MODELLING REAL GDP PER CAPITA IN THE USA: COINTEGRATION TESTS

Ivan O. **KITOV**, Oleg I. **KITOV**, Svetlana A. **DOLINSKAYA**
Russian Academy of Sciences, Moscow, **RUSSIA**
Institute for the Geospheres' Dynamics
ikitov@mail.ru

Abstract

A two-component model for the evolution of real GDP per capita in the United States is presented and tested. First component of the growth rate of GDP represents the growth trend and is inversely proportional to the attained level of real GDP per capita, with the nominator being constant through time. Second component is responsible for the fluctuations around the growth trend and is defined as a half of the growth rate of the number of 9-year-olds. This nonlinear relationship between the growth rate of real GDP per capita and the number of 9-year-olds in the US is tested for cointegration. For linearization of the problem, the population time series is predicted using the relationship. Both single year of age population time series, the measured and predicted one, are shown to be nonstationary and integrated of order 1 – the original series have unit roots and their first differences have no unit root. The Engel-Granger procedure is applied to the difference of the measured and predicted time series and to the residuals of a linear regression. Both tests show the existence of a cointegrating relation. The Johansen test results in the cointegrating rank 1. Since the cointegrating relation between the measured and predicted number of 9-year-olds does exist, the VAR, VECM, and linear regression are used in estimation of the goodness of fit and root mean-square errors, (RMSE). The highest R^2=0.95 and the lowermost RMSE is obtained in the VAR representation. The VECM provides consistent, statistically reliable, and significant estimates of the slope in the cointegrating relation. Econometrically, the tests for cointegration show that the deviations of real economic growth in the US from the growth trend, as defined by constant annual increment of real per capita GDP, are driven by the change in the number of 9-year-olds.

Keywords: real GDP per capita, population estimates, cointegration, VAR, VECM, USA

JEL Classification: E32, E37, C53, O42, O51

1. Introduction

There are several macroeconomic variables, which are crucial for both theoretical consideration and practical usage. Undoubtedly, real economic growth is the most important among them. It defines the rate of economic evolution as associated with the increasing volume and quality of goods and services available for a society as a whole and for every member of the society in particular. Conventional economic concepts assume that the growth rate of real GDP reflects routine efforts of each and every economically active person, including those involved in the process of design and control of economic environment. Also, the interactions between economic agents are considered as partly controllable by economic authorities, which base their short-run actions and long-run approaches in the state of the art theories and experience. Such theories have to describe numerous aspects of the interactions between regular agents, and between the agents and the authorities as well. The literature devoted to various problems of real economic growth is extensive. A modern and almost comprehensive review of the achievements in the mainstream economics is available in the *Handbook of Economic Growth* [Agnion and Durlauf, (2005)].

There is an alternative, but simple and natural explanation using a sole cause for real economic growth [Kitov, (2006)]. Under the framework of the economic concept we have been developing since 2005, the only force driving macroeconomic evolution must be associated with some population group of specific (but constant over time) age. The intuition behind this concept is inherently related to the observation of personal income distribution (PID) in the United States. During the years of continuous and relatively accurate measurements of PID between 1960 and 2007, there was practically no change in the distributions, when they are normalized to the total population of 15 years of age and above (i.e. the working age population) and nominal per capita GDP [Kitov, (2005)]. This normalization reduces the PIDs to the portion of total income obtained by a given portion of the working age population. Some minor changes observed in the normalized PIDs are likely explained by the change in the age structure of the US society and the increase of the period when age-dependent average income grows with work experience [Kitov, (2005)]. Effectively, the PIDs demonstrate a rigid hierarchy completely reproduced by every new cohort and also by immigrants. The cohort independence is supported by the absence of any significant change with time in the normalized PID in all age groups defined by the US Census Bureau (2002) as reported in [Kitov, (2006); Kitov, (2008)].

In the economic models developed in econophysics (a branch of statistical physics) there has been a severe constrain and concern related to "conservation of energy" in actual economies [Gallegatia *et al.*, (2006)]. In reality, the gross income, as driven by the production of goods and services, is changing over time. The "frozen" hierarchy of personal incomes resolves the contradiction between the production and exchange in physical models of economy - no change in total income can affect fundamental properties of the economy as a physical system. The rigidity of the overall and age-dependent PIDs does not permit any age group of the population to improve or to lose relative income position in the economic system as a whole. Nominal changes in the absolute level of income are possible, however. In relative terms, a closed economic system has a constant structure.

In physics, there are many similar systems, where distribution of sizes is characterized by a mixture of quasi-exponential and power law distributions, as it is observed in the PIDs measured in the US [Yakovenko and Dragulescu, (2001)]. For example, in seismology the frequency distribution of seismic magnitudes, i.e. the recurrence curve introduced by Guttenberg and Richter, has these two braches – an exponential and a power law ones. Similarly to that in the Earth, any developed (there are no reliable data for developing economies or economies in transition to make any conclusion) economic system reacts to the influx of external "energy" (which is obviously not an equivalent to physical energy but is related to it) and develops the observed hierarchy of personal income distribution. The influx is provided by the existing internal economic agents and also by those who join the economy, i.e. is represented by a net sum of personal productive efforts or energy input. In a stationary case, when the number and age distribution of people is fixed and, hence, the influx is constant, there exists a nonzero economic growth trend (economic potential), which is described by a constant annual increment of real GDP per capita, as actually observed in developed countries [Kitov, (2005)]. Because the increment is constant through years, the growth rate is inversely proportional to the attained level of real GDP per capita.

In a non-stationary case, when the influx of "energy" is disturbed by the changes in the number of people joining the economy, one observes some fluctuations around the nonzero growth trend. It has been found in [Kitov, (2005)] that these fluctuations of real GDP per capita around some constant annual increase are normally distributed. Our model [Kitov, (2006)] assumed that there are no endogenous economic sources of these fluctuations, such as changes in demand and supply, inspirited or/and internally controlled by some economic agents or authorities. These fluctuations, which look like pure random innovations, are defined by the only *external* (exogenous) force. (We would like to stress again that the growth trend is of the endogenous nature.) For real economic growth, this force is the change in a single year of age population. This age is a country-specific one. In the USA and the UK, it makes nine years of age. In other European countries and Japan the age is eighteen years [Kitov, (2006)].

Therefore, one can explicitly formulate a two component model of real economic growth. Empirically, it is based on the observations of the PID in the USA and the normal distribution of annual increments of real GDP per capita in developed countries. This model is absolutely parsimonious since includes only one variable and one constant explaining the whole evolution of an economy, as expressed in monetary units. The model has described the evolution of real GDP per capita in the USA, the UK, France [Kitov, (2006)], and Japan [Kitov, (2006)].

Physics and economics both require any quantitative model to be validated by standard statistical and econometric procedures. Juselius and Franchi [Juselius, and Franchi, (2007)] have proposed the cointegrated vector auto-regression (VAR) as an adequate framework of such validation. The principal idea behind their approach consists in the estimation of statistical properties of the variables defining the models as themselves and in combinations in order to distinguish between probable and unlikely theoretical assumptions. They have also carried out an important initial analysis of conventional theoretical models of real economic growth, RBC and DSGE, and found that some principal assumptions underlying the models are not empirically supported. In a sense, we follow their procedure and also some statistical procedures developed in [Kitov, Kitov, and Dolinskaya, (2007)].

