On the Flow of a Compressible Fluid Past a Circular Cylinder, I

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The M^2–expansion method is more suitable than any other methods for the calculation of the steady two-dimensional irrotational flow of a compressible fluid past a body with blunt-nose and with fairly large thickness-ratio. In this paper, a fifth approximate solution correct to the order of M^{10} is obtained for the flow without circulation past a circular cylinder, by applying Imai's new M^2–expansion method[1)], where M is the free-stream Mach number. The analytical formulae for the fluid velocity on the surface of the body are given. Also, the local sound velocity on the surface and the critical sound velocity as well as the critical Mach number are given. Numerical computations have been carried out for the case of flow of air in four cases in which M is equal to 0.40, 0.45, 0.50 and 0.55 respectively. Thus, it has been found that the critical Mach number for the circular cylinder seems to converge to 0.40 and that the potential flow may exist up to M=0.50.

1. Introduction

The thin-wing expansion method and the hodograph method are both useful for dealing with the steady two-dimensional irrotational flow of a compressible fluid past a cylindrical obstacle. However, they are not always convenient for the body with blunt-nose and with finite thickness-ratio.

On the other hand, the M^2–expansion method is more suitable for any cylindrical obstacle with arbitrary cross-section. Recently, this method has been improved by Dr. I. Imai in several features[1), 2), 3), 4)]. Especially the method used in the present paper seems to be the most convenient for a circular cylinder placed in a uniform flow of a compressible fluid. Introducing the conjugate complex variables, he has established an ingeneous method to calculate the complex velocity potential under appropriate boundary conditions. As for a circular cylinder without circulation, Imai[5)] himself has obtained the approximate solution correct to the order of M^6. The present author has also calculated to the order of M^8 by use of the original method of Janzen[6)] and Rayleigh[7)], though the results are unpublished.

In 1928, G. I. Taylor[8)] has remarked that the regular solution of the above problem may exist when the Mach number is equal to 0.4, but not equal to 0.5 and the limit of the Mach number for the potential flow may be near to 0.45. This so-called Taylor's problem or the mixed region problem, however, has not come to a satisfactory conclusion. Thus it is required to examine the convergency of the M^2–expansion series more precisely. For this purpose, we have improved the approximate solution so far correct to

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the order of M^{10} by making use of Imai's new M^2-expansion method[1]. The velocity potential newly obtained is in exact agreement with Imai's to the order of M^6 and with the unpublished one of the author to the order of M^8. Numerical computations have been carried out for the case of air ($\gamma=1.405$) in four cases in which M is equal to 0.40, 0.45, 0.50 and 0.55 respectively. From the results of them, it has been found that the critical Mach number for the circular cylinder seems to converge to 0.40 and that the potential flow may exist up to $M=0.50$.

2. Outline of Imai's method

According to Imai's method, the equations of motion and that of continuity of the steady two-dimensional irrotational flow of a compressible fluid can be reduced to a single equation as

$$\frac{\partial \bar{f}}{\partial z} = \left(1-\frac{\rho}{\rho_\infty}\right)\frac{\partial \varphi}{\partial z}, \tag{2.1}$$

where φ is the velocity potential; ρ and ρ_∞ are respectively the density at any point in the fluid flow and at infinity in the undisturbed flow; $\bar{f}$ and $\bar{z}$ are the conjugate ones of the complex velocity potential f and the complex coordinate z respectively.

If we assume that f, φ, $1-\frac{\rho}{\rho_\infty}$ and the magnitude of the fluid velocity q can be expanded in the ascending powers of M^2 as follows:

$$\left.\begin{aligned}
f &= f_0+f_1M^2+f_2M^4+\cdots,\\
\varphi &= \varphi_0+\varphi_1M^2+\varphi_2M^4+\cdots,\\
1-\frac{\rho}{\rho_\infty} &= \rho_0+\rho_1M^2+\rho_2M^4+\cdots,\\
q^2 = 4\frac{\partial\varphi}{\partial z}\frac{\partial\varphi}{\partial\bar{z}} &= 1+q_0+q_1M^2+q_2M^4+\cdots,
\end{aligned}\right\} \tag{2.2}$$

