



Optimal Delegation in Nash Bargaining

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Abstract

When appointing a representative in negotiations, the principal can offer his agent a contract that promises a percentage of the bargaining result, and a bonus payment (or penalty) if bargaining fails. Conventional wisdom of contract theory seems to suggest that the share should be as great as possible to provide proper incentives for a risk-neutral agent, while the bonus should be small or even negative. Drawing on the symmetric Nash bargaining solution, this paper argues that the optimal share is rather small, whereas the optimal bonus is rather large.

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1 Introduction

This paper analyses negotiations between a principal and a third party when the principal may employ a risk-neutral agent as his representative. To solve the bargaining problems with and without delegation I use the symmetric Nash bargaining solution.¹ An optimal contract consists of a moderate share for the agent, and a high bonus payment in case the negotiations fail.

The intuition behind these results is based on the structure of the Nash bargaining solution. The fixed bonus payment drives up the threat point of the agent, which increases the Nash bargaining result. A moderate share for the agent would have the same effect: The lower a bargainer's marginal valuation of the bargaining result, the higher is this party's share according to the Nash bargaining solution.² When trying to exploit these two effects, the principal has to obey three constraints: If the agent has an outside option, then his share must not be lowered to zero; an increased threat point must not distract him from concluding an agreement; and the delegation must be profitable for the principal. This paper discusses these constraints, shows the conditions under which a non-empty set of mutually agreeable contract parameters exists, and derives the optimal contract.

The results derived here are in contrast to the basic intuition of economic contract theory, according to which it would be recommendable to offer a risk-neutral agent the full returns on his (unobservable) work and to charge him a fixed fee. Such a recommendation would also be in line with the results of Bester and Sákovics (2001). In their model, it is optimal to put a bargaining delegate into the position of a residual claimant. The authors distinguish a case in which the delegation contract is irrevocable, and a case where renegotiation is possible. The first case is rather close to my model, but they have applied the non-cooperative bargaining model of Rubinstein (1982) instead of the Nash bargaining solution. In their model, the principal optimally uses a "sell the shop" contract: The agent pays a fixed amount, and receives the full bargaining result. Moreover, the agent has a zero outside option. In their model, as in my model, the principal can capture the whole bargaining rent through the optimal delegation contract.

A non-cooperative bargaining model is applicable to negotiations in which the rules for making offers are explicitly given (e.g., the Rubinstein model allows the parties to make alternating offers for an infinite time).³ Threat points, i.e., the parties' payoffs in case bargaining breaks up, play no role in Rubinstein's model, as it consists of an infinite number of rounds.⁴ A delegate's decision on

¹See Nash (1950). The application of this solution concept requires the delegation contract to be observable for the other party; on unobservable contracts as commitment devices see Katz (1991).

²This effect was discussed in another context in Kirstein, Kirstein, and Gerhard (2008).

³Lammers (2008) analyzes which impact fairness on the part of the delegate may have on ultimatum bargaining with a third party.

⁴See, however, Binmore/Shaked/Sutton (1989) who distinguish between "impasse" and "breakdown" payoffs. While the former play no role in an infinite game, the latter may have

how much effort to spend in order to promote his principal's goals can easily be integrated into a non-cooperative model.

The Nash bargaining solution is more adequate if negotiations between the two parties lack a clear structure. The Nash solution expressly acknowledges threat points and, thereby, allows for the analysis of their strategic impact on the bargaining outcome. Effort of the bargainer (the principal or his delegate) can not be taken into account in the framework of the Nash solution – the classical moral hazard problem plays, therefore, no role in the subsequent analysis.

The results derived below may shed some light on a puzzle discussed in Levitt and Dubner (2005, 73)⁵ and in Bazerman/Neale (1993). Real-estate agents normally receive only 6 % commission. Hence, it does not really pay for the agent to engage in tough negotiations with a customer. If she manages to drive up the price of a house from, say, \$ 420.000 to \$ 430.000, this would only benefit her client, as she receives only \$ 600 of the incremental \$ 10.000. It makes little sense for her to risk the whole deal for such a small additional profit. In the terminology of principal-agent theory, the agent receives a small variable pay and a zero fixed wage. If the agent is risk-neutral without limited liability constraints, this theory would recommend a very different contract structure (“sell the shop”): let the real-estate agent pay a fixed sum to the client and give her the residual rights, i.e., a commission of 100%. However, the real-estate agent represents his client in negotiations; according to the results derived here, the puzzling contract parameters make some sense.

