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Design Concept and Information Processing for Fundamental Design of Flexible Manufacturing Systems.

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This paper presents the design concept and the fundamental design procedure for the flexible manufacturing systems (FMS). The method is proposed to obtain the appropriate cellular structure of the FMS based on the Group Technology (GT) so as to attain the high productivity and flexibility. A software system is developed to realize the proposed method in human-computer interaction, which is entitled CAFPLAN-I (Computer Aided Factory PLANning system – I). This system enables us to systematically find the satisfactory solution taking account of the number of cells, cell sizes, alternative machine tools, workload balance among machines and machine utilization by the efficient use of the designer's know-how.

1. Introduction

As variety in market needs increases and the life-cycle of products becomes shorter, the system configuration for manufacturing is evolutionally changed. One of the typical changes is the realization of the flexible manufacturing systems (FMS) instead of the manufacturing line in mass-production. Various types of the FMS have been developed to attain both high productivity and flexibility in the mid-variety, mid-volume manufacturing area1)2). These existing FMS's have been developed on the basis of the know-how stored up so far in the work shop, therefore, may suggest a guiding principle for designing the FMS in the future.

However, the systematic procedure is still lacking for designing the adequate FMS for the various kinds of jobs loaded. This paper proposes a design concept and a systematic procedure to design the FMS by the efficient use of the designer's know-how and the computer.

2. Planning Process and Design Concept for FMS

Figure 1 shows a stepwise planning process for the development of the highly automated factory. This process is based on the Group Technology Principle and may be accepted as a feasible one from an economic, social and technological points of view3).

In the figure, the FMC representing the Flexible Manufacturing Cell is defined by Spur4) as a single CNC machine tool with integrated tool and work handling and parts inspection. This cell is a basic component in a hierarchical structure constructing an automatic factory. Several such cells are linked and construct the group technology cell (GT cell for short) to produce a part family based on the group technology. Then, the GT cell is fully automated by DNC (Direct Numerical Control) to realize the FMS. Such FMS's are linked to produce various types of part families and, consequently an auto-

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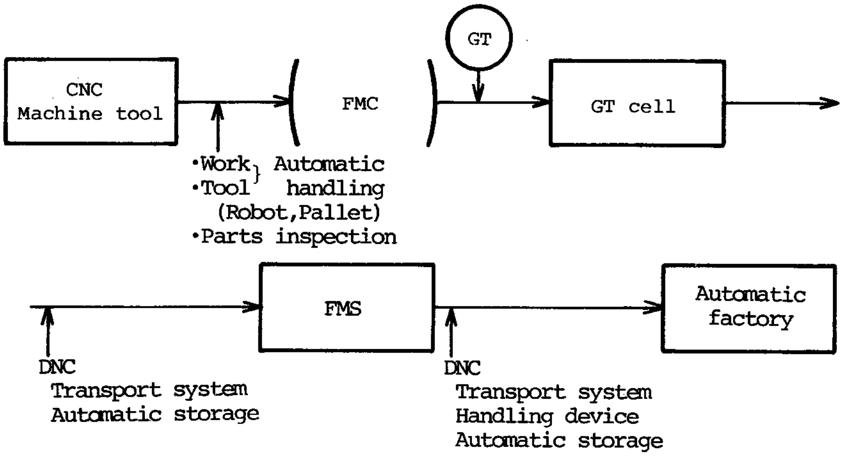


Fig. 1 Stepwise planning process for the development of the highly automated factory

matic factory is realized. In the stepwise planning process for the FMS, it is very important how to form the adequate GT cells for obtaining both high productivity and flexibility. This is a subject of this paper.

There exist, in general, several types of manufacturing systems as shown in Fig. 2 for the various kinds of production tasks$^{5)-7)}$. In the figure, three types of manufacturing systems, *i.e.*, FMC's with functional layout, the typical FMS and the flexible transfer line (FTL for an acronym) are in practice recognized as the FMS. These can be defined in various ways, but are usually thought of as follows.

FMC's with functional layout:

One which consists of several FMC's clustered and allocated according to their manufacturing function to perform.

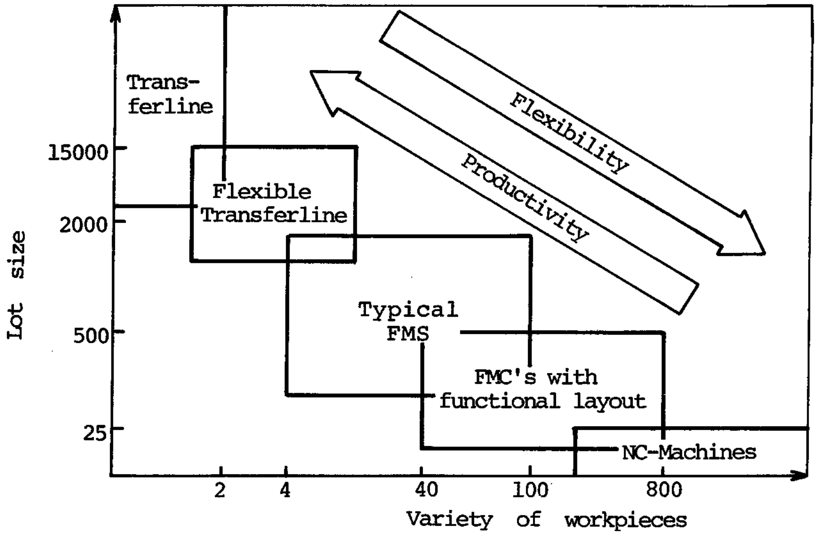


Fig. 2 Various types of manufacturing systems and application range

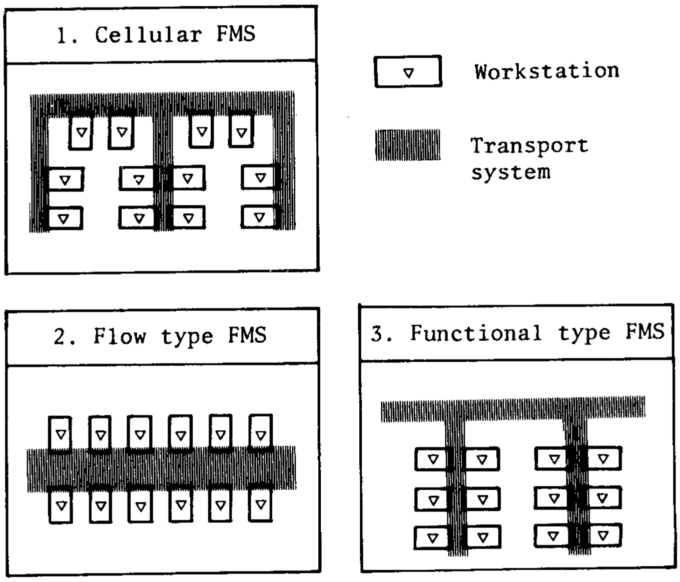


Fig. 3 Design concepts of FMS

Typical FMS:

One which consists of several GT cells linked by a transport system with different paths.

