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A Simplified Integral Equatin of Three-Dimensional Wire Structure
—— Application to Top-Loaded Antenna

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A general formulation is given for the current distributions on the wire elements comprising an antenna or a scatterer. Hallen's type integral equation containing the integral kernel is derived. The kernel is transformed into a closed form, which greatly facilitates the numerical calculations for many antenna and scatterer problems. The validity of the theory proposed here is checked by comparing the calculated results with the experimental data for the top-loaded antennas.

1. Introduction

The problem of the antenna structures assembled from arbitrarily located straight wires is of considerable practical importance. Not only is it important in itself, but also it enters as an element in solving more complicated problems, such as those of the V antenna, the top-loaded antenna, an antenna with bend or junction point located along its length, etc.

In recently distributed research repots, many computer programs are presented and described for analysis of arbitrarily bent wire antennas and scatterers. Many of these are based on a generalized Hallen's type integral equation. However, unlike Hallen's equation for a straight wire problem, the bend or junction problem leads to coupled integral equation with the kernel that has an integral form. This, of course, possesses analytical as well as numerical difficulties. In this paper, therefore, a method is proposed in which the integral kernel is reduced to a closed form.

The presented formulation can be applied without any formal restriction on the dimension of the wires. For the scattering problem an intergral equation is derived for the total current distribution induced under arbitrary illumination of incident wave. For transmitting antenna problem the integral equation is derived for an idealized δ-function excitation.

To illustrate the validity of the present formulation the problems of the inverted L, T and four elements top-loaded antenna are treated by using this formulation.

2. Integral Equation

Consider an antenna structure as shown in Fig. 1. The structure may be three dimen-

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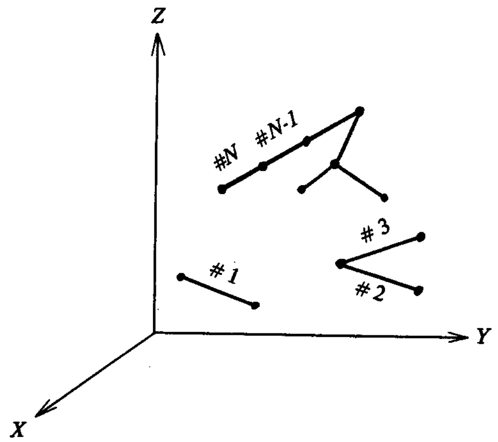


Fig. 1. Antenna structure assembled from arbitrarily located straight wires.

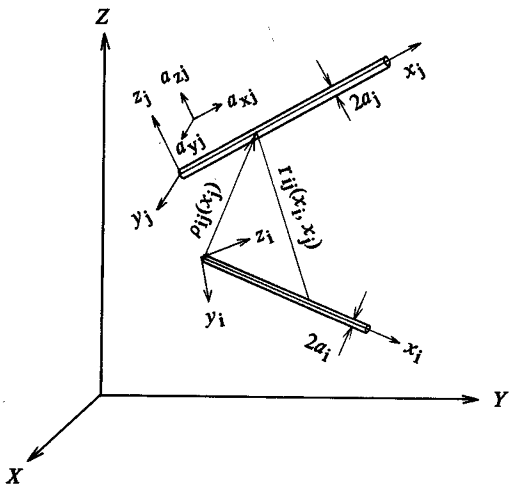


Fig. 2. Coordinate system used to analyze wire structure.

sional, and it is constructed of straight wire segmets. If any segment is much longer than $\lambda/4$, it is subdivided into shorter sections. Fig. 2 shows the coordinate system for arbitrarily located two wire elements (the i-th and the j-th) of N wires structure. Let the origin of Cartesian coordinate (x_i, y_i, z_i) be defined at an end of the i-th element. Let $(\boldsymbol{a}_{xi}, \boldsymbol{a}_{yi}, \boldsymbol{a}_{zi})$ denote unit vector directed along the axes (x_i, y_i, z_i).

