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The structure of the optimal combined sourcing policy using capacity reservation and spot market with price uncertainty

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Abstract: This contribution focuses on the cost-effective management of the combined use of two procurement options: the short-term option is given by a spot-market with random price, whereas the long-term alternative is characterized by a multi period capacity reservation contract with fixed purchase price, reservation level and capacity reservation cost. Considering a multi-period problem with stochastic demand, the structure of the optimal combined purchasing policy is derived using stochastic dynamic programming.

Key words: Capacity reservation, spot market, purchasing policy, supply contracts, stochastic inventory control

1. Introduction

This contribution focuses on the cost-effective management of the combined use of a long-term and a short-term procurement option. In our case, the short-term option is given by a spot-market with a random spot-market price (which is independent of the quantity procured), whereas the long-term alternative is characterized by a simple wholesale price contract with a capacity reservation level and a downward flexibility to order at most the reservation level in each period. The planning situation we consider gains further complexity in the fact that in addition to the stochastic spot-market procurement option, the demand for the procured goods is also random. The management task is to fix a long-term capacity reservation level and to decide period-by-period how to combine the two supply options in order to profit from the cost savings of long-term procurement while still remaining flexible. Concerning the price variations on the spot market, this flexibility can be used to benefit from low short-term price levels while the long-term contract is a means to hedge the risk of high spot market prices.

This paper is related to a variety of research streams in the operations management literature. A topic of recent interest is the study of the supply chain procurement strategies combining spot market purchases with purchases made in advance from a specific long-term supplier. Henig et al. (1997) derived a three-parameter optimal policy without the consideration of uncertainty on the procurement side, which is a critical factor in practice. Bonser and Wu (2001) study the fuel procurement problem for electric utilities in which the buyer can use a mix of long-term and spot purchases. Our problem was first defined and studied in the inventory literature in Serel et al. (2001). They considered the simple (R,S) capacity reservation – order up policy, but they disregarded the spot market price uncertainty. Wu, Kleindorfer, and Zhang (2002) consider uncertainty in spot market prices and analyze the contracts for non-storable goods involving options executable at a predetermined price. Using a similar single-period model, Spinler and Huchzermeier (2006) show that, mainly due to the decrease in the supplier's production costs when an options contract is used, the combination of an options contract and a spot market is Pareto improving with respect to the other alternative market structures. Seifert et al. (2004) also analyzed a single-period problem from the buyer's standpoint with changing levels of buyer's risk preferences. Kleindorfer and Wu (2003) linked this literature to evolving B2B exchanges on the Internet. Serel (2007) considered spot market uncertainty with respect to available quantity but didn't consider price uncertainties. In Sethi et al. (2004) a situation with both demand and price uncertainty is taken into consideration, and a quantity flexibility contract is employed; however, no capacity reservation takes place.

Various types of supply contracts involving advance capacity purchases have been investigated, generally based on a single-period framework. Erkoc and Wu (2005) model the negotiations between a manufacturer and a supplier when the supplier has to make a costly investment in additional production capacity. Jin and Wu (2001) analyze capacity reservation contracts between a single supplier and multiple buyers with reservation fees deductible from the purchase price paid in delivery. Deng and Yano (2002) study the contracts between a component supplier and a manufacturer involving a fixed wholesale price for advance purchases and a spot price determined and charged for purchases after the demand is realized. Burnetas and Ritchken (2005) look into the impact of option contracts on the wholesale and retail prices under price-dependent demand in a manufacturer–retailer chain. Other papers on the use of options in supply chains include Kamrad and Siddike (2004).

In our paper, we consider both demand and spot market price uncertainty in a multi-period framework. Our focus is on investigating the structure of the optimal purchasing and capacity reservation policy. To this end we use a stochastic dynamic programming approach for analyzing and solving the problem. It turns out that the optimal structure is given by a quite complex three-parameter decision policy with a price-dependent order-up-to level for short-term procurement.

