# THE ADVANTAGE OF HIDING BOTH HANDS:
# FOREIGN EXCHANGE INTERVENTION, AMBIGUITY AND
# PRIVATE INFORMATION

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**Abstract**: This paper analyzes a dynamic exchange rate policy game in which the central bank has private information about its short-term exchange rate target, on the one hand, and in which the market is faced with a certain degree of ambiguity concerning the actual intervention volume, on the other. Sterilized interventions are shown to derive their effectiveness from the fact that they transmit information about the short-term exchange rate target to the market. In this respect, we provide an explanation for the presumed inconsistency between intervention secrecy and the effectiveness of the signalling channel since our model predicts that interventions will be more effective on average if the central bank retains some degree of ambiguity. Moreover, it is also shown that sterilized interventions will not exert a lasting effect on the exchange rate. Finally, we have also investigated the extent to which some political and economic parameters determine the size of the intervention bias.

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# 1. Introduction

The demise of the Bretton Woods system has produced a wide variety of research in the field of exchange rate management. One of the important questions academics have tried to answer in this respect is whether or not the central bank will be able to pursue an *independent* exchange rate target when at the same time it also uses the instruments of monetary policy to keep inflation under control. For many large industrial countries it is a reasonable approximation to assume that, should a conflict between these two objectives arise, the central bank will always give priority to the latter. Consequently, the central bank will need to have at least one additional *effective* instrument at its disposal if it is to be capable of pursuing some exchange rate objective. Since the short-term domestic interest rate cannot be used to influence the external value of the currency authorities have frequently used *sterilized* foreign exchange interventions for the latter purpose.

As documented by Almekinders and Eijffinger (1991) and Edison (1993) these sterilized interventions may derive their effectiveness from two sources. First of all, provided otherwise identical domestic and foreign assets are imperfect substitutes, the exchange rate may be affected via the *portfolio balance channel*. It is not likely, however, that central banks can induce a significant imbalance in investors' portfolios since the amount of official reserves is dwarfed by the daily turnover in the foreign exchange markets[1]. Hence, if the central bank is to have any hope of pursuing an independent exchange rate target it will have to rely exclusively on the *signalling* or *expectations channel*. The idea behind this is that sterilized interventions can have a direct impact on exchange rate expectations if they transmit hitherto privately held information to the market.

One approach to study the attempts on the part of the central bank to exploit the possible effectiveness of this channel is to model intervention policy as a game between speculators, on the one hand, and the central bank, on the other. In this respect, Almekinders (1994, 1995) has developed a static exchange rate policy game of symmetric information in which the central bank's attempts to exploit the signalling channel will always be futile. The basic reason for this is that the central bank has no private information because of which interventions will not provide the market with information it did not have beforehand. A certain degree of *policy*

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[1] It should be mentioned that the evidence on this presumed ineffectiveness is not clear cut. Dominguez and Frankel (1993b) have provided new evidence on the statistical significance of the portfolio balance effect. Nevertheless, because this paper focusses on the signalling channel we will assume the portfolio balance channel to be completely ineffective.

*secrecy* thus seems to be crucial in rendering effectiveness to sterilized interventions. This observation is central to the model constructed by Bhattacharya and Weller (1992) who interpret this policy secrecy as private information about the central bank's (short-term) exchange rate target. In their static model speculators do not extract information from the intervention volume as such. Rather, they will use the current spot rate, which is under full control of the central bank, to revise their prior information about the target.

The aim of this paper is obtain a better understanding of observed intervention behaviour by extending the theoretical insights outlined above. First of all, we feel that policy secrecy in the context of foreign exchange interventions actually consists of two components. In addition to the afore-mentioned asymmetric information concerning the central bank's short-run exchange rate target, we will also assume that the market is faced with a certain degree of *ambiguity* about the actual intervention volume. It is a well-known fact that, apart from the exchange rate target, central banks also prefer to keep their intervention data secret[2]. On the other hand, central banks' foreign exchange dealings rarely go unnoticed. Therefore, the market's *perception* of the intervention volume (and not the actual volume as such) will play an explicit role in our model.

Secondly, to fully capture the effects of these two components of policy secrecy[3] we will present a *dynamic* intervention model by assuming that the stage game under consideration will be repeated an infinite number of times. Since foreign exchange interventions take place rather frequently we feel that repeated interaction is indispensable in this respect. Even more so because this setting will be shown to imply a *learning process* on the part of the speculators which will, in turn, induce the central bank to take future consequences of current intervention policy into account as well. Analogous to monetary policy games, this learning process will mitigate the familiar time-inconsistency problem. Incidentally, in doing so we also take up the challenge of Bhattacharya and Weller (1992, p.26) who conclude their paper as follows: '...*A third line of work would be to make the model dynamic. In our view, that would be the most fruitful extension, and need we mention it, the most difficult...*'

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[2] Nowadays, the Federal Reserve system makes the intervention data available to researchers on request with a delay of one year. However,the Bank of Japan still preserves full confidentiality regarding its intervention data.

