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INTEREST RATE RISK AND OTHER DETERMINANTS OF POST-WWII U.S. GOVERNMENT DEBT/GDP DYNAMICS

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ABSTRACT

This paper uses the sequence of government budget constraints to motivate estimates of interest payments on the U.S. Federal government debt. We explain why our estimates differ conceptually and quantitatively from those reported by the U.S. government. We use our estimates to account for contributions to the evolution of the debt to GDP ratio made by inflation, growth, and nominal returns paid on debts of different maturities.

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1 Introduction

This paper shows the contributions that nominal interest payments, the maturity composition of the debt, inflation, and growth in real GDP have made to the evolution of the U.S. debt to GDP ratio since World War II. Among the questions we answer are these. Did the U.S. inflate away much of the debt by using inflation to pay negative real rates of return? Sometimes, but not usually. Did high net-of-interest deficits send the debt-GDP ratio upward? Substantially during World War II, but not too much after that. How much did growth in GDP help contribute to holding down the debt GDP ratio? A lot. How much did the variation in returns across maturities affect the evolution at the debt to GDP ratio? At times substantially, but on average not much since the end of World War II.

Our answers to these questions rely on our own accounting scheme and not the U.S. government's.[1] Our accounting emerges from a decomposition of the government's period-by-period budget constraint, to be described in section 2 and justified in detail in appendix A. We use prices of indexed and nominal debt of each maturity to construct one-period holding period yields on government IOU's of various maturities. Multiplying the vector of returns by the vector of quantities outstanding each period then gives us the appropriate concept of interest payments that appears in the government budget constraint.

Unfortunately, the government's interest payments series fails to measure the concept that appears in the government budget constraint, the equation that determines the evolution of the debt to GDP ratio. Appendix B describes how the government's measure ignores capital gains and losses on longer term government obligations, an error that is revealed by the absence of holding period yields for longer maturity government obligations in what we think is the government's formula for interest payments. The government creates its estimate of interest by summing all coupon payments and adding to that sum a one-period holding period yield times all principal repayments. The government could drive that measure of interest payments to zero every period by perpetually rolling over, let us say, zero-coupon 10 year bonds. Such bonds would never pay coupons and never mature: each period they would be repurchased as nine year zero-coupon bonds and reissued as fresh 10 year zero-coupon bonds. Though the government accounts would put interest payments at zero, the government would still pay interest in the economically relevant sense determined by the government budget constraint.

2 Interest payments in the government budget constraint

Let Y_t be real GDP at t and let B_t be the real value of government IOU's owed the public. That least controversial equation of macroeconomics, the government budget constraint, accounts for how a nominal interest rate $\tilde{r}_{t-1,t}$, net inflation $\pi_{t-1,t}$, net growth in real GDP $g_{t-1,t}$, and the net-of-interest deficit def_t combine to determine the evolution of the government debt-GDP ratio:

$$\frac{B_t}{Y_t} = (\tilde{r}_{t-1,t} - \pi_{t-1,t} - g_{t-1,t})\frac{B_{t-1}}{Y_{t-1}} + \frac{\mathrm{def}_t}{Y_t} + \frac{B_{t-1}}{Y_{t-1}}. \tag{1}$$

[1] Meaning the Treasury and the NIPA.

The appropriate concept of a nominal return $\tilde{r}_{t-1,t}$ is one that verifies this equation.[2]

The nominal yield $\tilde{r}_{t-1,t}$ and the real stock of debt B_t in equation (1) are averages of pertinent objects across terms to maturity. To bring out some of the consequences of *interest rate risk* and the maturity structure of the debt for the evolution of the debt-GDP ratio, we refine equation (1) to recognize that the government pays different nominal one-period holding period returns on the IOUs of different maturities that compose B_t. Thus, let B^j_{t-1} and $\bar{B}^j_{t-1}$ be the real values of nominal and indexed zero coupon bonds of maturity j at $t-1$, while $B_{t-1} = \sum_{j=1}^n B^j_{t-1}$ and $\bar{B}_{t-1} = \sum_{j=1}^n \bar{B}^j_{t-1}$ are the total real values of nominal and indexed debt at $t-1$; let $\tilde{r}^j_{t-1,t}$ be the net *nominal* holding period yield between $t-1$ and t on nominal zero-coupon bonds of maturity j; let $\bar{r}^j_{t-1,t}$ be the net *real* holding period yield between $t-1$ and t on inflation indexed zero coupon bonds of maturity j.[3] Then the government budget constraint expresses the following law of motion for the debt-GDP ratio:

$$\begin{aligned}
\frac{B_t + \bar{B}_t}{Y_t} &= \sum_{j=1}^n \tilde{r}^j_{t-1,t} \frac{B^j_{t-1}}{Y_{t-1}} - (\pi_{t-1,t} + g_{t-1,t}) \frac{B_{t-1}}{Y_{t-1}} \\
&+ \sum_{j=1}^n \bar{r}^j_{t-1,t} \frac{\bar{B}^j_{t-1}}{Y_{t-1}} - g_{t-1,t} \frac{\bar{B}_{t-1}}{Y_{t-1}} \\
&+ \frac{\mathrm{def}_t}{Y_t} \\
&+ \frac{B_{t-1} + \bar{B}_{t-1}}{Y_{t-1}}. \qquad (2)
\end{aligned}$$

Notice how equation (2) distinguishes contributions to the growth of the debt-GDP ratio that depend on debt maturity j from those that don't. Thus, $\pi_{t-1,t}$ and $g_{t-1,t}$ don't depend on j and operate on the *total* real value of debt last period; but the holding period yields $\tilde{r}^j_{t-1,t}$ and $\bar{r}^j_{t-1,t}$ do depend on maturity j and operate on the real values of the maturity j components B^j_{t-1} and $\bar{B}^j_{t-1}$.

Section 3 describes the behavior of holding period yields across maturities. Section 4 then displays the outcome of our accounting exercise by decomposing the evolution of the ratio of debt to GDP into components coming from inflation, growth in real GDP, nominal yields, and the maturity composition of the debt. Appendix A describes the data and the theory of the term structure of interest rates that we use to construct components of the government budget constraint (2). Appendix B compares our estimates of interest payments on the U.S. government debt to quite different estimates reported by the Federal government. Appendix B also reverse

[2]The nominal value of interest payments from the government to the public is $\frac{\tilde{r}_{t-1,t} p_{t-1} B_{t-1}}{Y_{t-1}}$ where p_{t-1} is the price level at $t-1$. Unfortunately, the U.S. government reports something that it calls 'interest payments' as a component of Federal expenditures, but this is not the same as the term $\frac{\tilde{r}_{t-1,t} p_{t-1} B_{t-1}}{Y_{t-1}}$ that belongs in (1). As we discuss in appendix B, what the government reports presumably was designed to answer *some* question, but that question is not "what is the appropriate interest payment to record in order to account properly for the motion through time of the real government debt owed to the public?"

[3]In a *nonstochastic* version of the standard growth model that is widely used in macroeconomics and public finance, the net holding period yield on debt is identical for zero-coupon bonds of all maturities (e.g., see Ljungqvist and Sargent, 2011, chapter 11). The presence of risk and possibly incomplete markets changes that.

engineers the *question* that the official government interest series seemingly answers, though we confess limited success in making sense of that question. To help bring out the quantitative significance of the interest rate risk that confronts the government and its creditors, appendix C constructs counterfactual series for the evolution of the debt/GDP series under alternative hypothetical debt-management policies the choice among which would have no impact on the evolution of the debt-GDP ratio in a world without interest rate risk.

3 Risk and return across maturities

To set the stage for the role that interest rate risks will play in our story, for various maturities j measured in years, figure 1 shows contributions to the propulsion of B/Y in formula (1) from nominal interest payments $\tilde{r}^j_{t-1,t}\frac{B^j_{t-1}}{Y_{t-1}}$. The figure shows that volatility of nominal interest rate payments has been larger for longer horizons. For the period 1942-2008, figure 2 plots average one-year real holding period while figure 3 plots the standard deviation of the one-year real holding by maturity.[4,5] Figure 2 reveals that while longer maturities have generally been associated with higher and more volatile returns, returns on bonds maturing in 15, 20 and 30 years were on average lower than those for adjacent maturities. This outcome reflects investors' preferences for newly issued or so-called 'on the run' securities.

Figure 4 plots the average maturity, in years, of the Treasury debt held by the public. Immediately after World War II, the average maturity of the government portfolio was approximately 7 years. Over the next three decades, it fell steadily, reaching a trough in the mid-1970s at around 2 years. In the 1960s and early 1970s this fall is partly the consequence of federal legislation, repealed in 1975, which prevented the Treasury from issuing long-term instruments paying interest above a threshold rate, that market rates exceeded during that period. As we shall see, by causing the Treasury to shorten the average maturity of its debt during the high inflation years of the 1970s, this law prevented the government from fully benefiting from the negative implicit real interest it managed to pay through inflation. Since the repeal of this restriction, the Treasury has lengthened the maturity the average maturity to between 3 and 4 years.

4 Contributions to the evolution of the U.S. debt-GDP ratio

Figure 5 reports the ratio of the market value of U.S. Treasury debt to GDP from 1941 to 2008.[6] In 1941, this ratio was 29.6 and in 1945 it stood at 66.2 percent. It fell steadily over the next

[4] A principal aim of stochastic discount factor models like the one proposed by Piazzesi and Schneider (2006) is to capture how means and standard deviations of one-period holding period yields depend on maturity.

[5] TIPS are not included in the holding period yields in these graphs.

[6] Figure 5 plots the ratio of the end of the calendar year total market value of interest-bearing marketable Treasury securities held by private investors to GDP. This measure of government debt is narrower than other Federal debt series sometimes reported. It does not include nonmarketable securities (e.g. savings bonds, special issues to state and local governments), securities held by other government entities (e.g. the Federal Reserve or the Social Security Trust Fund), or agency debt (e.g. Tennessee Valley Authority). Further each security is valued at market prices rather than par values. See appendix A for a complete description of how this series was constructed.