We assume that (1) the wire radii are very small compared with the wire length and the wavelength, (2) currents are taken to flow along the wires only in a longitudinal direction and (3) all wire segements are perfect conductors.

The intergral equation for the current of two demensional structure has been discussed in the previous papers[1),2)]. Extending the equation, we obtain the Hallen's type integro-differential equation for the current on the element of three dimensional wire structure as follows:

$$B_i \cos\beta x_i + C_i \sin\beta x_i + v_i(x_i) = \sum_{j=1}^{N} \int_0^{h_j} I_j(x'_j) G_{ij}(x_i, x'_j) \mathrm{d}x'_j \qquad (i=1,2,\cdots,N) \tag{1}$$

where

$$v_i(x_i) = \int_0^{x_i} \overset{0}{E}_{xi}(\eta_i) \sin\beta(x_i - \eta_i) \mathrm{d}\eta_i \tag{2}$$

$$G_{ij}(x_i, x'_j) = j30\left\{\boldsymbol{a}_{xi}\cdot\boldsymbol{a}_{xj}\psi_{ij}(x_i, x'_j) + \boldsymbol{a}_{yi}\cdot\boldsymbol{a}_{xj}\overset{c}{g}_{ij}(x_i, x'_j) + \boldsymbol{a}_{zi}\cdot\boldsymbol{a}_{xj}\overset{c}{f}_{ij}(x_i, x'_j)\right\} \tag{3}$$

$$\overset{c}{g}_{ij}(x_i, x'_j) = \int_0^{x_i} \frac{\partial\psi_{ij}(\xi_i, x'_j)}{\partial y_i} \cos\beta(x_i - \xi_i) \mathrm{d}\xi_i \tag{4}$$

$$\overset{c}{f}_{ij}(x_i, x'_j) = \int_0^{x_i} \frac{\partial\psi_{ij}(\xi_i, x'_j)}{\partial z_i} \cos\beta(x_i - \xi_i) \mathrm{d}\xi_i \tag{5}$$

$$\psi_{ij}(x_i, x'_j) = e^{-j\beta r_{ij}(x_i, x'_j)} / r_{ij}(x_i, x'_j) \tag{6}$$

$$r_{ij}(x_i, x'_j) = \left[(x_i + \boldsymbol{\rho}_{ij}\cdot\boldsymbol{a}_{xi})^2 + (\boldsymbol{\rho}_{ij}\cdot\boldsymbol{a}_{yi})^2 + (\boldsymbol{\rho}_{ij}\cdot\boldsymbol{a}_{zi})^2 + a_i^2\right]^{\frac{1}{2}} \tag{7}$$

in which $\boldsymbol{\rho}_{ij}(x'_j)$ is the vector representing the distance from the point x'_j to the point $x_i = 0$, as shown in Fig. 2, and B_i and C_i are arbitrary constants.

This formulation of the integral equation can be applied without any formal restriction on the dimensions of wire segments. For the scattering problem, the current distribution is induced under arbitrary illumination. Then the impressed electric field $\overset{0}{E}_{xi}(x_i)$ is expressed by continuous function. For the transmitting atntenna problem, the wire segment is considered to be drived from the ideal δ-function source. Then the impressed electric field component is

$$\overset{0}{E}_{xi}(x_i) = V_i \delta(x_i - x_{i0}) \tag{8}$$

where V_i is the driving voltage and x_{i0} is the position of the source on the i-th element.

