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Extension of N-continuous OFDM Precoder Matrix for Improving Error Rate

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Abstract—The modulation technique of N-continuous orthogonal frequency division multiplexing (OFDM) has lower sidelobes than those of the original OFDM as a result of the continuous connection with higher-order derivatives between the OFDM symbols. Compared with the other sidelobe suppression schemes, for example, cancellation carrier insertion, and windowing, N-continuous OFDM does not require an extended guard interval nor the insertion of a wide guard band or cancellation carriers. However, N-continuous OFDM requires an iterative algorithm to remove the correction symbols from received symbols at the receiver and the large power of the correction symbols in the high-frequency area results in unavailable bandwidth. Here, we propose an extension of the N-continuous OFDM precoder matrix that flattens the bias power caused by the correction symbols in all frequency areas. Numerical experiments show that the proposed method improves the error rate performance without degrading the sidelobe suppression performance. The proposed method can also reduce the unavailable bandwidth.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been adopted in several telecommunications technologies owing to the advantages of fast data transmission and robustness against multipath fading. However, one problem associated with OFDM is that high sidelobes arise from the discontinuity between adjacent OFDM symbols. Various methods of sidelobe suppression have been proposed. Windowing [1] in the time domain and adaptive symbol transition [2] reduce data transmission speeds due to the associated extended guard interval or non-data time domain blocks for the sidelobe suppression. The insertion of a wide guard band or cancellation carriers [3] is a useful approach for avoiding interference with adjacent bands, but these methods decrease the spectral efficiency. Subcarrier weighting [4] is a method for sidelobe suppression in OFDM systems, but it causes an increased bit error rate, and compared with cancellation carriers, the interference reduction is not significant. Lower sidelobes can be achieved with N-continuous OFDM [5], which is a modulation technique in which the OFDM symbols are continuously connected with higher-order derivatives. This technique does not require an extended guard interval nor the insertion of a wide guard band or cancellation carriers.

The N-continuous OFDM transmitter adds a correction symbol to the data symbol to achieve a smooth connection between each OFDM symbol, and this means that the receiver must remove the bias caused by this correction symbol before demodulation. Van de Beek and Berggren [5] proposed an

iterative algorithm method to remove the bias, but large biases cannot be completely removed, and as a result, the error rate performance is degraded. The power per subcarrier of the bias is proportional to the power of the precoder matrix row vector, and the power in the high-frequency area is exceptionally large and thus severely degrades the error rate performance [6].

In this paper, we propose an extension of the N-continuous OFDM precoder matrix to improve the error rate performance by flattening the bias power of all subcarriers.

We explain the proposed method in section II, present the results of numerical experiments in section III, and analyze computational complexity in section IV. We conclude the paper in section V.

II. PROPOSED METHOD

The transmitter and receiver for our proposed method are designed to be the same as those of the conventional N-continuous OFDM except for a precoder matrix. In subsection A, the extended precoder matrix is introduced to the conventional N-continuous OFDM transmitter. The receiver using the matrix and the bias causing degradation of the error rate performance are defined in subsection B. A method for computing the matrix to flatten the bias is explained in subsection C.

A. Transmission

In the conventional N-continuous OFDM, the i-th OFDM symbol of the proposed method is

$$s_i(t) = \sum_{k \in \mathcal{K}} \bar{d}_{i,k} e^{j2\pi kt/T_s}, \quad -T_g \leq t < T_s, \quad (1)$$

where $\bar{d}_{i,k}$ are the results of the precoding information symbols $d_{i,k}$, $\mathcal{K} = \{k_0, \ldots, k_{K-1}\}$ is a set of data subcarrier indices, K is the number of data subcarriers, T_s is the OFDM symbol duration and T_g is the guard interval length.

