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ADVANCES IN DECISION ANALYSIS. EFFICIENT METHODS IN FINANCE

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Abstract:

Decision analysis offer workable solutions in domain such as the environment, health and medicine, engineering and operations research and finance. In finance area we can observe a big variety of method and techniques for research fluctuates from economy and financial date.

Most economic decisions are related to monetary variables. Since money has a capacity to connect the present and the future, conflicts among long–run and short–run economic goals and uncertainties of the future make economic decisions very complicated.

In this paper we will introduce some models to show how monetary variables can be investigated real aspects of economic development. We discusses problems encountered in financial models, describes efficient method and show how to apply them to practical problems in finance.

Key words: decision analysis, mathematical modeling, dynamic models, money.

JEL Classification: C6, E4

1. Decision analysis and mathematical modeling

The length or weight of the decisional problems is large enough, from quotidian ones of everyone, so with a particular value, to those of major importance, which are confronted by the ones skilled to lead and coordinate high level activities. In the last time, the efforts of the researchers are concentrated over the problems referring to the codification of the information and to structuralize a theory capable of to contribute to successful knowing and solving the complex decisional aspects, specific to the modern society. An unforgettable advantage can be that *the methodology of the decisional analysis* obligates the decider to see the problem as an organic whole and to surprise the information connections and fluxes between its elementary components, favoring the communication between specialists of diverse domains (engineers, economists, mathematicians, sociologists.) implicated in evaluating and solving particular segments of the problem, in which purpose they use a similar language and an advantaging ground or environment for obtaining realizable compromises between their opinions, which aren't always identical. More than that, the methodology mentioned before requests the decomposition of the problem examined, with precedence to the complex ones, in other more simple ones, concomitant with the logic synthesizing of the results obtained concerning the conception, projection and constructing an unitary and realist action program, viable under the report of the economical, technological, ecological and, why not, juridical economy.

Besides, such an analysis needs responses more clear and correct answers, without equivoque to the elementary or hypothetical problems. Sometimes, these are hard enough to solve, needing the explicitly of the points of view until the smallest details, confirming the conclusion in conform to which the "bet", the ambition of *the theory of decision* is that of quantifying the incertitude and the risks, the preferences and accepting the consequences in any situation, the efforts done by the researchers in the last years being a proof more than obvious, in this sense.

The continuous growth of diversity and complexity of calculation and modelation problems that were requested by the research activity, as a domain of avant–grade, in front of the scientists, has leaded lately to an explosive evolution of methods and ways of calculation. So, the dynamic

modeling today is a discipline in full progress from the fame of the mathematic modeling and computer science but also a component that we find more and more in the everyday life, implicated in the industrial and social technology. It represents a distinctive, mature chapter of the competitional sciences which, by mathematical fine concepts and advanced informatics instruments with which they operate constitute a production force that is manifested with pregnancy in all the techno–scientific domain and economic activities.

Generally, as much as complex the activities are, in the same way the planning, the search for formal and systematic strategies and actions grow. The monetary domain is a domain in which the degree of incertitude and the risk is very high and where the planning plays a very important role in trying to reduce this incertitude. In essence, the elaboration of strategies in this domain purposes a clear and systematic structuring of the modes by which the followed objectives may be reached by a judicious allocation of resources on medium of high ground. In the frame of any development of this type there must be considered the most important aspects of planning.

Preoccupations of the economists concerning the international commerce and finances have appeared since the 14th century. These domains of economy, private in strengthened independency have stood in the view of some eminent scientific personalities, like: Adam Smith, David Hume and John Stuart Mill, which theories and concepts continue to be, in a large part, available today.

The analysis of economic relations, including those from the international currency transactions, by the prism of the creative methods (statistic, mathematical), concerning assuring a scientific rigor in the economic area as well as in the nature sciences, has started in the same century [Matei, (2004)].

The representative of the mathematical–political school (W. Petty, Gr. King, J. Grount etc.) has approached by the prism of the quantitative problems concerning the tax rate and money in the international commerce and finance. Later, in the 18th and 19th centuries, there are approached, under the quantitative aspect concerning the import and the export (Fr. Quesnay) or the request in report with the determinant facts (Ernst Engel).