The scalar potential can also be expressed in terms of the currents along the elements as follows:

$$\phi_i(x_i) = -B_i \sin\beta x_i + C_i \cos\beta x_i + \frac{1}{\beta}\cdot\frac{\partial v_i(x_i)}{\partial x_i} + \sum_{j=1}^{N} \int_0^{h_j} I_j(x'_j) H_{ij}(x_i, x'_j) \mathrm{d}\,x'_j \qquad (i=1,2,\cdots,N) \tag{9}$$

where

$$H_{ij}(x_i, x'_j) = j30\left\{\boldsymbol{a}_{yi}\cdot\boldsymbol{a}_{xj}\,\overset{s}{g}_{ij}(x_i, x'_j) + \boldsymbol{a}_{zi}\cdot\boldsymbol{a}_{xj}\,\overset{s}{f}_{ij}(x_i, x'_j)\right\} \tag{10}$$

$$\overset{s}{g}_{ij}(x_i, x'_j) = \int_0^{x_i} \frac{\partial\psi_{ji}(\xi_i, x'_j)}{\partial y_i} \sin\beta(x_i - \xi_i) \mathrm{d}\xi_i \tag{11}$$

$$\overset{s}{f}_{ij}(x_i, x'_j) = \int_0^{x_i} \frac{\partial\psi_{ij}(\xi_i, x'_j)}{\partial z_i} \sin\beta(x_i - \xi_i) \mathrm{d}\xi_i. \tag{12}$$

It is now quite evident that the integral equation (1) possesses an integral form kernel which greatly complicates numerical computation. However, it is possible to rewrite Eqs. (4), (5), (11) and (12) in a closed form whithout any approximation. By using the similar transformation as those in the previous papers[1),2)], for the three dimensional problem, these equations may be transformed into

$$g^{c}_{ij}\left(x_i, x'_j\right) = -\boldsymbol{\rho}_{ij}\cdot\boldsymbol{a}_{yi}\, K^{c}_{ij}\left(x_i, x'_j\right) \tag{13}$$

$$f^{c}_{ij}\left(x_i, x'_j\right) = -\boldsymbol{\rho}_{ij}\cdot\boldsymbol{a}_{zi}\, K^{c}_{ij}\left(x_i, x'_j\right) \tag{14}$$

$$g^{s}_{ij}\left(x_i, x'_j\right) = -\boldsymbol{\rho}_{ij}\cdot\boldsymbol{a}_{yi}\, K^{s}_{ij}\left(x_i, x'_j\right) \tag{15}$$

$$f^{c}_{ij}\left(x_i, x'_j\right) = -\boldsymbol{\rho}_{ij}\cdot\boldsymbol{a}_{zi}\, K^{s}_{ij}\left(x_i, x'_j\right) \tag{16}$$

$$K^{c}_{ij}\left(x_i, x'_j\right) = \frac{1}{(\boldsymbol{\rho}_{ij}\cdot\boldsymbol{a}_{yi})^2 + (\boldsymbol{\rho}_{ij}\cdot\boldsymbol{a}_{zi})^2 + a_i^2}\Big[(x_i + \boldsymbol{\rho}_{ij}\cdot\boldsymbol{a}_{xi})\,\psi_{ij}\left(x_i, x'_j\right) - (\boldsymbol{\rho}_{ij}\cdot\boldsymbol{a}_{xi})\cos\beta x_i \psi_{ij}\left(0, x'_j\right) + j\sin\beta x_i\, e^{-j\beta r_{ij}(o, x'_j)}\Big] \tag{17}$$

$$K^{s}_{ij}\left(x_i, x'_j\right) = \frac{1}{(\boldsymbol{\rho}_{ij}\cdot\boldsymbol{a}_{yi})^2 + (\boldsymbol{\rho}_{ij}\cdot\boldsymbol{a}_{zi})^2 + a_i^2}\Big[-(\boldsymbol{\rho}_{ij}\cdot\boldsymbol{a}_{xi})\sin\beta x_i\, \psi_{ij}\left(x_i, x'_j\right) + j\, e^{-j\beta r_{ij}(x_i, x'_j)} - j\cos\beta x_i\, e^{-j\beta r_{ij}(o, x'_j)}\Big]. \tag{18}$$

The closed form kernels facilitate the numerical calculations for many antenna and scatterer problems.

