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Author(s)	Watanabe, Koji; Ikai, Takeo; Fukunaga, Kunio
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A Method for Minimal Realizations of Finite Automata via Input-output Responses

Koji WATANABE*, Takeo IKAI** and Kunio FUKUNAGA**

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Representing finite automata (FAs) by means of bilinear state space models over $B(=\{0,1\})$, a linear algebraic approach for FAs is made possible and some concepts of the field of dynamical systems and controls can be introduced to that of FAs.

In this paper, we propose a minimal realization algorithm of deterministic finite automata (DFAs) using Silverman's minimal realization algorithm. Since Silverman's algorithm is built for linear state space models over the real numbers, we extend this algorithm to apply it to FAs over B. Furthermore, we derive a minimal partial realization algorithm of DFAs from finite partial input-output responses of FAs.

1. Introduction

Regarding finite automata (FAs) as discrete time dynamical systems, bilinear state space models of FAs can be obtained by means of the vectorization of states and symbols over $B(=\{0,1\})$ and the parameterization of system characteristics (the state transition function etc.). Based on the state space model representation, a linear algebraic approach for FAs is made possible as well as linear systems over the real numbers (R) in the field of dynamical systems and controls[1]. The concepts such as reachability, observability and canonical decomposition, for example, can be defined for state space models of FAs over B[2,3].

An approach to construct system parameters of a minimal state space model from input-output responses of a dynamical system, called the realization theory, was begun by Kalman[4] in the field of linear dynamical systems over R and the relations among reachability, observability and minimal realizations were clarified. At present, there are many algorithms for minimal realizations. Since these algorithms are built for linear state space models over R, however, some extensions are needed to apply them to FAs. One minimal realization method for deterministic finite automata (DFAs) based on an extended Attasi's algorithm[5] has already proposed[2]. But system parameters of minimal DFAs cannot be obtained directly by that algorithm.

In this paper, we shall propose a novel minimal realization method of DFAs using Silverman's algorithm[6] which is one for linear systems over R and by which system parameters are given directly without solving matrix equations.

In chapter 2, for a given DFA, we show how to construct its state space model over B. Next we define the reachability, observability, characteristic responses and the Hankel matrix for this model.

In chapter 3, we derive a method for minimal realizations of DFAs via Hankel matrices using Silverman's algorithm. Since the models of FAs are built over B as against the Silverman's algorithm over R, extensions of the algorithm are necessary. Then, we propose three algorithms for minimal realizations of FAs which are selected according to the type of a given Hankel matrix. Furthermore, we propose a minimal partial realization algorithm via subHankel matrices constructed from a finite part of input-output responses. In this minimal partial realizations, minimal DFAs are realized in the sense of the identification-in-the-limit defined by Gold[7,8].

* Graduate Student, Department of Computer and Systems Sciences, College of Engineering.
** Department of Computer and Systems Sciences, College of Engineering.

2. State Space Models of Finite Automata

2. 1 State space models

We first introduce an algebraic system called the boolean semiring $(B, +, \cdot)$ (where $B=\{0,1\}$) to derive state space models of FAs. Table 1 shows the addition and multiplication of boolean semiring. These are ordinary addition and multiplication in integers except $1+1=1$.

Table 1 Boolean semiring

addition	multiplication
$0+0=0$	$0 \cdot 0=0$
$0+1=1$	$0 \cdot 1=0$
$1+0=1$	$1 \cdot 0=0$
$1+1=1$	$1 \cdot 1=1$

A FA is formally represented by 5-tuple,

$$M=(Q, \Sigma, \delta, p_0, F) \tag{1}$$

where Q is the set of states, Σ is the set of input symbols, δ is the state transition function, p_0 is the initial state, and F is the set of accepting states. The number of states denoted by $|Q|$ is n and that of symbols $|\Sigma|$ is m.

