Dynamics of neighborhood formation and segregation by income

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Abstract

This paper analyzes some determinant conditions under which neighborhood formation gives rise to segregation by income. In contrast to the literature, we explore the *sequential* arrival of poor and rich individuals to neighborhoods exploited by oligopolistic land developers. These developers try to maximize a discounted flow of lot prices during neighborhood formation, taking advantage of the local externalities generated by the rich and the poor. Under a speedy arrival of new potential inhabitants and/or low discount rates, competing developers are more likely to concentrate rich people in the same neighborhood. This happens because the benefits from early agglomeration are outweighed by a more profitable matching of rich neighbors within nearby lots.

1. Introduction

The choice of neighborhood is one of the most determinant decisions in the social and economic life of any individual. That is especially significant when segregation (by race, income, etc) has long-lasting effects in the dynastic trajectories of those favored and disfavored by the environment they live in. In this paper we propose a framework in which, from a purely positive – as opposed to normative – point of view, we can analyze the *dynamic* conditions in which *new* integrated and segregated neighborhoods arise.

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As it is well-known, the endogenous processes of socioeconomic sorting or scrambling have been already extensively studied by the profession. Early pioneers, like Schelling, were able to vividly describe the conditions for integrated equilibria to become unstable. The main difference between Schelling (1978)'s study and Becker and Murphy (2000)'s is that the former analyzes a process by which the members of two (or more) groups want to congregate with other members of *their own kind*; whereas in the latter case there is a unique desirable type ('the high', or 'the rich'), and everybody wants to be close to them. In principle, we may think that segregated equilibria are more plausible in Schelling's environment; though De Bartolome (1990) and Benabou (1993) show that – under reasonable conditions – segregation is also excessive from a normative viewpoint in the second case.

In our model, as in the cases of Schelling (1978) or Becker and Murphy (2000), externalities within neighborhoods are the driving force of individuals' welfare and willingness to pay. For instance, anybody would like to live close to well reputed judges or doctors who could help you in case of need. Therefore, people will tend to receive higher positive externalities from high ('rich') rather than low ('poor') types.[3] In their model (like in ours) people derive utility both from local externalities and from housing facilities (amenities)[4].

However, their starting point is a fully integrated neighborhood where everybody has a residence from the beginning. In that context, they analyze the conditions under which the initially integrated configuration evolves towards a different one (either partially or fully segregated). On the contrary, in our model we portray new neighborhoods that are filled step by step with the *sequential arrival* of poor and rich residents. Also unlike Becker and Murphy (2000), which includes a *single* developer and *competitive bidding* by households (those with the highest willingness to pay always obtain the desired residence); we consider a situation of duopolistic competition between developers. That competition (together with the fact that people arrive sequentially and henceforth cannot directly compete with each other) prevents the extraction of the whole willingness to pay from buyers.

[3] Strictly speaking, in our model there is nothing specific about segregation *by income*. Our framework can be equally applied to racial segregation phenomena, etc. However, we preferred the former interpretation for being more universally applicable and, perhaps, more intuitive to our minds.

[4] In order to focus better on the main point, the quality of amenities is assumed here to be identical for rich and poor households.

The main features of our model are its *dynamic nature* (because of the sequential arrivals) and the *strategic interaction* between developers. What does the dynamic nature of our model add? It adds the possibility of understanding how residents' higher (or lower) arrival rates determine the final degree of neighborhood segregation. Why? Because, in this case, even though the rich had a uniformly higher (marginal) willingness to pay to live with other rich people, this would not guarantee segregation. Since there are *agglomeration gains* from accepting immediately the poor who come, under a very slow arrival rate it may be in the developers' interest to offer them a cheap lot. Therefore, we can observe the implications of lower interest rates (when waiting for the rich is not costly) and higher/lower arrival rates of the rich (dependent on income distribution, demographics, migration...) for the degree of segregation.[5]

In this sense, in order to provide some *motivation* for that result, we want to emphasize the faster urbanization in most areas of the New World (America) relative to the Old Continent (see e.g. Puga (1998)), and the potential connections between this fact and the higher levels of segregation we can observe today in many New World's cities.