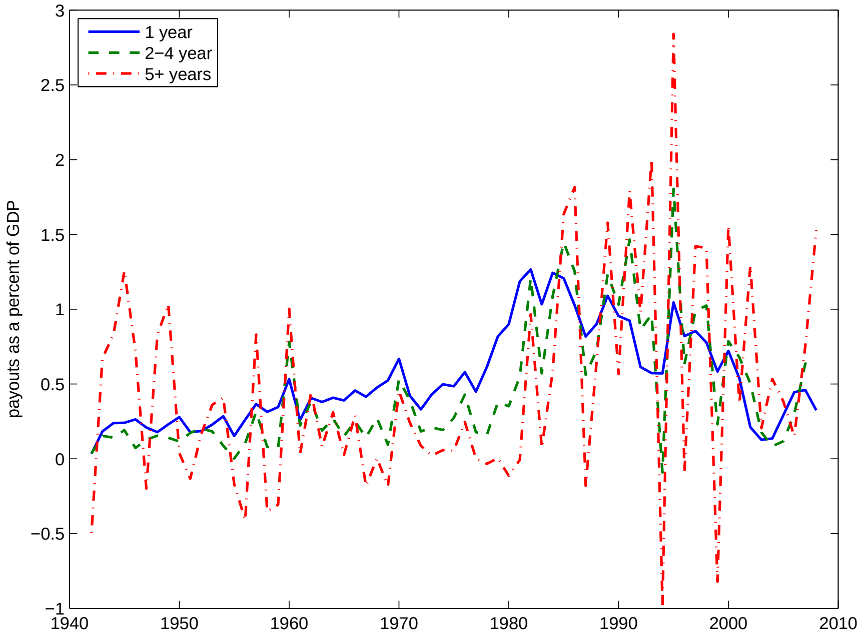

Figure 1: Decomposition of the Nominal Payouts by Maturity of Obligation

The line labeled '1 year' is $100 \times \tilde{r}^1_{t-1,t}\frac{B^1_{t-1}}{Y_{t-1}}$; the line labeled '2-4 years' is $100 \times \sum_{j=2}^{4} \tilde{r}^j_{t-1,t}\frac{B^j_{t-1}}{Y_{t-1}}$; and the line labeled '5+ years' is $100 \times \sum_{j=5}^{n} \tilde{r}^j_{t-1,t}\frac{B^j_{t-1}}{Y_{t-1}}$

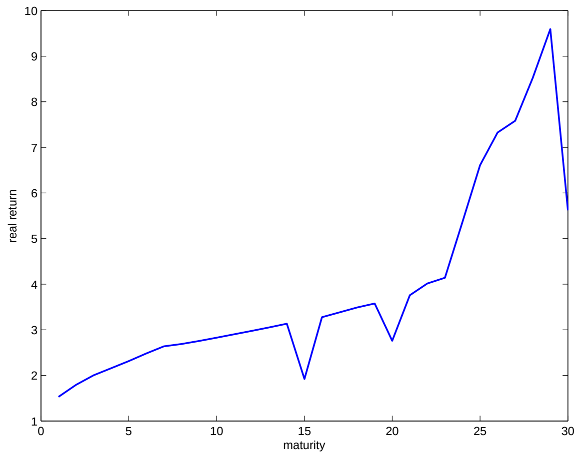

Figure 2: Mean Real Holding Period Returns by Maturity

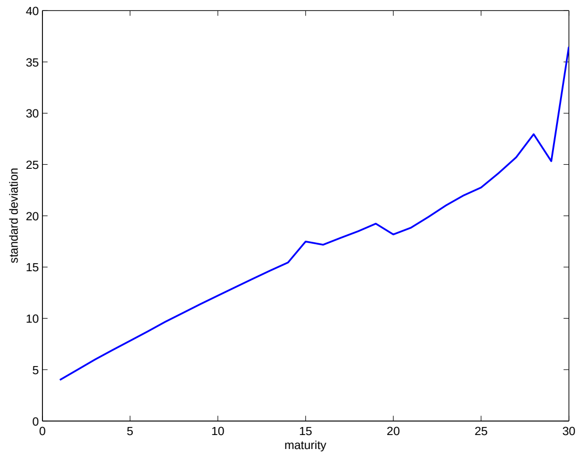

Figure 3: Standard Deviation of the Real Holding Period Returns by Maturity

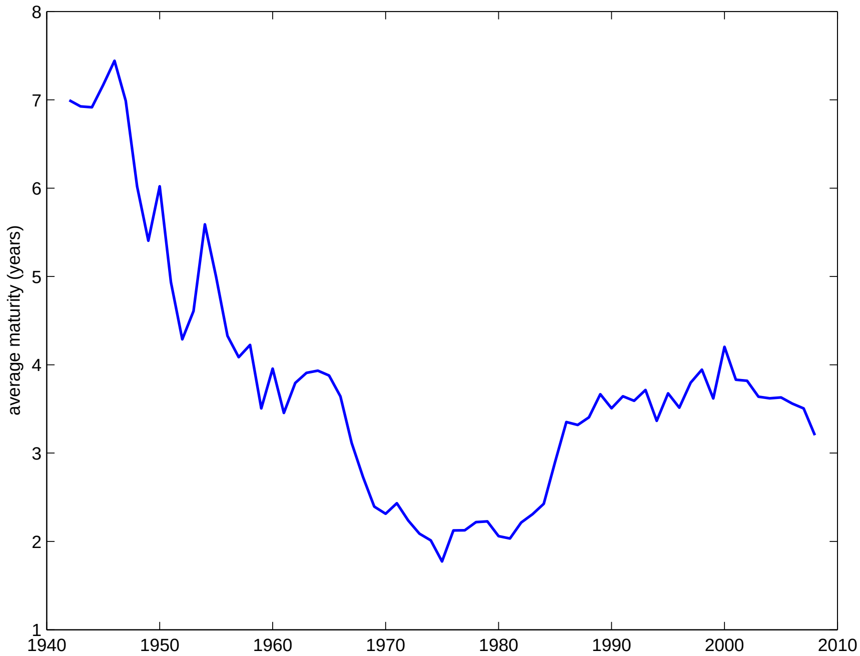

Figure 4: Average maturity, in years, for the Federal debt

three decades, reaching a trough in 1974 at 11.3 percent. After the deficits of the 1980s, it peaked again in 1993 at 42 percent. It fell below 30 percent during the Clinton administration, but by December 2008 it had climbed back to 37.8 percent.

What contributions did inflation, growth, and compound interest make to the evolution of the debt-GDP ratio depicted in figure 5? To prepare the decompositions reported in table 1, we take $\frac{B_{t-\tau}+\bar{B}_{t-\tau}}{Y_{t-\tau}}$ as an initial condition at time $t-\tau$ and iterate on (2) to arrive at:

$$\begin{aligned}\frac{B_t+\bar{B}_t}{Y_t} &= \sum_{s=0}^{\tau-1}\left[\sum_{j=1}^{n}\tilde{r}^j_{t-s-1,t-s}\frac{B^j_{t-s-1}}{Y_{t-s-1}} - (\pi_{t-s-1,t-s}+g_{t-s-1,t-s})\frac{B_{t-s-1}}{Y_{t-s-1}}\right.\\ &+ \left.\sum_{j=1}^{n}\bar{r}^j_{t-s-1,t-s}\frac{\bar{B}^j_{t-s-1}}{Y_{t-s-1}} - g_{t-s-1,t-s}\frac{\bar{B}_{t-s-1}}{Y_{t-s-1}} + \frac{\text{def}_{t-s}}{Y_{t-s}}\right] + \frac{B_{t-\tau}+\bar{B}_{t-\tau}}{Y_{t-\tau}}. \qquad (3)\end{aligned}$$

Figure 6 and table 1 reports elements of a decomposition based on equation (3). In particular, for various values of t and τ, table 1 reports decompositions of the debt-GDP increments $\frac{B_t+\bar{B}_t}{Y_t} - \frac{B_{t-\tau}+\bar{B}_{t-\tau}}{Y_{t-\tau}}$ into components attributable to (i) the maturity structure of interest payments, (ii) inflation, (iii) GDP growth, and (iv) the primary deficit.

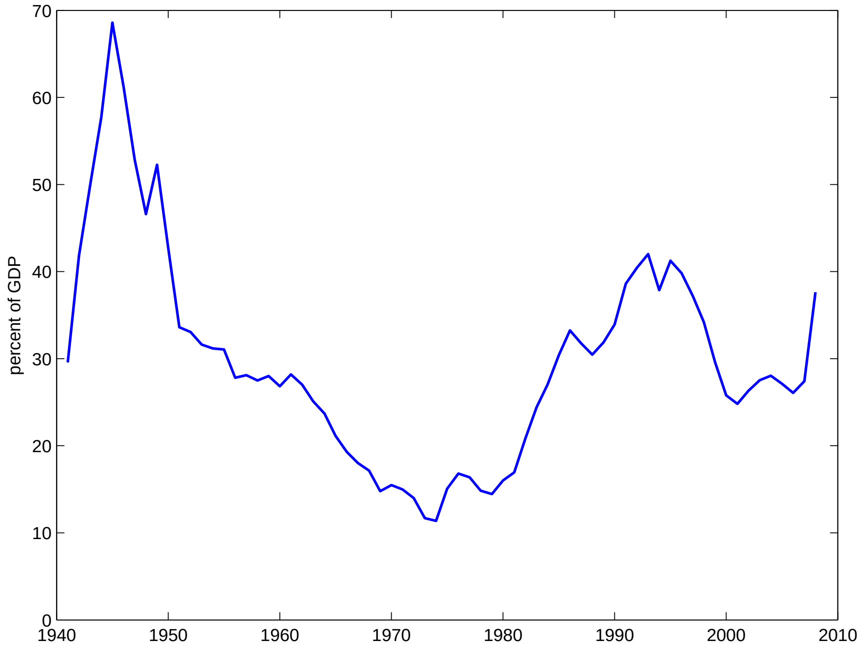

Figure 5: Ratio of Market Value of Public Debt Held by Private Investors to GDP

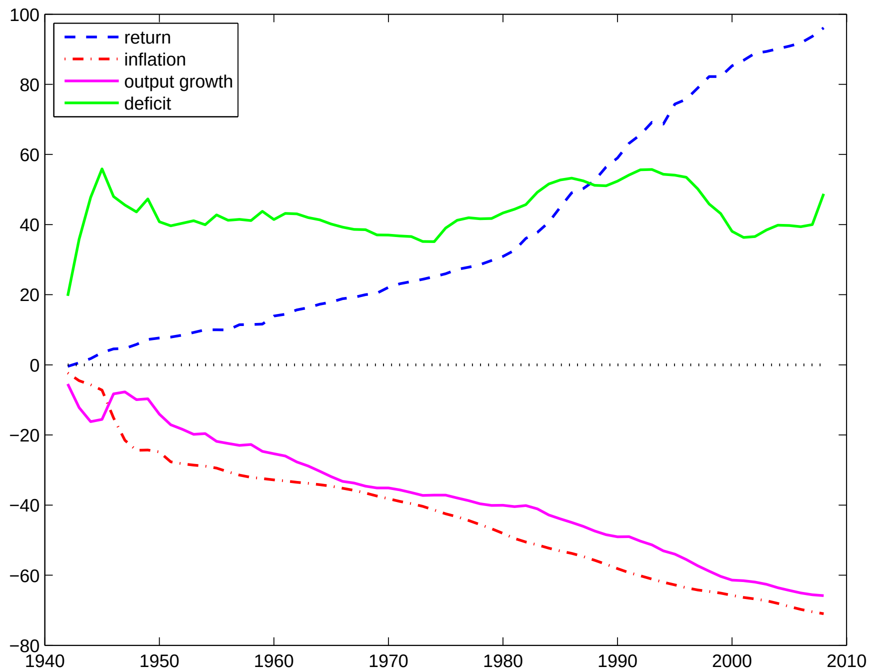

Figure 6: Cumulative sum of the components of the change in the debt to GDP ratio