In the starting of the 20th century, eminent scientists have given the economists methods of measuring and analyzing the level and intensity of the economic processes, like the grade of dependency between phenomenons. We remind, in the same sense K.Pearson, W. Pearson, I.Fisher, I. Hooker, M. Yule and M. Benini. The quantitative approaches purposed have vised the price dynamics domain, the imports and exports but also that of measuring connections between the economic processes in evolution (the correlation, the business cycle, the late effect, etc.)

Once the econometric society appeared (in 1930), this kind of preoccupations concerning the measuring and rigorous analyzing in economy by statistic and mathematical methods have took amplitude. The first models appeared (J. Tinbergen, L. Klein, J. Koopman etc.) which describe by an equation, or, especially, by an equation system of simultaneous equations of economical cause–effect type of relations. These type of models show schematically a sector of economy or national economy in ensemble (macroeconomic models), making possible the influence analysis, economic processes development prognosis, monetary politics simulation, representing an expression of progresses realized in analyzing and economic foresight in the modern and contemporary époque.

2. Efficient methods in finance

Just like the new telescopes enlarge our horizons without invalidating the discoveries did before from the close part of the universe, the mathematics show also new views, while it develops on existent knowledge. Perspectives may change, but not truths.

The new territories rediscover the reality of change. Time no longer represents the eternal repeat of some identical things, but becomes wearer of differences. This means that the status of a system at an ordinary moment is not contained in a previous state: between the first and second

state there's a qualitative change. The sudden transactions, "catastrophic" bifurcations are not produced in an uncertain manner. On the contrary, they result from the conjunction of a multitude of facts that lead the system in one direction instead of another.

The dynamic models follow to make understood the temporal relations. The model operates with events and states that express the value of an attribute by which is identified the apparition of events. With the help of the data structures there are constructed transition diagrams of the states that indicate all the opinions specific to every type of object and corresponding class.

On this changing background of research facilities, the econometric studies have reoriented the analysis of periods by the dynamic models that can be studied by the new theories.

The model, as an instrument of scientific knowledge, is used in numerous theoretic and practice disciplines. The knowledge obtained from the work with methods and the try to apply them may discover new valuable concepts about a certain problem and with the type of decisions that are necessary. Simply knowing of the decision zones may be a major progress in some situations. Plus, using models, there can be recognized variables tat may be controlled for influencing the system's performance, the relevant costs and their dimension as well as the correlation between costs and variables, including the options of important costs.

The modeling of an economic process constitutes a scientific way of unfolding the determinant factors that interfere in the frame of the respective phenomenon. In order to see the importance of these factors for the considered process it is necessary to introduce in the model constructed of the most important factors, so that, every time it is possible, there must be assured their possible quantification, which will permit the mathematical treatment.

Starting from the idea that any model is based on real data and parameters there is necessary to consider the fact of obtaining trustful data which will permit a good representation of the reality by the model. This way, when this is the case, the clinical or periodical aspect of the studied phenomenon is identified, implicitly the horizon of time it refers to.

From an econometric point of view, the classical methods based on continuity, linearity and stability have been proven unstable for representing economic phenomenon and processes with a higher degree of complexity. The researchers are obligated to follow these processes in a dynamic way, to study qualitatively the changes that interfere with the economic variables implicated as well as the results obtained with their help. Besides other characteristics, the mathematical models permit the introducing of a new isomorphism between the real economic system and the ideal one, represented as model. With their help it becomes possible to approach the instable components of different economic nonlinear systems accentuating more often the fact that linearity and stability are particular cases of economic growth.

The dynamic modeling is based on the fact that the functionality of a system is represented by the knowledge of interactions between the fluxes of information, commands, human resources, material resources etc. a dynamic model surprises the behavior of complex systems showing how their structure determinates the trajectory, respectively the behavior in time.

