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Nested Information and Manipulation in Financial Markets[1]

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Abstract

We construct a model of trading in a financial market with an insider who may or may not be informed about the fundamentals. Rational traders called followers possess part of the private information of the insider (which the market makers do not possess) and trade on this information. This increases the competitive pressure on the informed insider and, when his private information is sufficiently long-lived, leads him to manipulate in every equilibrium. The presence of the followers also enables the uninformed trader to profitably manipulate by using the followers to generate "momentum" in the price process. We show that when there are many followers and a large number of trades before all private information is revealed, *each type of the insider will manipulate in every equilibrium.* The results are related to the literature on disclosure of insider trades and to dual trading.

1 Introduction

In this paper we consider a model of strategic trading by an insider trader. The insider may have long-lived private information about the expected future returns of the asset being traded. However he may also be uninformed and may not possess any such private information about the expected future returns of the asset.

We assume that no trader in the market knows if the insider is informed about the fundamentals or if he is uninformed. Further, the market makers or price-setters also do not know if the insider has traded at all i.e., if the insider exists. In contrast to the market makers, we suppose that there are a large number of other traders in the market who we will call followers[1]. These traders have superior information compared to the market makers in that they know if the insider has traded although, like the market makers, they do not know if the insider has any information and what the nature of his information is. Due to this informational advantage over the market makers, the followers will find it profitable to mimic the trades of the insider.

This has two important effects in our model. First, this creates competitive pressure on the informed insider and creates an incentive for him to manipulate the market by sometimes trading in the wrong direction (e.g., buying when he has bad information). We show that, regardless of the strategy of the uninformed insider, when there are sufficiently many periods of trading, the informed insider will manipulate in every equilibrium. Second, because the followers' optimal strategy is to mimic the trades of the insider even though they do not know if the insider is informed or uninformed, the uninformed insider has an incentive to manipulate the market. For example he may buy low and the followers, by mimicking that buy, will drive the price up (without the insider himself taking a large position on the asset) whereupon the uniformed insider can sell at a high price and make a profit. We show, that when there are sufficiently many periods of trading and when follower orders are a sufficiently large proportion of the total order flow, then the uninformed insider will manipulate in every equilibrium, and make strictly positive profits.

[1] In the trading game we will consider the followers will try to "follow" the trades of the insider, hence the name.

The conclusions of this model adds to those obtained from the extant literature on market manipulation (see the Literature Review section) in a number of ways. First, we look at the possibilities of market manipulation by a trader who potentially has inside information, without placing exogenous restrictions on the strategy space[2]. We analyze the properties of the optimal strategies of both the informed and the uniformed insider. Interestingly, we show that the incentives and the ability for each type of the insider to manipulate is adversely affected by the extent to which the other type manipulates. More precisely, if the uninformed insider manipulates then he provides liquidity to the market and lowers the adverse selection pressure on the informed trader, if he trades on his information. This lowers the incentives of the informed insider to undertake costly manipulation. Conversely, when the informed insider manipulates, the incentives of the followers to mimic the trades of the insider are lower, as the trade contains less information. This lowers the ability of the uninformed insider to use the followers to move the prices and profitably manipulate. Nevertheless, we show that each type of insider will manipulate in every equilibrium if there are sufficiently many periods of trading and follower orders are a sufficiently large proportion of the total order flow.

There is a literature on the effect of disclosure laws on market manipulation (see, for example, Fishman and Hagerty (1995), John and Narayanan (1997): see Literature Review), which shows that mandatory disclosure laws increase the incentives to manipulate. Our information structure essentially implies that there is mandatory disclosure of insider's information not to the market as whole, but to the followers who are a subset of active traders in the market. Therefore, the second important contribution of this paper, to the general conclusions obtained from the extant literature on manipulation, is that mandatory disclosure to a large enough subset of the market, instead of the entire market, is sufficient to increase the incentives to manipulate.

The assumption that the followers have superior information relative to the market makers' information can be justified in a number of ways. We may think of the followers as brokers who also trade on their own account, i.e., dual trading brokers. The reason why they might know that the insider has traded when the market makers do not, is

[2]For example, by postulating that the informed trader always trades in the direction of his information and concentrating exclusively on uninformed manipulation.

because the insider's trade may be actually executed by them. They do not precisely know the information content of their client's order but they are at an informational advantage over the market makers who do not know if a given trade is coming out of the brokers' own account or from the brokers' client. In particular, they do not know if the insider has submitted any order at all. Unlike the existing literature on dual trading (for example, Fishman and Longstaff (1992), Roell (1990): see Literature Review) this paper considers the possibility that dual trading may create incentives for the broker's client to manipulate.

We may also think of the insider's information as arising out of learning the future prospects of the firm whose stock is the asset under consideration. The insider may then decide (or pretend) to try and acquire a toehold because the future prospects are good. We may then think of the followers as speculators who know of the impending takeover, before the market makers know that fact. In that sense, the followers could also be thought of as informed insiders, where their information consists of the nature of the large trader's (impending) trades. This raises the share price and allows the insider to unload his position at a high price.

Manipulation increases the informed insider's profits because it makes prices less responsive to his trades. Manipulation increases the uninformed insider's profits because it makes prices respond to his trades. Manipulation has two opposing effects on follower profits: it lowers the precision of their information arising out of observing the insider's trades. However, by lowering the responsiveness of prices to trades it lowers the adverse price movement when they trade. The market makers, break even in expectation, by assumption.

