The Rodney L. White Center for Financial Research

Strategic Voting and Proxy Contests

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Abstract

We analyze proxy fights where privately informed shareholders are uncertain about the management ability of the raider. We show that the shareholders vote towards compensating for the initial bias formed by supermajority amendments and the shares that the incumbent controls. Consequently, the amount of support given to the raider is an increasing function of the initial bias favoring the incumbent. This compensating behavior may reverse the effects of the incumbent's defensive strategies. More specifically, we show that anti-takeover measures may increase the likelihood of a takeover by an inferior raider. In contrast to earlier sincere voting models we also show that simultaneous (strategic) voting can serve as a Pareto dominant voting mechanism for aggregating dispersed information.

1 Introduction

The effects of strategic behavior of shareholders during the tendering stage of a takeover process have been extensively analyzed since the pivotal work of Grossman and Hart (1980). However, the literature lacks a similar analysis of proxy fights. The aim of this paper is therefore to analyze the consequences of the strategic behavior of privately informed shareholders in proxy fights for corporate control. Nevertheless, the results established here could easily be applied to any proxy contest or other collective decision making process where there is private information.

We assume identical preferences for the non-contender shareholders with each shareholder preferring the better manager as long as he himself is not a candidate.[1] The only thing that distinguishes one shareholder from another is the nature of his private information. In this sense our model is similar to that of Harris and Raviv (1988a) where the authors assume that "each (shareholder) will vote for the candidate he believes is best". However, we will show that such behavior, which we call sincere[2] voting, is often irrational.

To understand the logic behind rational voting it is useful to consider briefly the winner's curse in common value auctions. In first price sealed bid auctions where each bidder receives a private signal about the common value prior to bidding, if every bidder offers his expected value given his private signal, then the winner loses money in expected terms. This follows from the fact that the winner has the most optimistic and hence most likely to be overvalued signal. The solution to this problem is for every bidder to condition his bid not only on his private signal but also on what must be true about the value of the object if his bid is the highest.

Strategic voting has a similar feature; namely, one's decision has an effect on his utility only in certain state(s) of the world, i.e., when the agent is pivotal. Therefore, each strategic shareholder conditions his decision on the information which must be true when he is pivotal as well as on his private signal.

The following example illustrates why sincere voting -i.e., voting on the basis only of one's private information- is irrational.

[1] The analysis could easily be extended into models with a conflict of interest among shareholders. See Maug and Yılmaz (1998).

[2] This kind of behavior is also called naive voting.

Consider a proxy fight between a raider with negligible holdings against an incumbent with 10% of the voting rights under simple majority rule. The remaining 90% voting rights are distributed among many small shareholders. Furthermore, we will assume that a contender always supports himself in the proxy contest due to private benefits of control. Each small shareholder receives a noisy signal about the management ability of the contenders. For the sake of argument let's assume that every non-contender shareholder vote sincerely. Consider a shareholder with a signal favoring the incumbent. Is it optimal for this shareholder to vote for the incumbent given everyone else votes sincerely? If he is pivotal, what then must be true about signals the others have received? Considering the fact that the raider owns a negligible amount of shares but yet still receives half of the proxies, five out of every nine small shareholders must have supported him in comparison to four out of nine for the incumbent. This means that there are more signals favoring the raider. Therefore, it is optimal to vote for the raider if everyone else is behaving sincerely. Hence, sincere voting is not an equilibrium.

It turns out that in equilibrium shareholders optimally compensate for the *initial bias* formed by initial holdings and the supermajority rule used in the contest. Each shareholder knows that a candidate who is favored by the initial bias does not need as many votes as his opponent does in order to win. Thus, one should not vote for such a candidate unless he has a strong positive signal, since there is a higher risk of affecting the outcome in a negative way and choosing him even if he is inferior. After all, there will be sufficient number of shareholders with a strong positive signal to secure a victory for such a candidate as long as he is the better manager. Therefore, the amount of support a candidate receives is positively correlated with the initial bias against him.

A natural concern regarding this kind of strategic behavior is its informational efficiency. In other words, how much of the privately held information is aggregated and gets reflected in stock prices following an announcement of a proxy contest? Clearly, the best possible outcome can be achieved if every shareholder can have access to all of the information.

This brings us to the topic of SEC regulations about shareholder communications prior to a proxy contest. In recent years, corporate ownership became more concentrated in large pension funds and in the hands of professional investment managers.[3] This creates a window of opportunity for shareholder activism since these larger investors have the correct incentives for monitoring. However, earlier SEC rules severely restricted shareholder communications and thus privately informed shareholders could not aggregate the information and act upon it.[4] This and other restrictive regulations caused the California Public Employees' Retirement System (Cal PERS) and other institutional investors and shareholders' rights groups such as United Shareholder Association (USA) to demand a policy change in a wide range of issues about proxy rules and shareholder rights.

We focus on shareholder communication prior to a proxy contest and analyze which level of information aggregation is possible in an environment in which such communication is very limited. In this paper we show that even in the absence of such communication proxy fights aggregate information almost perfectly as long as shareholders (privately informed or not) act rationally.

In fact, we show that the strategic behavior of shareholders increases efficiency so that the equilibrium behavior is the optimal voting rule (for shareholders) for any ownership structure. Furthermore, we prove that the probability of choosing the inferior management is arbitrarily close to zero for widely held firms regardless of which supermajority rule is used. Therefore, an inferior incumbent can guarantee his control only if he owns the necessary fraction of voting rights, since otherwise the probability of staying in office is zero for him. This contradicts Harris and Raviv's claim that guaranteeing control does not require an inferior incumbent to own the majority of the equity of a widely held firm. Although we agree with Harris and Raviv that an announcement of a proxy fight increases the stock price, since there is a possibility of an improvement in value, we show that this increase is higher in more widely held firms with privately informed shareholders, and is also affected by the particular supermajority rule and the quantity of shares held by the contenders.

We also show that strategic behavior a shareholder varies with the amount of his shares; larger shareholders have a tendency to vote more sincerely. In other words, smaller shareholders more aggressively compensate for the initial bias favoring one of the contenders.

[3]See Gilson and Kraakman (1991).

[4]See Black (1990) and Pound (1991)

Next we discuss defense mechanisms and anti-takeover measures. If strategic shareholders over-compensate for the initial bias, then it may not be optimal for the incumbent to have a bias favoring him. There are two kinds of errors the shareholders can make; keeping an inferior incumbent and replacing a superior incumbent with an inferior contender. Obviously, an incumbent would like to minimize the probability of the former and maximize the probability of the latter. We demonstrate that anti-takeover measures may have perverse effects on both kinds of errors. Specifically, we show that dual class shares, higher supermajority amendments and incumbent holdings may increase the probability of removing a superior incumbent or decrease the probability of keeping an inferior incumbent due to the shareholder's "overreaction" to the initial bias.