[3] The combination of private information and ambiguity *in monetary policy making* was first introduced by Cukierman and Meltzer (1986).

Next, we will also show that there is no inconsistency between the effectiveness of thesignalling channel and the existence of ambiguity about the intervention volume[4]. On the contrary, we find that interventions will, on average, have a *larger* impact on the exchange rate if the central bank transmits a noisier report of the actual intervention volume to the market. This result is closely connected to the exchange rate model used, a basic feature of which is that the link between exchange rates and underlying fundamentals is very weak in the short run. The idea is that in the presence of a large degree of uncertainty concerning future fundamentals, it is rational for speculators to anchor their expectations to past exchange rate movements, on the one hand, and to the (often) imprecisely stated and revealed preferences of the central bank, on the other. In this respect it does not matter whether these (short-term) preferences are in line with the underlying fundamentals or not. After all, the fact that exchange rates tend to be fully determined by the fundamentals *in the long run* is not particularly relevant for the majority of traders since they are mainly concerned with *short-term* profits. In our model sterilized interventions will be shown to contain an ambiguous signal of the central bank's short-term exchange rate target. In other words, by the very act of intervening the central bank will transmit some of its hitherto privately held information concerning its own preferences to the markets. This will provide the market with a relevant anchor on which it can partly base short-term exchange rate expectations.

It turns out that our answer to the question why central banks frequently conduct sterilized interventions is a simple one. In the short run sterilized interventions may be partly successful in achieving the desired exchange rate target but the central bank can never be sure of this ex ante. On the other hand, *on average* (i.e. in the long run) sterilized interventions will be shown to have no effect on the exchange rate. This result should not be very surprising in view of the fact that exchange rates tend to be determined by the underlying fundamentals *on average*. The latter will, of course, remain unaffected if the intervention is completely sterilized.

Finally, we also uncover some of the *political* and *institutional* factors which determine the size of the intervention bias. This concept was first introduced by Almekinders (1994) and refers to that part of the total intervention volume which has no impact on the exchange rate and which should, therefore, be avoided. However, analogous to the well-known inflationary bias, any precommitment by the central bank to eliminate the intervention bias will be time-inconsistent.

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[4] While Bhattacharya and Weller (1992) provide an explanation for private information about the exchange rate target, they do not explicitly deal with the frequently addressed presumed inconsistency between secrecy about the intervention *volume* as such and the effectiveness of the signalling channel.

The remainder of this paper is organized as follows. Section 2 will outline the model and present the sequence of events in the form of a stage game. Section 3 will uncover the central bank's reaction function and will pay explicit attention to the effect of current interventions on future expectations, on the one hand, and the central bank's trade-off between present and future costs of undesired exchange rate levels, on the other. In Section 4 we will assess the effect of asymmetric information on the equilibrium intervention volume in general, and on the intervention bias in particular. Furthermore, we will provide an investigation of the political and institutional factors which determine the size of this bias. The shocks which determine the effectiveness of a given intervention operation will be examined in Section 5 where we will also provide a rationale for ambiguity. Finally, Section 6 will summarize our main conclusions.

## 2. The exchange rate policy game

In this section we will extend the static exchange rate policy game developed by Almekinders (1994,1995) in three ways. First of all, we will introduce *asymmetric information* by assuming that the central bank does not reveal its short-term exchange rate target to the market. Moreover, we will also introduce *ambiguity* by postulating that the current exchange rate is influenced by the reported intervention volume as perceived by speculators in stead of the actual intervention volume set by the central bank. Finally, we will extend the stage game thus obtained in a *dynamic setting* which will allow reputational forces to play a role.

The loss function of the central bank ($L_t^{CB}$) which expresses the trade-off between undesired exchange rate levels and intervention costs reads as follows:

$$L_t^{CB} = \frac{1}{2}( kINV_t )^2 + \frac{\varphi}{2}( s_t - T_t )^2 \tag{1}$$

The central bank will incur a loss whenever the (log of the) spot rate ($s_t$) differs from the current exchange rate target ($T_t$). The parameter $\varphi$ denotes the central bank's relative weight on exchange rate stabilization. Undesired exchange rate levels can be mitigated by means of sterilized interventions ($INV_t$)[5] which induce a cost of k per unit of foreign exchange traded.

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[5] A positive (negative) value of $INV_t$ denotes a purchase (sale) of foreign exchange by the central bank.