FTL:

One which consists of flexible machine tools with multiple functions linked by rigid transport system.

Analyzing the fundamental structure of these three types of FMS's, three types of design concepts are correspondingly derived as shown in Fig. 3[8]. Generally speaking, the typical FMS corresponds to the cellular FMS, the FTL to the flow type FMS, and the FMC's with functional layout to the functional type FMS.

In the following sections, the systematic method will be discussed to determine the fundamental structure of such an FMS on the basis of a product analysis.

3. Mathematical Foundation for Cell Formation

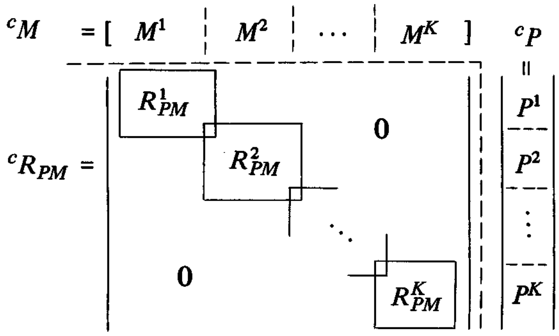
The basis for a selection of the suitable structure of the FMS is a detailed analysis of the production task, which yields the information on the number of different kinds of parts to be produced, production quantity of each part and the process route given in the form of the machine sequence for each process and the operation time on each machine.

Based on such information, the required capacity of each machine m_j is calculated for each part p_i, which is denoted by t_{ij} and called a workload. According to the workload, the relationship between parts and machines can be represented in the form of a relation matrix R_{PM} as shown in Eq. (1).

$$\begin{array}{rl|l} M & = [\, m_1, \ \cdots m_j, \ \cdots m_{NM} \ ] & P \\ & & \parallel \\ \hline R_{PM} & = \begin{bmatrix} t_{1,1} & \cdots t_{1,j} & \cdots t_{1,NM} \\ \vdots & \vdots & \vdots \\ t_{i,1} & \cdots t_{i,j} & \cdots t_{i,NM} \\ \vdots & \vdots & \vdots \\ t_{NP,1} & \cdots t_{NP,j} & \cdots t_{NP,NM} \end{bmatrix} & \begin{bmatrix} p_1 \\ \vdots \\ p_i \\ \vdots \\ p_{NP} \end{bmatrix} \end{array} \qquad (1)$$

where NM is the number of different kinds of machines used, m_j is the machine number ($j = 1, 2, ..., NM$), NP is the number of different parts to be processed, p_i is the part number ($i = 1, 2, ..., NP$), $t_{i,j}$ is the workload given by the ratio of the operation time of the part p_i on the machine m_j to the available time of the machine m_j. In Eq. (1), the vectors M and P will be called the machine vector and part vector, respectively.

This relation matrix R_{PM} may be transformed into the matrix ${}^cR_{PM}$ in the following Eq. (2) by appropriate rearrangements of machine vector M and part vector P in Eq. (1)$^{9)-12)}$.



(2)

In this case the vector M^i ($i = 1, 2, ..., K$) indicates the GT cell corresponding to the part family indicated by the vector P^i, therefore the vectors M^i and P^i will be called the cell vector and part family vector, respectively, and the number K gives the number of cells constructed. The matrix R^i_{PM} indicates the relationship between the cell vector M^i and the corresponding part family vector P^i. Therefore, the matrix R^i_{PM} will be called the cellular matrix and the matrix ${}^cR_{PM}$ the structural matrix.

Equation (2) will present the well insights with respect to the structures of manufacturing system. If any two cellular matrices R^i_{PM}'s are not overlapping, all cells are entirely independent each other, *i.e.*, every machine group can be formed as a complete isolated cell so that each part family P^i can be fully processed in only one cell M^i. In other case where a small overlap between any two cellular matrices exists, the entirely independent cells cannot be constructed, but every machine group can be also formed as an almost isolated cell. These two cases result in the structure of cellular FMS in Fig. 3.

On the contrary, it is impossible to form any machine group as an isolated cell in such a case where a large overlap between any two cellular matrices exists; that is, many common machines appear in the process routes of many different kinds of parts. In such a case, two typical structures of manufacturing systems can be formed based on the similarity associated with the process routes of parts; *i.e.*, the sequence of machines used. One refers to the structure of flow type FMS in Fig. 3, like a flexible transfer line, which can be employed in case that all parts have similar process routes. The other refers to the structure of functional type FMS in Fig. 3, where the machines are grouped according to functions, in case that process routes are different for every part and hence sequence of machines varies part by part.

Now, to determine the design concept for the FMS, the problem to be solved is how to construct the suitable cellular structure. The basic idea is to form the adequate structural matrix in Eq. (2) so as to maximize the correlation between parts and machines[13]. The quantification theory III on the correlation analysis can be applied to solve this problem. Then the maximum value of the correlation coefficient ρ can be equal to the maximum eigenvalue except for 1 of the following characteristic equation.

$$\sum_{j=1}^{NM} c_{kj} z_j = \rho^2 z_k \ (k = 1, 2, ..., NM) \tag{3}$$

where the coefficients $c_{k,j}$'s are determined by using the workload in Eq. (1) as follows.

$$c_{kj} = \frac{1}{\sqrt{b_k}\ \sqrt{b_j}} \sum_{n=1}^{NP} \frac{1}{L_i} t_{ik}\, t_{ij} \tag{4}$$

$$b_j = \sum_{i=1}^{NP} t_{ij} \tag{5}$$

$$L_i = \sum_{j=1}^{NM} t_{ij} \tag{6}$$

If Eq. (3) has more eigenvalues than one, whose values are equal to 1, there exist the same number of independent cellular matrices in Eq. (2) as the number of such eigenvalues. On the other hand, in case that Eq. (3) has some eigenvalues nearly equal to 1 including 1 (there necessarily exists an eigenvalue with the value of 1 in Eq. (3)), there exist the same number of nearly independent cellular matrices in Eq. (2) as the number of such eigenvalues. In such cases, the cellular FMS with adequate structure may be constructed according to the resulting structural matrix in Eq. (2). On the contrary, if there are no eigenvalues nearly equal to 1 except for one with the value of 1, any adequate cellular structure cannot be constructed. In such a case, the flow type FMS or the functional type FMS may be suitably constructed according to the similarity in the process routes of parts. Thus, the eigenvalues ρ^2 of the characteristic equation (3) or the correlation coefficient ρ between parts and machines plays an important role for determining the cellular structure.

Now, the method based on the quantification theory III is given to form the cellular structure as shown in Eq. (2). First, similarity indices v_j's among machines are determined by using the eigenvector (z_j) $(j = 1, 2, ..., NM)$ corresponding to the maximum eigenvalue except for 1 in Eq. (3), *i.e.*,

$$v_j = z_j \,/ \sqrt{b_j} \qquad (j = 1, 2, ..., NM) \tag{7}$$

The similarity indices u_i's among parts are also determined as follows.