The apparition of the nonlinear dynamics theory has enabled the understanding and developing of some processes and methods that approach us more to reality. The development of the theory of singularities and the theory of bifurcation has completed the multitude of ways by which we dispose for analyzing and representing more and more complex, dynamics, giving the possibility of analyzing some systems which were hard, if not impossible to approach by traditional methods. The study of nonlinear dynamics is of maximum interest because the economical systems are by excellence nonlinear systems. Many of these contain multiple discontinuities and incorporate inherent instability being permanently under shock actions, extern and intern perturbations.

The application of dynamic methods represents mainly an exercise of simulation. Indifferent if someone studies the clinic behavior, alternative politics, history interpretation, model error evaluation or anything else, the numeric simulation is the correct instrument for this.

For the small or linear systems, to many of these questions can be directly answered in the frame of the analytic work, many times by forms including mathematical expressions, but for the general cases, especially in the case of the large proportion used today, the numeric simulation represents the only method that is possible to use. However, "of all the types of simulation of the model, one is considered "queen" – presage"[Ungureanu, (2004)].

3. Same mathematical models used in finance

In economic science, especially in the organizational and leading disciplines, the models are used in all the diversity of types that exist. In the last decades however, there is a more and more tendency of using, in these disciplines, the mathematical type models, especially because of their capacity to condensate rigorously the essential, and also their possibility to be programmed with the help of computational techniques, forming together an instrument of scientific investment of an unknown power until the present, a prodigious "extension" of human intelligence.

3.1 Theoretic model of fiscal evasion (Allingham and Sandmo, 1972)

The high tax rates, the controller fragility and the gentle fines in the case of fiscal fraud unveiling has contributed to stimulating people to try their chances of not being discovered when a fiscal evasion strategy is applied. The tax and tax rates payer's problem is in maximizing the total income waiting utilities in case he adopts the fiscal evasion.

In the frame of this model [Albu, (2002)], it is considered the situation in which a tax payer with risk aversion receives the possibility to declare to the fiscal a smaller sum than its real income, X. The declared income $X_n > 0$ is imposited with a constant rate, $\theta > 0$, while the undeclared income $X - X_n$ is taxed, if detected, with a larger rate π. The contributable chooses X_n^* for maximizing his expected utility.

$$E[U] = (1-p)U(Y) + pU(Z)$$

where p is the possibility of detection (hexogen data) and $Y = X - \theta X_n$, $Z = X - \theta X_n - \pi(X - X_n)$ represents its income in case of detected and respectively undetected. The ordinal condition first for maximizing E[U] is

$dE[U]/dX_n = -\theta(1-p)U'(Y) + (\pi - \theta)pU'(Z) = 0$ from where the response of the tax payer at a change of θ may be determined by the derivation of

$$dX_n^*/d\theta = -D^{-1}(1-p)U'(Y)\{\theta X_n[R_A(Z) - R_A(Y)] - \pi/(\pi - \theta)\}$$

where $D = \theta^2(1-p)U''(Y) + (\pi - \theta)^2 pU''(Z) < 0$ represents the second order condition for maximization and $R_A(I) = -U''(I)/U'(I)$ is the Arrow–Pratt measure of the absolute aversion to risk. The model is discussed depending on the $dX_n^*/d\theta$ sign, considering the cases where the aversion with absolute risk is in diminution, constant or growing.

3.2 The dynamic model of Dornbusch – the exchange rate and monetary politics (1975)

This model is given by [Dameron, (2001)]

$$p_{t+1} - p_t = \pi\delta(e - p_t)$$

$$e_{t+1} - e_t = \frac{\alpha}{\beta}e_t - \left(\frac{\alpha}{\beta} - \frac{1}{\lambda}\right)p_1 - \frac{1}{\lambda}m - i$$

Where e_t represents the logarithm of the p_t exchange rate, it is the price index number logarithm, i is the intern rate of the mortgage, m the logarithm of the quantity of monetary offer, and π is the adjusting coefficient, and the parameters verify the inequalities $\alpha > 0$, $\beta > 0$, $\lambda > 0$, $\alpha > 0, \beta > \alpha\lambda$.