The present analysis is related to those contributions where optimal policies are derived in the context of combined sourcing and capacity reservation. In Henig et al. (1997) and Serel et al. (2001) a simple three-parameter policy is shown to be optimal when no supply-side uncertainty exists. Sethi et al. (2004) consider additional price uncertainty for the short-term procurement option, but instead of a capacity reservation scheme they deal with a quantity flexibility contract. Their procurement policy is a price-contingent order-up-to policy (like ours). However, it is only proved for the single-period case and a simple geometric distribution of the random spot market price. The analysis in Serel (2007) is the closest to ours. The main difference is that they consider a spot market with random capacity at a given price instead of a random price without capacity restriction. Furthermore, they assume that the spot capacity is not known when the ordering decision is made so that procurement decisions will not depend on the respective capacity level at the spot market. Under these circumstances, the optimal policy in Serel (2007) has a simple three-parameter structure, but is not capacity-contingent.

2. The Structure of the Optimal Policy

2.1 Problem Description and Notation

We assume that for the random stationary demand, ξ, per period and random spot market price, π, per period we know the following characteristics:

$F(x), f(x), \mu_x, \sigma_x$	cdf., pdf ., expected value and standard deviation of **demand** and
$G(p), g(p), \mu_p, \sigma_p$	the same distribution characteristics for the **spot market price**.

Both demand and price are assumed to be identically and independently distributed.

We consider a sequential decision process involving different level of knowledge in time. The first decision before any inventory is observed and ordering takes place is on

R	the capacity reservation quantity

that must be fixed for a longer time horizon based on the random demand and spot market price distribution and the following stationary cost factors:

c	the unit price charged by the long-term supplier,
r_0	the capacity reservation price per period for a unit of capacity reserved,
h	the inventory holding cost per unit and period,
v	the shortage cost per unit and period.

Next, the decision at the beginning of each time period is

Q_L order quantity from the long-term supplier, and

Q_K order quantity from the spot market

knowing

I_t inventory level and p_t the realized spot market price

at the beginning of each period, t, of a given (finite) planning horizon T ($t=1,\ldots T$), but without knowing the realized demand for the period. The shipments are assumed to arrive before the period demand is realized. Unsatisfied demand is backordered. The final cost and finishing inventory level is only known after the realization of the demand at the end of the period.

Future cost is discounted by

β the single-period discount factor.

The overall objective is to choose the long-term capacity reservation level before the first period starts and after that in each period of the planning horizon select the spot market and reservation based order quantities in such a way that the total expected cost is minimized.

2.2 Optimal Capacity Reservation and Procurement Policy

For the problem under consideration, the structure of the optimal policy can be evaluated by using a stochastic dynamic programming approach. The result is given in *Proposition 1*.

Proposition 1: For a finite planning horizon, T, the optimal policy structure for the above combined ordering decision process, is an ($R,S_L,S_K(p)$) policy, characterized by the constant capacity reservation quantity, R, and constant base stock level S_L for long-term supplier and a price-dependent base stock $S_K(p)$ for spot market. The order policy in each period t:

(a) If $p_t < c$, order only from spot market up to base stock level $S_K(p)$.

(b1) If $p_t \geq c$, order from long term supplier only up to base stock level S_L if the reserved capacity, R, is sufficient.

(b2) If $p_t \geq c$ and the reserved capacity is not sufficient order from spot market up to level $S_K(p)$ as long as this level is not yet exceeded.

More formally, this policy can be described in the following way

(a) If $p_t < c$:

$$Q_L = 0 \quad \text{and} \quad Q_K = \begin{cases} S_K(p_t) - I_t & \text{if} \quad I_t \le S_K(p_t) \\ 0 & \text{if} \quad I_t \ge S_K(p_t) \end{cases}$$

(b) If $p_t \ge c$:

$$Q_L = \begin{cases} S_L - I_t & \text{if} \quad I_t \le S_L \;\text{ and }\; S_L - I_t \le R \\ R & \text{if} \quad I_t \le S_L \;\text{ and }\; S_L - I_t \ge R \\ 0 & \text{if} \quad I_t \ge S_L \end{cases} \quad \textbf{and} \quad Q_K = \begin{cases} S_K(p_t) - R - I_t & \text{if} \quad I_t \le S_K(p_t) - R \\ 0 & \text{if} \quad I_t \ge S_K(p_t) - R \end{cases}$$

For a finite horizon problem the order-up-to levels S_L and $S_K(p)$ vary from period to period. For this order-up-to policy, the total expected cost is a convex function of the capacity reservation level R.