Delegation in bargaining has been discussed in several papers. Cai and Cont (2004) also discuss the impact of a bargainer's marginal valuation of the bargaining result, but do not use this parameter as a strategic variable. Their model rather addresses the problem of moral-hazard on the side of the agent. Agent's effort and type are excluded from consideration in my model, as there is little scope for parties' effort in the framework of Nash bargaining. Harris (1990) also manipulates the marginal valuation and the threat point of one party in Nash bargaining, but does not systematically analyze the effect derived here. Burtraw (1992) focuses (the strategic use of) risk-aversion in bargaining, as the curvature of the agent's utility function may also influence his marginal valuation of the outcome. The canonical work on the manipulation of bargaining threat points is the book by Schelling (1960).⁶ Putnam (1988) demonstrates how domestic constitutional constraints at home can make an international bargainer tougher.⁷ Bilateral delegation in bargaining is analyzed in Segendorff (1998) and in Jones (1989). The latter also introduces a payment scheme based on shares of the outcome, but in his model, the reason for low shares is competition among agents.

an impact on the bargaining outcome.

⁵See also Levitt and Syverson (2005).

⁶Kirstein and Rickman (2005), Kirstein (2000) present litigation and settlement models which exploit this effect.

⁷Such a commitment has been analyzed in a more general framework by Dewatripont (1988).

2 The Model

2.1 Setup

Consider two risk-neutral parties, P and T, who negotiate over the distribution of a certain amount of money, which can be normalized to one (dollar) without loss of generality. We assume that, in case of a disagreement, P would receive a payment $\pi \geq 0$, while T receives nothing.⁸ Let ψ denote P's share (measured in money) if the parties negotiate directly. Then, the symmetric Nash bargaining solution ψ^* maximizes the Nash product:

$$\psi^* = \arg \max [\psi - \pi][1 - \psi] = \frac{1 + \pi}{2}. \quad (1)$$

It is a well explored feature of this solution that it increases in P's threat point π , which gives room for strategic manipulations. If P, before the negotiations start, makes an investment to increase his threat point, this would strategically exploit his opponent. With $\pi = 1$, P could even capture the whole bargaining rent (and drive his opponent down to zero). This makes sense if the cost of the strategic move is smaller than the additional rent, namely $1/2$. Moreover, the strategic move is only effective if it is irrevocable - it has to be a credible commitment. This prerequisite may be hard to satisfy if it is P himself who takes part in the negotiations.

2.2 Delegation

This section argues how P can credibly manipulate the bargaining situation by appointing a delegate. Moreover, it shows that, besides the manipulation of the threat point, a second way exists to influence the bargaining outcome. Assume that P may appoint a risk-neutral agent A as his representative in the negotiations with T, but it is impossible to impose binding restrictions (such as "do not accept less than 60 cents!"). Agent A can earn an alternative monetary income, denoted $u \geq 0$ if he declines P's contract offer. P offers a share α of the bargaining result to A and, should the negotiations fail, a bonus payment $\beta \in \mathbb{R}$ (if $\beta < 0$, then it is a penalty). If A accepts the contract (α, β) , he bargains in lieu of P, i.e., A decides on his own whether to accept a bargaining result. The outcome of bargaining under delegation, denoted ϕ , maximizes a Nash product in which the first factor is now governed by A's marginal valuation of the bargaining result, and by A's threat point. Hence,

$$\phi^* = \arg \max [\alpha\phi - \beta][1 - \phi].$$

This leads to the first-order condition $\alpha(1 - \phi) - (\alpha\phi - \beta) \stackrel{!}{=} 0$. The necessary condition for an internal maximum is satisfied if $\alpha > 0$, and the solution is

$$\phi^*(\alpha, \beta) = \frac{\alpha + \beta}{2\alpha} \quad (2)$$

⁸The latter assumption keeps the following derivations simpler. T's threat point is irrelevant for the argument made in this paper and, thus, can be normalized to zero.

