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Comparison of Deterioration Factor of 6-th order moment and Cumulant in a Broad Sense

Kazuhiro Takeyasu • Yuki Higuchi

Abstract: So far, many signal processing methods for machine diagnosis have been proposed such as Kurtosis and Bicoherence. But these methods have to be calculated using computers which are not always easily available at the maintenance site. In the field of plant maintenance, data should be handled in the simplest manner possible, and machine diagnosis should be done in the quickest way. In this paper, n-th normalized cumulant is considered so as to intensify the sensitivity of diagnosis. Higher order cumulant is expressed as the combination of the same order moment and the moments less than that order. New indices would be introduced because they are the combinations of plural order moments. By the way, the theoretical value of n-th moment divided by n-th moment calculated by measured data would behave in the same way. We name this factor as absolute n-th moment deterioration factor. In this paper, a simplified calculation method of this factor is introduced. This method is also applied to n-th normalized cumulant. The concept of absolute deterioration factor of n-th order moment in a broad sense is introduced and the analysis is executed. From the result of comparison, absolute deterioration factor of 6-th order moment including 6-th order cumulant in a broad sense is better than other factors in the viewpoint of sensitivity and practical use. Thus, n-th order moment including n-th order cumulant in a broad sense was examined and evaluated, and we obtained practical good results. The proposed method enables failure detection of equipment in a simple and quick way. This method is applicable in many data handling fields.

Keywords: impact vibration, kurtosis, cumulant

1. INTRODUCTION

In mass production firms such as steel making that have big equipments, sudden stops of production processes of machine failure cause great damages such as shortage of materials to the later processes, delays to the due date and the increasing idling time. To prevent these troubles, machine diagnosis techniques play important roles. So far, Time Based Maintenance (TBM) technique has constituted the main stream of the machine maintenance, which makes checks for maintenance at previously fixed time. But it has a weak point that it makes checks at scheduled time without taking into account whether the parts still keeping good conditions or not. On the other hand, Condition Based Maintenance (CBM) makes maintenance by watching the condition of machines. Therefore, if the parts are still keeping good condition beyond its supposed life, the cost of maintenance may be saved because machines can be used longer than planned. Therefore, the use of CBM has become dominant. Therefore the purpose of latter one needs less cost of parts, less cost of maintenance and leads to lower failure ratio.

However, it is mandatory to catch a symptom of the failure as soon as possible of a transition from TBM to CBM is to be made. Many methods are developed and examined focusing on this subject. In this paper, we propose a method for the early detection of the failure on rotating machines which is the most common theme in machine failure detection field.

So far, many signal processing methods for machine diagnosis have been proposed (Bolleter, 1998; Hoffner, 1991). As for sensitive prameters, Kurtosis, Bicoherence, Impact Deterioration Factor (ID Factor) were examined (Yamazaki, 1988; Maekawa *et al.*, 1997; Shao *et al.*, 2001; Song *et al.*, 1998; Takeyasu, 1989).

Kurtosis is one of the sophisticated inspection parameter which calculates normalized 4-th moment of probability density function. In the field of plant maintenance, data should be handled in the simplest manner possible, and machine diagnosis should be done in the quickest way.

In this paper, we consider the case such that impact vibration occurs on the gear when the failure arises. Under normal condition, probability density function of amplitude signal would be Normal Distribution in general (Our experiment in the past (Takeyasu, 1987, 1989) shows that it is a Normal Distribution). New index would be introduced using the relation that higher order cumulants more than 3rd order are 0 under Normal Distribution. Higher order cumulant is expressed as the combination of the same order moment and the moments less than that order. New indices would be introduced because they are the combinations of plural order moments. If they are calculated into simple formed equations and their indices are sensitive, new indices would be expected to be useful.

Furthermore, the absolute deterioration factor such as Bicoherence would be much easier to handle because it takes the value of 1.0 under the normal condition and tends to be 0 when damages increase. Cumulants more than 3rd order are 0 under normal condition and when failure increases, the value grows big. Therefore, inverse number of the sum of calculated value of cumlants plus 1 would behave as an absolute index. New index shows that it is 1.0 under normal condition and tends to be 0 when failure increases. In this paper, we introduce a simplified calculation method to this new index and name this as a simplified absolute index of higher order cumulant. As n-th cumulant is expressed with the combination of n-th order moment and the moment less than that order, n-th cumulant is said to be one of the n-th order moment in a broad sense. Furthermore, as Bicoherence can be considered to be a kind of 6-th order moment, several factors concerning absolute deterioration factor of 6-th order moment are compared and evaluated.

