Contentious Contracts[¤]

Ulrich Hege
Tilburg University and CEPR[y]

Pascale Viala
Université de Montréal and CIRANO[z]

September 1997

[¤] We would like to thank Georges Dionne, René Garcia, David Margolis, Michel Poitevin, Jacques Robert, Ilya Segal and Bernard Sinclair-Desgagné for many helpful suggestions and seminar audiences at the Université de Montreal and the 1997 Econometric Society North American Summer Meeting in Pasadena for useful comments. This project was begun while the second author was visiting INSEE-CREST and …nished while the …rst author was visiting the Université de Montreal. We are grateful to both institutions and HEC for …nancial support.

[y] Address: Tilburg University, Department of Finance, PO Box 90153, NL-5000 LE Tilburg, Netherlands. E-mail hege@kub.nl.

[z] Address: Department of Economics, Université de Montréal, C.P. 6128, succ. Centre Ville, Montreal, H3C 3J7, Canada. E-mail viala@eib.org.

Abstract

This paper offers an explanation of rationally incomplete contracts where incompleteness refers to unforeseen contingencies. Agents enter a relationship with two-sided moral hazard in which a commitment to discard parts of the joint resources may be ex ante efficient. This happens through costly legal dispute which arises when contract terms are missing for the undesirable outcomes. We show that an optimal contract needs only to specify the obligation for the more litigious party to assure a certain output level - the threshold between foreseen and unforeseen contingencies - and a linear sharing rule for the foreseen contingencies. If litigation reveals some information about the effort levels of the agents, less costly dispute is typically needed and the allocation will improve.

Key words: incomplete contracts, unforeseen contingencies, burning money, team production, contract law.

JEL classification: D82, K12.

1. Introduction

A puzzling aspect about the simplicity of many contracts is their de…ciency or incompleteness: terms are missing or contracts are silent about some contingencies. Furthermore, incompleteness frequently leads to costly legal dispute. For legal scholars, the phenomenon that legal contest arises because a contingency has not been addressed in su¢ ciently clear terms is the essence of contractual incompleteness. Like other aspects of contract simplicity, this begs for an explanation. Why are so many contracts open to con‡ict even though litigation-proof contracts are not hard to write? This question seems pertinent to many types of contracts: to contracts about commercial transactions like sales, franchises, patent leases and joint ventures; to labor and executive compensation contracts; …nally, to contracts in private life like marriage contracts and to many other situations where explicit contracts are used. Take the case of a patent lease as an illustration. At the outset of their relationship, the lessor and the tenant of a patent lease usually set up a contract stating a …xed royalty and a (linear) user fee, but remain silent about many contingencies. As an example for an unforeseen contingency, consider the following event: the lessor sells a similar, but technically di¤erent device to a competitor and the tenant unilaterally reduces the fee after the infringement. It is startling that the parties do not choose to make provisions eliminating any ensuing con‡ict in this case. It is not hard to come up with su¢ ciently general clauses encompassing all possible contingencies, e.g. a provision that assigns all such risk to the tenant.

In this article, we propose an explanation of these phenomena stipulating that there is an implicit agreement between the parties to remain silent about bad outcomes. Undesirable outcomes are omitted because this raises the potential for con‡ict, thereby serving as an incentive device against careless behavior or free-riding.

The observation that unforeseen contingencies are typically undesirable outcomes is a key element of this mechanism. If such a genuinely undesirable contingency occurs, the question arises whether all parties have done enough to avoid it. This question provides the backdrop against which con‡ict ensues. The dispute is about how to split the bill for the negative consequences. Dispute tends to be wasteful, as has been shown in theory[1]; in practice, the single most prominent form under which dispute destroys resources is through the legal system and its costs.

The fact that destroying resources or "burning money" can be desirable for incentives purposes has been recognized in the literature. This is in particular the case in situations of team production or double moral hazard which is the framework of the present paper. Incidentally, team production has played a prominent role in incomplete contracts theory since the seminal contribution by Alchian and Demsetz (1972). Alchian and Demsetz have argued that incentive problems emanating from joint production are easier to solve within an organization than via market-based contract solutions. Theoretical work since, however, has shown that creating a common organization is not sufficient to solve the team production problem. Holmström (1979), Legros and Matsushima (1991) and Williams and Radner (1988) show that the dilemma remains if the organization has to split the joint surplus among the agents and their monitors. Therefore, one solution which Holmström envisions is to discard a fraction of the surplus in some states. Our model can be viewed as a direct follow-up on Holmström's suggestion. The original contribution of the present paper is to link the burning money motive to contractual incompleteness.

To this end, we propose a general model of team production where parties address their free-riding problem in contractual form. We show that the optimal contract is characterized by (a) a linear sharing rule for good outcomes which are the foreseen contingencies, and (b) a threshold between foreseen and unforeseen contingencies and omission of the latter. In the patent lease example, an optimal contract would be a contract which specifies fee schedules if the contract is used, but remains silent about the case where the contract is repudiated (the patent is altered, is infringed or is insufficiently exploited). Implicitly, the contract contains a break point, i.e. rational agents are aware of the fact that the contract will not always be honored smoothly.

[1] Theoretical models demonstrating bargaining inefficiencies without refering to legal costs are based on imperfections like bargaining externalities (e.g. Jehiel and Moldovanu (1994)) and notably, asymmetric information (see the survey by Kennan and Wilson (1993)).

The role of a court of law in this interpretation is to correct for the deficiency of a contract. By rendering a verdict on the conflict, the court "fills in the contract". In doing so, the court "verifies" the state of nature by establishing performance, compliance, breach or negligence of the disputing parties. In our model, the court tries to establish the unobservable effort decision of the agents. This comes close to what courts actually do when they "fill in" an incomplete contract. To give some examples, in labor contracts, "in an absence of a waiver of the breach, the employer may recover damages from his employee ... for involving his employer in loss through his negligence or wrongful act"[2]. Similarly, the "respondeat superior" rule governing the liability of an employer, requires to establish whether or not the employer was in control of the employee. Cooter and Ulen (1994) stipulate in their textbook that for efficiency reasons, liability should be assigned "to the party that was the cheaper preventer of, or insurer against, the contingency that frustrated the contract".[3]

In this regard, our model captures an element frequently overlooked in principal-agent theory: unobservable actions or parameters need not automatically be excluded from contracting. However, if the agents decide to conditionalize their contract on an unobservable variable, they implicitly leave it to the court to "fill in" the facts about the unobservables. Whether or not the court reveals information about the liability of the parties is of secondary importance in our model. We investigate the extreme case where the court takes random decisions with respect to establishing effort, in order to emphasize that it is the costliness rather than the informativeness of the court decisions that constitutes the basis of the incentive mechanism proposed here. This should not be misunderstood as a claim that courts of law are ineffective when it comes to establishing the facts. On the contrary, our paper shows that information production of the court is desirable. This emerges from an extension to informative litigation. We show that the more accurate the information that litigation reveals, the better for the contracting parties because the discovery in court will be anticipated in the optimal contract and enhance the incentive effect of litigation.

An obvious question is why the constrained efficient arrangement should be

[2] 56 CJS 500.

[3] Cooter and Ulen (1988), p. 281.

burning money in a legal dispute rather than, say, transferring them to a third party. We suggest the following reason: the use of contentious contracts renders the disposal of resources irrevocable, whereas allocating revenues to a third party is vulnerable to coalition formation or renegotiation.

Besides the explicit contracts mentioned earlier, joint production or double moral hazard is present in economic partnerships like law …rms and accounting …rms, in relationships with two-sided speci…c investments like upstream-downstream relationships[4], in employment contracts and in …nancial contracts[5]. In all these cases, the following conditions seem to hold: (a) elements of two-sided moral hazard are present; (b) there is a positive probability of the relationship breaking up or costly con‡ict ensuing; (c) the threshold where such con‡ict is expected to occur depends on the contract; (d) the consequences in this case are not clearly speci…ed. These four properties are the basic ingredients of our model. Whenever they occur jointly, then the implementation device analyzed out here should be present in practice, consciously or unconsciously.

The comparison of our explanation of unforeseen contingencies to various strands of the contracting literature reveals similarities and di¤erences. Incomplete contracts are often de…ned as contracts that do not conditionalize on "observable, but not veri…able" states of nature. The accepted explanation for this type of contractual incompleteness is based on prohibitive transaction costs to writing complete contracts or, which is equivalent, on bounded rationality.[6] A number of papers have formally endogenized the choice of incomplete contracts as a rational response to transactions cost problems, notably by invoking complexity costs.[7]

In our model, contingencies are both observable and veri…able. Veri…cation costs are avoidable, but they occur as an artefact of the optimal contract. The

[4] For example, Hart and Moore (1988). The ensuing literature is surveyed in Hart (1995).

[5] For example, securities issues involve various parties (issuer, underwriter, rating agency). Joint stockholdings of a family-dominated company is another example.

[6] Three di¤erent forms of transactions costs are generally invoked: …rst, complexity costs in discerning large sets and intricately de…ned states of nature; second, legal veri…cation costs in …guring out what the actual state is; third, costs of forecasting all possible contingencies. See Grossman and Hart (1986) for the seminal contribution and Hart (1995) and Tirole (1994) for surveys.

[7] Notably, Anderlini and Felli (1994)(1996) and MacLeod (1996).

conventional view on incomplete contracts is one of bounded rationality (parties know that a complete contract would serve them better). The papers explaining incompleteness by means of complexity costs o¤er a boundedly rational explanation of incomplete contracts. In our explanation, contractual incompleteness is unboundedly rational: parties can deal with any complexity of the situation, but they know that a complete contract (which they could draw up at no cost) would be worse.[8] Thus, the recent criticism of incomplete contracts models based on unveri…ability[9] does not extend to our model.[10]

There are some similarities with the costly state veri…cation (CSV) and auditing literature.[11] In this work as well as in our model, veri…cation costs are avoidable, but they occur for incentive reasons and only for bad outcomes. However, in the CSV literature, contracts are complete and there are adverse selection problems about the outcome. In our model, contracts are incomplete and there is no lack of observability of the outcome. The relationship to the literature on the breach of contracts and breach remedies is similarly complex.[12] On the one hand, breach of contract is frequently a special case of the legal con‡icts that our model addresses.[13] On the other hand, not every breach of contract leads to con‡ict, particularly not if the contract is su¢ciently complete about the breach remedies.

The paper is organized as follows. The model is laid out in Section 2. Section 3 introduces to the role of dispute as an implementation device. In Section 4, the optimal contracts are developed. In Section 5, we introduce informative litigation. In Section 6, we discuss the robustness of our mechanism. Section 7 concludes.

---

[8] Segal (1995) is another paper where incompleteness is unboundedly rational.

[9] The criticism is whether unveri…ability, if modelled in a rational choice model, is a su¢cient condition to explain incompleteness. See e.g. Tirole (1994).

[10] A similar di¤erence arises with respect to the dynamic properties of incomplete contracts: within the transaction costs view, contracts which are initially incomplete may be dynamically completed in a time consistent manner as events evolve and therefore not lead to a di¤erent allocation than a complete contract. A formalization of this idea is in Maskin and Tirole (1997). See also Hart (1987), p. 753.. In our model, there is no time consistent completion of incomplete contracts.

[11] Townsend (1979), Diamond (1984) and Gale and Hellwig (1985).

[12] See Shavell (1984), Edlin and Reichelstein (1996) and Che and Chung (1996).

[13] Breach of contract is typically one-sided, the break-up of relationships often two-sided and con‡ict may not lead to a break-up at all.

2. The model

The model depicts two agents 1 and 2 concluding a contract about a joint production effort. There are two dates. At date 0, they sign the contract. The joint output is determined by the agents' efforts between date 0 and date 1. At date 1, the joint output is realized and distributed according to their agreement.

