Title	A Nonlinear Process Generating Tollmien-Schlichting Waves
Author(s)	Asai, Masahito; Nishioka, Michio
Editor(s)	
Citation	Bulletin of University of Osaka Prefecture. Series A, Engineering and natural sciences. 1990, 38(2), p.13-19
Issue Date	1990-03-15
URL	http://hdl.handle.net/10466/8482
Rights	

A Nonlinear Process Generating Tollmien-Schlichting Waves

Masahito ASAI* and Michio NISHIOKA*

(Received November 15, 1989)

Our ability to predict laminar-turbulent transition in boundary layer heavily depends on our understanding of the so-called receptivity, namely the process by which external disturbances are internalized as eigenmodes, i.e. Tollmien-Schlichting waves. In this paper, a nonlinear process generating Tollmien-Schlichting waves, found in our recent experiment of the ribbon-induced transition in plane Poiseuille flow, is reported and discussed.

1. Introduction

Recently there has been a great progress in the study of laminar to turbulent transition, in particular for the case of boundary layers and plane Poiseuille flow initially controlled by the spatial growth of Tollmien-Schlichting (T-S) waves. Indeed, we are now able to predict the threshold amplitude of T-S waves, which is required for the secondary instability to operate to lead to the transition through the process of wall turbulence generation; see reviews by Nishioka[1] and Asai and Nishioka[2]. The progress has crucially been dependent on experimental observations obtained by introducing well-controlled T-S waves by means of the so-called vibrating ribbon. The present note is concerned with the important ribbon-induced T-S wave and an observed nonlinear process for its generation. The reason why we take up the present problem is due to the fact that it is closely related to the problem of receptivity[3)4)], a process by which external disturbances are internalized as eigenmodes, namely T-S waves.

2. Vibrating Ribbon Technique

First of all the vibrating ribbon technique is explained briefly. Originally the ribbon technique is due to Schubauer and Skramstad[5]. In our experiment[1)2)] on the transition in plane Poiseuille flow (realized in a rectangular channel whose width (span), depth and length are respectively 400 mm, 14.6 mm and 6000 mm), a phospher bronze ribbon (of 0.05 mm in thickness and 4 mm in width) is placed near the channel lower wall (at a height near the critical layer) and stretched across the full span, under tension by means of weights. The segment inside the channel is free

* Department of Aeronautical Engineering, College of Engineering.

to vibrate normal to the wall, when driven by a sinusoidal electric current in a steady magnetic field produced by permanent magnets attached to the opposite side of the wall. The exciting current is supplied by a sine-wave generator through a power amplifier.

Throughout our experience of transition experiments made thus far by using the ribbon technique, we always check and confirm that the T-S wave introduced has the same frequency as that of the driving ribbon current, without any appreciable harmonic content. Of course, the wave amplitude is proportional to that of the ribbon current. This means that the vibrating ribbon is a linear system.

In spite of the linear character, we quite recently found that the flow forced by the vibrating ribbon can generate two systems of T-S waves. One is of course the wave with the forcing frequency f, and the other with $2f$. It is rather surprising that the latter is not of the second harmonic of the T-S mode with f but itself of the T-S mode with $2f$. This finding is quite important and will be described in more detail as follows.

3. Nonlinear Process Generating T-S waves

The observations described here are made in an effort of investigation into the secondary instability[6)~8)] at a subcritical Reynolds number (based on the channel half depth h=7.3 mm and the centre-line velocity U_c=9.8 m/s) R=5000: The critical Reynolds number for the linear instability is 5772.

Figs. 1(a) to (d) demonstrate ribbon-induced u-fluctuations at a fixed excitation

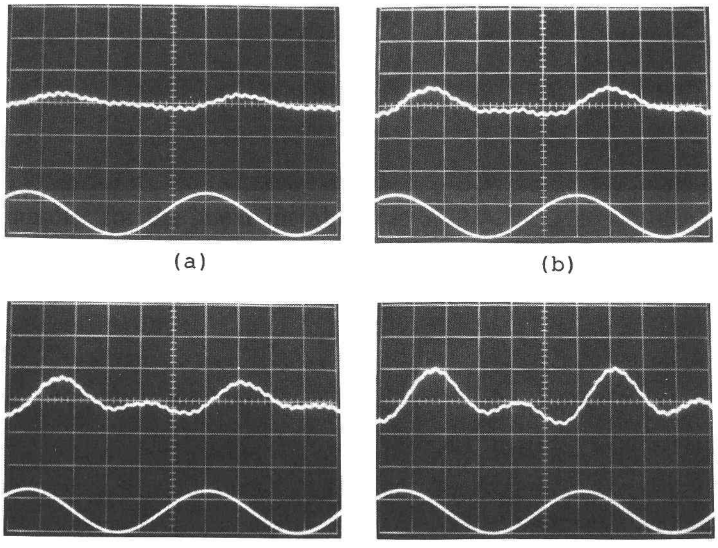

Fig. 1 Wave-forms of u-fluctuation at various excitation intensities (y/h=0.18, z=6.5cm). Upper trace; u-fluctuation (vertical scale; $0.02U_c$/div), lower trace; ribbon current (36.5Hz).

