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Jakas, Vicente

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# Theory and Empirics of an Affine Term Structure Model Applied to European Data

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## Abstract

The basic asset pricing equation is adapted to include the effects of unemployment, consumers' expectations, the price level and money supply on money market rates and government bond yields. Expected consumption growth is modelled using European unemployment figures and Eurostat Consumer Confidence Index. The price level is incorporated in the aggregate marginal utility function using production price index (PPI) as a proxy. An affine term structure model is derived using a state space system with an observation equation which links observable yields to these macroeconomic variables and a state equation which describes the dynamics of these variables. Unemployment and consumer confidence index will have a shift and a slope effect on the yield curve, for front-end yields moving faster than in the long end. Production price index exhibits a twist effect (flattening or steepening of the curve) which results in front-end yields shifting in opposite directions to the long end of the curve. This empirical work shows that yields are negatively correlated to money supply, as expected in classical IS-LM models. And that money supply exhibits a slope effect, with the front-end of the curve shifting faster than the longer end.

**Keywords:**
Macroeconomic releases, Term structure of interest rates; Dynamic factors; Affine term structure models.

***JEL classification:***
E12; E43; E44; E52; G12.

*Vicente Jakas ✉ Deutsche Bank AG, Grosse Gallusstrasse 10-14, 60594 Frankfurt am Main, Germany. E-Mail: Vicente.Jakas@db.com, Tel. ++(0)49(0)69 910 30786.* *Special thanks to Prof. Dr. Ashok Kaul, for all his valuable comments and suggestions. I would also like to thank my father Prof. Dr. M. M. Jakas for making my notation clearer.*

## 1. Introduction

This paper is motivated by the fact that consumption based asset pricing models despite being robust theoretically, they perform poorly empirically.

To overcome this, the model has been rearranged slightly replacing the usual consumption data by unemployment data and a consumer confidence index to better account for expectations. By doing so the model remains as a consumption based model with all its theoretical properties. Another aspect is that the price level has been added into the aggregate marginal utility function. Finally, the model also includes the effects of changes in monetary aggregates on asset prices. An affine term structure model is derived in this paper, using a state space system with an observation equation which links observable yields to these macroeconomic variables and a state equation which describes the dynamics of these variables. Expected aggregate marginal utility growth is modelled using unemployment figures, consumer confidence index and production price index. Money supply is modelled using monetary aggregate M3.

The rest of the paper is organised as follows: In section 2 we recall the basic prising equation to remind the reader how this looks like from a yield perspective. In section 3 we discuss our model and show how unemployment, consumer confidence index and the price level enter the aggregate marginal utility function and how money supply is added to the basic pricing equation. Section 4 outlines the empirical process and we show how we go about testing our model. Sections 5 and 6 we discuss the data and present the results. Section 7 we present some of the related literature and in section 8 we conclude and present our final remarks.

## 2. The Basic Asset Pricing Equation

The literature extensively documented the poor empirical results on consumption based asset pricing models. A useful summary can be seen on Guvenen and Lustig (2007), Piazzesi (2003) and Cochrane (2001).

It would be useful to start by recalling the basic asset pricing equation as in Cochrane (2001), which shows that asset prices are a function of expected future aggregate marginal utility growth, which boils down to:

$$P_t = E_t\left[\left(\beta\frac{u'(c_{t+1})}{u'(c_t)}\right)x_{t+1}\right] \quad (1)$$

For $P_t$ being the present value at time $t$, of an asset paying off $x_{t+1}$, $E_t$ is the expectations operator. $\beta$ the subjective discount factor and $u'(c_{t+1}) / u'(c_t)$ being the aggregate marginal utility growth which encapsulates investor's first order condition with respect to consumption and thus together with $\beta$ comprises the intertemporal marginal rate of substitution (Rubinstein, 1976; Lucas, 1978; Breeden, 1979).

According to the evidence, it appears that aggregate marginal utility growth using solely consumption data appears to have poor explanatory power on asset prices. For example, the arithmetical expectation of consumption growth as in [1] does not necessarily mean that it encapsulates all information about consumers' expectations and their behaviour. The intention is to deal with this problem, so it will be assumed that expected aggregate marginal utility growth is a function of current unemployment plus another variable which encapsulates consumers' expectations. In addition, the "new" aggregate marginal utility growth will be adjusted for the price level and for changes in monetary aggregates.

Recalling the basic pricing equation and assuming a 1 year zero coupon bond which pays out 1 monetary unit at maturity, the investor's first order condition for $E_t$ conditional to an information set at time $t$, would yield:

$$E\left[m_{t+1}|I_t\right] = E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)}\right] = e^{-y_t^{(N)}}$$

For $y_t^{(N)}$ representing the bond yields with maturity $N$ at time $t$ and $I_t$ being agents' information set at time $t$. Which for convenience it will be transformed to:

$$-logE\left[m_{t+1}|I_t\right] = y_t^{(N)} \tag{2}$$

Equation [2] basically shows that the stochastic discount factor is the inverse of the observable yields.