We discuss the related literature in the following section. In Section 3, we introduce the model. We analyze the equilibrium and present the results in Sections 3-7. We include our concluding remarks in the last section.

2 Related Literature

The literature on voting rules and information aggregation during a proxy contest is mostly empirical. Pound (1991) analyzes earlier regulations of the SEC and concludes that they have created inefficiencies. Strickland, Wiles and Zenner (1996) analyze the empirical evidence concerning how united small shareholders can aggregate information and enhance the shareholder value. Pound (1988) and Brickley, Lease and Smith (1988) show that ownership structure has a direct effect on the outcome of the proxy contests. There is also a ever growing literature on how voting rights and proxy contest themselves influence the stock prices.[5]

Harris and Raviv (1988a) is the first to model the proxy contest under asymmetric information. Bhattacharya (1997) models the proxy contest as one (pivotal) shareholder's strategic decision about information acquisition. Grossman and Hart (1988) and Harris and Raviv (1988b) offer a theoretical treatment of voting rights, dual class shares, supermajority amendments and their effects on efficiency. Yılmaz (1996) analyzes how supermajority amendments may increase the shareholder value in a model of

[5]See Bhagat and Brickley (1984), Jarrel and Poulsen (1987 and 1988) and Bhagat and Jefferis (1992).

incomplete information. Stulz (1988) investigates the effects of the incumbent's voting rights on the probability of a successful takeover.

Our paper is also closely related to the literature on jury decision making with private information and the Condorcet Jury Theorem. The Condorcet Jury Theorem states that a majority based rule is less likely to make a "mistake" than any single voter. Several proofs have been offered for variations and extensions of this claim.[6] One shortcoming of these papers is that they implicitly assume that each voter behaves sincerely.[7] Recently, several studies showed that such a "naive" voting is often irrational.[8] Among these, Austin-Smith and Banks (1996) and Feddersen and Pesendorfer (1997b) are more closely related to our model, since they also consider identical preferences. The former paper was the first to point out that naive voting is often irrational in this setting and therefore previous proofs of the Condorcet Jury Theorem do not necessarily hold. On the other hand, Feddersen and Pesendorfer analyze the asymptotic features of information aggregation and show that the probability of each type of error, i.e., convicting an innocent defendant or acquitting a guilty defendant is bounded away from zero under unanimity rule. In contrast, they show that both of these probabilities converge to zero under any other supermajority rule when the number of jurors get arbitrarily large. Hence, they conclude that requiring unanimity in jury verdicts is inferior to requiring any other supermajority rule. In addition to these theoretical models there is also recent experimental evidence which finds support to the strategic voting hypothesis.[9]

We differ from the literature on the Condorcet Jury Theorem, in a number of ways. First, we analyze an informationally richer environment allowing some shareholders to be more informed than others. Particularly, we do not restrict ourselves with the binary signal assumption which is used in the existing literature and show that such an addition to the model changes the nature of the unique symmetric equilibrium. Second, we analyze the non-asymptotic features of strategic voting as well as its asymptotic properties. [In a simultaneous paper Maug (1998) shares our concerns about non-asymptotic

[6]See Ladha (1992), Miller (1986), Young (1988).

[7]Sincere voting means that a voter behaves as if his vote alone determines the outcome.

[8]See Austin-Smith and Banks (1996); Feddersen and Pesendorfer (1996a, 1996b, 1997a, 1997b); Myerson (1994b) and McLennan (1996).

[9]See Ladha, Miller and Oppenheimer (1997).

properties of strategic voting using Feddersen and Pessendorfer's (1997b) setting. Unlike Feddersen and Pesendorfer (1997b), he analyzes the set of asymmetric equilibria in addition to the symmetric equilibrium.] Third, we investigate the effects of strategic voting on anti-takeover mechanisms and stock prices. Finally, we are concerned with strategic behavior of larger shareholders in contrast to smaller shareholders.

Our analysis is complemented by Maug and Yılmaz's (1998) analysis on strategic voting in the presence of conflicts of interest. They search for the optimal voting mechanism to resolve the problem stated by Feddersen and Pesendorfer (1997a) that voters with heterogenous preferences waste their private information in a standard voting game. Maug and Yılmaz (1998) show that a multi-class voting mechanism (similar to the practice under Chapter 11) aggregates the dispersed information optimally once there is a conflict of interest among shareholders.

3 The Model

Let $\omega \in \Omega = \{0, 1\}$ be the true state of the world. There are n small shareholders each with a single share and two candidates, namely the incumbent, I, and the raider, R. The fraction of shares held by a contender is denoted by α_j, where $j \in \{I, R\}$. Given the true state of the world, small shareholders have identical preferences represented by $u(., \omega)$. Identical preferences mean that each non-contender shareholder prefers the better manager, i.e., the one who is capable of supplying higher cash flows. For simplicity, we assume that the absolute value of the difference between cash flows under two candidates is constant over the states of the world. Furthermore, each small shareholder prefers the incumbent if the true state of the world is 0 and prefers the raider otherwise, i.e., $u(R, 1) - u(I, 1) = u(I, 0) - u(R, 0) > 0$. However, due to the private benefits of control a candidate always prefers his/her own management. The shareholders do not know the true state of the world, but instead share the same prior belief. Let the common prior be a uniform distribution. That is $\pi(0) = \pi(0) = \frac{1}{2}$. Furthermore, conditional on the true state of the world each small shareholder, i, receives a signal $s_i \in [0, 1]$ independently drawn from an identical distribution. Let $f(s|\omega)$ be the density function for the probability of receiving signal s, if ω is the true state of the world. $F(.|\omega)$ stands for the cumulative probability function. We assume that $f(.|1)$ is continuous and strictly increasing in the $[0, 1]$ interval with $f(0|1) = 0$.

This means that if the true state of the world is 1 then receiving a lower signal is less likely than receiving a higher signal. For simplicity, we will take $f(s|0) = f(1-s|1)$ for all s. Each small shareholder updates his belief given his signal s and let $\beta(\omega|s)$ stand for this posterior belief. Therefore, a shareholder with a signal $s = \frac{1}{2}$ has a posterior belief identical to the common prior, since receiving such a signal is equally probable in both states of the world. This means that $s = \frac{1}{2}$ is an uninformative signal. Similarly, a shareholder with a signal very close to 1 is very informed and is almost sure that the true state of the world is 1, since receiving a signal that high is almost impossible if the true state of the world is 0, i.e., $f(1|0) = 0$.