On the other hand, the strategic (oligopolistic) interaction between developers is useful to introduce another crucial implication of this framework: *vacant lots* may appear in certain periods, since *interactions* drive developers to *wait*, in order to *relax competition* and make larger profits in later stages. In reality, these vacant lots are easily observed in many neighborhoods. If carefully developed, this idea may have strong policy implications: certain forms of competition among developers may slow down the process of lot filling, leading to possible dynamic losses in the housing markets.

There are abundant samples in the recent literature analyzing the interaction between inequality and segregation. The main piece of evidence that they were trying to explain is the coexistence of a declining trend for US racial inequality in the United States, and the continuity of extreme levels of urban racial separation. Both Sethi and Somanathan (2004) and Bayer, Fang and McMillan (2005) took inequality levels as a primitive for their models[6], as we do. However, the former authors obtained non-monotonic

[5] Our model is also similar to Henderson and Thisse (2001), in the sense that they model the competition between developers to attract high-income segments of the population. However, they endogenize the number and size of developments and do not consider externalities as the main determinant of willingness to pay. Furthermore, their model is static and focuses on the efficiency in the provision of a public good, determined by the number of emergent communities.

[6] Our framework also takes as given not only the magnitude of income inequality, but also the precise order of arrival of different neighbor-types to different developments. This seems to be a specially rigid and unrealistic

segregation levels as a response to growing inter-racial disparities, by introducing an individual concern for the racial composition of the community. In turn, Bayer, Fang and McMillan incorporated an endogeneity in the number of communities, created by homogeneous segments of the household population (as in Durlauf (1996)) to explain how richer middle-income blacks had a new incentive to separate from their low-income counterparts, which aggravated segregation.

To the best of our knowledge, our paper is the first to study the connection between developer competition, inequality (as a determinant of arrival rates) and urban segregation. Nevertheless, our project is still in such a preliminary stage that we do not know yet whether we will be able to replicate the stylized facts with this original approach.

Finally, some recent and very interesting papers are providing new insights on the simultaneous (joint) determination of inequality and segregation, by means of asymmetric information considerations among members of different racial groups (see e.g. Lundberg and Startz (2007)). Although we aim to generate segregation in the absence of asymmetric information, these papers are especially relevant to us, since they use search models as a tool to model interactions in which the sequence of arrivals is stochastic, and not strictly predetermined. This sort of technique seems to be crucial to extend our research in the future.

2. The Model: Main Assumptions and Basic Structure

In order to derive our main insights with a model as simple as possible, we are going to consider two given neighborhoods (developments), each of which will be run by a developer (duopolist). For simplicity, every neighborhood will just consist of two lots. Moreover, a predetermined and certain sequence of four people (two of them will be rich, two of them poor) will arrive to these neighborhoods in four different time periods.

Residents will derive utility both from within-neighborhood externalities and from residential facilities, and (again, for simplicity) will be attached to the same lot from their arrival *ad infinitum*. Since our main purpose is eliciting the determinants of

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feature of this preliminary work. We would like to relax it, by means of the introduction of a stochastic process of arrival to the different neighborhoods. At the present moment, our main doubt is whether we should also dispense with the strategic interaction among developers (by introducing a continuum of them in a standard search model) or we should preserve such oligopolistic structure, with the interesting implications underlined in the main body of the text.

segregation in terms of both within-group and between-group externalities, we will consider that per-period residential utility (*u*) will be identical for rich and poor individuals. On the contrary, the magnitude of the momentary externalities will differ according to their origin and destination. That is, we will differentiate between the externality generated by a rich to another rich (x_{hh}), by a rich to a poor (x_{hl}), by a poor to a rich (x_{lh}) and by a poor to another poor (x_{ll}). The terms rich and poor will be henceforth equivalent to high (*h*) and low (*l*).