Manipulation reduces the information content of each trade. In particular, because of the possibility that manipulation is driving the volume and/or direction of trading, prices will be less responsive to trading in the sense that a given number of buy orders will raise prices by less than what it would in markets where such manipulative trading is known to be unlikely. Similarly, if one looks at the history of prices just before a public announcement of information in the market (e.g., an earnings announcement), prices will be more volatile and could be quite far away from the price which prevails just after the announcement.

The Securities Exchange Act of 1934 has provisions which make manipulation based on actions or announcements difficult to implement. Firms are required to issue information regularly so that it is difficult to spread rumors. However we consider a situation where the only instrument open to the insider for manipulating is his trading strategy. In markets with anonymous trading, it is potentially very difficult to directly detect purely trade-based manipulative trading[3]. Moreover, since in our model the possibility of manipulation arises directly out of the information held by different traders in the market, attempts to regulate such manipulation might have adverse effects on the incentives of traders to acquire information and to trade on information.

In the next subsection we discuss and relate to the existing literature on manipulation. In section 2 we discuss our model and results informally. In section 3 we set up the model formally and define our notion of manipulation. We prove first the existence of equilibrium. Then we show that when the private information of the insider is potentially long-lived, the informed insider will manipulate regardless of what thew uninformed insider does. Finally, we show that when there are enough followers in the market the uninformed insider will also manipulate in every equilibrium and make strictly positive profits, if the private information is sufficiently long-lived. The Appendix contains the proofs of the main results.

1.1 Review of the Literature

A number of other authors have considered the definition and possibility of manipulation. Jarrow (1992) formulates sufficient conditions for manipulation to be unprofitable. These sufficient conditions are properties of the "reduced-form" price function[4] which implicitly incorporates the beliefs of the market about the equilibrium strategy used by the large trader. In contrast to Jarrow, we set up an explicit model of strategic trading where the dependence of prices on the market's beliefs about the insider's strategy is made explicit.

[3]Although there are restrictions on how the directors and officers of a firm can trade the securities of their own firm.

[4]A reduced form price function is one in which the response of the market to the large trader's trades has already been incorporated, so that it is only a function of the large trader's trades.

Allen and Gale (1992) also consider an explicit model of strategic trading in which some equilibria involve manipulation. Like our model, they consider a situation in which the market does not know if the potential manipulator is informed or uninformed. However, manipulation by the informed trader is not profitable in their model. Manipulation by an uninformed trader is profitable, in the presence of certain restrictions on the strategy of the informed trader.

Allen and Gorton (1992) consider a model of uninformed manipulation in which an asymmetry in buys and sells by noise traders trades creates the possibility of manipulation. In every equilibrium of their model, the manipulator makes zero profits. Kumar and Seppi (1992) show how an uninformed trader makes profits when an information event takes place. The manipulator, by a sequence of trades in the spot and futures markets, is able to profit knowing that informed traders are about to trade based on the private information they receive. This example requires a "cash-settled" futures contract along with the underlying asset.

Fishman and Hagerty (1995) provide a one-period equilibrium model of profitable manipulation when an uninformed insider successfully misleads the market into thinking he is informed. The profitable opportunity arises due to the mandatory disclosure of the insider's trades. John and Narayanan (1997) and Huddart, Hughes and Levine (1999) also look at the effect of mandatory disclosure laws on the insider's incentive to manipulate. The present paper is complementary to this literature because it shows that even if the insider's trades are revealed not to the market as whole but to a section of the market who trade on this information, then the insider will manipulate[5].

Chakraborty and Yılmaz (1999) show that the every type of the informed trader manipulates in an economy with only informed and noise traders, and uncertainty about the existence of an informed trader. The results in Chakraborty and Yılmaz (1999) holds in market order models as well as bid-ask models.

Kyle (1984) considers a model of market manipulation in futures markets where a large trader can undetectably acquire a large position and manipulate by implementing a profitable squeeze. Benabou and Laroque (1992) consider a reputational model of

[5]In contrast to these papers where the trader actually makes costless announcements, in our paper, the only action is the potentially costly trades which the followers observe.

manipulation in which an insider sometimes tells the truth to build a reputation and sometimes lies and profitably manipulates. They assume that he is a price-taker and can trade without being detected so that his trades do not affect prices and only his announcements do. Therefore, they do not consider a pure trade based model of manipulation in which no announcements are made or actions (other than trading) taken. Bagnoli and Lipman (1996) develop a model of action-based manipulation where the manipulator pools with someone who can take an action that alters the value of the firm. In their model, the manipulator takes a position, announces a takeover bid and unwinds his position. Vila (1989) presents another model of action-based manipulation where the manipulator pools with someone who is buying stock prior to a takeover bid in which the value of the firm will be increased.

The followers in the present paper can be identified as dual trading brokers (see above). There is a literature on the effect of dual trading brokers on the performance of the market. Roell (1990), Fishman and Longstaff (1992) are examples. The general conclusion of the literature is that dual trading lowers the profits of the informed traders by increasing competition. However, this literature does not consider the possibilities of manipulation arising out of dual trading.

A recent paper by Avery and Zemsky (1998) investigate conditions on the structure of information that would lead to herd behavior and manipulation in financial markets. While the behavior of the followers outlined above looks similar to herd behavior, we would like to emphasize that this is not a model of herd behavior in the formal sense. Formally, herd behavior is identical to an informational cascade when successive traders trade ignoring their private information. In our model, it is crucial that the followers' positive feedback herd-like trading contain information for the market makers[6]. However, this work is complementary to the work of Avery and Zemsky in that both papers try to isolate properties of the information structure that could lead to manipulation in financial markets.