This cost can be explained by transaction costs and the fact that the central bank may incur a loss on its purchases (sales) of foreign exchange if these turn out to be unsuccessful in preventing the domestic currency from appreciating (depreciating).

The central bank's short-term exchange rate target ($T_t$) will be determined as follows:

$$T_t = A + p_t \quad \text{with} \quad p_t = \rho p_{t-1} + v_t \quad , \quad 0 < \rho < 1$$
$$\text{and} \quad v_t \sim N(0, \sigma_v^2) \tag{2}$$

The parameter A represents the expected long run fundamental exchange rate which is based on a rational expectation over the distribution of the future fundamentals. In this paper we will assume that the underlying fundamentals do not change over time which allows us to treat this parameter as given and known to all participants in the foreign exchange market[6]. Furthermore, in each period the short-term target will be subjected to a stochastic shock ($p_t$). To capture the notion that speculators will usually have some intuition about the position of the short-term target (i.e. whether the central bank pursues a target below or above the fundamental value A) we assume that $p_t$ follows an AR(1) process where the parameter $\rho$ measures the degree of *target persistence*. However, because the unconditional expectation of $p_t$ is equal to zero it follows that the central bank cannot *systematically* defy the underlying fundamentals.

Since foreign exchange interventions take place relatively frequently, it is vital to have a clear understanding of the process that drives the spot rate in the short run. Taking domestic and foreign interest rates as given, this boils down to understanding the way in which short-term exchange rate expectations are formed[7]. In this respect there are, at least, two important differences between the present free float and the Bretton-Woods era. First of all, both the degree of capital mobility and (as a consequence) the amount of speculative capital have increased dramatically. Consequently, the spot rate has become increasingly determined by speculators with a relatively short horizon which results from the fact that they are assessed on

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[6] This assumption can be justified on the grounds that fundamentals pertain to low frequency (i.e. yearly or quarterly) data from the perspective of this model in which the time span is very short (inter-and intradaily). At any rate, this assumption is not crucial because a change in the fundamentals simply implies a shift of the distribution of the future fundamentals. The important aspect for our purposes is that the mean of the distribution is commonly known at all times.

[7] For simplicity we will assume that otherwise completely identical domestic and foreign assets are perfect substitutes. This implies the absence of risk premia in the foreign exchange market as a result of which unconvered interest parity will hold at all times.

their short-term performance. Secondly, in the absence of an explicit commitment on the part of monetary authorities to a particular exchange rate, agents will have to look for some other device to anchor their expectations. Most exchange rate models assume these expectations to be fully determined by expected future fundamentals (i.e. the parameter A in our model).

However, based on the poor empirical performance of these models in the short run (see e.g. Meese and Rogoff (1983)), some authors (e.g. Goodhart (1988) and DeGrauwe (1994)) have questioned the ability of these theories to describe short-term exchange rate behavior. In stead, they argue that expectations which drive the spot rate are determined by the interplay between fundamentalist and non-fundamentalist analysis. The basic reason for this is that future fundamentals are very hard to predict in an increasingly interdependent world which lacks international policy coordination. Therefore, the rational exchange rate expectation based on the fundamentals (A) will not provide much information about the likely future course of the spot rate since the concomitant variance of the forecast error will be very large. As a result, speculators will start looking for other non-fundamental criteria on which to base their expectations. Of course, the past behavior of exchange rates then becomes an obvious anchor to resort to in this respect. This explains the widespread popularity and use of technical analysis in the foreign exchange markets (see e.g. Taylor and Allen (1992)). In addition, speculators' expectations have become highly sensitive to the (often) imprecisely stated and revealed preferences of the central bank. It is well-known that *announcements* or 'cheap talk' by central bankers may have a substantial effect on the spot rate (see Stein (1989)). Complementary to this, we should expect any *actions* of the central banker which implicitly convey information about (short-term) preferences to affect the spot rate as well.

According to De Grauwe (1994), the influence of fundamentalists will be particularly weak when the spot rate is relatively close to its expected fundamental value (since the current spot rate is then as good a guess of the unknown actual fundamental value than the fundamentally expected value itself). Consequently, the link between the spot rate and the underlying fundamentals will usually be very weak in the short run[8]. This theory is consistent with the notion that the bulk of foreign exchange trading is the result of short-term speculation. The mere fact that all traders know that the spot rate will be fully determined by underlying

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[8] In this respect De Grauwe (1994) notes: *'...in 1994 when the Federal Reserve started to raise domestic interest rates and the Bundesbank initiated a policy of lower domestic rates, most analysts predicted that the dollar would increase in value. Exactly the opposite happened....it turns out that conditional forecasts are as difficult to make as unconditional ones...'.* Hence, there must have been some other factors at work which caused the expected future value of the dollar in terms of Deutschmarks to *decline* even though the interest differential exerted *upward* pressure on the value of the dollar. This example supports the contention that (seemingly) non-fundamental expectations tend to dominate fundamental forces in the short run.

fundamentals in the long run is simply *irrelevant* for most of them[9]. After all, given the afore-mentioned uncertainty about the fundamentals and given the fact that all other traders base their expectations on non-fundamental anchors, any trader who purely acts on these fundamentalist views will soon be out of business.