Proof:

We introduce the following additional notation:

$D_t(I_t,R,p_t)$: minimum expected cost from period t to T for a starting inventory I_t and a given capacity reservation R, after realization of spot market price p_t

$C_t(I_t,R)$: minimum expected cost from period t to T for a starting inventory I_t and a given capacity reservation R, before spot market price p_t realizes

$C_0(R)$: minimum expected cost from period 1 to T for a given capacity reservation level R before any spot market price realizes and starting inventory I_1 is known.

The state transformation for the backorder situation is $I_{t+1} = I_t+Q_{L,t}+Q_{K,t}-x_t$ for $t=1,2,\ldots,T$ and with I_1 as given initial inventory.

For sake of simplicity the time index is suppressed for all variables in the following expressions.

The minimum cost depending on the choice of the capacity reservation quantity R is given by taking the expected value over all possible initial inventory levels

$$C_0(R) = \underset{I}{E}\left[C_1(I,R)\right]$$

where we assume that this initial inventory is not yet known when the capacity reservation decision is made.

For the procurement decisions in period *t=1,…,T* , the dynamic programming recursive relations can be expressed by

$$C_t(I,R) = \int_0^\infty D_t(I,R,p)g(p)dp \quad \text{and}$$

$$D_t(I,R,p) = \min_{R \ge Q_L \ge 0, Q_K \ge 0} \left\{ cQ_L + pQ_K + L(I+Q_L+Q_K) + \beta \cdot \int_0^\infty C_{t+1}(I+Q_L+Q_K-x,R)f(x)dx \right\}$$

with $C_{T+1}(I,R) \equiv 0$ as final condition for all I and R.

The **major steps of the proof** include

- proving the optimality of *(R,S_L,S(p))* policy by complete induction
- proving that this policy holds for any t if $C_{t+1}(I,R)$ is convex
- proving that $D_t(I,R,p)$ is convex if this policy is applied
- proving that this holds for the final period $t=T$
- proving that $C_0(R)$ is a convex function.

The optimization problem in period *t* can be reformulated as

$$D_t(I,R,p) = \min_{R \ge Q_L \ge 0, Q_K \ge 0} \left\{ cQ_L + pQ_K + H_t(I+Q_L+Q_K,R) \right\}$$

with $H_t(I+Q_L+Q_K,R) \equiv L(I+Q_L+Q_K) + \beta \cdot \int_0^\infty C_{t+1}(I+Q_L+Q_K-x,R)f(x)dx$

and $L(I) = h \cdot \int_0^I (I-x)f(x)dx + v \cdot \int_I^\infty (x-I)f(x)dx$.

By assumption $C_{t+1}(I,R)$ is convex in *I* and *R, thus* $H_t(I,R)$ is also convex in *I* and *R* due to well-known convexity of *L*(*I*). So we can analyze the properties of minimum cost functions $D_t(I,R,p)$ und $C_t(I,R)$

(i) in case of $p \le c$:

$$D_t(I,R,p) = \begin{cases} p \cdot (S_K(p) - I) + H_t(S_K(p),R) & \text{if} \quad I \le S_K(p) \\ H_t(I,R) & \text{if} \quad I \ge S_K(p) \end{cases}$$

(ii) in case of $p \geq c$:

$$D_t(I,R,p) = \begin{cases} c \cdot R + p \cdot (S_K(p) - I - R) + H_t(S_K(p), R) & \text{if} \quad I \leq S_K(p) - R \\ c \cdot R + H_t(I+R, R) & \text{if} \quad S_K(p) - R \leq I \leq S_L - R \\ c \cdot (S_L - I) + H_t(S_L, R) & \text{if} \quad S_L - R \leq I \leq S_L \\ H_t(I,R) & \text{if} \quad I \geq S_L \end{cases}$$

we can easily show that $D_t(I,R,p)$ is twice continuously differentiable in *I* and *R*. Due to convexity of $H_t(I,R)$ we have:

$$\frac{\partial^2}{\partial I^2} H_t(I,R) \geq 0\,, \quad \frac{\partial^2}{\partial R^2} H_t(I,R) \geq 0 \quad \text{and} \quad \frac{\partial^2}{\partial I^2} H_t(I,R) \cdot \frac{\partial^2}{\partial R^2} H_t(I,R) - \frac{\partial^2}{\partial I \partial R} H_t(I,R) \cdot \frac{\partial^2}{\partial R \partial I} H_t(I,R) \geq 0$$