[6]Otherwise prices will not move when followers trade.

2 Heuristic Description of the Results

We consider a model where the dynamic trader may have good news or may not have any news. Further we suppose that the dynamic trader may not also exist in the market.

We suppose that there exists traders called followers who know if the dynamic trader is in the market and also observe his first trade. For example, we may interpret the followers who are dual trading brokers who have better (but not perfect) information about the dynamic trader, compared to the market at large, because his trades are executed by them.

The market structure we look at is in the spirit of Glosten and Milgrom (1985). In particular, in each period market makers set bid and ask prices and one trader can buy one unit or sell one unit or choose not to trade at all. We assume that the market makers are competitive and in any period set prices equal to the expected value of the asset conditional on the observed history of trades up to and including that period.

The dynamic trader, in choosing his trade in any period, has to trade-off the short term profits from that trade with the long term effects that trade has on the market makers beliefs and future prices. We say that the dynamic trader manipulates if he undertakes a trade with a short term loss. If such a strategy is employed in equilibrium, then the dynamic trader undertakes the short term loss making trade only to "destabilize the auction" and earn higher long term profits[7]. We call such a strategy a manipulative strategy.

Our main contribution is to show that the uninformed dynamic trader will manipulate in every equilibrium when the number of period of trading is large enough and the dynamic trader is small[8] relative to the number of followers.

Because there are many followers and because the followers have superior information than the market makers about the dynamic trader's trades, small trades by the dynamic trader will have a large impact on prices in the long run, because of the relatively large number of followers trading on their superior information.

More precisely, suppose that the market makers believe that the uninformed trader is

[7]Kyle (1985)shows that such a strategy is not profitable in his model. See pp 1323, Kyle (1985).

[8]Even though not infinitesimally so, so that his trades still affect prices.

not manipulating. Then if he incurs a loss and buys at a price higher than the expected value of the asset, the followers will conclude that he is the informed trader with good news. As a result, they will follow the initial buy, and start to buy.

The market makers, upon observing the large number of buy orders, will start rasing the price as they will think that there is good news about the asset in the market. The uninformed trader can now trade against the trend and sell at high prices. The occasional sell order will be ignored by the market makers given the large number of buy orders coming in from the followers.

Therefore when the market makers believe that the uninformed trader will not manipulate then the uninformed trader can profitably manipulate, provided there are a large number of possible trading periods. He does this by buying once which leads to the followers (rationally) buying. This leads to the price converging to one and staying there even though he sells. This convergence of Bayesian learning by the market makers to a "falsehood" with probability 1 shows that there cannot be an equilibrium with no manipulation by the uninformed trader.

The idea can be illustrated by considering the following simple Bayesian updating problem. Suppose a decision maker is updating (like the market makers) on the true state of the world. In state of the world 1 (in which case the value of an asset is high) a coin with a probability of heads equal to $\frac{2}{3}$ will be tossed repeatedly (and independently, for simplicity). In state of the world 2 (in which case the value of an asset is low) a coin with a probability of heads equal to $\frac{1}{3}$ will be tossed, similarly. The decision maker puts positive probability on only these two states of the world and observes the infinite history of coin tosses. But suppose the true state of the world is not either state 1 or state 2 but a third state with the relevant coin having a probability of heads equal to p. Now provided that the decision maker puts 0 probability on this state, his posterior beliefs will converge to state 1 if $p > \frac{1}{2}$ and to state 2 if $p < \frac{1}{2}$ and will stay at his priors if $p = \frac{1}{2}$. This coin corresponds to the manipulative strategy of the uninformed trader. We show that when there exists many followers, by transferring part of the costs of manipulation to the followers, the uninformed trader has a profitable, manipulative, deviation strategy which keeps the posteriors of the market makers fixed at a certain state of the world (State 1, for example).

Therefore, when there are sufficiently many periods of trading, the uninformed trader will manipulate and make greater profits than not manipulating, when market makers' beliefs put zero probability on the uninformed trader manipulating. In other words, for many periods of trading, there does not exist an equilibrium of our trading game where the uninformed trader does not manipulate (Proposition 3 and corollary). We prove that an equilibrium exists (Proposition 1). Therefore, for many periods of trading, every equilibrium involves manipulation by the uninformed trader.

Notice that the argument above suggests that the uninformed trader will like to mimic the informed trader. We also show that the informed trader would like to mimic a noise trader and, as a result, will also manipulate when there are many period of trading (Proposition 2).

The existence of the followers allows the uninformed trader to profitably mimic the informed trader and manipulate the market. The existence of the followers creates adverse selection pressure on the informed trader and creates the incentives for him to manipulate and "mislead" the followers. Both types will manipulate in every equilibrium if there are sufficiently many periods of trading before all private information is revealed.

3 A Model of Manipulation

3.1 Notation and Definitions

3.1.1 The Asset

We consider a market for one asset. The long-term return or the fundamental value of the asset, v, is not known to all the participants in the market. In particular, we assume $v \in V = \{0, 1\}$, with the prior probability that $v = 1$ equal to $\overline{v} \in (0, 1)$.

3.1.2 The Traders

There are three kinds of traders in the market. The first is a long-lived trader, who we will call the dynamic trader, $i = D$. The dynamic trader does not necessarily come to the market. Further, even if he comes he may or may not have private information about the value of the asset.

Apart from the dynamic trader there is also a second class of rational informed trader in the market. For reasons which will be specified below, we will call these traders followers and denote them by $i = F$.