Period		Debt to GDP			Nominal Return on Debt					Inflation				GDP Growth					Deficit to GDP
					Nominal Bonds				TIPS	Nominal Bonds				Nominal Bonds				TIPS	
		start	end	diff	all	1	2-4	5+		all	1	2-4	5+	all	1	2-4	5+		
1941	1945	29.6	66.2	36.6	3.5					-7.2				-15.6					55.9
						0.7	0.5	2.3			-1.6	-1.2	-4.4		-3.3	-2.6	-9.7		
1945	1974	66.2	11.3	-54.9	21.7					-34.2				-21.6					-20.8
						10.1	6.1	5.5			-11.4	-7.0	-15.8		-8.6	-6.7	-6.3		
1974	1993	11.3	42.0	30.7	44.0					-19.7				-14.2					20.6
						16.7	14.7	12.6			-9.3	-6.6	-3.8		-6.4	-4.8	-3.0		
1993	2000	42.0	25.9	-16.2	16.1				0.1	-4.6				-10.0				-0.1	-17.7
						5.4	5.4	5.3			-1.7	-1.7	-1.2		-3.8	-3.6	-2.7		
2000	2008	25.9	37.8	11.9	10.9				0.2	-5.2				-4.5				-0.2	10.7
						2.5	3.1	5.2			-2.1	-1.6	-1.5		-1.8	-1.4	-1.3		
1972	1981	13.9	16.6	2.8	8.9					-9.9				-4.0					7.8
						6.0	2.7	0.2			-5.4	-3.1	-1.4		-2.2	-1.3	-0.5		
1981	1993	16.6	42.0	25.4	36.5					-11.6				-10.9					11.3
						11.6	12.4	12.5			-4.9	-4.0	-2.7		-4.6	-3.8	-2.6		
1981	2008	16.6	37.8	21.1	63.5				0.4	-21.4				-25.4				-0.3	4.4
						19.6	20.9	23.0			-8.7	-7.3	-5.5		-10.1	-8.7	-6.6		
1941	2008	29.6	37.8	8.2	96.1				0.4	-71.0				-65.9				-0.3	48.8
						35.4	29.8	30.9			-26.1	-18.1	-26.8		-23.7	-19.1	-23.0		
1945	2008	66.2	37.8	-28.4	92.6				0.4	-63.8				-50.3				-0.3	-7.1
						34.7	29.3	28.7			-24.5	-16.9	-22.4		-20.5	-16.5	-13.3		

Table 1: Contributions to Changes in the Debt-to-GDP Ratio

Figure 7 shows the contributions to $\frac{B_t+\bar{B}_t}{Y_t} - \frac{B_{t-1}+\bar{B}_{t-1}}{Y_{t-1}}$ depicted on the right side of (2). The top left panel shows $100 \times \sum_{j=1}^{n} \tilde{r}^j_{t-1,t} \frac{B^j_{t-1}}{Y_{t-1}}$; the top right panel shows $-100 \times \pi_{t-1,t} \frac{B_{t-1}}{Y_{t-1}}$; the bottom left panel shows $-100 \times g_{t-1,t} \frac{B_{t-1}}{Y_{t-1}}$; and the bottom right panel shows $100 \times \frac{\text{def}_t}{Y_t}$. The nominal returns series plotted in the top left panel is the sum of the three series plotted in figure 1.

Figure 8 plots the inflation rate, the growth rate of real GDP, and the value weighted return on the government's debt portfolio. For the first half of the sample, the growth rate of GDP exceeded the return on the debt, while in the second half of the sample, the return on the government debt exceeded the growth rate.

Table 1 and figures 6, 7, and 8 reveal the following patterns in the way that the U.S. grew, inflated, and paid its way toward higher or lower B/Y ratios:

1. From 1945 to 1974, the debt to GDP ratio fell from 66.2 to 11.3. Of this 54.9 percentage drop,
   (a) 12.5 was due to negative real returns on debt via inflation. This largely (10.3 out 12.5) hit the long-term bond holders. The average maturity of the debt was around 7 years right after WWII.
   (b) 21.6 was due to growth in real GDP.
   (c) 20.8 was due to running primary surpluses
2. During the 1970s, the U.S. continued to inflate away part of the debt, but the magnitudes were small.
   (a) Long term bond holders received negative real returns, but since there was not much debt outstanding (B/Y was less than .2) and the average maturity of the debt was low (around 2 years), the government was unable to nail the long-term bond holders as it had done immediately after WWII.
   (b) B/Y continued to grow during the 1970s in spite of the government inflating away part of the debt. The causes were insufficiently rapid real GDP growth and primary deficits.
3. During the Reagan-Bush (41) years (1981-1992),[7] the debt to GDP ratio grew from 16.6 in 1982 to 42.0 in 1993– an increase of 25.4 percent.
   (a) Almost half of this increase (11.3) came from primary deficits.
   (b) Despite strong GDP growth, B/Y grew by more than the primary deficits due to large real returns paid to bond holders. Returns to long-term bond holders account for nearly 10.0 of the 25.4 increase. Thus, while long-term bondholders were heavily taxed by inflation after WWII, they did very well when Volcker brought inflation down during the early 1980s.

[7] The 41 stands for the forty-first president.

Figure 7: Contributions to Changes in the Debt to GDP Ratio

In this figure, the top left panel shows $100 \times \sum_{j=1}^{n} \tilde{r}_{t-1,t}^{j} \frac{B_{t-1}^{j}}{Y_{t-1}}$; the top right panel shows $-100 \times \pi_{t-1,t} \frac{B_{t-1}}{Y_{t-1}}$; the bottom left panel shows $-100 \times g_{t-1,t} \frac{B_{t-1}}{Y_{t-1}}$; and the bottom right panel shows $100 \times \frac{\mathrm{def}_t}{Y_t}$.

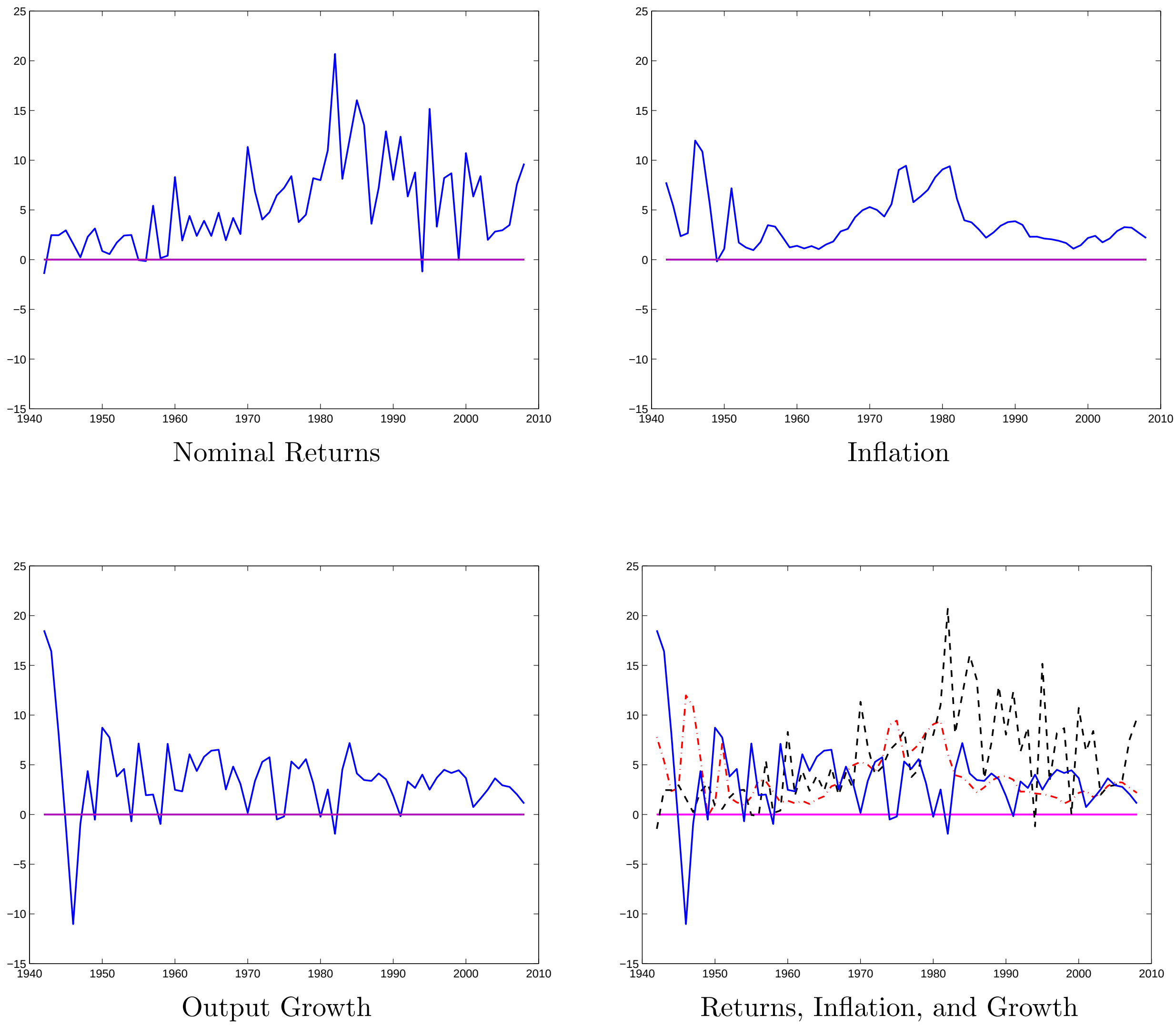

Figure 8: Return on Government Debt, Inflation, and GDP Growth Rate

Top left panel, value weighted return on the government's debt portfolio; top right, the inflation rate, bottom left, the growth rate of real GDP. In the bottom right plot, the solid blue line is the growth rate in real GDP, the red dot-dashed lined is the inflation rate, and the black dashed line is the value-weighted nominal return on the government's portfolio of debt.