Let a_i denote the level of effort of agent i; $i \in \{1, 2\}$, which is chosen from the convex set A_i. The cost of effort is expressed by the cost function $c(a_i)$ which is increasing and strictly convex. The function $c(a_i)$ is the same for both agents. The two agents are risk-neutral and utility is transferable. $x \in X = [0; \bar{x}]$ will denote a generic level of output. The joint output function is stochastic and characterized by the cumulative distribution function $F(\cdot|a_1; a_2)$ and density $f(\cdot|a_1; a_2)$, with $f(x|a_1; a_2) > 0$ over X for all $(a_1; a_2) \in A_1 \times A_2$. $F_i(x|a_1; a_2) = \partial F(x|a_1; a_2)/\partial a_i$ and $f_i(x|a_1; a_2) = \partial f(x|a_1; a_2)/\partial a_i$ denote the partials. Let $E[x|a_1; a_2] = \int_X x\, f(x|a_1; a_2)dx$ denote the expected output. We assume that $E[x|a_1; a_2]$ is concave in $(a_1; a_2)$. With transferable utility, the efficient action profile $(a_1^e; a_2^e)$ solves:

$$(a_1^e; a_2^e) \in \arg\max_{(a_1;a_2)} E[x|a_1; a_2] - c(a_1) - c(a_2):$$

Moreover, we assume:

Assumption 1. The joint output distribution function satisfies

$$f_1(x|a_1; a_2) = k(a_1; a_2)\ f_2(x|a_1; a_2)$$

for all $x \in X$ and $(a_1; a_2) \in A_1 \times A_2$, where $k(a_1; a_2)$ is a single-valued and positive function.

This assumption says that the likelihood ratios of any effort profile $(a_1; a_2)$ are collinear.[14] Then, there is no way to infer the contribution of each agent in terms of effort from a particular level of output. This has been identified as the

[14] This assumption comprises many standard production function with conventional specifications of stochastic shocks, including the class of functions of the form $x = Q[g(a_1; a_2); \varepsilon]$, where ε is an additive or a multiplicative productivity shock.

key condition for a team problem to prevail.[15]

Assumption 2. Effort ameliorates the distribution function in the sense of the monotone likelihood ratio property (MLRP):

$$\frac{\partial \left(\frac{f_1(x|a_1;a_2)}{f(x|a_1;a_2)} \right)}{\partial x} > 0; \quad \forall x \in X; \; (a_1;a_2) \in A_1 \times A_2:$$

MLRP is a standard assumption in principal-agents models, which is typically made to demonstrate the monotonicity of incentives contracts. It implies first-order stochastic dominance of the output with respect to effort.

The joint output of production is verifiable, but effort is only privately observable by each of the agents. Contracts may be contingent on the output alone or contain performance requirements. In the first case, contracts are enforceable at no cost. In the second case, agents must rely on a mechanism which enforces the revelation of the unobservable information. The only option[16] is to legally enforce provisions about the unobservable effort level. If there is legal action, the role of the court is to sort out whether there has been fault of the parties. The court has to render a verdict, but is effectively impeded from establishing the facts as the actions are unobservable. We capture this by the assumption that the verdict of a court is stochastically independent from the agents' true choices of effort, i.e. agents have no impact on their chances to prevail in court if they increase or reduce their effort levels. This extreme case implies that the specified "required" effort level plays no role for the allocation because the true effort level remains as unobservable in court as out-of-court.[17]

With the required effort level being irrelevant, the contractual choice concerning legal enforcement is about the states where contest is possible, e.g. states where a performance requirement applies. For example, contest can be excluded

[15] Whenever Assumption 1 holds, then a balanced sharing rule leading to $(a_1^e; a_2^e)$ does not exist. See Williams and Radner (1988). A similar condition for the discrete case is contained in Legros and Matsushima (1991).

[16] A self-enforcing contract giving incentives for voluntary revelation of private information is not possible if Assumption 1 holds.

[17] This provides an additional insight on contractual incompleteness, because the parties will do as well by remaining silent about required performance and to leave it to the court to "fill in" this requirement according to the law.

by a water-tight provision for a certain outcome, like a waiver of one of the two parties to ever claim damages under a certain outcome. Contest can also be excluded by a clause stating that a certain output level x is regarded as sufficient proof that both parties met their performance requirements. On the other hand, contest can be included if the contract is insufficiently specific about a certain state or is not tight enough to exclude litigation. Let $S_1 \subset X$ denote the set of states where agent 1 can invoke a contestable performance requirement binding agent 2 and bring an action against agent 2 for the payment of damages, and vice versa for $x \in S_2$.[18] If $x \in S_1 \cap S_2$; then both agents could bring an action. We assume that in this case, only one law suit is accepted in court, depending on a chance move by nature: either law suit is accepted with probability $\frac{1}{2}$.

Let $S = S_1 \cup S_2$ denote the set of all states where at least one agent can bring an action. We say that if $x \in S$, then x is a contestable state. A contract where $S = \emptyset$ is called a complete or litigation-proof contract. Whenever S non-empty, the contract is called a contentious contract.

For any outcome state $x \in S_i$, the contract may specify the damages $D_i(x)$ that the plaintiff (agent i) recovers from the defendant (agent j) if the court rules that performance was insufficient. We assume that there are legal or institutional bounds to applicable damages which we denote by $D^{max}(x)$: For example, under United States commercial law, punitive damages in contract disputes (damages exceeding the monetary loss of the victim) are routinely denied in court even if the contract expressly contains provisions for higher damages. We simply assume that an effective bound on damages exists somewhere, with $\partial D^{max}(x)/\partial x \leq 0$.

Among the many prior models on pre-trial settlement and litigation, we choose to adapt Schweizer's (1989) because it is the simplest model with two-sided asymmetric information.[19] Only the essential features are summarized here, leaving a complete account of this model to Appendix A. In any contestable state $x \in S$, both parties have private information regarding the merit of the case: each agent

[18] Note that, even if the contract is silent about the required performance, there will usually be a commercial law imposing performance standards, either statutory law like the Uniform Commercial Code or judicial precedents. This permits to contest each others effort whenever the contract does not exclude so, for example via waiver clauses.

[19] Two-sided asymmetric information is desirable because we want to endogenize the choice of defendant and plaintiff as a function of their litigiousness.

observes a signal which has two possible outcomes, "strong" or "weak". The signals are obtained at the same time when agents choose their actions. They are independently distributed. Whether the case is won or not depends on the pair of signals. After having received their signals, parties have the opportunity to settle their dispute.[20] There is no cost to settlement bargaining. The defendant makes a settlement offer and the plaintiff decides whether to accept or to reject the offer. If she rejects, the case is going to court, at a cost which is a deadweight loss. We assume that this cost is linear in $D_i(x)$ and denote it by $l \cdot D_i(x)$. $l \cdot D_i(x)$ is split according to the English Rule, i.e. the loser pays all.

Of the equilibria of this game, we consider only one, the least-cost fully revealing equilibrium.[21] The logic of this separating equilibrium is that the plaintiff uses the probability to reject an offer as a screening device inducing the two types of the defendant to make truthful settlement offers. In this equilibrium, only the offer of a "strong" defendant is sometimes rejected while the offer of a "weak" defendant is always accepted. Let $p(1)$ ($p(2)$) denote the probability that agent 1 (agent 2) receives the "strong" signal. The higher $p(1)$ or $p(2)$, the more likely is pretrial settlement bound to fail. Therefore, we refer to $p(1)$ and $p(2)$ as measuring how litigious the agents are. Let $p(i)$ ($p(j)$) denote the litigiousness of the agent who is designated as plaintiff (defendant). Let $q(i)$ denote the (endogenous) probability of acceptance by plaintiff i of a settlement offer proposed by the "good" defendant j. The expected payoffs of plaintiff and defendant for a case brought by agent i in state $x \in S_i$ will be denoted as $\pi^p(x;i)$ and $\pi^d(x;i)$, respectively, where $\pi^p(x;i) \geq 0 \geq \pi^d(x;i)$. $C(x;i)$ will denote the expected net cost of litigation in this case. Then:

$$C(x;i) = -\left(\pi^p(x;i) + \pi^d(x;i)\right) = p(j)(1 - q(i))\, l\, D_i(x) \geq 0. \qquad (2.1)$$

[20] We exclude renegotiation prior to reception of the signals. The idea is that the signals are a reduced form which really tries to capture pretrial discovery efforts. If agents can and will acquire information prior to litigation, they can and will do so also prior to settlement bargaining. It can be shown that asymmetric information obtains as an endogenous outcome of costly discovery, but this would come at the expense of a considerably more complicated model structure.

[21] This is the equilibrium where the offer of the defendant is fully revealing (concerning his type) and where the probability of the plaintiff accepting the offer is maximized (Riley outcome). This is also the single outcome surviving all standard refinements developed for signaling games (universal divinity or stable outcome).

In the expression of $C(x;i)$, all transfers between the agents cancel out and only the deadweight cost remains. This cost is equal to $l \cdot D_i(x)$ times the probability that the settlement offer is rejected which is $p(j)(1 - q(i))$. A useful observation is that $C(x;i)$ is (linearly) increasing in $D_i(x)$. In Appendix A, we show that $C(x;i)$ is more sensitive to the litigiousness of the defendant than to the litigiousness of the plaintiff. The intuition is simple: the plaintiff adopts a mixed strategy making the "weak" defendant indifferent between a truthful offer and mimicking a strong type. The more likely it is that the defendant is strong, the more often must a strong offer be rejected to keep the weak defendant's incentives in balance. Hence, if $D_1(x) = D_2(x)$, we obtain

$$C(x;i) \gtreqless C(x;j) \quad \Leftrightarrow \quad p(i) \lesseqgtr p(j)$$

We summarize the instruments available for contracting. Recall that parties can choose the set of contestable states S as well as a sharing rule and a function of damages for contestable states. Thus, any feasible contract can be represented as $\{\beta(x); D_1(x); D_2(x); S_1; S_2\}$, where $\beta(x)$ is a sharing rule of the joint output, $D_i(x)$ is the (contingent) amount of damages that can be demanded by agent i in state $x \in S_i$, and S_i is the set of contestable states x where agent i is designated as plaintiff.