We will assume that there are also noise traders in the market and denote them by $i = N$. The market price in each period will be set by competitive market makers, $i = MM$.

3.1.3 The Trading Game and the Market Structure

The trading game we consider is in the spirit of Glosten and Milgrom (1985) where the market makers post bid and ask prices and in each period a trader, chosen randomly, gets to trade at these prices.

Let $t = 1, .., T$ index the periods of trading. In each period a buy order (of size 1) or a sell order (of size 1) or a no-trade can be observed. Let $E = \{b, n, s\}$ denote the set of possible trade-events observed in any period, with e the generic element. Thus $e = b$ denotes a buy order, $e = s$ a sell order and $e = n$ a no-trade.

Let E^t denote the $t-$fold Cartesian product of E. This the set of possible $t-$period histories of trades. Let e^t denote the generic element of E^t and let $e_{t'}(e^t)$ the $t^{'}-$th element of e^t, $t^{'} \leq t$. Denote by $E^0 = \{e^0\}$ the null history.

Let $\overline{E} = \cup_{t=0}^{\infty} E^t$. Let $H = E^{\infty}$ denote the set of infinitely long histories with h being its generic element. Denote by $e^t(h)$ the first $t-$elements of h and by $e_t(h)$ the $t-$th element of h.

We suppose that the dynamic trader trades in the first period if he exists. Recall that followers know if the dynamic trader exists or not. This implies that they get a signal of the dynamic trader's information from the first period trade. This creates the competitive pressure on the dynamic trader when he trades on his information because followers also trade in the same direction. Conversely, this allows the dynamic trader to "use" the followers when he manipulates. In equilibrium, since the followers know that the dynamic trader is manipulating they know that the reliability of the first period trade as an informative signal is low, which in turn relieves the competitive pressure on the dynamic trader.

In every subsequent period after the first period, with probability α_D the dynamic trader trades, with probability α_F a follower trades and with probability α_N a noise trader trades, $\alpha_D + \alpha_F + \alpha_N = 1$.

3.1.4 The Information Structure: Types & Information Sets

We will denote by $\theta \in \Theta = \{\theta_I, \theta_U, \theta_N\}$ the type of the dynamic trader $i = D$. Denote by $\mu(\theta|v)$ the probability that the dynamic trader is of type θ conditional on the value of the asset being v.

When $\theta = \theta_I$ the dynamic trader is informed. We will assume that with positive probability that the dynamic trader is informed when there is good news; and that if there is no good news in the market, then the dynamic trader cannot be informed, i.e.,

$$\begin{aligned} \mu(\theta &= \theta_I|v = 1) \equiv \mu_I \in (0, 1) \\ \mu(\theta &= \theta_I|v = 0) = 0. \end{aligned}$$

When $\theta = \theta_U$ the dynamic trader is uninformed. Accordingly, we assume that

$$\mu(\theta = \theta_U|v = 1) = \mu(\theta = \theta_U|v = 0) \equiv \mu_U \in (0, 1)$$

Notice that $\Pr[v = 1|\theta = \theta_U] = \overline{v}$, the prior, so that θ_U indeed corresponds to the uninformed type of the dynamic trader.

When $\theta = \theta_N$ the dynamic trader does not exist and is replaced (at each node in the trading game to be described below) by a noise trader. We suppose that $\mu(\theta = \theta_N|v) \in (0, 1)$ for all v.Notice that

$$\Pr[v = 1|\theta = \theta_N] = \frac{(1 - \mu_I - \mu_U)\overline{v}}{1 - \mu_I - \mu_U\overline{v}} \equiv \underline{v} < \overline{v}$$

The situation we have in mind can be described as follows. We are thinking of $v = 0$ as the "baseline" expected value of the asset. With a certain probability $\overline{v}$ there may be some good news about the asset i.e., $v = 1$. With a certain probability $(1 - \mu(\theta_N|v)$, which depends on the realization of v), there may be a dynamic trader in the market. If there is good news ($v = 1$), the dynamic trader may know it with a certain probability ($\mu(\theta_I | v = 1)$). However he may also be uninformed about the value of the asset.

We suppose that the followers know if the dynamic trader exists, i.e., $\theta \in \Theta/\{\theta_N\}$, or not, $\theta = \theta_N$. However, they do not know if he is informed or uninformed. Let $\Theta_F = \{\{\Theta/\{\theta_N\}\}, \{\theta_N\}\}$ with generic element $\theta_F(\theta)$,where the dependence of the followers' information on the dynamic trader's information has been made explicit. Market makers do not know if the dynamic trader exists or not, and if he does if he is informed or not.

Note that this is the information structure that the traders have prior to the start of trading and after all private information has been learned. All traders and the market makers observe the history of trades and update on these beliefs. The information of a trader also consists of information of the past trades of the trader as well as information on prices and strategies but we ignore that from our description.

3.1.5 Strategies, Probability Spaces and Manipulation

A strategy for the dynamic trader is a function $\sigma_D : \Theta_D \times \overline{E} \to \Delta(E)$. A strategy for a follower is a function $\sigma_F : \Theta_F \times \overline{E} \to \Delta(E)$, and that for a noise trader is a function $\sigma_N : \overline{E} \to \Delta(E)$. Let $\sigma_D(\theta, e^t)(e)$ be the probability that the strategy σ_D assigns to action e after history e^t conditional on θ. Analogous notation will be used for σ_F and σ_N. Let Σ_D and Σ_F be the set of strategies for $i = D, F$. We suppose that

$$\sigma_D(\theta_N, e^t)(e) = \sigma_N(e^t)(e) = \frac{1}{3} \text{ for all } e \in E, \text{ for all } e^t \in \overline{E}$$

The market makers choose a price function $p : \overline{E} \to \mathbb{R}$. We suppose that the market makers are "competitive", i.e.,

$$p(e^t) = E[v|e^t] = \Pr[v = 1|e^t]$$

Note that for all e^t we must have $p(e^t) \in [\underline{v}, 1]$.