## 3. The Model

In an attempt to depict the idea that aggregate marginal utility growth is determined by consumers' confidence, unemployment, the price level and a monetary aggregate, we would then propose the following identity:

$$y_t^{(N)} = -logE_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{M_{t+1}^S}{M_t^S}\right] \tag{3}$$

For consumption being:

$$c_t = \frac{(1-U_t)wC_t^e}{\Pi_t} \tag{4}$$

For $U_t$ representing the unemployment rate, which should not get confused with the utility function which is noted in this paper as $u(c_t)$. $w$ is the nominal wage level, which for the sake of simplicity it will be assumed to be constant but it does not have to be that way. $C_t^e$ represents consumers' expectations on their future consumption growth determined by the consumer confidence index, $\Pi_t$ is the price level and $M_t^s$ represents a monetary aggregate to account for money supply, for this case M3 (M4 for the UK equivalent). For our analysis it will be assumed that preferences follow a log utility function with constant relative risk aversion as follows:

$$u(c_t)=\frac{c_t^{(1-\gamma)}}{(1-\gamma)} \tag{5}$$

Conditions [3] to [5] have the following implications:

1. We will start describing what we believe the effects of $U_t$ – unemployment – have on the yield curve. Unemployment is expected to have a negative relationship to risk-free assets' returns. And this is because an increase in unemployment results in a decrease in expected future consumption growth and therefore, results in an increase in aggregate marginal utility with the subsequent increase in the discount factor for assets which are uncorrelated with consumption growth. From [3] we can see that an increase in aggregate marginal utility results in an increase in the discount factor for risk-free assets, and as a consequence, yields are expected to fall.

2. The improvements in consumers' expectations about the future of the economy should have a positive relationship to government yields. An improvement in consumers' expectations about the future results in an increase in expected future consumption growth and therefore, results in a decrease in aggregate marginal utility with the subsequent decrease in the discount factor for assets which are uncorrelated with consumption growth such as government bonds. An improvement in consumers' expectations results in a decrease in the discount factor for risk-free assets and as a consequence risk-free assets' yields are expected to rise.

3. Points (1) and (2) have been relatively straight forward. However, the effects of the price level on the yield curve are rather ambiguous. Hence, these effects are governed by: a) two effects which move in opposite directions. Thus, the effect of the price level on expected aggregate marginal utility growth and the effect of central banks reaction as a consequence of increases in the price level. And b) these effects will be determined by the segment or maturity we are at in the curve, as central banks have only greater influence on the lower end of the curve, thus the money market curve and lesser effects in the longer end, the capital markets curve. To describe this idea better: an increase in the price level is expected to have a negative effect in expected future consumption growth and is expected to increase future aggregate marginal utility with the subse-

quent increase in the discount factor for assets which are uncorrelated with consumption growth such as government bonds. As in Piazzesi (2006), it is assumed throughout the paper that unfavourable change in the price level is always bad news for consumption. Therefore an increase in the price level is expected to result in a fall in e.g. government bond yields. However if the central bank responses to increases in the price level is to increase interest rates, then yields are expected to be positively correlated with the price level but only in the lower end of the curve. Simply because central banks are more efficient in influencing the money market curve than the capital markets. The effect throughout the curve will depend on the elasticity of substitution between money and capital market instruments. Thus, for those parts of the curve where money markets are perfect substitutes of capital markets, i.e. presumably up to the 2 to 3 year maturities, changes in the yield curve are expected to be more influenced by monetary policy reaction to changes in the price level rather than changes in expected aggregate marginal utility growth. The elasticity of substitution is expected to dissipate as maturity increases and money market instruments elasticity of substitution become inelastic. As elasticity of substitution between money markets and capital markets become more inelastic the longer the maturities, the yield curve is governed rather by changes in expected aggregate marginal utility growth than by central banks policy reaction to expected changes in the price level.

4. By adjusting the stochastic discount factor to a monetary aggregate indicator we intend to account for these effects described in Turnovsky (1989). Assuming that the price level remains unchanged, an expansionary monetary policy will result in increases in asset prices with the subsequent fall in yields, as a consequence of an increase in real money balances. For central banks, to be able to apply an expansionary monetary policy they need to either, relax reserves requirements, increase money supply via open market operations, acquire government debt securities and therefore inject additional quantity of money in the financial system, all this drives to asset price increases particularly government bonds. An increase in real money balances increases the quantity of money available for speculative purposes, which results in rising asset prices. Risk-free assets such as government bonds are expected to be affected up to the level that the elasticity of substitution between money markets and capital markets allows it to. On the longer end of the government curve, movements in monetary aggregates are expected to dissipate. This is because in our model monetary aggregates do not enter into the aggregate marginal utility function and therefore, have little effects on risk-free assets which are expected to act as hedges during bad times, when consumption growth is expected to be low. In this model, if an expansionary monetary policy results in an increase in consumption, due to a rise in real money balances, this would have no effect on asset prices, as these increases are assumed to be offset by increases in the consumer price levels. In fact, with this model we are able to show that if the price level is independent to

increases in monetary aggregates, it would mean that an expansionary monetary policy affect asset prices.

The model now asserts that if the price level is expected to remain unchanged and there is an expected increase in money supply, asset prices are expected to rise. This is because there is an increase in real money balances which results in a decrease in yields. However this could be seen from a different perspective: if the quantity of money increases, and this increase results in increases in real money balances for speculative purposes rather than for increases in consumption growth, money could be seen as to have lost value against other assets (particularly risky assets) as the quantity of money has increased at a greater rate than the quantity of risky assets. The paradigm here is that the price level will not always necessarily increase due to increases in the quantity of money and as long as the increase in the quantity of money has no effect in consumption growth, there is no reason to believe that could have an effect in the price level measured by consumer price indices. Therefore, money has lost value against other assets, whilst maintaining its purchasing power against consumption goods.