The concept of absolute deterioration factor of n-th order moment in a broad sense is introduced and the analysis is executed. From the result of comparison, absolute deterioration factor of 6-th order moment including 6-th order cumulant in a broad sense is better than other factors in the viewpoint of sensitivity and practical use. Thus, n-th order moment including n-th order cumulant in a broad sense was examined and evaluated, and we obtained practical good results. The proposed method enables failure detection of equipment in a simple and quick way. This method is applicable in many data handling fields.

The rest of the paper is organized as follows. We show the fundamental relationship with moments and cumulants in section 2. We affirm simplified calculation method of kurtosis we proposed before in section 3. In section 4, simplified absolute index of higher order cumulants are introduced. Numerical examples are presented and they are compared with Bicoherence in section 5 which are followed by remarks of section 6. Section 7 is a summary.

2. MOMENT AND CUMULANT

Here, we review moment, cumulant and Bicoherence for the preparation of 3.

2.1 MOMENT

In cyclic movements such as those of bearings and gears, the vibration grows larger whenever the deterioration becomes bigger. Also, it is well known that the vibration grows large when the setting equipment to the ground is unsuitable (Yamazaki, 1977). Assume the vibration signal is the function of time as $x(t)$. And also assume that it is a stationary time series with mean $\bar{x}$. Denote the probability density function of these time series as $p(x)$. When mean $\bar{x}$ is denoted by

$$\bar{x} = \int_{-\infty}^{\infty} xp(x)dx \tag{1}$$

and variance σ^2 is denoted by

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\bar{x})^2 p(x)dx \tag{2}$$

3rd order moment $MT(3)$, 4th order moment $MT(4)$ are denoted by

$$MT(3) = \int_{-\infty}^{\infty} (x-\bar{x})^3 p(x)dx \tag{3}$$

$$MT(4) = \int_{-\infty}^{\infty} (x-\bar{x})^4 p(x)dx \tag{4}$$

Normalized index of 3rd order moment and 4-th order moment is known as Skewness (SK), Kurtosis (KT) for each in the following definition.

$$SK = \frac{\int_{-\infty}^{\infty} (x-\bar{x})^3 p(x)dx}{\left[\int_{-\infty}^{\infty} (x-\bar{x})^2 p(x)dx\right]^{\frac{3}{2}}} \tag{5}$$

$$KT = \frac{\int_{-\infty}^{\infty} (x-\bar{x})^4 p(x)dx}{\left[\int_{-\infty}^{\infty} (x-\bar{x})^2 p(x)dx\right]^{2}} \tag{6}$$

Discrete time series are stated as follows.

$$x_k = x(k\Delta t) \quad (k = 1, 2, \cdots)$$

where Δt is a sampling time interval. Mean $\bar{x}$, variance σ^2, Skewness (SK) and Kurtosis (KT) are stated as follows under discrete time series.

$$\bar{x} = \lim_{M\to\infty} \frac{1}{M} \sum_{i=1}^{M} x_i \tag{7}$$

$$\sigma^2 = \lim_{M\to\infty} \frac{1}{M-1} \sum_{i=1}^{M} (x_i-\bar{x})^2 \tag{8}$$

$$MT(3) = \lim_{M\to\infty} \frac{1}{M-1} \sum_{i=1}^{M} (x_i-\bar{x})^3 \tag{9}$$

$$MT(4) = \lim_{M\to\infty} \frac{1}{M-1} \sum_{i=1}^{M} (x_i-\bar{x})^4 \tag{10}$$

$$SK = \lim_{M \to \infty} \frac{\frac{1}{M-1} \sum_{i=1}^{M} (x_i - \bar{x})^3}{\left[\frac{1}{M-1} \sum_{i=1}^{M} (x_i - \bar{x})^2 \right]^{\frac{3}{2}}} \tag{11}$$

$$KT = \lim_{M \to \infty} \frac{\frac{1}{M-1} \sum_{i=1}^{M} (x_i - \bar{x})^4}{\left[\frac{1}{M-1} \sum_{i=1}^{M} (x_i - \bar{x})^2 \right]^{2}} \tag{12}$$

Under the following Gaussian distribution

$$\phi(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2} \tag{13}$$

its moment is described as follows which is well known (Hino, 1977)

$$\overline{x^{(2n-1)}} = 0 \tag{14}$$

$$\overline{x^{(2n)}} = \prod_{k=1}^{n} (2k-1)\sigma^{2n} \tag{15}$$

If we divide Eq. (15) by σ^{2n}, we can obtain normalized moment.