then, comparing the values of the same order of M, we have

$$\left.\begin{aligned}
q_0 &= 4\frac{\partial\varphi_0}{\partial z}\frac{\partial\varphi_0}{\partial\bar{z}}-1\\
q_1 &= 4\frac{\partial\varphi_0}{\partial z}\frac{\partial\varphi_1}{\partial\bar{z}}+\text{conjugate complex},\\
q_2 &= 4\frac{\partial\varphi_0}{\partial z}\frac{\partial\varphi_2}{\partial\bar{z}}+2\frac{\partial\varphi_1}{\partial z}\frac{\partial\varphi_1}{\partial\bar{z}}+\text{conjugate complex},\\
q_3 &= 4\frac{\partial\varphi_0}{\partial z}\frac{\partial\varphi_3}{\partial\bar{z}}+4\frac{\partial\varphi_1}{\partial z}\frac{\partial\varphi_2}{\partial\bar{z}}+\text{conjugate complex},\\
q_4 &= 4\frac{\partial\varphi_0}{\partial z}\frac{\partial\varphi_4}{\partial\bar{z}}+4\frac{\partial\varphi_1}{\partial z}\frac{\partial\varphi_3}{\partial\bar{z}}+2\frac{\partial\varphi_2}{\partial z}\frac{\partial\varphi_2}{\partial\bar{z}}+\text{conjugate complex},\\
&\cdots\cdots\cdots\cdots\cdots\cdots,
\end{aligned}\right\} \tag{2.3}$$

$$\begin{aligned}
&\rho_0 = 0,\quad \rho_1 = \frac{1}{2}q_0,\quad \rho_2 = \frac{1}{2}q_1-\frac{1}{8}(1-k)q_0^2,\\
&\rho_3 = \frac{1}{2}q_2-\frac{1}{4}(1-k)q_0q_1+\frac{1}{48}(1-3k+2k^2)q_0^3,
\end{aligned}$$

$$
\left.\begin{aligned}
\rho_4 &= \frac{1}{2}q_3-\frac{1}{8}(1-k)(2q_0q_2+q_1^2)+\frac{1}{16}(1-3k+2k^2)\,q_0^2q_1 \\
&\quad -\frac{1}{384}(1-6k+11k^2-6k^3)\,q_0^4\,, \\
\rho_5 &= \frac{1}{2}q_4-\frac{1}{4}(1-k)(q_0q_3+q_1q_2)+\frac{1}{16}(1-3k+2k^2)(q_0^2q_2+q_0q_1^2) \\
&\quad -\frac{1}{96}(1-6k+11k^2-6k^3)\,q_0^3q_1+\frac{1}{3840}(1-10k+35k^2-50k^3+24k^4)\,q_0^5\,, \\
&\quad \cdots\cdots\cdots\cdots\cdots\cdots\,,
\end{aligned}\right\} \tag{2.4}
$$

and

$$
\bar{f}_N=\int\left(\rho_1\frac{\partial\varphi_{N-1}}{\partial z}+\rho_2\frac{\partial\varphi_{N-2}}{\partial z}+\cdots+\rho_N\frac{\partial\varphi_0}{\partial z}\right)dz+\bar{G}_N(\bar{z})\,,\quad (N=0,1,2,\cdots)\,, \tag{2.5}
$$

where the adiabatic index γ is equal to $k+1$ and $\bar{G}_N(\bar{z})$ are the complementary functions which are analytic functions of $\bar{z}$ only and should be determined by the appropriate boundary conditions.

3. Determination of the complementary functions $\bar{G}_N(\bar{z})$

Here we consider the case of a circular cylinder without circulation. We put a circular cylinder with unit radius normally to the uniform flow of a compressible fluid whose direction coincides with that of x-axis and whose speed is unit.

We assume

$$
\left.\begin{aligned}
u &= u_0+u_1M^2+u_2M^4+\cdots\,, \\
v &= v_0+v_1M^2+v_2M^4+\cdots\,, \\
\psi &= \psi_0+\psi_1M^2+\psi_2M^4+\cdots\,,
\end{aligned}\right\} \tag{3.1}
$$

where u and v are the x- and y-components of the fluid velocity respectively and ψ is the stream function. Then the boundary conditions become as follows:

$$
u_0=1\,,\quad v_0=0\,;\quad u_N=v_N=0\,,\quad (N=1,2,\cdots)\quad \text{at infinity,} \tag{3.2}
$$

$$
\psi_N=0\,,\quad (N=0,1,2,\cdots)\quad \text{on the unit circle.} \tag{3.3}
$$

Here the first and the second conditions of (3.2) correspond to the flow of an incompressible fluid.

