A two-supplier inventory model

Fred Janssen*
Ton de Kok†

October 3, 1996

Abstract

In this paper we consider an inventory system with two suppliers. A supply agreement is made with one of the suppliers, to deliver a fixed quantity Q every review period. The replenishment decisions for the other supplier are governed by a (R, S) replenishment policy; that is, when the inventory position at a review period is below the order-up-to level S, an order is placed at the second supplier such that the inventory position is raised up to S. In this paper an algorithm is developed for the determination of the decision parameters S and Q such that the total relevant costs are minimized, subject to a service level constraint; these costs are defined as the sum of the holding, purchasing, and ordering costs. Based on the numerical results, conclusions follow about the division of the purchase volume among the two suppliers.

*Co-operation center Tilburg and Eindhoven Universities, Tilburg University, P.O. Box 90153, B929, 5000 LE Tilburg, The Netherlands, E-mail: F.B.S.L.P.Janssen@kub.nl

†Department of Technology Management, Eindhoven University of Technology, P.O. Box 513, Paviljoen F4, 5600 MB Eindhoven, The Netherlands

1 Introduction

When setting up an inventory policy, first of all it has to be decided whether to source all of the replenishments from one supplier, or to divide the orders among two or more sources. Both single sourcing and multiple sourcing have advantages and disadvantages (see, for example, Fearon [4]). The selection of suppliers heavily depends on the purchase price (in addition to possible discounts) and the lead time characteristics of a supplier. Adapting to discounts or other supply agreements often implies that the timing and sizes of future replenishment orders are more or less predetermined. Hence the choice of adapting to discounts is a trade off between purchase price and ordering flexibility. Often a choice is made for either a flexible but expensive supplier or a rigid but cheap supplier. Yet, it may be profitable to use two suppliers as follows. A rigid supplier is used to obtain discounts or a low purchase price for the majority of the purchase volume, and a flexible supplier is used to react to short term changes in demand. For example, the largest share of the purchase volume is purchased at a manufacturer, and the remaining part at a distributor or wholesaler.

In this paper we consider such a multiple sourcing purchasing strategy. General supply agreements are made with the main supplier to deliver a fixed quantity Q, every review period. It is assumed that the lead time is deterministic. At review epochs the inventory position (defined as the stock on hand plus outstanding orders minus backorders) is evaluated. When the inventory position is below the order-up-to level S, an order is placed at the second supplier, such that the inventory position is raised to the order-up-to level. Also the replenishment order from the second supplier will arrive after a deterministic lead time.

Notice that this multiple sourcing strategy is a combination of a push system (the main supplier delivers every review period a predetermined quantity) and a pull system (the replenishment orders placed at the second supplier are governed by an (R, S) replenishment policy). When using more than one source, one must decide how to divide the purchase volume. In this paper we develop an algorithm for the determination of the decision parameters, that is, the order-up-to level S and the fixed order quantity Q ordered from the main supplier, such that the total relevant costs (the sum of the holding, purchasing, and ordering costs) are minimized, subject to a service level constraint. Note that Q determines the partitioning of the purchase volume.

In the literature much attention is paid to multiple sourcing models (also known as order splitting); see, for example, Sculli and Wu [8], Hong and Hayya [5], and Lau and Zhao [6]. The main idea of order splitting is the reduction of lead time uncertainties by splitting the replenishment orders over more than one supplier, each time a replenishment is initiated. Hence, the order splitting strategy differs from the two-supplier strategy as defined above, in the sence that in order splitting each supplier is used every time a replenishment is placed, whereas in the two-supplier strategy the second supplier is used only when necessary.

The rest of this paper is organized as follows. In section 2 the two-supplier model is defined in more detail, and a method is presented to compute the optimal decision parameters. In section 3 the proposed method is verified by a number of simulation experiments. Furthermore, for a number of situations the optimal values for the decision parameters are computed by the algorithm; moreover, the shape of the total relevant cost function is analysed. In section 4 conclusions are presented, and lines of future research are indicated.