The high standard introduced in [Juselius, and Franchi, (2007)] establishes that any economic model should come from and be justified by empirical data, not from "the easiness of mathematical formulation". At least, the involved variables should meet minimal requirements established by models themselves. Such an approach has been successfully applied in hard sciences and brought a well-recognized reliability of scientific knowledge and technical inventions such as aircrafts, bridges, and so on. The reliability follows from an extensive statistical test of each and every parameter, variable, empirical relationship or fundamental law. Obviously, any physical (and economic) model is actually an approximation to a finite set of statistical links (or scatter plots) between measured variables [Ormerod, (2005)].

Our model describes the measured time series of real GDP per capita in the USA between 1960 and 2002 and allows predictions of the growth of real GDP per capita at various time horizons. The accuracy of these predictions depends on the accuracy of relevant population estimates. In this paper, we test the model (and corresponding data) in econometric sense and demonstrate the existence of a (nonlinear) cointegrating relation between real economic growth and population. The level of confidence associated with the obtained cointegrating relation is high as supported by various statistical tests. The model also involves the lowermost possible number of variables and does not contain any structural breaks. We consider a developed economy as a natural (in sense of physics) system, which evolves according to its own strict laws. Because the system is characterized by a rigid structure of personal income distribution no internal part, including economic authorities, can accelerate the evolution of the system as a whole by economic means. Of course, any part of the system can hamper or stop the evolution, as demonstrated by socialist and developing countries. The predictability and controllability (through demography) of real economic growth are important features of our model, which are wrongly denied by some (econo-) physicists [Gallegatia *et al.*, (2006), Kitov, and Kitov, (2008)].

The remainder of the paper is organized as follows. Section 2 presents a two-component model for real economic growth and the data used in the study. The model is reversed in order to obtain the number of 9-year-olds from measured economic growth, as expressed by real GDP per capita. Section 3 is devoted to the estimation of basic statistical properties of the variables, including the order of integration. Section 4 contains three different tests for cointegration between the measured number of 9-year-olds in the USA and that predicted from the measured GDP – two associated with the Engle-Granger approach and also the Johansen test. Section 5 presents a number of VAR and vector error correction (VEC) models as well as some estimates of root mean square errors (RMSE) and goodness-of-fit. Section 6 discusses principal results and concludes.

2. Model and data

There is a measured macroeconomic variable characterized by a long-term predictability for a large developed economy. This is the annual increment of real GDP per capita [Kitov, (2006)]. One can distinguish two principal sources of the intensive part of real economic growth, i.e. the evolution of real GDP per capita, G: the change in the number of 9-year-olds, and the economic growth trend associated with per capita GDP, G_t. The trend has the simplest form – no change in mean annual increment, as expressed by the following relationship:

$$dG_t(t)/dt = A \quad (1)$$

where G(t) is the absolute level of real GDP per capita at time t, A is an empirical and country-specific constant. The solution of this ordinary differential equation is as follows:

$$G_t(t) = At + B \quad (2)$$

where $B=G_t(t_0)$, t_0 is the starting time of the studied period. Then, the relative growth rate (or economic growth trend) of real GDP per capita is:

$$g_{trend}(t) = dG_t/G_tdt = A/G \quad (3)$$

which indicates that the (trend) rate is inversely proportional to the attained level of the real GDP per capita and the growth rate should asymptotically decay to zero.

One principal correction has to be applied to the per capita GDP values published by the Bureau of Economic Analysis (2006). This is the correction for the difference between the total population and the population of 15 years of age and above, as discussed by Kitov (2006). Our concept requires that only this economically active population should be considered when per capita values are calculated.

Following the general concept of the two principal sources of real economic growth [Kitov, (2006)] one can write an equation for the growth rate of real GDP per capita, $g_{pc}(t)$:

$$g_{pc}(t) = dG(t)/(dt\Box G(t)) = 0.5dN_9(t)/(dt\Box N_9(t)) + g_{trend}(t) \quad (4)$$

where $N_9(t)$ is the number of 9-year olds at time t. One can obtain a reversed relationship defining the evolution of the 9-year-old population as a function of real economic growth:

$$d(\ln N_9(t)) = 2(g_{pc} - A/G(t))dt \quad (5)$$

Equation (5) defines the evolution of the number of 9-year-olds as described by the growth rate of real GDP per capita. The start point of the evolution has to be characterized by some (actual) initial population. However, various population estimates (for example, post- and intercensal one) potentially require different initial values and coefficient A.

Instead of integrating (5) analytically, we use the annual readings of all the involved variables and rewrite (5) in a discrete form:

$$N_9(t+\Delta t) = N_9(t)[1 + 2\Delta t(g_{pc}(t) - A/G(t))] \quad (6)$$

where Δt is the time unit equal to one year. Equation (6) uses a simple representation of time derivative of the population estimates, where the derivative is approximated by its estimate at point t. The time series g_{pc} and N_9 are independently measured variables. In order to obtain the best prediction of the $N_9(t)$ by the trial-and-error method one has to vary coefficient A and (only slightly in the range of the uncertainty of population estimates) the initial value - $N_9(t_0)$. The best-fit parameters can be obtained by some standard technique minimising the RMS difference between predicted and measured series. In this study, only visual fit between curves is used, with the average difference minimised to zero. This approach might not provide the lowermost standard deviation.

Equation (6) can be interpreted in the following way – the deviation between the observed growth rate of GDP per capita and that defined by the long-tern trend is completely defined by the change rate of the number of 9-year olds. A reversed statement is hardly to be correct - the number of people of some specific age can not be completely or even in large part defined by contemporary real economic growth. Specifically, the causality principle prohibits the present to influence the birth rate nine years ago. Econometrically speaking, the number of 9-year olds has to be a weakly exogenous variable relative to contemporary economic growth. This property of the variables is used in the VAR models in Section 5.

In fact, Eq. (6) provides an estimate of the number of 9-year-olds using only independent measurements of real GDP per capita. Therefore, the amplitude and statistical properties of the deviation between the measured and predicted number of 9-year olds can serve for the validation of (4) and (5). In Sections 3 through 5 we use the predicted number of 9-year-olds for statistical estimates instead of the real GDP per capita readings themselves. The link between population and economic growth is effectively nonlinear and there would be difficult to study it in a linear representation. Since both involved variables are measured with some uncertainty and probably are nonstationary, the cointegrated VAR analysis should be an appropriate one.