The raider, R, needs the support of fraction $\rho \in [\frac{1}{2}, 1]$ of the outstanding shares to get elected. We say that ρ is a *nominal supermajority rule* if $\rho > \frac{1}{2}$. Let r stand for the *effective (supermajority) rule* where $r = \frac{\rho - \alpha_R}{1 - \alpha_R - \alpha I}$. Note that r is the fraction of the independent votes the raider needs to receive to replace the incumbent, since we assume that no candidate votes for another contender. Obviously the requirement that at least rn shareholders should support the raider means that the number of votes the raider needs is $\| rn \| + 1$, i.e. the smallest integer greater than or equal to rn. We will slightly abuse the notation and use rn instead of $\| rn \| + 1$.

Each shareholder takes one of the two actions: voting for the raider or for the incumbent.[10] Let $\sigma_i : [0, 1] \to \{I, R\}$ stand for a (pure) strategy of voter i. A mixed strategy $\bar{\sigma}_i$ is a probability distribution over pure strategies. Let $\bar{\sigma}_{-i}$ denote the profile of all such strategies except $\bar{\sigma}_i$. We will analyze the symmetric Nash equilibria of this game.

The careful reader may have already noticed that we do not model a tender offer stage. This is a model of proxy contest stage only.[11] Thus, the results of this model can be applied to any voting contest in corporate finance as long as members are privately informed.[12]

We should note that several assumptions of this model could be relaxed without changing the results qualitatively at the cost of using more notation in the proofs. These

[10] Note that abstaining and supporting the incumbent have the same affect on the outcome since the raider needs to receive a certain fraction of the outstanding shares. Therefore, we can simplify the model by not allowing abstention as a choice.

[11] However, one could assume that a hostile tender offer has been unsuccessful previously.

[12] By members we mean shareholders in a proxy contest, board members in a meeting, creditors in a bankruptcy situation, etc.

assumptions include uniform priors, constant (in absolute value) difference between cash flows and symmetric probability density functions. In order to have a tractable model all we need to assume is to have the probability density functions satisfy the monotonic likelihood ratio property.[13]

4 Equilibrium

By an equilibrium we mean a non-trivial (or responsive) symmetric Nash equilibrium. In all voting models including the one presented above there are trivial equilibria of this game in which each shareholder votes for the same candidate irrespective of his private information. These equilibria are not only uninteresting but also fail to exist if the model is perturbed slightly. If we assume that there is a possibility of each shareholder behaving like noise, the trivial equilibria dissappear since with noise, there is a positive probability of being pivotal for every candidate. In fact, in an earlier version we show that trivial equilibria are ruled out even in the absence of a potential noise as long as there is a small possibility of having fully informed[14] shareholders.[15]

Proposition 1 *In the unique equilibrium there exists a cut-off signal s^* such that $\sigma_i^*(s) = I$ for all $s < s^*$ and $\sigma_i^*(s) = R$ for all $s > s^*$.*

Proof: See appendix. $\diamond$

As we have seen above, each shareholder conditions his decision (i) on his private signal and (ii) on what must be true whenever he is pivotal. However, in equilibrium, everyone extracts the same information (i.e., the probability of the realization of each state) from being pivotal. Therefore, the only reason we see different shareholders behaving differently is that they receive different private signals.[16] A higher signal

[13] Yılmaz (1998) presents the generalized version of Proposition 1 below.

[14] By a small probability of having fully informed shareholders, we mean there exist $1 > \bar{s} > \underline{s} > 0$ such that $f(s|1) = 0$ for $s \in [0, \underline{s}]$ and $f(s|0) = 0$ for $s \in [\bar{s}, 1]$.

[15] In the earlier version we use a more refined equilibrium concept in which no voter uses a weakly dominated strategy.

[16] Note that if the shareholders have different preferences, then their action may not be identical even if they share the same private signal. In other words, conditioning on what must be true if the race is very close may result in different actions for different shareholders.

means a greater probability of having a superior raider. Consequently, the shareholders with low signals vote for the incumbent whereas the higher signals result in voting for the raider.

Naively, this segregated behavior resembles sincere voting. A shareholder with a signal $s = \frac{1}{2}$ is indifferent between two candidates given his information, since his posterior beliefs suggest that each state of the world is equally probable. Thus, a shareholder with a signal $s < \frac{1}{2}$ thinks that having a superior incumbent is more likely. Therefore, if a shareholder behaves sincerely, i.e., acts as if he is the only decision maker in this process, then he votes for the incumbent whenever $s < \frac{1}{2}$ and similarly votes for the raider whenever $s > \frac{1}{2}$. Therefore, the cut-off signal for sincere voting is $s = \frac{1}{2}$. Note that the cut-off signal for sincere voting does not depend on the supermajority rule r; it is always $\frac{1}{2}$.[17] This is the point where strategic behavior departs from sincere voting. The following proposition characterizes this deviation from sincere voting.

Proposition 2 *The cut-off signal s^* is a decreasing function of r. Therefore, the expected number of votes given to the raider increases as r increases.*

Proof: See appendix. $\diamond$

The equilibrium cut-off signal s^* is an outcome of a strategic behavior rather than an outcome of Bayesian updating alone. Let's consider a shareholder with a signal less than $s = \frac{1}{2} - \epsilon$ where ϵ is a very small positive number. Given his signal, this shareholder thinks that having a superior incumbent is slightly more likely than having a superior raider. Therefore, his sincere choice must be the incumbent. However, if there is a very high supermajority requirement, this shareholder may find it in his interest not to vote for the incumbent. Given that the supermajority requirement is high (for the raider), the incumbent needs a small fraction of votes in order to stay in office. Furthermore, every shareholder knows that if the incumbent is the better candidate, then there are many shareholders with a signal close to 0. Therefore, it is clear to everyone that the support given by the shareholders with signals around 0 will be sufficient to secure a victory for him provided he is the superior candidate. Hence a shareholder with a

[17] After all, your posterior beliefs about the true state of the world should not depend on what the requirements are for a successful takeover.

signal $s = \frac{1}{2} - \epsilon$ votes for the raider. Clearly, this compensating behavior is more severe as the initial bias increases in the favor of the incumbent.

One can easily see why the equilibrium cut-off signal changes as we use different supermajority requirement levels. For the sake of the argument, assume that the equilibrium cut-off signal does not depend on r. In that case, being pivotal implies that the probability of the raider being the better manager is greater for a higher r, since the number of votes given to the raider is greater. Thus, there must exist a group of shareholders who switch sides under a higher r and vote for the raider due to this new "information". However, this contradicts the assumption of a constant equilibrium cut-off.