We show the method to construct state space models of DFAs. First, the state $q_i (\in Q)$ (where $i=1, \cdots, n$) is represented by an n-dimensional unit vector e_i (only i-th component is 1, the rest are 0 and called i-unit vector). The initial state p_0 is expressed by x_0.

Second, when the states q' and q of the state transition function $\delta(q, a_k)=q'$ for an input symbol a_k $(\in \Sigma)$ are represented by i- and j-unit vectors respectively, the j-th column of a square matrix A_{a_k} of order n over B is made of an i-unit vector. A_{a_k} is called a state transition matrix and abbreviated as A_k.

The state equation of a FA over B is defined as,

$$x(t+1)=\sum_{k=1}^{m} u_k(t) A_k x(t) \tag{2}$$

where $x(0)=x_0$, and $u_k(t)$ expresses an input symbol to FAs as follows.

The strings over Σ are discrete time sequences and are inputed to FAs from the time 0 in order. When a_k is inputed at the time t, $u_k(t)$ of Eq.(2) becomes 1 otherwise 0. An input symbol a_k of time t and $u_k(t)$ are considered to be identical hereafter. $x(t+1)$ is given from the state equation (2) by operations over B. In the case of DFAs, however, an operation $1+1=1$ does not occur and then the state vector $x(t)$ is an n-dimensional unit vector.

Next, the set F of accepting states is represented by an n-dimensional vector c. That is, if q_i is a member of F, an i-th component of c is 1 and otherwise 0.

The output equation using the vector c is defined as,

$$y(t)=c^t x(t) \tag{3}$$

where c^t means the transposed vector of c. If the state vector $x(t)$ coincides with one of unit vectors representing accepting states, $y(t)$ becomes 1 which means the acceptance of a input string.

From described above, the state space model of a FA is obtained as follows:

$$\begin{cases} x(t+1)=\sum_{k=1}^{m} u_k(t) A_k x(t) \\ y(t)=c^t x(t) \end{cases} \tag{4}$$

Thus, FAs are expressed by bilinear systems over B. The parameter representations are denoted by $(\{A_k\}, c, x_0)$ (where $k=1, \cdots, m$) corresponding to symbolic representations (1) and A_k's, c and x_0 are called system matrices or system parameters.

Let an input string $w=a_{k_{t-1}} a_{k_{t-2}} \cdots a_{k_1} a_{k_0} (\in \Sigma^*)$ be applied to a FA from right side symbol, then the transition matrix $A(w)$ which corresponds to w is defined as,

$$A(w)=A_{k_{t-1}} A_{k_{t-2}} \cdots A_{k_1} A_{k_0} \tag{5}$$

where a_{k_t} $(\in \Sigma)$ is an input symbol at the time t and A_{k_t} $(\in \{A_1, \cdots, A_m\})$ is the transition matrix for a_{k_t}.

2. 2 Reachability and observability

For FAs considered as bilinear systems, the reachability and observability can be defined in the same manner as linear discrete time systems.

2. 2. 1 Reachability

When there are input strings which cause transitions from the initial state to some state q, the state q is called reachable. That is, when such an input string w as $\delta(p_0, w)=q$ exists, the state q is reachable. If all states are reachable, system parameters $(\{A_k\}, \boldsymbol{x}_0)$ are called completely reachable.

The reachability matrix $R_{n,\infty}$ is constructed by putting n-dimensional vectors $A(w)\boldsymbol{x}_0$ in alphabetical order as follows:

$$R_{n,\infty}=[\boldsymbol{x}_0,\ A_1\boldsymbol{x}_0,\ \cdots,\ A_m\boldsymbol{x}_0,\ A_1A_1\boldsymbol{x}_0,\ \cdots,\ A_mA_1\boldsymbol{x}_0,\ \cdots,\ A(w)\boldsymbol{x}_0,\ \cdots]\ (w\in\Sigma^*). \quad (6)$$

The column $A(w)\boldsymbol{x}_0$ is denoted by the column label w. The number of non-zero row vectors of $R_{n,\infty}$ shows that of reachable states. System parameters $(\{A_k\}, \boldsymbol{x}_0)$ are completely reachable if there is no zero row vector in $R_{n,\infty}$.