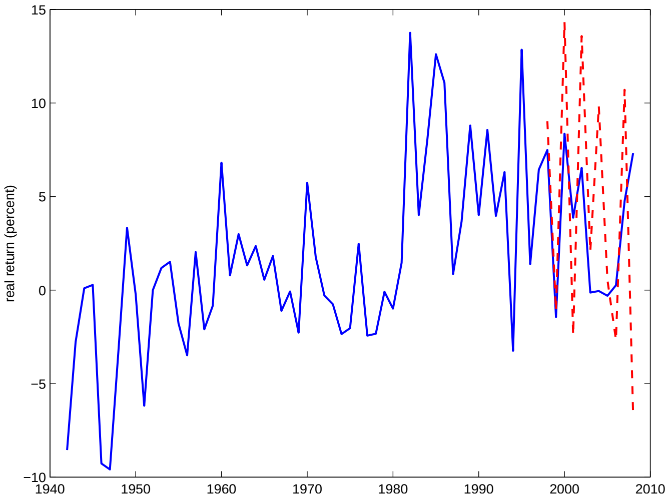

Figure 9: One Year Holding Period Real Valued-Weighted Returns of Nominal Debt and Inflation-Protected Debt

Note: The solid blue line is the value-weighted average return on the nominal portion of the debt, namely,

$$\frac{\sum_{j=1}^{n}(\tilde{r}_{t-1,t}^{j} - \pi_{t-1,t})B_{t-1}^{j}}{\sum_{j=1}^{n} B_{t-1}^{j}}.$$

The dashed red line is the value-weighted average return on the TIPS portion of the debt, namely,

$$\frac{\sum_{j=1}^{n} \bar{r}_{t-1,t}^{j}\bar{B}_{t-1}^{j}}{\sum_{j=1}^{n} \bar{B}_{t-1}^{j}}.$$

4. The reduction in B/Y that occurred during the Clinton years (1993-2000) was largely driven by primary surpluses. Real returns to bond holders approximately offset the contribution from GDP growth.

5. During the Bush (43) years (2001-2008),[8] primary deficits largely fueled growth in B/Y. As in the previous decade, real returns to bond holders approximately offset GDP growth.

Figure 9 plots, $\frac{\sum_{j=1}^{n}(\tilde{r}_{t-1,t}^{j}-\pi_{t-1,t})B_{t-1}^{j}}{\sum_{j=1}^{n} B_{t-1}^{j}}$ and $\frac{\sum_{j=1}^{n} \bar{r}_{t-1,t}^{j}\bar{B}_{t-1}^{j}}{\sum_{j=1}^{n} \bar{B}_{t-1}^{j}}$, which are the value-weighted real one-year holding period returns on the government's portfolio of nominal and inflation-protected debt, respectively. These two series are quite volatile. The average annual return on the nominal portion of the debt over the entire time period from 1942 to 2008 was 1.7 percent with a standard deviation of 4.9 percent. Figure 9 reveals three especially striking outcomes:

1. There were large negative returns immediately after World War II.

[8] The 43 stands for the forty-third president.

Variable	Mean	Std Dev
Nominal Return on Nominal Debt	5.47	4.53
Real Return on Nominal Debt	1.69	4.87
Inflation	3.77	2.67
Nominal GDP growth	7.09	3.77
Real GDP growth	3.28	3.43
100× Deficit to GDP Ratio	1.07	5.40
Real Return on TIPS (1998-2008)	4.29	7.36
Real Return on Nominal Debt (1998-2008)	3.33	3.74

Table 2: Means and Standard Deviations of Components to Debt-to-GDP Dynamics: 1942-2008

2. There were large positive returns in the early 1980s after Volcker brought down inflation.[9]

3. Annual real returns were considerably more volatile in the period between 1980 and 2006 – a period of low volatility in GDP growth often described as the Great Moderation.

We see in table 2 that the average growth rate of the real GDP exceeds the sum the average real return paid to the government's creditors and the average deficit-to-GDP ratio. Finally it is interesting to note that since the introduction of TIPS, their returns have on average exceeded those of the nominal debt. For the TIPS the real return for the period from 1998 to 2008 is 4.3 percent with a standard deviation 7.4. For the nominal portion of the debt over this ten-year period, the real return was 3.3 percent with a standard deviation 3.7.

5 Conclusion

From 1946 to 1974, the United States reduced the debt to GDP ratio far below what it had been immediately after World War II. During these three decades, the U.S. managed to reduce its debt-to-GDP ratio by 85 percent by a mixture of negative real returns on its bonds, primary surpluses, and rapidly growing real income. Only about 20 percent of the decline in the debt-GDP came from using inflation to deliver negative returns to bond-holders. The remaining 80 percent was split about equally between growth in GDP and running net-of-interest surpluses.

The mix has changed. Since 1974, for the most part, government creditors have on average been paid positive, though highly volatile, real returns. In particular, during the early 1980s, when, perhaps unexpectedly, Paul Volcker brought down inflation, government bondholders earned positive real returns that outpaced the growth in real GDP, increasing the debt-to-GDP ratio beyond what would have been implied by the Reagan-era primary deficits themselves. The debt paydown

[9]It is interesting to compare these outcomes with predictions of Lucas and Stokey's (1983) model of tax smoothing, according to which government debt pays low returns when there are high government expenditure shocks. See Berndt, Lustig, and Yeltekin (2009) for an empirical study and also Lustig, Sleet, and Yeltekin (2008).

of the 1990s was largely driven by years of primary surpluses with the positive real interest paid to bondholders roughly offsetting growth in real GDP.

We get these conclusions by manipulating the government's period-by-period budget constraint. Directly computing real returns on government debts of various maturities lets us accurately measure contributions to the evolution of the debt-to-GDP ratio. In addition, we claim to have reverse engineered the question answered by the government's series on interest payments, whose intertemporal properties differ from ours substantially.

The Congressional Budget Office estimates that the debt-to-GDP ratio will return to World War II levels by the end of 2011 as a consequence of recent large primary deficits and drops in GDP growth.[10] This has reawakened concerns that rising government interest payments could eventually unleash inflation or other painful fiscal readjustments via 'unpleasant monetarist arithmetic' (Sargent and Wallace (1981)).[11] A key element of that unpleasant arithmetic is the ratio of interest payments on the government debt to government expenditures or GDP. So to frame the tradeoffs and risks facing the United States, it is important to account appropriately for the substantial interest rate risks that the U.S. government shares with its creditors.

[10]See table 1.1 on page 2 of "The Budget and Economic Outlook: An Update," *Congressional Budget Office*, August 2009.

[11]See for example, Edward Andrews' article in the November 22, 2009 New York *Times* "Payback Time: Wave of Debt Payments Facing U.S. Government," and Martin Feldstein's op-ed in the April 19, 2009 *Financial Times* "Inflation is Looming on America's Horizon."

A Good accounting

At each date t, we compute the number of dollars the government has promised for each date $t+j$ in the future. We regard a coupon bond consisting of a stream of promised coupons plus an ultimate principal payment as a bundle of zero coupon bonds of different maturities. We price it by unbundling it into a set of component zero-coupon bonds, one for each date at which a coupon on the bond is due, valuing each such component individually, then adding up the value of the components. In other words, we strip the coupons from the bond and price the bond as a weighted sum of zero coupon bonds of maturities $j = 1, 2, ..., n$. The market and the government already do this. Prestripped coupon bonds are routinely traded.

We treat nominal bonds and inflation-indexed bonds separately. For nominal bonds, let s^t_{t+j} be the number of time $t+j$ dollars that the government has promised to deliver, as of time t. To compute s^t_{t+j} from historical data, we add up all of the dollar principal-plus-coupon payments that the government has promised to deliver at date $t+j$ as of date t. We let $\bar{s}^t_{t+j}$ be the number of inflation protected $t+j$ dollars, or time $t+j$ goods, that the government has promised to deliver, as of time t.

Since zero-coupon bond prices were not directly observable until prestripped coupon bonds were introduced in 1985, we extract the nominal implicit forward rates from government bond price data. We then convert these nominal forward rates on government debt into prices of claims on future dollars. Let q^t_{t+j} be the number of time t dollars that it takes to buy a dollar at time $t+j$:

$$q^t_{t+j} = \frac{1}{(1+\rho_{jt})^j} \approx \exp(-j\rho_{jt})$$

where ρ_{jt} is the time t yield to maturity on bonds with j periods to maturity. The *yield curve* at time t is a graph of yield to maturity ρ_{jt} against maturity j. The vector $\{q^t_{t+j}\}^n_{j=1}$, where n is the longest maturity outstanding, prices all nominal zero coupon bonds at t. To convert t dollars to goods we use

$$v_t = \frac{1}{p_t}$$

where p_t is the price level in base year 2005 dollars, and v_t is the value of currency measured in goods per dollar.

For inflation-protected bonds (TIPS), let $\bar{s}^t_{t+j}$ be the number of time $t+j$ goods that the government promises to deliver at time t. For indexed debt, when we add up the principal and coupon payments that the government has promised to deliver at date $t+j$ as of date t, we must adjust for past realizations of inflation in ways consistent with the rules governing TIPS.

To compute a real price of a promise, sold at time t, of goods at time $t+j$, $\bar{\rho}_{jt}$, we construct a real yield to maturity as

$$\bar{\rho}_{jt} = \rho_{jt} - \pi_{t-1,t}$$

where $\pi_{t-1,t}$ is the inflation rate from $t-1$ to t realized at t. We appeal to a random walk model for inflation to justify this way of estimating real yields to maturities.[12] We then compute $\bar{q}^t_{t+j}$, the number of time t goods that it takes to purchase a time $t+j$ good, by:

$$\bar{q}^t_{t+j} \quad = \quad \frac{1}{(1+\bar{\rho}_{jt})^j} \approx \exp(-j\bar{\rho}_{jt}).$$

The total real value of government debt issued in period t equals

$$v_t \sum_{j=1}^{n} q^t_{t+j} s^t_{t+j} + \sum_{j=1}^{n} \bar{q}^t_{t+j} \bar{s}^t_{t+j}.$$

The first term is the real value of the nominal debt, computed by multiplying the number of time $t+j$ dollars that the government has sold, s^t_{t+j}, by their price in terms of time t dollars, q^t_{t+j}, summing over all outstanding bonds, $j = 1, \ldots, n$, and then converting from dollars to goods by multiplying by v_t. The second term is the value of the inflation-protected debt, computed by multiplying the number of time $t+j$ goods that the government has promised, $\bar{s}^t_{t+j}$, by their price in terms of time t goods, $\bar{q}^t_{t+j}$, and then summing over $j = 1, \ldots, n$.