In general, normalized n-th moment $Q(n)$ is stated as follows in continuous time system.

$$Q(n) = \frac{\int_{-\infty}^{\infty} (x-\bar{x})^n p(x)\,dx}{\left[\int_{-\infty}^{\infty} (x-\bar{x})^2 p(x)\,dx\right]^{\frac{n}{2}}} \tag{16}$$

In discrete time system, $Q(n)$ is stated as follows.

$$Q(n) = \lim_{M \to \infty} \frac{\frac{1}{M-1} \sum_{i=1}^{M} (x_i - \bar{x})^n}{\left\{ \frac{1}{M-1} \sum_{i=1}^{M} (x_i - \bar{x})^2 \right\}^{\frac{n}{2}}} \tag{17}$$

We describe $Q(n)$ as $Q_N(n)$ if it is calculated by using N amount of data.

$$Q_N(n) = \frac{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^n}{\left\{ \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2 \right\}^{\frac{n}{2}}} \tag{18}$$

Then absolute n-th moment deterioration factor $z_N(n)$ is described as follows.

$$Z_N(n) = \frac{\prod_{k=1}^{n/2}(2k-1)}{\dfrac{\frac{1}{N-1}\sum_{i=1}^{N}(x_i-\bar{x})^n}{\left\{\frac{1}{N-1}\sum_{i=1}^{N}(x_i-\bar{x})^2\right\}^{\frac{n}{2}}}} \tag{19}$$

Under the normal condition, $Z_N(n) \to 1$ $(N \to \infty)$, and if failure becomes larger, $Z_N(n) \to 0$.

2.2 CUMULANT

Characteristic function $\varphi(u)$ is defined as follows (Hino, 1977).

$$\varphi(u) = \int_{-\infty}^{\infty} e^{iux} p(x)dx \tag{20}$$

After taking Taylor expansion of the characteristic function, we can obtain coefficient $c(n)$ as:

$$\ln \varphi(u) = \sum_{n=1}^{\infty} c(n)\frac{i^n}{n!}u^n \tag{21}$$

$$c(n) = \left[\frac{(-i)^n d^n \ln \varphi(u)}{du^n}\right]_{u=0} \tag{22}$$

This $c(n)$ is called cumulant. There exists following relation between cumulant and moment.

$$MT(n) = \sum_{\lambda^{(1)}+\cdots+\lambda^{(q)}=v} \frac{1}{q!}\frac{v!}{\lambda^{(1)}!\cdots\lambda^{(q)}!}\prod_{n=1}^{q} c(n) \tag{23}$$

$$c(n) = \sum_{\lambda^{(1)}+\cdots+\lambda^{(q)}=v} \frac{(-1)^{q-1}}{q!}\frac{v!}{\lambda^{(1)}!\cdots\lambda^{(q)}!}\prod_{n=1}^{q} MT(n) \tag{24}$$

When $\bar{x} = 0$, Eq. (24) becomes as follows for $n = 1, 2, 3, 4, 6, 8$.

$$c(1) = \bar{x} = 0 \tag{25}$$

$$c(2) = \sigma^2 \tag{26}$$

$$c(3) = MT(3) \tag{27}$$

$$c(4) = MT(4) - 3\{MT(2)\}^2 \tag{28}$$

$$c(6) = MT(6) - 15MT(4)MT(2) - 10\{MT(3)\}^2 + 30\{MT(2)\}^2 \tag{29}$$

$$\begin{aligned} c(8) = {} & MT(8) - 28MT(6)MT(2) - 56MT(5)MT(3) - 35\{MT(4)\}^2 \\ & + 420MT(4)\{MT(2)\}^2 + 560\{MT(3)\}^2 MT(2) - 630\{MT(2)\}^4 \end{aligned} \tag{30}$$

where $MT(6)$, $MT(8)$ are 6-th moment, 8-th moment for each. It is well known that cumulant is 0 for the higher order cumulant more than 3 when probability density function is a normal distribution.

Eq.(28) is a power spectrum. Power spectrum is a Fourier Transform of Autocorrelation function (Tokumaru *et al.*, 1982). Therefore it is a kind of second order moment in a broad sense. Watching at the denominator of Eq. (24), a square root is taken for the triple products of power spectrum. Normalization is executed by this item. That is, Bicoherence is equivalent to the square root of normalized 6-th order moment in a broad sense. Therefore, Bicoherence can considered to be a kind of an absolute deterioration factor of normalized 6-th order moment in a broad sense.