2. 2. 2 Observability

When there are strings which cause transitions from some state q to an accepting state, the state q is called observable. That is, when such a string w as $\delta(q, w)\in F$ exists, the state q is observable and, if all states are observable, system parameters $(\{A_k\}, \boldsymbol{c})$ are called completely observable.

The observability matrix $O_{\infty,n}$ is defined as well as the reachability matrix as follows,

$$O_{\infty,n}=\begin{pmatrix} \boldsymbol{c}^t \\ \vdots \\ \boldsymbol{c}^tA(w) \\ \vdots \end{pmatrix}\ (w\in\Sigma^*). \quad (7)$$

The row $\boldsymbol{c}^tA(w)$ of $O_{\infty,n}$ is denoted by the row label w. The number of non-zero column vectors of $O_{\infty,n}$ shows that of observable states. System parameters $(\{A_k\}, \boldsymbol{c})$ are completely observable if there is no zero column vector in $O_{\infty,n}$.

Next we introduce the distinguishability. For some states q_i and q_j, if there is a string w such that $\delta(q_i, w)\in F$ and $\delta(q_j, w)\notin F$, then q_i and q_j are called distinguishable. Since each state q_i corresponds to the i-th column of $O_{\infty,n}$, the states which have different column vectors in $O_{\infty,n}$ are distinguishable.

2. 3 Characteristic responses and Hankel matrix

The general solution of state space models (4) for some input string w is written as follows:

$$\boldsymbol{x}(t)=A(w)\boldsymbol{x}_0,\ y(t)=\boldsymbol{c}^tA(w)\boldsymbol{x}_0 \quad (8)$$

where $w=\varepsilon$ (ε :empty string) at the time 0 and $A(\varepsilon)=E_n$ (E_n: the unit matrix of order n).

Let

$$h(w)=\boldsymbol{c}^tA(w)\boldsymbol{x}_0, \quad (9)$$

then the output sequence:

$$\{h(\varepsilon), h(a_1),\ \cdots,\ h(a_m),\ \cdots,\ h(w), \cdots\}\quad ({}^\forall w\in\Sigma^*) \quad (10)$$

is called the characteristic responses for state space models (4).

We construct an infinite matrix $\mathcal{H}_{\infty,\infty}$ called Hankel matrix from characteristic responses (10) as follows,

$$\mathcal{H}_{\infty,\infty}=\begin{array}{c} \\ \varepsilon \\ \vdots \\ r \\ \vdots \end{array}\begin{array}{c} \begin{array}{cccc} \varepsilon & \cdots & v & \cdots \end{array} \\ \begin{pmatrix} h(\varepsilon) & \cdots & h(v) & \cdots \\ \vdots & & \vdots & \\ h(r) & \cdots & h(rv) & \cdots \\ \vdots & & \vdots & \ddots \end{pmatrix} \end{array} \quad (11)$$

where, $r\ (\in\Sigma^*)$ is a row label and $v\ (\in\Sigma^*)$ is a column label which are arranged in alphabetical order.

The Hankel matrix $\mathcal{H}_{\infty,\infty}$ can be decomposed into the observability matrix $O_{\infty,n}$ and reachability matrix $R_{n,\infty}$ as follows,