3. Dispute as an implementation device

Let $R^1(x)$ denote agent 1's and $R^2(x)$ denote agent 2's ex-ante expected litigation payoff in state x: That is, $R^1(x) = \Lambda^p(x;1)$ and $R^2(x) = \Lambda^d(x;1)$ if $x \in S_1 \setminus S_2$ (agent 1 is plaintiff) and $R^1(x) = \Lambda^d(x;2)$ and $R^2(x) = \Lambda^p(x;2)$ if $x \in S_2 \setminus S_1$ (agent 2 is plaintiff). Moreover, $R^1(x) = \frac{1}{2}\left[\Lambda^p(x;1) + \Lambda^d(x;2)\right]$ and $R^2(x) = \frac{1}{2}\left[\Lambda^d(x;1) + \Lambda^p(x;2)\right]$ if $x \in S_1 \cap S_2$. Of course, $R^1(x) = R^2(x) = 0$ for $x \in X \setminus S$ since contest is excluded for these states. We denote agent 1's and agent 2's (date 0) expected utility by $V^1(K;(a_1;a_2))$ and $V^2(K;(a_1;a_2))$ respectively, where $K = \{\beta(x); D_1(x); D_2(x); S_1; S_2\}$ is the contract. Taking into account budget balancing, we have:

$$\begin{aligned} V^1(K;(a_1;a_2)) &= \int_X \beta(x) f(x|a_1;a_2)dx - c(a_1) + \int_X R^1(x) f(x|a_1;a_2)dx \\ V^2(K;(a_1;a_2)) &= \int_X (1 - \beta(x)) f(x|a_1;a_2)dx - c(a_2) + \int_X R^2(x) f(x|a_1;a_2)dx \end{aligned}$$

Incentive compatibility of an action profile $(a_1; a_2)$ requires that

$$a_i \in \arg\max_{\hat{a}_i} V^i(K; (\hat{a}_1; a_2)); \text{ for } i = 1; 2:$$

It is convenient to apply the first-order approach (FOA) to our analysis. The FOA approach allows us to replace the set of incentive compatibility constraints by a pair of first-order conditions.[22] Technically speaking, this approach requires additional assumptions ensuring that the expected utility function $V^i(K; (a_1; a_2))$ is strictly concave in agent i's action[23]. The following first order conditions are then necessary and sufficient for interior solution to the agents' effort problems:

$$\int_X \beta(x)\, f_1(x|a_1; a_2)dx - c_1(a_1) + \int_X R^1(x) f_1(x|a_1; a_2)dx = 0 \quad (3.1)$$

$$\int_X (1 - \beta(x))\, f_2(x|a_1; a_2)dx - c_2(a_2) + \int_X R^2(x) f_2(x|a_1; a_2)dx = 0 \quad (3.2)$$

It is useful to begin with a complete contract as a benchmark. Let $(a_1^c; a_2^c)$ denote the action profile which is attainable under a complete contract. Recall that for a complete contract, $S = \emptyset$; and hence, $\int_X R^i(x) f_i(x|a_1; a_2)dx = 0$; $i = 1; 2$: The first order equations (3.1) and (3.2) show then that $(a_1^c; a_2^c)$ is determined as the solution to the first-order conditions $\int_X \beta(x)\, f_1(x|a_1^c; a_2^c)\, dx - c_1(a_1^c) = 0$ and $\int_X (1 - \beta(x))\, f_2(x|a_1^c; a_2^c)\, dx - c_2(a_2^c) = 0$. The inefficiency of this allocation can be seen from the fact that the optimal allocation $(a_1^e; a_2^e)$ is determined by the first-order conditions $\int x f_i(x|a_1^e; a_2^e)\, dx - c_i(a_i^e) = 0$; $i = 1; 2$. Whatever the splitting rule $\beta(x)$; these conditions are incompatible. This is the well-known result of the team production literature that a balanced sharing rule does not allow to accomplish this task for both agents simultaneously if Assumption 1 holds[24]. The attainable action profile $(a_1^c; a_2^c)$ is inferior to the first best allocation.

---

[22] Regarding this approach, consult Mirrlees (1979), Rogerson (1985) and Jewitt (1988) for one-dimensional principal agent models and Sinclair-Desgagné (1994) for multi-dimensional principal agent problems. Sufficient conditions for the validity of this approach for the partnership problem are also provided by Williams and Radner (1988).

[23] If $p(1) = p(2)$, a sufficient condition for the FOA to be valid here is the Mirrlees (1979) - Rogerson (1985) convexity of the distribution function condition (CDFC), which says that $F_i(x|a_1; a_2)$ is strictly increasing with a_i: If $p(1) \neq p(2)$, then an additional boundary condition on the slope of $D_i^{max}(x)$ is needed. For example, the following condition is sufficient: $\sigma < \frac{1}{2}\left[1 - \frac{c_1(\bar{a}) - \underline{k} c_1(0)}{E_1[x|\bar{a}; \bar{a}]}\right]$; where $\sigma \geq -\partial D^{max}(x)/\partial x$; $\bar{a} < a^e$ is the highest implementable level of effort and $\underline{k}$ is the minimum value of function $k(a_1; a_2)$ over $[0; \bar{a}] \times [0; \bar{a}]$:

[24] Using Assumption 1 and adding up the FOC, one can see that the attainable allocation solves $\int x f_1(x|a_1^c; a_2^c)\, dx - c_1(a_1^c) = k c_2(a_2^c)$ and $\int x f_2(x|a_1^c; a_2^c)\, dx - c_2(a_2^c) = c_1(a_1^c)/k$.

What is the role of legal dispute? Suppose that the parties want to implement an action profile $(a_1; a_2) > (a_1^c; a_2^c)$: A contentious contract can achieve this by introducing an additional marginal punishment into at least one of the two first order conditions: either $\int_X R^1(x) f_1(x|a_1; a_2)dx > 0$; or $\int_X R^2(x) f_2(x|a_1; a_2)dx > 0$ or both.

It turns out that only the net costs of litigation are relevant for implementation. The reason for this is simple: since there is no way to determine the agents' relative effort levels from the observation of the joint output, all that matters for incentives purposes is the sum of the punishment that can be inflicted to the parties and hence damages transfers between the agents cancel out. Let $C(x)$ denote this net cost of litigation which is equal to:

$$C(x) = \begin{cases} 0 \text{ for } x \in X \setminus S \\ C(x; 1) \text{ for } x \in S_1 \setminus S_2 \\ C(x; 2) \text{ for } x \in S_2 \setminus S_1 \\ (\frac{1}{2}C(x; 1) + \frac{1}{2}C(x; 2)) \text{ for } x \in S_1 \cap S_2: \end{cases}$$

The additional punishment can be positive by an appropriate choice of the set S. In short, litigation plays the role of a "budget breaker" allowing to impose penalties for both agents simultaneously.

4. Optimal contracts

In this section, we characterize the optimal contracts. The relaxed optimization problem can be written as:

$$\max_{\beta(x);\, D_1(x);\, D_2(x); S_1; S_2} V^1(K; (a_1; a_2)) + V^2(K; (a_1; a_2)) \tag{4.1}$$

s.t.

$$\int_X \left(\beta(x) + R^1(x)\right) f_1(x|a_1; a_2)dx - c_1(a_1) \geq 0 \tag{4.2}$$

$$\int_X \left(1 - \beta(x) + R^2(x)\right) f_2(x|a_1; a_2)dx - c_2(a_2) \geq 0 \tag{4.3}$$

$$D_i(x) \in [0; D^{max}(x)]; i = 1; 2; \quad \forall x \tag{4.4}$$

$$S_i \subset X; \; i = 1; 2; \tag{4.5}$$

where constraints (4.2) and (4.3) are the incentive compatibility constraints for the two agents and constraint (4.4) recalls the existence of legal limits on damages. Moreover, individual rationality constraints of the form $V^1(K;(a_1;a_2)) \geq 0$ and $V^2(K;(a_1;a_2)) \geq 0$ must hold. These can w.l.o.g. be assumed to be satisfied because there are no limited liability constraints.[25] Note that the objective function (4.1) is equal to the net surplus:

$$V^1(K;(a_1;a_2)) + V^2(K;(a_1;a_2)) =$$
$$E[x|a_1;a_2] - \sum_{i=1}^{2} c(a_i) - \int_X C(x) f(x|a_1;a_2)dx:$$

We denote by $\hat{x}(a_1;a_2)$ the (unique) output level such that $f_1(x|a_1;a_2) \leq 0$ for all $x \leq \hat{x}$ and $f_1(x|a_1;a_2) > 0$ otherwise[26]. The basic properties of the solution are contained in Lemma 1:

Lemma 1. The following contract $K^* = \{\beta^*(x); D_1^*(x); D_2^*(x); S_1^*; S_2^*\}$ is optimal and leads to an action profile $(a_1^*;a_2^*)$ such that $a_1^* > a_1^c$ and $a_2^* > a_2^c$ for $S^* \neq \emptyset$:

1. The sharing rule is linear: $\beta^*(x) = \beta^* x + B^*; \forall x$, where $B^*;\beta^* \in \mathbb{R}$:
2. Dispute occurs for all states below some threshold of dispute x^* : $S^* = \{x \in X : x < x^*\}$, where x^* is such that $0 \leq x^* < \hat{x}(a_1^*;a_2^*)$.
3. A performance requirement applies only to the more litigious agent: $S_1^* = \emptyset$ if $p(1) > p(2)$; or $S_2^* = \emptyset$ if $p(2) > p(1)$. If $p(1) = p(2)$, then either of the agents or both can be assigned the performance requirement.
4. In each contestable state $x \in S^*$, damages are at the maximum feasible level: $D_i^*(x) = D^{max}(x)$.

Proof. See the Appendix. ■

The characteristics of this contract are closely linked to the collinearity of the likelihood ratios. This implies that there is no way to determine ex post which agent has been more responsible for an observed output. On the one hand, this is the reason why a linear sharing rule can do as well as any other (non-linear)

[25] Individual rationality can always be satisfied by adding or subtracting a constant to $\beta(x)$: The value of this constant will depend on the bargaining power of the agents.

[26] If MLRP holds, then there exists, for each action profile $(a_1;a_2)$, a unique output level $\hat{x}(a_1;a_2)$; $\underline{x} < \hat{x}(a_1;a_2) < \overline{x}$, such that $f_i(x|a_1;a_2) \leq 0 \ \forall x \leq \hat{x}(a_1;a_2)$.

balanced splitting rule.[27] On the other hand, the impossibility to tell who has been the likely deviator is also the reason why an optimal incentive scheme relies on extra punishment via a costly dispute. The most effective impact on incentives is brought about by invoking the dispute option for those states whose probability of occurrence is most drastically increasing if one of the agents provides too little effort. Under MLRP, this is true for the outcomes in the lower tail of the distribution. This explains why the contestable states should be chosen to be the worst outcomes of the joint production effort.

An important feature of the optimal contract K^* is that the conflict threshold x^* is inferior to the value $\hat{x}$, the point where the maximal increase in the cumulative distribution induced by an agent's deviation occurs: Intuitively, if the dispute threshold were any higher than $\hat{x}$; the incentive effect would be lower and the deadweight cost higher than at $\hat{x}$; which cannot be optimal. This result confirms our interpretation of contestable states as undesirable outcomes, where undesirable has two meanings: these outcomes represent the worst outcomes of the joint production function and the probability of these outcomes increases if an agent deviates. This corresponds to how legal scholars think about unforeseen contingencies: they are described as outcomes which the parties should have tried to avoid. Hence, in contract law, the response to an unforeseen contingency is to search for the agent who caused the unwanted outcome or who would have been best placed to avoid it. Our analysis vindicates this view.

The improvement in the effort allocation depends on how much of the joint product can be destroyed in each contestable state, i.e. the net cost of litigation. This net cost should be maximized in order to keep the threshold x^* as low as possible. This explains the last two items of Lemma 1. On the one hand, the probability of a costly litigation increases with the contractual damages. Hence, maximal net costs of litigation are achieved by imposing maximal penalties, $D_i(x) = D^{max}(x)$: On the other hand, the probability of a litigation increases with the chances to face a litigious defendant, i.e. a defendant more likely to receive a "strong" signal. This explains why the assignment of the performance

[27] Intuitively, any attempt to raise the slope of the incentive schedule for one agent with a non-linear contract comes at the cost of symmetrically weakening incentives for the other agent. See Bhattacharyya and Lafontaine (1995) for a demonstration for an output function which is a special case of ours.

requirement is asymmetric: differences in the attitude of agents towards litigation are optimally exploited. If such differences do not exist, i.e. if $p(i) = p(j)$, then the choice of the defendant is indeterminate.[28] It is always sufficient to make just a single agent liable, even though both parties are perfectly aware of the two-sidedness of the moral hazard problem. Thus, one insight of our model is that one-sided performance requirements do not mean that in reality, the moral hazard problem is one-sided. It simply means that the contract is optimized by exploiting perceived differences.