3.3 The deflation spiral (Groth (1993) and Krugman (1999))

The model presented forward constitutes a try to model the economic recession from Japan and SUA from the last years. All the variables from the model, excepting the inflation and mortgage rates are under the logarithmic form. The model is described by equation [Krugman, (1999)].

$$\begin{aligned} c &= a + b(1-t_1)y, \\ i &= i_0 - h(r - \pi^e), \\ y &= c + i + g, \\ m^d &= ky - ur, \\ m^s &= m - p, \\ m^d &= m^s, \\ \pi &= \alpha(y - y_n) + \pi^e, \\ \dot{\pi}^e &= \beta(\pi - \pi^e) \end{aligned}$$

where c is the real consummation, y the real income, $_i$ real investments, r nominal mortgage rate, π^e prognosed inflation, g governmental spending, m^d real request of money, m^s real money offer, m the nominal stock of money, p the price level, g, y_n, a, i_0 and m are supposed constants, the parameters b, t_1, h, k, u, α and β are positive, and $bt_1 < 1$. The dynamic of the model is analyzed in report with the real money offer and the forecasted inflation rate.

3.4 IS–LM model with tax rate incomes (Cesare and Sportelli, 2005)

We present a model where the real money offer m^s is variable in time, so $m^s = m(t)$, and which considers, at the same time, the incomes from taxes.

In formulating the model, it is purposed for the incomes from taxes $T(t)$ to represent the sum between a component from the current income and one from the past income, so that

$$T(t) = (1-\varepsilon)\tau Y(t) + \varepsilon\tau Y(t-\theta),\ 0 \le \varepsilon < 1,$$

where τ is the medium rate of tax rates, and θ is a constant lateness in collecting taxes.

The IS–LM model with a collection lateness of tax rate payments purposed in [Cesare, and Sportelli, (2005)] is written under this form

$$\begin{aligned} \dot{Y}(t) &= \alpha\left[I(Y(t), r(t)) + G - S(Y^d(t)) - T(t)\right], \\ \dot{r}(t) &= \beta\left[L(Y(t), r(t)) - M(t)\right], \\ \dot{M}(t) &= G - T(t). \end{aligned}$$

α and β are coefficients of positive adjustment, $Y^d(t) = Y(t) - T(t)$ is the real income available.

The third equation refers to the governmental budget constraint and expresses the necessity for the government to supply money for financing its deficit or to retreat money in case of surplus from taxes.

The system's dynamic is very complex, putting in evidence stable or unstable regions, limited cycles generated by the Hopf bifurcation, limited cycles existent simultaneously.

4. Study of case. The inflation evolution in Romania

In this section we extend the Tobin's model [Zhang, (1990)]. We will study the case in which the markets are cleared and expectations are fulfilled in every period of time.

The model is given by

$$\begin{cases} \dot{x} = x\{z - \alpha[x - g(q)]\} \\ \dot{q} = \beta\{\alpha[x - g(q)] - q\} \end{cases}$$

where x is real per capita money holdings, q is the expected inflation rate, z is the constant proportional rate of increase in the nominal stock of money (is fixed by the government), β is the "expectation coefficient" (introduced by Cagan, 1956) and α is a positive constant parameter.

It is assumed that the expected inflation rate may be different from the actual inflation rate.

4.1 Equilibrium points

The equilibrium for this model is given by

$$\begin{cases} x\{z - \alpha[x - g(q)]\} = 0 \\ \beta\{\alpha[x - g(q)] - q\} = 0 \end{cases}$$

whose solution is denoted $(x_0, q_0) = \left(\frac{z}{\alpha} + g(z), z\right)$. The Jacobian matrix is given by $A = \begin{pmatrix} -\alpha x_0 & \alpha x_0 g'(q_0) \\ \alpha\beta & -\alpha\beta g'(q_0) - \beta \end{pmatrix}$ and we obtain $\det A = \alpha\beta x_0$, $trA = -\alpha\beta\left(\frac{x_0}{\beta} + \frac{1}{\alpha} + g'\right)$.