The idea that money supply is a determinant of asset prices is not new, see for example Kindleberger (1978) and Allen and Gale (2000). In recent cases, asset prices have risen due to for what it appears to have been an expansion in credit, e.g. following financial liberalisation. Another example is the rise in real estate and stock prices that occurred in Japan in the late 1980's. Once the BoJ decided to tackle inflation with sharp increases in interest rates – with the subsequent reduction in money supply by imposing reserves requirements –, asset prices fell dramatically. Similar effects were observed in Scandinavian countries.

These examples suggest a relationship between credit or money supply and the rise in asset prices.

Therefore, in the wake of these events it seems plausible to take monetary aggregates into account. So recalling [1] for logarithmic utility function and log prices, nominal yields can be specified as:

$$y_t^{(N)} = -\frac{1}{N} p_t^{(N)} = -\frac{1}{N} ln\, E_t \left[ \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} \frac{M_{t+1}^s}{M_t^s} \right] \quad (6)$$

Transforming [6] for all state space components discussed from [3] to [5] into their natural logarithms, and assuming normality we obtain:

$$y_t^{(N)} = -\frac{ln\beta}{N} - \frac{1}{N} E_t \left[ \Delta m_{t+1}^s \right] + \frac{1}{N} \gamma E_t \left[ -\Delta u_{t+1} + \Delta c_{t+1}^e - \Delta \pi_{t+1} \right] - \frac{1}{N} \frac{\gamma^2}{2} Var \left[ -\Delta u_{t+1} + \Delta c_{t+1}^e - \Delta \pi_{t+1} \right] \quad (7)$$

For $\Delta m^{s}_{t+1}$ , $\Delta u_{t+1}$ , $\Delta c^{e}_{t+1}$ and $\Delta \pi_{t+1}$ being the rate of change in monetary supply, unemployment, consumer expectations and inflation for the period $t$ to $t$+1 or more precisely: $ln(M^{s}_{t+1}/M^{s}_{t})$, $ln(U_{t+1}/U_{t})$, $ln(C^{e}_{t+1}/C^{e}_{t})$ and $ln(\Pi_{t+1}/\Pi_{t})$.

Equation [7] shows how an increase in the volatility results in falling yields for a risk-free asset, which is supposed to hedge in times of low consumption growth. This can be seen in the last term. Hence, if any components comprising the marginal utility growth become more volatile and result in a higher volatility for expected future consumption growth, risk-free assets are expected to experience a fall in yields as a result of the precautionary savings effect seen on the variance term from [7].

Notice that changes in the price level can be positive or negative depending on the level of substitution between money market and capital market instruments.

Now from a recursive perspective we need to analyse the *beta* coefficients that we expect for different maturities. Hence, let us assume there is a sequence of weights $\beta_i$ assumed to converge to a constant value as maturity increases. The coefficients $\beta_i$ should not get confused with $\beta$ which is the subjective discount factor.

A possible specification could be:

$$\beta_i^{(N)} = \frac{\sum_{i=1}^{N} \lambda^i}{\sum_{i=0}^{N} \lambda^i}$$

Notice that by specifying as above all we want to do is to show that the coefficients $\beta_i$ are a function of $N$ (maturity) and converge to a constant value as the maturity increases. We will define $\lambda^i$ as a specific discount factor, hence specific to maturity $i$, which for simplicity it will be fixed to a time horizon $N$. If $\lambda^i$ <1, the weight $\beta_i^{(N)}$ converges to $\beta_i$ as the horizon $N$ becomes larger. However, notice that if $\lambda^i$ >1, the weight $\beta_i^{(N)}$ converges faster to 1 as the horizon $N$ becomes larger. We present a recursion, including the behaviour of coefficients $\beta_i$ above and including the elasticity of substitution discussed in 3), and assuming a recursion that starts for $N$=1 to $N$ under normality,

$$y_t^{(N)} = -\frac{ln\beta}{N} - \frac{E_t\left(\sum_{i=0}^{N=T-t} \beta_{t,1+i}^{(N)} \Delta m^{s}_{t+1+i}\right)}{N} + \frac{\gamma E_t\left(\sum_{i=0}^{N=T-t} \beta_{t,1+i}^{(N)} \left[-\Delta u_{t+1+i} + \Delta c^{e}_{t+1+i} - \left(1-\varepsilon_{t,1+i}^{(N)}\right)\Delta\pi_{t+1+i}\right]\right)}{N} - \frac{\gamma^2 Var \sum_{i=0}^{N=T-t} \beta_{t,1+i}^{(N)} \left[-\Delta u_{t+1+i} + \Delta c^{e}_{t+1+i} - \left(1-\varepsilon_{t,1+i}^{(N)}\right)\Delta\pi_{t+1+i}\right]}{2N} \quad (8)$$

The elasticity of substitution $\varepsilon_{t+1}$ which affects the the price level coefficient term under $(1-\varepsilon^{(N)})$ measures the degree of substitution between two instrument types: the money market instruments and capital market instruments (e.g. 2 year maturity repos and 2 year government securities). As maturities become longer, the elasticity of substitution becomes inelastic so that the sign changes as $\varepsilon_{t+1}$ gets closer to 0. If elasticity equals 1, instruments become perfect substitutes (perfectly elastic) and the price level is neutral because the effect of the central bank reaction to changes in the price level is exactly off-set by the effect resulting from shocks from the aggregate utility function. If elasticity is greater than 1, elasticity is said to be very elastic and therefore, the curve movements will be mainly govern by central bank policy and less by aggregate marginal utility growth.