Since we will be interested in the asymptotic properties of the market makers' beliefs about the state of the world as they update from the observed history of trades, and as the observed history of trades depends on the strategy used by the dynamic trader, we will now construct the probability space over observables given any particular strategy for the dynamic trader, in the usual fashion.

Let $\mathcal{C}^0$ be a collection of all (finite) cylinders. Then $\mathcal{C}^0$ is a field. Let $\mathcal{C}$ be the sigma-field generated by $\mathcal{C}^0$. Each strategy pair $\sigma = (\sigma_D, \sigma_F)$ induces a probability measure on $\mathcal{C}^0$. This has an unique extension to the generated sigma field $\mathcal{C}$. Denote this by $Q(\ \sigma)$. Whenever we talk about asymptotic properties of the price function e.g., its convergence to a specified value, we will have in mind the underlying probability space $\{H, \mathcal{C}, Q(\sigma)\}$ for some specified σ.

In a dynamic trading setting, when a long-lived trader is trading in any period, he trades-off his gains from that particular trade with the effect that the trade has on the market's beliefs about his expectation and future prices. For example, in Kyle (1985), the insider engaged in stealth trading, and traded more aggressively whenever the noise in the market order was high, so that the effect of his trades on the price was low. However, in Kyle the insider always traded in the direction of his information and never made a short term loss. In the unique linear equilibrium of Kyle (1985) it was not in the insider's interest to undertake trades so as to destabilize prices "with unprofitable trades made at the nth auction, then recouping the losses and much more with profitable trades made at future auctions" (pp.1323, Kyle (1985)). What we mean by manipulation is precisely to make such short term losses in order to make larger long term profits. Therefore, manipulation must involve at least one period in which the dynamic trader intentionally makes a loss, for example by buying an asset which is already overvalued given his information, in expectation of recouping the losses by "destabilizing" the prices and selling at a higher price in the future. Now we formally state what we mean by a manipulative strategy:

Definition 1 *A strategy σ_D is non-manipulative if for all $\theta \neq \theta_N$, for all $e^t \in \overline{E}$*

$$\begin{array}{ll} (i) & \sigma_D(\theta, e^t)(b) > 0 \Rightarrow E[v|\theta] \geq p(e^t b) \\ (ii) & \sigma_D(\theta, e^t)(s) > 0 \Rightarrow E[v|\theta] \leq p(e^t s) \end{array}$$

Otherwise it is manipulative.

We should note that the conditions a non-manipulative strategy must satisfy according to above definition are very weak[9]. For example, not buying with probability one

[9]In fact, note that not every mixed strategy is a manipulative strategy: there exists non-manipulative

even if the price is lower than its expected value is not considered manipulative. Similarly not trading even if it is profitable to do so is non-manipulative. In other words, it is very easy for a strategy to qualify to be a non-manipulative one. In return a manipulative strategy is very restricted. Considering the fact that our main result will state that every equilibrium involves manipulation, working with this definition of manipulation makes our results only stronger in terms of characterization of equilibrium behavior.

3.1.6 Equilibrium and Existence

Definition 2 *An equilibrium consists of the tuple*

$$< p(.), \sigma_D, \sigma_F >$$

such that:

(i) $p[e_t(h)] = E[v|e_t(h)] \, \forall h, \ \forall t$ *is derived from* (σ_D, σ_F) *according to Bayes' Rule.*

(ii) σ_D *and* σ_F *are sequentially rational given* $p(.)$.

Proposition 1 *For all finite* T *an equilibrium exists.*

Proof. Since for finite T we have a game of incomplete information with compact action spaces, the proof of existence follows from Kakutani's Fixed Point Theorem by standard arguments. □

3.1.7 Manipulation by the Informed Type

We first analyze the equilibrium behavior of the informed trader. The following proposition states that in every equilibrium the informed trader plays a strictly mixed action in the first period, as long as his information is sufficiently long-lived.

Before we state and prove our result we discuss the intuition behind it. Suppose that the market makers believe that the informed trader is playing a non-manipulative

mixed strategies. Further, a non-manipulative strategies is not merely the myopic optimal strategy. The strategy of not trading ever is non- manipulative and is clearly not optimal.

strategy. Therefore they do not expect him to sell. If he deviates and sells then the market makers put zero weight on the dynamic trader being informed and, as a result, the price will always be bounded above by $\overline{v}$ no matter what history of trades they observe[10]. In contrast if he plays non-manipulatively then the price cannot stay bounded above by $\overline{v}$ and may even converge to the true value. As a result, with a sufficiently large number of periods, the informed trader will find it profitable to manipulate.

Proposition 2 *Suppose $\alpha_D < \alpha_F$. Then there exists $\overline{T}$ such that if $T > \overline{T}$, there exists no equilibrium where* $\min[\sigma_D(\theta_I, e^0)(b), \sigma_D(\theta_I, e^0)(s)] = 0$.