Now we denote

$$
\bar{f}_N=\bar{\chi}_N(z,\bar{z})+\bar{G}_N(\bar{z})\,, \tag{3.4}
$$

then, if no circulation, $\bar{\chi}_N(z,\bar{z})$ are expanded in the form:

$$
\bar{\chi}_N(z,\bar{z})=\sum_{m,n}a_{mn}z^m\bar{z}^n\,, \tag{3.5}
$$

where a_{mn} is generally complex number.

When the values of z and $\bar{z}$ on the surface ($z\bar{z}=1$) are denoted by t and $\bar{t}$ respectively, we can deform (3.5) in the form:

$$
\bar{\chi}_N(t,\bar{t})=\sum_m\frac{a_m}{t^m}+\sum_n\frac{b_n}{\bar{t}^n}\,. \tag{3.6}
$$

The condition (3.3) can be written in the form:

$$
\Im\bar{G}_N(\bar{t})=-\Im\bar{\chi}_N(t,\bar{t})\,, \tag{3.7}
$$

where $\Im$ denotes the imaginary part. It is obvious that the signs of the imaginary parts of $\bar{\chi}_N(t, \bar{t})$ do not change by the following transformation:

$$\bar{\chi}_N(t, \bar{t}) \longrightarrow -\sum_m \frac{\bar{a}_m}{\bar{t}^m} + \sum_n \frac{b_n}{\bar{t}^n}, \tag{3.8}$$

where $\bar{a}_m$ is the conjugate complex value of a_m. Thus the required analytic functions $\bar{G}_N(\bar{z})$ which satisfy the condition (3.7) are given in the form:

$$\bar{G}_N(\bar{z}) = \sum_m \frac{\bar{a}_m}{\bar{z}^m} - \sum_n \frac{b_n}{\bar{z}^n}. \tag{3.9}$$

And these procedures are very simple and easy ones.

Now, f_0 and $\bar{f}_0$ correspond to an incompressible fluid flow under the conditions (3.2) and (3.3) and it is well known that

$$f_0 = z + \frac{1}{z}, \qquad \bar{f}_0 = \bar{z} + \frac{1}{\bar{z}}. \tag{3.10}$$

By making use of these fundamental complex velocity potentials, we can proceed the subsequent evaluations as follows:

$$\begin{aligned}
4\bar{\chi}_1(z, \bar{z}) &= z\left(-\frac{1}{\bar{z}^2}\right) + \frac{1}{z}\left(1 - \frac{2}{\bar{z}^2}\right) + \frac{1}{z^3}\left(-\frac{1}{3} + \frac{1}{3\bar{z}^2}\right), \\
4\bar{\chi}_1(t, \bar{t}) &= -\frac{1}{\bar{t}^3} + \frac{1}{t} - \frac{2}{\bar{t}} - \frac{1}{3t^3} + \frac{1}{3t}, \\
4\bar{G}_1(\bar{z}) &= \frac{1}{\bar{z}^3} + \frac{1}{\bar{z}} + \frac{2}{\bar{z}} - \frac{1}{3\bar{z}^3} + \frac{1}{3\bar{z}}, \\
&= \frac{10}{3}\frac{1}{\bar{z}} + \frac{2}{3}\frac{1}{\bar{z}^3}.
\end{aligned}$$

and so on. Thus the required integrals $\bar{f}_N$ are given successively from the small number of the suffix N.

4. Surface velocity

Following the above analysis, we have obtained the complex velocity potential correct to the order of M^{10}. And the velocity potential and the stream function are also reduced so far correct to the same order of M. And these results agree with those due to Imai[5] to the order of M^6.

However, the analytical expressions of the complex velocity potentials: f_1, f_2, f_3, f_4 and f_5; the velocity potentials: φ_1, φ_2, φ_3, φ_4 and φ_5 and the stream functions: ψ_1, ψ_2, ψ_3, ψ_4 and ψ_5 are all omitted here for the limitation of space. So, in this paper we shall report the velocity distribution on the cylindrical surface.