Why are we differentiating only for the case of the price level? Because of central banks open market operations. For the cases of unemployment and consumer confidence we assume that there is rather no effective intervention adding extra shocks into the model. Now for the case of the price level it is virtually impossible to ignore money market shocks resulting from these interventions and spreading throughout the curve.

Clearly, as maturity increases, the variance of the discount factor is expected to fall by $1/(2N)$ as shown in [8]. Another aspect from [7] and [8] is that a volatile monetary policy is expected to have negative effect on risk-free-assets' yields. Recalling that risk-free assets act as a hedge to consumption growth, it could be understood from this that a volatile monetary policy could have a negative effect on consumption growth, and as a result risk-free asset yields are expected to fall with the subsequent increase in risk-free asset prices.

One of the shortcomings seen in [7] and [8] is the assumption of investors' homoscedastic expectations. This is only for convenience, as it does not necessarily have to be this way.

## 4. The Empirics

For empirical analysis, equations [3] to [8] can be summarised together into a state space system similar to Piazzesi (2003). These specifications are put together with an observation equation which links observable yields to the state vector and a state equation which describes the dynamics of the state as follows:

$$y_t^{(N)} = A(N) + B(N)^T x_t + \varepsilon_t^{(N)} \tag{9}$$

$$x_t = \rho x_{t-1} + u_t \tag{10}$$

For $x_t$ being the state space vector with macroeconomic data used in [8] comprising unemployment, consumer confidence index, production price index and monetary aggregate. Coefficients $A(N)$ and $B(N)$ depend on yields' maturities $N$. $\varepsilon_t^{(N)}$ and $u_t$ are measurement errors and for the sake of simplicity are assumed to be normally distributed and i.i.d. Substituting [10] into [9] gives us the equation to be used to estimate parameters $A(N)$ and $B(N)$ by means of OLS which can be written as:

$$y_t^{(N)} = A(N) + B(N)^T x_{t-1} + v_t^{(N)} \tag{11}$$

In which $y_t^{(N)}$ is a $N$ dimensional vector of observed yields and $x_{t-1}$ is an $N$ dimensional vector of observed macroeconomic data, $A(N)$ is a $N$ dimensional vector of constants, $B(N)^T$ is a $N \times N$ matrix of parameters. $B(N)^T$ includes $\rho$ which is the lag coefficient on the AR(1) from [10]. $v_t^{(N)}$ is a $N$ dimensional vector of disturbance terms independently and normally distributed with zero mean and constant variance. This is because if $\varepsilon_t^{(N)}$ and $u_t$ are measurement errors assumed to be independently and normal so that the sum $v_t^{(N)}$ is also independent and normally distributed.

## 5. The Data

This empirical work is based on monthly European macroeconomic data particularly from the ECB and Eurostat available in Bloomberg. EONIA, Euribor and German government yields have been obtained from Bloomberg. Most of the data series is only available since 1998. This makes this analysis difficult, hence for lack of longer time series. The data points for the macroeconomic data are assumed to be released at every end of month. The day of the month at which the data is released is not relevant on a monthly basis analysis. We have tested this and results remain unaffected. The period considered is from January 1999 until July 2008. This results in 114 observations and 9 regressions for each of the yields comprising EONIA for the overnight rate, Euribor 3 months, Euribor 6 months and the German government securities for 2, 5, 10, 15, 20 and 30 years.

## 6. Empirical Results

Table 1 below shows the regression results from [11]. Strikingly, almost all coefficients are significantly different from zero and the model also appears to perform well in predicting interest rates and yields. R-Squared falls as maturities increases. On average, we observed an upward sloping yield curve starting with an average overnight rate of the period of 3.06% up to 4.66% for a 30 year German government bond. Results also show that the lower end of the curve is far more volatile that the longer end. The lower end exhibiting a standard deviation of 1.20 compare to 0.61 and 0.63 for 15 and 30 year

## Table 1. EONIA, Euribor Rates, and the German Government Yield Curve as Proxies for the Term Structure of Interest Rates.