We spell out right now our condition on the rankings of x_{hh}, x_{hl}, x_{lh} and x_{ll}, by introducing our first assumption, which broadly conforms to Luttmer (2005)[7]'s empirical results:

$$x_{hh} > x_{hl} > x_l \equiv x_{lh} = x_{ll} \quad (1)$$

Since individuals (or dynasties) will remain in their lot for an infinite number of periods after arrival, the total willingness to pay exhibited by any resident will be represented by the (discounted) *cumulative externality* (*cx*) enjoyed since his arrival to the neighborhood till the end of his infinite life. Time is going to be discrete, and our measure of the arrival rate is going to be the discount factor between two periods ($\frac{1}{1+r}$). That is, the smaller is *r*, the shorter will be the time period between two arrivals. Therefore, a decline in the interest rate of the economy will be equivalent to a faster arrival rate of new neighbors for our analysis of segregation. Furthermore, we assume that our duopolists cannot differentiate the product to target any specific population group. However, they are allowed to price-discriminate over time to extract as much of the residents' cumulative externality as possible. Both developers have full information and know the certain sequencing of arrivals.

Finally, let us emphasize that our analysis is going to be purely *positive* instead of normative. This means that we will not try to determine the optimal allocation of residents for a given arrival rate, but just the predictable pattern of segregation – or integration - according to the parameter values, which are *r*, *x's* and *u*.

3. Integrated and segregated equilibria

Since neighborhoods just have two lots, the only possible configurations are fully integrated (with a high-type and a low-type in each neighborhood) or fully segregated

[7] This author reports that, in general, people care about their relative position, and "lagging behind the Joneses" decreases their well-being. He finds that, controlling for an individual's own income; higher earnings of neighbors are associated with lower levels of self-reported happiness. That justifies our crucial assumption that $x_{hh} > x_{hl}$.

(with two highs with one of the developers and two lows with the other). Buyers will arrive sequentially to the neighborhoods, where both duopolists will compete for them: the sequence begins with a high-type in period 1 (h_1), and continues with a low type in period 2 (l_2), with both types alternating subsequently. The developer who attracts h_1 in period 1 will be called "rich developer" (**RD**), whereas the other duopolist will be called "poor developer" (PD).

It is immediately obvious that, in period 1, the newcomer high-type (h_1) is indifferent as to which neighborhood to choose. This is true because neither of the developers has any initial advantage over the other (yet) to attract h_1, and therefore both of them will quote the same price and offer h_1 identical utility. However, from period 2 onwards, the neighborhood owned by **RD** will be more attractive to new residents in terms of future externalities. This fact will allow **RD** to (be able to) capture those residents he is really interested in; to the extent that *the nature of the configuration* (either segregated or integrated) *will only depend on* ***RD****'s willingness to attract* l_2 *or not*. That is, our configuration will be integrated (segregated) if and only if **RD** accepts (rejects) l_2 in his neighborhood.

The extensive form of the game that determines the nature of the equilibrium can be found in the appendix (see the decision tree). In principle, we can guess that **RD** will decide to take l_2 (and generate an integrated configuration) if the level of residential utility (u) and/or the discount rate (r) are high enough, since in that case he would need to wait a long time for h_3 and would also miss a high potential price on the resident l_2. However, we need to be more systematic and solve the game backwards, starting by the last arrival (l_4), in order to find out the different parameter values that give rise to integrated/segregated equilibria.

To that purpose, we will move a few nodes down the decision tree, by assuming first that the configuration is segregated (i.e. **RD** decides to reject l_2) and PD chooses not to wait and takes immediately l_2. Under those conditions, we can establish the preliminary result specified in Lemma 1. But before stating such a lemma, let us before anticipate that the duopolists' strategies are characterized by *'limit pricing'* (i.e. the competitor with a lowest minimum price will (roughly) be able to attract the buyer, and will charge him a price equal to the minimum price of his rival, minus an epsilon). Moreover, such strategies are subject to a *participation constraint*, by which no resident will pay a price superior to the future cumulative externality he will enjoy living in the lot.