$$\begin{aligned} \mathcal{H}_{\infty,\infty} &= \begin{array}{c} \\ \varepsilon \\ \vdots \\ r \\ \vdots \end{array}\begin{array}{c} \begin{array}{cccc} \varepsilon & \cdots & v & \cdots \end{array} \\ \begin{pmatrix} \boldsymbol{c}^t\boldsymbol{x}_0 & \cdots & \boldsymbol{c}^tA(v)\boldsymbol{x}_0 & \cdots \\ \vdots & & \vdots & \\ \boldsymbol{c}^tA(r)\boldsymbol{x}_0 & \cdots & \boldsymbol{c}^tA(r)A(v)\boldsymbol{x}_0 & \cdots \\ \vdots & & \vdots & \ddots \end{pmatrix} \end{array} \\ &= \begin{array}{c} \varepsilon \\ \vdots \\ r \\ \vdots \end{array}\begin{pmatrix} \boldsymbol{c}^t \\ \vdots \\ \boldsymbol{c}^tA(r) \\ \vdots \end{pmatrix} \begin{array}{c} \begin{array}{cccc} \varepsilon & \cdots & v & \cdots \end{array} \\ \begin{pmatrix} \boldsymbol{x}_0 & \cdots & A(v)\boldsymbol{x}_0 & \cdots \end{pmatrix} \end{array} \\ &= O_{\infty,n}R_{n,\infty}. \end{aligned} \quad (12)$$

If a DFA of n states is completely reachable and completely observable and moreover all states are distinguishable (completely distinguishable), that is, there are all kinds of n-dimensional unit column vector in $R_{n,\infty}$ and all column vectors of $O_{\infty,n}$ are non-zero and different from each other, then the DFA is minimal. The Hankel matrix $\mathcal{H}_{\infty,\infty}$, in this case, consists of only n kinds of non-zero column vector including zero column vectors. The number n of different non-zero column vectors of $\mathcal{H}_{\infty,\infty}$ represents that of states of a minimal DFA.

We next define a matrix $\mathcal{H}^{(k)}_{\infty,\infty}$. The (r_i, c_j) component (where r_i and c_j are a row label and a column label respectively) of $\mathcal{H}^{(k)}_{\infty,\infty}$ is the output of FAs for the input string $r_i a_k c_j$. $\mathcal{H}^{(k)}_{\infty,\infty}$ is decomposed as follows,

$$\mathcal{H}^{(k)}_{\infty,\infty} = O_{\infty,n} A_k R_{n,\infty} \tag{13}$$

where A_k is the transition matrix of a DFA of n states for an input symbol a_k and $O_{\infty,n}$ and $R_{n,\infty}$ are its observability and reachability matrices respectively.

2.4 Dual automata

When a FA accepts reverse strings which some FA M accepts, the FA is called a dual automaton of M and denoted by M_{dual}. M_{dual} is ordinarily a non-deterministic finite automaton (NFA) even if M is a DFA. System parameters of M_{dual} are $(\{A_k^t\}, \boldsymbol{x}_0, \boldsymbol{c})$ for $(\{A_k\}, \boldsymbol{c}, \boldsymbol{x}_0)$ of M and the Hankel matrix of M_{dual} is a transpose of that of M.

A DFA which is obtained by the subset construction[9] from the dual automaton of a completely reachable DFA is a minimal DFA.

3. Minimal Realizations and Minimal Partial Realizations of Deterministic Finite Automata

In the field of dynamical systems and controls, constructing system parameters from input-output responses of linear systems is called realizations. In particular, the construction of minimal dimensional system parameters is called minimal realizations.

There are many algorithms of minimal realizations over R, and some minimal realization algorithms for DFAs represented by the state space models over B have already proposed[2,3,10,11,12]. One of them is an extended Attasi's algorithm[5] which uses commutative Hankel matrices (the Hankel matrices whose row and column labels consist of commutative strings) to non-commutative Hankel matrices[13]. But, this algorithm does not give system parameters of a minimal DFA directly. That is, singular matrix equations over B must be solved to obtain system parameters.

In this chapter, we apply Silverman's algorithm to DFAs which gives system parameters of a minimal DFA directly.

Next, we derive a minimal partial realization algorithm of DFAs via finite subHankel matrices constructed from a finite partial sequence of characteristic responses.