The proposed method satisfies the constraint

$$\left. \frac{d^n}{dt^n} s_i(t) \right|_{t=-T_g} = \left. \frac{d^n}{dt^n} s_{i-1}(t) \right|_{t=T_s}, \quad n = 0, \ldots, N. \quad (2)$$

For the OFDM symbols of (1), the constraint (2) becomes

$$\sum_{k \in \mathcal{K}} k^n \bar{d}_{i,k} e^{-j2\pi kT_g} = \sum_{k \in \mathcal{K}} k^n \bar{d}_{i-1,k}. \quad (3)$$

This has an equivalent matrix form of

$$\mathbf{A}\mathbf{\Phi}\bar{\mathbf{d}}_i = \mathbf{A}\bar{\mathbf{d}}_{i-1}, \qquad (4)$$

where $\bar{\mathbf{d}}_i = [\bar{d}_{i,0}, \ldots, \bar{d}_{i,K-1}]^T$ is the result of the precoding data symbol $\mathbf{d}_i = [d_{i,0}, \ldots, d_{i,K-1}]^T$,

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ k_0 & k_1 & \cdots & k_{K-1} \\ \vdots & \vdots & \ddots & \vdots \\ k_0^N & k_1^N & \cdots & k_{K-1}^N \end{bmatrix}, \qquad (5)$$

$$\mathbf{\Phi} = \mathrm{diag}(e^{j\phi k_0}, \ldots, e^{j\phi k_{K-1}}), \qquad (6)$$

and $\phi = -2\pi T_g/T_s$.

To extend the conventional N-continuous OFDM precoding, we introduce an arbitrary regular matrix $\mathbf{U}$ into (4) such that

$$\mathbf{A}\mathbf{\Phi}\mathbf{U}\mathbf{U}^{-1}\bar{\mathbf{d}}_i = \mathbf{A}\bar{\mathbf{d}}_{i-1}. \qquad (7)$$

Assuming that $K > N$, (7) gives many solutions for the precoded symbol $\bar{\mathbf{d}}_i$. We choose the correction symbol $\mathbf{w}_i$ that minimizes $\|\mathbf{U}^{-1}\mathbf{w}_i\|_2^2$ so that

$$\bar{\mathbf{d}}_i = \mathbf{d}_i + \mathbf{w}_i, \qquad (8)$$

satisfies (7). Specifically, $\mathbf{w}_i$ is found to be

$$\mathbf{w}_i = -\mathbf{Q}\mathbf{d}_i + \mathbf{Q}\mathbf{\Phi}^H\bar{\mathbf{d}}_{i-1}, \qquad (9)$$

where the precoder matrix $\mathbf{Q}$ is defined as

$$\mathbf{Q} = \mathbf{U}[\mathbf{A}\mathbf{\Phi}\mathbf{U}]^{\dagger}\mathbf{A}\mathbf{\Phi}, \qquad (10)$$

and $[\cdot]^{\dagger}$ is the Moore–Penrose pseudoinverse of $[\cdot]$. Here, $\mathbf{Q}$ coincides with the $\mathbf{P}$ in Ref.[5] when $\mathbf{U} = \mathbf{I}$.

B. Reception

The OFDM signal of the proposed method is received in a frequency-selective fading channel and additive white Gaussian noise. The i-th received symbol after the FFT-demodulation of the receiver is modelled as $\mathbf{r}_i = \mathbf{H}_i\bar{\mathbf{d}}_i + \mathbf{n}_i$, where $\mathbf{H}_i$ is a $K \times K$ matrix with complex-valued channel attenuations and $\mathbf{n}_i$ is a complex-valued zero-mean Gaussian noise vector. Assuming that the channel is known at the receiver, we obtain

$$\tilde{\mathbf{r}}_i = \mathbf{H}_i^{-1}\mathbf{r}_i, \qquad (11)$$

by the zero-forcing equalizer.