3.1. Segregated configuration; PD decides not to wait

Lemma 1: *Under the assumptions specified in (1) and in any segregated configuration,* ***RD*** *will be able to attract h_3 (for all positive values of u and r), provided that PD decided to accept l_2 in period 2.*

Proof: In order to prove it, let us start solving the game by the last arrival (l_4). When either PD or **RD** receive l_4, there will be no competition between them, since there will be just one empty lot in one of the neighborhoods. Therefore, both developers would charge a monopoly price over l_4. That is, they will be able to extract the whole cumulative externality plus the residential utility enjoyed by this buyer (l_4). More formally,

$$p_{l4}^{PD}(l,l|h,h) = cx_{l4}^{PD}(l,l|h,h);\ p_{l4}^{\boldsymbol{RD}}(l,h|h,l) = cx_{l4}^{\boldsymbol{RD}}(l,h|h,l) \quad (2)$$

In expression (2) above, $p_{l4}^{PD}(l,l|h,h)$ denotes the price set by PD to resident l_4 when we have segregation, i.e. when the neighborhood administered by **RD** concentrates both high types and PD hosts the low types: $(l,l|h,h)$. Similarly, $cx_{l4}^{PD}(l,l|h,h)$ stands for the cumulative externality enjoyed by l_4 in PD's neighborhood under the same circumstances, *including the residential utility* available in every period after arrival.

However, we still do not know which of the two developers will finally receive l_4. Therefore, in order to be sure about the issue, we need to find out the prices they will set on h_3: if **RD** attracts h_3, l_4 will go for sure to the only empty lot in PD's neighborhood. The minimum price that **RD** would set on h_3 is one leaving him indifferent between attracting h_3 or not. This price is:

$$p_{h3(l|h)}^{\boldsymbol{RD}min} = \frac{p_{l4}^{\boldsymbol{RD}}(l,h|h,l)}{(1+r)} = \frac{cx_{l4}^{\boldsymbol{RD}}(l,h|h,l)}{(1+r)} \quad (3)$$

Expression (3) implies that **RD** is indifferent between capturing h_3 at a price $p_{h3(l|h)}^{\boldsymbol{RD}min}$ and waiting until period 4 to extract a monopoly price from buyer l_4. On the other hand, the minimum price for PD will be:

$$p_{h3(l|h)}^{PDmin} = \frac{cx_{l4}^{PD}(l,l|h,h)}{(1+r)} \quad (4)$$

Who will attract h_3 when both developers are competing for it? The winner of the competition will be the developer showing the lowest minimum price, *once this price* *has been adjusted for the difference in cumulative externalities enjoyed by h_3 in each neighborhood.* Subsequently, the winner of the competition for h_3 will set an equilibrium-price equal to the minimum price of the other developer (possibly minus an epsilon). In this particular case, using (3) and (4) we can obtain that **RD** will attract h_3 in scenario (l_2-,h_1 -) if and only if

$$\frac{cx_{l4}^{\boldsymbol{RD}}(l,h|h,l)}{(1+r)} < \frac{cx_{l4}^{PD}\,(l,l|h,h)}{(1+r)} + (cx_{h3(l,l|h,h)}^{\boldsymbol{RD}} - cx_{h3(l,h|h,l)}^{PD}) \quad (5)$$

The last term in brackets reflects the adjustment in terms of the difference in cumulative externalities for h_3 between both neighborhoods. If we express condition (5) in terms of the underlying parameters, we can conclude that **RD** will capture h_3 if and only if[8]

$$(1+r) > \frac{x_{hl}-x_l}{x_{hh}-x_l} \quad (6)$$

And it is easy to observe that this condition (6) immediately holds as a result of our assumption (1). That is, provided that we are in a segregated configuration and PD did not postpone the attraction of l_2, **RD** will always be able (and willing) to capture h_3. This completes the proof.

It is noticeable that we have not considered the participation constraints of resident h_3 during the previous proof. The reason for that is the following: the prices finally quoted by both developers are inferior to the cumulative externalities that this buyer would enjoy in the respective neighborhoods. However, so far we only know their minimum prices. Which are the prices they would actually set on h_3?