Let q_s be the magnitude of the fluid velocity along the surface of the cylinder and put

$$\begin{aligned}
q_s &= \left[-\frac{1}{r}\frac{\partial\varphi}{\partial\theta}\right]_{r=1} \\
&= q_{s00} + q_{s10}M^2 + (q_{s20} + q_{s21}k)M^4 + (q_{s30} + q_{s31}k + q_{s32}k^2)M^6 \\
&\quad + (q_{s40} + q_{s41}k + q_{s42}k^2 + q_{s43}k^3)M^8 + (q_{s50} + q_{s51}k + q_{s52}k^2 + q_{s53}k^3 + q_{s54}k^4)M^{10} \\
&\quad + \cdots
\end{aligned} \tag{4.1}$$

where r and θ are the polar coordinates, namely $z=re^{i\theta}$ and k is the modified adiabatic index, namely $k=\gamma-1$. Then we have the following formulae:

$$q_{smn}=A\sin\theta+B\sin 3\theta+C\sin 5\theta+D\sin 7\theta+E\sin 9\theta+F\sin 11\theta\,, \tag{4.2}$$

where the coefficients A, B, C, D, E and F are shown in Table 1.

Table 1

$\sin l\theta$ \ q_{smn}	Coeff. of $\sin\theta$ (A)	Coeff. of $\sin 3\theta$ (B)	Coeff. of $\sin 5\theta$ (C)	Coeff. of $\sin 7\theta$ (D)	Coeff. of $\sin 9\theta$ (E)	Coeff. of $\sin 11\theta$ (F)
q_{s00}	2					
q_{s10}	$\frac{2}{3}$	$-\frac{1}{2}$				
q_{s20}	$\frac{37}{40}$	$-\frac{25}{24}$	$\frac{3}{8}$			
q_{s21}	$\frac{23}{120}$	$-\frac{11}{40}$	$\frac{1}{8}$			
q_{s30}	$\frac{139}{84}$	$-\frac{2467}{1008}$	$\frac{9503}{6048}$	$-\frac{37}{96}$		
q_{s31}	$\frac{2813}{3360}$	$-\frac{2\,3441}{1\,6800}$	$\frac{1591}{1680}$	$-\frac{25}{96}$		
q_{s32}	$\frac{103}{840}$	$-\frac{57}{280}$	$\frac{11}{84}$	$-\frac{1}{24}$		
q_{s40}	$\frac{25\,3657}{7\,2576}$	$-\frac{126\,6251}{20\,1600}$	$\frac{349\,1743}{63\,5040}$	$-\frac{17\,9777}{7\,2576}$	$\frac{59}{128}$	
q_{s41}	$\frac{89\,2277}{30\,2400}$	$-\frac{284\,6929}{50\,4000}$	$\frac{351\,8521}{70\,5600}$	$-\frac{1\,5719}{6720}$	$\frac{15}{32}$	
q_{s42}	$\frac{15\,4567}{17\,2800}$	$-\frac{1103}{640}$	$\frac{8\,2601}{5\,6448}$	$-\frac{84\,2011}{120\,9600}$	$\frac{39}{256}$	
q_{s43}	$\frac{1889}{2\,0160}$	$-\frac{601}{3360}$	$\frac{95}{672}$	$-\frac{37}{576}$	$\frac{1}{64}$	
q_{s50}	$\frac{144\,2407}{17\,2800}$	$-\frac{241\,8311\,9119}{13\,9708\,8000}$	$\frac{362\,8569\,0591}{19\,5592\,3200}$	$-\frac{116\,6270\,1403}{10\,0590\,3360}$	$\frac{1285\,1477}{319\,3344}$	$-\frac{1543}{2560}$
q_{s51}	$\frac{40\,0014\,5887}{3\,9916\,8000}$	$-\frac{499\,7183\,1509}{23\,2848\,0000}$	$\frac{247\,6809\,7777}{10\,8662\,4000}$	$-\frac{80\,4515\,8271}{5\,5883\,5200}$	$\frac{101\,8967}{19\,7120}$	$-\frac{157}{192}$
q_{s52}	$\frac{500\,6675}{106\,4448}$	$-\frac{187\,9341\,0241}{18\,6278\,4000}$	$\frac{270\,0651\,5257}{26\,0789\,7600}$	$-\frac{543\,1018\,7443}{83\,8252\,8000}$	$\frac{635\,0543}{266\,1120}$	$-\frac{6173}{1\,5360}$
q_{s53}	$\frac{67\,1731}{66\,5280}$	$-\frac{602\,4791}{282\,2400}$	$\frac{183\,9553}{88\,7040}$	$-\frac{312\,6721}{249\,4800}$	$\frac{20\,6161}{44\,3520}$	$-\frac{323}{3840}$
q_{s54}	$\frac{4657}{5\,5440}$	$-\frac{1273}{7392}$	$\frac{1145}{7392}$	$-\frac{35}{396}$	$\frac{56}{1760}$	$-\frac{1}{160}$