A natural way to interpret this result is to say that the shareholders are compensating for the initial bias. Thus, all of the factors which play a role in the initial bias must have the same effect as the following corollary points out:

Corollary 1 *The number of votes given to the raider is an increasing function of both the incumbent's holdings α_I and the nominal supermajority rule ρ and a decreasing function of the raider's holdings α_R.*

Each shareholder is minimizing the negative effects of the initial bias and thus maximize his share's expected value. However, this is a non-cooperative action on the behalf of the shareholders, since they do not communicate and coordinate their actions prior to the proxy contest. Therefore, it is interesting to see how well the information is aggregated via non-cooperative behavior. There are three natural benchmarks to compare with: first there is sincere voting; second is coordination via a contingent plan or mechanism. That is, each shareholder is dictated (by a plan) what to do given his signal.[18] Last is perfect communication after receiving private information. We consider both asymptotic efficiency and non-asymptotic efficiency properties with respect to each benchmark.

Definition 1 *An outcome is called a mistake if the inferior candidate wins the proxy fight.*

It is known that strategic behavior results in asymptotically (i.e., in the limit when number of voters goes to infinity) more efficient outcomes than the sincere voting does in

[18]For the sake of the argument we are ignoring incentive compatibility considerations.

various different environments.[19] We also know that when the number of shareholders with identical preferences gets arbitrarily large and the probability of making a mistake converges to zero under sincere voting, then there must exist a strategic equilibrium in which the probability of making a mistake converges to zero as well.[20] Here, we are especially interested in comparing the efficiency of strategic and sincere voting. Considering the fact that each shareholder's action depends heavily on the assumption that his vote may have an effect on the outcome, analyzing this strategic behavior in a small electorate is not only interesting but also necessary.[21]

As we will see in the following propositions and corollaries, strategic behavior does very well in terms of information aggregation. First, it aggregates information better than does sincere voting, both asymptotically and non-asymptotically. It does at least as well as any coordination through contingent plans -both asymptotically and non-asymptotically- and as well as full communication, asymptotically. Furthermore, depending on the signal space and information structure it may aggregate information non-asymptotically as well as full communication.

Now we would like to formally define what we mean by a voting rule. This class of rules is of special interest because it includes not only sincere and strategic voting but also any contingent plan the shareholders can employ in order to coordinate their actions.

Definition 2 *Call σ a voting rule if it specifies a choice for each shareholder given his signal, i.e., $\sigma_i : [0,1] \to \{I, R\}$ for all $s \in [0,1]$.*

Proposition 3 *For any n and r, no voting rule increases the shareholder value more than the strategic behavior characterized by Proposition 1.*

Proof: See appendix. $\diamond$

For the intuition behind this result consider a social planner choosing an optimal cut-off signal. Whenever the race is not close, the effect of a small perturbation in s

[19] See Feddersen and Pesendorfer (1994, 1996 and 1997) and Myerson (1994b).

[20] See McLennan (1996).

[21] However, so far in the literature the non-asymptotic properties of the strategic voting have not been analyzed in comparison to other voting behavior even in the political economy contexts.

is negligible compared to that of a close race. Therefore, the optimization problem is identical to that of an individual considering the case in which he is pivotal.[22]

Given that sincere voting is also a voting rule, we have the following as a corollary to the above proposition.

Corollary 2 *The probability of mistake in the unique strategic equilibrium is always (weakly) lower than the probability of mistake under naive voting.*[23]

Before we proceed with asymptotic properties of this information aggregation and prove that this strategic behavior of shareholders aggregates information perfectly in a widely held firm, we would like to present an example to show that how much of the available information is aggregated in relatively small environments. To see this consider the example presented in Table 1. In this example, we take $n = 25$, $\alpha_I = \alpha_R = 0$ and $f(s|1) = 2s$. Here, P_I denotes the probability of keeping an inferior incumbent. Similarly, P_R stands for the probability of electing an inferior raider.

the (supermajority) rule ρ	50%	60%	67%	75%	90%	100%
P_I	0.0032	0.0036	0.0042	0.0057	0.0191	0.0808
P_R	0.0032	0.0035	0.0041	0.0052	0.0151	0.0503

Table 1

To see how these probabilities are calculated, let's consider the last column of Table 1, namely the unanimity rule. Under the unanimity rule the raider needs everyone's support to win the proxy contest. Consequently, the incumbent needs only one vote to stay in office. In other words, a shareholder who votes for the incumbent knows that his vote is sufficient for the incumbent to win even if everyone else votes for the raider. Therefore, a shareholder votes for the incumbent if and only if he thinks that

[22] I am grateful to Wolfgang Pesendorfer for suggesting this intuition.

[23] Another implication of our results is in the Condorcet Jury Theorem literature. As we have mentioned above, the earlier proofs of this Theorem involve sincere voting only. Austin-Smith and Banks (1996) show that sincere behavior may not be rational. Furthermore, they argue that the outcome in the equilibrium may contradict the Condorcet Jury Theorem itself. Contrary to their claim, the Proposition 3 shows that strategic voting strengthens the argument for the Condorcet Jury Theorem.

there is a sufficiently high probability that the incumbent is the superior candidate. The cut-off signal is hence very small and equal to 0.058. If the true state of the world is 1, i.e., the raider is the better manager, then the probability of choosing the inferior candidate is the probability of having at least one shareholder voting for the incumbent. The probability of having only one out of 25 shareholders voting for the incumbent is $25(\int_0^{0.058} 2sds)(\int_{0.058}^1 2sds)^{24} = 0.0776$. Similarly, we can calculate the probability of having only two shareholders supporting the incumbent and so on. $P_I = 0.0808$, the probability of choosing the inferior candidate, is the sum of these probabilities.

It is important to see that with a simple majority or a mild supermajority rule ($\rho = 0.6$), strategic behavior of shareholders reduces the expected probability of making a mistake to (approximately) 0.3% even with relatively imprecise signals. Given the imprecision of the signals it is clear that the probability of making a mistake must be bounded away from zero even with full communication prior to making a decision.

We will discuss this example in greater detail in Section 7, when we analyze the effects of a higher supermajority rule on both the shareholder value and the incumbent's total benefits. However, now we would like to continue our discussion of the asymptotic properties of information aggregation under strategic voting.

Proposition 4 *For any supermajority rule r, the equilibrium probability of a mistake converges to zero as n gets large.*

Proof: See appendix. $\diamond$

Feddersen and Pesendorfer (1997) show that the probability of a mistake is bounded away from zero for $r = 1$ in a very similar setting with binary signals. The difference between Proposition 4 and their result is due to the assumptions of continuum signal space and convergence of the likelihood ratio $\frac{f(s|1)}{f(s|0)}$ to zero. However, in both settings unanimity is a "bad" supermajority rule.[24]

Let's now analyze how the appearance of the raider and hence the announcement of a proxy contest affects the prices. For simplicity, we assume that existing shareholders are risk neutral and the stock price is equal to the expected value. Then we can make the following observation about the shareholder value:

[24] As we see in Table 1, the probability of making a mistake reaches a maximum at $r = 1$, i.e., at unanimity.