Ref.[5] proposed detecting $\mathbf{d}_i$ from $\tilde{\mathbf{r}}_i$ by using an iterative algorithm, and our proposed method uses the same iterative algorithm. In the mth receiver iteration, the decision variable produced is

$$\bar{\mathbf{r}}_i^{(m)} = (\mathbf{I} - \mathbf{Q})\tilde{\mathbf{r}}_i + \mathbf{Q}\hat{\mathbf{d}}_i^{(m-1)}, \qquad (12)$$

where $\hat{\mathbf{d}}_i^{(m)}$ denotes a vector with symbol decisions $\hat{d}_{i,k}^{(m)}$, which are determined from $\hat{\mathbf{d}}_i^{(0)} = \mathbf{0}$,

$$\hat{d}_{i,k}^{(m)} = \arg\min_{d \in \mathcal{C}}\{|\bar{r}_{i,k}^{(m)} - d|^2\}, \qquad (13)$$

where, $\mathcal{C}$ is a complex-valued symbol constellation. In this manner, the decisions for each subcarrier (13) are used to determine the data symbol.

In this iterative method, information symbols are more likely to be correctly demodulated with a lower power for each element in the bias symbol $\mathbf{Q}\mathbf{d}_i$. The reason is the followings. From (11),

$$\tilde{\mathbf{r}}_i = \mathbf{H}_i^{-1}\mathbf{r}_i = \mathbf{d}_i + \mathbf{w}_i + \tilde{\mathbf{n}}_i, \qquad (14)$$

where $\tilde{\mathbf{n}}_i = \mathbf{H}_i^{-1}\mathbf{n}_i$. After the first receiver iteration, the decision variable is

$$\bar{\mathbf{r}}_i^{(1)} = (\mathbf{I} - \mathbf{Q})\tilde{\mathbf{r}}_i = \mathbf{d}_i - \mathbf{Q}\mathbf{d}_i + \bar{\mathbf{n}}_i, \qquad (15)$$

where $\bar{\mathbf{n}}_i = (\mathbf{I} - \mathbf{Q})\tilde{\mathbf{n}}_i$. Note that $(\mathbf{I} - \mathbf{Q})\mathbf{w}_i = \mathbf{0}$ since $\mathbf{Q}^2 = \mathbf{Q}$. From (15) and (13), it can be seen that a lower power for the bias $(\mathbf{Q}\mathbf{d}_i)_k$ will result in a more accurate demodulation of $d_{i,k}$.

C. Grinder Matrix $\mathbf{U}$

The proposed method flattens the power per subcarrier of the bias by the parameter $\mathbf{U}$, named *grinder matrix*, that minimizes

$$J(\mathbf{U}) = \sum_{k \in \mathcal{K}}\left(V_k(\mathbf{U}) - \sum_{m \in \mathcal{K}}V_m(\mathbf{U})/K\right)^2. \qquad (16)$$

Here $V_k(\mathbf{U})$, the expectation of the power per subcarrier of the bias symbol $\mathbf{Q}\mathbf{d}_i$, is defined as follows:

$$V_k(\mathbf{U}) \equiv E[|(\mathbf{Q}\mathbf{d}_i)_k|^2] = \sigma^2\sum_{l=0}^{K-1}|q_{k,l}|^2, \qquad (17)$$

where $\sigma^2 = E[|d_{i,k}|^2]$, and $q_{k,l}$ is the (k,l)-element of the matrix $\mathbf{Q}$.

We determine

$$\mathbf{U} = \mathrm{diag}(\mathbf{u}), \qquad (18)$$

$$\mathbf{u} = [u_0, \ldots, u_{K-1}]^T, \qquad (19)$$

and $\mathbf{u}$ is obtained by the steepest descent method such as

$$\mathbf{u}^{(j+1)} = \mathbf{u}^{(j)} - \varepsilon\left[\frac{\partial J}{\partial u_0}, \ldots, \frac{\partial J}{\partial u_{K-1}}\right]^T, \qquad (20)$$

where $\mathbf{u}^{(j)}$ is $\mathbf{u}$ in the jth iteration, $\mathbf{u}^{(0)} = [1, \cdots, 1]^T$, and ε is a positive constant. We then normalize $\mathbf{u}$ such that $E[\|\mathbf{u}\|_2^2] = K$.