The answer comes from Lemma 1: since **RD** is going to be the winner for sure, his unconstrained [9] optimal price will be $p_{h3(l|h)}^{\boldsymbol{RD}} = \frac{cx_{l4}^{PD}\,(l,l|h,h)}{(1+r)} + (cx_{h3(l,l|h,h)}^{\boldsymbol{RD}} - cx_{h3(l,h|h,l)}^{PD}) < cx_{h3(l,l|h,h)}^{\boldsymbol{RD}}$, since $\frac{cx_{l4}^{PD}\,(l,l|h,h)}{(1+r)} = \frac{x_l+u}{r} < cx_{h3(l,h|h,l)}^{PD} = \frac{x_l+u}{r}(1+r)$.

[8] Notice that the residential utilities (u) that are on the left-hand side and the right-hand side of (5) finally wash away.

[9] The term unconstrained means here that this is **RD**'s optimal price if we disregard the resident's participation constraint.

By the same token, since PD will be for sure the loser in the competition, he will just quote his minimum price, i.e. $p^{PD}_{h3(l|h)}$= $p^{PDmin}_{h3(l|h)} = \frac{cx^{PD}_{l4}(l,l|h,h)}{(1+r)} < cx^{PD}_{h3(l,h|h,l)}$, as checked just three lines above.

Our next questions are: which will be the final PD's payoff from accepting immediately l_2 in period 2? And which are the conditions (in terms of the parameters) for **RD** to prefer segregation to integration? Both will be clarified in the following proposition.

Proposition 1: *Our configuration will be segregated (by **RD**'s decision) if and only if*

$$(1+r)\big(x_{hh} - x_{hl}(1+r)\big) - (x_{hl} - x_l) > ru$$

Moreover, PD's final payoff from accepting immediately l_2 is equal to

$$Min\left\{\frac{2u}{r(1+r)} + \frac{2x_l}{r(1+r)} + \frac{x_{hh}}{r} - \left(\frac{1+r}{r}\right)x_{hl}\ ,\frac{2u}{r(1+r)} + \frac{2x_l}{r(1+r)} + \frac{(2+r)u}{(1+r)}\right\} \quad (7)$$

Proof: Let us first determine under which conditions **RD** will capture l_2. If PD were the loser in the competition for this resident, he would be able to charge later pure monopoly prices to h_3 and l_4. In that case, PD's profits from period 2 onwards would be

$$\pi^{PDloser}_{2(-|h_1)} = \frac{cx^{PD}_{h3(h|h,l)}}{(1+r)} + \frac{cx^{PD}_{l_4(h,l|h,l)}}{(1+r)^2} \quad (8)$$

Alternatively, we know from Lemma 1 that if PD were the winner in the competition, he would not be able to gain h_3, and therefore his revenue from period 2 onwards would be

$$\pi^{PDwinner}_{2(-|h_1)} = p^{PD}_{l2(l2,l4|h1,h3)} + \frac{cx^{PD}_{l4(l,l|h,h)}}{(1+r)^2} \quad (9)$$

By equating (8) and (9), we are able to obtain the minimum price $p^{PDmin}_{l2(l2,l4|h1,h3)}$ as follows:

$$p^{PDmin}_{l2(l2,l4|h1,h3)} = \frac{cx^{PD}_{h3(h|h,l)}}{(1+r)} + \frac{cx^{PD}_{l_4(h,l|h,l)}}{(1+r)^2} - \frac{cx^{PD}_{l4(l,l|h,h)}}{(1+r)^2} \quad (10)$$

On the other hand, if **RD** chose to reject l_2 the subsequent configuration would be segregated, since he would be able to attract h_3 with certainty. Furthermore, h_3 would be charged a price given by (5), i.e.

$$p^{\boldsymbol{RD}}_{h3(l,h)}=\frac{cx^{PD}_{l4}\ (l,l|h,h)}{(1+r)}+(cx^{\boldsymbol{RD}}_{h3(l,l|h,h)}-cx^{PD}_{h3(l,h|h,l)})$$

Therefore, the minimum price **RD** would require to capture l_2 would be

$$p^{\boldsymbol{RD}min}_{l2(h3,l4|h1,l2)}=\frac{cx^{PD}_{l4}\ (l,l|h,h)}{(1+r)^2}+\frac{\left(cx^{\boldsymbol{RD}}_{h3(l,l|h,h)}-cx^{PD}_{h3(l,h|h,l)}\right)}{(1+r)} \quad (11)$$

If we compare (10) and (11), (expressing all equations in terms of the parameters) taking into account the necessary adjustment in terms of the different cumulative externalities enjoyed by l_2, we come to the conclusion that **RD** will win the competition for l_2 if and only if

$$(1+r)\big(x_{hh}-x_{hl}(1+r)\big)-(x_{hl}-x_l)>ru \quad (12)$$

This completes the first part of the proof.