2 The two supplier model

We address an inventory replenishment strategy which is a combination of a pull and a push system. The main supplier, denoted as supplier 1, will deliver each review period a fixed quantity of size Q. We assume that the lead time is deterministic. Hence, the interarrival times of the replenishment orders are equal to the length of the review period. Furthermore, each review period the inventory position is monitored, in order to make a replenishment decision for the second supplier. When the inventory position, say x, is below the order-up-to level, denoted by S, an order of size $S - x$ is placed at supplier 2. The lead times of replenishment orders from supplier 2 are deterministic and equal to L. Note that the actual lead time of the first supplier is not relevant for the reordering decision concerning the second supplier. The reason for this is that the ordering decisions for supplier 2 are based on the inventory position and hence only the moments at which this inventory position is changed are relevant. Therefore we can, without loss of generality, choose the length of the lead time of supplier 1 also equal to L, implying that the arrivals of replenishment orders from the two suppliers coincide in time. In summary, at each review epoch first the inventory position is raised with size Q (because a replenishment order at supplier 1 is booked, which will arrive L periods later), and secondly the inventory position is compared with the order-up-to level S in order to make a replenishment decision for supplier 2.

The time axis is divided into time units (e.g. days), and the demands per time unit are independent and identically distributed (i.i.d.) random variables. Since the demand process is a discrete time process, we assume that L is an integral number of time units. Furthermore, it is assumed that customer orders are handled at the end of a day just before the replenishment orders are handled.

Customer orders which cannot be delivered directly from stock will be backordered. As performance criterion the P_2-service measure is used (see Silver and Peterson [9]), which is defined as the long-run fraction of demand delivered directly from shelf, denoted by $\beta(S,Q)$.

In determining the total average relevant costs per review period (TRC), we distinguish between ordering, purchase, and holding cost. The ordering costs are proportional to the number of replenishment orders, but independent of the size of a replenishment order. The purchase costs are proportional to the size of the replenishment order. Both the ordering costs and the purchase costs may depend on the supplier. The holding costs are proportional to the size of the physical stock level. However, in spite of the difference in purchase costs of the products, all the items in stock have the same unit value (for example the market value). Notice that in case one would like to differentiate between holding costs for products of different suppliers, a specification of the customers delivery rule is required (for example first deliver products with the largest purchase price). In order to derive an expresion for TRC, we define:

Variables describing the underlying stochastic processes:

D_n	demand during the n-th period;
$D(t_1, t_2)$	the total demand during the interval $(t_1, t_2]$;
L	the deterministic lead time of both suppliers;

Cost parameters:

m_i	the purchase costs per unit at supplier i, $(i \in \{1, 2\})$;
K_i	the ordering costs per order at supplier i, $(i \in \{1, 2\})$;
h	the holding costs per unit per period;
r	the opportunity factor (\$/\$/year);

Decision variables:

R	the length of the review period;
$\beta(S, Q)$	the customers service level;
Q	the fixed order quantity delivered each review period by supplier 1;
S	the order-up-to level for supplier 2;

Performance measures:

$\Pi(S, Q)$	the probability that an order is placed at supplier 2 during an arbitrary review period;
β_{target}	the target service level;
$\psi(S, Q)$	the average expected physical stock level at the beginning of an arbitrary time period;
X_n	the inventory position at the n-th review period immediately after a replenishment order at supplier 2 is placed, if any;
W_n	$= X_n - S$.

The total relevant cost during a review period R can now be written as

$$TRC(S, Q) \;=\; h\psi(S, Q) + m_1 Q + m_2 \mathbb{E}(D(0, R) - Q)^+ + K_1 I_{Q>0} + \Pi(S, Q) K_2 \quad (1)$$

where I_B denotes the indicator function of set B, and $x^+ = \max\{0, x\}$.
Note that $TRC(S, Q)$ has a discontinuity at $Q=0$, which corresponds to the single source situation in which only supplier 2 is used.