Corollary 3 *Assume that* $\alpha_I < 1 - \rho$ *and* $\alpha_R < \rho$, *so that we have a non-trivial setting. Then, the probability of choosing the better management is arbitrarily close to* 1 *in a widely held firm. Furthermore, announcement of a proxy fight increases the share price. This increase is larger for more widely held firms.*

The first part of the above corollary contradicts Harris and Raviv's (1988) claim that in a widely held firm guaranteeing control does not require an incumbent to own $1 - \rho$ or more of the equity. On the other hand, they also argue that the announcement of a proxy fight will increase the value of the firm but they do not find any relation between this increase and the number of the shareholders. Although our model of proxy contests is similar to that of Harris and Raviv, our results differ because we consider the strategic behavior of shareholders whereas Harris and Raviv assume sincere behavior parallel to the earlier analysis in the voting theory literature.

In a companion paper (Yılmaz, 1998), we extend our information aggregation results and show that there exists an optimal supermajority rule which maximizes the shareholder value. In fact, that paper also shows that there exists a sequence of probability density functions $f(s|1)$ and $f(s|0)$ satisfying monotonic likelihood property such that the first best is achieved in the limit by a symmetric pure strategy equilibrium.

In an earlier version, we extended our analysis into strategic voting in presence of some irrational shareholders. We showed that existence of irrational voters increased the initial bias and hence resulted in more aggressive strategic behavior.

5 Larger Shareholders

In this section we will try to answer the following two questions. First, is it optimal (for the shareholder value) not to allow a large shareholder to split his votes? Second, is the strategic behavior of a large shareholder any different than that of a small shareholder? If we expect any behavioral differences it must be due to the information one extracts from being pivotal. The probability of being pivotal is greater for a larger shareholder. Hence, being pivotal reveals less information about the true state of the world. Consequently, a larger shareholder places a greater weight on his private signal in making his voting decision.

One can formalize this intuition in two different models. For the first, consider two firms where one is more widely held than the other. That is, each shareholder holds a smaller stake in the more widely held firm. We want to compare the behavior of a representative shareholder of each firm because this will enable us to answer the second question we raised above: is the strategic behavior of a large shareholder any different than that of a small shareholder? This is relatively easy given that we have already solved the model for any n. Therefore, we have the following proposition:

Proposition 5 *For any supermajority rule r, the cut-off signal s^* is a decreasing function of n. Therefore, a smaller shareholder of a more widely held firm behaves less "sincerely" than a larger shareholder of a less widely held firm.*

Proof: See appendix. $\diamond$

Note that, for any supermajority rule the sincere voting cut-off signal is $\frac{1}{2}$ whereas equilibrium cut-off s^* is less than $\frac{1}{2}$. Therefore, a lower equilibrium cut-off signal means that the difference between the strategic and sincere cut-off is increasing. Hence, as we increase the number of shareholders, n thus making each shareholder smaller, the probability of voting insincerely increases.[25]

In order to answer the first question we have to relax the one shareholder-one share assumption.[26] In this case, equilibrium behavior changes for large shareholders with several votes. In the new equilibrium, larger shareholders may divide their votes among two contenders. This division of votes is more egalitarian for larger shareholders with less informative signals. This behavior resembles the abstention of swing voters characterized by Feddersen and Pesendorfer (1996). What the large shareholders are doing here by dividing their votes is an attempt not to have too much influence on the outcome, and to allow the optimal aggregation of information.

It is easier to see this behavior in an extreme example. Consider a shareholder with 2 shares and a smaller shareholder with 1 share in an otherwise symmetric setting, i.e., $r = \frac{1}{2}$. Furthermore, for tractability reasons take $f(s|1)$ to be linear. In equilibrium, the smaller shareholder votes for the incumbent for $s < \frac{1}{2}$ and for the raider for $s > \frac{1}{2}$. Note that the cut-off signal $s = \frac{1}{2}$ would be the cut-off signal for each shareholder if

[25]Naturally, a similar argument would be true for an asymmetric case (i.e., s^* is greater than the naive voting cut-off) where s^* is an increasing function of n.

[26]A natural extension of this model is to allow larger shareholders to have a higher possibility of receiving more precise signals. However, this does not change the strategic behavior of a large shareholder.

we had 3 shareholders holding 1 share each. Therefore, the behavior of the smaller shareholder does not seem to be affected by having a larger shareholder. However, the larger shareholder has two equilibrium cut-off signals; $s = \frac{1}{4}$ and $s = \frac{3}{4}$. He gives both of his votes to the incumbent for $s < \frac{1}{4}$ and splits his votes for $s \in (\frac{1}{4}, \frac{3}{4})$. For $s > \frac{3}{4}$, the raider receives his full support. In short, the larger shareholder allows the smaller shareholder to cast the decisive vote if his own signal is not very informative, thinking that the smaller shareholder is more likely to be better informed. Therefore, requiring a large shareholder to vote for one candidate only (by simply asking him to give a proxy to one of the contenders) is sub-optimal for the shareholder value.

6 Anti-Takeover Measures

In Section 4, we show that the shareholders will attempt to compensate for the initial bias. This raises a natural question: Do anti-takeover measures truly work in the favor of the incumbent? In other words, is it a good idea for an incumbent to create a bias seemingly favoring himself given that strategic shareholders will be then more inclined to vote for the raider? The answer to this question is not immediate given the nature of the strategic behavior we analyze in this paper.

Grossman and Hart (1988) and Harris and Raviv (1988a) prove that one share-one vote rule is optimal for the shareholders. In other words, they show that dual class shares may result in undesirable outcomes for the value of the firm and thus for shareholders. The beneficiary of such a capital structure is thought to be the contender holding shares with larger voting rights. Therefore, introducing dual class shares and holding the securities with more voting rights is accepted as an anti-takeover measure. There are several other methods which can be employed by an incumbent as a defensive strategy. Larger management ownership and higher supermajority requirements for a control change are generally considered as strategies strengthening the management's control.

Clearly, all of these measures create an initial bias against the raider in our model as well, since they increase the parameter r, the effective supermajority rule. However, each non-contender shareholder votes in order to maximize the expected value of his share. In other words, his goal is to minimize the probability of choosing the inferior candidate. Therefore, shareholders may find it in their interest to "overreact" to an anti-takeover measure so that the probability of replacing an incumbent increases as long as the probability of making a mistake is minimized. This leads to the following proposition.