5. Velocity distribution on the surface for air ($\gamma=1.405$)

For the air ($\gamma=1.405$), the distribution of the fluid velocity q_s on the surface is given by the following formulae. Let

$$q_s=q_{s0}+q_{s1}M^2+q_{s2}M^4+\cdots\,, \tag{5.1}$$

we have

$$q_{s0}=2\sin\theta\,,\qquad q_{s1}=\frac{2}{3}\sin\theta-\frac{1}{2}\sin 3\theta\,,$$

$$q_{s2} = 1.0026\ 250 \sin\theta - 1.1530\ 417 \sin 3\theta + 0.4256\ 250 \sin 5\theta\,,$$

$$q_{s3} = 2.0139\ 415 \sin\theta - 3.0459\ 070 \sin 3\theta + 1.9762\ 873 \sin 5\theta - 0.4977\ 198 \sin 7\theta\,,$$

$$q_{s4} = 4.8430\ 097 \sin\theta - 8.8632\ 869 \sin 3\theta + 7.7674\ 316 \sin 5\theta - 3.5428\ 827 \sin 7\theta + 0.6768\ 074 \sin 9\theta\,,$$

$$q_{s5} = 13.2466\ 849 \sin\theta - 27.8026\ 880 \sin 3\theta + 29.6236\ 312 \sin 5\theta - 18.5731\ 055 \sin 7\theta + 6.5411\ 796 \sin 9\theta - 1.0055\ 818 \sin 11\theta\,.$$

Table 2

$\theta°$	q_{s0}	q_{s1}	q_{s2}	q_{s3}	q_{s4}	q_{s5}
0	0	0	0	0	0	0
10	0.34730	−0.13423	−0.07637	−0.12702	−0.29288	−0.76483
20	0.68404	−0.20500	−0.23649	−0.32269	−0.64732	−1.66580
30	1.00000	−0.16667	−0.43892	−0.80193	−1.46343	−3.11937
40	1.28558	−0.00449	−0.49966	−1.52907	−3.73036	−8.39427
50	1.53209	0.26070	−0.20842	−1.75086	−6.72866	−21.64992
60	1.73205	0.57735	0.49970	−0.39843	−5.60085	−29.39677
70	1.87939	0.87646	1.44477	2.69099	4.24297	−0.33426
80	1.96962	1.08955	2.25954	6.06174	18.64981	62.17342
90	2.00000	1.16667	2.58129	7.53386	25.69342	96.79287

The values of q_{s0}, q_{s1}, q_{s2}, $\cdots$ are shown in Table 2 and Fig. 1.

Next, by using Table 2 we can evaluate the velocity on the surface for any Mach number corresponding to each step of approximations. Here we shall define the new notations U_0, U_1, U_2, $\cdots$ by the following expressions:

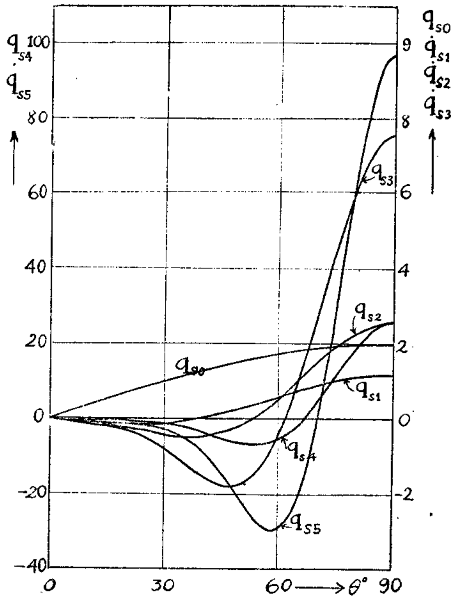

Fig. 1

$$\left.\begin{aligned} U_0 &= q_{s0}\,, & U_1 &= U_0 + q_{s1}M^2\,, \\ U_2 &= U_1 + q_{s2}M^4\,, & U_3 &= U_2 + q_{s3}M^6\,, \\ U_4 &= U_3 + q_{s4}M^8\,, & U_4 &= U_4 + q_{s5}M^{10}\,. \end{aligned}\right\} \quad (5.2)$$