| | EONIA | Euribor 3M | Euribor 6M | 2 Years | 5 Years | 10 Years | 15 Years | 20 Years | 30 Years |
|---|---|---|---|---|---|---|---|---|---|
| | Coefficient<br>Standard Error<br>T-Ratio [Prob] | Coefficient<br>Standard Error<br>T-Ratio [Prob] | Coefficient<br>Standard Error<br>T-Ratio [Prob] | Coefficient<br>Standard Error<br>T-Ratio [Prob] | Coefficient<br>Standard Error<br>T-Ratio [Prob] | Coefficient<br>Standard Error<br>T-Ratio [Prob] | Coefficient<br>Standard Error<br>T-Ratio [Prob] | Coefficient<br>Standard Error<br>T-Ratio [Prob] | Coefficient<br>Standard Error<br>T-Ratio [Prob] |
| Intercept | 15.61 | 0.70 | -14.46 | -11.90 | 6.85 | 26.11 | 32.90 | 39.96 | 40.10 |
| | 4.64 | 5.15 | 5.91 | 5.45 | 5.70 | 5.30 | 5.93 | 5.74 | 5.69 |
| | 3.36[.001] | 0.14[.892] | -2.45[.016] | -2.18[.031] | 1.23[.221] | 4.92[.000] | 5.55[.000] | 6.96[.000] | 7.05[.000] |
| Log Confidence Index | 1.02 | 0.58 | 1.39 | 4.79 | 3.31 | 1.42 | 0.01 | -0.51 | -0.02 |
| | 0.44 | 0.48 | 0.56 | 0.51 | 0.52 | 0.50 | 0.56 | 0.54 | 0.54 |
| | 2.34[.021] | 1.20[.231] | 2.49[.014] | 9.34[.000] | 6.31[.000] | 2.84[.005] | -0.010[.988] | -0.95[.346] | -0.03[.977] |
| Log Unemployment | -9.72 | -9.63 | -8.05 | -5.50 | -4.30 | -3.59 | -3.66 | -3.77 | -3.45 |
| | 0.52 | 0.58 | 0.66 | 0.61 | 0.63 | 0.60 | 0.67 | 0.64 | 0.64 |
| | -18.65[.000] | -16.66[.000] | -12.14[.000] | -8.98[.000] | -6.87[.000] | -6.04[.000] | -5.50[.000] | -5.84[.000] | -5.41[.000] |
| Log Euro-Zone M3 | -7.25 | -8.36 | -8.98 | -4.94 | -2.38 | -1.15 | -1.28 | -0.37 | -0.60 |
| | 0.66 | 0.74 | 0.84 | 0.78 | 0.80 | 0.76 | 0.85 | 0.82 | 0.81 |
| | -10.93[.000] | -11.37[.000] | -10.65[.000] | -6.34[.000] | -2.99[.003] | -1.52[.131] | -1.52[.132] | -0.46[.648] | -0.74[.463] |
| Log Production Prices | 14.65 | 20.45 | 23.43 | 10.63 | 2.63 | -2.28 | -2.04 | -4.72 | -4.94 |
| | 1.85 | 2.05 | 2.35 | 2.17 | 2.22 | 2.11 | 2.36 | 2.28 | 2.26 |
| | 7.93[.000] | 9.98[.000] | 9.97[.000] | 4.90[.000] | 1.19[.237] | -1.08[.282] | -0.87[.338] | -2.07[.041] | -2.18[.031] |
| R-Squared | 0.94 | 0.93 | 0.91 | 0.90 | 0.82 | 0.76 | 0.64 | 0.61 | 0.69 |
| R-Bar-Squared | 0.94 | 0.93 | 0.91 | 0.89 | 0.82 | 0.75 | 0.63 | 0.60 | 0.68 |
| S. E. of Regression | 0.29 | 0.32 | 0.37 | 0.34 | 0.35 | 0.33 | 0.37 | 0.36 | 0.36 |
| F-Stat: F( 4; 117) | 485.06[.000] | 398.17[.000] | 290.10[.000] | 271.61[.000] | 135.48[.000] | 90.70[.000] | 52.10[.000] | 45.48[.000] | 65.36[.000] |
| Average Yields | 3.06 | 3.19 | 3.27 | 3.21 | 3.68 | 4.16 | 4.42 | 4.60 | 4.66 |
| Standard Deviation | 1.20 | 1.21 | 1.20 | 1.07 | 0.81 | 0.66 | 0.61 | 0.56 | 0.63 |
| Residual Sum of Squares | 9.85 | 12.13 | 15.94 | 13.59 | 14.18 | 12.86 | 16.05 | 15.08 | 14.78 |
| Durbin Watson Statistic | 1.22 | 0.59 | 0.67 | 0.47 | 0.41 | 0.33 | 0.32 | 0.31 | 0.27 |
| A: Serial Correlation | 18.03[.000] | 59.79[.000] | 53.25[.000] | 70.47[.000] | 75.88[.000] | 82.93[.000] | 84.88[.000] | 85.03[.000] | 88.369[.000] |
| B: Functional Form | 3.46[.019] | 9.45[.000] | 8.41[.000] | 11.92[.000] | 12.73[.000] | 16.75[.000] | 37.95[.000] | 33.63[.000] | 31.74[.000] |
| C: Normality | 2.57[.277] | 5.29[.071] | 18.93[.000] | 1.28[.528] | 1.29[.524] | 1.21[.545] | 2.67[.263] | 4.41[.110] | 2.224[.328] |
| D: Heteroscedasticity | 0.34[.853] | 6.61[.010] | 5.44[.020] | 24.04[.000] | 45.63[.000] | 59.93[.000] | 69.38[.000] | 77.76[.000] | 78.02[.000] |

*A: Breusch-Godfrey, Lagrange multiplier test of residual serial correlation, hence to test the null that the disturbances follow an AR(p). B: Ramsey's RESET test using the square of the fitted values, hence to test the null that the model has no omitted variables. C: Based on Jarque-Bera test of skewness and kurtosis of residuals to test the null hypothesis of normally distributed disturbances. D: Based on LM test for ARCH effects, test the null of homoscedasticity. Note: OLS regression computed with observed yields and observed state space variables. Variables being the Eurostat Consumer Confidence Index, Unemployment Rate, Monetary Aggregate M3, Production Prices for the Euro-Zone. (Source: Eurostat, ECB and Bloomberg).*

government bonds. Most importantly, the majority of the coefficients confirm our expectations as in [7] and [8] as well as discussions in section 3 points 1) to 4).

Figure 1 shows the observed versus the estimated values for EONIA rate, the Euribor three months, as well as German Government bonds 5 years and 30 years respectively (GBRD 5Yr and GBRD 10Yr). The model predicts fairly well the lower end of the curve and, with less accuracy but still with success, the longer end. Notice, that we are not using the lagged value of any endogenous variable to obtain these yields hence, we are inferring these values purely from our macro data via OLS. This is important, because most of the models until now have included and AR(1) with a lag of the endogenous variable, this gave these authors good results for obvious reasons.