3.1 Minimal realizations of DFAs

Silverman's algorithm needs the rank of a given Hankel matrix. But Hankel matrices of FAs are ones over B, then the ordinal definition of the rank over R cannot be applied. Then, since the number of different non-zero column vectors of Hankel matrix is that of states of a minimal DFA, we now define the column rank of a Hankel matrix of a FA as the number of different non-zero column vectors of it. Furthermore, we define the row rank of a Hankel matrix denoted by rank_{row} as the number of different non-zero row vectors of it. Under these definitions, the column rank and row rank do not agree generally. Then, we derive two kinds of algorithms which are selected according to the column rank and the row rank of a given Hankel matrix.

<u>**Algorithm 1**</u>

1. Compute the column rank of a subHankel matrix in turns $\mathcal{H}_{1,1}$, $\mathcal{H}_{2,2}$, $\cdots$ in order to obtain the minimal c which satisfies the following equation,

$$\mathrm{rank}\ \mathcal{H}_{c,c} = \mathrm{rank}\ \mathcal{H}_{c+1,c+1} = \cdots = \bar{n} \tag{14}$$

where $\bar{n}$ is the number of states of a minimal DFA. Next, compute the row rank of the Hankel matrix as well as the column rank of it in order to obtain the minimal r satisfying the following equation,

$$\mathrm{rank}_{row}\ \mathcal{H}_{r,c} = \bar{n}. \tag{15}$$

If the row rank is less than the column rank, r cannot be obtained. In such a case, the realizations are performed by algorithms 2 or 3 described later on.

2. Construct an $r \times c$ subHankel matrix $\mathcal{H}_{r,c}$.
3. $P_{r,\bar{n}}$ is obtained by pulling out different non-zero column vectors from $\mathcal{H}_{r,c}$ in order from the first column of it.
4. Construct $P^{(k)}_{r,\bar{n}}$ from $\mathcal{H}^{(k)}_{r,c}$ by taking out the corresponding column vectors to $P_{r,\bar{n}}$.
5. $\bar{P}_{\bar{n},\bar{n}}$ is obtained by pulling out different row vectors from $P_{r,\bar{n}}$.
6. Construct $\bar{P}^{(k)}_{\bar{n},\bar{n}}$ from $P^{(k)}_{r,\bar{n}}$ by taking out the corresponding column vectors to $\bar{P}_{\bar{n},\bar{n}}$.
7. System parameters of a minimal DFA are obtained as follows:

$$\bar{A}_k = \bar{P}^{-1}_{\bar{n},\bar{n}} \bar{P}^{(k)}_{\bar{n},\bar{n}} \tag{16}$$

$$\bar{\boldsymbol{c}} = (1 \quad 0 \quad \cdots \quad 0)\bar{P}_{\bar{n},\bar{n}} \tag{17}$$

$$\bar{\boldsymbol{x}}_0 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}. \tag{18}$$

Since $\bar{P}^{-1}_{\bar{n},\bar{n}}$ is needed in this algorithm, $\bar{P}_{\bar{n},\bar{n}}$ is regarded as a matrix over R.

Example 1

Let the input-output data(characteristic responses) of a DFA of Fig. 1 be given. This DFA is minimal.

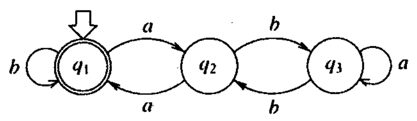

Fig. 1 3 states DFA

1. The Hankel matrix $\mathcal{H}_{\infty,\infty}$ of the DFA of Fig. 1 is as follows,

$$\mathcal{H}_{\infty,\infty} = \begin{array}{cc} & \begin{array}{cccccccc} \varepsilon & a & b & aa & ba & ab & bb & \cdots \end{array} \\ \begin{array}{c} \varepsilon \\ a \\ b \\ aa \\ ba \\ ab \\ bb \\ \vdots \end{array} & \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 1 & \cdots \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & \cdots \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & \cdots \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & \cdots \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \cdots \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \end{array}. \tag{19}$$