The currents can then be found numerically by using a method of moment or collocation of matrix equation with the boundary conditions. As is obvious from Eq. (1), there are $2N$ auxiliary unknown coefficients B_i, C_i (i = 1, 2, ... , N), in addition to the unknown current distributions I_j (x_j) (j = 1, 2, . . . , N). These $2N$ coefficients are determined by the end conditions. The $2N$ additional constraints may be obtained by (1) the continuity of currents and (2) the continuity of the scalar potentials or (3) the currents vanishment at wire extremities.

3. Numerical Results

As a numerical example, we choose the top-loaded antenna. The top-loaded antenna is a suitable structure to check the validity of the theory, because it contains a typical discontinuity of the wire antenna. Simpson has investigated fully the properties of top-loaded antenna, and has given experimental results for various configuration of top-loading[3),4)].

The top-loaded antenna under study consists of a vertical monopole of length h_1 and N-1 loading elements. The i-th loading element is of length h_i and placed at an angel θ_i

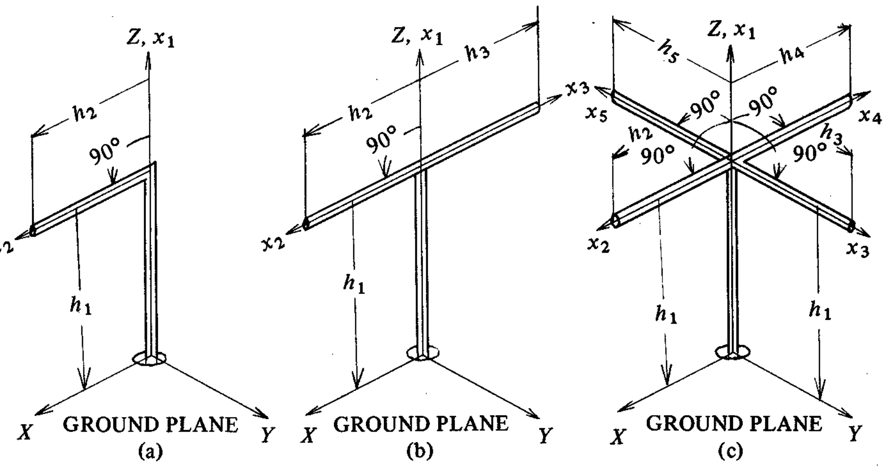


Fig. 3. Three types of top-loaded antennas. (a) Inverted L antenna. (b) T antenna. (c) Four elements top-loaded antenna.

with respect to the axis of the monopole ($i = 2, \ldots, N$). Three simple top-loading structures are indicated in Fig. 3. It is excited by a δ-function voltage V_e above an infinite and perfectly conducting ground plane. The antenna together with its image is modeled by $2N$ segemnts of the wire.

The integral equations derived in the preceding section may be solved by using the so-called method of collocation[4]. It has been shown that the polynomial expansion for the current distribution in the method of collocation provides a rapidly convergent solution. According to this technique the current is expressed by

$$I_j\left(x_j'\right)=\sum_{m=1}^{M} I_{jm}\, S_{jm}\left(x_j'\right) \tag{19}$$

where

$$S_{jm}\left(x_j'\right)=\left(1-\frac{x_j'}{h_j}\right)^{m-1} \tag{20}$$

and I_{jm}'s are complex coefficients to be determined.

Using the equation of continuity and requiring a zero current at the junction point and end point, we have

$$I_i(h_i)=\begin{cases}\sum_{j=2}^{N} I_j(0) & (i=1)\\ 0 & (i=2,\cdots,N).\end{cases} \tag{21}$$

Also requiring continuity of scalar potential at the junction point yields

$$\phi_1(h_1)=\phi_i(0) \qquad (i=2,\cdots,N). \tag{22}$$

Of course, the driving-point bounday condition is

$$\phi_1(+0) = V_e \tag{23}$$

as required by the idealized source.

Substituting the current expansion equation (19) into the integral equation (1) for the current and forcing the resulting equation to be satisfied at a suitable set of points yelds a system of linear equation for the expansion co-efficients. Also, from boundary conditions on the current and scalar potential, Eqs. (21) – (23), must be satisfied.