The bargaining solution under delegation is increasing in β if $\alpha > 0$. It is decreasing in A's share α if $\beta > 0$.⁹ Hence, to achieve the greatest possible bargaining result, P should offer a contract in which A's share α is as small as possible (yet positive), while the bonus β is as great as possible. In doing so, P has to obey three constraints:

- An incentive compatibility constraint (ICC): If the bonus β is too high, this could distract A from concluding an agreement with T. Hence, β must not exceed the maximum possible amount A could possibly acquire, i.e., α times the whole dollar. Hence the ICC can be written as $\beta \leq \alpha$.
- The agent's participation constraint (PCA): The share α must be high enough to induce A to accept the contract (α, β) with P. Hence, $\alpha\phi^* \geq u$.
- The principal's participation constraint (PCP): The bargaining result under the contract has to be profitable for P, thus $(1 - \alpha)\phi^* \geq \psi^*$.

2.3 Mutually acceptable contracts

Substituting ϕ^* into the PCA leads to $\beta \geq 2u - \alpha$. The PCA and the ICC (i.e., $\beta \leq \alpha$) imply a set of contract parameters (α, β) which are acceptable for the agent, and which yield the delegated bargaining outcome $\phi^*(\alpha, \beta)$. In Figure 1, the ICC is obeyed by (α, β) combinations below the upwards sloped line through the origin. The PCA is obeyed by combinations above the downwards sloped diagonal line.

The PCP leads to $(1 - \alpha)[\alpha + \beta]/2\alpha \geq (1 + \pi)/2$. For $0 < \alpha < 1$, this can be rearranged to

$$\beta \geq \frac{\alpha(\pi + \alpha)}{1 - \alpha}. \quad (3)$$

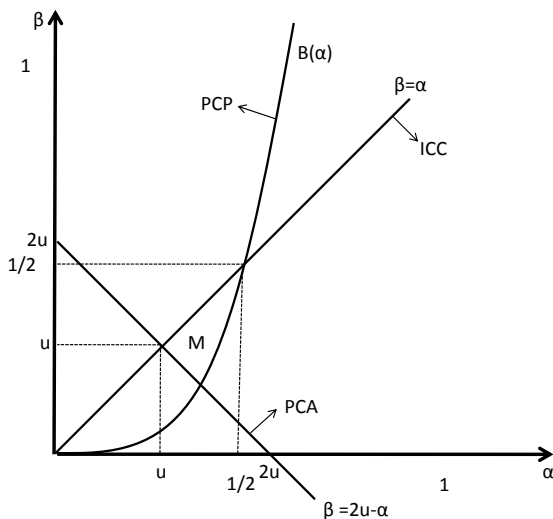
Denote the right hand side of this inequality as $B(\alpha)$. It is easy to show that $B(0) = 0$ and $B(\alpha) \rightarrow \infty$ for $\alpha \rightarrow 1$. Moreover, $\partial B(\alpha)/\partial \alpha > 0$ and $\partial^2 B(\alpha)/\partial \alpha^2 > 0$ for $0 < \alpha < 1$. Hence, the equation $\beta = B(\alpha)$ has a positive slope and is convex over the relevant interval $0 < \alpha < 1$. Combinations of (α, β) which simultaneously satisfy ICC, PCA, and PCP are mutually acceptable for P and A - the parties are better off with such a contract than without it. Denote the set of these combinations as M . It is visualized in Figure 1 (for the case $u < 1/2$). This leads to the following result:

Lemma: *The set of mutually acceptable (α, β) combinations is nonempty if, and only if, $(1 - \pi)/2 > u$.*

Proof: It has already been shown that $B(\alpha)$ is continuous and - for $0 < \alpha < 1$ - increasing and convex. What remains to be demonstrated is $B(u) \leq u \Leftrightarrow u < (1 - \pi)/2$ (for $u \geq 0$). $B(u) \leq u$ can be expanded to $u(\pi + u) \leq (1 - u)u$, which is equivalent to $u \leq (1 - \pi)/2$: q.e.d. \square

⁹With $\beta = 0$, the modified bargaining solution coincides with the previously derived solution $\psi^* = \phi^*(\alpha, 0)$ for any $\alpha > 0$.