The problem can now be formulated as

$$\min\{TRC(S, Q) \;\; | \;\; \beta(S, Q) \geq \beta_{target},\, 0 \leq Q \leq \mathbb{E}D(0, R),\, S \geq 0\}. \quad (2)$$

Clearly, in order to solve (2) we need expressions for $\beta(S, Q)$, $\psi(S, Q)$, and $\Pi(S, Q)$. In the two supplier model it is possible that the inventory position at a review period is larger than the order-up-to level S. To find an expression for $\beta(S, Q)$ we use the renewal reward theorem, which justifies the focus on a single tagged replenishment cycle (see, for example Tijms [10]). In Figure 1 we assume that 0 is an arbitrary review moment in time. Furthermore, we consider the first complete replenishment cycle after 0, where a replenishment cycle is defined as the time interval between two successive arrivals of replenishment orders of supplier 1. In Figure 1 we have chosen L such that $2R \leq L < 3R$. Hence, just before a replenishment decision is made at epoch 0 there are two outstanding orders at supplier 1 (each of size Q), and there are at most two outstanding orders at supplier 2 (denoted by Q_0 and Q_1). At the review moment 0 the inventory position is raised with a size of Q due to replenishments of supplier 1, and since the inventory position still is below S, a replenishment order of size Q_2 is placed at supplier 2. Let I_1 be the net stock (defined as the physical stock minus backorders) at the beginning of the tagged replenishment cycle (time period L), and I_2 the net stock at the end of the tagged replenishment cycle (time period $R+L$). By tracing the sample path of the inventory position from 0 to $R+L$ neglecting all the replenishments that are made after time epoch 0 (denoted by the tagged inventory position) there is a clear relation between the inventory position just after time epoch 0 and the net stock at the beginning and end of the tagged replenishment cycle. Using the fact that the expected backlog at the beginning and at the end of the replenishment cycle are equal to $\mathbb{E}(-I_1)^+$ and $\mathbb{E}(-I_2)^+$ respectively, we can derive the following expression for the service level

$$1 - \beta(S,Q) = \frac{\mathbb{E}(-I_2)^+ - \mathbb{E}(-I_1)^+}{\mathbb{E}(I_1 - I_2)}. \tag{3}$$

In order to derive expressions for $\mathbb{E}(-I_1)^+$ and $\mathbb{E}(-I_2)^+$ we have to distinguish between the situation $X_0 = S$ and $X_0 > S$. Then it is easy to see that

$$\begin{aligned} \mathbb{E}(-I_1)^+ &= \Pi(S,Q)\mathbb{E}((D(0,L) - X_0)^+ | X_0 = S) \\ &\quad + \big(1 - \Pi(S,Q)\big)\mathbb{E}((D(0,L) - X_0)^+ | X_0 > S) \end{aligned} \tag{4}$$

and

$$\begin{aligned} \mathbb{E}(-I_2)^+ &= \Pi(S,Q)\mathbb{E}((D(0,R+L) - X_0)^+ | X_0 = S) \\ &\quad + \big(1 - \Pi(S,Q)\big)\mathbb{E}((D(0,R+L) - X_0)^+ | X_0 > S) \end{aligned} \tag{5}$$

For $\mathbb{E}(I_1 - I_2)$, we find

$$\mathbb{E}(I_1 - I_2) = \mathbb{E}D(L, R+L) \tag{6}$$

By substituting (4-6) into (3), we find

$$\begin{aligned} 1 - \beta(S,Q) &= \Pi(S,Q)\frac{\mathbb{E}(D(0,R+L) - S)^+ - \mathbb{E}(D(0,L) - S)^+}{\mathbb{E}D(L,R+L)} \\ &\quad + \big(1 - \Pi(S,Q)\big)\frac{\mathbb{E}((D(0,R+L) - X)^+ | X > S) - \mathbb{E}((D(0,L) - X)^+ | X > S)}{\mathbb{E}D(L,R+L)}. \end{aligned} \tag{7}$$