MACHINE NUMBER

PARTS NUMBER

	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010
1001	A	.	A	.	.	A	.	.	.	A
1002	.	.	.	A	.	.	.	A	A	.
1003	.	A	.	.	A	.	A	.	.	.
1004	A	.	A	.	.	A	.	.	.	.
1005	.	A	.	.	A	.	A	A	.	.
1006	.	A	.	.	.	.	.	A	A	.
1007	A	.	A	A	.	A	.	.	.	A
1008	A	.	.	.	.	A	.	.	.	A
1009	.	.	.	A	.	.	.	A	A	.
1010	.	.	.	A	.	.	.	A	A	.
1011	.	.	.	A	.	.	.	.	A	.
1012	A	.	A	.	.	.	.	.	.	A
1013	.	.	.	.	A	.	A	.	.	.
1014	.	A	.	.	A	.	A	.	.	.
1015	.	A	.	.	.	.	A	.	.	.

(a) Parts-machines relation matrix

MACHINE NUMBER

PARTS NUMBER

	1010	1001	1006	1003	1004	1009	1008	1002	1007	1005
1008	A	A	A	.	.	.	.	.	.	.
1001	A	A	A	A	.	.	.	.	.	.
1012	A	A	.	A	.	.	.	.	.	.
1004	.	A	A	A	.	.	.	.	.	.
1007	A	A	A	A	A	.	.	.	.	.
1011	.	.	.	.	A	A	.	.	.	.
1009	.	.	.	.	A	A	A	.	.	.
1010	.	.	.	.	A	A	A	.	.	.
1002	.	.	.	.	A	A	A	.	.	.
1006	.	.	.	.	.	A	A	A	.	.
1005	.	.	.	.	.	.	A	A	A	A
1015	.	.	.	.	.	.	.	A	A	.
1014	.	.	.	.	.	.	.	A	A	A
1003	.	.	.	.	.	.	.	A	A	A
1013	.	.	.	.	.	.	.	.	A	A

(b) Structural matrix

Fig. 4 Cell formation by the quantification theory III

$$u_i = \frac{1}{L_i}\sum_{j=1}^{NM} t_{ij}v_j \qquad (i=1,2,\dots,NP) \tag{8}$$

According to the quantification theory III, as any two parts p_i and p_k have more similar process routes, the corresponding two similarity indices u_i and u_k have closer values. And as any two machines m_j and m_l process more common parts, the corresponding two similarity indices v_j and v_l have closer values.

Next, the part vector P and machine vector M in Eq. (1) are rearranged in the decreasing order of the similarity indices u_i and v_j, respectively. The rows and columns of the relation matrix R_{PM} in Eq. (1) are simultaneously rearranged in the corresponding orders. As a result, the structural matrix ${}^cR_{PM}$ in Eq. (2) can be obtained.

As an example, suppose the relation matrix between parts and machines shown in

PARTS NUMBER

	1001	1002	1003	1004	1005	1006
1001	A	A	A	A	.	A
1002	A	.	.	.	.	A
1003	A	.	A	.	.	A
1004	.	A	.	A	A	.
1005	.	.	.	A	A	.
1006	A	.	A	.	.	A
1007	A	A	A	.	.	A
1008	.	A	A	A	A	.
1009	.	A	.	A	A	.
1010	A	.	.	.	.	A
1011	A	.	A	.	.	A
1012	A	A	A	A	A	.
1013	.	A	.	A	A	.
1014	.	.	.	A	A	.
1015	.	A	A	A	A	.
1016	A	.	.	.	.	A

(a) Relation matrix

MACHINE NO PARTS NO	1005	1004	1002	1003	1001	1006
1014	0.50	0.10	0.	0.	0.	0.
1005	0.60	0.20	0.	0.	0.	0.
1004	0.40	0.30	0.30	0.	0.	0.
1009	0.40	0.30	0.40	0.	0.	0.
1013	0.40	0.30	0.50	0.	0.	0.
1015	0.40	0.40	0.30	0.20	0.	0.
1008	0.30	0.40	0.40	0.30	0.	0.
1012	0.30	0.60	0.50	0.40	0.60	0.
1001	0.	0.50	0.50	0.40	0.40	0.30
1007	0.	0.	0.30	0.40	0.40	0.40
1011	0.	0.	0.	0.60	0.40	0.40
1006	0.	0.	0.	0.50	0.50	0.50
1003	0.	0.	0.	0.50	0.30	0.50
1010	0.	0.	0.	0.	0.30	0.40
1002	0.	0.	0.	0.	0.20	0.50
1016	0.	0.	0.	0.	0.20	0.60
TOTAL WORKLOAD	3.30	3.10	3.20	3.30	3.30	3.60
NUMBER OF MACHINES	4	4	4	4	4	4

(b) Structural matrix

Fig. 5 Cell formation by the quantification theory III

```
                 MACHINE NUMBER

             1 1|1 1 1 1 1 1|1 1 1
             0 0|0 0 0 0 0 0|0 0 0
             0 0|0 0 0 0 0 0|0 0 0
             5 4|5 4 2 3 1 6|3 1 6
            CELL 1
PARTS NUMBER
      1014   A A . CELL 2 . . . . .
      1005   . . A A . . . . . . .
      1004   . . A A A . . . . . .
      1009   . . A A A . . . . . .
      1013   . . A A A . . . . . .
      1015   . . A A A A . . . . .
      1008   . . A A A A . . . . .
      1012   . . A A A A A . . . .
      1001   . . . A A A A A . . .
      1007   . . . . A A A A CELL 3
      1011   . . . . . . . . A A A
      1006   . . . . . . . . A A A
      1003   . . . . . . . . A A A
      1010   . . . . . . . . . A A
      1002   . . . . . . . . . A A
      1016   . . . . . . . . . A A
```

(a) Structural matrix obtained in 1st iteration

```
*** CELL 1 ***
  ( PROCESSED PARTS )
        1014
  ( MACHINE )       ( NUMBER )       ( LOAD )       ( LOAD RATE )
     1005               1               0.5             50.0 %
     1004               1               0.1             10.0 %

*** CELL 2 ***
  ( PROCESSED PARTS )
        1005    1004    1009    1013    1015
        1008    1012    1001    1007
  ( MACHINE )       ( NUMBER )       ( LOAD )       ( LOAD RATE )
     1005               3               2.8             93.3 %
     1004               3               3.0            100.0 %
     1002               4               3.2             80.0 %
     1003               2               1.7             85.0 %
     1001               2               1.4             70.0 %
     1006               1               0.7             70.0 %

*** CELL 3 ***
  ( PROCESSED PARTS )
        1011    1006    1003    1010    1002
        1016
  ( MACHINE )       ( NUMBER )       ( LOAD )       ( LOAD RATE )
     1003               2               1.6             80.0 %
     1001               2               1.9             95.0 %
     1006               3               2.9             96.7 %
```