A.1 Constraint on debt management (or open market) policy

At a given date t, the government faces the following constraint on *debt management* or *open market* operations

$$v_t \sum_{j=1}^{n} q^t_{t+j}(s^t_{t+j} - \hat{s}^t_{t+j}) + \sum_{j=1}^{n} \bar{q}^t_{t+j}(\bar{s}^t_{t+j} - \tilde{s}^t_{t+j}) = 0, \tag{4}$$

where $\{\hat{s}^t_{t+j}\}^n_{j=1}$ is an alternative portfolio of nominal claims and $\{\tilde{s}^t_{t+j}\}^n_{j=1}$ is an alternative portfolio of real claims. This equation expresses the restriction that the total value of government debt is set by the government's funding requirements, which are determined by obligations stemming from its borrowing in the past and from its current net-of-interest deficit.

A.2 Government budget constraint, a.k.a. the law of motion for debt

Let def_t be the government's real net-of-interest budget deficit, measured in units of time t goods. The government's time t budget constraint is:

$$v_t \sum_{j=1}^{n} q^t_{t+j} s^t_{t+j} + \sum_{j=1}^{n} \bar{q}^t_{t+j} \bar{s}^t_{t+j} = v_t \sum_{j=1}^{n} q^t_{t+j-1} s^{t-1}_{t+j-1} + \sum_{j=1}^{n} \bar{q}^t_{t+j-1} \bar{s}^{t-1}_{t+j-1} + \text{def}_t, \tag{5}$$

where it is understood that $q^t_t = 1$ and $\bar{q}^t_t = v_t$. The left hand side of equation (5) is the real value of the interest bearing debt at the end of period t. The right side of equation (5) is the sum of the real value of the primary deficit def_t and the real value of the outstanding debt that the government owes at the beginning of the period, which in turn is simply the real value this period

[12] See Atkeson and Ohanian (2001) and Stock and Watson (2006) for evidence that a random walk model is a good approximation to inflation in the U.S. since WWII.

of outstanding promises to deliver future dollars s^{t-1}_{t-1+j} and goods $\bar{s}^{t-1}_{t-1+j}$ that the government issued last period.

To attain the government constraint in the form of equation (2) in the body of our text, we simply rearrange (5)

$$\begin{aligned}
\frac{\sum_{j=1}^{n} v_t q^t_{t+j} s^t_{t+j} + \sum_{j=1}^{n} \bar{q}^t_{t+j} \bar{s}^t_{t+j}}{Y_t} &= \sum_{j=1}^{n} \Big(\frac{v_t}{v_{t-1}} \frac{q^t_{t+j-1}}{q^{t-1}_{t+j-1}} - 1 - g_{t-1,t} \Big) \frac{v_{t-1} q^{t-1}_{t+j-1} s^{t-1}_{t+j-1}}{Y_{t-1}} + \\
&+ \sum_{j=1}^{n} \Big(\frac{\bar{q}^t_{t+j-1}}{\bar{q}^{t-1}_{t+j-1}} - 1 - g_{t-1,t} \Big) \frac{\bar{q}^{t-1}_{t+j-1} \bar{s}^{t-1}_{t+j-1}}{Y_{t-1}} \\
&+ \frac{\text{def}_t}{Y_t} \\
&+ \frac{\sum_{j=1}^{n} v_{t-1} q^{t-1}_{t+j-1} s^{t-1}_{t+j-1} + \sum_{j=1}^{n} \bar{q}^{t-1}_{t+j-1} \bar{s}^{t-1}_{t+j-1}}{Y_{t-1}}. \qquad (6)
\end{aligned}$$

To see that this equation is equivalent with (2), we use the definitions

$$v_{t-1} q^{t-1}_{t+j-1} s^{t-1}_{t+j-1} = B^j_{t-1} \qquad (7)$$

$$\bar{q}^{t-1}_{t+j-1} \bar{s}^{t-1}_{t+j-1} = \bar{B}^j_{t-1} \qquad (8)$$

$$B_{t-1} = \sum_{j=1}^{n} B^j_{t-1} \qquad (9)$$

$$\bar{B}_{t-1} = \sum_{j=1}^{n} \bar{B}^j_{t-1} \qquad (10)$$

$$\Big(\frac{v_t}{v_{t-1}} \frac{q^t_{t+j-1}}{q^{t-1}_{t+j-1}} - 1 - g_{t-1,t} \Big) = \tilde{r}^j_{t-1,t} - \pi_{t-1,t} - g_{t-1,t} \qquad (11)$$

$$\Big(\frac{\bar{q}^t_{t+j-1}}{\bar{q}^{t-1}_{t+j-1}} - 1 - g_{t-1,t} \Big) = \bar{r}^j_{t-1,t} - g_{t-1,t} \qquad (12)$$

$$\frac{v_t}{v_{t-1}} = 1 + \pi_{t-1,t} \qquad (13)$$

Here B^j_{t-1} and $\bar{B}^j_{t-1}$ defined in (7) and (8) are the real values of nominal and indexed zero-coupon bonds (strips) of maturity j at $t-1$; B_{t-1} and $\bar{B}_{t-1}$ defined in (9) and (10) are the total real values of nominal and indexed debt at $t-1$; $\tilde{r}^j_{t-1,t}$ defined in (11) is the net *nominal* holding period yield between $t-1$ and t on nominal strips of maturity j; $\bar{r}^j_{t-1,t}$ defined in (12) is the net *real* holding period yield between $t-1$ and t on inflation indexed strips of maturity j, and $\pi_{t-1,t}$ is the inflation rate between $t-1$ and t.

A.3 Data

The price and quantity data for nominal bonds are in the CRSP Monthly Government Bond File.[13] The quantity outstanding of the Treasury inflation-protected securities (TIPS) are in December

[13] In the CRSP data set the quantity of publicly held marketable debt only goes back to 1960. We extended this series using data from the *Treasury Bulletin*.

issues of the U.S Treasury's *Monthly Statement of the Public Debt.* For the pre-1970 period, we fit a zero-coupon forward curve from the coupon bond price data via Daniel Waggoner's (1997) cubic spline method. Waggoner fits the zero-coupon one-period forward-rate curve with a cubic spline employing a set of roughness criteria to reduce oscillations in the approximated curve. For 1970 to 2008, we use the nominal and real zero-coupon yield curves computed by Gurkaynak, Sack and Wright (2006, 2008). The value of currency v_t is the inverse of the fourth quarter observation of the GDP price deflator.

As mentioned above, the left side $v_t \sum_{j=1}^{n} q_{t+j}^t s_{t+j}^t + \sum_{j=1}^{n} \bar{q}_{t+j}^t \bar{s}_{t+j}^t$ of equation (5) is the real *market value* of the interest bearing debt at the end of period t. It excludes nonmarketable securities (e.g. savings bonds, special issues to state and local governments), securities held by other government entities (e.g. the Federal Reserve or the Social Security Trust Fund), or agency debt (e.g. Tennessee Valley Authority). While the Treasury typically reports the *par value* of the debt, Seater (1981), Cox and Hirschhorn (1983), Eisner and Peiper (1984), Cox (1985), and Bohn (1992) have calculated series on the market value of the Treasury's portfolio. Our debt series most closely aligns with Seater's (1981) MVPRIV2 series (see his Table 1) and Cox and Hirschhorn's (1983) series "Market value of privately held treasury debt" (see their Table 6).

Eisner and Pieper (1984), Eisner (1986), and Bohn (1992) computed similar measures of the government's interest payments. Rather than computing the terms on the left side of (5) directly, they exploited the inter-temporal budget constraint (1) to compute interest payments $\tilde{r}_{t-1,t}B_{t-1}$ as the change in the market value of debt minus the primary deficit. An advantage of that approach is that there no need to construct the pricing kernels and stripped series; the market value of the debt can be computed directed from the observed prices and quantities outstanding. An advantage of our approach is that since we compute the returns on the debt directly our measure is not sensitive to how the primary deficit is measured. Furthermore, our arithmetic also allows us (a) to account for the different holding period yields on obligations of different maturities and thereby form the decompositions of interest payments in table 1 and figures 1 and 6, (b) to execute the counterfactual debt management experiments described in appendix C, and (c) to dissect the difference between our estimates of the interest costs and those reported by the Treasury. We turn to this last task in appendix B.

B Bad accounting: what the government instead reports as interest payments

As documented earlier by Hall and Sargent (1997), our estimates of the interest paid on U.S. government debt differ substantially from those reported by the government. In this section, we attempt to track down the sources of differences between our way of accounting for interest and the government's. Since they give different answers, these two interest payment series must be asking different questions. Our series answers the question "what interest payments appear in the law of motion over time of real government indebtedness?"[14] What question does the

[14]The law of motion of real government indebtedness is also known as the government budget constraint.

government's interest payment series answer? And how can we compute it in terms of the objects $q, \bar{q}, s, \bar{s}$ defined in appendix A?

We won't get much of an answer by reading government reports. Issues of the Treasury *Bulletin* from 1957 to 1982 contained the following concise description of the government's method for computing its interest expenses:

> The computed annual interest charge represents the amount of interest that would be paid if the interest-bearing issue outstanding at the end of each month or year should remain outstanding for a year at the applicable annual rate of interest. The charge is computed for each issue by applying the appropriate annual interest rate to the amount of the security outstanding on that date.

We interpret these statements to mean that the government computes interest expenses by adding next year's coupon payments to the product of the outstanding principal due next year and the one-period holding period yield on *one-period* pure discount bonds. In this appendix, we shall verify this interpretation by executing this computation with the $q, \bar{q}, s, \bar{s}$ series from appendix A and show that it produces a time series that closely approximates the government's reported series on interest payments. Then we'll try to reverse engineer a question that this series is designed to answer. Finally, we'll describe in detail how the government's concept of interest costs fails to match the concept required by the government's budget constraint (1) or (2) because it treats coupon payments and capital gains improperly.

To cast the government's computations in terms of our notation, it is useful to define the decomposition $s_t^{t-1} = s_t^{t-1}(tb) + s_t^{t-1}(p) + s_t^{t-1}(c)$ where $s_t^{t-1}(tb)$ represents the par value of one-period pure discount treasury bills and notes, $s_t^{t-1}(p)$ denotes the contribution to s_t^{t-1} coming from principal due on longer term bonds that mature at t, and $s_t^{t-1}(c)$ represents coupon payments on longer term bonds accruing at time t.