The exchange rate theories outlined here can be summarized by the following expression:

$$s_t = A + \delta(INV_t^R - INV_t^e) + \varepsilon_t \qquad with \qquad \varepsilon_t = \zeta\varepsilon_{t-1} + \mu_t \quad ; \quad 0 < \zeta < 1$$
$$\mu_t \sim N(0,\sigma_\mu^2) \qquad (3)$$

As noted before, the parameter A denotes the mean of the publicly known distribution of the future fundamentals. Next, it is assumed that the wide-spread use of technical analysis (and the trading strategies which result from that) produce an exchange rate shock ($\varepsilon_t$) in every period. To model the fact that (partly as a result of technical analysis) short-term exchange rate expectations often entail bandwagon-effects[10] we assume that this exchange rate shock follows and AR(1) process where the parameter $\zeta$ denotes the degree of *exchange rate persistence*. Put differently, equation (3) describes the spot rate as being constantly subjected to *rational speculative bubbles* which cause it to deviate from its fundamental solution A[11]. Nevertheless, *on average* the spot rate will be fully determined by the underlying fundamentals as indicated by the mean-reverting behaviour of the exchange rate shock[12].

Finally, the second term on the RHS of equation (3) describes the signalling channel of sterilized interventions. It is a fact of observation that central banks typically do not reveal the exact magnitude of their intervention operations. On the other hand, as documented by Dominguez and Frankel (1993a), the central bank's presence in the foreign exchange market

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[9] Any foreign exchange trading that results from, for instance, long-term foreign direct investment decisions will be much more influenced by fundamental analysis. However, the magnitude of these long-term investments is dwarfed by the vast amount of short-term capital movements at any given moment.

[10] These bandwagon expectations are based on backward-looking behaviour by chartists and should not be confused with the well-known overshooting phenomenon (see e.g. Dornbusch (1976)). The latter is accompanied by *regressive* expectations and is based on fundamental (i.e. forward-looking) analysis. Consequently, in our model these regressive expectations are incorporated in the 'fundamental parameter' A.

[11] In this respect Blanchard and Fisher (1989, p. 223) have noted: *'...Thus one would generally expect bubbles when fundamentals are difficult to ascertain, such as in the gold, art, or foreign exchange markets...'*

[12] In this paper we will not be concerned with the exact specification of the mathematical technicalities of rational bubbles (see Blanchard and Fisher (1989)). In stead, we argue that the exchange rate behaviour which results from such bubbles can be described by equation (3) and that the economic intuition as to why these bubbles arise is captured by the kind of exchange rate theories outlined in this section.

rarely goes unnoticed. From this it can be inferred that it is not the actual intervention volume ($INV_t$) itself but the market's *perception* of this volume which is relevant for the signalling channel. This can be modelled by assuming that market observes a *report*[13] on the intervention operation after the actual intervention has taken place. This reported intervention volume ($INV_t^R$) consists of the actual intervention volume ($INV_t$) augmented by a white noise error term ($\eta_t$). This implies that the market will not make *systematic* forecast errors, even though its perception of the actual intervention volume will be distorted in every single period:

$$INV_t^R = INV_t + \eta_t \qquad with \qquad \eta_t \sim N(0,\sigma_\eta^2) \tag{4}$$

The exact working of the signalling channel can now be understood as follows. It is well-known that speculators in the foreign exchange markets are heavily engaged in central bank watching. Presumably, this is because central banks show a tendency to retain private information about their preferences, on the one hand, and because knowledge of these preferences provides speculators with a useful benchmark for short-term exchange rate expectations, on the other[14]. Consequently, every time the central bank 'releases' some of its private information, speculators will adjust their perception of these preferences and will change their exchange rate expectations accordingly. In this respect, the effect of mere statements by central bankers on the foreign exchange markets is notorious. Incidentally, this also provides an informal argument for central bank secrecy because if the central bank's preferences were common knowledge it would lose a potentially powerful instrument to influence the markets. In other words, uncertainty about the central bank's preferences will give speculators a permanent incentive to closely watch the words and actions of the central bank.

In this paper we will show that intervention operations are (partly) motivated by the central bank's current short-term exchange rate target. Hence, upon observing the difference between the reported intervention volume ($INV_t^R$) and the ex ante expectation of this volume ($INV_t^e$), the market receives new (albeit noisy) information about the state of the short-term target ($p_t$).