(b) Contents of the obtained cells

Fig. 6 Reconstruction of the cellular structure (1st iteration)

Fig. 4 (a), where the elements are, for convenience, given with "A"s in place of actual workloads. The structural matrix in Fig. 4 (b) can be obtained by carrying out the procedure mentioned above. It can be seen that there exist three almost independent cells in the structural matrix. In this case, the values of the eigenvalues are $\rho = 1, 0.97, 0.85, 0.11, 0.11, 0.08, 0.06, 0.03, 0.02$ and 0.01. The existence of three almost independent cells can be confirmed by the first three eigenvalues which are nearly equal to 1 including 1.

```
                         MACHINE.NUMBER
           1 1 1 1|1 1 1 1|1 1 1 1 1|1 1 1
           0 0 0 0|0 0 0 0|0 0 0 0 0|0 0 0
           0 0 0 0|0 0 0 0|0 0 0 0 0|0 0 0
           5 4 2 5|4 2 3 1|4 2 3 1 6|3 1 6
          CELL 1
      1014 A A . . . . . . . . . . . . . .
      1005 A A . . . . . . . . . . . . . .
      1004 A A A . . . . . . . . . . . . .
      1009 A A A CELL 2. . . . . . . . . .
PARTS 1013 . . . A A A . . . . . . . . . .
NUMBER1015 . . . A A A A . . . . . . . . .
      1008 . . . A A A A . . . . . . . . .
      1012 . . . A A A A A  CELL 3 . . . .
      1001 . . . . . . . . A A A A A . . .
      1007 . . . . . . . . . A A A A CELL 4
      1011 . . . . . . . . . . . . . A A A
      1006 . . . . . . . . . . . . . A A A
      1003 . . . . . . . . . . . . . A A A
      1010 . . . . . . . . . . . . . . A A
      1002 . . . . . . . . . . . . . . A A
      1016 . . . . . . . . . . . . . . A A
```

(a) Structural matrix obtained in 4th iteration

```
*** CELL 1 ***
  ( PROCESSED PARTS )
         1014    1005    1004    1009
  ( MACHINE )      ( NUMBER )       ( LOAD )         ( LOAD RATE )
    1005               2               1.9               95.0 %
    1004               1               0.9               90.0 %
    1002               1               0.7               70.0 %

*** CELL 2 ***
  ( PROCESSED PARTS )
         1013    1015    1008    1012
  ( MACHINE )      ( NUMBER )       ( LOAD )         ( LOAD RATE )
    1005               2               1.4               70.0 %
    1004               2               1.7               85.0 %
    1002               2               1.7               85.0 %
    1003               1               0.9               90.0 %
    1001               1               0.6               60.0 %

*** CELL 3 ***
  ( PROCESSED PARTS )
         1001    1007
  ( MACHINE )      ( NUMBER )       ( LOAD )         ( LOAD RATE )
    1004               1               0.5               50.0 %
    1002               1               0.8               80.0 %
    1003               1               0.8               80.0 %
    1001               1               0.8               80.0 %
    1006               1               0.7               70.0 %

*** CELL 4 ***
  ( PROCESSED PARTS )
         1011    1006    1003    1010    1002
         1016
  ( MACHINE )      ( NUMBER )       ( LOAD )         ( LOAD RATE )
    1003               2               1.6               80.0 %
    1001               2               1.9               95.0 %
    1006               3               2.9               96.7 %
```

(b) Contents of the obtained cells

Fig. 7 Reconstruction of the cellular structure (4th iteration)

4. Human-Computer Interactive Procedure for Cell Formation

In the previous section, the basic algorithm for the cell formation was shown, but there remain several problems to be solved in order to apply the algorithm in practice. Those are quantitative ones such as how many machines are required at least, how the machines should be allocated to each cell, how good workload balance could be achieved, and so on. In this section, the human-computer interactive procedure is proposed to solve such problems.

Now, consider the relation matrix shown in Fig. 5 (a). The structural matrix, whose elements are described in workloads, can be obtained as shown in Fig. 5 (b) in the previous manner, in which two cells may be found. In this case, however, it cannot be expected to entirely gain the advantage of cellular structure due to the large cell sizes and large interdependency between two cells$^{14)}$.

Then, the cells may be divided into the independent cells with smaller sizes and good workload balances$^{13)}$. The subdivision of cells can be performed by reallocating the machines to cells so as not to increase the total number of machines. The algorithm is carried out by testing from the top of rows to the bottom in the structural matrix in Fig. 5 (b) if it is able to divide the cells in no violation of the constraint on the total number of machines. As a result, the structural matrix and the contents of the resulting cells are obtained as shown in Fig. 6 (a) and (b), respectively. However, the result cannot be regarded as an acceptable one due to the unbalance between the first two cells with their cell sizes and also the unbalance of the workload on the machine 1004 between these two cells. Thus, the designer points out a cell to be modified, *i.e.*, cell 1 in this case. Then, all the cells which follow the cell pointed out are reconstructed according to the algorithm mentioned before. Such an interactive procedure between the designer and the computer is iterated to get the satisfactory solution.

After four iterations, the structural matrix shown in Fig. 7 (a) can be obtained. Figure 7 (b) shows the contents of cells formed. According to the results shown in Fig. 7, it can be concluded that this cell structure is acceptable one with regard to cell size and workload balance among machines. Figure 8 schematically shows a block layout of the cellular manufacturing system formed in this case study.

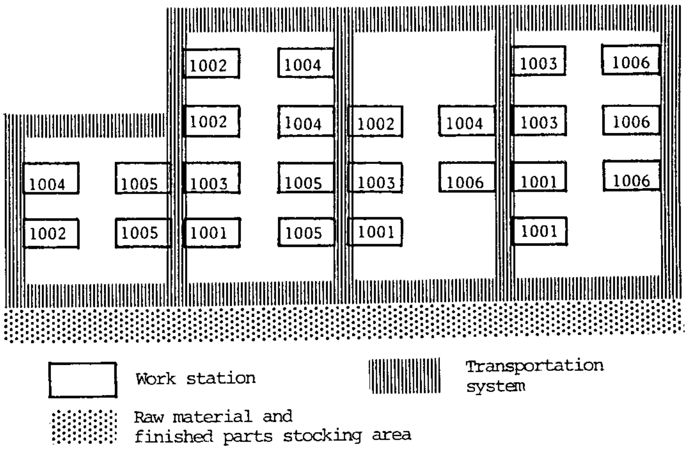


Fig. 8 Block layout of the cellular structure in Fig. 7

5. Cell Formation Problem with Alternative Machines

There, in practice, often exist alternative machines which can perform the required operations at each processing stage of parts. In such cases, by selecting an appropriate one among alternative machines, it is possible to reduce the number of required machines, increase machine utilization and eliminate interdependencies among cells. Hence, it is very important to consider the existence of alternative machines for cell formation. In this section, this problem is formulated and solved by the branch and bound technique.