2.3 BICOHERENCE

Bicoherence shows the relationship between two frequencies and is expressed as:

$$Bic,_{xxx}(f_1, f_2) = \frac{B_{xxx}(f_1, f_2)}{\sqrt{S_{xx}(f_1) \cdot S_{xx}(f_2) \cdot S_{xx}(f_1+f_2)}} \tag{31}$$

Here

$$B_{xxx}(f_1, f_2) = \frac{X_T(f_1) \cdot X_T(f_2) \cdot X_T^*(f_1+f_2)}{T^{\frac{3}{2}}} \tag{32}$$

means Bispectrum and

$$X_T(t) = \begin{cases} x(t) & (0 < t < T) \\ 0 & (else) \end{cases}$$

T : Basic Frequency Interval

$$X_T(f) = \int_{-\infty}^{\infty} X_T(t) e^{-j2\pi ft} dt \tag{33}$$

$$S_{xx}(f) = \frac{1}{T} X_T(f) X_T^*(f) \tag{34}$$

Range of Bicoherence satisfies

$$0 < Bic,_{xxx}(f_1, f_2) < 1 \tag{35}$$

When there exists a significant relationship between frequencies f_1 and f_2, Bicoherence is near 1. Otherwise, the value of Bicoherence comes close to 0.

Eq.(34) is a power spectrum. Power spectrum is a Fourier Transform of Autocorrelation function (Tokumaru *et al.*, 1982). Therefore it is a kind of second order moment in a broad sense. Watching at the denominator of Eq. (31), a square root is taken for the triple products of power spectrum. Normalization is executed by this item. That is, Bicoherence is equivalent to the square root of normalized 6-th order moment in a broad sense. Therefore, Bicoherence can considered to be a kind of an absolute deterioration factor of normalized 6-th order moment in a broad sense.

3. SIMPLIFIED CALCULATION METHOD OF MOMENT

Assume that we get N amount of data and then newly get l amount of data. Let mean, variance, $MT(3)$, $MT(4)$ of $1 \sim N$ data state as $\bar{x}_N$, σ_N^2, $MT_N(3)$, $MT_N(4)$ for each. And as for $N+1 \sim N+l$ data, we state above items as

$\bar{x}_{N/l}$, $\sigma_{N/l}^2$, $MT_{N/l}(3)$, $MT_{N/l}(4)$

For example, $MT_N(4)$ and $MT_{N/l}(4)$ are stated as follows.

$$MT_N(4) = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^4 \tag{36}$$

$$MT_{N/l}(4) = \frac{1}{l-1} \sum_{i=N+1}^{N+l} (x_i - \bar{x})^4 \tag{37}$$

When there arise failures on bearings or gears, peak value arise cyclically. In the early stage of the defect, this peak signal usually appears clearly. Generally, defects will injure another bearings or gears by contacting the covering surface as time passes. Assume that the peak signals of S times magnitude from normal signals arise during m times measurement of samplings. As for determining sampling interval, sampling theorem is well known (Tokumaru *et al.*, 1982). But in this paper, we do not pay much attention on this point in order to focus on our proposal theme. Let $\sigma_{N/l}^2$ and $MT_{N/l}(4)$ of this case denote $\bar{\sigma}_{N/l}^2$, $\overline{MT}_{N/l}(4)$, then we get the following equations.

$$\begin{aligned} \bar{\sigma}_{N/l}^2 &= \frac{1}{l-1} \sum_{i=N+1}^{N+l} (x_i - \bar{x})^2 \\ &\cong \frac{l - \frac{l}{m}}{l} \sigma^2 + \frac{\frac{l}{m}}{l} S^2 \sigma_N^2 \\ &= \sigma_N^2 \left(1 + \frac{S^2 - 1}{m}\right) \end{aligned} \tag{38}$$

$$\begin{aligned} \overline{MT}_{N/l}(4) &= \frac{1}{l-1} \sum_{i=N+1}^{N+l} (x_i - \bar{x})^4 \\ &\cong \frac{l - \frac{l}{m}}{l} MT_{N/l}(4) + \frac{\frac{l}{m}}{l} S^4 MT_{N/l}(4) \\ &= \left(1 + \frac{S^4 - 1}{m}\right) MT_{N/l}(4) \end{aligned} \tag{39}$$