Then we believe that the government reports the following object as its nominal interest payments at time t:

$$v_t \Big\{ s_t^{t-1}(c) + \Big(1 - q_t^{t-1}\Big)\Big(s_t^{t-1} - s_t^{t-1}(c)\Big)\Big\} + \Big\{ \bar{s}_t^{t-1}(c) + \Big(1 - \bar{q}_t^{t-1}\Big)\Big(\bar{s}_t^{t-1} - \bar{s}_t^{t-1}(c)\Big)\Big\} \tag{14}$$

or

$$v_t \Big\{ s_t^{t-1}(c) + \tilde{r}_{t-1,t}^{1}\Big(s_t^{t-1}(tb) + s_t^{t-1}(p)\Big)\Big\} + \Big\{ \bar{s}_t^{t-1}(c) + \bar{r}_{t-1,t}^{1}\Big(\bar{s}_t^{t-1}(tb) + \bar{s}_t^{t-1}(p)\Big)\Big\} \tag{15}$$

The term $v_t s_t^{t-1}(c)$ in the first expression is the real value of the coupon payments on nominal bonds, while $\bar{s}_t^{t-1}(c)$ is the real value of coupon payments on indexed bonds. The term $v_t\Big(1 - q_t^{t-1}\Big)\Big(s_t^{t-1} - s_t^{t-1}(c)\Big)$ is the real value of all payments *except* coupons, $v_t\Big(s_t^{t-1} - s_t^{t-1}(c)\Big) = v_t\Big(s_t^{t-1}(tb) + s_t^{t-1}(p)\Big)$, promised at $t-1$ to be paid at t multiplied by the one-period holding period yield $\tilde{r}_{t-1,t}^{1} = \Big(1 - q_t^{t-1}\Big)$ on one-period pure discount bills. The two terms in the second pair of braces are the indexed debt counterparts to the preceding two terms.

In figure 10 we plot the government's official interest payments series and our concept (15). We divide both series by the market value of debt. The two series track each other quite closely; the correlation coefficient for the two series is 0.97.

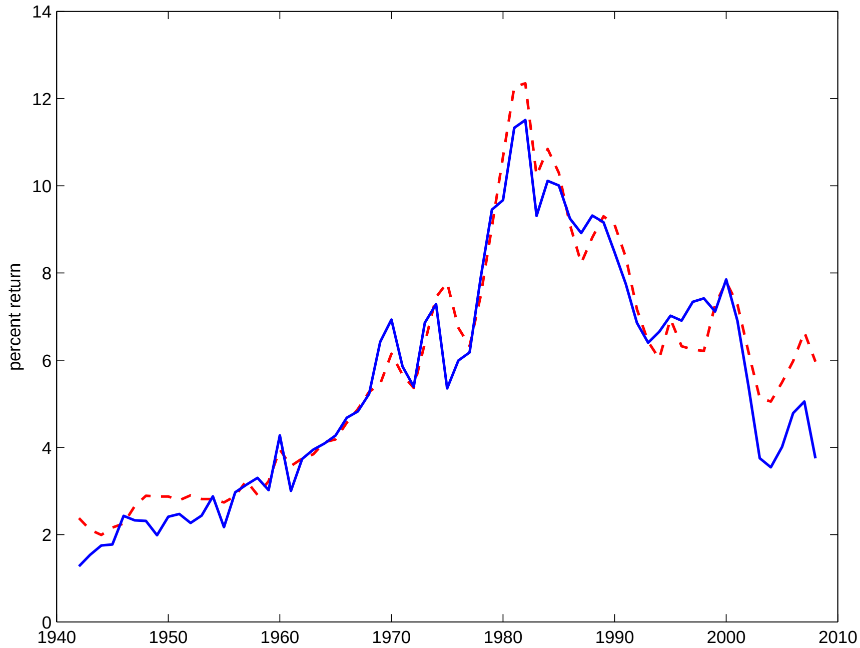

Figure 10: Our Reverse Engineered Estimates of the Government's Reported Interest Payments

The dashed-red line in the officially reported interest costs divided by the market value of debt. The solid blue line is equation (14) divided by the market value of debt.

In Figure 11 we contrast the Federal Government official interest payment series with our interest payment series using annual end of the year data from 1941 to 2008. [15] In this graph, we report both our measure of interest paid (the solid blue line) and the government's reported interest payments (dashed green line) as percentages of the market value of debt. As can be seen in this figure, our series is lower on average and considerably more volatile than the government's. As we report in table 3 the official interest payments average 5.79 percent of the debt while our measure of the real return on the debt averages 1.69. We then subtract the inflation rate from officially reported interest payments (dashed red line). The two series have roughly the same mean (2.02 versus 1.69). Up until the 1980s it appears that much of the difference between the reported series and our series is due to inflation. Post-1980 something else is going on, namely, nominal interest rate risk that, in a lower and less volatile inflation environment, translated into real interest rate risk.

[15]The Federal Government reports its interest payments in two places: the annual budget issued by the Treasury and the NIPA. We use fiscal data from the NIPA Table 3.2 rather than budget data from the Treasury for two reasons. First, the Treasury's reports data for the fiscal year, which runs from October to September while we measure the cost of funds on a calendar year basis. Second, NIPA interest payments (NIPA table 3.2, line 28) exclude interest paid to other government trust funds, such as the Social Security trust fund. Interest on the public debt reported by the Treasury includes interest paid to these trust funds. NIPA interest payments do include interest paid to the Military and Civil Service retirement funds, because the NIPA shows the assets for these funds in the household sector. We net out these payments using data on NIPA table 3.18B, line 24.

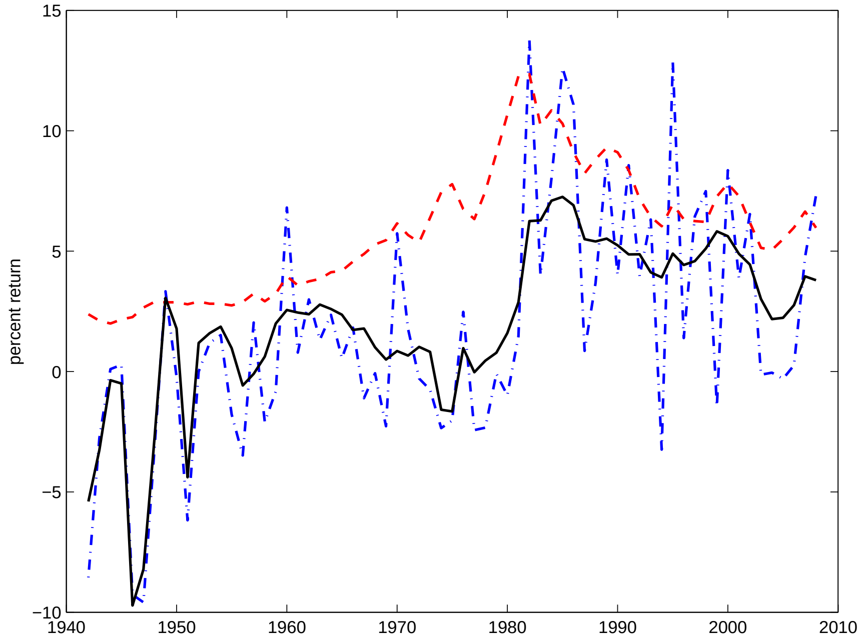

Figure 11: A Comparison of the Official Interest Payments and Our Estimates of Interest Payments

The dot-dashed-blue line is our computed value weighted return of the debt. The dashed-red line is the officially reported interest payments divided by the market value of debt. The solid-black line is the dashed-red line minus the inflation rate.

Variable	Mean	Std Dev
Official Interest/Debt	5.79	2.66
Inflation	3.77	2.67
Official Interest/Debt - inflation	2.02	3.32
Real Return on Nominal Debt	1.69	4.87

Table 3: Means and Standard Deviations of Percent Returns: 1942-2008

B.1 What question is being answered?

To what question is the object computed in (15) the answer? Frankly, this has us stumped. Object (15) mixes apples and oranges – imputed interest and principal repayments – in peculiar ways. The spirit of the calculation seems to be, "let's calculate the total funds the government must devote at time t to servicing its debt," where the devil resides in the details of what one means by *servicing*. An interesting question, might be "given the structure of debt in place at $t-1$, how many dollars of principal plus coupons fall due at t?" But that is not what the government computes either. The real value of the *total* funds that the government has to come up to honor its promises at t is evidently $v_t\Big\{s_t^{t-1}(c) + \Big(s_t^{t-1}(tb) + s_t^{t-1}(p)\Big)\Big\} + \Big\{\bar{s}_t^{t-1}(c) + \Big(\bar{s}_t^{t-1}(tb) + \bar{s}_t^{t-1}(p)\Big) = v_t s_t^{t-1} + \bar{s}_t^{t-1}$. The government seems to want to exclude repayments of principal and to record only enough interest payments to roll over its outstanding principal. Thus, the quantity $v_t s_t^{t-1}(c) + \bar{s}_t^{t-1}(c)$ measures the quantity of real resources that the government has to come up with to make *coupon payments* (but not principal repayments) to bond holders. The government also has to come up with $v_t\Big(s_t^{t-1}(tb) + s_t^{t-1}(p)\Big) + \Big(\bar{s}_t^{t-1}(tb) + \bar{s}_t^{t-1}(p)\Big)$ in repayments of *principal* at time t. Apparently, the government multiplies these principal payments by the appropriate one-period holding period yields for *one-period* zero-coupon bonds $\tilde{r}^1_{t,t-1}, \bar{r}^1_{t-1,t}$, sums the outcomes, and counts that sum as a contribution to its total interest payments.

B.2 Pinpointing the government's accounting error

A symptom of the conceptual error in the government's interest payment series is the absence of yields $\tilde{r}^j_{t-1,t}, \bar{r}^j_{t-1,t}$ for holding periods j exceeding one. Rewriting the government's concept of interest payments (15) as

$$\Big\{v_t s_t^{t-1}(c) + s_t^{t-1}(c)\Big\} + \Big\{v_t \tilde{r}^1_{t-1,t}\Big(s_t^{t-1}(tb) + s_t^{t-1}(p)\Big) + \bar{r}^1_{t-1,t}\Big(\bar{s}_t^{t-1}(tb) + \bar{s}_t^{t-1}(p)\Big)\Big\} \tag{16}$$

can help isolate the source of the government's error in computing its interest costs. This expression reveals the following misconceptions in the government's way of estimating its interest payments:

1. The first term in braces is total coupon payments. But coupon payments should not be viewed as pure interest payments: they are part principal repayments, part interest payments.

2. The second term in braces correctly measures what is properly a *part* of government interest payments according to our budget-constraint-driven definition (1) or (2), namely, the capital gains or losses that the government pays on its *one-period* zero coupon bonds; but ...

3. Expression (16) evidently contains *no* accounting for the capital gains or losses that the government pays on its zero coupon bonds of maturities longer than one-period. One-period holding period yields $\tilde{r}^j_{t-1,t}, \bar{r}^j_{t-1,t}$ and promised coupon payments for maturities j exceeding 1 do not appear in (15).