There are numerous revisions and vintages of the population estimates. Figure 1 compares post- and intercensal population estimates of the number of 9-year olds between 1960 and 2002 [U.S. Census Bureau, (2007)]. The error of closure, i.e. the difference between the census count and the postcensal estimate at April 1, 2000, is 57233. The error of closure for the population group between 5 and 13 years of age is 1309404, however, i.e. approximately twice as large for every single year of age as that for the 9-year-olds. For the intercensal estimate, this error of closure is proportionally distributed over the 3653 days between April 1, 1990 and April 1, 2000 [U.S. Census Bureau, (2004)]. Hence, the level of the intercensal estimate is represented by the level of the postcensal one plus corresponding portion of the error of closure. The curves in Figure 1 demonstrate a growing divergence between these two estimates. There are also some non-zero corrections between adjacent years of birth in wider age groups. After April 2000, both estimates in Figure 1 are apparently postcensal with different bases in 2000. Even this minor deviation between the estimates might be of importance for statistical tests and inferences and both are analyzed in this study.

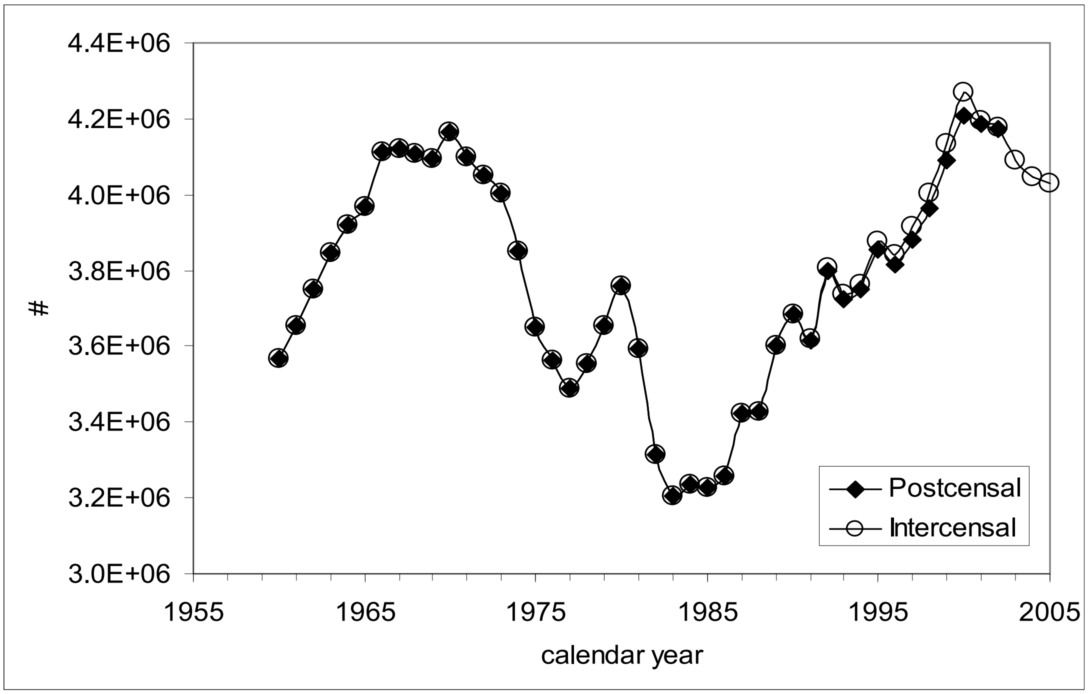

Figure 1. Comparison of the postcensal and intercensal estimates of the number of 9-year olds reported by the US Census Bureau (2007). The difference is observed only during the years between 1990 and 2002.

Real GDP per capita is estimated using total real GDP and the number of people of 15 years of age and above. This excludes from the macroeconomic consideration those who do not add to real economic growth [Kitov, (2006)]. Figure 2 depicts the growth rate of real GDP per capita in the USA between 1960 and 2002 used in the study. In average, the growth rate is 0.020 with standard deviation of 0.022. There are seven negative readings coinciding with the recession periods defined by the National Bureau of Economic Research (2007).

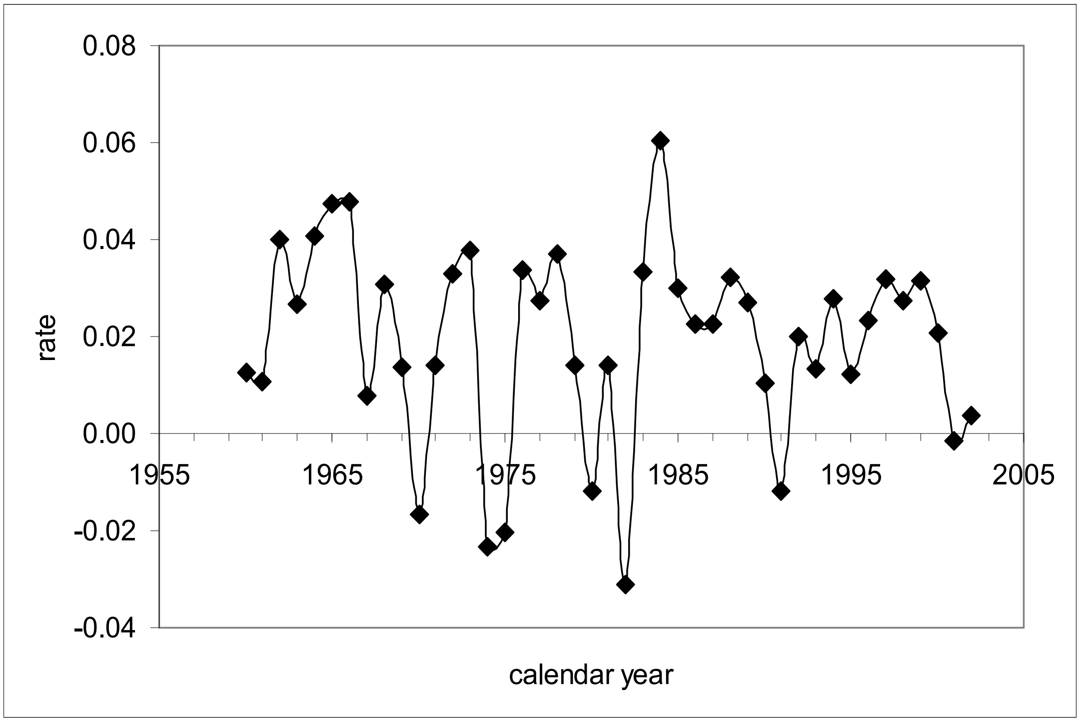

Figure 2. The growth rate of real GDP per capita in the USA between 1960 and 2002. The growth rate is corrected for the difference between total population and that above 15 years of age [Kitov, I., (2006)].

The period between 1960 and 2002 has been chosen by the following reasons. Before 1960, the single year of age population estimates are not reliable and might introduce a significant distortion in statistical estimates and inferences. After 2002, the GDP values are prone to comprehensive NIPA revisions of unknown amplitude, which historically occurred about every 5 years [Fixler, and Green, (2005)]. The most recent comprehensive revision was in 2003 and spanned the years between 1929 and 2002.

3. Unit root tests

The technique of linear regression for obtaining statistical estimates and inferences related to time series is applicable only to stationary series, as Granger and Newbold showed [Granger, and Newbold, (1967)]. Two or more nonstationary series can be regresses only in the case when there exists a cointegrating relation between them [Hendry, and Juselius, (2001)], with several precautions discussed in [Engle, and Granger, (1987)]. Therefore, the first step in any econometric analysis of time dependent data sets is currently consists is the estimation of the order of integration of involved series. Unit root tests applied to original series and their first and higher order differences are a useful tool to determine the order of integration.