Now we are going to look at the profitability for PD of accepting l_2 without delay in case of segregation. Under such circumstances, PD's revenue from period 2 onwards would be

$$Min\left\{\, p^{\boldsymbol{RD}min}_{l2(h3,l4|h1,l2)}-(cx^{\boldsymbol{RD}}_{l2}-cx^{PD}_{l2}),cx^{PD}_{l2}\right\}+\frac{cx^{PD}_{l4}}{(1+r)^2} \quad (13)$$

We can observe that expression (13) incorporates the *participation constraint* in the price charged to l_2 by PD[10], since this resident can never be charged a price higher than his cumulative externality received in the lot (otherwise he would refuse to locate there). Now, using (11) and (13), we can restate the last expression as

$$\pi^{PD}{}_{\substack{segreg\\ no\ waiting}}=Min\left\{\frac{2u}{r(1+r)}+\frac{2x_l}{r(1+r)}+\frac{x_{hh}}{r}-\left(\frac{1+r}{r}\right)x_{hl}\ ,\frac{2u}{r(1+r)}+\frac{2x_l}{r(1+r)}+\frac{(2+r)u}{(1+r)}\right\}$$

This finalizes the proof of Proposition 1.

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[10] Rigorously speaking, the pricing strategies followed by **RD** and PD in period 2 can be specified as follows:

$p^{\boldsymbol{RD}}_{l2}$(-|h_1)=max[min($p^{PDmin}_{l2}+(cx^{\boldsymbol{RD}}_{l2}-cx^{PD}_{l2}),cx^{\boldsymbol{RD}}_{l2}$), $p^{\boldsymbol{RD}min}_{l2(h3,l4|h1,l2)}$]

p^{PD}_{l2}(-|h_1)=max[min($p^{\boldsymbol{RD}min}_{l2}-(cx^{\boldsymbol{RD}}_{l2}-cx^{PD}_{l2}),cx^{PD}_{l2}$), $p^{PDmin}_{l2(h3,l4|h1,l2)}$]

We also need to examine the situation in which PD postpones the attraction of l_2 in case of segregation. He may do it in order to avoid the competition from **RD** and to extract pure monopoly profits from both $l_{2[3]}$ and l_4 (henceforth we will denote by $l_{2[3]}$ the poor resident who arrives in period 2 and has to wait until period 3 to be assigned a lot in PD's neighborhood). The comparison of PD's gains in both situations will allow us to elucidate for which parameter values it is in the interest of this developer to postpone or not.

3.2. Segregated configuration; PD decides to postpone the attraction of $l_{2[3]}$

When PD decides to keep l_2 out of his neighborhood until period 3, he will do it either because he is aiming to capture h_3, or because he prefers charging both low types ($l_{2[3]}$ and l_4) a monopoly price. We are going to rule out the first possibility by proving that PD's payoff of postponing and getting h_3 is lower than that of accepting l_2 in period 2. Intuitively, in order to attract h_3, PD would need to quote such a low price that this would not compensate him given the relatively weak inter-group externalities. We can summarize this statement as follows:

Lemma 2: *If PD decides to postpone (under our conditions in (1) and provided that 0<r<1), it will never be in his interest to capture h_3. Therefore, PD will only consider charging a pure monopoly price to $l_{2[3]}$ and l_4.*

Proof: Suppose for the moment that PD is able to get h_3 in period 3. Under this premise, we are going to get to a contradiction.