And as for $N+1 \sim N+l$ data, we get the following equation.

$$\overline{MT}_{N/l}(4) = \frac{N}{N+l} MT_N(4) + \frac{l}{N+l} \overline{MT}_{N/l}(4)$$
$$\cong \left(1 + \frac{l}{N+l} \cdot \frac{S^4-1}{m}\right) \cdot \overline{MT}_{N/l}(4) \tag{40}$$

Let mean, variance and n-th moment calculated by using $1 \sim N$ data state as:

$\bar{x}_N,\ \sigma_N^2,\ M_N(n)$

And as for $N+1 \sim N+l$, let them state as:

$\bar{x}_{N/l},\ \sigma_{N/l}^2,\ M_{N/l}(n)$

Where

$$M_N(n) = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^n \tag{41}$$

$$M_{N/l}(n) = \frac{1}{l-1} \sum_{i=N+1}^{N+l} (x_i - \bar{x})^n \tag{42}$$

Therefore, Eq. (18) is stated as:

$$Q_N(n) = \frac{M_N(n)}{\sigma_N^n} \tag{43}$$

Assume that the peak signal which has S times impact from normal signals arises in each m times samplings. As for determining the sampling interval, the sampling theorem which is well known can be used (Tokumaru *et al.*, 1982). But in this paper, we do not pay much attention on this point in order to focus on the proposed theme.

Let $\sigma_{N/l}^2$ and $M_{N/l}$ of this case, of $N+1 \sim N+l$ be $\bar{\sigma}_{N/l}^2$, $\bar{M}_{N/l}$, then we get the following equations.

$$\bar{\sigma}_{N/l}^2 = \frac{1}{l-1} \sum_{i=N1}^{N+l} (x_i - \bar{x})^2$$
$$\cong \frac{l - \frac{l}{m}}{l} \sigma_N^2 + \frac{\frac{l}{m}}{l} S^2 \sigma_N^2$$
$$= \sigma_N^2 \left(1 + \frac{S^2-1}{m}\right) \tag{44}$$

$$\overline{M}_{N/l}(n) = \frac{1}{l-1} \sum_{i=N+1}^{N+l} (x_i - \bar{x})^n$$
$$\cong \frac{l - \frac{l}{m}}{l} M_{N/l}(n) + \frac{\frac{l}{m}}{l} S^n M_{N/l}$$
$$= \left(1 + \frac{S^n-1}{m}\right) M_{N/l}(n) \tag{45}$$

From these equations, we obtain $\overline{Q}_{N+l}(n)$ as $Q_{N+l}(n)$ of the above case

$$\overline{Q}_{N+l}(n) \cong \frac{\dfrac{N}{N+l}M_N(n)+\dfrac{l}{N+l}\left(1+\dfrac{S^n-1}{m}\right)M_N(n)}{\left\{\dfrac{N}{N+l}\sigma_N^2+\dfrac{l}{N+l}\sigma_N^2\left(1+\dfrac{S^2-1}{m}\right)\right\}^{\frac{n}{2}}}$$

$$=\frac{1+\dfrac{l}{N+l}\cdot\dfrac{S^n-1}{m}}{\left(1+\dfrac{l}{N+l}\cdot\dfrac{S^2-1}{m}\right)^{\frac{n}{2}}}\cdot\frac{M_N(n)}{\sigma_N^2}$$

$$=\frac{1+\dfrac{l}{N+l}\cdot\dfrac{S^n-1}{m}}{\left(1+\dfrac{l}{N+l}\cdot\dfrac{S^2-1}{m}\right)^{\frac{n}{2}}}Q_N(n) \tag{46}$$

While $Q_{N+l}(n)$ is Kurtosis when $n=4$,

$Q_N(4)=KT$

We assume that time series are stationary as is stated before in 2. Therefore, even if sample pass may differ, mean and variance are naturally supposed to be the same when the signal is obtained from the same data occurrence point of the same machine.

We consider such case when the impact vibration occurs. Except for the impact vibration, other signals are assumed to be stationary and have the same means and variances. Under this assumption, we can derive the simplified calculation method for machine diagnosis which is a very practical one.