So the Hessian of $D_t(I,R,p)$ is nonnegative definite for each *p,* $D_t(I,R,p)$ is convex in *I* and *R* for each *p*, and $C_t(I,R) = \int_0^\infty D_t(I,R,p) g(p) dp$ is convex in *I* and *R* due to $g(p) \geq 0$

Steps of induction:

For each *t<T* the following holds: From convexity of $C_{t+1}(I,R)$ it follows that also $C_t(I,R)$ is convex in *I* and *R, so* $H_{t-1}(I,R)$ is also convex in *I* and *R and consequently* (*R,S_L,$S_K(p)$*) policy is optimal also for *t*-1.

For *t=T* (start of induction) we have: $H_T(I,R) = L(I)$ independent of *R* *thus* $H_T(I,R)$ is convex in *I* and (*R,S_L,$S_K(p)$*) policy is optimal for *t=T.*

General Conclusions

Policy Structure: (*R,S_L,$S_K(p)$*) policy is optimal for each $1 \leq t \leq T$

- Policy parameter $S_{L,t}$ is calculated from: $\dfrac{\delta H_t(S,R)}{\delta S} + c = 0$ for each *R*
- Policy parameter $S_{K,t}(p)$ is calculated from: $\dfrac{\delta H_t(S,R)}{\delta S} + p = 0$ for each *R* and p
- Policy parameter *R* is calculated from: $\dfrac{\delta C_0(R)}{\delta R} = 0$
- Functions $C_0(R)$ and $H_t(I,R)$ are convex.

From unconstrained optimisation we get as optimal inventory levels

- after Q_L-optimization : $S_{L,t}(R)$ from: $\dfrac{\delta H_t(S,R)}{\delta S} + c = 0$
- after Q_K-optimization : $S_{K,t}(p_t, R)$ from: $\dfrac{\delta H_t(S,R)}{\delta S} + p_t = 0$.
-

Due to $\dfrac{\delta}{\delta Q_L} H_t(I + Q_L + Q_K, R) = \dfrac{\delta}{\delta Q_K} H_t(I + Q_L + Q_K, R)$,

and due to restrictions $0 \le Q_L \le R$ and $0 \le Q_K$ we get the policy structure described in *Proposition 1*.

Order-up-to levels $S_{L,t}$ and $S_{K,t}(p_t)$

From convexity of $H_t(S,R)$ it immediately follows that

$$S_{K,t}(p_t) \begin{cases} > S_{L,t} & \text{if } p_t < c \\ = S_{L,t} & \text{if } p_t = c \\ < S_{L,t} & \text{if } p_t > c \end{cases}$$

The convexity properties described can also be exploited for simplifying the numerical computation of the optimal policy parameters.

3. Extensions

The analysis given in Section 2 will be extended in three directions:

- considering the initial inventory information in the capacity reservation decision,
- a lost-sales environment for unsatisfied demand, and
- the case of an infinite planning horizon.

In Section 2, we modeled a situation where the negotiation about the contract with the long-term supplier takes place before the starting inventory of the first period is known. If the decision on the capacity reservation level can be postponed until this inventory is known the following proposition holds.

Proposition 2: Knowing the initial inventory at the time of the capacity reservation, the optimal ordering policy is identical to that in *Proposition 1* , while the optimal capacity reservation quantity will become a function of the initial inventory I_1.

Proof:

The only difference to the optimization problem formulated in Section 2 is that for the total cost impact of the capacity reservation level we now have

$$C_0(R) = C_1(I, R)$$

which due to convexity of $C_1(I, R)$ is a convex function for each possible inventory level I. All other cost functions for periods $t=1,\ldots,T$ are not affected by the change of information at the beginning of the planning period.