C Counterfactual debt management

As a way to bring out quantitatively the possibilities that interest rate risk has presented debt management authorities, we perform some artificial counterfactual experiments that study what the time path of B/Y would have been under alternative settings for $\{s^t_{t+j}\}$, *on the assumption* of unchanged $\{v_t, q^t_{t+j}\}$.[16]

We consider alternative debt-management policies in which the government issues only nominal interest bearing debt. The class of feasible financing rules is

$$\frac{q^t_{t+j} s^t_{t+j}}{\text{def}_t + \sum_{j=1}^n q^t_{t+j-1} s^t_{t+j-1}} = f^t_{t+j} \tag{17}$$

$$\sum_{j=1}^n f^t_{t+j} = 1. \tag{18}$$

Evidently f^t_{t+j} is the fraction of the outstanding debt at time t that is due at time $t+j$. Restrictions (17) and (18) follow from the government budget constraint (5).

Let $v_t \sum_{j=1}^n q^t_{t+j-1} s^{t-1}_{t+j-1} \equiv V_t$ be the value of interest bearing government debt at the beginning of period t. Given a policy $f^t_{t+j}, j = 1, \ldots n$, together with observed interest rates, equation (17) can be solved for $s^t_{t+j}, j = 1, 2, \ldots n$:

$$s^t_{t+j} = \left(\frac{f^t_{t+j}}{q^t_{t+j}} \right) (\text{def}_t + V_t).$$

This equation can be solved recursively to build up paths for s^t_{t+j} and decompositions of interest cost under alternative hypothetical debt management rules. For time invariant policies, let

$$f^t_{t+j} = f_j \ \forall\ t.$$

We consider four policies:

1. Bills only: Set $f^t_{t+1} = f_1 = 1$ and $f^t_{t+j} = f_j = 0$ for all $j \neq 1$
2. Tens only: Set $f^t_{t+10} = f_{10} = 1$ and $f^t_{t+j} = f_j = 0$ for all $j \neq 10$.
3. Minimum Variance: Set f_1, f_5 and f_{10} to minimize

$$\sum_{j=1,5,10} \sum_{i=1,5,10} f_j f_i \rho_{i,j} \sigma_i \sigma_j$$

where $\rho_{i,j}$ is the correlation coefficient between the holding period returns of i and j period zero-coupon bonds, and σ_i is the standard deviation of the holding period return for an i period zero-coupon bond.

[16] Aizenman and Marion (2009) run a series of counterfactual experiments holding the maturity structure of the debt fixed at it historical values and varying the inflation rate. In contrast, we hold the inflation rate at its historical rates and vary the maturity structure.

4. Clairvoyance: Set s^t_{t+j} to minimize

$$\sum_{j=1}^{n}\Big(\frac{v_{t+1}}{v_t}\frac{q^{t+1}_{t+j}}{q^t_{t+j}} - 1 - g_{t,t+1}\Big)\frac{v_t q^t_{t+j} s^t_{t+j}}{Y_t}$$

each period. We assume the government knows v_{t+1} and q^{t+1}_{t+j} prior to choosing s^t_{t+j}.

For the first two policies, the entire debt is purchased and resold each period make sure *all* the debt is held in either one-year bills or ten-year zero-coupon bonds (depending on the experiment). For the third and fourth policies, we assume the government can see into the future. Under the third policy, we assume the government knows the variances and covariances of the holding period returns of the different maturities over the sample period (1942-2008) when it computes the weights of the minimum variance portfolio. For our sample period, the time-invariant portfolio shares that minimize the variance of the holding period returns are: $f_1 = 1.26$, $f_5 = -0.35$, $f_{10} = 0.09$. Under the clairvoyance policy, we not only assume that the Treasury can fully anticipate movements in the term structure one year in advance but can also act on this foresight by freely shifting the government policy to minimize its interest costs. While it is unrealistic to expect that the Treasury could ever implement either of these last two polices, they provide 'best-case' or 'upper bound' benchmarks, to compare to the alternative policies.

For all four experiments, we start with the market value of debt at the end of 1941. In figures 12, 14, 16, and 18, we plot the holding period returns for each of these counterfactual portfolios. In figures 13, 15, 17, and 19, we plot the time path of the debt-to-GDP ratio for each portfolios. In table 4 we report for each portfolio the mean and standard deviation of the holding period returns and the terminal debt-to-GDP ratio.

Prior to the Treasury-Federal Reserve Accord in 1951 the path of returns is close to invariant to the choice of portfolio. Examination of figures 12, 14, 13, and 15 indicates that debt management policies weighted toward *longer* maturities would have led to lower interest costs and less accumulation of debt over the period from the Accord until the early 1980s. After the early 1980s, debt-management policies weighted toward *shorter* maturities would have generally lowered interest costs and led to less accumulation of debt. From figure 4 it is clear that the Treasury and Federal Reserve steadily reduced the average maturity of outstanding debt from the 1940s until the early 1970s; they then increased the average maturity during the late 1970s and throughout the 1980s. Our analysis indicates that to have minimized its borrowing costs, the government should have engaged in the opposite strategy.

Over the entire sample, a portfolio of shorter maturity debt would have generated lower borrowing costs, a reduction in the variance of the returns, and a lower 2008 debt-to-GDP ratio than a portfolio of longer maturity debt. Consistent with the plot of the nominal payouts reported in figure 1, we see in figures 12 and 14 that the high volatility of returns in the post-1980 period was largely concentrated in the longer-term securities. The weights for the minimum variance portfolio suggest that the bills-only policy is close the minimum variance portfolio. This suggestion is verified in figures 12 and 16.[17]

[17] Given the ability of nominal debt to act as a hedge to fiscal shocks, it may not be optimal for the government

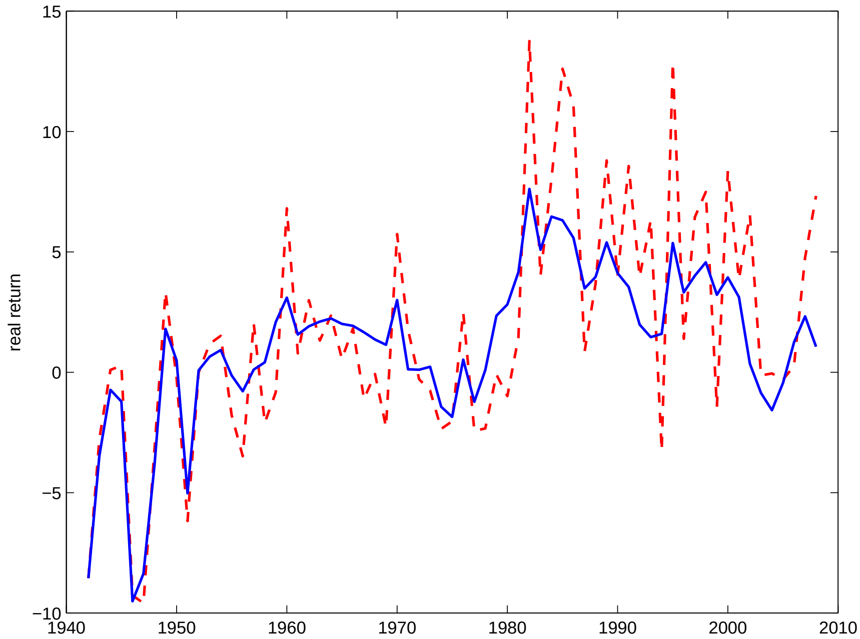

Figure 12: Counterfactual: Ones Only, Returns

Red-dashed line = Historical; Blue-solid line = Counterfactual

Policy	Mean Return	Std Dev Return	Dec 2008 Debt-to-GDP
Actual	1.69	4.87	37.8
Ones Only	1.18	3.27	27.3
Tens Only	2.70	11.43	68.5
Minimum Variance	0.98	3.13	24.4
Clairvoyance	-5.90	10.76	8.9

Table 4: Counterfactual Debt Management Policies

Under the clairvoyance policy we see that even if the Treasury could have minimized its interest costs, returns would have been considerably more volatile than we observed and we would still have a positive debt-to-GDP ratio. The illustrates the limits of debt management to offset deficit spending. Even with the 20/20 hindsight, it would not have been possible to shift all the burden of deficit spending onto bondholders.

to minimize the volatility of returns.

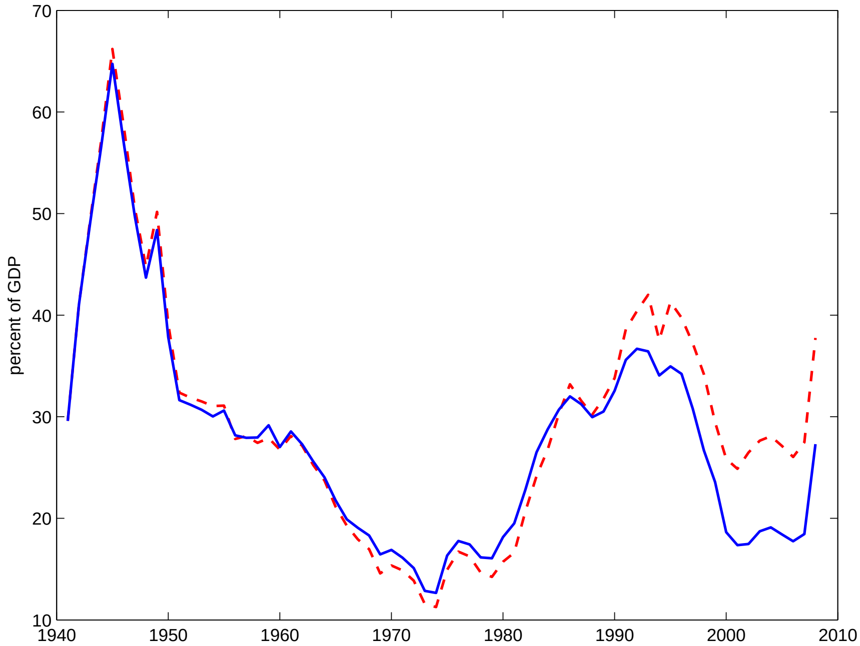

Figure 13: Counterfactual: Ones Only, Debt-to-GDP
Red-dashed line = Historical; Blue-solid line = Counterfactual