III. Numerical Experiments

To evaluate the performance of the proposed method, we conducted numerical experiments with 16-QAM, 512 FFT-points, $\mathcal{K} = \{-150, \ldots, -1, +1, \ldots, +150\}$, $K = 300$, $T_s = 1/15$ ms, $T_g = 9T_s/128$, and $N = 3$. We also compared the proposed method with two other extensions of the N-continuous scheme proposed in [7] and [8], which are referred to here as "Method 1" and "Method 2" respectively. Before the experiments, the grinder matrix $\mathbf{U}$ was computed by (20), where the number of iterations was 250.
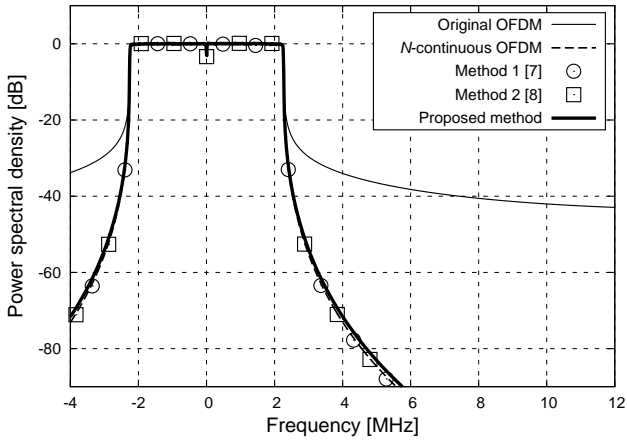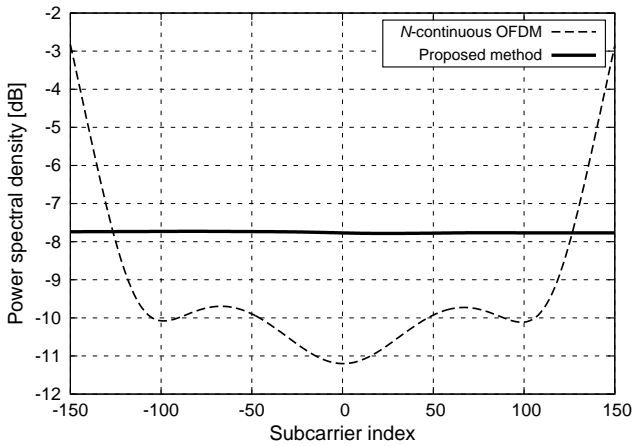
Fig. 1. Power spectral density ($N = 3$).
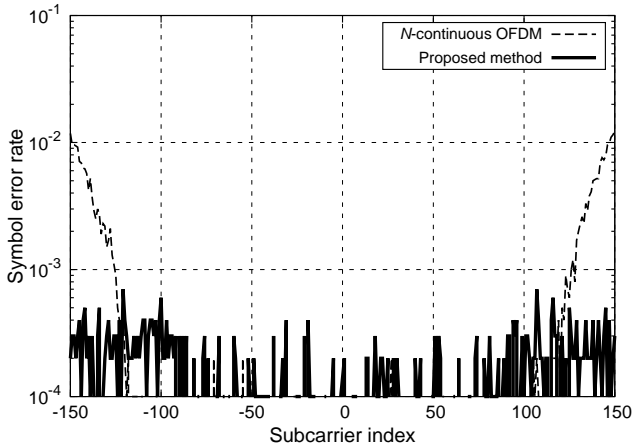

Fig. 2. Power of bias symbol ($N = 3$).

Fig. 3. SER per subcarrier at SNR = 14 dB ($N = 3$, $M = 2$).