Let $M_{i,s}$ the group of alternative machines that can perform the operation for the s-th process of part p_i. S_i denotes the processing sequence of part p_i, and it is given as follows.

$$S_i = [\,M_{i,1} \cdots M_{i,s} \cdots M_{i,n_i}\,] \tag{9}$$

Based on the primary machine with the shortest processing time among alternatives in each process of each part, the alternative machine groups to be used in each process are expressed in Eq. (10).

$$\begin{array}{ll}
M^1 & = [\,m_1^1, \quad \cdots, m_j^1, \quad \cdots, m_{NM}^1 \quad] \qquad P \\
 & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \parallel \\
AM^1 & = \left(\begin{array}{lll} M_{1,1}^1 & \cdots M_{1,j}^1 & \cdots M_{1,NM}^1 \\ \vdots & \vdots & \vdots \\ M_{i,1}^1 & \cdots M_{i,j}^1 & \cdots M_{i,NM}^1 \\ \vdots & \vdots & \vdots \\ M_{NP,1}^1 & \cdots M_{NP,j}^1 \cdots & M_{NP,NM}^1 \end{array}\right) \left(\begin{array}{c} p_1 \\ \vdots \\ p_i \\ \vdots \\ p_{NP} \end{array}\right)
\end{array} \tag{10}$$

In each alternative machine group, a machine is selected so as to minimize the total number of machines required.

For example, suppose the simple case with two parts and two processes. The alternative machine group in each process and corresponding workload are given in Eqs. (11) and (12), respectively.

$$\begin{array}{ll}
M^1 & = [\,m_1^1 = 1001, \quad m_2^1 = 1002\,] \qquad P \\
 & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \parallel \\
AM^1 & = \left[\begin{array}{ll} M_{1,1}^1 = \begin{pmatrix} 1001 \\ 1002 \end{pmatrix} & M_{1,2}^1 = 1002 \\ M_{2,1}^1 = \begin{pmatrix} 1001 \\ 1002 \\ 1003 \end{pmatrix} & M_{2,2}^1 = 1002 \end{array}\right] \left[\begin{array}{l} p_1 = 1001 \\ \\ p_2 = 1002 \end{array}\right]
\end{array} \tag{11}$$

$$AT^1 = \left[\begin{array}{ll} T_{1,1}^1 = \begin{pmatrix} 1.0 \\ 1.3 \end{pmatrix} & T_{1,2}^1 = 0.4 \\ T_{2,1}^1 = \begin{pmatrix} 0.5 \\ 0.6 \\ 0.7 \end{pmatrix} & T_{2,2}^1 = 0.9 \end{array}\right] \tag{12}$$

For the purpose of machine selection, new variables $x_{i,j}$'s are introduced which correspond to the alternative machine groups $M_{i,j}^1$'s. That is,

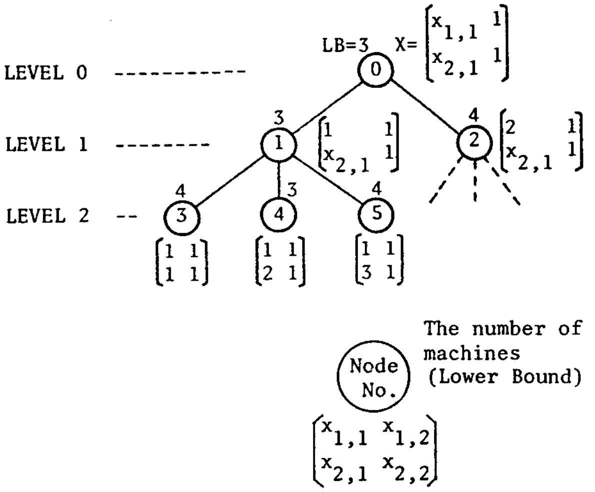


Fig. 9 Search procedure of the optimum alternative machines by Branch and Bound method

$$x = \begin{bmatrix} x_{1,1} & x_{1,2} \\ x_{2,1} & x_{2,2} \end{bmatrix} \tag{13}$$

In Eq. (13), for instance, if $x_{1,1}$ is equal to 1, it means that the Machine 1001 should be selected out of the alternative machine group $M^1_{1,1}$ in Eq. (11), on the contrary, if $x_{1,1}$ is equal to 2, the Machine 1002 is selected. In Eq. (13), it should be noted that $x_{1,2}$ and $x_{2,2}$ are definitely equal to 1, as both machine groups $M^1_{1,2}$ and $M^1_{2,2}$ in Eq. (11) each include only one machine.

With these variables, the search procedure for the optimum combination of alternative machines by the branch and bound method is illustrated in Fig. 9. In the figure, Node 0 denotes the initial state where no variables of $x_{1,1}$ and $x_{2,1}$ are determined. The lower bound of the total number of machines at this node is calculated by supposing that the primary machines are selected for all operations and all the machines are capable of substitution for one another. Thus, the lower bound is obtained from Eq.(12) as follows,

$$\begin{aligned} LB_0 &= [\ 1.0 + 0.5 + 0.4 + 0.9\ ]^+ \\ &= [\ 2.8\ ]^+ \\ &= 3 \end{aligned}$$

Here, the symbol $[x]^+$ means the minimum integer not less than the value of x.

Then the branching procedure is carried out to yield two Nodes of 1 and 2 corresponding to the possible values 1 and 2 of the variable $x_{1,1}$. The lower bound at each node is evaluated on the assumption that the primary machine is selected for each process of each part except for the process where the machine to be used is already determined, *i.e.*,

$$\begin{aligned} LB_1 &= [\ 1.0 + 0.5 + 0.4 + 0.9\ ]^+ \\ &= [\ 2.8\ ]^+ \\ &= 3 \end{aligned}$$

$$\begin{aligned} LB_2 &= [\ 1.3 + 0.5 + 0.4 + 0.9\ ]^+ \\ &= [\ 3.1\ ]^+ \\ &= 4 \end{aligned}$$

Thus, Node 1 gives the smallest lower bound between these nodes. Hence, Node 1 is selected as the new branching node, which yields three new Nodes 3, 4 and 5 corresponding to the possible values 1, 2 and 3 of the variable $x_{2,1}$ in Eq.(13). At these nodes, all the machines to be used are determined, hence the number of machines required at each node can be calculated from Eqs. (11) and (12), that is,

At Node 3: Machine 1001: $[1.0 + 0.5]^+ = 2$
Machine 1002: $[0.4 + 0.9]^+ = 2$
Total number of machines required: 4
At Node 4: Machine 1001: $[1.0]^+ = 1$
Machine 1002: $[0.6 + 0.4 + 0.9]^+ = 2$
Total number of machines required: 3
At Node 5: Machine 1001: $[1.0]^+ = 1$
Machine 1002: $[0.4 + 0.9]^+ = 2$
Machine 1003: $[0.7]^+ = 1$
Total number of machines required: 4

Consequently, it can be found that Node 4 gives the optimum solution where a Machine of 1001 and two Machines of 1002 are required. It should be noted that it is required no longer to branch at Node 2 in Fig. 9, because the lower bound at the node is greater than the number of machines required in the optimum solution.