Standard econometric package Stata9 provides a number of appropriate procedures implemented in an interactive form. The Augmented Dickey-Fuller (ADF) and the modified DF t-test using a generalized least-squares regression (DF-GLS) are used in this study. Potentially, the tests provide adequate results for the available short series consisting of only 41 annual readings - the real GDP per capita and the number of 9-year olds. Small samples are usually characterized by a limited reliability of statistical inferences.

There are four original time series tested for unit roots - the measured and predicted according to (6) number of 9-year-olds between 1962 and 2002. Each of the series contains two versions - a postcensal and intercensal one (for the period between 1990 and 2000, i.e. between two decennial censuses). The difference is minor, as Figure 1 demonstrates, but the intercensal series potentially contains such artificial features as autocorrelation introduced by the Census Bureaus during the revision associated with the error of closure. Statistically, the postcensal time series might be less "contaminated" than the intercensal one.

Some results of the unit root tests for the four original series are listed in Table 1. All these series are characterized by the presence of unit roots - the test values are significantly larger than the 1% critical values. In the ADF tests, trend specification is *constant* and the maximum lag order is 3. In the DF-GLS tests, the maximum lag is 4 and the same trend specification is used. Hence, one can conclude that the studied time series are nonstationary. The order of integration is not clear, however.

Table 1. Unit root tests for the measured and predicted number of 9-year-olds. Trend specification is constant

Test	Lag	Intercensal		Postcensal		1% critical
		predicted	measured	predicted	measured	
ADF	0	-1.50	-0.72	-1.51	-0.70	-3.65
	1	-2.10	-1.39	-2.10	-1.40	-3.66
DF-GLS	1	-2.34	-1.52	-2.35	-1.55	-2.63
	2	-1.82	-1.60	-1.82	-1.62	-2.63

The first differences of the measured and predicted number of the 9-year-olds (the postcensal version) between 1962 and 2002 (the reading for 1961 is also used in the difference) are presented in Figure 3. There is no visible trend in the data and one can presume a constant as trend specification. The average value is 9600 and 12625, and standard deviation is 152487 and 105287 for the measured and predicted time series, respectively.

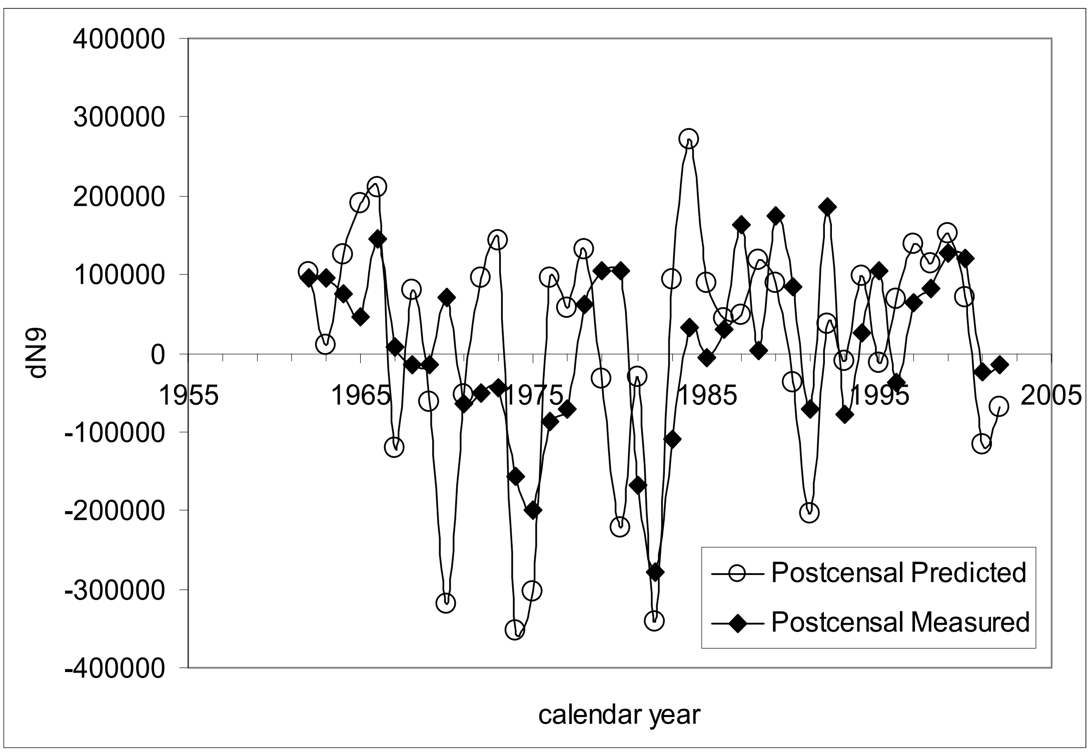

Figure 3. The first differences of the measured (postcensal) and the predicted number of 9-year-olds. There is no visible trend in the time series with average values 9600 and 12625, respectively. Standard deviation is 152487 and 105287.

Table 2 summarizes some results of the unit root tests as applied to the first differences. The predicted time series are definitely characterized by the absence of unit roots, as the ADF and DF-GLS both demonstrate for the maximum lag order 2. For lag 3, the ADF gives values just marginally below the 1% critical value. The measured time series have specific autoregressive properties intrinsically related to the methodology of population revisions and are characterized by mixed results for the unit root tests. The DF-GLS test rejects the null hypothesis of the presence of a unit root for all lags from 1 to 3. The ADF rejects the null only for lag 0. Bearing in mind the shape of the measured original curves in Figure 1, which demonstrate a quasi-sinusoidal behaviour without any significant linear trend; one can assume that their first differences are stationary. In this study, the absence of unit roots in all the first difference series is accepted.

Table 2. Unit root tests for the first differences of the measured and predicted number of 9-year-olds. Trend specification is constant. The maximum lag order is 3.

Test	Lag	Postcensal		Intercensal		1% critical
		predicted	measured	predicted	measured	
ADF	0	-4.86*	-4.22*	-4.87*	-4.27*	-3.65
	1	-4.66*	-3.37	-4.67*	-3.39	-3.66
	2	-3.86*	-2.80	-3.86*	-2.80	-3.66
	3	-3.44	-3.22	-3.44	-3.20	-3.67
DF-GLS	1	-4.64*	-3.01*	-4.54*	-3.02*	-2.63
	2	-3.67*	-2.48	-3.67*	-2.48	-2.63
	3	-3.12*	-2.84*	-3.12*	-2.84*	-2.63

The presence of unit roots in the original series and the absence of unit roots in the first differences evidences that the former series are integrated ones of order 1. This fact implies that cointegration analysis has to be carried out before any linear regression because the latter is potentially a spurious one.