Proposition 6 *Introducing dual class shares and a supermajority amendment and increasing management ownership may each increase the probability of a control change.*

The example presented in Table 1 suffices to demonstrate that the effects of anti-takeover measures not only are weakened but also may be reversed by the strategic behavior of shareholders. In the last row of the Table 1, we see that the probability of replacing a superior incumbent is 0.32% under simple majority rule. If a supermajority amendment is accepted, then there will be "too much" bias for the incumbent since the probability of choosing the raider increases as a result of overreacting shareholders. Therefore, the optimal rule for a superior incumbent is around $\rho = 0.5$ which is less than $\rho = 1$. However, one should note that although higher ρ is unfavorable for the superior incumbent, it is beneficial for the inferior incumbent. As we see in the second row of Table 1, the optimal level for an inferior incumbent is $\rho = 1$.

One should note that the optimal supermajority rule for an incumbent depends on other factors such as initial holdings and voting rights. Specifically, simple majority rule may not be optimal for the superior incumbent. In fact, he may prefer a higher supermajority requirement. To see this consider the following example, where $n = 15$, $\alpha_I = 0.10$, $\alpha_R = 0.15$ and $f(s|1) = 2s$.[27]

the (supermajority) rule ρ	50%	55%	60%	70%	80%	90%
P_I	0.0175	0.0173	0.0179	0.0219	0.0348	0.1025
P_R	0.0179	0.0173	0.0175	0.0203	0.0298	0.0701

Table 2

As we see in Table 2, the optimal supermajority rule is $\rho = 0.55$ for a superior incumbent. Another observation one can make from Table 2 is that the inferior incumbent does not always benefit from a higher supermajority rule. In the second row of

[27] Obviously, a larger n reduces the probability of both types of errors for any supermajority rule as stated in Proposition 4. We know from Table 1 that for $n = 25$, both P_I and P_R are considerably lower.

Table 2, the probability of keeping an inferior incumbent falls as we move from simple majority rule to $\rho = 0.55$.

Although our primary purpose here is to analyze how each type of incumbent is affected by so called anti-takeover strategies, we would like to point out that the shareholder value is maximized at $\rho = 0.5$ in the first example whereas at $\rho = 0.55$ in the second example. Note that in the second example the raider has larger initial holdings (15% vs 10%) resulting an advantage for him. Nevertheless, $\rho = 0.55$ requires both of the contenders to receive an additional 40% of the outstanding votes. Thus the shareholder value is maximized whenever there is no initial bias.[28]

7 Conclusion

We investigate the strategic behavior of privately informed shareholders in a proxy contest. Each shareholder conditions his decision not only on his private information but also on what must be true if his decision affects the outcome. Consequently, less informed shareholders vote against the candidate favored by the initial bias, since such a candidate does not need many votes to be successful and the support of the more informed shareholder will be sufficient as long as he is the superior candidate. Therefore, under private information and common values, shareholders' voting behavior compensates for the initial bias formed by both the supermajority rule and voting blocks held by each contender.

Our second set of results shows that simultaneous voting can serve as a natural mechanism for aggregating dispersed information for two reasons: (i) for any ownership structure and supermajority rule there does not exist a (cooperative or non-cooperative) voting strategy which can increase the shareholder value more than the strategic behavior; (ii) in equilibrium the probability of choosing an inferior candidate gets arbitrarily small as the number of shareholders increase. Therefore, we find no support for the argument that shareholder communication prior to a proxy contest can increase the shareholder value drastically. Furthermore, we show that our results are robust to existence of irrational shareholders.

We show that the size of a shareholder plays an important role in his strategic behavior, since being pivotal carries less information for a large shareholder. Thus,

[28]For a generalization of this observation see Yılmaz (1997).

larger shareholders tend to act more sincerely in the sense that their decision is mostly affected by their private information. We also demonstrate that requiring a large shareholder to vote as a block reduces the shareholder value.

Finally, we investigate the outcome of defense mechanisms. We show that taking anti-takeover measures have ambiguous effects. There is some empirical evidence suggesting that these measures have a tendency to reduce shareholder value.[29] However, we show that anti-takeover measures may increase the stock price if there is an initial bias favoring the raider. Moreover, we also show that the incumbent may not benefit by taking such anti-takeover measures.

Although this paper considers identical preferences, we realize that there are several other collective decision making processes involving environments where people have different preferences. The voting which takes place under Chapter 11 bankruptcy is a good example of such processes since the security holders in each class have different preferences determined by their priority. An analog of this analysis is presented by Maug and Yılmaz (1998) in order to shed light on strategic behavior with more heterogeneity across security holders.

[29] See Section 2.

Appendix

Proposition 1 *In the unique equilibrium there exists a cut-off signal s^* such that $\sigma_i^*(s) = I$ for all $s < s^*$ and $\sigma_i^*(s) = R$ for all $s > s^*$.*

Proof: Existence of an equilibrium follows from Theorem 1 of Milgrom and Weber (1985). Let $p(\omega, \sigma^*)$ stand for the probability that a voter is pivotal in state ω. In equilibrium, $p(\omega, \sigma^*)$ is always strictly positive. Now let's define $W(s, \sigma^*)$ as

$$\sum_{\omega \in \Omega} [u(R, \omega) - u(I, \omega)] \frac{\beta(\omega|s)p(\omega, \sigma^*)}{\sum_{\omega \in \Omega} \beta(\omega|s)p(\omega, \sigma^*)} \tag{1}$$

Clearly, there exists an $\epsilon > 0$ such that for $s \in [0, \epsilon]$, $W(s, \sigma^*) < 0$ and for $s \in [1-\epsilon, 1]$, $W(s, \sigma^*) > 0$. Furthermore, $W(s, \sigma^*)$ is strictly increasing in s for all $s \in (0, 1)$ given that $\beta(1|s)$ is strictly increasing and $\beta(0|s)$ is strictly decreasing in the same interval. By continuity, there exists a s^* such that $W(s, \sigma^*) = 0$.

In fact, given that $\mid u(R, \omega) - u(I, \omega) \mid$ is constant, if we attempt to solve for the cut-off signal, Equation 1 becomes:

$$\beta(1|s)p(1, \sigma^*) - \beta(0|s)p(0, \sigma^*) = 0$$

If we divide both sides of the equation by $f(s|1) + f(s|0)$ then we have

$$f(s|1)p(1, \sigma^*) - f(s|0)p(0, \sigma^*) = 0,$$

since $\pi(0) = \pi(1) = \frac{1}{2}$.

Let $K(s) = \int_0^s f(t|1)\,dt$ and $L(s) = \int_s^1 f(t|0)\,dt$ stand for the probability of voting for the "wrong candidate" given the true state of the world. Then the above equation becomes:

$$K^{n-rn}(s)(1 - K(s))^{rn-1} f(s|1) - L^{rn-1}(s)(1 - L(s))^{n-rn} f(s|0) = 0 \tag{2}$$

Hence, the cut-off signal s^* is the solution of the above equation.