PD's payoff resulting from postponing and obtaining h_3 will consequently be $p^{PD}_{h3(l|h)} = p^{\boldsymbol{RD}min}_{h_3} - (cx^{\boldsymbol{RD}}_{h3(l,l|h,h)} - cx^{PD}_{h3(l,h|h,l)})$. Furthermore, subsequently PD would compete (under equality of conditions!) with **RD** for $l_{2[3]}$ *and* l_4, and hence we can predict that both residents could be captured (by both duopolists) with a probability of ½. All in all, PD's payoff from postponing and obtaining h_3 (and either $l_{2[3]}$ or l_4) would be:

$$\frac{cx^{\boldsymbol{RD}}_{l4}(l,h|h,l)}{(1+r)} - (cx^{\boldsymbol{RD}}_{h3(l,l|h,h)} - cx^{PD}_{h3(h,l|h,l)}) + \frac{cx^{PD}_{l4}(h,l|h,l)}{(1+r)} =$$

$$= 2\frac{cx^{\boldsymbol{RD}}_{l4}(l,h|h,l)}{(1+r)} - (cx^{\boldsymbol{RD}}_{h3(l,l|h,h)} - cx^{PD}_{h3(h,l|h,l)}) \quad (15)$$

On the other hand, we know from Proposition 1 that PD's payoff (in terms of the parameters) from accepting immediately l_2 would be

$$Min\left\{\frac{2u}{r(1+r)}+\frac{2x_l}{r(1+r)}+\frac{x_{hh}}{r}-\left(\frac{1+r}{r}\right)x_{hl}\quad,\qquad\frac{2u}{r(1+r)}+\frac{2x_l}{r(1+r)}+\frac{(2+r)u}{(1+r)}\right\}\ (16)$$

By expressing (15) in terms of the parameters and comparing the outcome with (16), we come to the conclusion that the payoff from accepting immediately l_2 exceeds that from waiting for h_3 (i.e. (16) is higher than (15)) if

$$x_{hh} > x_{hl}; x_{hh} > x_l \ and\ 0 < r < 1\ \ (17)$$

And (17) is guaranteed for sure, given (1), if we also impose as a reasonable condition that 0<r<1. This completes the proof.

The next step will be finding out a sufficient condition under which PD will never postpone the acceptance of l_2, in case of segregation. We will obtain here a remarkable result: *for all parameter values that allow for segregation, PD could never decide to postpone.* In other words, segregation excludes postponing. Why is this intuitive? Because segregation itself requires a low residential utility, since otherwise **RD** would never forego the immediate possible charges on l_2 (relative to those on h_3, *which are discounted*); but, precisely, a *high* residential utility is a prerequisite for postponing, since all that utility can be extracted from $l_{2[3]}$ with monopoly power. That is the reason why both phenomena are incompatible. All this is summarized in the following proposition.

Proposition 2: *A segregated configuration excludes postponing on the part of the PD.*

Proof: By expression (12), segregation implies that

$(1+r)\big(x_{hh}-x_{hl}(1+r)\big)-(x_{hl}-x_l) > ru$. Therefore, since (by (1)) $x_{hl} > x_l$, then segregation also implies that

$$ru < (1+r)\big(x_{hh}-x_{hl}(1+r)\big)\ \ (18)$$

Now, as (by Lemma 2) PD can never attain h_3 by postponing, his payoff derived from that option will come as a result of the monopoly power exercised on $l_{2[3]}$ and l_4. Those returns from postponing can be expressed as follows[11]:

$$\frac{cx_{l2[3]}^{PD}}{(1+r)}+\frac{cx_{l4}^{PD}}{(1+r)^2}=\frac{2x_l}{r(1+r)}+\frac{(2+r)u}{r(1+r)}\ \ (19)$$

[11] That payoff is expressed in period-two monetary units.

We know from (16) which the returns are from accepting immediately l_2. Notice that the term on the right of expression (16) is clearly bigger than (19). Therefore, the only chance we have that PD could postpone is that the term on the left of (16) is both smaller than the term on the right of (16) and smaller than expression (19). Is that possible? We will see that it is not.