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[13] This reported intervention volume can be understood as the intervention data reported by the financial press and 'rumours' that an intervention has taken place in dealing rooms across the world.

[14] Hence, in a sense one player, the central bank, wants to retain its private information (so as to be able to use this information strategically) while the other player, the market, contantly tries to narrow the 'information gap' between itself and the central bank.

The concomitant readjustment of short-term exchange rate expectations will produce a change in the spot rate. The parameter $\delta$ in equation (3) then simply represents the extent to which the market reacts this new information.

Finally, the *sequence of events* in the (infinitely repeated) stage game runs as follows:

Stage 1: nature draws the realization of the exchange rate shock ($\varepsilon_t$) which is subsequently observed by all players.

Stage 2: speculators in the foreign exchange market form expectations about the volume of interventions ($INV_t^e$).

Stage 3: nature draws the current state of the short-term exchange rate target ($p_t$) which is revealed to the central banker but not to speculators.

Stage 4: the central bank sets the actual volume of interventions ($INV_t$) which is kept secret.

Stage 5: nature draws the realization of the control error in the reported volume of intervention ($\eta_t$) and, thereby, the reported intervention volume ($INV_t^R$) itself. The latter is subsequently revealed to speculators.

Stage 6: the spot exchange rate ($s_t$) is realized.

## 3. The dynamically consistent solution

To simplify the calculations it will be assumed that speculators have a perfect observation on the state of the central bank's target realized two periods earlier ($p_{t-2}$)[15]. This means that the current intervention volume ($INV_t$) will only influence the expected volume in the next period ($INV_{t+1}^e$)[16]. In reality the learning process involved will probably extend to more than one period and will fade out gradually (in the sense the public will place a higher weight on more recent periods relative to less recent periods). Nevertheless, the main implications of asymmetric information and the essence of the learning process can also be demonstrated by

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[15] Cukierman and Meltzer (1986) present a reputational model for a monetary policy game in which the public is never informed about the precise nature of the innovation to the policymaker's objectives. However, they have incorporated a *linear* term for the deviation of output from society's bliss point because of which current inflationary expectations do not influence the current inflation rate. Unfortunately, it is impossible to incorporate the linear deviation of the change in the exchange rate from its target in this case. In contrast to output for which it arguably holds that 'more is always better', both positive and negative deviations of the exchange rate from its target will cause a loss. For this reason this deviation appears as a quadratic term in equation (6)

[16] Hence, the following condition will hold: $\partial INV_{t+i}^e / \partial INV_t = 0$ for $i \geq 2$.

means of a short-lived information advantage. The equilibrium concept[17] we will use is of the Nash-variety. Analogous to Cukierman (1992) it can be formulated as follows:

* In every period the central bank selects the intervention volume so as to minimize its *intertemporal* loss function given the exchange rate constraint (3) and its perception of the market's expectations formation process. The intertemporal loss function reads as follows:

$$L^{CB} = E_0 \left[ \sum_{t=0}^{\infty} \beta^t L_t^{CB} \right] \tag{5}$$

* Given their perception of the policy rule followed by the central bank and the information currently available, speculators form their expectations about the intervention volume so as to minimize the conditional mean forecast error ($E[\,(INV_t - INV_t^e)^2 \mid I_t\,]$) in each period.
* The policy rule as perceived by speculators is identical to the policy rule that comes about in equilibrium and, conversely, the perceptions of the central bank concerning the expectations formation process is identical to the actual process used by speculators in equilibrium.

From this it is clear that the actual volume of interventions ($INV_t$) and the expectations concerning this volume ($INV_t^e$) will be determined simultaneously. Substituting equations (1) to (4) into equation (5), it can be seen that $INV_t$ will be affected by $\varepsilon_t$, $p_t$, $INV_t^e$ and, because of the link between periods, also on $E_t[p_{t+1}]$, $E_t[INV_{t+1}^R - INV_{t+1}^e]$ and $E_t[\varepsilon_{t+1}]$[18]. Since the loss function of the central bank is *quadratic* in both terms we will postulate the following *linear* central bank intervention reaction function in which $D_i$, $i=1,..,6$ are the coefficients to be determined[19]:

$$INV_t = D_1 \varepsilon_t + D_2 p_t + D_3 INV_t^e + D_4 E_t(p_{t+1}) + D_5 E_t(INV_{t+1}^R - INV_{t+1}^e) + D_6 E_t(\varepsilon_{t+1}) \tag{6}$$

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[17] The solution concept used is that of *dynamic consistency* which is weaker than the concept of subgame perfection (see Cukierman (1992), Chapter 11). Dynamic consistency requires that the player's actions be optimal at each point in time along the equilibrium path only while subgame perfection (or its equivalent in games of incomplete information) puts requirements on beliefs and actions off the equilibrium path as well.