frequency $f=36.5$ Hz, and at various excitation intensities; non-dimensional frequency $2\pi fh/U_c=0.17$. They are observed at a distance y from the lower wall, $0.18h$, at a streamwise distance, $48h$ downstream of the ribbon. It is clear that each oscilloscope trace of u-fluctuation contains not only the fundamental (f) but also the component with the second harmonic ($2f$) frequency together with a high-frequency ripple due to the fan noise (700 Hz). It should be stressed that in all the cases, the excitation is so weak that we do not usually expect nonlinear behaviour of fluctuations. In fact, the maximum in the y-distribution of the r. m. s. value of the fundamental u-fluctuation is 0.24, 0.40, 0.52 and 0.84 % of U_c in Figs. 1(a), (b), (c) and (d) respectively. Nevertheless, even in the case of the lowest intensity, Fig. (a), we see a clear evidence for the presence of the component with the second harmonic frequency. This feature is observed at various y and z (spanwise) positions. It is also noted that the phenomenon is observed only for a range of the frequency below 40 Hz. Beyond 40 Hz, what is observed at these excitation levels is the T-S wave of the forcing frequency f alone. The most unstable (actually, the least damping) frequency at $R=5000$ is 62Hz ($2\pi fh/U_c=0.30$) under the present experimental conditions, that is, the frequency range for the peculiar phenomenon to occur is much lower than the most unstable frequency. The reason why this has never been observed is that no one has ever worked in such a low frequency range.

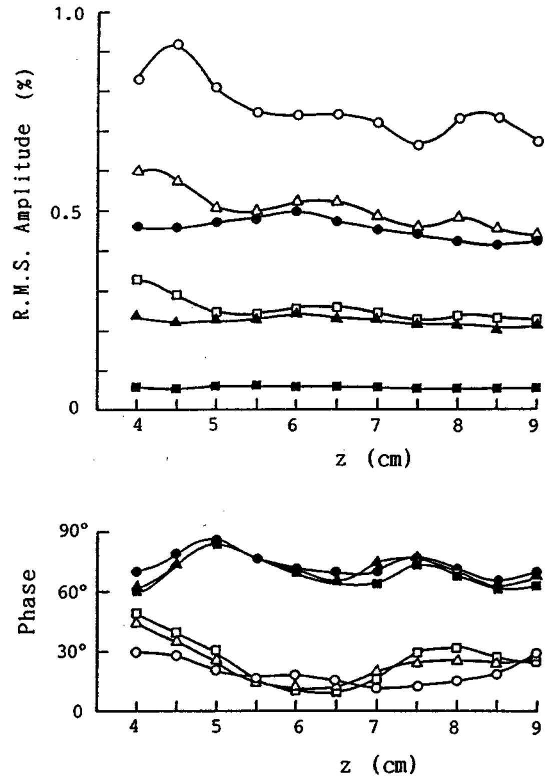

Fig. 2 Spanwise distributions of r.m.s. amplitude and phase of u-fluctuation at $y/h=0.18$. ○, △, □; fundamental frequency (36.5Hz), ●, ▲, ■; second harmonic frequency (73Hz).

Fig. 2 shows the spanwise distributions of the r. m. s. amplitude and the phase of the components with the fundamental and second harmonic frequencies, that are singled out from the total u-fluctuation at $y/h=0.18$. Comparisons are made between the results for three different excitation intensities corresponding to Figs. 1(a), (c) and (d) observed at $z=6.5$ cm. Both the amplitude and phase distributions show no large spanwise variations. Indeed, variations are within 20 % and 30 degrees in relative amplitude and phase respectively. Furthermore, we see no tendency of increases in their spanwise variations with increasing the forcing. These facts indicate that the wave system is essentially two-dimensional.