Proof. The proof is in the Appendix. □

Note that the informed trader may exist only if $v = 1$. Further he is fully informed, i.e., he knows that $v = 1$ whenever he exists. Therefore, if he is to maximize first period profits he has to buy with probability one, considering the fact that bid and ask prices are between zero and one. Similarly, selling results in a loss in the first period. Hence, playing a strictly mixed strategy means that the informed trader is manipulating the market.

Corollary 3 *In every equilibrium, the informed trader manipulates the market as long as his information is long lived and there are sufficiently many followers.*

The manipulation in the first period by the informed trader has different effects on each group. The followers information set allows them to detect the dynamic trader. However, they cannot differentiate between two dynamic traders as we will see next section the uninformed dynamic trader will also be playing a mixed strategy in the first period. Consequently, as the two dynamic strategies becomes more similar the informed type's information gets more "private". Hence, the informed trader benefits from an action shared by the uninformed dynamic trader. On the other hand, the market makers entertain a positive probability of having a noise trader in the first period. Therefore, by mixing the informed trader tries to keep that probability away from zero as his "random" trade resembles that of noise. Obviously, keeping the makers doubtful of his existence becomes quite profitable in the long run.

[10]Note that noise trading can generate all finite histroies with positive probability.

3.1.8 Manipulation by the Uninformed Type

The uninformed trader would like to mislead the followers (therefore also the market makers) into believing that he is the informed trader. Hence, it is optimal for him to play a strategy similar to that of the informed trader. We have already established in Proposition 2 that the informed trader employs a strictly mixed strategy. The following proposition states that the uninformed trader will also use a mixed strategy under some mild conditions.

Before we state and prove our result we will discuss the intuition behind the result. Suppose that the market makers believe that the uninformed trader is behaving non-manipulatively and suppose that the first period ask price is above $\overline{v}$. Then the uninformed trader is not supposed to buy in the first period. But suppose he does. Then the followers believe that the dynamic trader is informed with probability 1. As a result the expected value of the good given their information is also equal to 1 and they will buy from here on. The market makers upon seeing lots of buy orders will think that the informed trader is in the market and as a result the price will rise and converge to 1. Even if the uninformed trader starts trading against the trend, the price will stay at the high level as long as the uninformed trader's trades are small compared to the buy orders submitted by the large number of followers. This is because, after a sufficiently large number of buy orders the market makers will ignore the sell order coming from the uninformed trader and ascribe them to noise. Therefore the uninformed trader will profit from such a deviation if the number of periods is large enough and hence there will not exist an equilibrium where the uninformed trader does not manipulate. Notice that the argument for manipulation is symmetric and applies equally to the case where the uninformed trader starts with a sell order, the followers follow by selling and then the uninformed trader buys the asset at a low price.

Proposition 4 *There exists* $\overline{\alpha}_D \in (0,1)$ *such that if* $\alpha_D < \overline{\alpha}_D$ *and* $\alpha_F + \alpha_N = 1 - \alpha_D$ *then there exists* $\overline{T}$ *such that if* $T > \overline{T}$ *, there exists no equilibrium where* $\min[\sigma_D(\theta_U, e^0)(b), \sigma_D(\theta_U, e^0)(s)] = 0$

Proof. The proof is in the Appendix. □

Recall that the uninformed trader has no private information about the fundamentals. Hence, given his information the expected value of the asset is $\overline{v}$, the prior. His equilibrium strategy dictates him to sell and buy in the first period randomly. Therefore, he is guaranteed to make a loss in this period if either the ask price is greater than the bid price or $\overline{v}$ is not between the two prices.

Corollary 5 *If the conditions in Proposition 4 are satisfied, the uninformed trader manipulates the market as long as the bid-ask spread is positive.*

Proposition 4 and its corollary form a stronger result than that of the existing literature in the sense that even if the informed type manipulates (i.e. we do not impose exogenous restrictions on his strategy), the uninformed type will also manipulate in every equilibrium.

We should note that the bid-ask spread can be negative only if the informed trader's manipulation is too extreme, namely he sells with a probability higher than the noise traders. Clearly, such a behavior is sub-optimal for the informed trader considering the fact that this strategy reveals his identity more than just imitating the noise and is more costly.

Even if the informed trader manipulates too much, this does not mean that the uninformed trader stops manipulating as a best response. In fact, he continues copying the informed trader, i.e., playing a mixed strategy. The only difference is that this strategy is profitable for him even in the short run given his information set.

4 Concluding Remarks

In this paper, we look at a model where the information sets of the different market participants are nested. This increases the competitive pressure on the insider when he trades on his information and also allows him to manipulate the market by trading against his information and "misleading" a large section of the market. The informed trader benefits from manipulation because it lowers the impact of his trades on prices. This is achieved by playing mixed strategies so that it is difficult to detect his identity.

On the other hand, the uninformed trader wants to imitate the informed trader in order to start a buying streak which he can later sell against. He accomplishes this task by employing a strategy similar to that of the informed trader. This strictly mixed strategy may be unprofitable in the short run (given his information set) since it involves random buys and sells. However, in the long run unloading position will be profitable once the market entertains a high probability of an informed trader's existence.

We show that manipulation by each type of informed trader adversely affects the incentives of the other type to manipulate. Nevertheless both types of the insider will manipulate in every equilibrium, if their private information is long-lived.

The crucial feature of the model above which allows the dynamic trader to manipulate is the fact that his trades are observable to some traders in the market. Thus trading is not completely anonymous. In fact, it might be in the interest of both the informed trader and the uninformed trader to sometimes deliberately disclose their trades[11]. Such strategic disclosure of trading positions is not discussed in this paper. However, it goes hand in hand with manipulative trading and so it would be interesting to study the interaction between strategic disclosure and manipulative trading.