[18] Throughout this paper we will use the following convention: expectations conditioned on the central bank's information set in time t will be denoted as $E_t[...]$ while expectations conditioned on the information set of the speculators in time t will be denoted as $E[...|I_t]$.

[19] The calculations in this paper follow the *method of undetermined coefficients* as, for example, used by Cukierman (1992), Chapter 15.

**Establishing the link between current interventions and future expectations**

It is important to note that the connection between expected shocks in period t+1 and the current intervention volume ($INV_t$) arises exclusively on account of the fact that current interventions will affect future intervention expectations. In Appendix A it is shown that a unit increase in the intervention volume will increase next period's expectations by:

$$\frac{\partial INV_{t+1}^e}{\partial INV_t} = \frac{\rho\,\theta}{1-D_3} \qquad (7)$$

In period t+1 speculators will have to make a conjecture about the state of the central bank's target ($p_{t+1}$) to form a rational expectation about the intervention volume in that period. They know that $p_{t+1}$ will be affected by the innovation realized in period t ($v_t$) because of the afore-mentioned persistence property. What's more, speculators actually have a distorted observation of this innovation because the reported level of intervention in period t ($INV_t^R$) is contained in their information set in period t+1 ($I_{t+1}$). Appendix A shows that they can use this to calculate the following *intervention residual* ($INV_t^R$ - g(t)):

$$INV_t^R - g(t) = (D_2 + \rho D_4)v_t + \eta_t \qquad (8)$$

The term g(t), which appears on the LHS and which is defined in Appendix A, consists solely of shocks which are contained $I_{t+1}$. Speculators cannot decompose the intervention residual into its constituent shocks which appear on the RHS. Consequently, they will use this residual in conjunction with their knowledge of the economy to make an optimal forecast for $v_t$. Obviously, the link between different periods will be stronger, the better the accuracy of the forecast. The reason is that an increase in $INV_t$ will cause a ceteris paribus increase in the intervention residual which in the presence of a more accurate forecast will feed through into expectations to a greater extent.

The accuracy of the forecast, in turn, crucially depends on two parameters. First of all, an increase in the persistence in the central bank's target ($\rho$) means that the there is an increase in the degree to which innovations in the current target will feed through into next period's target. Naturally, this will imply a stronger link between periods for any *given* value of $\theta$.

Secondly, we can investigate the effect of $\theta$, hereafter to be referred to as the *speed of learning* parameter, which is defined as follows:

$$\theta \equiv \frac{(D_2 + \rho D_4)^2 \sigma_v^2}{(D_2 + \rho D_4)^2 \sigma_v^2 + \sigma_\eta^2} \tag{9}$$

From this equation it is clear that the speed of learning is actually a measure of the degree of informativeness of the intervention residual. An increase in this parameter implies that a larger proportion of the *average* intervention residual (the denominator in equation (12)) can be attributed to innovations in the target rather than to misperception errors. Needless to say that this will serve to strengthen the link described by equation (10).

**The derivation of the respective reaction functions**

The coefficients in equation (6) can be determined by writing out the central bank's intertemporal objective function (5) for periods t and t+1, taking expectations conditional on the central bank's information set in period t. Computing the first-order condition of the resulting equation, using the expression obtained in equation (7) and rearranging, we obtain the following expressions for the coefficients in equation (6):

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\begin{aligned}
D_1 &= -\frac{\varphi \delta}{(k^2 + \varphi \delta^2)} \\
D_2 &= \frac{\varphi \delta}{(k^2 + \varphi \delta^2)} = -D_1 \\
D_3 &= \frac{\varphi \delta^2}{(k^2 + \varphi \delta^2)} \\
D_4 &= -\frac{\beta \varphi \delta \rho \theta}{k^2} \\
D_5 &= \frac{\beta \varphi \delta^2 \rho \theta}{k^2} = -\delta D_4 \\
D_6 &= \frac{\beta \varphi \delta \rho \theta}{k^2} = -D_4
\end{aligned}
\tag{10}
$$

The model thus yields explicit solutions for $D_1$, $D_2$ and $D_3$. Through the dependence of $\theta$ on the coefficient $D_4$ it also yields an implicit solution for $D_4$ and, thereby, also for $D_5$ and $D_6$. Using equation (9) this implicit solution can be written as follows:

$$D_4 = -\frac{\beta \varphi \delta \rho [\varphi \delta/(k^2 + \varphi \delta^2) + \rho D_4]^2 \sigma_v^2}{k^2([\varphi \delta/(k^2 + \varphi \delta^2) + \rho D_4]^2 \sigma_v^2 + \sigma_\eta^2)} \equiv F(D_4) \tag{11}$$