4. Cointegration test

The assumption that the measured number of 9-year-olds in the USA, $N_{9m}(t)$, and that predicted from the real economic growth, $N_{9p}(t)$, are two cointegrated non-stationary time series is equivalent to the assumption that their difference, $\varepsilon(t)=N_{9m}(t) - N_{9p}(t)$, is a stationary or I(0) process. The predicted and measured series corresponding to the post- and intercensal population estimates are shown in Figures 4 and 6, and their differences in Figures 5 and 7, respectively.

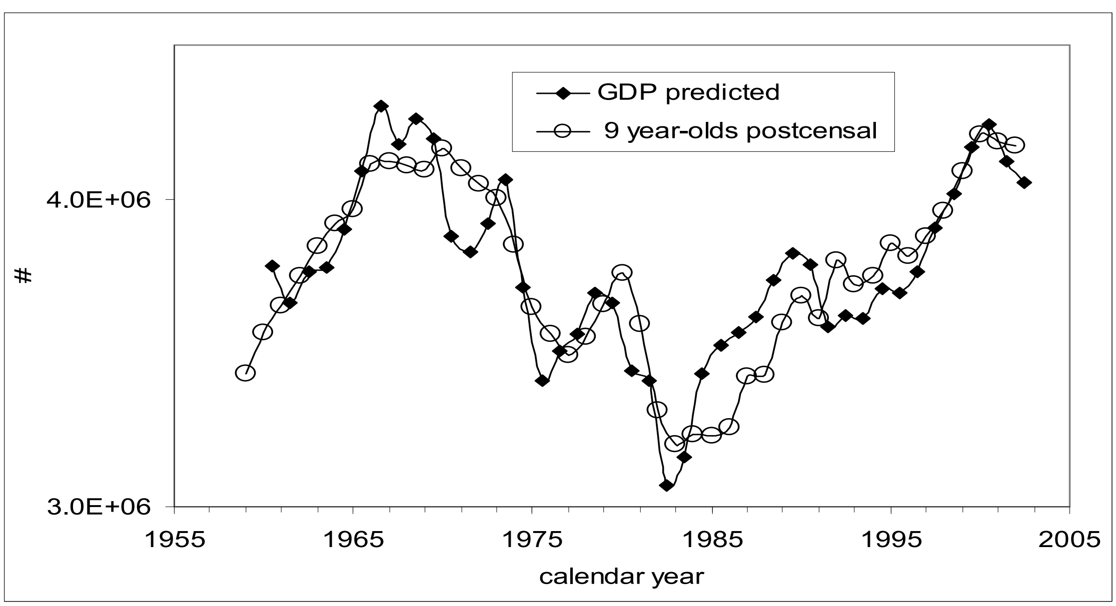

Figure 4. Comparison of the measured and predicted postcensal population estimates between 1960 and 2002.

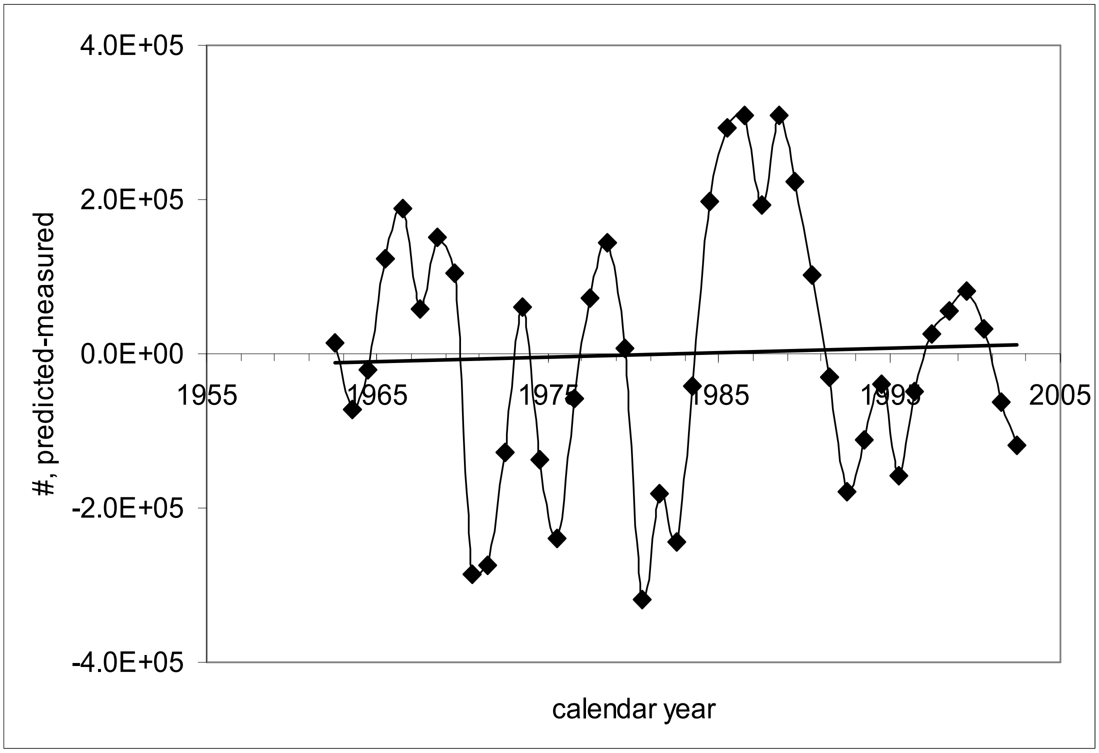

Figure 5. The difference between the measured and predicted population estimates presented in Figure 4. For the period between 1962 and 2002, the average difference is 0 and standard deviation is 164926 for coefficient A=547.1325 and the initial value for the population of 3900000 in 1959. Linear regression is represented by a bold straight line.