The accuracy of results and computation times depend on the choice of the order M in the polynomial approximation in Eq. (19). As noted by Popović[5], in the case of halfwave and fullwave isolated dipoles, the reasonable accuracy can be expected from the 3rd-order approximation. However, the higher order approximation may be needed for the top-loaded antennas. This is because the top-loaded antennas have a discontinuity like a bend or junction point. For each of the following examples, therefore, the current distribution is approximated by the 4th-order (M = 5) polynomials.

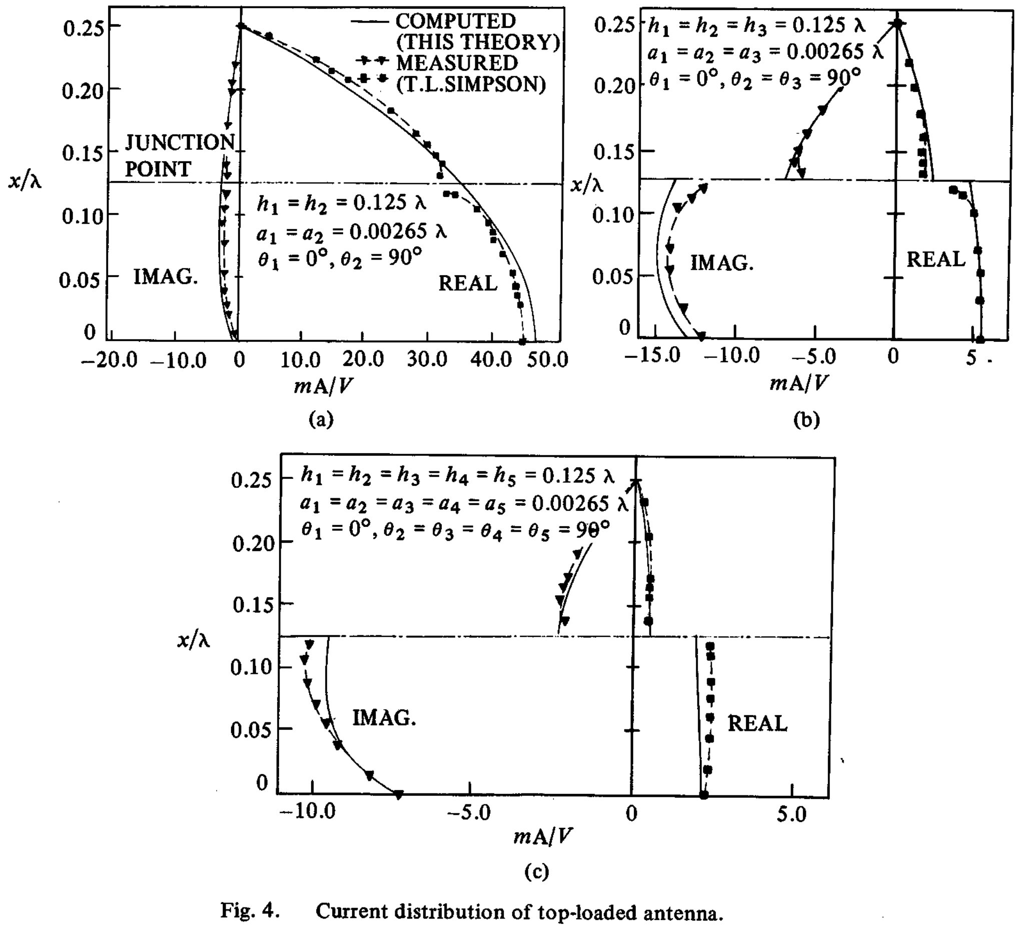


Fig. 4. Current distribution of top-loaded antenna.
(a) Inverted L antenna (N = 2). (b) T antenna (N = 3).
(c) Four elements top-loaded antenna (N = 5).