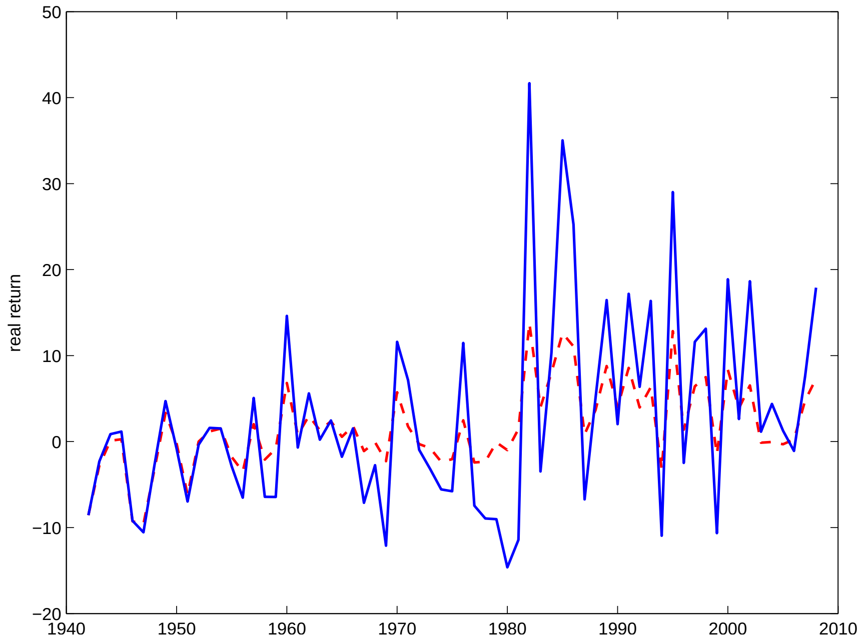

Figure 14: Counterfactual: Tens Only, Returns
Red-dashed line = Historical; Blue-solid line = Counterfactual

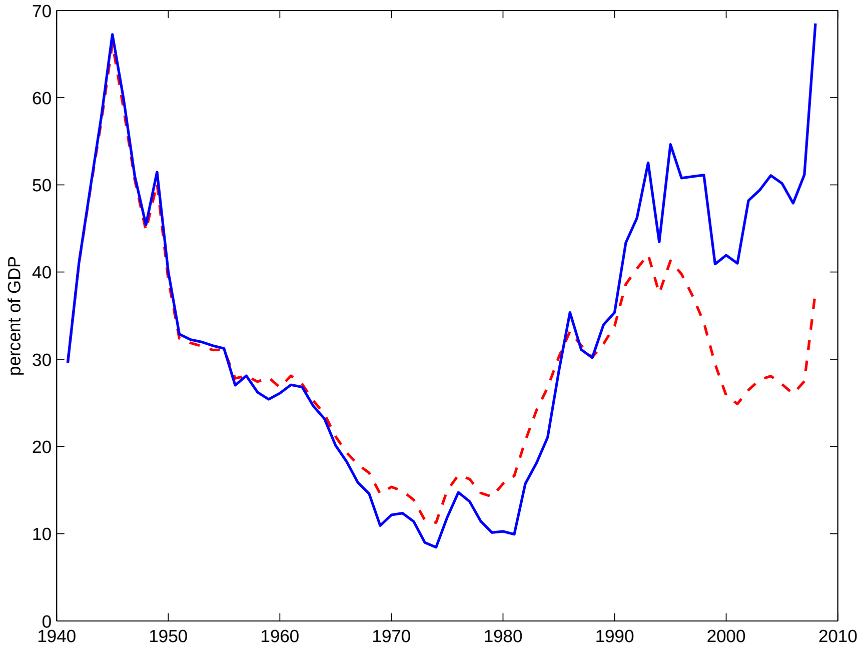

Figure 15: Counterfactual: Tens Only, Debt-to-GDP
Red-dashed line = Historical; Blue-solid line = Counterfactual

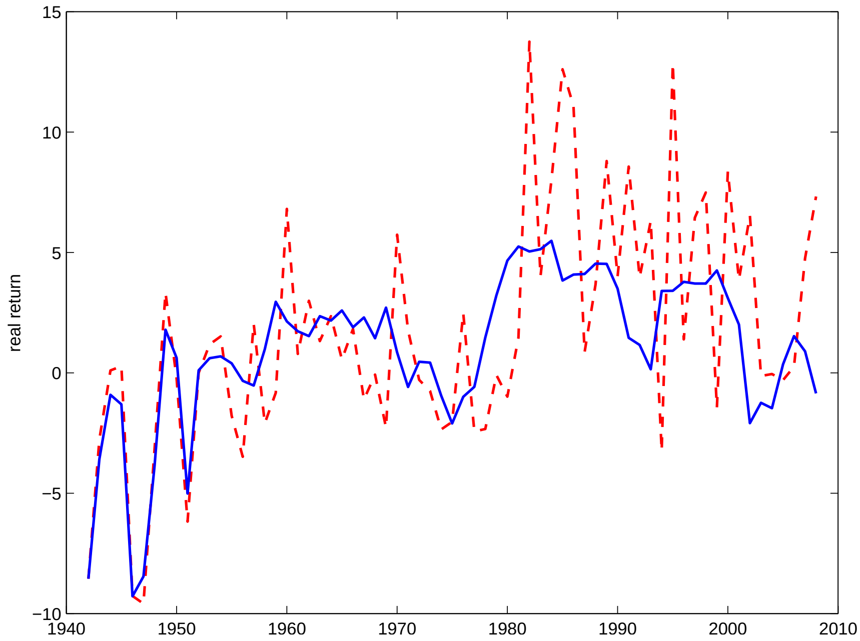

Figure 16: Counterfactual: Minimum Variance, Returns
Red-dashed line = Historical; Blue-solid line = Counterfactual

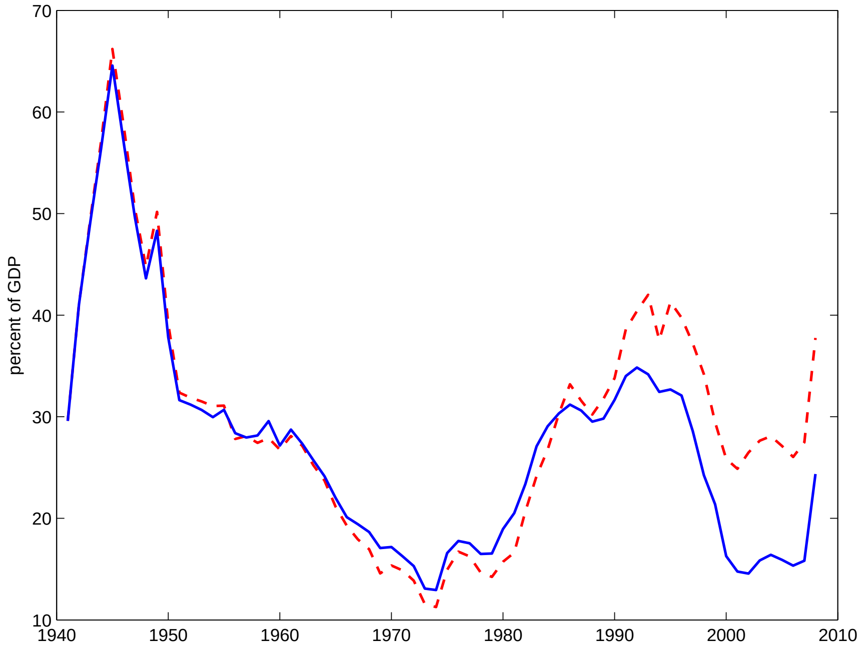

Figure 17: Counterfactual: Minimum Variance, Debt-to-GDP

Red-dashed line = Historical; Blue-solid line = Counterfactual

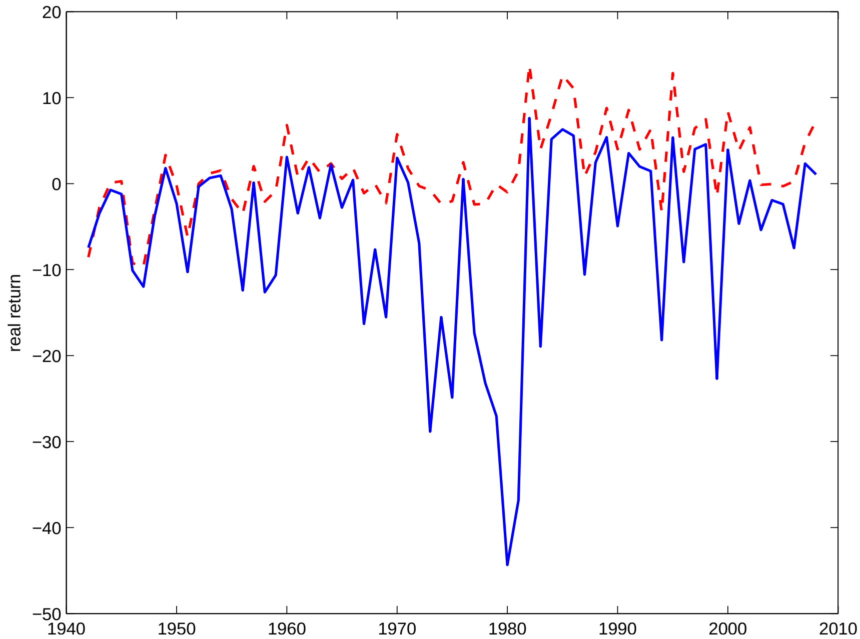

Figure 18: Counterfactual: Clairvoyance, Returns

Red-dashed line = Historical; Blue-solid line = Counterfactual

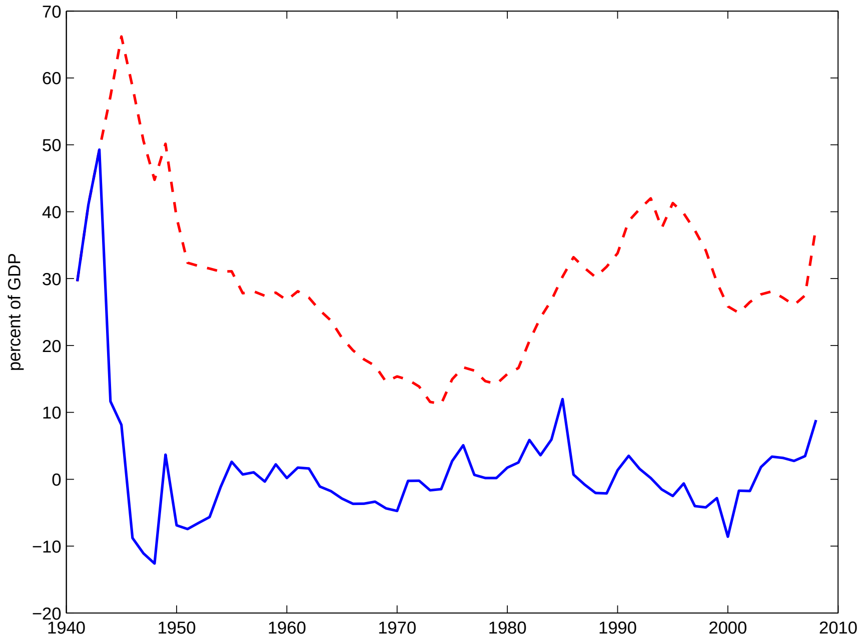

Figure 19: Counterfactual: Clairvoyance, Debt-to-GDP
Red-dashed line = Historical; Blue-solid line = Counterfactual

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