We now turn to our main result which simplifies the provisions for contestable states:

Proposition 1. Let $\hat{D}_i(x)$ be the damages awarded if the plaintiff wins. Suppose that $\hat{D}_i(x) = \min\{D_i(x), D^{max}(x)\}$ if $D_i(x)$ is specified in the contract and $\hat{D}(x) = D^{max}(x)$ if not. Then, the optimal contract K^* is equivalent to a contract containing only the following provisions:

1. If $p(1) \neq p(2)$, the more litigious of the agents commits to deliver an output of x^* or more. If $p(1) = p(2)$, either of the agents or both commit to deliver an output of x^* or more.
2. A linear sharing rule for all $x \geq x^*$.

Proof. See the Appendix. ■

The additional element of Proposition 1 over Lemma 1 is that, when maximal damages are unaffected by the terms of a contract, the optimal contract can remain silent about contestable states altogether and contestable states can be viewed as truly unforeseen contingencies. In practice, many contracts exhibit features like this: they impose a performance level for the agents and take satisfactory performance for granted by not specifying what happens if the defendant does not deliver. An optimal contract corresponding to Proposition 1 can obviously be written in a very simple form, for example like this:

[28] The plaintiff who is liberated from a performance requirement receives in turn incentives by receiving a higher share of the joint surplus. There must be a compensation for this differential treatment: usually, the designated plaintiff will make a lump-sum payment B to the designated defendant.

"Agent 2 has to deliver an output of x^* (or better). After fulfillment, agent 1 makes a lump-sum payment of B^* and retains a share of β^* of the output x."

No mention is made what happens if $x < x^*$. The rationale for this omission is that filling in by the court will be just as good as explicit penalties. This result is based on the following two insights.

First, this conclusion depends of course on the assumption that the damages will be $D^{max}(x)$ if the contract is silent about contentious states. A rational plaintiff will always seek the maximum damages. This leads us to conclude that nothing is to be gained by explicitly providing applicable penalties. For, if $D_i(x) < D^{max}(x)$, then the contract is not optimal. If $D_i(x) = D^{max}(x)$, then the penalty need not be mentioned in the contract. It follows that the optimal contract can be silent about the function $D_i(x)$:

We illustrate the plausibility of the condition in Proposition 1 by means of two examples. In commercial contracts, punitive damages are routinely denied in court, even if a contract expressly grants higher damages, setting the maximum amount which can be obtained at the full restitution of the defendant's loss. In terms of our model, this would amount to $D^{max}(x) = x^* - x$. But then, the plaintiff can and will seek full restitution even if fines are not mentioned in the contract. Divorce law is the other example. There is an obvious limit on the compensation that spouses can demand, namely fifty percent of their joint wealth. Marriage contracts (like separation of goods) can only limit this amount and thus reduce the potential for conflict, but not increase it.

Second, we consider what the contract should determine concerning the splitting rule $\beta(x)$ in case of a bad outcome $x < x^*$: When rendering a verdict, the court fixes also a splitting rule $\beta(x)$: either by confirming the rule in place, or by modifying it, or by filling in a splitting rule in case the contract does not mention one. Recall that we defined the damages to be the difference in the plaintiff's total revenue if she wins the trial as compared to the case where she loses it. This difference will be fixed at $D^{max}(x)$. It is straightforward to show that the inefficiency in the settlement bargaining game depends only on the difference in the plaintiff's payoff between a won and a lost case. The penalty depends on what is at stake for the parties in the dispute which is the difference between the payoffs in both cases, not their absolute level.[29]

To complete our analysis, it is of interest to know when litigation would actually be part of an optimal contract, i.e. when $x^* > 0$. In fact, this is the case whenever the joint output distribution function is such that the likelihood ratio $\frac{f_1(0|a_1^c,a_2^c)}{f(0|a_1^c,a_2^c)}$ is small. More precisely, we find:

Corollary 1. The optimal contract will be contentious if

$$\frac{\int_x x\, f_1(x|a_1^c;a_2^c)dx - c_1(a_1)}{\int_x x\, f_{11}(x|a_1^c;a_2^c)dx - c_{11}(a_1)}\,\frac{f_1(0|a_1^c;a_2^c)}{f(0|a_1^c;a_2^c)} > 1. \tag{4.6}$$

Proof. See the Appendix. ■

In other words, if condition (4.6) holds, then there exists a non-empty set $S \subset [0;\hat{x}(a_1^c;a_2^c)]$ for which the marginal return of an increase in effort with respect to saved litigation costs outweigh its marginal cost.

5. Informative litigation

In the model discussed so far, we have assumed that the prospect of agents to prevail in court is independent of their effort. This abstraction was made for simplicity. Often, the court can reconstruct at least some indications about the effort. In short, the effort choice should influence the probability with which the agents expect to prevail in court.

Recall that agents' chances to prevail on court depend on private signals. In the basic model, the signals were uncorrelated with the true performance levels (see Appendix 9.9). By contrast, in this section, we capture the idea that the court is partially successful in retrieving information by assuming that the probabilities of the signals "strong" and "weak" depend on the agents' unobservable actions. Hence, the merit of the case is expected to be weaker for an agent who has deviated. The signal probabilities are then functions of the effort choices. We denote by $p^l(a_i)$ for agent i (the designated plaintiff) and by $p^l(a_j)$ for agent

[29] There might be an additional reason to remain silent about damages, which is that it creates uncertainty about what the parties perceive would be likely or realistic claims for damages. Thus, a second element of asymmetric information about the amount of damages may come into the play which, in a separating equilibrium, could increase the probability of a failure of pre-trial settlement bargaining.

j (the designated defendant) the probability to observe the strong signal if she takes action a_i and a_j, respectively. We say that litigation is informative if the signal $p^I(a_i)$ is such that:

$$\frac{dp^I(a_i)}{da_i} > 0 \quad \forall a_i;\ i \in \{1;2\}$$

If litigation is informative, the conditions for incentive compatibility of a certain effort level change.[30] In fact, an agent who deviates to a worse action $a_i < a_i^*$ can expect to receive a worse signal that is indicative for the likely cost to be borne by her. A deviation inflicts an expected punishment upon the deviator. Therefore, one may suspect that an increase in the correlation of the signals with actions will make the use of contentious contracts a more efficient instrument. We restrict attention to symmetric models, i.e. $f(x|a;a') = f(x|a';a)\ \forall (a;a') \in A_1 \times A_2$ and $p^I(a_1) = p^I(a_2)$ if $a_1 = a_2$. For our comparison in Proposition 2, we relate the signal probabilities $p^I(a_i)$ ($p^I(a_j)$) to corresponding probabilities in a model which is identical except that signals are uninformative. For the latter, we keep the notation $p(i)$ ($p(j)$). We use the following notation: $(a^I;a^I)$ denote the optimal allocation in the informative case, $(a^*;a^*)$ in the uninformative case, and S^I and S^* denote the corresponding optimal sets of dispute states.

Proposition 2. Suppose that $p^I(a^*) = p(i) = p(j)$. Then, the following results hold for the comparison of the symmetric allocations $(a^I;a^I)$ and $(a^*;a^*)$:

1. The set of dispute states S^I needed to implement $(a^*;a^*)$ is smaller, $S^I < S^*$.

2. The allocation is pareto-superior in the informative case.

Proof. See the Appendix. ■

In short, having informative litigation is unambiguously good news if the qualification in the Proposition hold. Note that, under these conditions, both agents are equally litigious at the optimal solutions. Then, informed litigation decreases the necessary scope of dispute states and makes implementation less costly. As a consequence, higher effort levels will be implemented.

[30] See Appendix.

The intuition for the impact of information in litigation can also be explained by the analogy to monitoring. An informed court retrieves information about the effort levels, which were hitherto unobservable. Because the verdict is conditional on this information, the outcome compares to a situation where a monitor (as in Alchian and Demsetz' proposal) obtains information on the effort levels and rewards or punishes the agents accordingly. Obviously, this can improve the situation even if monitoring is not very accurate, as long as the monitor obtains some information in a statistical sense. The accuracy of jurisdiction is reflected in the present model by the functions $p^l(a_j)$ and $p^l(a_i)$[31]. Suppose for a moment that the court is a perfectly informed monitor. Relaxing the independency assumption, it could then adjudicate as follows:

$$p^l(a_1) = \begin{cases} 1 & \text{if } a_2 < a^l \\ 0 & \text{if } a_2 \geq a^l \quad \text{if agent 1's effort is less than } a^l \\ \frac{1}{2} & \text{if } a_2 \geq a^l \quad \text{if agent 1's effort is } a^l \text{ or higher} \end{cases}$$

and correspondingly concerning agent 2.

In other words, the adjudication, as measured by these functions would be discontinuous around the targeted effort levels, for example the efficient levels a^l. It is not hard to see that this adjudication can implement the efficient allocation, provided that damages $D^{max}(x)$ are large enough. This is of course only possible if the court were a perfect monitor which is quite unrealistic. But the same logic carries over: the better the court is informed, the steeper the expected punishments and rewards that can be inflicted upon agents as a statistical function of their true effort levels. It can be shown that the efficiency gain of the allocation depends monotonically on feasible damages $D^{max}(x)$.

Another comparative statics question is how the efficiency gain depends on the quality of information, i.e. on the slope of the functions $p^l(a_1)$ and $p^l(a_2)$? This amounts to the comparative statics analysis of the impact of an increase in the of $p^l(a_1)$ and $p^l(a_2)$. We add an informal discussion of this question. It turns out that an increase in the slope of $\frac{dp^l(a_i)}{da_i}$ is not sufficient to get a monotonicity result. Similar to the condition stated in Proposition 2, an additional assump-

[31] To be precise, it is actually also measured by α_{dp} which is, for simplicity, kept constant throughout the paper. Extending informativeness to α_{dp} would not change the results qualitatively.

tion concerning the absolute values of signal probabilities around a^* is needed. With this qualification, the comparative statics is actually monotonic. That is, the higher the slope of the functions $p^l(a_j)$ and $p^l(a_i)$ etc., the smaller the necessary set of contestable states, the higher the implementable effort allocation and welfare.[32]

6. The robustness of litigation

Dispute is an inherently wasteful implementation device. The reader is probably wondering if there is not a less expensive way to achieve the same goal, for example by transferring the resources to a third party. In this section, we propose an explanation why wasteful legal dispute may be preferred. We argue that any attempt to transfer these outlays may not be robust against renegotiation or collusion. By contrast, the burning money mechanism created by contract incompleteness appears to be well suited to withstand strategic opportunism. We discuss renegotiation, coalition formation and finally corruption of the judiciary.

6.1. Renegotiation-proofness

Imagine that agents envision the following solution. Instead of wasting surplus in a costly dispute, they write a complete contract including the following provision: an amount $D^{max}(x)$ is paid to a third person like a charitable fund whenever the joint output x is less than the threshold x^*: Hence, if the expected donation is equivalent to the wasted resources through litigation in the contentious contract, then incentives should be the same. The important drawback, however, is that the contractual promise would not be renegotiation-proof. Once a bad outcome $x < x^*$ is realized, parties would quickly agree to renege on the promised donation. Because the contribution is a gift, the beneficiary has no legal title to sue.

To show this formally, one simply supposes that renegotiation is possible after the actions $(a_1; a_2)$ are sunk and agents observe their signals. An equilibrium is renegotiation-proof if the initial contract remains in place after the renegotiation stage, for all states x: However, for all states $x < x^*$, whoever is making the last offer will find it beneficial to propose a split of $D^{max}(x)$ rather than letting the

[32] A formal condition behind this comparative statics analysis, called co-monotonicity, can be added and corresponding results are straightforward extensions of the proof of the Proposition.

initial contract in place. Since the state is perfectly observable, the other party will always accept.

By contrast, the separating equilibrium of the pre-trial settlement game withstands renegotiation, because the contract incompleteness forces renegotiation to take place in a situation of asymmetric information. To see this, simply note that the pre-trial settlement game is in itself a renegotiation stage. Because of asymmetric information about the merit of a court case, ex post inefficiency is unavoidable in a separating equilibrium.