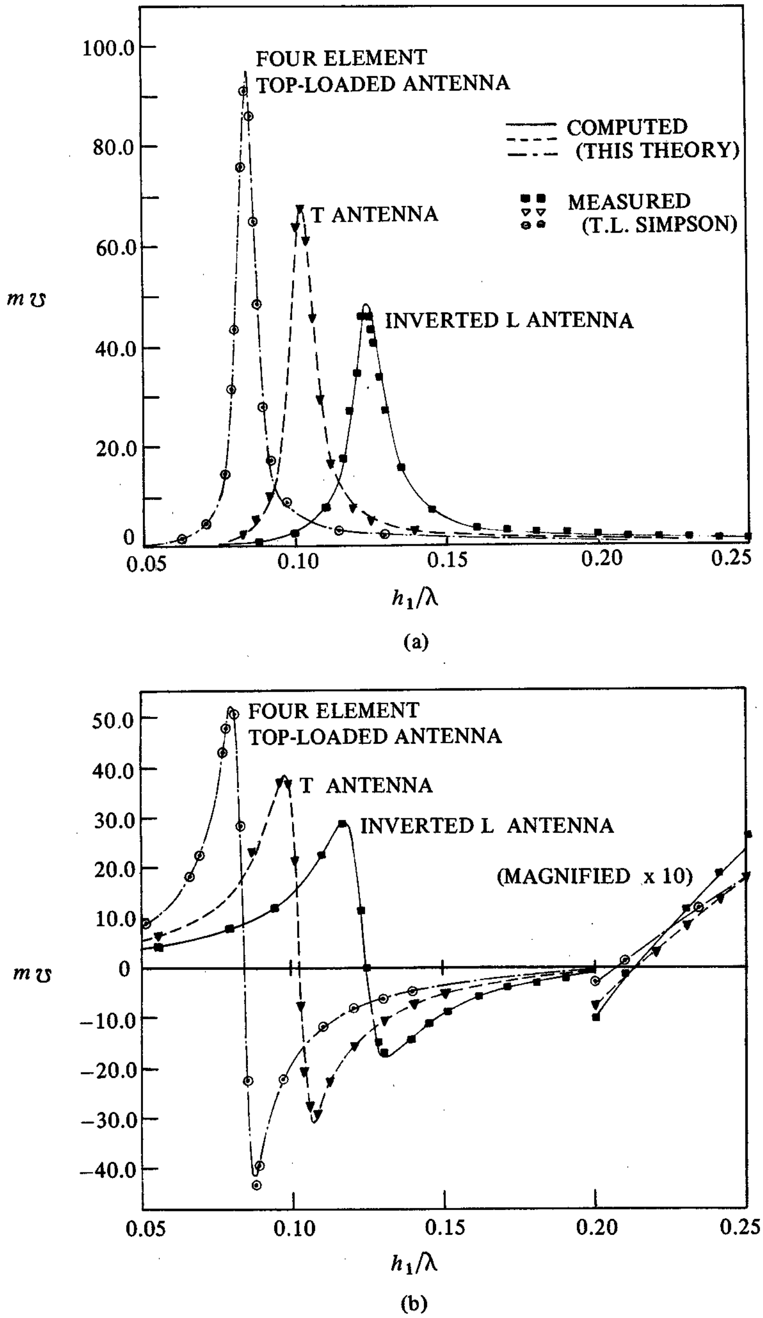


Fig. 5. Input admittance versus element length h_1 for $h_1 = h_i$ $(i = 2, \ldots, N)$. (a) Input conductance. (b) Input susceptance.