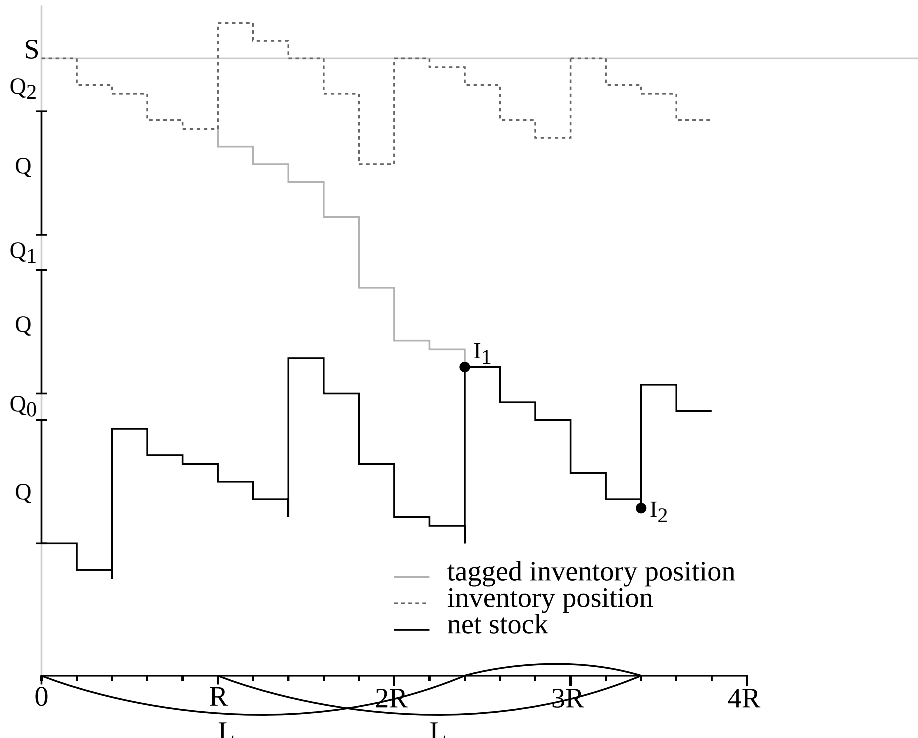

Figure 1: Evolution of the net stock and inventory position during a tagged replenishment cycle

A widely used method to approximate expressions of the form $\mathbb{E}(A-B)^+$, is to approximate the distribution functions of A and B by that of mixture of Erlang distribution. In that case a closed form expression for $\mathbb{E}(A-B)^+$ exists, (see, for example, Tijms and Groenevelt [10]). In order to fit a mixture of Erlang distribution function only the first two moments of the associated stochastic random variable are required. Using this technique (7) can be computed when (besides an expression for $\Pi(S,Q)$) expressions for $\mathbb{E}D(0,L)$, $\mathbb{E}D(0,L)^2$, $\mathbb{E}D(0,R+L)$, $\mathbb{E}D(0,R+L)^2$, $\mathbb{E}(X|X>S)$, and $\mathbb{E}(X^2|X>S)$ are available.

The first two moments of $D(0,L)$ and $D(0,R+L)$ can be obtained by using well-known results for the first two moments of a the sum of a fixed number of i.i.d. random variables:

$$\mathbb{E}D(0,L) = L\mathbb{E}D \tag{8}$$
$$\mathbb{E}D(0,L)^2 = L\sigma_D^2 + (L\,\mathbb{E}D)^2. \tag{9}$$

Analoguously expressions for the first two moments of $D(0,R+L)$ are obtained.

Consider the inventory positions at successive review epochs (immediately after a replenishment order at supplier 2 is placed, if any). Then it is easy to see that the following relation holds:

$$X_{n+1} = \max\{S, X_n + Q - D(nR,(n+1)R)\}. \tag{10}$$

Then using the relation between W_n and X_n gives

$$W_{n+1} = \max\{0, W_n + Q - D(nR,(n+1)R)\}. \tag{11}$$

Relation (11) is equivalent to the relation for the waiting times of two successive customers in a $GI|D|1$ queue with the distribution of the interarrival times equal to $F_{D(0,R)}(.)$ and with deterministic service of length Q.