The values of U_1, U_2, U_3, U_4 and U_5 are given in Table 3, Table 4, Table 5 and Table 6 and are shown by Fig. 2, Fig. 3, Fig. 4 and Fig. 5 in four cases in which M is equal to 0.40, 0.45, 0.50, and 0.55 respectively. The dotted lines in Figs. 2, 3, 4 and 5 are the curves of the sound velocity on the surface and consequently their intersections with the velocity curves will give the positions of the sonic points on the surface. And from these figures, it will be seen that the potential flow may exist up to M=0.50.

Table 3. $M=0.40$

$\theta°$	1st Approx. U_1	2nd Approx. U_2	3rd Approx. U_3	4th Approx. U_4	5th Approx. U_5
0	0	0	0	0	0
10	0.3258	0.3239	0.3233	0.3232	0.3231
20	0.6512	0.6452	0.6439	0.6434	0.6433
30	0.9733	0.9621	0.9588	0.9579	0.9575
40	1.2849	1.2721	1.2658	1.2634	1.2625
50	1.5738	1.5685	1.5613	1.5569	1.5546
60	1.8244	1.8372	1.8356	1.8319	1.8288
70	2.0196	2.0566	2.0676	2.0704	2.0704
80	2.1439	2.2018	2.2266	2.2388	2.2454
90	2.1867	2.2527	2.2836	2.3004	2.3106

Table 4. $M=0.45$

$\theta°$	1st Approx. U_1	2nd Approx. U_2	3rd Approx. U_3	4th Approx. U_4	5th Approx. U_5
0	0	0	0	0	0
10	0.3201	0.3170	0.3159	0.3154	0.3152
20	0.6425	0.6328	0.6301	0.6291	0.6285
30	0.9662	0.9483	0.9416	0.9391	0.9381
40	1.2847	1.2642	1.2515	1.2452	1.2423
50	1.5849	1.5763	1.5618	1.5505	1.5431
60	1.8490	1.8695	1.8661	1.8567	1.8467
70	2.0569	2.1161	2.1385	2.1456	2.1455
80	2.1903	2.2829	2.3332	2.3646	2.3858
90	2.2363	2.3421	2.4047	2.4479	2.4808

Table 5. $M=0.50$

$\theta°$	1st Approx. U_1	2nd Approx. U_2	3rd Approx. U_3	4th Approx. U_4	5th Approx. U_5
0	0	0	0	0	0
10	0.3137	0.3090	0.3070	0.3058	0.3051
20	0.6328	0.6180	0.6130	0.6104	0.6088
30	0.9583	0.9309	0.9184	0.9127	0.9096
40	1.2845	1.2532	1.2293	1.2148	1.2066
50	1.5973	1.5842	1.5569	1.5306	1.5095
60	1.8764	1.9076	1.9014	1.8795	1.8508
70	2.0985	2.1888	2.2308	2.2474	2.2471
80	2.2420	2.3832	2.4779	2.5508	2.6115
90	2.2917	2.4530	2.5707	2.6711	2.7656

Table 6. $M=0.55$

$\theta°$	1st Approx. U_1	2nd Approx. U_2	3rd Approx. U_3	4th Approx. U_4	5th Approx. U_5
0	0	0	0	0	0
10	0.3067	0.2997	0.2962	0.2937	0.2918
20	0.6220	0.6004	0.5915	0.5860	0.5818
30	0.9496	0.9094	0.8872	0.8750	0.8671
40	1.2842	1.2385	1.1962	1.1649	1.1437
50	1.6110	1.5919	1.5434	1.4871	1.4322
60	1.9067	1.9524	1.9414	1.8945	1.8200
70	2.1445	2.2767	2.3512	2.3867	2.3859
80	2.2992	2.5060	2.6738	2.8299	2.9874
90	2.3529	2.5891	2.7977	3.0128	3.2580