Figure 1. **Comparison of benchmark model using OLS versus observed values.**

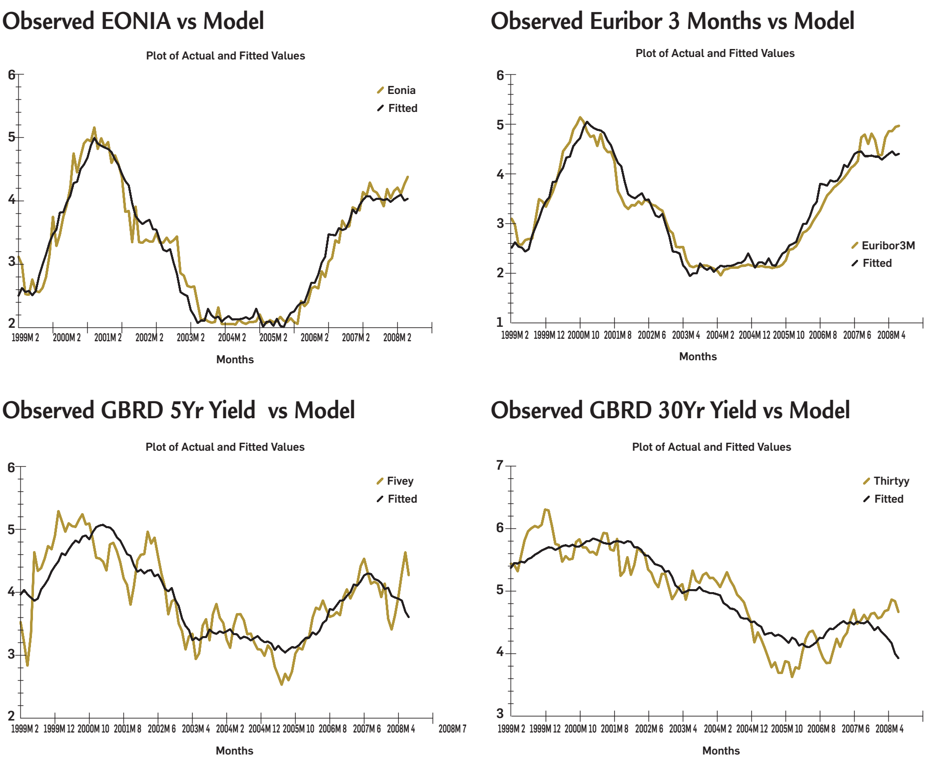
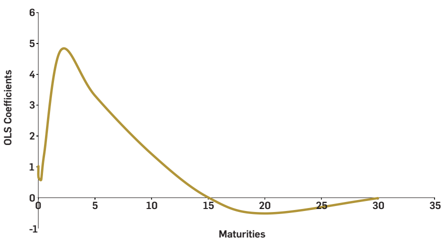


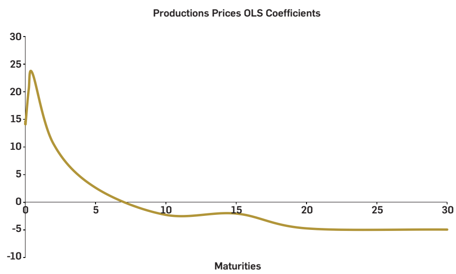

Figure 2 shows the coefficients from the explanatory variables plotted as a function of the maturities. Notice how the coefficients for the consumer confidence index and monetary aggregate M3 dissipate away as maturity increases. In fact, coefficients become less significant for the longer end of the curve, as seen in Table 1. Thus, as the time horizon increases, the effects of changes in consumer confidence and the quantity of money have lower predictive power. This makes sense, as not many of us make consumption decisions under an investment horizon greater than 10 years. Similarly, monetary shocks fall dramatically during the first 5 years maturities confirming that monetary aggregates will have only effects on the lower end of the curve.

**Figure 2. Estimated OLS beta coefficients using equation [11]. Impulse Response Analysis**

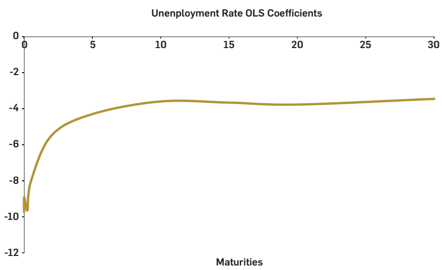
**Consumer Confidence Betas vs. Maturities**



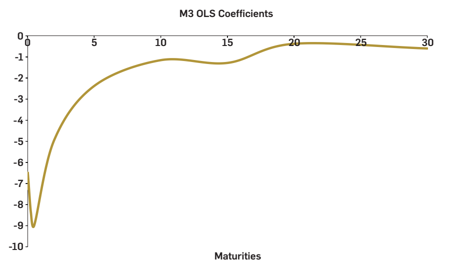

**Production Price Index Betas vs. Maturities**




**Unemployment Betas vs. Maturities**




**Monetary Aggregate M3 Betas vs. Maturities**




The price level, which is in this case measured by the Production Price Index, exhibits a positive relationship to yields in the lower end of the curve. Not surprisingly, this relationship reverts as maturities become longer. This is in line with our discussion as in [7] and [8] as well as in section 3 points 3) to 4). Notice how the five year maturity is not affected by the price level at all. Again, as maturities greater than the 5 years are less affected by monetary policy shocks and more influenced by movements in aggregate marginal utility growth. Below the 5 years maturities, changes in the price level are positively correlated to yields. This is because asset prices are here rather influenced by monetary policy shocks than changes in aggregate marginal utility growth. This could also mean simply that interest rates or yields in the lower end of the curve are rather influenced by central bank policy responses to this particular index (PPI). Yields or interest rates on the longer end exhibit a negative relationship to current price level data, because the market knows that central bank will increase interest rates and as result expected future changes in the price level will fall with the subsequent decrease in yields. This is important, because contrary to the controversial results presented by Cook and Hahn (1989) this evidence predicts that increases in the central bank policy rates as a consequence of a contractionary monetary policy would immediately lower long-term nominal interest rates. In general, the effects of the price level on the term structure of interest rates appears to us, at least for the period analysed, to be in line with discussions outlined under section 3, points 3 and 4.