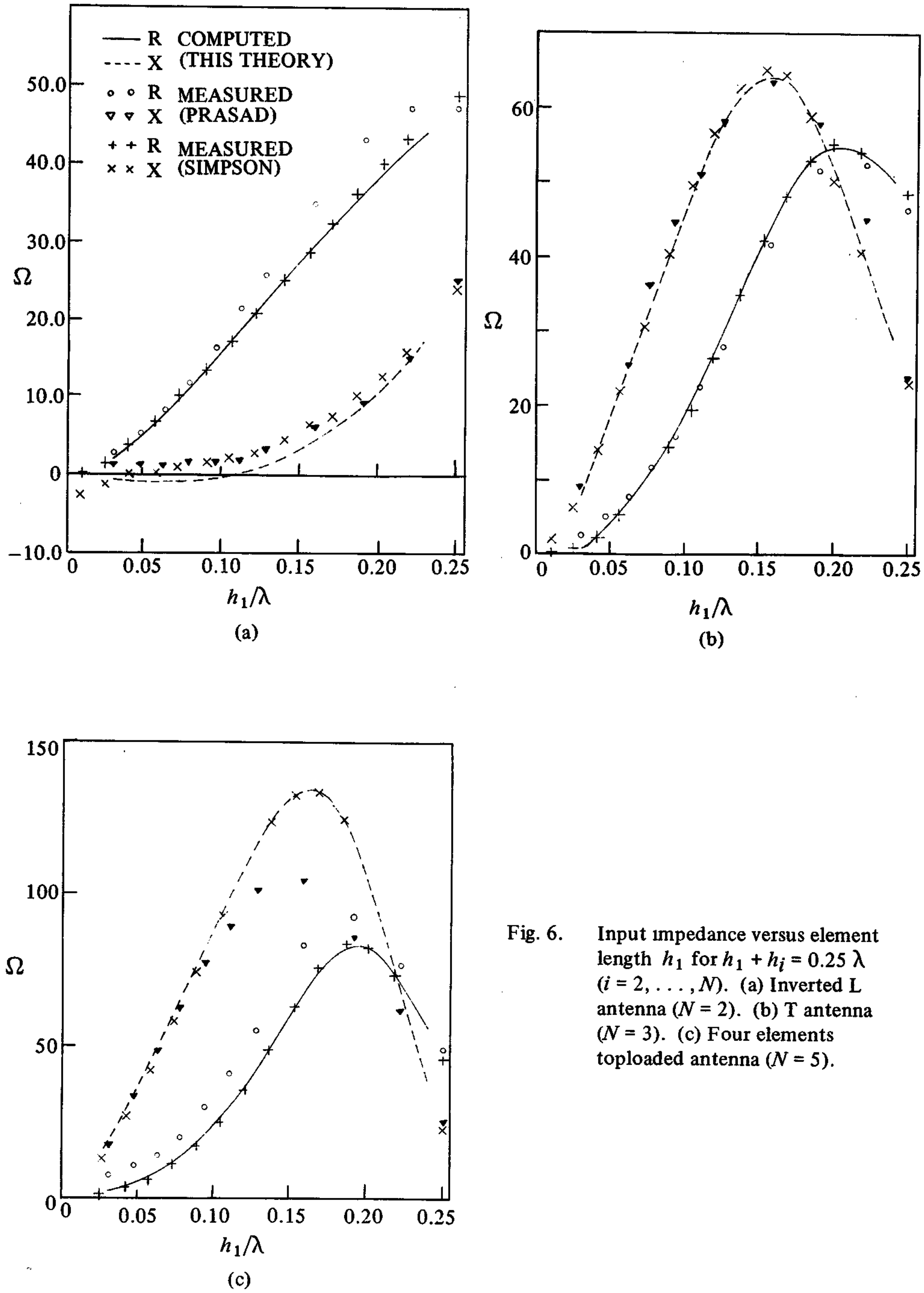
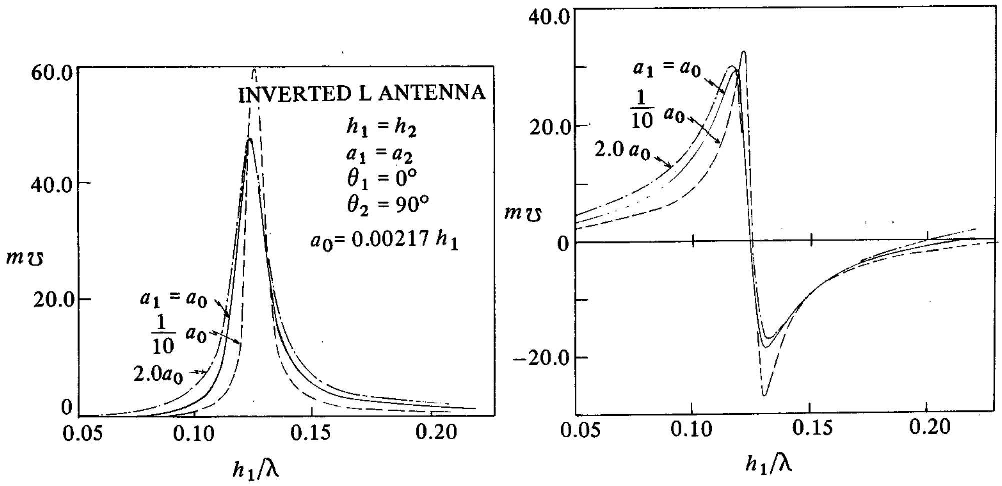