5 Appendix

As it will be useful in the proof below, we first write formally the information held by the different traders in the market. Let $\Omega = V \times \Theta \times H$ with generic element ω. At each date t, for a state of the world $\omega = \{v, \theta, h\}$ let $\mathcal{F}_t^i(\omega)$ be the information of $i \in \{D, F, MM\}$. We suppose that $\mathcal{F}_t^D(\omega) = \{\theta, e^t(h)\}$. Further, for the followers $\mathcal{F}_t^F(\omega)$ is $\{\theta_F(\theta), e^t(h)\}$. Finally, $\mathcal{F}_t^{MM}(\omega) = \{e^t(h)\}$.

Proof of Proposition 2 Suppose we have an equilibrium where $\sigma_D(\theta_I, e^0)(s) = 0$. If, in contrast to the prescribed strategy, the informed trader, $\theta = \theta_I$, deviates and sells in the first period then the market makers believe that $\theta \in \{\theta_U, \theta_N\}$ with probability one. Since all finite histories can be generated with positive probability if $\theta = \theta_N$, no

[11] Even when not required to do so.

sequence of trades to follow can change their beliefs. Since $\Pr[v = 1|\theta = \theta_U] = \overline{v}$ and $\Pr[v = 1|\theta = \theta_N] = \underline{v} < \overline{v}$, we must have $p(e^t) \in [\underline{v}, \overline{v})$ for all e^t, following this deviation. To prove the result it suffices to show therefore that if the informed trader sticks to his candidate equilibrium strategy the price will converge to something in $[\overline{v}, 1]$ with probability one.

Convergence of the price with probability one is guaranteed by the martingale convergence theorem. We now show that the price cannot converge to something less than $\overline{v}$.

Suppose not. Then the set

$$A = \{h \in H | \exists \overline{T}(h) \text{ s.t. } \forall t \geq \overline{T}(h), p(e^t(h)) < \overline{v}\}$$

has $Q(\sigma(\theta_I))-$probability 1[12].

For any e^t and any $t' \leq t$ let $\#b_{t'}(e^t)$ be the number of buys in e^t after date t'. Consider the set

$$B = \{h \in A| \lim_{t\to\infty} \frac{\#b_{\overline{T}(h)}(e^t(h))}{t - \overline{T}(h)} \geq \frac{\alpha_N}{3} + \alpha_F\}$$

Clearly $B \in \mathcal{C}$. Further, since $\theta = \theta_I$, the followers put zero weight on $\theta = \theta_N$ and so $\Pr[v = 1|\ \mathcal{F}_t^F] \geq \overline{v}$. Therefore, for all $h \in A$, for all $t \geq \overline{T}(h)$, the followers will always buy after history $e^t(h)$. Hence, for all $h \in A$, for all $t \geq \overline{T}(h)$, $Q(\sigma(\theta_I), e^t(h))[e_t = b] \geq \frac{\alpha_N}{3} + \alpha_F$. Therefore, by the SLLN B has $Q(\sigma(\theta_I))-$probability 1. On the other hand, if $\theta = \theta_N$, the followers know it and so $\Pr[v = 1|\ \mathcal{F}_t^F] = \underline{v}$. Therefore, for all $h \in A$, for all $t \geq \overline{T}(h)$, the followers will always sell after history $e^t(h)$. Hence, for all $h \in A$, for all $t \geq \overline{T}(h)$, $Q(\sigma(\theta_N), e^t(h))[e_t = b] = \frac{\alpha_N+\alpha_D}{3} \leq \frac{\alpha_N}{3} + \alpha_F$. Therefore, by the SLLN B has $Q(\sigma(\theta_N))-$probability 0.

But then, $\Pr[\theta = \theta_N|e^t]$ will converge to zero $Q(\sigma(\theta_I))-$a.s. and so

$$p(e^t) = \Pr[\theta = \theta_I|e^t] + \overline{v}\Pr[\theta = \theta_U|e^t] + \underline{v}\Pr[\theta = \theta_N|e^t]$$

must converge to something weakly greater than $\overline{v}$, $Q(\sigma(\theta_I))-$a.s. in contradiction to our hypothesis.

The proof of the case where $\sigma_D(\theta_I, e^0)(b) > 0$ is analogous.□

[12] Where we have supressed notation to write $\sigma(\theta_I)$ instead of $(\sigma_D(\theta_I), \sigma_F(\theta_F(\theta_I)))$.

We first define some new entities that will be useful in the proof of Proposition 3. For any θ, and associated strategies $(\sigma_D(\theta,.), \sigma_F(\theta_F(\theta),.))$ and any e^t, we suppress notation slightly and let $Q^0(\theta, e^t) \in \Delta(E)$ denote the probability distribution over trades in period $t+1$, conditional on θ and e^t. Let $Q^0(\theta) = \{Q^0(\theta, e^t)\}_{e^t}$ be the collection of all of these conditional distributions, one for each e^t. Let $\Pr[e^t|Q^0(\theta)]$ denote the probability of a history e^t being generated by $Q^0(\theta)$:

$$\Pr[e^t|Q^0(\theta)] = \prod_{t'=0}^{t-1} Q^0(\theta, e^{t'}(e^t))(e_{t'+1}(e^t))$$

Let

$$L[e^t;\theta,\theta'] = \frac{\Pr[e^t|Q^0(\theta)]}{\Pr[e^t|Q^0(\theta')]}$$

This is well-defined as $\alpha_N > 0$. We now start with the proof of the proposition.