2. From Eqs. (14) and (15), $\bar{n}=3$, $r=6$ and $c=5$ are obtained.

3. The subHankel matrix $\mathcal{H}_{6,5}$ is as follows,

$$\mathcal{H}_{6,5} = \begin{array}{cc} & \begin{array}{ccccc} \varepsilon & a & b & aa & ba \end{array} \\ \begin{array}{c} \varepsilon \\ a \\ b \\ aa \\ ba \\ ab \end{array} & \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}. \tag{20}$$

4. Next $P_{6,3}$ is constructed from $\mathcal{H}_{6,5}$ as,

$$P_{6,3} = \begin{array}{cc} & \begin{array}{ccc} \varepsilon & a & ba \end{array} \\ \begin{array}{c} \varepsilon \\ a \\ b \\ aa \\ ba \\ ab \end{array} & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}. \tag{21}$$

5. $\mathcal{H}^{(a)}_{6,5}$ and $\mathcal{H}^{(b)}_{6,5}$ are as,

$$\mathcal{H}^{(a)}_{6,5} = \begin{array}{cc} & \begin{array}{ccccc} a & aa & ab & aaa & aba \end{array} \\ \begin{array}{c} \varepsilon \\ a \\ b \\ aa \\ ba \\ ab \end{array} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}. \tag{22}$$

$$\mathcal{H}^{(b)}_{6,5} = \begin{array}{cc} & \begin{array}{ccccc} b & ba & bb & baa & bba \end{array} \\ \begin{array}{c} \varepsilon \\ a \\ b \\ aa \\ ba \\ ab \end{array} & \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \end{array}. \tag{23}$$

then, $P^{(a)}_{6,3}$ and $P^{(b)}_{6,3}$ are obtained as follows,

$$P^{(a)}_{6,3} = \begin{array}{c} \\ \varepsilon \\ a \\ b \\ aa \\ ba \\ ab \end{array} \begin{array}{c} \begin{array}{ccc} a & aa & aba \end{array} \\ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array} \tag{24}$$

$$P^{(b)}_{6,3} = \begin{array}{c} \\ \varepsilon \\ a \\ b \\ aa \\ ba \\ ab \end{array} \begin{array}{c} \begin{array}{ccc} b & ba & bba \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \end{array}. \tag{25}$$

6. $\bar{P}_{3,3}$, $\bar{P}^{(a)}_{3,3}$, and $\bar{P}^{(b)}_{3,3}$ are constructed as follows,

$$\bar{P}_{3,3} = \begin{array}{c} \\ \varepsilon \\ a \\ ab \end{array} \begin{array}{c} \begin{array}{ccc} \varepsilon & a & ba \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array} \tag{26}$$

$$\bar{P}^{(a)}_{3,3} = \begin{array}{c} \\ \varepsilon \\ a \\ ab \end{array} \begin{array}{c} \begin{array}{ccc} a & aa & aba \end{array} \\ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array} \tag{27}$$

$$\bar{P}^{(b)}_{3,3} = \begin{array}{c} \\ \varepsilon \\ a \\ ab \end{array} \begin{array}{c} \begin{array}{ccc} b & ba & bba \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \end{array}. \tag{28}$$

7. From Eqs. (16) ~(18), System parameters of a minimal DFA are obtained as follows:

$$\bar{A}_a = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \ \bar{A}_b = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{29}$$

$$\bar{c}^t = (1 \quad 0 \quad 0), \ \bar{x}_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \tag{30}$$

There are such Hankel matrices as their row ranks are less than their column ranks. If such a Hankel matrix is given, r of Eq.(15) cannot be obtained. Then, minimal realizations are performed by next algorithms 2 or 3.

Algorithm 2

$P_{r,\bar n}$ and $P^{(k)}_{r,\bar n}$ can be constructed even if the row rank of a Hankel matrix is less than the column rank of it by redefining r of Eq.(15) as a minimal r satisfying the following equation,

$$\mathrm{rank}_{row}\mathcal{H}_{r,c} = \mathrm{rank}_{row}\mathcal{H}_{r+1,c} = \cdots = n_{row} \tag{31}$$

where n_{row} is the row rank of Hankel matrices.