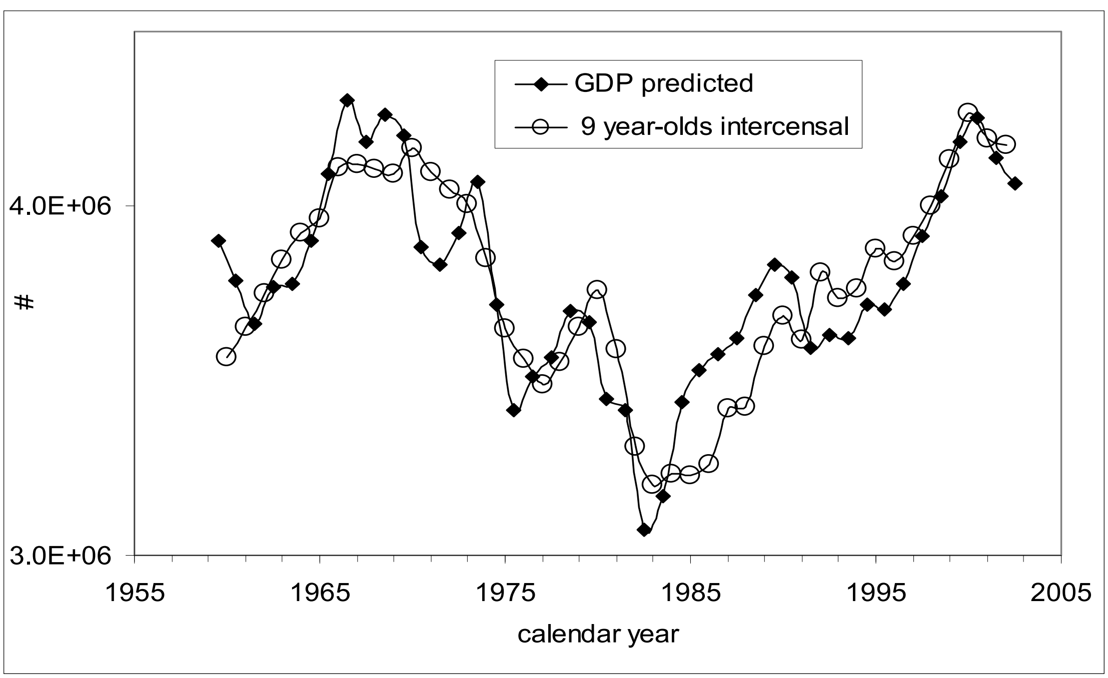

Figure 6. Comparison of the measured and predicted intercensal population estimates between 1960 and 2002.

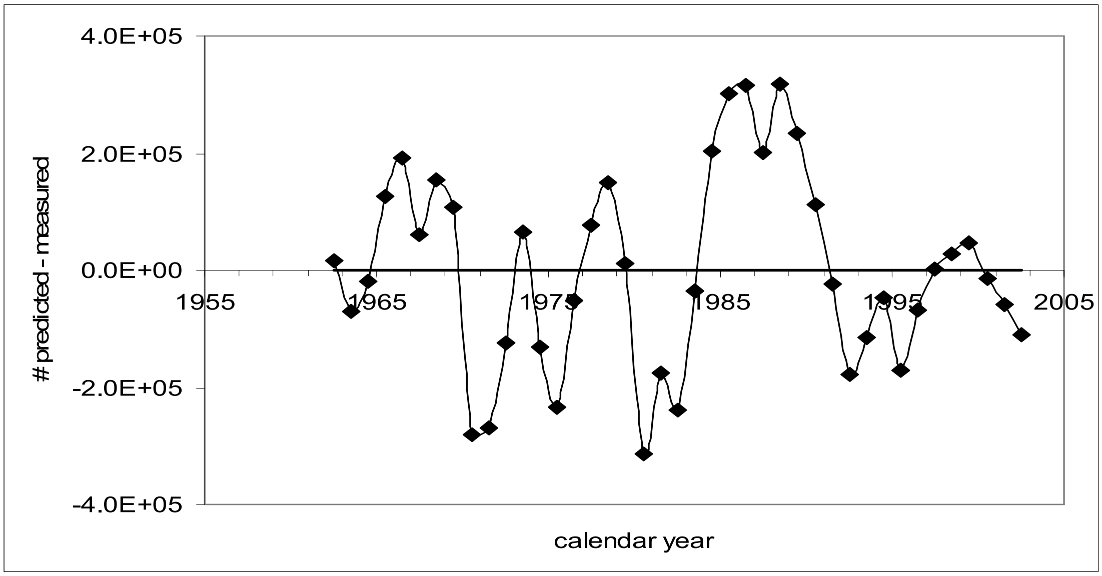

Figure 7. The difference between the measured and predicted population estimates presented in Figure 6. For the period between 1962 and 2002, the average difference is -1 and standard deviation is 165744 for coefficient A=546.079 and the initial value of population of 3900000 in 1959.
Linear regression is represented by a bold straight line.

It is natural to start with unit root tests in the difference. If ε(t) is a non-stationary variable having a unit root, the null hypothesis of the existence of a cointegrating relation can be rejected. Such a test is associated with the Engle-Granger approach [Engle, and Granger, (1987)], which requires the $N_{9m}(t)$ to be regressed on the $N_{9p}(t)$ as the first step, however. It is worth noting, that the predicted variable is obtained by a procedure similar to that of linear regression and provides the best visual fit between corresponding curves. The Engle-Granger approach is most reliable and effective when one of the two involved variables is weakly exogenous, i.e. is driven by some forces not associated with the second variable. This is the case for the GDP per capita and the number of 9-year-olds. The latter variable is hardly to be driven by the former one. The existence of an opposite causality direction is the main object of this study.

The results of the ADF and DF-GLS tests, listed in Table 3, demonstrate the absence of a unit root in the measured-predicted difference series for both the post- and intercensal population estimates. Since the predicted series are constructed in the assumption of a zero average difference, trend specification in these tests is "*none*". The maximum lag order in the tests is 3. These results give strong evidences in favor of the existence of a cointegrating relation between the measured and predicted time series. Therefore, from the econometric point of view, it is difficult to deny that the number of 9-year-olds is *the only* defining force behind the observed fluctuations of the real economic growth. These fluctuations are observed around the growth trend defined by constant annual increment, A, of the real GDP per capita.

Table 3. Unit root tests for the differences between the measured and predicted number of 9-year-olds. Trend specification is constant. The maximum lag order is 3.

Test	Lag	Time series		1% critical
		postcensal	intercensal	
ADF	0	-2.87*	-2.85*	-2.64
	1	-3.67*	-3.59*	-2.64
	2	-2.99*	-3.92*	-2.64
	3	-2.90*	-2.83*	-2.64
DF-GLS	1	-3.55*	-3.47*	-2.64
	2	-2.98*	-2.92*	-2.64
	3	-2.92*	-2.85*	-2.64

The next step is to use the Engle-Granger approach again and to study statistical properties of the residuals obtained from linear regressions of the measured and predicted single year of age populations. A pitfall of the regression analysis consists in a slight time shift between the measured and predicted series – the former variable is assigned to July 1 (averaged population) and the latter to December 31 (cumulative GDP increase) of the same year. Such a phase shift, apparently, results in a deterioration of regression results but can not be recovered since only annual population estimates are available before 1980.

Table 4 presents a summary of relevant unit root tests with the same specifications as accepted for the difference of the same series. The null hypothesis of a unit root presence is rejected for both time series and all time lags. Therefore, the residuals of the regression build an I(0) time series, and the Engle-Granger tests proves that the predicted and measured variables are cointegrated.

Table 4. Unit root tests for the residual time series of a linear regression of the measured series on the predicted one. The measured and predicted series are the numbers of 9-year-olds. Trend specification is *none* (zero average value of the residuals) and maximum lag order 3.

Test	Lag	Time series		1% critical
		postcensal	intercensal	
ADF	0	-3.03*	-3.02*	-2.64
	1	-3.88*	-3.86*	-2.64
	2	-3.15*	-3.13*	-2.64
	3	-3.05*	-3.01*	-2.64
DF-GLS	1	-3.71*	-3.69*	-2.64
	2	-3.06*	-3.04*	-2.64
	3	-2.98*	-2.95*	-2.64

The Johansen [Johansen, (1988)] approach is based on the maximum likelihood estimation procedure and tests for the number of cointegrating relations in the vector-autoregressive representation. The Johansen technique allows simultaneous testing for the existence of cointegrating relations and determining their number (rank). For two variables, only one cointegrating relation is possible. When cointegration rank is 0, any linear combination of the two variables is a non-stationary process. When the rank is 2, both variables have to be stationary. When the Johansen test results in rank 1, a cointegrating relation between the involved variables does exist.