Suppose that expression (19) represents a bigger number than the term on the left of (16). Then,

$$\frac{(2+r)u}{r(1+r)}+\frac{2x_l}{r(1+r)}>\frac{2u}{r(1+r)}+\frac{2x_l}{r(1+r)}+\frac{x_{hh}}{r}-\left(\frac{1+r}{r}\right)x_{hl}$$

And rearranging and simplifying

$$ru>(1+r)\big(x_{hh}-x_{hl}(1+r)\big) \quad (20)$$

And (20) is incompatible with segregation because it contradicts (18). This completes the proof of Proposition 2.

What do we know about the relation between an *integrated* configuration and the possible existence of vacant lots? The truth is that we have not explored formally that issue yet. However, the coexistence of integrated neighborhoods and vacant lots looks plausible, since high levels of residential utility give rise (intuitively) to both phenomena at the same time; something that did not happen in the segregated case.

We are about to show now the crucial result of the paper, related to the impact of different arrival rates / discount factors on the nature of the equilibrium.

4. Influence of the Arrival Rate on the Nature of the Equilibrium

Proposition 3: *If the (necessary and sufficient) condition* $(x_{hh}+x_l)>x_{hl}(2-r^2)$ *is satisfied, then the following couple of statements are true:*

- *The slower the arrival rate (represented by a higher value of r) the more likely will be the equilibrium configuration to be integrated.*
- *Identically, the faster is the arrival rate (represented by a lower value of r), the more likely will be the configuration to be segregated.*

Proof: Here we have to start from condition (12) and rewrite such inequality in this way:

$$\left(\frac{1+r}{r}\right)x_{hh}-\left(\frac{1+(1+r)^2}{r}\right)x_{hl}+\frac{x_l}{r}>u \quad (21)$$

Now, this inequality (21) that guarantees segregation should be easier to be fulfilled when the derivative of the left-hand side with respect to r is negative. In that case, a reduction in r (i.e. a faster arrival rate) would facilitate segregation. It is straightforward to check that the derivative of the left-hand side with respect to r is equal to

$$\frac{-1}{r^2}(x_{hh}+x_l)+\left(\frac{2-r^2}{r^2}\right)x_{hl} \quad (22)$$

And then, our desired effect takes place if and only if

$$x_{hh}+x_l>x_{hl}(2-r^2)$$

This completes the proof of Proposition 3.

Notice from (21) that the parameters facilitating a segregated configuration are those related to within-group externalities (x_{hh} and x_l), whereas strong between-group externalities (x_{hl}) and a high residential utility (u) tend to promote integration. That is nothing new. Our point is emphasizing the role played by the arrival rate / discount factor to select the equilibrium configuration. It would be also very stimulating to introduce variations in residents' (stochastic) income distribution (not just our predetermined sequencing) to study how the developers' competition tends to generate more or less segregation as a response.

5. Conclusions and Possible Extensions

In this paper we have studied the conditions under which a low enough discount rate and / or a high arrival rate of new rich residents can result in higher levels of segregation in new neighborhoods. One natural extension seems to be exploring the constrained-optimal allocation of residents by a monopolist that can not differentiate the product. Then, we could compare this constrained-efficient outcome with that resulting from our duopoly model, in order to obtain some normative implications: are the conditions for segregation more stringent under monopoly or under duopoly? Are poor people worse-off under monopoly or under duopoly? And the rich people?

We have clearly reached a bifurcation in which we have to decide how to pursue our future work in the issue. One possibility is forgetting about the implications about vacant lots and dynamic inefficiency, which is likely to arise under integrated (but never under segregated) configurations. In that case, we could proceed by using a search model in which continuums of developers (each one being infinitely small) compete without strategic considerations. That framework is specially suited for the introduction of stochastic sequences of neighbors, and variations in their prior distribution of income should have a clear impact on segregation measures; at least as clear as it is now the influence of the discount factor.

Another possibility is undertaking some empirical work, since the implications of our paper are clearly testable. We could measure the impact of urban demographic growth and variations in the interest rates as determinants of segregation levels. Another interesting sequel would be checking whether integrated neighborhoods are more likely to exhibit high proportions of vacant lots, unlike segregated ones.

6. References

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7. Appendix

Decision Tree