Now we are left to prove the uniqueness. From Lemma 1 (in the appendix) we know that every equilibrium is a pure strategy equilibrium. Suppose that the equilibrium with cut-off signal s^* is not unique and there exists another equilibrium σ' with a cut-off signal s'. Without loss of generality let us assume that $s' < s^*$. Therefore, the shareholder with a signal s^* votes for R in σ', i.e., $W(s^*, \sigma') > 0$. However, we also have $\frac{p(0,\sigma^*)}{p(1,\sigma^*)} > \frac{p(0,\sigma')}{p(1,\sigma')}$, due to the monotonic likelihood ratio of $\frac{f(s|1)}{f(s|0)}$. But this implies that $W(s^*, \sigma') < 0$. Hence, s' cannot be a cut-off signal. $\diamond$

Proposition 2 *The cut-off signal s^* is a decreasing function of r. Therefore, the expected number of votes given to the raider increases as the requirement goes up.*

Proof: Suppose that s^* is non-decreasing in r. For any $s \in (0,1)$ we have $\frac{K(s)}{1-L(s)} < 1$ and $\frac{1-K(s)}{L(s)} > 1$. As r gets larger, for any cut-off signal s both $(\frac{K(s)}{1-L(s)})^{n-rn}$ and $(\frac{1-K(s)}{L(s)})^{rn}$ increases. This means that $\frac{p(0,\sigma^*)}{p(1,\sigma^*)}$ decreases as the fraction r gets larger. But this contradicts with $W(s^*, \sigma^*) = 0$. $\diamond$

Definition 3 *We call σ a segregated voting rule, if there exists a $s' \in [0,1]$ such that $\sigma_i(s) = I$ for all $s < s'$ and $\sigma_i(s) = R$ for all $s > s'$.*

Lemma 1 *For any non-segregated voting rule, there exists a segregated voting rule which is (weakly) more efficient.*

Proof: Take any non-segregated voting rule. Let K' be the probability of voting for the wrong candidate given the true state of the world is 1. Similarly define L' as the probability of voting for the wrong candidate given the true state of the world is 0. Therefore, the probability of a mistake is

$$\frac{1}{2}[(K')^n + ... + \frac{n!}{(n-rn+1)!(rn-1)!}(K')^{n-rn+1}(1-K')^{rn-1}] + \frac{1}{2}[\frac{n!}{(n-rn)!rn!}(L')^{rn}(1-L')^{n-rn} + ... + (L')^n] \quad (3)$$

By definition of a non-segregated voting rule there exists two subsets S_I and S_R of $[0,1]$ such that $\sigma(s_R) = R$ for $s_R \in S_R$ and $\sigma(s_I) = I$ for $s_I \in S_I$ and $s_R < s_I$ for every $s_R \in S_R$ and $s_I \in S_I$. If there are no S_I and S_R with non-zero measure then the above Lemma holds trivially. Therefore, assume otherwise. Given $f(s|1)$ is an increasing function of s, one can increase both $1 - K'$ and $1 - L'$ by a voting rule imposing $\sigma(s) = I$ for some $s \in S_R$ and $\sigma(s) = R$ for some $s \in S_I$. However, such a modification decreases the probability of making a mistake, since it reduces both of the terms between brackets in the above equation. Therefore, this non-segregated voting rule cannot be optimal. $\diamond$

Proposition 3 *For any n and r, no voting rule increases the shareholder value more than the strategic behavior characterized by Proposition 1.*

<u>*Proof:*</u> Given Lemma 1, we can restrict attention to segregated voting rules when we calculate the optimal voting rule. Then the probability of making a mistake for the society as a function of a cut-off signal s is

$$M(s) = \frac{1}{2}[K^n(s) + ... + \frac{n!}{(n-rn+1)!(rn-1)!}K^{n-rn+1}(s)(1-K(s))^{rn-1}] + \frac{1}{2}[\frac{n!}{(n-rn)!rn!}L^{rn}(s)(1-L(s))^{n-rn} + ... + L^n(s)] \quad (4)$$

The term in the first bracket is $F_{n-rn+1}(s)$, the cdf of the $(n-rn+1)$th order statistic. Similarly, the second term is $1 - G_{n-rn+1}(s)$. Then the first order condition is

$$f_{n-rn+1}(s) - g_{n-rn+1}(s) = 0$$

Therefore, by Theorem 5.5.2 of Casella and Berger (1990) the first order condition becomes

$$\frac{n!}{(n-rn)!(rn-1)!}[K^{n-rn}(s)(1-K(s))^{rn-1}f(s|1) - L^{rn-1}(s)(1-L(s))^{n-rn}f(s|0) = 0$$

The solution of this equation will give us the optimal cut-off signal. However, from Equation 2 above, we already know that the solution of this equation is s^*. $\diamond$

Lemma 2 *For $r = 1$, s^*, the solution of the first order equation, converges to zero as n goes to infinity.*

<u>*Proof:*</u> Suppose that $\lim_{n\to\infty} s^* \neq 0$. This implies that $\frac{1-K(s^*)}{L(s^*)} > 1$. Note that the f.o.c. for $r = 1$ is

$$(1-K(s))^{n-1}f(s|1) - L^{n-1}(s)f(s|0) = 0$$

Therefore, we know that $[\frac{1-K(s^*)}{L(s^*)}]^{n-1} = \frac{f(s^*|0)}{f(s^*|1)}$. Given that $\frac{1-K(s^*)}{L(s^*)} > 1$ left hand side of the above equation gets arbitrarily large (i.e. goes to infinity) as n goes to infinity. But this means that right hand side of the equation must also go to infinity. However, this implies that $\lim_{n\to\infty} s^* = 0$. However, this contradicts our assumption. $\diamond$

Lemma 3 *If* $K^{1-r}(s)(1-K(s))^r = L^r(s)(1-L(s))^{1-r}$*, then for any* $r \in (0,1)$*, we have* $1-K(s) > r$ *and* $1-L(s) > 1-r$*.*

Proof: Suppose that Lemma 2 does not hold. Then there are three possible cases:

(i) $1-K(s) \leq r$ and $1-L(s) \leq 1-r$

(ii) $1-K(s) > r$ and $1-L(s) \leq 1-r$

(iii) $1-K(s) \leq r$ and $1-L(s) > 1-r$

Clearly, the first case cannot be a possibility since it implies that $L(s) > 1-K(s)$. To see that the other two cases fail to be correct we need to analyze the function $x^r(1-x)^{1-r}$. Note that, this is a single peaked function reaching the maximum at $x = r$. But this implies that $1-K(s) = L(s)$ in order to have $K^{1-r}(s)(1-K(s))^r = L^r(s)(1-L(s))^{1-r}$. However, $1-K(s) = L(s)$ contradicts with the fact that $1-K(s) > L(s)$ for all $s \in (0,1)$. $\diamond$