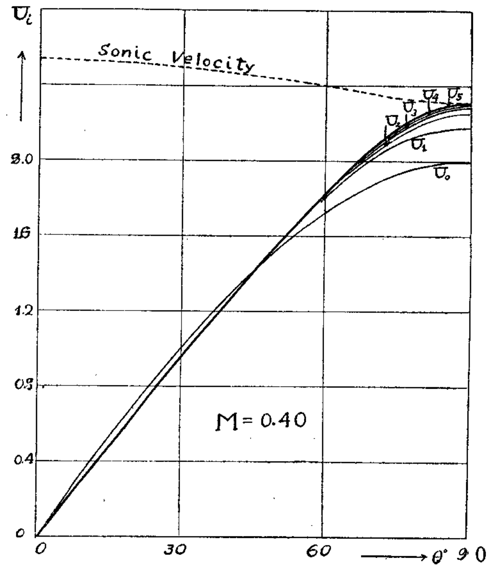

Fig. 2

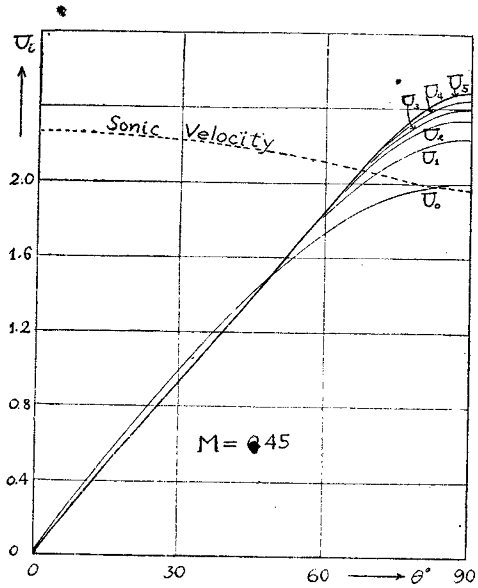

Fig. 3

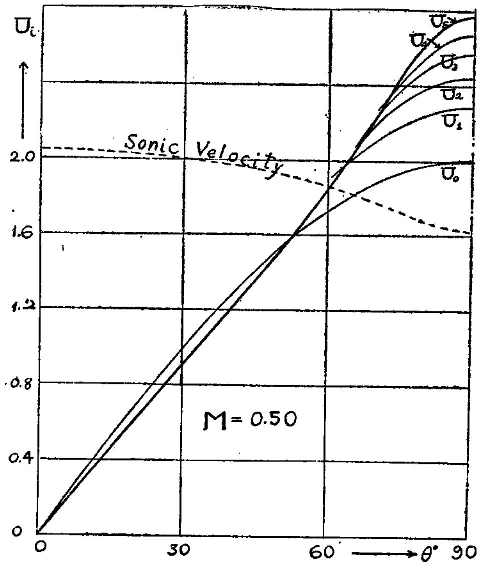

Fig. 4

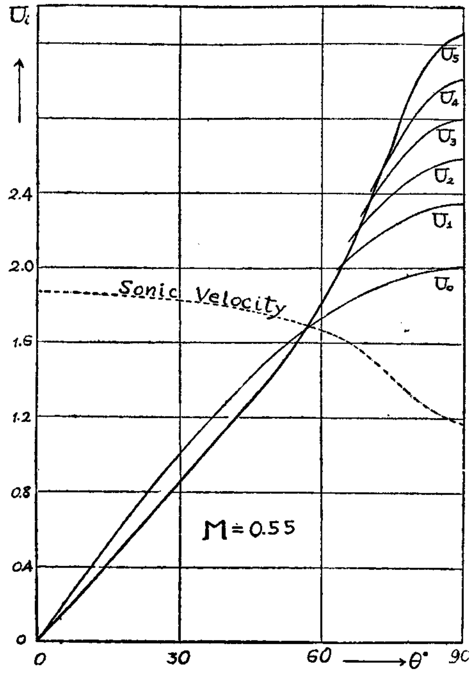

Fig. 5

6. Critical Mach number for air

Owing to Taylor's[8),9)] and Imai's[4),5)] results, the potential flow round a cylindrical obstacle seems to have its maximum fluid velocity on the surface of the obstacle. And such characteristic features have been also verified by numerical calculations. So, the critical Mach number which indicates the first appearance of the sonic point in the entire region of the flow, may reasonably be given from the maximum velocity $q_{\max}$ on the surface by making use of the well-known relation:

$$\frac{1}{M_*^2}=\frac{k+2}{2}q_{\max}^2-\frac{k}{2}. \qquad (6)$$

Here M_* is the so-called critical Mach number which is usually regarded as a very important factor to estimate the compressible effects in the aerodynamics.