In the Johansen approach, one has first to analyze some specific properties of the underlying VAR model for the two variables. Table 5 lists selection statistics for the pre-estimated maximum lag order in the VAR. Standard trace statistics is extended by several useful information criteria: the final prediction error, FPE; the Akaike information criterion, AIC; the Schwarz Bayesian information criterion – SBIC; and the Hannan and Quinn information criterion, HQIC. All tests and information criteria in Table 5 indicate the maximum pre-estimated lag order 1 for VARs and vector error-correction models, VECMs. Therefore, the maximum lag order 1 was used in the Johansen tests along with *constant* as the trend specification.

Table 5. Pre-estimation lag order selection statistics. All tests and information criteria indicate the maximum lag order 1 as an optimal one for VARs and VECMs.

	Lag	LR	FPE	AIC	HQIC	SBIC
postcensal	1	63.03*	5.8e+09*	25.31*	25.36*	25.44*
intercensal	1	61.63*	6.1e+09*	25.38*	25.42*	25.51*

FPE - the final prediction error, AIC - the Akaike information criterion, SBIC - the Schwarz Bayesian information criterion, HQIC - the Hannan and Quinn information criterion

The properties of the VAR error term have a critical importance for the Johansen test [Hendry, and Juselius, (2001)]. A number of diagnostic tests was carried out for the VAR residuals. The Lagrange multiplier test for the postcensal time series resulted in χ^2 of 0.34 and 0.09 for lags 1 and 2, respectively. This test accepts the null hypothesis of the absence of any autocorrelation at these lags.

The Jarque-Bera test gives χ^2=7.06 (Prob>0.03) with skewness=0.96 and kurtosis=3.77, the skewness being of the highest importance for the normality test and the validity of statistical inference. Hence, the residuals are probably not normally distributed, as expected from the artificial features of the measured population time series. The VAR model stability is guaranteed by the eigenvalues of the companion matrix, which are lower than 0.63. As a whole, the VAR model accurately describes the data and satisfies principal statistical requirements applied to the residuals.

Table 6 represents some results of the Johansen tests. In both cases the cointegrating rank is 1. Hence, there exists a long-run equilibrium relation between the measured and predicted number of 9-year-olds in the USA. The predicted number is obtained solely from the readings of real GDP per capita measured and reported by the BEA (2007). We do not test for the causality direction between the variables because the only possible way of influence, if it exists, is absolutely obvious.

Table 6. Johansen test for cointegration rank for the measure and predicted time series. Trend specification is constant. Maximum lag order is 2.

Time series	Rank	Eigenvalue	SBIC	HQIC	Trace statistics	5% critical value
postcensal	1	0.397	52.48*	52.23*	2.198*	3.76
intercensal	1	0.379	52.55*	52.30*	2.117*	3.76

In this Section, three different tests have demonstrated at a high level of confidence that the measured and predicted number of 9-year-olds in the USA are cointegrated. One can use the cointegrating relation for a reliable prediction of real economic growth in the USA. This finding proves that the evolution of a developed economy is predictable in principle.

5. VAR, VECM, and linear regression

Now, it is proved by standard econometric tools that the measured and predicted single year of age population series are cointegrated. Therefore, the estimates of the goodness-of-fit, R^2, and RMSE in various statistical representations have to be valid and can provide important information on the accuracy of relevant population and economic measurements, and the relation itself.

The VAR representation provides a good estimate of R^2 and RMSE due to strong noise suppression. In practice, AR is a version of a weighted moving average, which optimizes noise suppression throughout the whole series. Two VAR models are possible, however, with the predicted time series used as an exogenous predictor and as an endogenous variable. Table 7 summarizes some results of the VAR models and demonstrates that the goodness of fit is excellent, with the highest R^2~0.95 and the lowermost RMSE near 72000 corresponding to the exogenous predicted time series for the postcensal population estimates. This version of VAR uses the maximum lag order 2, and the Table confirms that coefficient L2 is not significant in line with the previous estimates of the maximum lag. The coefficient for the predictor is significant.

Table 7. VAR models for the measured and predicted number of 9-year-olds for the postcensal and intercensal estimates. Maximum lag order is 2. Two cases for the predicted time series are considered - endogenous and exogenous one.

Measured-Predicted VAR	RMSE	R^2	Measured		Predicted		
			L1	L2	L0	L1	L2
exogenous - postcensal	71645	0.9489	0.82* [0.13]	-0.12 [0.12]	0.34* [0.06]	-	-
endogenous - postcensal	89300	0.9229	0.85* [0.19]	-0.17 [0.15]	-	0.33* [0.11]	-0.03 [0.12]
exogenous - intercensal	73954	0.9474	0.82* [0.13]	-0.11 [0.12]	0.35* [0.06]	-	-
endogenous - intercensal	92440	0.9202	0.88* [0.19]	-0.17 [0.16]	-	0.33* [0.11]	-0.05 [0.12]

The VECM representation uses information additional to that provided by the VAR models due to separation of noise and equilibrium relation. So, it potentially provides an improvement on the VAR models. Table 8 lists some results obtained in the VECM (cointegrated VAR) representation. Coefficient β, defining the link between the measured and predicted series, is significant in both cases confirming the existence of a cointegrating relation. Coefficients α_1 and α_2 define the input of the cointegrating relation to the I(0) time series of lagged first differences of the measured and predicted series. Their estimates are significant and show a relatively large error correction effect. Coefficients of the LD terms are both insignificant as corresponded to the largest lag order 2. The values of R^2 are relatively high (0.34 and 0.32) and RMSE is ~90000 and 130000 for the postcensal and intercensal series, respectively. The RMSE values are slightly larger than those from the VAR models.

Table 8. VECM for the postcensal and intercensal estimates of the number of 9-year-olds. The maximum lag is 2. Cointegrating rank 1 for the relationship between the measured and predicted time series.

Measured-Predicted VECM	RMSE	R^2	β	α_1	α_2	Measured LD	Predicted LD
postcensal	89839	0.3446	-1.21* [0.11]	-0.24* [0.10]	0.28 [0.17]	0.11 [0.16]	0.06 [0.13]
intercensal	93007	0.3181	-1.24* [0.12]	-0.22* [0.10]	0.29 [0.16]	0.11 [0.16]	0.08 [0.13]

Finally, Table 9 is representing the results of linear regressions. These results are biased by the time shift between the series and are inferior to those obtained using VAR and VECM. The moving average technique, however, provides a slight improvement in the statistical estimates. This effect is inherently related to noise suppression in the time series.

Table 9. Results of linear regression of the measured time series on the predicted one.