From the above equation, we obtain $\overline{KT_{N+l}}$ in the following way.

$$\overline{KT} \cong \frac{1+\dfrac{l}{N+l}\cdot\dfrac{S^4-1}{m}}{\left(1+\dfrac{l}{N+l}\cdot\dfrac{S^2-1}{m}\right)^2}\times 3.0 \tag{47}$$

Consequently, we obtain $\overline{Z}_{N+l}(n)$ as of Eq. (19) as:

$$\overline{Z}_{N+l}(n)=\frac{\prod_{k=1}^{n/2}(2k-1)}{\overline{Q}_{N+l}(n)}$$

$$\cong\frac{\prod_{k=1}^{n/2}(2k-1)}{\dfrac{1+\dfrac{l}{N+l}\cdot\dfrac{S^n-1}{m}}{\left(1+\dfrac{l}{N+l}\cdot\dfrac{S^2-1}{m}\right)^{\frac{n}{2}}}Q_N(n)} \tag{48}$$

Under the normal condition,

$$Q_N(n) \cong \prod_{k=1}^{n/2}(2k-1) \tag{49}$$

Therefore, we get the following equations.

$$\bar{Z}_{N+l}(n) \cong \frac{\left(1+\frac{l}{N+l}\cdot\frac{S^2-1}{m}\right)^{\frac{n}{2}}}{1+\frac{l}{N+l}\cdot\frac{S^n-1}{m}} \tag{50}$$

Here we introduce the following number. Each index is compared with the normal index as follows.

$$F_a(P_{abn}) = \frac{P_{abn}}{P_{nor}}$$

P_{nor} : Index at normal condition

P_{abn} : Index at abnormal condition

In Eq. (29), F_a becomes as follows.

$$F_a(\bar{Q}_{N+l}(n)) \cong \frac{1+\frac{l}{N+l}\cdot\frac{S^n-1}{m}}{\left(1+\frac{l}{N+l}\cdot\frac{S^2-1}{m}\right)^{\frac{n}{2}}} \tag{51}$$

Correlation between $\bar{Z}_{N+l}(n)$ and F_a is as follows.

$$\bar{Z}_{N+l}(n) = \frac{1}{F_a(\bar{Q}_{N+l}(n))} \tag{52}$$

In the same way, let n-th cumulant in the above case state as $\bar{c}(n)$.

4. SIMPLIFIED ABSOLUTE INDEX OF HIGHER ORDER CUMULANT

Utilizing the relation of cumulant and moment stated in 2.2 and the simplified calculation method of moment stated in 3, we introduce simplified absolute index of higher order cumulant.

4.1 ABSOLUTE INDEX OF HIGHER ORDER CUMULANT

The absolute deterioration factor such as Bicoherence would be much more easy to handle because it takes the value of 1.0 under the normal condition and tends to be 0 when damages increase. Cumulants more than 3rd order are 0 under normal condition and when failure increases, the value grows big. Therefore, inverse number of the sum of calculated value of cumlants plus 1 would behave as an absolute index. New index shows that it is 1.0 under normal condition and tends to be 0 when failure increases. In this paper, we introduce a simplified calculation method to this new index and name this as a simplified absolute index of higher order cumulant.

Set this new index as $\overline{Zc}_{N+l}(n)$.

Then, simplified absolute index of higher order cumulant is described as follows.

$$\overline{Zc}_{N+l}(n) = \frac{1}{1+\bar{c}_{N+l}(n)} = \frac{1}{1+\sum_{\lambda^{(1)}+\cdots+\lambda^{(q)}=n} \frac{(-1)^{q-1}}{q!} \frac{n!}{\lambda^{(1)}!\cdots\lambda^{(q)}!} \prod_{k=1}^{q} \overline{MT}_{N+l}(\lambda(k))} \tag{53}$$

This is an absolute deterioration factor of which range is 1 to 0.

4.2 SIMPLIFIED CALCULATION METHOD FOR HIGHER ORDER CUMULANT

3rd order cumulant is 3rd order moment itself as is in Eq. (20). Therefore, we can obtain the following equations. When peak signals arise, we denote $c_{N+l}(3)$ to be $\bar{c}_{N+l}(3)$ as before for $N+l$ data.

$$\begin{aligned}\bar{c}_{N+l}(3) &= \overline{MT}_{N+l}(3) \\ &\cong \frac{1}{N+l} MT_N(3) + \frac{l}{N+l}\left(1+\frac{S^3-1}{m}\right) MT_N(3) \\ &= \left(1+\frac{l}{N+l}\frac{S^3-1}{m}\right) MT_N(3)\end{aligned} \tag{54}$$

Under normal condition or probability density function has the symmetric form for right and left even if impact vibration occurs, $MT_N(3)$ is 0. Therefore $\bar{c}_{N+l}(3)$ is also 0. We can obtain following 4-th cumulant from Eq. (21).