Proof of Proposition 3 Suppose that $\sigma_D(\theta_U, e^0)(b) = 0$ in a candidate equilibrium. Suppose that the uninformed trader deviates and buys in period 1. Then, using the last proposition, the followers think $\theta = \theta_I$ with probability one and hence they will buy with probability one in every period.

Following the deviation, the MM think that $\theta \in \{\theta_I, \theta_N\}$ and they use the observed history of trades to update between these two distributions. If $\theta = \theta_I$, for all e^t such that $e_1(e^t) = b$,

$$\begin{aligned} Q^0(\theta_I, e^t)(b) &= \tfrac{\alpha_N}{3} + \alpha_F + \alpha_D\sigma_D(\theta_I, e^t)(b) \\ Q^0(\theta_I, e^t)(n) &= \tfrac{\alpha_N}{3} + \alpha_D\sigma_D(\theta_I, e^t)(n) \\ Q^0(\theta_I, e^t)(s) &= \tfrac{\alpha_N}{3} + \alpha_D\sigma_D(\theta_I, e^t)(s) \end{aligned}$$

On the other hand, if $\theta = \theta_N$, for all e^t such that $e_1(e^t) = b$,

$$\begin{aligned} Q^0(\theta_N, e^t)(b) &= \tfrac{\alpha_N}{3} + \tfrac{\alpha_D}{3} \\ Q^0(\theta_N, e^t)(n) &= \tfrac{\alpha_N}{3} + \tfrac{\alpha_D}{3} \\ Q^0(\theta_N, e^t)(s) &= \tfrac{\alpha_N}{3} + \tfrac{\alpha_D}{3} + \alpha_F \end{aligned}$$

Suppose that the uninformed trader does not trade following the deviation. Then the true distribution driving the trades, on which the MM put zero weight, is:

$$Q^*(b) = \frac{\alpha_N}{3} + \alpha_F$$
$$Q^*(n) = \frac{\alpha_N}{3} + \alpha_D$$
$$Q^*(s) = \frac{\alpha_N}{3}$$

Now, the posterior belief of the market makers will converge to $\theta = \theta_I$ (equivalently, the price will converge to 1) iff the likelihood ratio $L[e^t; \theta_N, \theta_I]$ converges to zero. Clearly,

$$L[e^t; \theta_N, \theta_I] \leq \widehat{L}(e^t) \equiv (\frac{\frac{\alpha_N}{3} + \frac{\alpha_D}{3}}{\frac{\alpha_N}{3} + \alpha_F})^{\#b(e^t)}(\frac{\frac{\alpha_N}{3} + \frac{\alpha_D}{3}}{\frac{\alpha_N}{3}})^{\#n(e^t)}(\frac{\frac{\alpha_N}{3} + \frac{\alpha_D}{3} + \alpha_F}{\frac{\alpha_N}{3}})^{\#s(e^t)}$$

where $\#b(e^t)$ is the number of buys in history e^t, $\#s(e^t)$ is the number of sells in history e^t, and $\#n(e^t)$ is the number of no-trades in history e^t. Now,

$$\widehat{L}(e^t) = [(\frac{\frac{\alpha_N}{3} + \frac{\alpha_D}{3}}{\frac{\alpha_N}{3} + \alpha_F})^{\frac{\#b(e^t)}{t}}(\frac{\frac{\alpha_N}{3} + \frac{\alpha_D}{3}}{\frac{\alpha_N}{3}})^{\frac{\#n(e^t)}{t}}(\frac{\frac{\alpha_N}{3} + \frac{\alpha_D}{3} + \alpha_F}{\frac{\alpha_N}{3}})^{\frac{\#s(e^t)}{t}}]^t$$

Therefore $\widehat{L}(e^t)$ converges to zero a.s. iff the term inside the square brackets converges to something less than 1, a.s. Given the true Q^*, this term converges to

$$l^*(\alpha) \equiv (\frac{\frac{\alpha_N}{3} + \frac{\alpha_D}{3}}{\frac{\alpha_N}{3} + \alpha_F})^{\frac{\alpha_N}{3}+\alpha_F}(\frac{\frac{\alpha_N}{3} + \frac{\alpha_D}{3}}{\frac{\alpha_N}{3}})^{\frac{\alpha_N}{3}+\alpha_D}(\frac{\frac{\alpha_N}{3} + \frac{\alpha_D}{3} + \alpha_F}{\frac{\alpha_N}{3}})^{\frac{\alpha_N}{3}}$$

Clearly, there exists $\overline{\alpha}_D \in (0, 1)$ such that if $\alpha_D < \overline{\alpha}_D$ and $\alpha_F + \alpha_N = 1 - \alpha_D$, then $l^*(\alpha) < 1$ so that the price will converge to 1 $Q^* - a.s.$. Further, if the uninformed trader switches to selling after some point even then the price will converge to one, a.s., by an argument similar to the one above.

To show that this deviation is profitable it suffices to show that in the candidate equilibrium, the price cannot converge to 1 a.s. But, for the price to converge to 1 a.s. in equilibrium when $\theta = \theta_U$ the MM must believe that $\theta = \theta_I$ with probability one in the limit. However it is well known that (see Bray and Kreps etc.) Bayesian learning implies that there is probability 0 that the MM will assign probability one in the limit on a false event.

This proves the first half of the result that $\sigma_D(\theta_U, e^0)(b) > 0$ in every equilibrium under the conditions of the theorem. The proof of $\sigma_D(\theta_U, e^0)(s) > 0$ is analogous. □

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