(a) $N = 3$

(b) $N = 4$
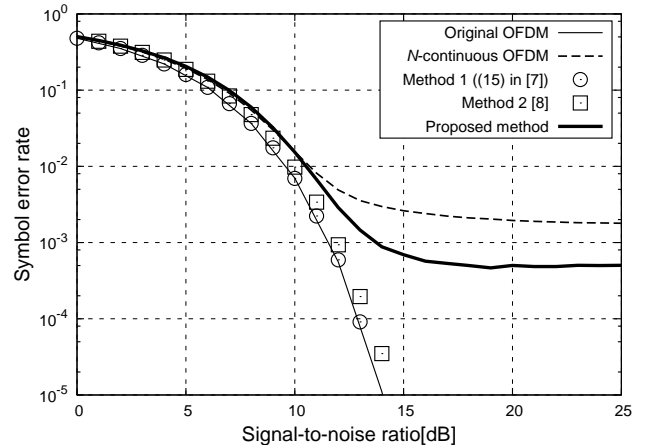
Fig. 4. SER performance in AWGN channel : (a) $N = 3$ and (b) $N = 4$ with 250 and 500 iterations, respectively.

Figure 1 shows the power spectral density (PSD) of the OFDM symbols. The PSD of the proposed method is almost identical to those of the conventional N-continuous OFDM, Method 1 and 2. Figures 2 and 3 show the average power per subcarrier of $10,000$ bias symbols and the symbol error rate (SER) per subcarrier at a SNR of 14dB after $M = 2$ receiver iterations, respectively. The proposed method yields a low error rate in all frequency areas by virtue of the flat bias, which is in contrast to the conventional N-continuous OFDM in which the error rate performance is degraded because of the large bias power in the high-frequency area.

Figure 4 shows the average SER of all subcarriers after $M = 2$. Although the SER of the proposed method is not at the same level as those of Methods 1 and 2, it is superior to the conventional N-continuous OFDM. Figure 5 shows the relationship between M and the SER for SNR = 14dB. The proposed method performs an accurate demodulation after fewer receiver iterations than the conventional method. Figure 6 shows the unavailable bandwidth for SNR = 14dB, which is defined as the number of subcarriers whose error rate is larger than 10^{-3}. The unavailable bandwidth of the proposed method is zero at $\mathrm{SNR} \geq 14$dB, whereas that of the conventional method is still finite for large SNRs.
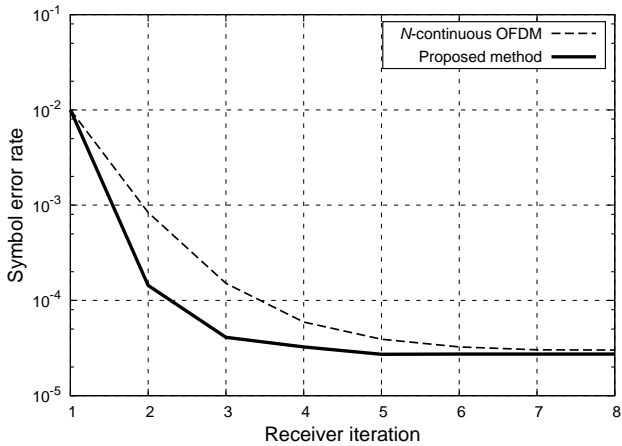
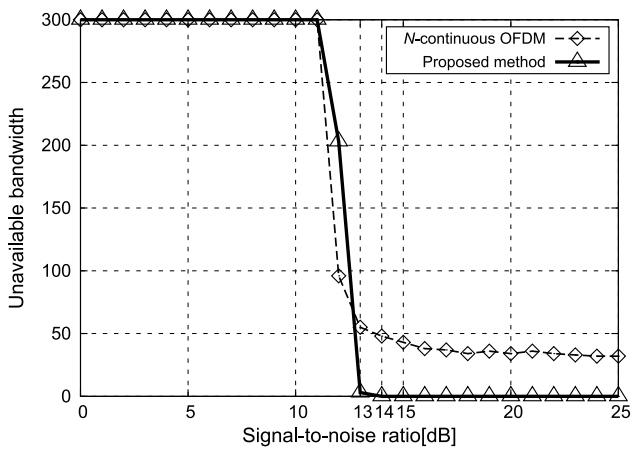Fig. 5.   SER performance against receiver iterations ($N = 3$).