6.2. Coalition-proofness

The renegotiation problem could be avoided by signing an explicit contract with the third party. For example, the agents might find a third party agreeing to pay them an amount of $\int^{x^*} C(x)f(x|a_1^*; a_2^*)dx$ up front, in exchange of the transfer of $C(x)$ in each state $x < x^*$. Not only is the $\int^{x^*} C(x)f(x|a_1^*; a_2^*)dx$ not wasted, it is also redistributed to the agents. Hence, they should prefer this to a contentious contract. The problem with this solution is that it is not coalitions-proof. Any of the two agents, say agent i, could approach the third party with the following proposal:

> "Agent i chooses a lower effort level than a_i^*; the probability of a bad outcome increases marginally, which will benefit the third party by $\int^{x^*} C(x)f_i(x|a_i^*; a_j^*)dx$. Both agree on a split of this additional transfer such that agent i is enticed to lower her effort below a_i^* and both parties are better off."

In other words, agent i and the third party can profitably collude at the expense of agent j.

To show this more rigorously, we invoke the concept of Coalition-Proof Nash Equilibrium (CPNE) (Bernheim, Peleg and Whinston (1987)). Loosely speaking, a Nash equilibrium is coalition-proof if no coalition of players would find it beneficial to undertake a joint deviation or if any such profitably deviating coalition would itself be undermined by a profitably deviating sub-coalition. The set of CPNE is a subset of the Nash equilibria of a game.

To apply this concept, we extend the game in the following way. We assume that the agents have the option to transfer resources either to "players" or to "sinks" which are assumed not to be players of the game[33]. Concerning the difference between both transfer options, we assume that ex ante contracts (of the sort that can contain a payment in exchange for the contingent transfer) can only be written with "players" and coalitions can only be formed with these agents. Note that coalitions will only be accepted if they are formed prior to taking actions a_i and a_j. After actions are sunk, the reason to form coalitions has gone. It is then possible to demonstrate that any equilibrium of the game where resources are transferred to strategic players and where $(a_1; a_2) > (a_1^c; a_2^c)$ is not a CPNE.

Thus, agents face the following dilemma: if they transfer to players in order not to waste resources, then the contract is prone to be undermined by collusion. If they transfer to sinks, then the resources are lost for the agents. Furthermore, in the latter case, the two agents are not better off than if they squander resources through costly legal dispute[34], even though there might be recipients benefiting from the transfer which is not necessarily the case for legal disputes. We conclude: there might be solutions which are socially preferable to the dispute solution (as someone benefits from the transferred resources), but they are not privately preferable for the agents. The court system may not be the socially optimal device to squander resources, but the agents have little incentive to look for other solutions.

[33] An example of sinks is the device the paper has focused on hitherto, legal fees and other direct court costs: these are resources which are squandered without benefiting anyone. Note that even if the costs $l \cdot D_i(x)$ increase the utility of someone, they fall in this category: judges are often assigned in an unpredictable way and, even if they are not, the state budget - not the judge - is recipient of $l \cdot D_i(x)$; a large organization like the state is not easily susceptible to a collusive suggestion. There could be other examples of sinks, for example if beneficiaries are randomly chosen ex post in a way that the agents cannot influence (by a lottery, for example). A transfer to a strategic player, on the other hand, is any payment made to a player who is ex ante identifiable, like the charitable fund introduced earlier.

[34] This is true as long as the transferable amount does not exceed $C(x; i)$ in state x. If more can be transferred, then a better solution than the legal dispute solution is feasible by reducing the size of S^*.

6.3. Corruption

Corruption is the attempt to buy the favor of the judge (or jury) and thus alter ex ante incentives through the manipulation of the trial outcome. Could our mechanism be undermined by this possibility? Corruption is distinct from collusion. The side payments flow in opposite directions in both cases: in a coalition, a benefiting third party bribes one of the agents to increase the probability of the bad outcome. In corruption, one of the agents pays the judge. Also, coalitions need to be formed before actions are taken. Corruption can be attempted before or after the action (and in fact there is no advantage to bribing a judge ex ante).

We want to argue that there is no reason why our mechanism should be any less effective if the judge is corrupt compared to a situation where she is not. On the contrary, corruption could even improve the allocation by adding to the net litigation costs. To see this, assume that one of the agents has access to bribing the judge. If this is beneficial for the agent, the agent will do so. Ex ante, the bribery is anticipated, and this will be built into the optimal contract: the agent who has access to the judge has a higher expected probability to receive a "strong" signal, or is more litigious. The other agent is more likely to receive a "weak" signal. The aggregate effect on the probability of litigation is ambiguous. However, the cost of legal dispute have now increased by the amount of the bribe which makes it likely that the net cost of litigation $C(x)$ increases. A similar reasoning applies if both agents are competing to bribe the judge: neither is necessarily more likely to win the judge's vote but both expect to spend on trying to gain the judges favor. Net litigation costs have increased and the overall effect is again ambiguous.

7. Conclusion

The main conclusion of our analysis is that the prospect of dispute can be interpreted as a deterrence device against lack of effort or care. We identify conditions where both parties are better off with a contentious contract compared to a litigation-proof contract. Even if parties are not fully aware of this side effect of unforeseen contingencies, this aspect could help to explain why incomplete contracts are perhaps less costly than it might appear and why there is frequently little effort to eradicate incompleteness.

In this paper, we propose a model of rational incompleteness of contracts, based on the idea that legal dispute after unwanted outcomes could be employed as an incentive device. Of course, this should not be misunderstood as an encompassing theory of incomplete contracts. There are several limitations to the model. First, our model applies only to joint production. Even though aspects of joint production are pervasive and certainly more important than is expressly acknowledged in contracts, incompleteness is not limited to these cases. Second, there are incompleteness phenomena which this model does not address, for example omitted favorable contingencies (windfalls). Finally, our contribution should not be misunderstood as saying that bounded rationality is not an important, and probably the most important, source of incompleteness. Many incomplete contracts may exhibit both sources of incompleteness: on the one hand, it is costly to foresee, to de…ne and to verify contingencies because agents are boundedly rational. On the other hand, the true costs of incompleteness may be lower because there is the aspect of rational deterrence which is highlighted in the present paper. That would explain why so often even the attempt of sorting out contingencies is lacking.

8. Appendix A: The pretrial settlement bargaining game

This Appendix documents the pretrial settlement game which is adapted from Urs Schweizer's model (1989).[35] While we document the details needed to understand the selected equilibrium, we refer to the original for other interesting details.

In any contestable state $x \in S$; each agent observes a signal which has two possible outcomes, "strong" or "weak". The signals are independently distributed. Recall that $p(i)$ will denote the probability that the plaintiff i observes the good signal, etc. The plaintiff's chances of winning a process in court is a function of the profile of signals for both agents. Let d and p denote the defendant's and the plaintiff's private information, respectively, and let α_{dp} denote the probability that the case is won depending on the pair of signals of defendant and plaintiff, with $d; p \in \{g; b\}$: For example, α_{bg} is the probability that litigation is won by the plaintiff if she observes the "good" signal and the defendant observes the "bad" signal. We have

$$\alpha_{gb} < \alpha_{bb} < \alpha_{bg} \text{ and } \alpha_{gb} < \alpha_{gg} < \alpha_{bg}:$$

Let

$$G_{dp}(x; i) = (\alpha_{dp}(1 + l) - l)\, D_i(x)$$

denote the expected gain of a plaintiff of type p against a defendant type d. Then, the plaintiff i's expected gain in court, if her type is p, is:

$$G_p(x; i) = (1 - p(j))G_{bp}(x; i) + p(j)G_{gp}(x; i)$$

Let

$$L_{dp}(x; i) = \alpha_{dp}(1 + l)D_i(x)$$

denote the expected loss of defendant type d against a plaintiff of type p. A defendant j of type d has then an expected loss in court of

$$L_d(x; i) = (1 - p(i))L_{db}(x; i) + p(i)L_{dg}(x; i)$$

[35] The only significant change with respect to Schweizer is that litigation costs are a function of damages. Any of the numerous models of pretrial settlement bargaining under one-sided or two-sided incomplete information, adapted to our model, would give analogous results, see for example Bebchuk (1984), Png (1983) or Spier (1992). See Cooter and Rubinfeld (1989) and Kennan and Wilson (1993) for surveys.

In the least-cost separating equilibrium, the "good" and the "bad" defendant make distinct offers. We describe next the least-cost separating equilibrium where the offer of a "strong" defendant is sometimes rejected while the offer of a "weak" defendant is always accepted. This is the outcome for a certain set of parameter values; the bounds for this solution are documented below. The weak defendant makes an offer of $G_{bg}(x; i)$ which is accepted because no type of the plaintiff i could receive more. Therefore, the strong defendant must offer a settlement which makes the weak plaintiff indifferent between accepting and rejecting: this amount is $G_{gb}(x; i)$; as the plaintiff infers (in the separating equilibrium) from the offer that she is confronted to a strong defendant. Let $q(i)$ denote the probability of acceptance of a settlement offer proposed by the defendant if agent i is the plaintiff.[36] Note that only the weak plaintiff mixes between accepting and rejecting the offer $G_{gb}(x; i)$; the strong plaintiff always rejects it. The key to establish separation between the defendant's types is that the weak defendant should have no incentive to mimic her strong counterpart. If she were to imitate a strong defendant, she would need to offer only $G_{gb}(x; i) < G_{bg}(x; i)$: If she were always rejected, she would expect to lose $L_b(x; i)$. However, with probability $q(i)$; her offer of $G_{gb}(x; i)$ is accepted; in this case, her marginal gain is $G_{gb}(x; i) - L_{bb}(x; i)$; i.e. her offer minus her loss if being rejected (taking into account that she is actually the weak type.). In the least-cost separating equilibrium, the weak defendant is just indifferent between both options, or

$$L_b(x; i) + q(i)\left[G_{gb}(x) - L_{bb}(x)\right] = G_{bg}(x)$$

Thus, the acceptance probability of a settlement out of court is:

$$\begin{aligned} q(i) &= \frac{L_b(x; i) - G_{bg}(x)}{L_{bb}(x) - G_{gb}(x)} \\ &= \frac{(1+\lambda)(1-p(i))(\alpha_{bb}-\alpha_{bg})+\lambda}{(1+\lambda)(\alpha_{bb}-\alpha_{gb})+\lambda} \end{aligned} \tag{8.1}$$

By calculating out the expectation over the possible matches, the ex-ante expected payoffs can be determined as:

$$\Pi^d(x; i) = \left[(1 - p(j) + p(j)\,q(i))\lambda - (1+\lambda)\Phi(i)\right] D_i(x)$$

[36] Equation (8.1) demonstrates that $q(i)$ is independent of x and $D_i(x)$.

$$\Pi^p(x,i) = ((1+l)\Phi(i) - l)\, D_i(x)$$

respectively, where $\Pi^p(x,i)$ and $\Pi^d(x,i))$ denote the plaintiff's and the defendant's expected profit, respectively, and where

$$\Phi(i) = p(j)((1-p(i)\alpha_{gb} + p(i)\alpha_{gg}) + (1-p(j))\alpha_{bg}.$$

Note that both functions are linear in $D_i(x)$. For the total litigation cost, one calculates:

$$\begin{aligned} C(x,i) &= -\Pi^d(x) - \Pi^p(x) \\ &= p(j)(1-q(i))\, l\, D_i(x). \end{aligned} \tag{8.2}$$

Note that if $p(i) = p(j)$, then $C(x,i) = C(x,j)$, i.e. the ex-ante expected payoffs of a dispute in state $x \in S$ are the same for both agents, and the expected costs of litigation is independent of the choice of the defendant. More generally, we have that

$$C(x,i) \gtrless C(x,j) \quad \Leftrightarrow \quad p(i) \lessgtr p(j)$$

as a straightforward consequence of (8.2) and (8.1).