Chaudry [1] gives an extensive overview of the available literature concerning the waiting times in a $GI|D|1$ queue. Most methods in literature require finding the roots of an equation (for example, in Chaudry [1] the roots of equation $\tilde{A}(s)e^{-s/\mu} = 1$ are required, where $\tilde{A}(s)$ is the Laplace transform of the interarrival times of customers). Although this method is exact, we do not use this approach. The reason for this is that the method is relatively hard to implement. We use the approximate, however straightforward to implement, moment-iteration method for the waiting times in the $GI|G|1$ queue (see De Kok [2]); this method computes values for $\mathbb{P}(W > 0)$, $\mathbb{E}(W|W > 0)$, and $\mathbb{E}(W^2|W > 0)$. Note that $\mathbb{P}(W > 0)$ is independent of S (see relation (11)). Using $X_n = W_n + S$ we get the following relations

$$\begin{aligned} \Pi(S,Q) &= 1 - \mathbb{P}(W > 0); && (12) \\ \mathbb{E}(X|X > S) &= \mathbb{E}(W|W > 0) + S; && (13) \\ \mathbb{E}(X^2|X > S) &= \mathbb{E}(W^2|W > 0) + 2S\mathbb{E}(W|W > 0) + S^2. && (14) \end{aligned}$$

To obtain an expression for $\psi(S,Q)$ we again consider a tagged replenishment cycle. Notice that lead times do not cross in time because they are deterministic. Therefore, all the outstanding orders, at the ordering epoch of the associated replenishment order of the tagged replenishment cycle, have arrived at the beginning of the tagged replenishment cycle. Hence the net stock (defined as the physical stock minus backlog) at the beginning of time epochs during the tagged replenishment cycle, $t \in \{L, R+L-1\}$, equals $X - D(0,t)$. Then using again the renewal reward theorem, it is easy to see that the expected average physical stock is given by

$$\begin{aligned} \psi(S,Q) &= \frac{1}{R}\sum_{t=L}^{R+L-1} \mathbb{E}(X - D(0,t))^+ \\ &= \frac{1}{R}\sum_{t=L}^{R+L-1} \Pi(S,Q)\mathbb{E}(S - D(0,t))^+ + (1 - \Pi(S,Q))\mathbb{E}(X - D(0,t)|X > S)^+. \quad (15) \end{aligned}$$

For an extensive exposition of the expected average physical stock in a (R,S) inventory model see, for example, De Kok [3].

Now the TRC can be calculated for given values of S and Q. Note that for any given Q, the minimal value for S can be determined by solving $\beta(S,Q) = \beta_{target}$. Let $S^*(Q, \beta_{target})$ denote the optimal value of S as function of Q and β_{target}. Then (2) can be reformulated into a one-dimensional optimization problem, namely

$$\min\{TRC(S^*(Q,\beta_{target}), Q) \quad | \quad 0 \le Q \le \mathbb{E}D(0,R)\}. \quad (16)$$

To solve (16) we used a local search procedure on the interval $(0, \mathbb{E}D(0,R)]$ (see, for example Press et al. [7]), and compared the solution with the single source situation: $TRC(S^*(0,\beta_{target}), 0)$. In our numerical investigations we did not find a counter example for the statement that the total relevant cost function is convex for $Q \in (0, \mathbb{E}D(0,R)]$.

3 Numerical experiments

In order to verify the algorithm which is developed in the previous section, and to investigate the shape of the total relevant cost function and the optimal values of the decision varaibles, we carried out three experiments. In experiment 1 we compared the target service level (β_{target}) with the realised service level based on the optimal value of S for given Q, where the realised service level is computed by discrete event simulation. Furthermore, in experiment 2 and 3 the total relevant costs and the optimal values for the decision variables are calculated for various values for the system and cost parameters.

In the three experiments that follow we take one week as the basic time unit, and one year equal to 48 weeks. The starting values of the system and cost parameters for each of the three experiments are given in Table 1. In the first experiment the algorithm as

Table 1: basic setting parameters for the experiments

	R (weeks)	$\mathbb{E}D$ (units/week)	c_D	L (weeks)	β_{target}	m_1 ($/unit)	m_2	K_1 ($/repl.)	K_2	r ($/$/year)
exp. 1	4	5	1	5	0.95	-	-	-	-	-
exp. 2	4	5	1	5	0.95	90	100	100	100	0.20
exp. 3	1	5	2	1	0.99	98	100	25	25	0.20

proposed is verified by simulation. We simulated the system during 500.000 time units. In Figure 2 we varied β_{target} as 0.90, 0.91, ..., 0.99, 0.995, 0.999, and fixed Q equal to 15; then the order-up-to level $S^*(15, \beta_{target})$ is computed, and the actual service β_{sim} associated with the computed order-up-to level $S^*(15, \beta_{target})$ is obtained by simulation. In Figure 3 we again varied β_{target} as 0.90, 0.91, ..., 0.990, 0.995, 0.999, and fixed Q equal to 15, however now we compared the expected average physical stock level computed by the algorithm (see formula (15)) with the average physical stock level computed by simulation. From both the experiments it is clear that the algorithm performs very well.