Under lost-sales conditions we can derive the following proposition regarding the optimal policy structure.

Proposition 3: If unsatisfied demand is lost in each period instead of being backordered, the structure of the optimal policy is still of the ($R,S_L,S_K(p)$) type.

Proof:

In the lost-sales case the state transformation is $I_{t+1} = \max\{I_t+Q_{L,t}+Q_{K,t}-x_t\,;\,0\}$ for $t=1,2,\ldots,T$.

Thus, cost function $H_t(I+Q_L+Q_K, R)$ changes to

$$H_t(I+Q_L+Q_K, R) \equiv L(I+Q_L+Q_K) + \beta \cdot \int_0^{\infty} C_{t+1}(\max\{I+Q_L+Q_K-x;0\}, R) f(x)dx \ .$$

It can easily be checked that convexity of $C_{t+1}(I, R)$ guarantees convexity of

$\int_0^{\infty} C_{t+1}(\max\{I+Q_L+Q_K-x;0\}, R) f(x)dx$, so that also $H_t(I, R)$ is convex in I and R. Thus, the proof by induction given for *Proposition 1* also holds in the lost-sales case.

When we face an infinite number of periods ($T \to \infty$), the following proposition holds for the optimal policy

Proposition 4: In the infinite horizon problem with discount factor $\beta<1$, a policy of the ($R,S_L,S_K(p)$) type with stationary parameter values is optimal.

Proof:

For the infinite horizon problem the functional equations of dynamic programming have to fulfil

$$C(I,R)=\int_0^\infty D(I,R,p)g(p)dp \text{ and}$$

$$D(I,R,p)=\min_{R\geq Q_L\geq 0, Q_K\geq 0}\left\{cQ_L+pQ_K+L(I+Q_L+Q_K)+\beta\cdot\int_0^\infty C(I+Q_L+Q_K-x,R)f(x)dx\right\}$$

Due to the stationary environment and the infinite horizon the decision problem for ordering is the same in each period.

Now, Theorem 8-14 of Heyman and Sobel (1984) can be used to prove that the above functional relationship is satisfied by

$C(I,R)=\lim_{T\to\infty} C_1(I,R)$ and $D(I,R,p)=\lim_{T\to\infty} \mathrm{D}_1(I,R,p)$ where $C_1(I,R)$ and $D_1(I,R,p)$ are defined as in Section 2. In the very same way as it is done in Serel (2007) for the three-parameter policy in case of spot market capacity uncertainty, it can be shown that the conditions a to d of Theorem 8-14 hold in our case because the single-period costs and optimal order levels are bounded.

It follows that all convexity properties of the respective cost functions also hold in the infinite horizon case. The order-up-to levels $S_{L,1}$ *and* $S_{K,1}(p)$ converge to the stationary ones S_L and $S_K(p)$ and can be calculated using the stationary cost function $H(I,R)$ and the optimality conditions from Section 2. The optimal capacity reservation level R is calculated from minimizing $C_0(R)=\underset{I}{E}\left[C(I,R)\right]$.

The optimality of a stationary ($R,S_L,S_K(p)$) policy for a discounted cost criterion in the infinite horizon case does not necessarily mean that this property also holds for an average cost criterion. From a practical point of view, however, the ($R,S_L,S_K(p)$) policy can also be applied to minimize average period cost since discount factor β can be chosen arbitrarily close to 1.

4. Conclusions

Although we can exploit the knowledge of the optimal policy structure for problem solving, the optimal parameters generally can only be calculated by elaborate numerical methods. This is also due to the fact a complete function $S_K(p)$, *for any* $p > 0$, has to be computed for the short-term procurement level. Thus for practical applicability we have the two main options. We can provide a simple heuristic approximation for the policy parameters or consider a simpler policy structure where the optimal parameters can be derived analytically.

The latter approach has been investigated in Inderfurth and Kelle (2008). There a simple base stock (R,S) policy is considered where both short-term, spot market based and long-term, capacity reservation based purchasing decisions follow a single order-up-to level S which does not depend on the spot market price p. The option of developing tractable heuristics for calculating the parameters of the optimal $(R,S_L,S_K(p))$ policy type is a field for further research.

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