Finally, we document the parameter restrictions necessary for this outcome to be feasible. These conditions are that $0 \le q(i) \le 1 - p(i)$ (see Schweizer (1989), p.166), or :

$$1 - \frac{l}{1+l}(\alpha_{bg} - \alpha_{bb}) \le p(i) \le \frac{1 + l(\alpha_{bg} - \alpha_{gb})}{l + (1+l)(\alpha_{bg} - \alpha_{gb})}. \tag{8.3}$$

If $p(i)$ is larger than the upper bound in (8.3), then the weak plaintiff will always accept the good offer while the strong plaintiff will mix between accepting and rejecting it. If $p(i)$ is below the lower bound in (8.3), then a fully separating equilibrium is not possible.

9. Appendix B: Proofs

9.1. Proof of Lemma 1.

Lemma (1) is proved by transforming the relaxed optimization problem into a control problem. To this end, we define control variables $\gamma_1(x)$, $\gamma_2(x)$ and $\gamma_3(x)$

as follows. $\gamma_1(x)$ ($\gamma_1(x) \in \{0; 1\}$) indicates whether or not the state x is included in S_1; while $\gamma_2(x)$ ($\gamma_2(x) \in \{0; 1\}$) indicates if the state x is included in S_2. Finally, $\gamma_3(x)$ indicates if the state x is contained in $S_1 \cap S_2$. That is, $S_1 = \{x \in X : \gamma_1(x) = 1\}$, $S_2 = \{x \in X : \gamma_2(x) = 1\}$ and $S_1 \cap S_2 = \{x : \gamma_3(x) = 1\}$. Thus, we write agent 1's and agent 2's ex-ante expected payoffs from a dispute in the state x as follows

$$\begin{aligned} R^1(x) &= \left(\gamma_1(x) - \frac{\gamma_3(x)}{2}\right)\Pi^p(x; 1) + \left(\gamma_2(x) - \frac{\gamma_3(x)}{2}\right)\Pi^d(x; 2) \\ R^2(x) &= \left(\gamma_1(x) - \frac{\gamma_3(x)}{2}\right)\Pi^d(x; 1) + \left(\gamma_2(x) - \frac{\gamma_3(x)}{2}\right)\Pi^p(x; 2) \end{aligned}$$

with the constraint that

$$\gamma_3(x) - \gamma_1(x)\gamma_2(x) = 0; \forall x. \tag{9.1}$$

The ex-ante expected costs of litigation in state x, i.e. $C(x)$, are given by

$$-(R^1(x) + R^2(x)) = \gamma_1(x)C(x; 1) + \gamma_2(x)C(x; 2) - \frac{\gamma_3(x)}{2}(C(x; 1) + C(x; 2))$$

and the Lagrangian for the relaxed optimization problem is

$$\begin{aligned} L = {} & E[x|a_1; a_2] - \sum_{i=1} c(a_i) f(x|a_1; a_2)dx - \int_X (R^1(x) + R^2(x)) f(x|a_1; a_2)dx \\ & + \lambda_1 \left\{\int_X \beta(x) f_1(x|a_1; a_2)dx + \int_X R^1(x) f_1(x|a_1; a_2)dx - c_1(a_1)\right\} \\ & + \lambda_2 \left\{\int_X (1 - \beta(x)) f_2(x|a_1; a_2)dx + \int_X R^2(x) f_2(x|a_1; a_2)dx - c_2(a_2)\right\} \\ & + \int_X \left\{\sum_{i=1} \{\delta_i(x)(D^{max} - D_i(x)) + \rho_i(x)D_i(x) + \psi_i(x)(1 - \gamma_i(x)) + \phi_i(x)\gamma_i(x)\}\right\} dx \\ & + \int_X \{\omega(x)(\gamma_3(x) - \gamma_1(x)\gamma_2(x))\} dx \end{aligned} \tag{9.2}$$

To analyze this problem, we proceed in several steps.

Step 1. In this step, we show that Item 1 of Lemma 1 must be true for any $S_1; S_2; D_1(x)$ and $D_2(x)$. This is shown from incentives constraints (3.1) - (3.2) and the first order conditions of (9.2) with respect to $\beta(x)$ and a_i:

$$\frac{\partial L}{\partial \beta(x)} = f(x|a_1; a_2)\left(\lambda_1 \frac{f_1(x|a_1; a_2)}{f(x|a_1; a_2)} - \lambda_2 \frac{f_2(x|a_1; a_2)}{f(x|a_1; a_2)}\right) = 0 \tag{9.3}$$

and

$$
\begin{aligned}
\frac{\partial L}{\partial a_i} &= E_i[x|a_1,a_2] - c_i(a_i) - \int_X (R^1(x) + R^2(x))\, f_i(x|a_1,a_2)dx \qquad (9.4)\\
&\quad + \lambda_1 \left\{ \int_X \beta(x)\, f_{1i}(x|a_1,a_2)dx + \int_X R^1(x) f_{1i}(x|a_1,a_2)dx - c_{1i}(a_1) \right\}\\
&\quad + \lambda_2 \left\{ \int_X (1-\beta(x))\, f_{2i}(x|a_1,a_2)dx + \int_X R^2(x) f_{2i}(x|a_1,a_2)dx - c_{2i}(a_2) \right\}\\
&= 0
\end{aligned}
$$

for $i = 1, 2$, with $c_{ji}(\cdot) = 0$ for $j \neq i$. Note that equation (9.3) implies that any solution must satisfy:

$$
\frac{f_1(x|a_1,a_2)}{f(x|a_1,a_2)} (\lambda_1 - k(a_1,a_2)\lambda_2) = 0 \text{ for all } x \in X \qquad (9.5)
$$

since $f_2(x|a_1,a_2) = k(a_1,a_2) f_1(x|a_1,a_2)$ under Assumption 1. Thus, we have the following restriction on the equilibrium values of the multipliers for the incentive-compatibility constraints:

Lemma 2. At any solution of the relaxed optimization problem, $\lambda_1 - k(a_1,a_2)\lambda_2 = 0$, with $k(\cdot)\lambda_2 > 0$.

Proof. By definition, $\lambda_i \geq 0$, $i = 1, 2$. Hence, condition (9.5) implies that one of two situations can occur: either $\lambda_1 = 0$ and $\lambda_2 = 0$, or $\lambda_1 - k(\cdot)\lambda_2 = 0$ with $k(\cdot)\lambda_2 > 0$.

Suppose that $\lambda_1 = 0$ and $\lambda_2 = 0$ at the optimal solution. Then, equation (9.4) reduces to for a_2:

$$
\frac{\partial L}{\partial a_2} = E_2[x|a_1,a_2] - c_2(a_1) - \int_X (R^1(x) + R^2(x))\, f_2(x|a_1,a_2)dx = 0 \qquad (9.6)
$$

Now, using the fact that

$$
-C(x) = R^1(x) + R^2(x), \ \forall x
$$

we can rewrite incentive constraint (3.2) as follows

$$
\int_X (1 - \beta(x))\, f_2(x|a_1,a_2)dx - c_2(a_2) - \int_X (C(x) + R^1(x)) f_2(x|a_1,a_2)dx \geq 0. \qquad (9.7)
$$

Then, substituting equation (9.6) into (9.7) and using Assumption 1 gives us

$$-\int_X \beta(x) f_1(x|a_1;a_2)dx - \int_X R^1(x) f_1(x|a_1;a_2)dx \geq 0 \tag{9.8}$$

which contradicts incentive constraint (3.1). ■

Lemma (2) and Assumption 1 allow us to establish the existence of a linear sharing rule (item 1 of Lemma 1). The proof is exactly analogous to the proof of Bhattacharyya and Lafontaine ((1995), Proposition 1) and is thus omitted.

Step 2. Next, we determine $S_1^*; S_2^*; D_1^*(x)$ and $D_2^*(x)$. Using the fact that

$$\mu_1 f_1(x|a_1;a_2) = \mu_2 f_2(x|a_1;a_2)$$

permits to write the first order conditions of the problem with respect to $\gamma_1(x); \gamma_2(x)$ and $\gamma_3(x)$ as follows:

$$\begin{aligned}
\frac{\partial L}{\partial \gamma_1(x)} &= -C(x;1)\left(1+\mu_1\frac{f_1(x|a_1;a_2)}{f(x|a_1;a_2)}\right) f(x|a_1;a_2) - \omega(x)\gamma_2(x) - \psi_1(x) + \phi_1(x) \\
&= 0. && (9.9) \\
\frac{\partial L}{\partial \gamma_2(x)} &= -C(x;2)\left(1+\mu_1\frac{f_1(x|a_1;a_2)}{f(x|a_1;a_2)}\right) f(x|a_1;a_2) - \omega(x)\gamma_1(x) - \psi_2(x) + \phi_2(x) \\
&= 0. && (9.10) \\
\frac{\partial L}{\partial \gamma_3(x)} &= \frac{1}{2}\left(C(x;1)+C(x;2)\right)\left(1+\mu_1\frac{f_1(x|a_1;a_2)}{f(x|a_1;a_2)}\right) f(x|a_1;a_2) + \omega(x) \\
&= 0. && (9.11)
\end{aligned}$$

The FOC with respect to $D_1(x)$ and $D_2(x)$ are:

$$\begin{aligned}
\frac{\partial L}{\partial D_1(x)} &= -\frac{\partial C(x;1)}{\partial D_1(x)}\left(\gamma_1(x)+\frac{\gamma_3(x)}{2}\right)\left(1+\mu_1\frac{f_1(x|a_1;a_2)}{f(x|a_1;a_2)}\right) f(x|a_1;a_2) - \delta_1(x) + \rho_1(x) \\
&= 0. && (9.12) \\
\frac{\partial L}{\partial D_2(x)} &= -\frac{\partial C(x;2)}{\partial D_2(x)}\left(\gamma_2(x)+\frac{\gamma_3(x)}{2}\right)\left(1+\mu_1\frac{f_1(x|a_1;a_2)}{f(x|a_1;a_2)}\right) f(x|a_1;a_2) - \delta_2(x) + \rho_2(x) \\
&= 0. && (9.13)
\end{aligned}$$

Note that $\mu_1 > 0$ and MLRP imply that $1+\mu_1\frac{f_1(x|a_1;a_2)}{f(x|a_1;a_2)}$ is increasing with x. Therefore, there exist a unique $x^* \geq 0$ such that $\left(1+\mu_1\frac{f_1(x|a_1;a_2)}{f(x|a_1;a_2)}\right) f(x|a_1;a_2) < 0$ for all $x < x^*$ and $\left(1+\mu_1\frac{f_1(x|a_1;a_2)}{f(x|a_1;a_2)}\right) f(x|a_1;a_2) \geq 0$ for all $x \geq x^*$. If $1+\mu_1\frac{f_1(0|a_1;a_2)}{f(0|a_1;a_2)} \geq 0$, then $x^* = 0$; otherwise, x^* solves $1+\mu_1\frac{f_1(x|a_1;a_2)}{f(x|a_1;a_2)} = 0$. Note also that $\frac{\partial C(x;1)}{\partial D_1(x)} = lp(2)(1-q(1))$ and $\frac{\partial C(x;2)}{\partial D_2(x)} = lp(1)(1-q(2))$ are always positive.

Hence, by complementary slackness, the first order conditions (9.12) and (9.13) imply that $D_i^*(x) = D^{max}(x)$ for all x such that $x < x^*$, and $D_i^*(x) = 0$ for all x such that $x > x^*$.