Eq.(16) can be transformed to

$$\bar{P}_{\bar n,\bar n}\bar{A}_k = \bar{P}^{(k)}_{\bar n,\bar n} \tag{32}$$

and next equation holds.

$$P_{r,\bar n}\bar{A}_k = P^{(k)}_{r,\bar n} \tag{33}$$

By solving this matrix equation, $\bar{A}_k$ can be obtained.

The algorithm using Eq.(33) is as follows.

1. c, r and $\bar n$ are obtained from Eqs.(14) and (31).
2. $P_{r,\bar n}$ and $P^{(k)}_{r,\bar n}$ are constructed in the same manner as the algorithm 1.
3. $\bar{A}_k$ is obtained from Eq.(33).
4. $\bar{c}$ and $\bar{x}_0$ can be computed from Eqs.(17) and (18).

Algorithm 3

If, in a given Hankel matrix, the row rank is less than the column rank, a minimal realization can be also performed by use of a dual automaton mentioned in **2.4**.

1. Transpose a given Hankel matrix.
2. Realize a minimal DFA from the transposed Hankel matrix by using the algorithm 1.
3. Construct a dual FA of the realized DFA.
4. Transform the dual FA to a DFA by the subset construction. The transformed DFA is a desired minimal DFA.

3.2 Minimal partial realizations of DFAs

In this section, we show a method of minimal partial realizations for DFAs from subHankel matrices.

At first, we construct a subHankel matrix from a partial sequence of characteristic responses of a FA which are given in alphabetical order. From this subHankel matrix, we get a DFA using algorithms above-mentioned. Next, this subHankel matrix is enlarged and a new DFA is realized from it. By repeating this procedure, a minimal DFA is realized in the limit.

Algorithm

1. Start from subHankel matrix $\mathcal{H}_{1,1}$ and let $N=1$.
2. Let Π be the set of column labels of different columns of the subHankel matrix. The column of the subHankel matrix is expanded until involving the column labels $w_1 w_2$ (where $w_1 \in \Sigma$, $w_2 \in \Pi$).
3. Step 2. is repeated until a new different column do not appear in expanded subHankel matrix.
4. Construct DFA$_N$ using Silverman's algorithm from the subHankel matrix.
5. Expand the Hankel matrix by a row. Let $N=N+1$ and return step 2.

By this algorithm, DFA$_N$ ($N=1, 2, \cdots$) are obtained in order. After some N, DFA$_N$=DFA$_{N+1}=\cdots$ holds. This means minimal DFA$_N$ is obtained in the limit.

Example 2

The object of realizations is the DFA of Fig. 1.

1. Let $N=1$. $\mathcal{H}_{1,1}$ is gotten as follows,

$$\mathcal{H}_{1,1} = \begin{array}{cc} & \varepsilon \\ \varepsilon & (1) \end{array}. \tag{34}$$

2. $\mathcal{H}_{1,1}$ is expanded into next $\mathcal{H}_{1,5}$,

$$\mathcal{H}_{1,5} = \begin{array}{cccccc} & \varepsilon & a & b & aa & ba \\ \varepsilon & (1 & 0 & 1 & 1 & 0) \end{array}. \tag{35}$$

3. Construct DFA$_1$ from $\mathcal{H}_{1,5}$.

(a) rank$\mathcal{H}_{1,5}=2$ and rank$_{row}\mathcal{H}_{1,5}=1$. We use algorithm 2 because of rank$_{row}\mathcal{H}_{1,5}<$rank$\mathcal{H}_{1,5}$.

(b) $c=2$ and $r=1$ are computed from Eqs.(14) and (31).