Now, consider the case where the processing data of each part are given as shown in Fig. 10. In the figure, the first column shows the part number to be processed and the top of the rows shows the primary machine number for each process of each part. The processing information for each part consists of three rows, where the top denotes the workload on the primary machine for each process of the corresponding part, the middle shows the alternative machine for the corresponding operation and the bottom gives the workload on the alternative machine.

First, consider the cell formation problem where alternative machines are not considered. Figure 11 (a) shows the part-machine relation matrix when primary machines are assigned to all processing stages of each part. Figure 11 (b) gives the result of cell formation, and (c) represents the elements of each cell. It is found that three cells are formed including one bottleneck machine (* in the figure) and three exceptional elements (# in the figure), and the total number of machine is 19.

Here, the bottleneck machine is defined as the machine which interferes with the cell formation, because the machine has to process many kinds of parts. In case that there exists such a bottleneck machine, the cell structure can be obtained by temporarily eliminating the bottleneck machine. And after the cell formation by the proposed method, the eliminated machine is again allocated to the resulting cells. In such a case,

```
---------- PROCESSING INFORMATION ----------

                MACHINE NO.

       1001 1002 1011 1004 1005 1006 1008 1009 1010 1003 1007
------------------------------------------------------------------
1001 I  0.2  0.1  0.3  0.4  0.   0.   0.   0.   0.   0.   0.
(AM) I 1002 1005 1006 1002    0    0    0    0    0    0    0
(AT) I  0.3  0.2  0.5  0.6  0.   0.   0.   0.   0.   0.   0.
------------------------------------------------------------------
1002 I  0.1  0.3  0.3  0.5  0.2  0.   0.   0.   0.   0.   0.
(AM) I 1002 1005 1010    0 1002    0    0    0    0    0    0
(AT) I  0.2  0.4  0.5  0.   0.3  0.   0.   0.   0.   0.   0.
------------------------------------------------------------------
1003 I  0.   0.   0.2  0.   0.   0.1  0.2  0.4  0.2  0.   0.
(AM) I    0    0 1006    0    0 1011 1007 1003 1006    0    0
(AT) I  0.   0.   0.3  0.   0.   0.2  0.4  0.6  0.4  0.   0.
------------------------------------------------------------------
1004 I  0.3  0.   0.3  0.   0.   0.   0.   0.   0.   0.2  0.1
(AM) I 1002    0 1006    0    0    0    0    0    0 1007 1008
(AT) I  0.5  0.   0.5  0.   0.   0.   0.   0.   0.   0.3  0.3
------------------------------------------------------------------
1005 I  0.2  0.2  0.3  0.2  0.   0.   0.   0.   0.   0.   0.
(AM) I 1002 1005 1006 1002    0    0    0    0    0    0    0
(AT) I  0.4  0.3  0.5  0.4  0.   0.   0.   0.   0.   0.   0.
------------------------------------------------------------------
1006 I  0.   0.2  0.3  0.   0.3  0.   0.   0.   0.   0.   0.
(AM) I    0 1005 1010    0 1002    0    0    0    0    0    0
(AT) I  0.   0.3  0.4  0.   0.5  0.   0.   0.   0.   0.   0.
------------------------------------------------------------------
1007 I  0.2  0.   0.   0.3  0.3  0.   0.   0.   0.   0.4  0.
(AM) I 1005    0    0 1001 1001    0    0    0    0    0    0
(AT) I  0.3  0.   0.   0.5  0.4  0.   0.   0.   0.   0.   0.
------------------------------------------------------------------
1008 I  0.   0.2  0.3  0.   0.3  0.   0.   0.   0.   0.   0.
(AM) I    0 1005 1006    0 1002    0    0    0    0    0    0
(AT) I  0.   0.3  0.4  0.   0.5  0.   0.   0.   0.   0.   0.
------------------------------------------------------------------
1009 I  0.2  0.   0.3  0.2  0.2  0.   0.   0.   0.   0.   0.
(AM) I 1002    0 1006 1002 1002    0    0    0    0    0    0
(AT) I  0.4  0.   0.5  0.4  0.3  0.   0.   0.   0.   0.   0.
------------------------------------------------------------------
1010 I  0.2  0.   0.   0.   0.   0.   0.   0.   0.   0.3  0.2
(AM) I 1002    0    0    0    0    0    0    0    0 1007 1008
(AT) I  0.3  0.   0.   0.   0.   0.   0.   0.   0.   0.5  0.4
------------------------------------------------------------------
1011 I  0.   0.   0.3  0.   0.   0.   0.1  0.3  0.   0.2  0.3
(AM) I    0    0 1010    0    0    0 1007 1003    0 1009 1003
(AT) I  0.   0.   0.5  0.   0.   0.   0.2  0.4  0.   0.4  0.5
------------------------------------------------------------------
1012 I  0.1  0.   0.3  0.   0.   0.   0.   0.   0.   0.4  0.2
(AM) I    0    0 1006    0    0    0    0    0    0 1007 1008
(AT) I  0.   0.   0.5  0.   0.   0.   0.   0.   0.   0.6  0.4
------------------------------------------------------------------
1013 I  0.   0.   0.2  0.   0.   0.4  0.2  0.3  0.1  0.   0.
(AM) I    0    0 1006    0    0 1010 1007 1003    0    0    0
(AT) I  0.   0.   0.3  0.   0.   0.6  0.4  0.4  0.   0.   0.
------------------------------------------------------------------
1014 I  0.   0.   0.2  0.   0.   0.   0.2  0.1  0.2  0.   0.
(AM) I    0    0 1010    0    0    0 1007 1008    0    0    0
(AT) I  0.   0.   0.3  0.   0.   0.   0.4  0.3  0.   0.   0.
------------------------------------------------------------------
1015 I  0.   0.   0.   0.   0.   0.2  0.2  0.2  0.2  0.   0.
(AM) I    0    0    0    0    0 1010 1007 1008    0    0    0
(AT) I  0.   0.   0.   0.   0.   0.4  0.4  0.3  0.   0.   0.
------------------------------------------------------------------
```

Fig. 10 Processing information with alternative machine data

the total number of machines may be increased.