Figure 1: Mutually acceptable contracts



The intuition behind this result is straightforward: Without delegation, the symmetric Nash bargaining solution assigns 50% of the bargaining rent $(1 - \pi)$ to both of bargaining parties, i.e., $(1 + \pi)/2$ to P and $(1 - \pi)/2$ to T. The latter equals the maximum additional payoff that P can capture by any strategic move, such as employing a delegate. If, however, A's outside option u exceeds this maximum additional gain, then it makes no sense for P and A to conclude a contract. In that case, the intersection of the line $\beta = 2u - \alpha$ with $\beta = \alpha$ would be situated to the right hand side of the curve $\beta = B(\alpha)$, hence $M = \emptyset$.

2.4 The principal's optimal contract offer

When appointing an agent with a mutually acceptable contract, P's profit accrues to $(1 - \alpha)\phi^*$. Within the framework of the Nash bargaining solution with the implicit assumption of perfect and complete information, the bonus β never has to be paid. Just as the total bargaining result $\phi^*(\alpha, \beta)$, P's share of it is decreasing in α and increasing in β . For P, any contract $(\alpha', \beta') \in M$ is, thus, dominated by another mutually acceptable contract in the north-west of (α', β') . In other words: an optimal contract offer can only be found on the north-western border of M , i.e., on the $\beta = \alpha$ line. If such a contract is chosen, the delegated bargaining result amounts to $\phi^*(\alpha, \alpha) = 1$: The delegate acquires the whole pie, of which P receives $(1 - \alpha)$. Obviously, P can maximize his share by choosing the minimal α which is on the $\beta = \alpha$ line and in M . Hence, the following proposition is proven:

Proposition: *If A's outside option is low, i.e., $u < (1-\pi)/2$, then the payoff maximizing contract that P offers as an ultimatum to A consists of $\alpha^* = \beta^* = u$. If $u \geq (1-\pi)/2$, then P does not employ A.*

If P offers the optimal contract, A accepts and concludes a bargaining result $\phi^* = 1$ with T. The subgame-perfect equilibrium payoffs are $1 - u$ for P, u for A, and zero for T, where $1 - u > (1 - \pi)/2$.

2.5 Discussion

As an alternative to delegation, P could try to exploit his opponent by a strategic move that increases his own threat point π . If his threat point is equal to one, P could even capture the whole pie. However, this increase might not be feasible or require an investment which exceeds the additional share of the bargaining rent. The solution derived in the Proposition does not rest on a threat point equal to one. To the contrary, if P would propose a contract with $\beta = 1$, then the ICC would require him to offer a share $\alpha = 1$.

According to the derived results, P can safely propose a contract to A with a much lower threat point, namely $\beta = u$, as long as it obeys the PCA. Then the ICC allows P to offer a relatively low share $\alpha = u$. If an agent is available whose outside option is smaller than $1/2$, strategic delegation pays for P (in other words: the PCP holds).

3 Conclusion

For negotiations between parties of identical bargaining power, the Nash bargaining result predicts an equal split of the bargaining rent. By appointing a risk-neutral delegate, a party can capture the whole rent, but has to share it with his delegate. This paper has analyzed a contract consisting of a share for the agent and a fixed payment if the delegate fails to conclude an agreement. Under the assumption that the agent has a positive outside option, it was shown that a small share and a high fixed payment may improve the principal's outcome in Nash bargaining. This is in contrast to the result of moral hazard models with risk-neutral agents, which recommend high shares and negative fixed payments. The stylized model introduced in this paper could also be applied to situations where both bargaining parties have non-zero threat points, such as settlement negotiations. Another extension of the model would be the analysis of two-sided delegation in Nash bargaining, or the introduction of asymmetric information between the litigants.

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