In the second experiment we computed $TRC(S^*(Q, \beta_{target}), Q)$, where Q is varied between 0 and $\mathbb{E}D(0, R)$. We define ρ as the fraction of the purchase volume delivered by supplier 1, that is $\rho := \frac{Q}{\mathbb{E}D(0,R)}$. In Figure 4 we considered $TRC(S^*(Q, \beta_{target}), Q)$ and varied r between 0.04, 0.20 and 0.40 ($/$/year). First of all we notice that the TRC is very high for ρ almost equal to one. In this situation the load of the associated queueing system is near to one, which means long waiting lines. Therefore, the expected inventory position at the beginning of a cycle is very high, which implies high average stocks. This is also intuitively clear, since for Q almost equal to $\mathbb{E}D(0, R)$ there is no ordering flexibility, which means that in periods when demand is low unnecessary supplies are pushed into the system. Furthermore, we considered situations in which the opportunity factor is extremely low ($r = 0.04$ $/$/year) and extremely high ($r = 0.40$ $/$/year). Under this variation the optimal ρ does not vary much. This means that independent of the opportunity factor a large share of the replenishments should be sourced by supplier 1.

Finally, these experiments indicate that the optimal value of ρ is rather high. The

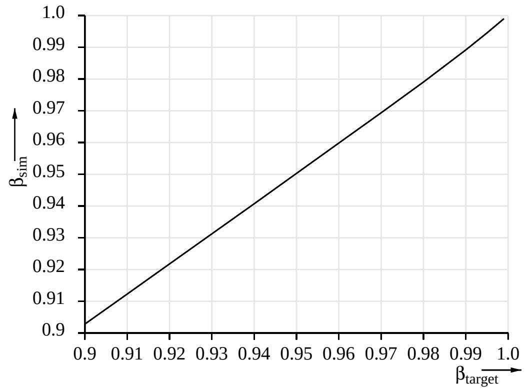

Figure 2: Actual service obtained by simulation

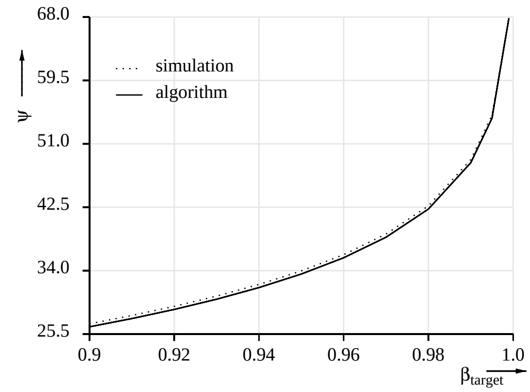

Figure 3: Average physical stock levels

reason for this is the large difference between m_1 and m_2, which results in a purchase cost difference of \$ 2400 per year; hence the expected average physical stock has to differ at least with 120 units, which is half the yearly demand.

Important parameters that determine the profitability of using two suppliers instead of one supplier are the fixed costs K_1 and K_2. In Figure 5 we computed the optimal value of ρ where we varied m_1. Furthermore we varied the fixed costs K_1 and K_2 as follows $\{K_1 = 200, K_2 = 100\}$, $\{K_1 = 0, K_2 = 0\}$, $\{K_1 = 100\,, K_2 = 200\}$ and $\{K_1 = 0\,, K_2 = 100\}$. Note that for $\{K_1 = 0, K_2 = 0\}$ the optimal value for ρ can take all values between 0 and 1, however only for small differences between m_1 and m_2 the optimal value for ρ is smaller than 0.8. When $K_1 > K_2$ the optimal value ρ drops to 0 before m_1 is equal to m_2, which means that only supplier 2 is used. Moreover, it is worthwhile to notice that when $K_1 < K_2$ it is even profitable to source products from supplier 1 even when $m_2 > m_1$. Hence, the increasing purchase cost is compensated by a decreasing ordering cost.