Now we have to describe the structure of each wave component; the fundamental f-component and the $2f$-component. It is first noted that the u-fluctuation of the f-component (T-S mode) is anti-symmetric with respect to the channel centre line. Then, the $2f$-component should be symmetric if it is of the second harmonic mode of the f-component as usually expected. Actually, however, the $2f$-component is anti-symmetric, suggesting that it is also of T-S mode just like as the f-component. This is unexpected and really interesting. To clarify the $2f$-component, we have to obtain the normal-to-wall (y-) distributions of the amplitude and phase of its u-fluctuation. To single out two-dimensional waves of f- and $2f$-frequencies with a good accuracy, we made double Fourier analysis with respect to the frequency and the spanwise wavenumber, which was used in our previous study[8] on the secondary instability. This analysis was done for the case (c). The u-fluctuations thus singled out are expressed as

$$u_n=A_n(y)\cos(2\pi nft+P_n(y)),$$

where A_n denotes the amplitude and P_n the phase, and $n=1$ and 2 stand for the f- and $2f$-components respectively.

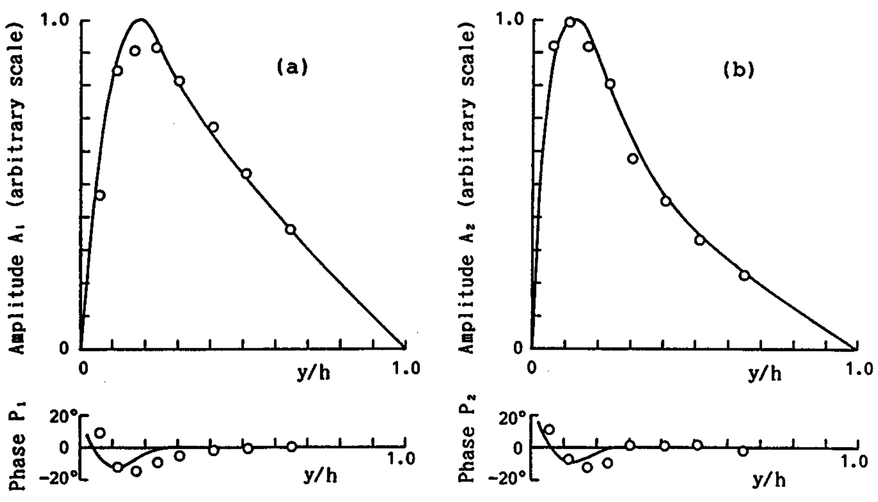

Fig. 3 Amplitude and phase distributions of u_1 and u_2. ○; experiment, ——; linear stability theory.

Figs. 3(a) and (b) plot the amplitude and phase distributions of u_1 and u_2 respectively. They are compared with the results calculated from the linear stability theory, i.e. those of the eigenmodes (i.e. T-S waves) of f- and $2f$-frequencies. The comparisons show that u_2 is really of the eigenmode as well as u_1. In other words, u_2 is not the second harmonic mode of u_1, but itself T-S wave. To clarify the result further, we measured the phase speed of u_1 and u_2. The measurement was made at $y/h=0.5$, 0.6 and 0.7 at $z=7.0$ cm, where three-dimensional components are all weak and have little effect on the measurement. The phase speed is found to be 0.22~0.23U_c for u_1 and 0.28~0.29U_c for u_2. Note that both of these values are in agreement with the corresponding eigenvalues calculated from the linear stability theory. Also note that if u_2 were the second harmonic of u_1, their phase speed would be equal to each other. As described thus far, it is verified that u_2 is a fundamental T-S wave and by no means the second harmonic mode of u_1. Then, is there no relation between the u_1 and u_2 ? This is the question we address in the following.

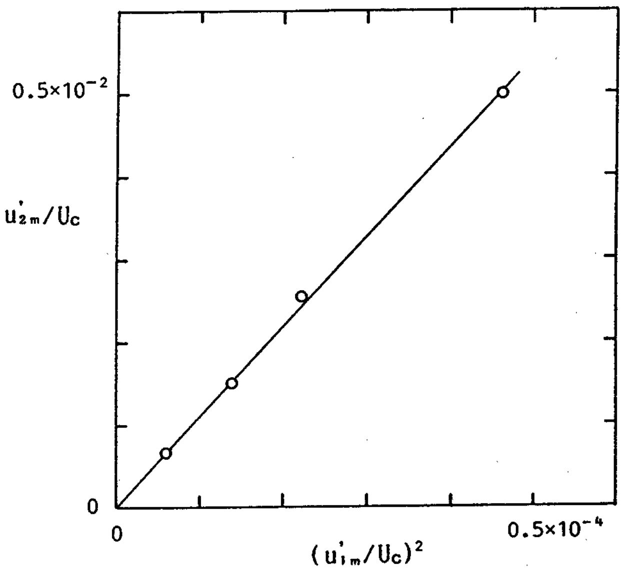

Fig. 4 Relationship between u'_{1m} andu'_{2m}.