(c) Contruct $\mathcal{H}_{1,2}$, $\mathcal{H}^{(a)}_{1,2}$ and $\mathcal{H}^{(b)}_{1,2}$,

$$\mathcal{H}_{1,2} = \begin{array}{ccc} & \varepsilon & a \\ \varepsilon & (1 & 0) \end{array}, \tag{36}$$

$$\mathcal{H}^{(a)}_{1,2} = \begin{array}{ccc} & a & aa \\ \varepsilon & (0 & 1) \end{array}, \quad \mathcal{H}^{(b)}_{1,2} = \begin{array}{ccc} & b & ba \\ \varepsilon & (1 & 0) \end{array}. \tag{37}$$

(d) $P_{1,2}$, $P^{(a)}_{1,2}$ and $P^{(b)}_{1,2}$ are obtained as follows,

$$P_{1,2} = \begin{array}{ccc} & \varepsilon & a \\ \varepsilon & (1 & 0) \end{array}, \tag{38}$$

$$P^{(a)}_{1,2} = \begin{array}{ccc} & a & aa \\ \varepsilon & (0 & 1) \end{array}, \quad P^{(b)}_{1,2} = \begin{array}{ccc} & b & ba \\ \varepsilon & (1 & 0) \end{array}. \tag{39}$$

(e) System parameters of DFA$_1$ are obtained as follows,

$$\overline{A}_a = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \overline{A}_b = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\overline{\boldsymbol{c}}^t = (1 \quad 0), \quad \overline{\boldsymbol{x}}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{40}$$

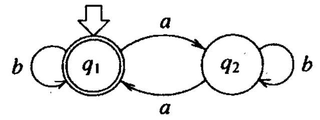

Fig. 2 DFA$_1$

4. Expand $\mathcal{H}_{1,5}$ by a row ($N=2$),

$$\mathcal{H}_{2,5} = \begin{array}{c} \\ \varepsilon \\ a \end{array} \begin{array}{c} \begin{array}{ccccc} \varepsilon & a & b & aa & ba \end{array} \\ \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \end{array}. \tag{41}$$

5. $\mathcal{H}_{2,5}$ is eventually expanded into $\mathcal{H}_{2,11}$ as follows,

$$\mathcal{H}_{2,11} = \begin{array}{c} \\ \varepsilon \\ a \end{array} \begin{array}{c} \begin{array}{cccccccc} \varepsilon & \cdots & ab & bb & aaa & baa & aba & bba \end{array} \\ \begin{pmatrix} 1 & \cdots & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & \cdots & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \end{array}. \tag{42}$$

6. Construct DFA$_2$ from $\mathcal{H}_{2,11}$,

$$\overline{A}_a = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \overline{A}_b = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{43}$$

$$\overline{\boldsymbol{c}}^t = (1 \quad 0 \quad 0), \quad \overline{\boldsymbol{x}}_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \tag{44}$$

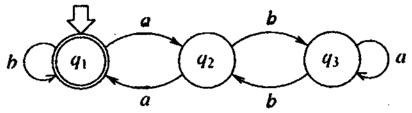

Fig. 3 DFA$_2$

7. Even if we repeat this procedure, an obtained DFA is as same as DFA$_2$. Then, DFA$_2$ is the minimal DFA which is identified in the limit.

4. Conclusions

There are two cases to obtain a minimal DFA. One is a reduction of states of a DFA given by the state transition diagram and so on[3,10,11,14]. The other is a minimal realization from the input-output data of a FA[2,7,8,12,15,16]. With respect to the latter, many algorithms such as Ho-Kalman's, Silverman's and Rissanen's[17] algorithms etc. are known for linear state space models of linear systems over R.

In this paper, we applied Siverman's algorithm to FAs. Since FAs are represented by the bilinear state space models over B, we expanded Silverman's algorithm in order to apply it to FAs. Then, we proposed various algorithms according to column ranks and row ranks of given Hankel matrices, where the column and the row rank of a matrix over B are defined respectively to be the number of distinct columns and rows. Moreover, we derived the method of a minimal partial realization algorithm based on Gold's identification-in-the-limit.

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