In Appendix B it is shown that there always exists at least one solution for $D_4$ for which it holds that:

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$$- \frac{1}{\rho} \frac{\varphi\delta}{(k^2+\varphi\delta^2)} < D_4 < 0 \qquad (12)$$

Using equations (6) and (10) and observing that $E_t(\varepsilon_{t+1}) = \zeta\varepsilon_t$ and $E_t(p_{t+1}) = \rho p_t$ , we can now derive the following expression for the central bank's reaction function[20]:

$$INV_t = - \frac{\varphi\delta}{(k^2+\varphi\delta^2)} [\varepsilon_t - p_t - \delta INV_t^e] - D_4 [\zeta\varepsilon_t - \rho p_t + \delta E_t(INV_{t+1} - INV_{t+1}^e)] \qquad (13)$$

The economic interpretation of this equation is straightforward. The first term between brackets represents the current gap between the spot rate and the short-term target $(s_t - T_t)$ under the condition that the central bank abstains from interventions. The absolute value of this term can be seen as an indicator of the current marginal loss induced by undesired exchange rate levels. Provided the current gap is strictly positive this marginal loss can be mitigated by *selling* foreign exchange $(INV_t < 0)$. However, as a side-effect this will also reduce future intervention expectations as can be seen from equation (10). The second term between brackets is equal to the expected future gap between the spot rate and the target $(E_t[s_{t+1} - T_{t+1}])$. Analogously, the absolute value of this term serves as a measure for expected future marginal loss due to undesired exchange rate levels. On the assumption that the this second term is strictly positive as well, the central bank has a ceteris paribus incentive to *buy* foreign exchange $(INV_t > 0)$. After all, the concomitant *increase* in next period's expected intervention volume will give the central bank more scope to the mitigate undesired exchange rate levels in the next period by means of *negative* surprise interventions. It can thus be concluded that the central bank is basically faced with an *intertemporal trade-off*. Lowering the present marginal loss of undesired exchange rate levels will raise the expected future marginal loss[21] through the effect of the present intervention volume on future expectations.

Having obtained the central banker's intervention reaction function we now derive the speculators' reaction function. In Appendix C it is shown that the latter is given by:

$$INV_t^e = - [\frac{\varphi\delta + \zeta D_4(k^2+\varphi\delta^2)}{k^2}] \varepsilon_t +$$
$$+ [\frac{\varphi\delta + \rho D_4(k^2+\varphi\delta^2)}{k^2}] [\rho^2 p_{t-2} + \rho\theta(v_{t-1} + \frac{(k^2+\varphi\delta^2)}{\varphi\delta + \rho D_4(k^2+\varphi\delta^2)} \eta_{t-1})] \qquad (14)$$

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[20] Since $E_t(\eta_{t+1}) = 0$ , the expression $E_t(INV_{t+1}^R - INV_{t+1}^e)$ has been replaced by $E_t(INV_{t+1} - INV_{t+1}^e)$.

[21] Note that present and future expected marginal losses are closely linked because of the persistence in exchange rate movements $(\zeta)$ and the persistence in the central bank's target $(\rho)$.

The first part of the RHS of this equation represents the market's ceteris paribus reaction to the current exchange rate shock ($\varepsilon_t$). It will be shown later that the concomitant coefficient is identical to the one that appears in the expression for the equilibrium intervention volume. This is a direct result of the fact that the realization of this shock is perfectly known by the market. The second term on the RHS basically concerns the market's expectation of the central bank's reaction to the current state of its target ($p_t$). First of all, due to the persistence property this expectation will be based on $p_{t-2}$ which is incorporated in the current information set of the speculators. Secondly, as discussed earlier, last period's intervention residual (which basically consists of the realizations of $v_{t-1}$ and $\eta_{t-1}$) will also affect the current expected intervention volume because of the fact that this residual contains information about last period's innovation to the state of the central bank's target.

## 4. The impact of asymmetric information on the equilibrium intervention volume and the intervention bias

Next, we can calculate the equilibrium volume of intervention by inserting equation (14) into (13) and using the expression obtained for $E_t(INV_{t+1}-INV_{t+1}^e)$ in Appendix D:

$$INV_t = - [\frac{\varphi\delta+\zeta D_4(k^2+\varphi\delta^2)}{k^2}]\varepsilon_t + [\frac{\varphi\delta+\rho D_4(k^2+\varphi\delta^2)}{k^2}]\rho^2 p_{t-2} +$$