A necessary and sufficient condition for stability of the equilibrium is that $\det A > 0$ and $traceA > 0$. We will do a local analysis for this model.

4.2 Local analysis. The stability of the model

Theorem 4.1 a) If both α and $\beta \to \infty$, then the model is locally unstable,

b) Even if neither α or $\beta \to \infty$, if money is a perfect substitute for capital, the model is locally unstable.

Theorem 4.2. a) The model is locally stable if and only if: $g/\beta + 1/\alpha + g' > 0$;

b) If $\beta \to \infty$, $\alpha < \infty$, then the condition of stability is: $\alpha < -1/g'$;

c) If $\alpha \to \infty$, $\beta < \infty$, then the condition of stability is: $\beta < -g/g'$;

We will determine the function g from the date who we found of the BNR site. We use the date of the December 2005 – June 2008 period.

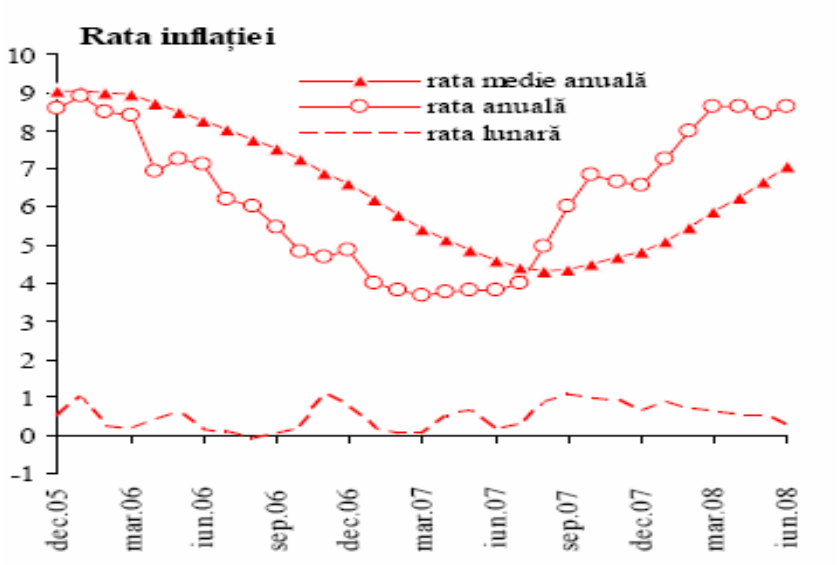

Figure 1. The inflation rate in December 2005 – June 2008 period **Source**: INS, National Bank of Romania

That observes *g* have a sinusoidal form, approaching or declining to the increase line witch represented the target of inflation ($d: 3{,}7x + 40y - 30 = 0$). We propose for the g function the form $g = \frac{\sin\frac{x}{2}}{x} + 8$. We deduced this expression utilized the classic method from prevision the method "the littlest squares". Its graphic is in figure 2.

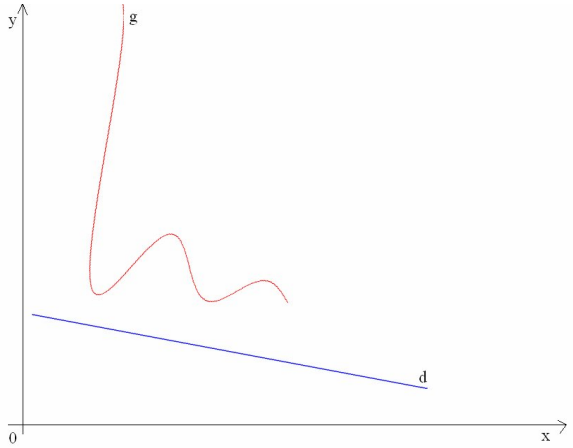

Figure 2. Graphic representation for curves g and d

Theorems 3.1 and 3.2 imply that the equilibrium may be either stable or instable; this is independent on the parameter values.