Time series	Regression	Tangent	Constant	R^2	RMSFE
postcensal	M vs. P	0.85* [0.09]	569325 [326128]	0.71	160000
	M vs. MA(2)	0.94* [0.07]	221197 [274652]	0.81	130000
	M vs. MA(3)	1.09* [0.06]	-318464 [231765]	0.89	99985
intercensal	M vs. P	0.86* [0.09]	511114 [330855]	0.72	160000
	M vs. MA(2)	0.96* [0.06]	167488 [281940]	0.81	130000
	M vs. MA(3)	1.04* [0.07]	-126233 [249095]	0.86	110000

M – measured time series
P – predicted time series
MA(N) – N-year moving average

Despite a very high goodness-of-fit, approaching 0.95, in the VAR representation, the RMSE estimates are relatively large. This severely complicates the usage of Eq. (4) for the prediction of real economic growth in the USA. The RMSEs are comparable in amplitude with the uncertainty of the population estimates, especially at younger ages [West, Robinson, (1999)]. In addition, a conservative estimate of the uncertainty of growth rate of real GDP is between 0.5 and 1 percentage point, which includes also the uncertainty associated with CPI and GDP deflator. In order to distinguish between these measurement errors and true deviations in the cointegrating relations one needs a substantial improvement in population estimates.

6. Conclusion

There is an equilibrium (nonlinear) long-run relation between the number of 9-year-olds and real GDP capita in the United States. This fact implies that real economic growth, as expressed in monetary units, is practically predetermined by the age structure of the US society. An increasing number of 9-year-olds would guarantee an accelerating growth, extra to that defined by the constant annual increment of real GDP per capita.

At low frequencies, the behavior of the number of 9-year-olds in the USA is characterized by a visible period of about 30 years, between the peaks in 1970 and 2000. Such long-period oscillations in economic evolution are well-know since the 1920s, when Russian economist Nikolai Kondratiev published his original analysis. Our model gives a natural explanation of the Kondratiev waves – they are related to the natural increases and decreases in birth rate (and/or migration). For numerous reasons, the birth rate fluctuates and cycles are observed at all frequencies.

A bad news for the USA is that the ten to fifteen years since 2000 will be probably associated with a decreasing branch of the K-wave. Taking into account the effect of the decreasing background growth rate associated with the increasing real GDP per capita in Eq. (3), one can expect a significant deceleration in the US economy as expressed by a lower growth rate of real GDP per capita. However, if the total population will continue to grow at an annual rate of 1 per cent, as has been observed in the USA during the last forty years, the negative effect of the N_9 decrease will be compensated. In developed European countries, the effect of the total population growth is practically negligible and they seemingly do not grow so fast as the USA does. There is just an illusion of an elevated growth rate, which disappears when one uses per capita GDP values.

The fluctuations of the annual increment of real GDP per capita around the average level represent a random process. This stochastic component is driven only by one force and can be actually predicted to the extent one can predict the number of 9-year-olds at various time horizons. The population estimates for younger ages in previous years provide an excellent source for this prediction. The growth rate of a single year population can be predicted with a higher accuracy because the levels of adjacent cohorts change proportionally. Therefore, the number of 7-year-olds today is a very good approximation to the number of 9-year-olds in two years. Theoretically, one can use the younger populations for an exact prediction. In practice, the current methodology of population estimates does not provide adequate precision and only long-term changes have a high enough signal (true change) to noise (measurement error) ratio to resolve of the link between real economic growth and population, as Figures 4 and 6 illustrate.

The concept we have been developing links the fluctuations of real growth rate to young people (9-year-olds) likely being outside the structure of economic production. However, they bring to the economic system a nonzero and changing input, which can be interpreted as demand for goods and services. Those economic agents who are currently inside the system can not change real demand per capita due to the rigid PID. Immigrants and the population decrease associated with deaths also cannot change per capita GDP values because the PID does not demonstrate any effect of these potential sources of changes. One can presume that the hierarchy of personal incomes momentarily recovers to its origin structure, when accommodating the disturbances induced by these two sources.

The model of real economic growth tested in this study is supported by the results reported in [Juselius and Franchi, (2007)] that the principal source of economic variations is the demand for consumption and for labor but not shocks to technology or total factor productivity. (Labor productivity in developed countries is driven only by real economic growth and labor force participation rate [Kitov and Kitov, (2008)]. The latter also is an unambiguous function of real economic growth, as expressed by real GDP per capita [Kitov and Kitov, (2008)].) Newcomers entering the economy, as represented by 9-year-olds, somehow bring and introduce their long-term demand for consumption into the economic system. This demand has been changing over time according to the variations in the number of 9-year-olds and induces relevant changes in the demand for labor. A complication to conventional models is the decelerating economic trend, as defined by Eq. (3).

Expenditures in developed economies cannot be separated into two distinct parts, which are usually described as saving (investment) and consumption, the former being the driving force of shocks to technology and total factor productivity. Many theories of endogenous economic growth, however, are based on this assumption and stress the importance of investment for the rate of economic growth. Under our framework, there is no direct link between real economic growth, as expressed in monetary units (per capita), and technological content. In other words, any set of technological breakthroughs achieved during a certain period, for example one year, has the same money valuation. What important for the monetary size is only changes in quantitative characteristics of population – the age structure. We also do not share the opinion or assumption that investments are made for the sake of economic growth *per ce*. One hardly can imagine that an owner, shear holder or manager who really wants an overall economic growth and decides what input s/he can bring to the process. Investment decisions are rather made for a sole purpose, which is psychologically and economically justified, one wishes by all means to elevate the current position in relevant PID.

Technological innovations (not only purely technological, but also cultural in a broader sense) have been stimulating the growth in the diversity of goods and services. At the same time, the innovations were helpful in creating new tools for deposing some people from their top positions in the PID. The rigidity of the PID does not allow joining the top positions – only deposing is possible (when working age population does not change). However, not all technologically excellent discoveries guarantee income increase.

Therefore, the main purpose to invest is to progress in the income pyramid to higher steps. This is a routine, strong and long-run interest and demand. Sometimes it uses not the best sides of human psychology and reflexes. But, in general, it makes what it should make – brings random and deterministic innovations in technologies. Juselius and Franchi [Juselius and Franchi, (2007)] justified our concept by empirical analysis. No technological innovations induce fluctuations in economic growth. (We do not consider here technical policy aimed at the selection of sound innovations, which can definitely bring a better result for the society as a whole. For example, investments in military technologies brought a large-scale profit to many areas of civil techniques.) The authors of [Juselius, and Franchi, (2007)] deny the possibility of technology, whatever it is, to drive monetary side of social life.

The Great Moderation is easily explained in our framework. Amplitude of the fluctuations of the defining age population around the constant level has been decaying since the 1980s, as Figure 5 and 7 demonstrate. The reasons behind the smoothing of the population changes are beyond the scope of this study but deserve a special attention. The economic growth trend, as a part of the growth rate of real GDP, has been also decreasing with increasing per capita GDP level as denominator. Inflation in the USA and other developed countries is driven by the change in the level of labor force [Kitov, (2006), (2007), Kitov, Kitov, and Dolinskaya, (2007)], which in turn, is defined by real GDP per capita and total population. Therefore, the observed decrease in the volatility of the GDP growth rate leads to lower fluctuations in inflation. The Great Moderation is not going to leave the scene in the future.

7. References

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