Fig. 6.   Unavailable bandwidth against SNR ($N = 3$).

IV. Complexity Analysis

Next, we discuss the computational complexity. From (8) and (9), the precoding is rewritten as

$$\bar{\mathbf{d}}_i = \mathbf{d}_i + \mathbf{Q}\mathbf{x}_i \qquad (21)$$

where $\mathbf{x}_i = -\mathbf{d}_i + \boldsymbol{\Phi}^H \bar{\mathbf{d}}_i$. Since the size of the matrix $\mathbf{Q}$ is $K \times K$, the computational complexity of $\mathbf{Q}\mathbf{x}_i$ is $O(K^2)$, which results in an enormous computational load. When the matrix $\mathbf{Q}$ is decomposed into $\mathbf{Q}_1 = \mathbf{U}[\mathbf{A}\boldsymbol{\Phi}\mathbf{U}]^\dagger$ and $\mathbf{Q}_2 = \mathbf{A}\boldsymbol{\Phi}$ from (10), the complexity reduces to $O(KN)$ because the sizes of $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are $K \times (N+1)$ and $(N+1) \times K$, respectively, and $\mathbf{Q}\mathbf{x}_i$ can be rewritten as $\mathbf{Q}_1(\mathbf{Q}_2)\mathbf{x}_i$. This decomposition is available in the conventional N-continuous OFDM. Table I shows a comparison of the number of multiplications for the precoding. The computational complexity of Method 1 is higher than the other methods, whereas that of the proposed method is identical to that of the conventional N-continuous OFDM with the decomposed matrix.

Lastly, we consider the spectral efficiency. Methods 1 and 2 sacrifice $N + 1$ and 2 data subcarriers for the precoding,

TABLE I
Comparison of computational complexity.

(a) Transmitter		
	Multiplications	Example
N-continuous OFDM	$K + 2K(N+1)$	2,700
Method 1	$(2K - N - 1)(K - N - 1)$	176,416
Method 2	$5K + 2K(N+1)$	3,900
Proposed method	$K + 2K(N+1)$	2,700

(b) Receiver		
	Multiplications	Example
N-continuous OFDM	$K + 2MK(N+1)$	19,500
Method 1	$K(K - N - 1)$	88,800
Method 2	$K + 2MK(N+1)$	19,500
Proposed method	$K + 2MK(N+1)$	19,500

(Example: $K = 300$, $N = 3$, $M = 8$(receiver iterations))

respectively. In Method 2 in particular, the SER performance can be degraded if the two subcarriers, known as "cancellation tones" , are severely damaged. The proposed method does not sacrifice any subcarriers.

V. Conclusion

We have proposed an extension of the N-continuous OFDM precoder matrix, and numerical experiments have shown that the proposed method improves the error rate performance without degrading the sidelobe suppression performance. Compared with the conventional method, the proposed method performs an accurate demodulation with a relatively low number of receiver iterations and reduces the unavailable bandwidth.

In future work, we will analyze the peak-to-average ratio (PAPR) performance, which is of concern because the precoder matrix $\mathbf{Q}$ is not Hermitian, although the precoder matrix $\mathbf{P}$ in Ref.[5] is Hermitian. Fortunately, the grinder matrix $\mathbf{U}$ can be designed freely as long as it is invertible. Therefore, the PAPR performance could be controlled by a well-designed grinder matrix.

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