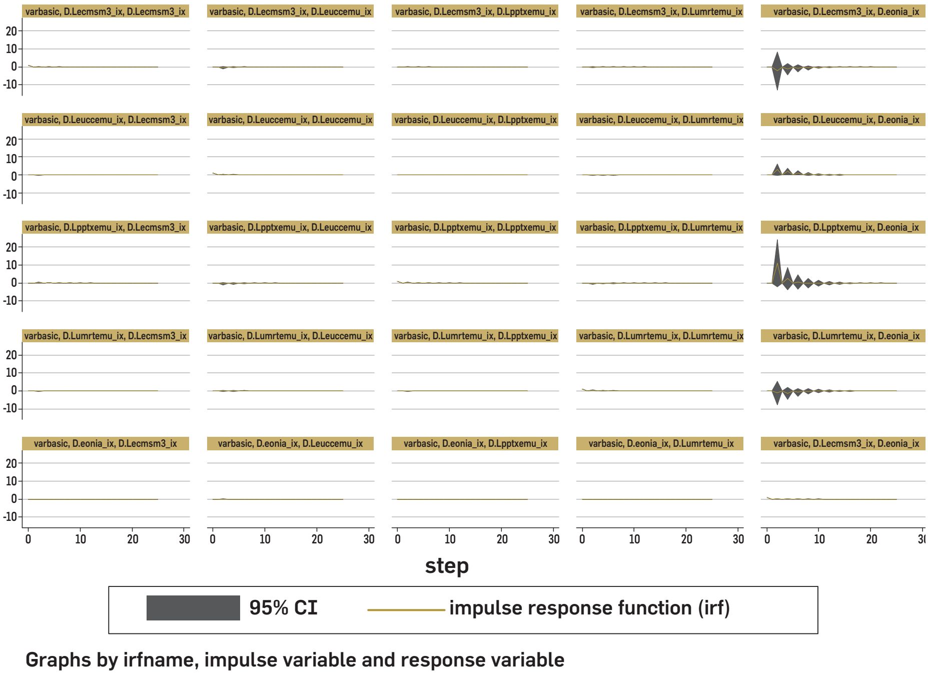
The monetary aggregate M3, the price level and unemployment account for the main drivers moving the lower end of the curve. However, only the price level and unemployment are persistent and their effects do not die away with longer maturities. A reason why these variables are persistent can be explained by unemployment and shocks in the price level having a persistent effect on aggregate consumption and investors being averse to persistence, as Piazzesi (2006) asserts, aversion to persistence generates concerns on future consumption when bad news arrives.

In general, our model predictions become poorer as maturity increases as seen in lower correlation coefficients as well as by increasing residual sum of squares, despite the fact that the standard deviation of the dependent variables decreases with longer maturities. The model, would on average, predict an upward sloping yield curve, as shown for higher observed mean of dependent variables. This is because the price level and unemployment coefficients are persistent throughout the yield curve and under normality these two effects account for an upward sloping yield curve, whereas consumer confidence and monetary aggregate shocks die away as maturities become longer.[1]

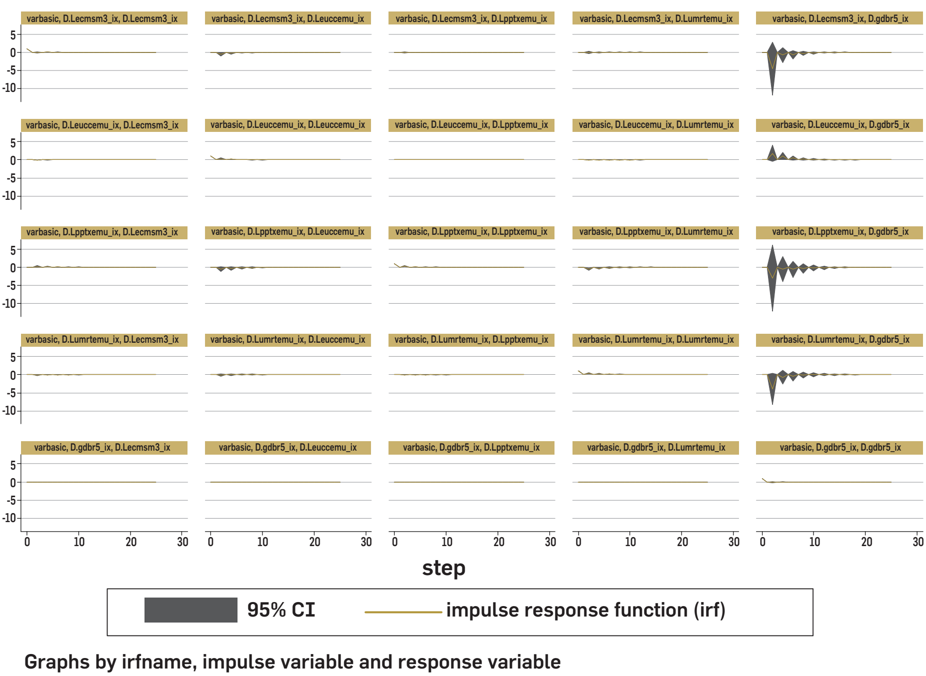
Figures 3 and 4 we analyse the impulse response function after running a VAR using 2 lags and 25 steps (each step represents an interval of 1 month) on the EONIA and the 5 Yr German Government bond yields. For example, it can we seen that an increase in the orthogonalised shock to the exogenous variable production prices (i.e. the proxy for the price level shown in the graph under the variable name D.pptxemu_ix) causes a short series of increases in the EONIA (shown in the graph under the variable name D.eonia_ix) that dies out in less than 5 months. Not surprisingly, the same increase in the orthogonalised shock to the price level –instead- generates a short series of decreases in the 5year Government bond yields that, similarly, dies out in less than 5 months. It can also be seen that ECB money supply M3 (D.Lecmsm3_ix) and EU unemployment have greater effect on the 5year German Government bond yields than on the EONIA rate. This is very important because it means that now we know that for a given macroeconomic environment, the yield curve is either expected to flatten, steepen or to twist which hence, has implications from a public debt policy as well as from a trading strategy point of view. As this can help reduce governments' debt roll-over risk, as well as shape sovereign debt portfolio trading or investment strategies.