$$\begin{aligned}\bar{c}_{N+l}(4) &= \overline{MT}_{N+l}(4) - 3\overline{MT}_{N+l}(2)^2 \\ &\cong \left(1+\frac{l}{N+l}\frac{S^4-1}{m}\right) MT_N(4) - 3\left(1+\frac{l}{N+l}\frac{S^2-1}{m}\right)^2 \sigma_N^4\end{aligned} \tag{55}$$

Under normal condition, following equation is derived because $MT_N(4) \cong 3\sigma_N^4$ by Eq. (35).

$$\bar{c}_{N+l}(4) \cong 3\cdot\frac{l}{N+l}\left(1-\frac{l}{N+l}\cdot\frac{1}{m}\right)\frac{(S+1)^2(S-1)^2}{m}\sigma_N^4 \tag{56}$$

Under normal condition, data can be normalized to be $\bar{x}=0$, $\sigma^2=1$ without loosing universal (Tokumaru *et al.*, 1982). Hence, we calculate using following equation hereafter.

$$\bar{c}_{N+l}(4) \cong 3\cdot\frac{l}{N+l}\left(1-\frac{l}{N+l}\cdot\frac{1}{m}\right)\frac{(S+1)^2(S-1)^2}{m} \tag{57}$$

In the same way, 6-th cumulant and 8-th cumulant are described as follows from Eq. (22), (23).

$$\bar{c}_{N+l}(6) \cong 15\cdot\frac{l}{N+l}\frac{1}{m}\left\{1+\frac{l}{N+l}\cdot\frac{1}{m}\left(-3+2\cdot\frac{l}{N+l}\frac{1}{m}\right)\right\}(S^2-1)^3 \tag{58}$$

$$\bar{c}_{N+l}(8) \cong 105\cdot\frac{l}{N+l}\frac{1}{m}\left[1-\frac{l}{N+l}\frac{1}{m}\left\{7-6\cdot\frac{l}{N+l}\frac{1}{m}\left(2-\frac{l}{N+l}\frac{1}{m}\right)\right\}\right](S^2-1)^4 \tag{59}$$

5. NUMERICAL EXAMPLES

If the system is under normal condition, we may suppose $p(x)$ becomes a normal distribution function. Under the assumption of 3, let $m=12$, considering the cases $S=2, 3, \cdots, 6$ for 3, and setting $l=N/10$, we obtain Table 1 from the calculation of $\overline{Zc}_{N+l}(n)$. Simplified absolute index of higher order moment $\overline{Z}_{N+l}(n)$ we previously proposed (Takeyasu *et al.*, 2004) is also added in Table 1 and Table 2. Conditions are the same.

Table 1: Transition of $\overline{Zc}_{N+l}(n)$, $\overline{Z}_{N+l}(n)$ $(m=12, l=\frac{N}{10})$

		S=1	2	3	4	5	6
$\overline{Zc}_{N+l}(n)$	n=4	1	0.8312581	0.4092465	0.1646119	0.0714708	0.0349284
	n=6	1	0.2500749	0.0172812	0.0026606	0.0006508	0.0002099
	n=8	1	0.0161136	0.0003237	0.0000262	0.0000039	0.0000008
		S=1	2	3	4	5	6
$\overline{Z}_{N+l}(n)$	n=4	1	0.9392393	0.7004002	0.4230091	0.2438672	0.148059
	n=6	1	0.7241338	0.1831208	0.0431292	0.0138286	0.0057131
	n=8	1	0.3731662	0.0249594	0.0030917	0.0006589	0.0002013

Next, setting $N \to 0$, $l \to N$ we obtain Table 2.

Table 2: Transition of $\overline{Zc}_{N+l}(n)$ $\overline{Z}_{N+l}(n)$ $(m=12,\ N=\varepsilon(\varepsilon \to 0),\ l=N)$

		S=1	2	3	4	5	6
$\overline{Zc}_{N+l}(n)$	n=4	1	0.3265306	0.0638298	0.0190250	0.0075188	0.0035495
	n=6	1	0.0376695	0.0020412	0.0003102	0.0000757	0.0000244
	n=8	1	0.0028335	0.0000561	0.0000045	0.0000006	0.0000001
$\overline{Z}_{N+l}(n)$		S=1	2	3	4	5	6
	n=4	1	0.6944444	0.3623188	0.2275280	0.1698113	0.1408441
	n=6	1	0.3125000	0.0750750	0.0332815	0.0207214	0.0154497
	n=8	1	0.1097261	0.0140889	0.0046920	0.0024882	0.0016812

As for higher order cumuulants cases such as $n = 6$, $n = 8$, sensitivity grow much better. But calculation becomes more complicated as n grows large.