In the third experiment we considered a basic setting in which the lead time of supplier 2 is small, the variance of the customers demand sizes are high, and the differences between m_1 and m_2 are small. In Figure 6 we again considered $TRC(S^*(Q, \beta_{target}), Q)$ and varied the coefficient of variation of the demand size (c_D) between 0.25, 1.00, and 2.00. The optimal value of ρ for $c_D = 0.25, 1.0$ and 2.0 is 0.980, 0.845 and 0.655 respectively. This indicates that the coefficient of variation of the demand size has a major impact on the value of the optimal ρ. For small values of c_D the optimal ρ is almost equal to one, and for large values of c_D the optimal value of ρ is relative small. This is to be expected, since one would use the flexible supplier whenever the demand process is erratic.

In Figure 7 we computed the optimal value of ρ where we varied m_1. Furthermore we varied the coefficient of variation of the demand size (c_D) between 0.25, 1.00 and 2.00. Again, we can conclude that the c_D has a major impact on the optimal value of ρ.

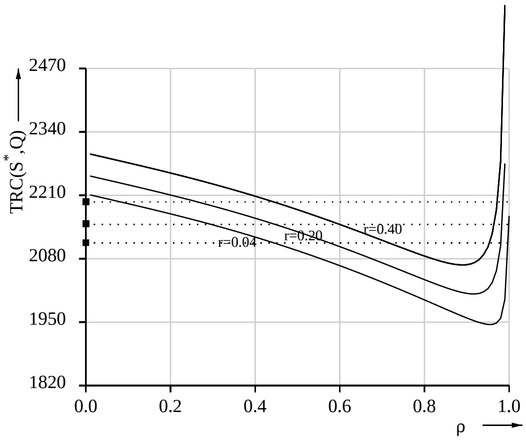

Figure 4: The total relevant cost as function of ρ for experiment 2

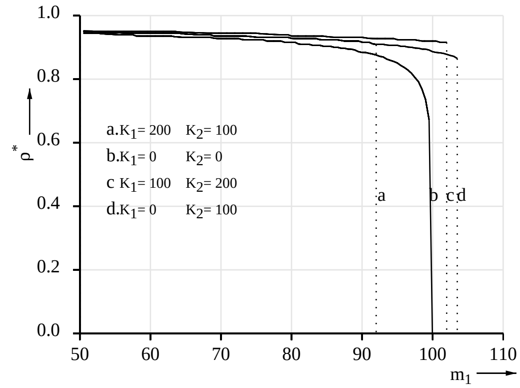

Figure 5: The optimal value of ρ as function of m_1 for experiment 2

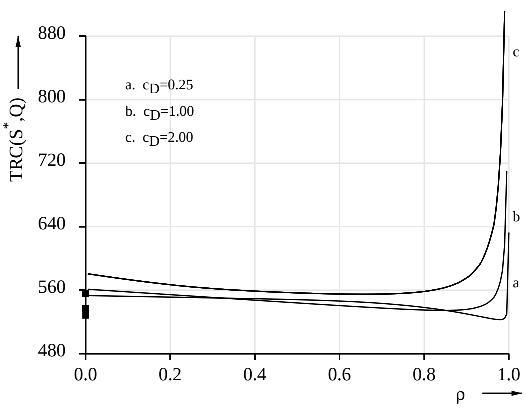

Figure 6: The total relevant cost as function of ρ for experiment 3

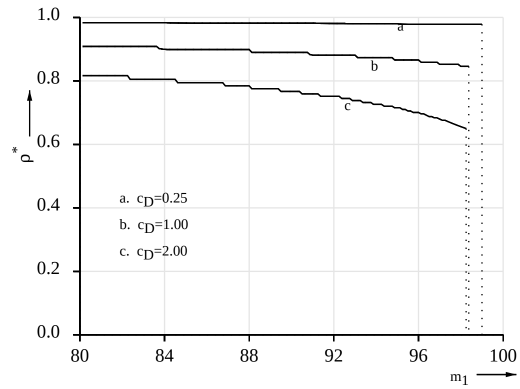

Figure 7: The optimal value of ρ as function of m_1 for experiment 3

4 Conclusions and future research

In this paper we considered an inventory policy with two suppliers. A general supply agreement is made with one of the suppliers to deliver a fixed quantity Q every review period, whereas the replenishment decisions for the other supplier are governed by the (R, S) replenishment policy. Hence, when the inventory position at a review period is below the order-up-to level, S, an order is placed at supplier 2, such that the inventory position is raised to S. An algorithm is derived for the determination of the decision parameters S and Q for which the total relevant costs are minimized subject to a service level constraint.

Through comparisons with simulation rusult, the algorithm developed in this article appeared to perform excellent for all the experiments we considered. Furthermore, the numerical results showed the effectiveness and profitability of the multiple sourcing strategy above the single sourcing strategy. It is clear that the profitability depends on the ratios between the ordering costs and purchase costs. However, the coefficient of variation of the demand turns out to be a determining factor for the optimal value of Q. Only for large values of c_D it is profitable to purchase a large share of the purchase volume from the flexible and expensive supplier (supplier 2).