Proposition 4 *For any supermajority rule* r*, the equilibrium probability of a mistake converges to zero as* n *gets large.*

Proof: The proof of this proposition is consist of two independent parts: For $r = 1$ and for $r < 1$. We start with the first case. For $r = 1$, the probability of a mistake is

$$\frac{1}{2}[K^n(s) + nK^{n-1}(s)(1-K(s)) + ... + nK(s)(1-K(s))^{n-1}] + \frac{1}{2}L^n(s).$$

From the f.o.c., we have $L^{n-1}(s^*) = (1-K(s^*))^{n-1}\frac{f(s^*|1)}{f(s^*|0)}$. Clearly, $(1-K(s^*))^{n-1}$ does not exceed 1. Therefore, $L^{n-1}(s^*) \leq \frac{f(s^*|1)}{f(s^*|0)}$. However, from Lemma 3, we know that

$\lim_{n\to\infty} \frac{f(s^*|1)}{f(s^*|0)} = 0$ which implies that $\lim_{n\to\infty} L^{n-1}(s^*) = 0$. Therefore, we are left to show that

$$\lim_{n\to\infty}[K^n(s^*) + nK^{n-1}(s^*)(1 - K(s^*)) + ... + nK(s^*)(1 - K(s^*))^{n-1}] = 0.$$

We should note that $K^n(s^*) + nK^{n-1}(s^*)(1 - K(s^*)) + ... + nK(s^*)(1 - K(s^*))^{n-1}$ is nothing but $[K(s^*) + (1 - K(s^*))]^n - (1 - K(s^*))^n$. Therefore, all we need to show is $\lim_{n\to\infty}(1 - K(s^*))^n = 1$, since $K(s^*) + (1 - K(s^*)) = 1$. First let $y = (1 - K(s^*))^n$ and then take the natural logarithm of both sides of the equation. Then we get $\ln y = \frac{\ln(1-K(s^*))}{\frac{1}{n}}$. If we take the limit, we get $\frac{0}{0}$ and therefore we can apply the l'Hôpital's rule. Clearly, $\lim_{n\to\infty} \frac{dK(s^*(n)))}{dn} = 0$, since we have $\lim_{n\to\infty} s^* = 0$ and $f(s = 0|1) = 0$. Therefore, $\lim_{n\to\infty} \frac{\ln(1-K(s^*))}{\frac{1}{n}} = 0$. But this implies that $\lim_{n\to\infty} \ln y = 0$. Thus we have $\lim_{n\to\infty}(1 - K(s^*))^n = e^0 = 1$.

Now we can proceed with the second case, namely $r < 1$. From the f.o.c. we have

$$K^{n-rn}(s)(1 - K(s))^{rn-1} f(s|1) = L^{rn-1}(s)(1 - L(s))^{n-rn} f(s|0)$$

If we take the n^{th} root of both sides of this equation we will have $K^{1-r}(s)(1-K(s))^r = L^r(s)(1 - L(s))^{1-r}$ in the limit where n goes to infinity. Then it follows from Lemma 3 that $1 - K(s) > r$ and $1 - L(s) > 1 - r$. Therefore, by invoking the law of large numbers we can conclude our proof. ◇

Proposition 5 *For any supermajority rule r, the cut-off signal s^* is a decreasing function of n. Therefore, a smaller shareholder of a more widely held firm behaves less "sincerely" than a larger shareholder of a less widely held firm.*

Proof: Take any two firms with different number of shareholders $\tilde{n} > \hat{n}$. Let $\tilde{s}$ and $\hat{s}$ be the equilibrium cut-off signals. From the f.o.c., we have

$$\frac{f(\hat{s}|0)}{f(\hat{s}|1)} = \frac{K^{\hat{n}-r\hat{n}}(\hat{s})(1 - K(\hat{s}))^{r\hat{n}-1}}{L^{r\hat{n}-1}(\hat{s})(1 - L(\hat{s}))^{\hat{n}-r\hat{n}}}.$$

For any supermajority rule r, we know that cut-off signal of equilibrium behavior, s^*, is less than that of sincere voting and thus less than $\frac{1}{2}$. Therefore, we have

$$\frac{f(\hat{s}|0)}{f(\hat{s}|1)} = \frac{K^{\hat{n}-r\hat{n}}(\hat{s})(1 - K(\hat{s}))^{r\hat{n}-1}}{L^{r\hat{n}-1}(\hat{s})(1 - L(\hat{s}))^{\hat{n}-r\hat{n}}} > 1.$$

Given that $\frac{1-K(s)}{L(s)} \geq 1$ for all $s \in [0,1]$

$$\frac{K^{\tilde{n}-r\tilde{n}}(\hat{s})(1-K(\hat{s}))^{r\tilde{n}-1}}{L^{r\tilde{n}-1}(\hat{s})(1-L(\hat{s}))^{\tilde{n}-r\tilde{n}}} > [\frac{K^{\hat{n}-r\hat{n}}(\hat{s})(1-K(\hat{s}))^{r\hat{n}-1}}{L^{r\hat{n}-1}(\hat{s})(1-L(\hat{s}))^{\hat{n}-r\hat{n}}}]^{\frac{\tilde{n}}{\hat{n}}} > \frac{K^{\hat{n}-r\hat{n}}(\hat{s})(1-K(\hat{s}))^{r\hat{n}-1}}{L^{r\hat{n}-1}(\hat{s})(1-L(\hat{s}))^{\hat{n}-r\hat{n}}}.$$

Therefore, we have $\frac{K^{\tilde{n}-r\tilde{n}}(\hat{s})(1-K(\hat{s}))^{r\tilde{n}-1}}{L^{r\tilde{n}-1}(\hat{s})(1-L(\hat{s}))^{\tilde{n}-r\tilde{n}}} > \frac{f(\hat{s}|0)}{f(\hat{s}|1)}$. This implies that

$$K^{\tilde{n}-r\tilde{n}}(\hat{s})(1-K(\hat{s}))^{r\tilde{n}-1}f(\hat{s}|1) - L^{r\tilde{n}-1}(\hat{s})(1-L(\hat{s}))^{\tilde{n}-r\tilde{n}}f(\hat{s}|0) > 0.$$

From Proposition 1, we know that s^* is unique in the $(0,1)$ interval. Furthermore, we also know that the identity $K^{n-rn}(s)(1-K(s))^{rn-1}f(s|1)-L^{rn-1}(s)(1-L(s))^{n-rn}f(s|0)$ is negative for $s < s^*$ and positive for $s > s^*$. Thus, the solution of the f.o.c. for $\tilde{n}$, $\tilde{s}$, is less than $\hat{s}$. Therefore, the probability of a shareholder behaving identical to a sincere voter is less for the more widely held firm. ◇

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