrium where either type of the informed trader trades non-manipulatively. Therefore, for T large enough every equilibrium involves manipulation by every type of the informed trader.

Proposition 4 *For $T < \infty$, an equilibrium always exists.*

Proof. Since for finite T we have a game of incomplete information with compact action spaces, the proof of existence follows from Kakutani's Fixed Point Theorem by standard arguments.□

Proposition 5 *There exists $\overline{T}(\mu_I)$ such that if $T > \overline{T}(\mu_I)$, there does not exist an equilibrium where any type of the informed trader play non-manipulative strategies.*

Proof. Suppose type I_1 of the informed trader uses non-manipulative strategies. For $\mu_I < 1$, it is straightforward to verify that the price after any history will be strictly positive and strictly less than one. Therefore, in the candidate equilibrium the type I_1 will never submit a sell order. Thus,

$$\min[\text{support}\{\zeta(I_1, e^t)\}] \geq -y_m \forall e^t \in \overline{E}$$

Suppose the type I_1 deviates and submits an order $x_n < 0$ every period until $y_t < -x_n - y_m$, following which he submits an order of $x_{n'} > 0$ every period. By the SLLN, for every $\varepsilon > 0$, there exists $\overline{T}(\varepsilon)$ such that $\Pr[y_t \geq -x_n - y_m \forall t \leq \overline{T}(\varepsilon)] < \varepsilon$. Therefore, all prices from the $\overline{T}(\varepsilon)$ onwards will be less than or equal to $\overline{v}$, with probability $1 - \varepsilon$. To complete the proof it suffices to show that the price cannot remain below $\overline{v}$, if he follows his candidate equilibrium strategy.

Consider the set $A = \{\omega | e_t(\omega) \notin \text{support}\{\zeta(I_1, e^{t-1}(\omega)\}^c \cap \text{support}\{\zeta(I_0, e^{t-1}(\omega)\} \forall t\}$. Clearly A is measurable. Furthermore, A has $Q(\sigma(I_1))-$probability 1. If A has positive probability under $Q(\sigma(I_0))$ we must have the set $A' = \{\omega | e_t(\omega) \in \text{support}\{\zeta(I_1, e^{t-1}(\omega)\} \cap \text{support}\{\zeta(I_0, e^{t-1}(\omega)\} \forall t\}$ also having positive probability under $Q(\sigma(I_0))$. But along any such path $\omega \in A$, since $\sigma(I_1)$ is non-manipulative, the trader I_0 always be submitting buy orders or not trading with positive probability. Therefore, along such a path, the pure strategy of never selling with positive probability has positive weight in the mixed strategy by the seller of type I_0. Since this does not yield him non-negative profits and since the strategy of selling just once yields him strictly positive profits, we have a contradiction with equilibrium in any finite version of the trading game. □

4 Concluding Remarks

The primary achievement of this paper is to show in Kyle (1985) type of models with one insider trading repeatedly, if the number of periods is large enough, then the equilibrium will involve a non-linear (in fact, non-monotonic) manipulative trading strategy of the insider. This is because, unlike in Kyle's model, the market faces uncertainty about the existence of the insider and noise trading is bounded. Due to bounded noise trading some of his trades will be revealed in the long run. If he is trading non-manipulatively this will also reveal his information and reduce his profits. This leads the insider to manipulate to try and signal that he is not trading on any information, provided there is uncertainty about his existence in the market place.

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