Although no numerical counter examples where found for the conjecture that the total relevant cost function is not convex in Q (neglecting the discontinuity at zero), a rigorous proof is needed to justify that $TRC(S^*(Q, \beta_{target}), Q)$ is indeed convex.

Several extensions are worthwhile to be considered. The generalization to stochastic lead times (with the non-overtaking restriction) will lead to complex expressions for the service equation. When the replenishment orders of supplier 2 are not restricted to arrive within a certain review period, the number of replenishments of supplier 1 within a replenishment cycle will not be fixed, which even complicates the determination of the service equation more.

Another important extension is to allow a stochastic replenishment quantity for supplier 1. The analysis is straightforward, when using the approximate methods, presented in this paper, for obtaining values for the inventory position at review epochs.

Furthermore, other inventory control systems could be used for supplier 2. However, the key step will be the determination of the inventory position at the possible replenishment epochs for the second supplier. For periodic review strategies this will again be possible using the approach presented in this paper; for continuous review replenishment policies the way to go is less clear.

References

[1] M. L. CHAUDRY, *Computing stationary queueing-time distributions of $GI/D/1$ and $GI/D/c$ queues*, Naval Res. Log., 39 (1992), 975–996.

[2] A. G. DE KOK, *A moment-iteration method for approximating the waiting-time characteristics of the $GI/G/1$ queue*, Prob. Engineer. Inform. Sci., 3 (1989), 273–287.

[3] A. G. DE KOK, *Basics of inventory management: part 2 The (R,S)-model*, Research memorandum, FEW 521, Department of Economics, Tilburg University, Tilburg, The Netherlands, 1991.

[4] H. E. FEARON, M. R. LEENDERS, *Purchasing and materials management*, Irwin, Homewood Ill., 1993.

[5] J. HONG, J. C. HAYYA, *Just-in-time purchasing: Single or multiple sourcing?*, Int. J. of Prod. Econ., 27 (1992), 175–181.

[6] H. LAU, L. ZHAO, *Optimal ordering policies with two suppliers when lead times and demands are all stochastic*, Eur. Jour. of Opl. Res., 68 (1993), 120–133.

[7] W. H. PRESS, B. P. FLANNERY, S. A. TEUKOLSKY AND W. T. VETTERLING, *Numerical Recipes in C*, Cambridge, England, 1992.

[8] D. SCULLI, S.Y. WU, *Stock control with two suppliers and normal lead times*, J. Opl. Res. Soc., 32 (1981), 1003–1009.

[9] E. A. SILVER, R. PETERSON, *Decision Systems for Inventory Management and Production Planning*, Wiley, New York, 1985.

[10] H. C. TIJMS, *Stochastic modelling and analysis: a computational approach*, John Wiley & Sons, Chichester, 1994.

[11] H. C. TIJMS, H. GROENEVELT, *Simple approximations for the reorder point in periodic and continuous review (s, S) inventory systems with service level constraints*, Eur. J. Opnl. Res. 17 (1986), 175–190.