Table 7

Degree of approx.	Maximum velocity q_{max}	Critical Mach number M_*
0	2.0000	0.4659
1	2.2064	0.4206
2	2.2673	0.4090
3	2.2926	0.4043
4	2.3053	0.4020
5	2.3120	0.4008

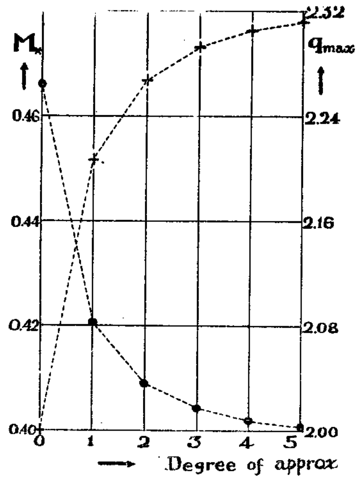

Fig. 6

$q_{\max}$: --+-- M_*: --●--

For the value of q_{max} we have used the present results to the order of M^{10} and evaluated it for each step of approximations. The maximum velocity as well as the critical Mach number M_* are shown in Table 7 and Fig. 6, where the dotted lines are drawn merely to estimate the convergency of approximations. From Fig. 6, it will be seen that the critical Mach number seems to converge to 0.40 and also that the convergency of the flow velocity is good when $M=0.40$.

7. Local and critical sound velocity for air

According to the results of the preceding paragraph, the critical Mach number M_* is nearly equal to 0.40, so that the flows for $M=0.45$, 0.50 and 0.55 may contain the so-called sonic lines which separate the flow field into subsonic and supersonic regions. And the starting points of such sonic lines are given by the intersections of the curve of the local sound velocity and the curve of the fluid velocity on the surface. For this purpose, we used the well-known relation:

$$C^2 = \frac{1}{M^2} + \frac{k}{2} - \frac{k}{2} q^2, \tag{7}$$

where C is the local sound velocity. If q is taken to the order of M^{10}, then the values of C on the cylindrical surface are obtained for $M=0.40$, 0.45, 0.50 and 0.55, and they are given in Table 8 and plotted in Figs. 2, 3, 4 and 5 by the dotted lines as already cited.

By using (7), we have also calculated the values of C to which the fluid velocity q on the cylindrical surface is equal in the above four cases. Here we shall call it the critical sound velocity C_* and its values are shown in Table 9.

Table 8. Local sound velocity (C) on the cylindrical surface.

$\theta°$ \ Mach number	0.40	0.45	0.50	0.55
0	2.5402	2.2673	2.0500	1.8730
10	2.5360	2.2629	2.0454	1.8684
20	2.5236	2.2496	2.0316	1.8547
30	2.5038	2.2277	2.0087	1.8319
40	2.4758	2.1971	1.9768	1.8009
50	2.4419	2.1584	1.9342	1.7444
60	2.4032	2.1098	1.8732	1.6845
70	2.3632	2.0500	1.7840	1.5371
80	2.3307	1.9970	1.6797	1.3043
90	2.3176	1.9734	1.6290	1.1657

Table 9. Critical sound velocity (C_*).

Mach number M	Critical sound vel. C_*
0.40	2.3164
0.45	2.0676
0.50	1.8694
0.55	1.7081

8. Conclusions

We have calculated the complex velocity potential to the order of M^{10} by the M^2-expansion method of Imai. For the air ($\gamma=1.405$), the numerical values of velocity on the circular cylinder surface are obtained for $M=0.40$, 0.45, 0.50 and 0.55. From these results, it has been concluded that the limiting Mach number for the potential flow seems to be near 0.50 and the critical Mach number to be near 0.40. Therefore, it is suggested that the so-called Taylor's mixed region may exist in the range of M between 0.40 and 0.50, The convergency of the M^2-expansion series will be discussed more fully in the following report.

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