Next, consider the cell formation problem taking account of the alternative machine tools. First, each machine is classified into one of the groups which are formed based on the mutual replacibility of machines. The result is shown in Table 1 where

```
               MACHINE NUMBER

           1 1 1 1 1 1 1 1 1 1 1
           0 0 0 0 0 0 0 0 0 0 0
           0 0 0 0 0 0 0 0 0 1 1
           1 2 3 4 5 6 7 8 9 0 1
PARTS NUMBER
     1001  A A . A . . . . . . A
     1002  A A . A A . . . . . A
     1003  . . . . . A . A A A A
     1004  A . A . . . A . . . A
     1005  A A . A . . . . . . A
     1006  . A . . A . . . . . A
     1007  A . A A A . . . . . .
     1008  . A . . A . . . . . A
     1009  A . . A A . . . . . A
     1010  A . A . . . A . . . .
     1011  . . A . . . A A A . A
     1012  A . A . . . A . . . A
     1013  . . . . . A . A A A A
     1014  . . . . . . . A A A A
     1015  . . . . . A . A A A .
```

(a) Part-machine relation matrix

```
                  MACHINE NUMBER

           1 1 1 1 1 1 1 1 1 1 1 1 1 1
           0 0 0 0 0 0 0 0 0 0 0 0 0 0
           0 0 0 1 0 0 0 0 1 0 0 0 1 1
           3 7 1 1 1 4 2 5 1 6 9 8 0 1
PARTS NUMBER
     1012  A A A * . . . . . . . . . .
     1010  A A A . . . . . . . . . . .
     1004  A A A * . . . . . . . . . .
     1005  . . . . A A A . * . . . . .
     1001  . . . . A A A . * . . . . .
     1009  . . . . A A . A * . . . . .
     1007  # . . . A A . A . . . . . .
     1002  . . . . A A A A * . . . . .
     1006  . . . . . . A A * . . . . .
     1008  . . . . . . A A * . . . . .
     1013  . . . . . . . . . A A A A *
     1015  . . . . . . . . . A A A A .
     1003  . . . . . . . . . A A A A *
     1011  # # . . . . . . . . A A . *
     1014  . . . . . . . . . . A A A *
```

(b) Structural matrix

cell No.	machine	number of machines	workload	load rate
1 (5 machines)	1003	2	1.5	75 %
	1007	1	0.8	80 %
	1001	1	0.6	60 %
	1011	1	0.6	60 %
2 (8 machines)	1001	1	0.9	90 %
	1004	2	1.6	80 %
	1002	1	1.0	100 %
	1005	2	1.3	65 %
	1011	2	1.8	90 %
3 (6 machines)	1006	1	0.7	70 %
	1009	2	1.3	65 %
	1008	1	0.9	90 %
	1010	1	0.7	70 %
	1011	1	0.9	90 %
	total number of machines	19		
	exceptional elements	3		

(c) Summary of cell formation

Fig. 11 Result of cell formation without alternative machines

three mutually independent groups of alternative machines are formed. Next, the optimum alternative plan for each group is obtained by using the branch and bound method. Results are also shown in Table 1. For example, the alternative machine Group 1 has an optimum solution range of $6 \leqslant n_{OPT} \leqslant 7$, and three optimum plans that attain $n_{OPT} = 6$ are found by using the proposed method. The first plan indicates that the only change required is to replace Machine 1005 with 1001 to process Part 1007. Then, the best plan for each machine group must be selected from the alternatives in the previous step. Finally, an optimum solution for the given problem will be obtained by combining the best alternatives for all machine groups.

Figure 12 (a) shows the part-machine relation matrix consisting of the optimum

Table 1 Alternative machine groups and optimum solutions

Machine group	Optimum solution	Plan No.	Parts	Primary machine → Alternative machine
No. 1		1	1007	1005 → 1001
	$6 \leqq n_{OPT} \leqq 7$	2	1001	1004 → 1002
1001			1007	1005 → 1001
1002			1009	1004 → 1002
1004	$n_{OPT}=6$	3	1001	1004 → 1002
1005			1005	1004 → 1002
			1007	1005 → 1001
No. 2		1	1013	1011 → 1006
	$5 \leqq n_{OPT} \leqq 6$		1014	1011 → 1010
1006		2	1003	1006 → 1011
1010			1008	1011 → 1006
1011	$n_{OPT}=5$		1014	1011 → 1010
		3	1003	1011 → 1006
			1014	1011 → 1010
No. 3		1	1013	1009 → 1003
	$5 \leqq n_{OPT} \leqq 6$	2	1011	1009 → 1003
1003		3	1011	1008 → 1007
1007			1013	1009 → 1003
1008	$n_{OPT}=5$	4	1011	1008 → 1007
1009				1009 → 1003

plans in each of the alternative machine groups, where Plan 1, Plan 1 and Plan 4 are selected for Groups 1, 2 and 3, respectively. The result of cell formation for this matrix in consideration of workloads is illustrated in Fig. 12 (b). This figure indicates that three cells were formed with one bottleneck machine and two exceptional elements. The elements of each cell are listed in Fig. 12 (c), which shows that the total of 17 machines is required. By using alternative machines, this case, compared to the one which uses only primary machines, reduces the total number of machines by two and the number of exceptional elements by one for the same number of cells. Note that the machine utilization in each cell is also higher than that in the other case. Figure 13 shows a block layout of the resulting cellular manufacturing system.

As it has been demonstrated in this case study, the use of alternative machines enables more appropriate and flexible cell formation than the case with only primary machines, by reducing the total number of machines required and overlaps between cells, and also increasing the machine utilization.

6. Development of Computer Aided Factory Planning System.

A software system is developed to realize the proposed method in human-computer interaction, which is entitled CAFPLAN-I (Computer Aided Factory PLANning system – I). The system consists of four subprograms listed in Table 2. The design procedure by CAFPLAN-I is shown in Fig. 14 and summarized as follows.

In case that alternative machines exist in manufacturing processes, the subprogram ALTSUB selects the machines among the alternatives to minimize the total number of machines required. The subprogram GPSUB forms part families and machine cells so as to maximize the correlation coefficient between parts and machines selected by

```
             MACHINE NUMBER

       1 1 1 1 1 1 1 1 1 1 1
       0 0 0 0 0 0 0 0 0 0 0
       0 0 1 0 0 0 0 0 1 0 0
       1 2 1 4 5 6 8 9 0 3 7

PARTS NUMBER
  1001 A A A A . . . . . . .
  1002 A A A A A . . . . . .
  1003 . . A . . A A A A . .
  1004 A . A . . . . . . A A
  1005 A A A A . . . . . . .
  1006 . A A . A . . . . . .
  1007 A . . A . . . . . A .
  1008 . A A . A . . . . . .
  1009 A . A A A . . . . . .
  1010 A . . . . . . . . A A
  1011 . . A . . . . . . A A
  1012 A . A . . . . . . A A
  1013 . . . . . A A A A . .
  1014 . . . . . . A A A . .
  1015 . . . . . A A A A . .
```

(a) Part-machine relation matrix

```
               MACHINE NUMBER

       1 1 1 1 1 1 1 1 1 1 1 1 1
       0 0 0 0 0 0 0 0 0 0 0 0 0
       0 0 1 0 1 0 0 0 0 0 0 0 1
       3 7 1 1 1 5 2 4 1 6 9 8 0

PARTS NUMBER
  1007 A . . * . . . # . . . . .
  1010 A A . * . . . . . . . . .
  1011 A A A . . . . . . . . . .
  1012 A A A * . . . . . . . . .
  1004 A A A * . . . . . . . . .
  1009 . . . . A A . A * . . . .
  1005 . . . . A . A A * . . . .
  1006 . . . . A A A . . . . . .
  1008 . . . . A A A . . . . . .
  1001 . . . . A . A A * . . . .
  1002 . . . . A A A A * . . . .
  1013 . . . . . . . . . A A A A
  1015 . . . . . . . . . A A A A
  1003 . . # . . . . . . A A A A
  1014 . . . . . . . . . . A A A
```