Now, we compare this new index with the one we proposed before (Takeyasu *et al.*, 2004). Obviously, newly proposed index is much more sensitive than previously proposed index. Each corresponding case of Table 2 shows much more proceeded value in deterioration than those of Table 1. It is because each case of Table 2 is occupied only by the data under irregular condition.

Subsequently, we examine Bicoherence. We made experiment in the past (Takeyasu, 1987, 1989). Summary of the experiment is as follows. Pitching defects are pressed on the gears of small testing machine.

Small defect condition: Pitching defects pressed on 1/3 gears of the total gear.

Middle defect condition: Pitching defects pressed on 2/3 gears of the total gear.

Big defect condition: Pitching defects pressed on whole gears of the total gear.

We examined several cases for the f_1, f_2 in Eq. (24). We got best fit result in the following case.

$$\begin{cases} f_1 : \text{peak frequency of power spectrum} \\ f_2 : 2\,f_1 \end{cases}$$

We obtained following Bicoherence values in this case (Table 3).

Table3: Transition of Bicoherence value

Condition	Normal	Small defect	Middle defect	Big defect
Bicoherence	0.99	0.38	0.09	0.02

Thus, Bicoherence proved to be a very sensitive good index. Bicoherence is an absolute index of which range is 1 to 0.

Therefore it can be said that it is a universal index.

In those experiment, small defect condition is generally assumed to be $S = 2$ and big defect condition is generally assumed to be $S = 6$ (Maekawa *et al.*, 1997). Therefore, approximate comparison may be achieved, though the condition does not necessarily coincide.

In the case of $n = 4$ in Table 2, the value is 0.327 at small defect condition and 0.019 at middle defect condition and 0.004 at big defect condition which show more sensitive behavior than Bicoherence. It could be said that the case $n = 4$ would be sensitive enough for practical use. Therefore, the case $n = 4$ would be recommended in this new method. This calculation method is simple enough to be executed even on a pocketsize calculator.

Sensitivity is better in $n = 6$ than that of $n = 4$. Sensitivity is much better in $n = 8$, but the value falls too fast therefore the judgment becomes hard. Therefore the case $n = 6$ is good for the practical use. As Bicoherence is one of the kind of 6-th order moment in a broad sense, these deterioration factors of 6-th order moment in a broad sense found to be sensitive and practical indices.

Compared with Bicoherence which has to be calculated by Eq. (24)~(27), proposed method is by far a simple one and easy to handle on the field defection.

6. REMARKS

The steps for the failure detection by this method are as follows.

1. Prepare standard $\overline{Zc}$ Table for each normal or abnormal level
2. Measure peak values by signal data and compare the peak ratio (S) to the normal data
3. Calculate $\overline{Zc}$
4. Judge the failure level by the score of $\overline{Zc}$

m is the value of each equipment. For example, it is the number of ball bearings or number of gears. Preparing standard Table of $\overline{Zc}$ for each normal and abnormal level, we can easily judge the failure level only by taking ratio of the peak value to the normal level and calculating $\overline{Zc}$. This calculation method is very simple and is very practical at the factory of maintenance site. This can be installed in microcomputer chips and utilized as the tool for early stage detection of the failure.

7. CONCLUSIONS

In this paper, n-th normalized cumulant was considered so as to intensify the sensitivity of diagnosis. Higher order cumulant was expressed as the combination of the same order moment and the moments less than that order. New indices would have been introduced because they were the combinations of plural order moments. By the way, the theoretical value of n-th moment divided by n-th moment calculated by measured data would behave in the same way. We named this factor as absolute n-th moment deterioration factor. In this paper, a simplified calculation method of this factor was introduced. This method was also applied to n-th normalized cumulant. The concept of absolute deterioration factor of n-th order moment in a broad sense was introduced and the analysis was executed. From the result of comparison, absolute deterioration factor of 6-th order moment including 6-th order cumulant in a broad sense was better than other factors in the viewpoint of sensitivity and practical use. Thus, n-th order moment including n-th order cumulant in a broad sense was examined and evaluated, and we obtained practical good results. The proposed method enables failure detection of equipment in a simple and quick way. This method is applicable in many data handling fields. The effectiveness of this method should be examined in various cases.

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