Fig. 4 shows the maximum in the y-distribution of r.m.s. u_2, namely u'_{2m} plotted against the square of the y-maximum u'_{1m}, both measured at $48h$ downstream of the ribbon. It is clear from Fig. 4 that u'_{2m} is proportional to the square of u'_{1m}. This clearly indicates a possibility that a certain nonlinear process is involved in generating u_2, considering the fact that the sinusoidal current through the ribbon is free of the second harmonic. Then, we took a close look at the ribbon vibration whether it exhibits nonlinear behaviour, but we could not find any such evidence. The only possibility of nonlinear process that we can imagin now is the flow itself. There is little doubt that the vibrating ribbon generates its own unsteady wake, which may contain not only the f-component but also $2f$-component. The latter may be produced as the second harmonic of the former in the wake. For the case of boundary layer, its receptivity to unsteady wake, that is, the possibility that an unsteady wake in the freestream can excite T-S waves has been observed by Nishioka and Morkovin[4]. In the present case, the wake develops at a height near the critical layer and the $2f$-component no doubt contains the necessary spatial scale for the T-S wave as the velocity in the wake is close to the phase speed of T-S waves. This is indeed a nonlinear process generating T-S waves, even though the process by which the $2f$-component in the ribbon wake generates them is linear. Another possible region where a periodic vorticity field of frequency $2f$ and of proper scale might appear is close to the wall below the ribbon. For instance, if the periodic wake of the ribbon develops the $2f$-component, it surely generates such a vorticity field close to the wall. However, it is reasonable to suppose that the periodic wake behind the ribbon itself is more efficient in generating the proper scale for the T-S wave.

As discussed above, the non-linearity seems to be involved in the flow near the ribbon. Thus it should be checked whether or not the f- and $2f$-components have such an amplitude relationship as to be judged as a combination of the fundamental and the second harmonic modes near the ribbon. This is rather easily examined as we know the damping rate α_i for the T-S waves of f-and $2f$-frequencies from the stability calculation. Namely, the calculation shows that $\alpha_i=0.04$ and 0.01 for the f-and $2f$-waves respectively. Therefore, the wave amplitude near the ribbon is about 6.8 and 1.6 times the amplitude at the present observation position ($48h$ downstream the ribbon) for the f-and $2f$-waves respectively. This fact and the data in Fig. 4 clearly indicate that the two components have a harmonic relationship; the $2f$-component is an order of magnitude smaller than the f-component.

4. Concluding Remarks

We have described a nonlinear process generating T-S waves. In fact, the present observation shows that the flow forced by a vibrating ribbon can excite two systems of T-S waves with the forcing frequency f and the second harmonic frequency $2f$. Importantly, both are of fundamental eigenmode. It is pointed out that the periodic wake behind the ribbon may introduce vorticity fields of f-and $2f$-components through its nonlinear development. When the frequency f is inside the amplification zone in the stability diagram, the $2f$-component usually damps rapidly. However, when f is below the neutral curve and $2f$ is inside it, the vibratig ribbon might induce growing T-S wave of $2f$-frequency as well as the decaying T-S wave of f-frequency. This possibility may be extended to the cases of freestream turbulence and sound unless they are of sufficiently low intensity. In other words, in the receptivity region, the nonlinear response of the flow to external disturbances can excite T-S waves of the second (or still higher) harmonic frequency if it is inside the amplification zone. We should not forget the possibility in transition prediction.

References

1) M. Nishioka, Recent Studies on Turbulent Phenomena (eds. T.Tatsumi, H. Maruo and H. Takami), Association for Science Documents Information, 193-203 (1985).
2) M. Asai and M. Nishioka, J. Fluid Mech. **208**, 1-23 (1989).
3) M. V. Morkovin, Air Force Flight Dyn. Lab. Rep. **AFFDL-TR-68-149** (1969).
4) M. Nishioka and M. V. Morkovin, J. Fluid Mech. **171**, 219-261 (1986).
5) G. B. Schubauer and H. K. Skramstad, NACA Rep. **909** (1947).
6) S. A. Orszag and A. T. Patera, J. Fluid Mech. **128**, 347-385 (1983).
7) Th. Herbert, Turbulence and Chaotic Phenomena in Fluids (ed. T. Tatsumi), North-Holland, 117-122 (1984).
8) M. Nishioka and M. Asai, Laminar-Turbulent Transition (ed. V. V. Kozlov), Springer, 173-182 (1985).