Now, substituting equation (9.11) into (9.9) and (9.10) gives

$$\left(\frac{\gamma_2(x)}{2}(C(x;1) + C(x;2)) - C(x;1)\left(1 + \mu_1 \frac{f_1(x|a_1;a_2)}{f(x|a_1;a_2)}\right) f(x|a_1;a_2) - \psi_1(x) + \phi_1(x)\right.$$
$$= 0 \qquad (9.14)$$

$$\left(\frac{\gamma_1(x)}{2}(C(x;1) + C(x;2)) - C(x;2)\left(1 + \mu_1 \frac{f_1(x|a_1;a_2)}{f(x|a_1;a_2)}\right) f(x|a_1;a_2) - \psi_2(x) + \phi_2(x)\right.$$
$$= 0 \qquad (9.15)$$

Recall that $C(x;1) = C(x;2) = 0$ for $D_1(x) = D_2(x) = 0$. Furthermore, $C(x;1) > C(x;2)$ when $D_1(x) = D_2(x) > 0$ if and only if $p(1) < p(2)$ and vice versa. Items 2 to 4 are therefore derived from conditions (9.14) and (9.15). First, $\gamma_1(x) = 0$ and $\gamma_2(x) = 0$ for all $x \geq x^*$ always solve these equations since $D_1^*(x) = D_2^*(x) = 0$ for all $x \geq x^*$.

Next, if $p(1) < p(2)$ (the case where $p(1) > p(2)$ is symmetric), by complementary slackness, these conditions imply that $\gamma_1(x) = 1$ and $\gamma_2(x) = 0$ for all $x < x^*$ since $C(x;1) > C(x;2)$ $\forall x < x^*$ (recall that $D_1^*(x) = D_2^* = D^{max}(x)$). Thus, $S_2^* = \emptyset$ while $S_1^* = S^* = \{x : x < x^*\}$, with maximum applicable damages ($D_1^*(x) = D^{max}(x)$, $\forall x \in S_1^*$).

If $p(1) = p(2)$, then $C(x;1) = C(x;2) = D^{max}(x)p(1)(1 - q(2)) > 0$ for all $x < x^*$. Hence, conditions (9.14) and (9.15) imply that we must have either $\gamma_1(x) = 1$ and $\gamma_2(x) = 0$, or $\gamma_1(x) = 0$ and $\gamma_2(x) = 1$; or $\gamma_1(x) = \gamma_2(x) = 1$, for all $x < x^*$. Thus, $S^* = \{x : x < x^*\}$, and any choice of S_1 and S_2 solve the problem.

Finally, to see that $x^* < \hat{x}(a_1;a_2)$, remember that $\hat{x}(a_1;a_2)$ is the (unique) value such that $f_1(x|a_1;a_2) < 0$ for $x < \hat{x}(a_1;a_2)$ and $f_1(x|a_1;a_2) \geq 0$ otherwise. Therefore, $1 + \mu_1 \frac{f_1(x|a_1;a_2)}{f(x|a_1;a_2)} > 0$ at $x = \hat{x}(a_1;a_2)$.

Step 3. Now, we show that the optimal contract leads to an action profile $(a_1^*;a_2^*)$ such that $a_1^* > a_1^c$ and $a_2^* > a_2^c$ for $S^* \neq \emptyset$. To this end, note that $\mu_1 = k\mu_2 > 0$ implies that incentive constraints (4.2) and (4.3) are binding at any solution. Thus, using Assumption 1 and adding up the two equations imply that the optimal action profile $(a_1^*;a_2^*)$ must solve

$$\int_X x\, f_1(x|a_1^*;a_2^*)dx - \int^{x^*} \overline{C}(x;1) f_1(x|a_1^*;a_2^*)dx - c_1(a_1^*) = c_2(a_2^*)/k(a_1^*;a_2^*)$$

for $p(1) \leq p(2)$, where $\overline{C}(x;1)$ corresponds to $C(x;1)$ evaluated at $D^{max}(x)$. Since $-\int^{x^*} \overline{C}(x;i) f_1(x|a_1^*, a_2^*)dx > 0$ for $x^* > 0$ at the optimal contract, the action profile (a_1^*, a_2^*) satisfies

$$\int_X x f_1(x|a_1^*, a_2^*)dx - c_1(a_1^*) < c_2(a_2^*)/k(a_1^*, a_2^*):$$

Hence, by concavity, we obtain that $a_1^* > a_1^c$ and $a_2^* > a_2^c$ in equilibrium.

Step 4. Finally, we check for the validity of the FOA. To do so, one must verify that each agent's effort problem is strictly concave at K^*. In other words, it is sufficient to show that

$$V_{11}^1(K^*, (a_1, a_2^*) < 0 \tag{9.16}$$

for all $a_1 \in A_1$, and

$$V_{22}^2(K^*, (a_1^*, a_2) < 0 \tag{9.17}$$

for all $a_2 \in A_2$. In fact, these conditions are always satisfied for $S_1^* = S_2^* = S^*$ under the Mirrlees-Rogerson convexity of the distribution function condition (CDFC). Without loss of generality, assume that $S_1^* = \emptyset$ and $S_2^* = S^*$, i.e. agent 1 is the defendant. We have:

$$V^1(K^*, (a_1, a_2)) = \int_X \beta^*(x) x f(x|a_1, a_2)dx - c(a_1) + \int^{x^*} \pi^d(x;2) f(x|a_1, a_2)dx \tag{9.18}$$

Integrating (9.18) by parts and differentiating twice gives

$$\begin{aligned} V_{11}^1(K^*, (a_1, a_2^*) &= -\beta^* \int_X F_{11}(x|a_1, a_2^*)dx + \pi^d(x;2)F_{11}(x^*|a_1, a_2^*) \\ &\quad - \int^{x^*} \frac{\partial \pi^d(x;2)}{\partial x} F_{11}(x|a_1, a_2^*)dx - c_{11}(a_1) \end{aligned}$$

Since $\pi^d(x;2) \leq 0$ for all $x \leq x^*$, and $\frac{\partial \pi^d(x;2)}{\partial x} \geq 0$ for $\frac{\partial D^{max}(x)}{\partial x} \leq 0$, $V_{11}^1(K^*, (a_1, a_2^*)$ is strictly negative if $F_{11}(x|.) \geq 0$ which is the Mirrlees-Rogerson condition (CDFC).

For agent 2, we have

$$V^2(K^*, (a_1, a_2)) = \int_X (1 - \beta^*(x)) x f(x|a_1, a_2)dx - c(a_2) + \int^{x^*} \pi^p(x;2) f(x|a_1, a_2)dx: \tag{9.19}$$

Integrating by parts this expression and differentiating twice gives:

$$
\begin{aligned}
V^2_{22}(K^*;(a_1^*,a_2) = \; & -(1-\beta^*)\int_X F_{22}(x|a_1^*,a_2)dx + \Pi^p(x^*,2)F_{22}(x^*|a_1^*,a_2) \\
& - \int^{x^*} \tfrac{\partial \Pi^p(x,2)}{\partial x} F_{22}(x|a_1^*,a_2)dx - c_{22}(a_2)
\end{aligned}
$$

Note that $\Pi^p(x,2) \geq 0$ for all x, and $\frac{\partial \Pi^p(x,2)}{\partial x} \leq 0$ for $\frac{\partial D^{max}(x)}{\partial x} \leq 0$. Therefore, in order to show that expression (9.17) is usually negative under CDFC, we must be more specific here about the legal bound on damages, $D^{max}(x)$. Let assume, for example, that punitive damages are denied in court. Then, the maximum applicable damages will cover the monetary loss of the plaintiff and (eventually) his legal expenses. We set $D^{max}(x) = \sigma(x^* - x)$. It implies that

$$
\begin{aligned}
V^2_{22}(K^*;(a_1^*,a_2) \;=\; & -((1-\beta^*) + \sigma(l - (1+l)\Phi(2)))\int^{x^*} F_{22}(x|a_1^*,a_2)dx \\
& -(1-\beta^*)\int_{x^*}^{\bar{x}} F_{22}(x|a_1^*,a_2)dx - c_{22}(a_2)
\end{aligned}
$$

since $\Pi^p(x^*,2) = 0$. The last expression is strictly negative for a wide range of values for σ. Note that incentives constraints (4.2) and (4.3) require that $(1-\beta^*) > \frac{1}{2}(1 - \frac{c_1(a_1^*) - k(.)c_2(a_2^*)}{E_1(x|a_1^*,a_2^*)})$ at any solution[37]. Thus, if $\sigma < \frac{1}{2}(1 - \frac{c_1(\bar{a}) - kc_2(0)}{E_1(x|\bar{a},\bar{a})})$ for example; then $((1-\beta^*) + \sigma(l - (1+l)\Phi(2)))$ is positive for all implementable (a_1,a_2) and $V^2_{22}(K^*;(a_1^*,a_2) < 0$. ■

Proof of Corollary 1.

Assume to the contrary that $S^* = \emptyset$ under condition (4.6). Then, incentives constraints (4.2) and (4.3) imply that (a_1^c,a_2^c) is the optimal action profile. Now, consider the first order conditions of problem (9.2). The FOC with respect to a_1, condition (9.3), reduces to

$$
\frac{\partial L}{\partial a_1} = E_1[x|a_1^c,a_2^c] - c_1(a_1^c) + \lambda_1 \left\{ \int_X x f_{11}(x|a_1^c,a_2^c)dx - c_{11}(a_1^c) \right\} = 0 \quad (9.20)
$$

since $\lambda_1 = k\lambda_2$ at any solution, which gives

$$
\lambda_1 = -\frac{E_1[x|a_1^c,a_2^c] - c_1(a_1^c)}{\int_X x f_{11}(x|a_1^c,a_2^c)dx - c_{11}(a_1^c)}.
$$

[37] To see that, substract equations (4.3) to (4.2).

But, FOC (9.9) through (9.13) are then violated at $x = 0$ since $1 + \ell_1 \frac{f_1(0|a_1^c, a_2^c)}{f(0|a_1^c, a_2^c)}$ is therefore negative. ■

Proof of Proposition 1.

To establish equivalence between K^* and the contract of Proposition 1, we need to show that the optimal contract does not need to specify $(D_i(x), \beta(x))$ for all $x < x^*$.

First, concerning $D_i(x)$, recall that K^* always picks $D_i(x) = D^{max}(x)$. By assumption, $D^{max}(x)$ is awarded if damages are not specified in the contract. It follows that not specifying $D_i(x)$ for $x < x^*$ is equivalent to K^*.

Second, concerning $\beta(x)$, recall that if $\beta(x)$ is not specified for some $x < x^*$ then it will be chosen by the court. We show the following claim: if the condition in Proposition 1 holds, then $C(x; i)$ is the same for contract K^* and an optimal contract which does not specify $(D_i(x), \beta(x))$.

Let the plaintiff's payoff be $w(x)$ if she wins and $\lambda(x)$ if she loses. By balancedness, we have that the defendant receives $x - w(x)$ if she loses and $x - \lambda(x)$ if she wins. Note that $w(x) - \lambda(x)$ is the amount of what is at stake in a dispute. By definition of maximum damages, it must be the case that:

$$w(x) - \lambda(x) \leq D^{max}(x)$$

Moreover, if $D_i(x)$ is not specified for some x, then $D^{max}(x)$ will be awarded, hence

$$w(x) - \lambda(x) = D^{max}(x)$$

for all $x \in \{x | D_i(x)$ is not specified for $x.\}$. Also, recall that $D^{max}(x)$ will be attributed under K^*. Thus, the contentious amount is the same in both cases, viz. $D^{max}(x)$. Recall that then litigation costs $lD^{max}(x)$ are also identical. With these results, it is easy to verify that $q(i)$ must be as defined in equation (8.1) and $C(x; i) = p(j)(1 - q(i))D^{max}(x)$ must be the same in both cases. Finally, from Lemma 1, it follows that the allocation (a_1^*, a_2^*) is fully explained by $\max_i C(x; i)$. ■

Proof of Proposition 2.