[1] *Due to the existence of autocorrelation and heteroscedasticity issues (see Table 1) autocorrelation and heteroscedasticity consistent regressions were performed. Results do not differ to those already presented in Table I, therefore have been omitted here.*

## Figure 3. Impulse response analysis on the EONIA rate.




## Figure 4. Impulse response analysis on the 5 year German government bond.



## 7. Related literature

So far most of the empirical literature has been focused on testing Taylor-Rules for the US Federal Reserve, as for example in Hamalainen (2004). See also Battini and Haldane (1999) and Clarida *et al* (1999, 2000).

Bernanke (2002) analyses the use broader set of information instead of the usual Taylor-Rules framework for analysing monetary policy, as most of the empirical literature has been mainly confined to a limited amount of information, merely output and the price level.

Studies on empirical reaction functions of the ECB can be seen in Hayo and Hofmann (2003), Gerdesmeier and Roffia (2003), Gerlach-Kirsten (2003) and Gerlach and Schnabel (1999). Most of these works lack of investigation of *real* ECB policy reaction function. Fendel (2007) studies the ECB reaction function using monthly data based merely on a Taylor-Rule framework. In his work, the ECB reaction function is confined to the European Overnight Index Average (EONIA) representing the overnight policy rate, the Harmonised Index for Consumer Prices (HICP) as the inflation for Euro-Zone and output proxy taken from the Industrial Production Index for the Euro Area. The EONIA is not the Policy Rate (PR) however it would be expected to fluctuate around the PR. Furthermore, it is reasonable to believe that it would not be optimum otherwise and hence, the ECB will have enough incentives to ensure that the EONIA fluctuates around its PR and ensure effectiveness of its monetary policy.

Hence, resent studies have been focused in analysing monetary policy by including the central bank reaction function in a small empirical macro model of inflation and output, most of it US data (see, for example also, Rudebusch, 2000, Rudebusch and Svensson, 1998, 2001 as well as Mc Callum and Nelson, 1999). Ang and Piazzesi (2002) show how macro-variables add to the understanding of yield curve movements. In fact, from a European perspective, the ECB (as well as the majority of central banks) is likely to analyse a significant number of time series not only confined to the above mentioned ones. This paper analyses a variety of macroeconomic data releases for the Euro-Zone and examines their effect on the EONIA rate via a reaction function for the ECB. The macroeconomic data analysed here is not confined only to inflation and output. We also include additional variables such as unemployment as well as consumer confidence indices.

## 8. Conclusions and Final Remarks

This paper presents slight adjustments to the classical consumption based asset pricing models by introducing unemployment data and survey data such as consumers' confidence index. The incorporation of a monetary aggregate has also been extensively discussed in this paper and its inclusion in the model appears to us to be robust enough. This paper has also tested empirically an affine term structure model which has been adapted to the above mentioned data. We have shown that using current theoretical developments and a few state space variables such as European unemployment data, the European Consumers' Confidence Index, European Production Price Index (PPI) and a monetary aggregate such as ECB M3 for Europe, it is possible to explain yield curve movements with strikingly very good results. Unemployment and consumer confidence index have exhibited a shift and a slope effect on the yield curve, for front-end yields moving faster than in the long end. Production price index has a twist effect on the yield curve (flattening or steepening of the curve) which results in lower-end yields shifting in opposite directions to the long end. This empirical work shows that yields are negatively correlated to money supply, as expected in classical IS-LM models. And that money supply exhibits a slope effect, with the lower end of the curve shifting faster than the longer end. In the light of these results, we suggest that further analysis is needed to understand what other variables or mechanisms are governing the longer end of the curve, as for the lower end our results show that the variables used have very high predictive ability already. In addition, we also suggest further research on cross-sectional data across countries. If further empirical results confirm our evidence then, it will be possible to advice governments on their macroeconomic risks from a term structure perspective. In this paper we have used macroeconomic data to predict yield curve movements. However, it could be interesting to see the extent to which this can contribute to the debt policy and debt management theories.

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