Consider the phase plane of this dynamics. First we can show that along $\dot{x} = 0$ the following result holds

$dx/dq = g' - 1/\alpha < 0$,

i.e. $\dot{x} = 0$ is a downward–sloping curve. Moreover, above the curve $\dot{x} = 0$, $\dot{x} < 0$ and below it $\dot{x} > 0$. We can similarly examine the properties of $\dot{q}$. The behavior is illustrated in Figure 3.

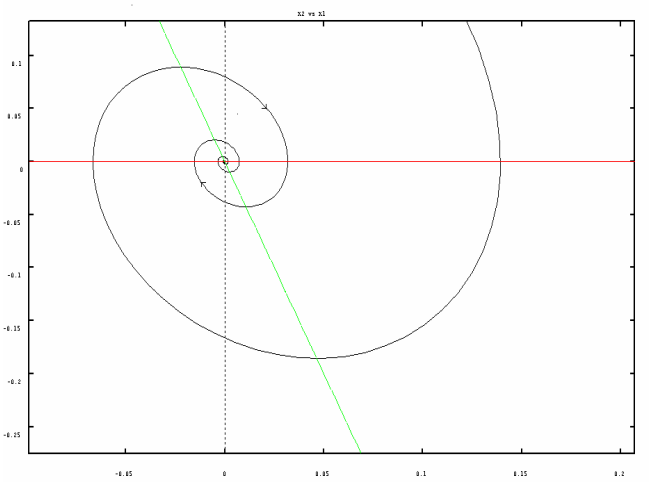

Figure 3. The Case of Stable Focus in x–q plane

5. Conclusion: recent and future development in the modeling of financial date

The relation between the degree of financial development of an economy (measured by the extent in which constraints to credit exist) and fluctuations affecting the trend of economic growth, is a relevant theme of discussion in macroeconomics.

The base element of a decision that determinates it's content, realism and efficiency, is constituted by its scientific fundament, no dissident will be able to adopt a correct decision basing only on intuition, by an empirical way, without a rigorous evaluation of the objective and subjective conditions of its implementation. The scientific foundation structure constitutes the result of some logic and exact methods to lead to the stabilizing of a more efficient decision.

In the last years, we can observe an impressive growth of interest of the scientific community for analyzing the nonlinear systems. The research of such systems, emerged by the studies realized by the mathematic and natural sciences domain researchers, has lead to the development of some new fundamental methods and concepts. Although their application in the frame of the economic science is still in the beginning phase, there have already been obtained some remarkable results of big interest for the economists. There are various economic domains and contexts in which the nonlinear methods can be proven very useful, as well as the behavior of the capital and exchange rate markets, the extern debt problems, economic depressions, hyperinflation and bank risk etc. It is clear that developing concepts and nonlinear methods represents a product of the computer era. Most of the studies from incipient phases have started with numeric analysis of very simple nonlinear methods, which today represent only the base of data editing and calculation by PC. This way it has been discovered that event the most simple nonlinear methods are capable to reproduce a large variety of properties. For example, there has been discovered that very small changes of the parameter values produce surprising results, like even the case of some classic simple methods, which in the past have been considered to have a clear cyclic behavior and easily predictable.

The open character of the national economy in the monetary politics context attest the fact that for reaching and maintaining successfully the macroeconomic stability, a multitude of

circumstances in the frame of which there are implemented the politics and financial–banking institutions are developed must be considered.

The monetary policy has a impact over the output and the workforce occupation on short term, but not for long term, being necessary here to contribute with the real economy. So, the monetary policy is a combination of a target–inflation chosen and a discrete response to certain shocks. These shocks are the ones to which the central bank can respond before the private sector adjusts its activity.

The monetary policy is under the influence of a multitude of hexogen factors, so any prognosis of it hare a high degree of incertitude. However, the monetary authority can purpose certain targets (inflation reduction, choosing the exchange rate course of the national coin) and model its policies in a manner to permit their reaching. Because the prices answer with a certain delay any economic shock, the objective of price stability implicates the debt rate growth immediately after the shock and not waiting for the prices to grow.

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