Step 1. We begin with the following crucial claim: the expression

$$\left.\frac{\partial \Pi^p(x,i)}{\partial p^I(a_i)}+\frac{\partial \Pi^d(x,i)}{\partial p^I(a_j)}\right|_{a_i=a_j}$$

is positive.

To prove this claim, note that:

$$\frac{\partial \Pi^p(x,i)}{\partial p^I(a_i)}=p^I(a_j)\left[\alpha_{gg}-\alpha_{gb}\right]D_i(x)(1+\lambda),\ i\neq j;$$

and

$$\frac{\partial \Pi^d(x,i)}{\partial p^I(a_j)}=-\left[p^I(a_i)\alpha_{gg}+(1-p^I(a_i))\alpha_{gb}-\alpha_{bg}\right]D_i(x)(1+\lambda)-\left[1-q^I(a_i)\right]\lambda D_i(x),\ i\neq j.$$

Thus :

$$\begin{aligned}\frac{\partial \Pi^p(x,i)}{\partial p^I(a_i)}+\frac{\partial \Pi^d(x,i)}{\partial p^I(a_j)}=\ &\left[\alpha_{bg}-\alpha_{gb}\right]D_i(x)(1+\lambda)-(1-q^I(a_i))\lambda D_i(x)\\&-D_i(x)(1+\lambda)\left[\alpha_{gg}-\alpha_{gb}\right]\left(p^I(a_i)-p^I(a_j)\right)\end{aligned}$$

and

$$\left.\frac{\partial \Pi^p(x,i)}{\partial p^I(a_i)}+\frac{\partial \Pi^d(x,i)}{\partial p^I(a_j)}\right|_{a_i=a_j}=\left[\alpha_{bg}-\alpha_{gb}\right]D_i(x)(1+\lambda)-(1-q^I(a_i))\lambda D_i(x)$$

Thus, to show that this expression is positive, we have to show that

$$\left[\alpha_{bg}-\alpha_{gb}\right](1+\lambda)>(1-q^I(a_i))\lambda.$$

After substituting for $q^I(a_i)$:

$$\left[\alpha_{bg}-\alpha_{gb}\right](1+\lambda)>\left[1-\frac{(1+\lambda)(1-p^I(a_i))(\alpha_{bb}-\alpha_{bg})+\lambda}{(1+\lambda)(\alpha_{bb}-\alpha_{gb})+\lambda}\right]\lambda$$

which is always true. This finishes the first step.

Step 2. Let $VI^i(K,(a_1,a_2))$ denote agent i's expected utility in the informative case. Let $P^i(x|a_1^I,a_2^I)$ denote i's payoff in a contestable state (in analogy to $R^i(x|a_1,a_2)$ in the model of Section 2.) To check for (1) in Proposition 2, consider the first-order conditions with respect to a_1 and a_2 which hold with equality for any optimal contract. If signals are correlated, then this condition takes the following form for agent 1:

$$\begin{aligned}VI_1^1(K,(a_1^I,a_2^I))=&\int_X \beta(x)f_1(x|a_1^I,a_2^I)dx+\int_{S^I}P^1(x|a_1^I,a_2^I)f_1(x|a_1^I,a_2^I)dx\\&-c_1(a_1^I)+\int_{S^I}\frac{\partial P^1(x|a^I,a^I)}{\partial p^I(a_1)}\frac{\partial p^I(a_1)}{\partial a_1}f(x|a_1^I,a_2^I)dx=0\end{aligned}\qquad(9.21)$$

and analogously for agent 2. Adding up the two first-order equations (9.21) and using the fact that $f_1(x|a;a) = f_2(x|a;a)$ under the assumptions of Proposition 2, we get for a symmetric action profile $(a^I;a^I)$:

$$
\begin{aligned}
&\int_X x f_1(x|a^I;a^I)dx + \int_{S^I}\left(P^1(x|a^I;a^I) + P^2(x|a^I;a^I)\right) f_1(x|a^I;a^I)dx - 2c_1(a^I) \\
&+ \int_{S^I}\left(\frac{\partial P^1(x|a^I;a^I)}{\partial p^I(a_1)}\frac{\partial p^I(a_1)}{\partial a_1} + \frac{\partial P^2(x|a^I;a^I)}{\partial p^I(a_2)}\frac{\partial p^I(a_2)}{\partial a_2}\right) f(x|a^I;a^I)dx = 0
\end{aligned}
\tag{9.22}
$$

Note that (9.22) is a necessary condition for the implementation of the action profile $(a^I;a^I)$. Of course, this expression depends on the properties of the optimal solution of the contracting problem in the informative case. In fact, one can show that, if the qualification in Proposition 2 holds and the allocation is symmetric, then the optimal set of dispute states satisfies $S^I = [0;x^I)$; $x^I < \hat{x}(a^I;a^I)$; with $S^I_1 = S^I_2$: Note also that the optimal $D_i(x) = D^{max}(x)$; as in the basic model. Therefore, we can rewrite equation (9.22) as follows:

$$
\begin{aligned}
&\int_X x f_1(x|a^I;a^I)dx + \int^{x^I} \overline{C}(x;\tfrac{1}{2}) f_1(x|a^I;a^I)dx - 2c_1(a^I) \\
&+ \int^{x^I} \frac{1}{2}\left(\frac{\partial \Pi^p(x;1)}{\partial p^I(a_1)} + \frac{\partial \Pi^d(x;1)}{\partial p^I(a_2)}\right)\frac{\partial p^I(a_1)}{\partial a_1} f(x|a^I;a^I)dx = 0
\end{aligned}
\tag{9.23}
$$

since $p^I(a_1)\frac{\partial p^I(a_2)}{\partial a_2} - p^I(a_2)\frac{\partial p^I(a_1)}{\partial a_1} = 0$ when $a_1 = a_2\ \forall a$. The first three terms on the RHS of (9.23) are the same as in the benchmark model (with uncorrelated signals) for $(a^I;a^I) = (a^*;a^*)$ and $p^I(a^*_1) = p(1)$ and $p^I(a^*_2) = p(2)$, respectively. Furthermore, we have shown that the last one is always positive. Hence,

$$
x^I < x^* \quad \text{at} \quad (a^I;a^I) = (a^*;a^*): \tag{9.24}
$$

Finally, Item 2 in the Proposition is an immediate consequence of (9.24). ■

References

[1] Alchian, Armen and Harold Demsetz (1972): "Production, Information Costs, and Economic Organization." American Economic Review, vol. 62, pp. 777 - 795.

[2] Anderlini, Luca and Leonardo Felli (1994): "Incomplete Written Contracts: Undescribable States of Nature." Quarterly Journal of Economics, vol. 109, pp. 1085-1124.

[3] Anderlini, Luca and Leonardo Felli (1996): "Costly Contingent Contracts." mimeo, Cambridge University and London School of Economics

[4] Bebchuck, Lucien Arye (1984), Litigation and Settlement under Imperfect Information, Rand Journal of Economics, vol. 15, pp. 404-415.

[5] Bhattacharyya, Sugato and Francine Lafontaine (1995): "Double-Sided Moral Hazard and The Nature of Share Contracts." Rand Journal of Economics, vol. 26, No. 4, pp. 761 -781.

[6] Che, Y. K and Tai-Yeong Chung (1996): "Contract Damages and Co-operative Investments." working paper, The University of Western Ontario.

[7] Cooter, Robert and Thomas Ulen (1988): Law and Economics. Harper Collins Publishers.

[8] Cooter, Robert and Daniel Rubinfeld (1989): "Economic Analysis of Legal Disputes and Their Resolution." Journal of Economic Literature, vol. 27(3), pp. 1067-1097.

[9] Diamond, Douglas (1984): "Financial Intermediation and Delegated Monitoring." Review of Economic Studies, vol.51, pp. 393-414.

[10] Edlin, Aaron S. and Stefan Reichelstein (1996): "Holdups, Standard Breach Remedies and Optimal Investment." American Economic Review, pp. 478-501.

[11] Gale, Douglas and Martin Hellwig (1985): "Incentive Compatible Debt Contracts: The One-Period Problem." Review of Economic Studies, vol. 52, pp. 647 - 663.

[12] Grossman, Sanford and Oliver D. Hart (1986): "The Costs and Bene…ts of Ownership: A Theory of Vertical and Lateral Integration." Journal of Political Economy, vol. 94, pp. 691 - 719.

[13] Hart, Oliver D. (1987): Entry "Incomplete Contracts" in New Palgrave, p. 753.

[14] Hart, Oliver D. and John H. Moore (1988): "Incomplete Contracts and Renegotiation." Econometrica, vol. 56, pp. 755 - 786.

[15] Hart, Oliver D. (1995): Firms, Contracts, and Financial Structure. Clarendon Lectures in Economics, Oxford University Press.

[16] Holmström, Bengt (1982): "Moral Hazard in Teams." Bell Journal of Economics, vol.13, pp. 324-340.

[17] Jehiel, Philippe and Benny Moldovanu (1994): "Cyclical Delay in Bargaining." Review of Economic Studies, vol. 62, pp. 619-638.

[18] Jewitt, Ian (1988): "Justifying the First-Order Approach to Principal-Agent Problems." Econometrica, vol. 56, pp. 1177-1190.

[19] Kennan, John and Robert Wilson (1993): "Bargaining with Private Information." Journal of Economic Literature, vol. 31(1), pp. 45-104.

[20] Legros, Patrick and Hitoshi Matsushima (1991): "E¢ciency in Partnerships." Journal of Economic Theory, vol. 55, pp. 296-322.

[21] Maskin, Eric and Jean Tirole (1997): "Unforeseen Contingencies, Property Rights, and Incomplete Contracts." Harvard University Institute of Economic Research DP No. 1796.

[22] MacLeod, W. Bentley (1996): "Complexity, Contract and the Employment Relationship." mimeo, Boston College.

[23] Masten, Scott E. (1993): "A Legal Basis for the Firm, in The Nature of the Firm." ed. by O. E. Williamson and S. G. Winter, Oxford University Press.

[24] Mirrlees, James (1979): "The Implications of Moral Hazard for Optimal Insurance." mimeo, Seminar given at the Conference held in honor of Karl Borch, Bergen, Norway.

[25] P'ng, I.P.L. (1983): "Strategic Behavior in Suit, Settlement, and Trial." Bell Journal of Economics, vol.14, pp. 539-550.

[26] Rogerson, William P. (1985): "The First-Order Approach to Principal-Agent Problems." Econometrica, vol.53, pp. 1357-1367.

[27] Schweizer, Urs (1989): "Litigation and Settlement under Two-Sided Incomplete Information." Review of Economic Studies, vol.56, pp. 163-178.

[28] Segal, Ilya (1995): "Complexity and Renegotiation: A Foundation of Incomplete Contracts." mimeo, Harvard

[29] Shavell, Steven (1984): "The Design of Contracts and Remedies for Breach." Quarterly Journal of Economics, vol. 99(1), pp. 121-148.

[30] Sinclair-Desgagné, Bernard (1994): "The First-Order Approach to Multi-Signal Principal-Agent Problems." Econometrica, vol. 62, pp. 459-465.

[31] Spier, Kathryn E. (1992): "Pretrial Bargaining and the Design of Fee-Shifting Rules." Rand Journal of Economics, vol. 25(2), pp. 197-214.

[32] Townsend, Robert (1979): "Optimal Contracts and Competitive Markets with Costly State Veri…cation." Journal of Economic Theory, vol. 21, pp. 265-293.

[33] Tirole, Jean (1994): "Incomplete Contracts: Where do we Stand ?", mimeo, IDEI.

[34] Williams, Steven R. and Roy Radner (1988): "E¢ciency in Partnerships when the Joint Output is Uncertain." mimeo, Northwestern University.