Fig. 6. Input impedance versus element length h_1 for $h_1 + h_i = 0.25\ \lambda$ ($i = 2, \ldots, N$). (a) Inverted L antenna ($N = 2$). (b) T antenna ($N = 3$). (c) Four elements toploaded antenna ($N = 5$).



(a) (b)

Fig. 7. Input admittance versus length of element with three different radii. (a) Input conductance. (b) Input susceptance.

Example 1 The current distributions for the inverted L (N = 2,), T (N = 3) and four elements (N = 5) top-loaded antennas are computed for h_i = 0.125λ, a_i = 0.00265λ (i = 1, 2, . . . , N) and θ_i = 90° (i = 2, . . . , N). They are plotted in Fig. 4. By comparing the results with the data measured by Simpson[3], it can be seen that the agreement is quite satisfactory. In the vicinity of the corner or junction region, there exists a discrepancy between the two results. However, as discussed in the Simpson's paper[3], this discrepancy is to be expected because of the difficulty of measurement and the failure of theoretical assumption to the axial current in this region.

Example 2 The real and imaginary components, G and B of antenna with $\theta_i = 90°$ and $h_1 = h_i$ (i = 2, . . . , N) are shown in Fig. 5 as a function of electrical length, h_1/λ, of vertical element. In the cases involving N= 2, 3 and 5, both experimental[3] and theoretical results are presented.

Example 3 Fig. 6 shows the input impedances of three top-loaded antennas (N = 2, 3 and 5) as a function of vertical element where $h_1 + h_2$ = 0.25λ, i.e. where the total axial path from the driving-point to the extremity of the top-loading element is maintained at a quater-wavelength[4].

Example 4 As a last example, the effect of the element radius upon the input admittance of the inverted L antenna has been investigated. Fig. 7 shows the real and imaginary components of inverted L antenna, G and B, as a function of electrical length $h_1(=h_2)$ with three element radii $a_1(=a_2) = 2a_0, a_0$ and $\frac{1}{10}a_0$ where $a_0 = 0.00217h_1$. This figure shows that the real component G increases with a decrease in radius a_1 and the resonant length (B = 0) is kept unchanged.

4. Conclusion

A simplified integral equation is given for the current distributions on the wire elements comprising an antenna or a scatterer. The integral kernel is transformed into closed one, which permits one to save troublesome numerical integration for any other conventional approach.

The integral equation presented in this paper may be solved numerically by using the method of moment or collocation of matrix equation with the suitable boundary conditions. This method of analysis permits one to obtain the current distributions not only about the antenna with ideal generator, but also on the scatterer excited by plane wave as well.

For top-loaded antenna containing a typical discontinuity of the wire antenna, the excellent agreement is obtained when theoretical results are compared with experimental data.

The same technique can also be used for analyzing other types of linear antenna such as Log-periodic array antennas.

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