(b) Structural matrix

cell No.	machine	number of machines	workload	load rate
1 (6 machines)	1001	2	1.2	60 %
	1003	2	1.9	95 %
	1007	1	1.0	100 %
	1011	1	0.9	90 %
2 (7 machines)	1001	1	0.7	70 %
	1002	1	1.0	100 %
	1004	2	1.6	80 %
	1005	1	1.0	100 %
	1011	2	2.0	100 %
3 (4 machines)	1006	1	1.0	100 %
	1008	1	0.8	80 %
	1009	1	1.0	100 %
	1010	1	1.0	100 %
	total number of machines	17		
	exceptional elements	2		

(c) Summary of cell formation

Fig. 12 Result of cell formation with alternative machines

ALTSUB. If necessary, the subprogram CUTSUB reforms the cellular structure constructed by GPSUB so as to realize the clear cut cell structure, suitable cell sizes and good workload balances among machines without increasing the total number of machines required. If a satisfactory result is obtained by CUTSUB, the procedure is terminated. In case that a large number of exceptional elements and/or bottleneck machines exist, they are temporarily removed by ELMSUB and the cell formation procedure is iterated. If an acceptable solution cannot be obtained at this step, another combination of the optimum alternatives obtained by ALTSUB may be selected, and again carried out the cell formation procedure.

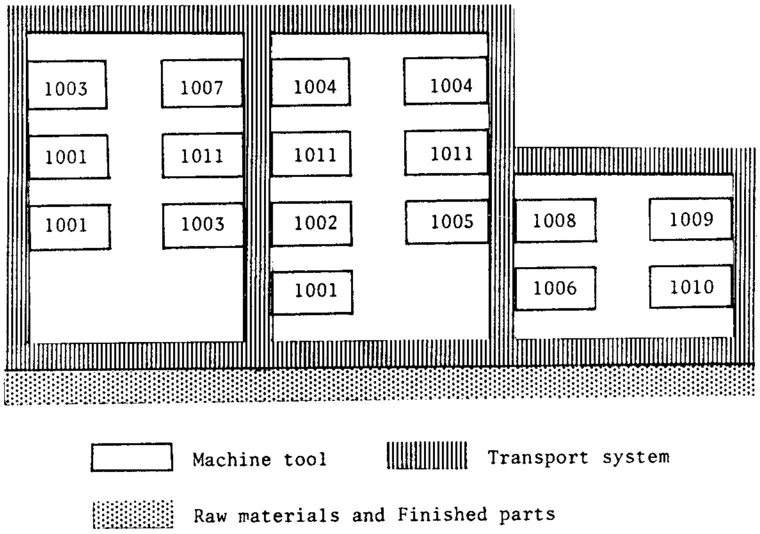


Fig. 13 Block layout of the cellular structure in Fig. 12

Table 2 Subprograms of CAFPLAN-I

Subprograms	Functions
ALTSUB	Solve the optimum alternatives for minimizing the total number of machines
GPSUB	Construct the structural matrix by applying a correlation analysis
CUTSUB	Generate the mutually independent cells by allocating the machines into cells
ELMSUB	Specify and remove the bottleneck machines or exceptional elements whose removal would result in the entirely independent cells

7. Concluding Remarks

The design concept of the FMS has been analyzed and the fundamental design procedure is proposed. The proposed method is successfully applied to the problem to obtain the appropriate cellular structure of the FMS based on the Group Technology so as to attain the high productivity and flexibility.

A computer software has been developed to realize the proposed method in human-computer interaction, which is entitled CAFPLAN-I. This system enables us to systematically find the satisfactory cellular structure taking account of the number of cells, cell sizes, alternative machine tools, workload balance among machines and utilization by the efficient use of the designer's know-how.

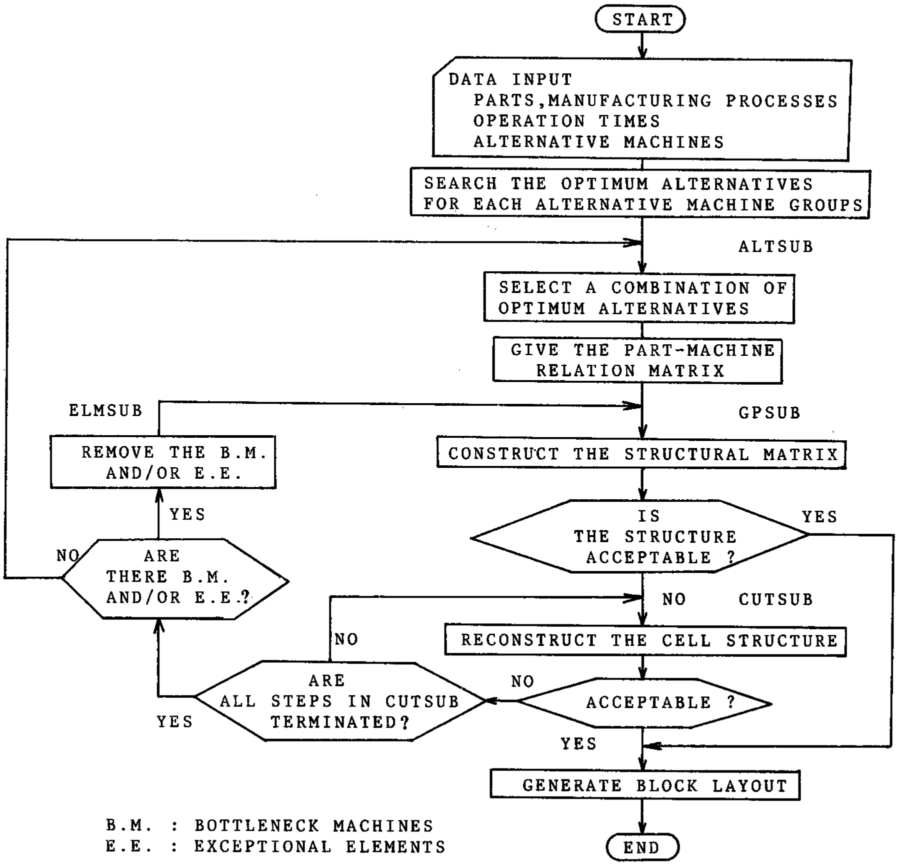


Fig. 14 Flow chart of CAFPLAN-I

The cellular structure constructed by this system can be regarded as an optimal one in the sense that the correlation coefficient between parts and machines is maximum and the total number of machines required is minimum.

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