$$+ [\frac{\varphi\delta}{(k^2+\varphi\delta^2)}+\rho D_4][1+\theta\frac{\varphi\delta^2}{k^2}]\rho v_{t-1} + [\frac{\varphi\delta}{(k^2+\varphi\delta^2)}+\rho D_4][1-\delta\rho D_4(1-\theta)]v_t + \frac{\varphi\delta^2}{k^2}\rho\theta\eta_{t-1}$$

$$(15)$$

To interpret this equation it will be instructive to compare it with the equilibrium intervention volume which will result in the presence of *symmetric* information. In that case the current stance of the central bank's short-term target ($p_t$) will be contained in the market's information set. Consequently, the game will be completely transparent for both players since speculators will be able to solve the central bank's problem without error. This will cause the concept of the *reported* intervention volume to lose its meaning:

$$INV_t^R = INV_t \quad \forall\ t \quad \leftrightarrow \quad \sigma_\eta^2 = 0 \qquad (16)$$

Furthermore, since speculators no longer need past intervention volumes to predict the current state of the target, the game will simplify into a string of unrelated one-period problems[22]. The central bank's reaction function can then be obtained by plugging (2) and (3) into the atemporal objective function (1). Taking the first order condition of the resulting expression and rearranging yields the following:

$$INV_t = - \frac{\varphi \delta}{(k^2 + \varphi \delta^2)}[\varepsilon_t - p_t - \delta INV_t^e] \qquad (17)$$

Since there is no information asymmetry, the market will always be able to predict the intervention volume without error:

$$INV_t^e = INV_t \qquad (18)$$

Substituting (21) into (20) and using the fact that $p_t = \rho^2 p_{t-2} + \rho v_{t-1} + v_t$ , we obtain the following expression for the (Nash) equilibrium intervention volume under symmetric information:

$$INV_t = - \frac{\varphi \delta}{k^2}\varepsilon_t + \frac{\varphi \delta}{k^2}[\rho^2 p_{t-2} + \rho v_{t-1} + v_t] \qquad (19)$$

When comparing the expressions obtained for the equilibrium intervention volume under asymmetric and symmetric information (equations (15) and (19) respectively) we can derive the following proposition:

**Proposition 1:**     The absolute value of the change in the equilibrium intervention volume as a result of a ceteris paribus mutation in either $\varepsilon_t$ , $p_{t-2}$ , $v_{t-1}$ or $v_t$ will be *strictly less* if the central bank retains *private information* about the short-term exchange rate target compared to the situation where the central bank chooses to reveal the target perfectly.

A proof of this proposition for each reaction coefficient is given in Appendix E. The intuition is that in the absence of private information the central bank will no longer react to the expected future gap between the spot rate and the short-term target ($E_t(s_{t+1}-T_{t+1})$) as can be seen from equation (20). The reason for this is simply that the central bank cannot manipulate future intervention expectations. Because of this and in contrast to the case where the central bank

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[22] Therefore, the following will hold: $\partial INV_{t+i}^e/\partial INV_t = 0 \quad \forall \ i \geq 0$.

has private information, these expectations will not exert a deterring effect on the current degree of activism. Hence, it can be concluded that central banks retain private information because this provides an instrument to mitigate the time-inconsistency problem in intervention policy. As pointed out by Almekinders (1994, 1995), in a symmetric world it would be optimal if the central bank were able to make a commitment not to intervene at all. However, since the central bank always faces the incentive to renege on this commitment ex post, it ends up in the Nash equilibrium described by equation (19) where the central bank buys or sells foreign exchange without any effect on the spot rate. Not revealing the short-term target will both reduce the degree of central bank activism and render effectiveness to intervention policy because of which the central bank will generally be better off than in a symmetric world.

Another interesting aspect of the introduction of asymmetric information is that it will induce the intervention volume to be sensitive to last period's misperception error ($\eta_{t-1}$). Again, this is a direct result of the dependence of current expectations on the past perceived intervention volume. Even though the central bank will never react *directly* to past misperceptions it does accommodate a change in the expected intervention volume ($INV_t^e$). Last period's misperception error will be reflected in $INV_t^e$ and will, therefore, affect the current intervention volume *indirectly* .

**The intervention bias under asymmetric information**

The analysis sofar has revealed that (apart from the contamination induced by last period's misperception error) the current intervention volume will basically react to the current exchange rate shock ($\varepsilon_t$) and the current stance of the short-term exchange rate target ($p_t$). Interventions derive their effectiveness precisely from the fact that the central bank has private information about this target. Nevertheless, the realization of the exchange rate shock itself does not create any scope for intervention surprises since it belongs to the information set of both the central bank and the speculators. The central bank's response to this shock will, therefore, always be futile as can clearly be seen from equations (14) and (15) which reveal that the central bank's reaction to $\varepsilon_t$ will be fully expected by the market ex ante. Consequently, the central bank would be better off if it were to avoid this part of the intervention volume altogether since it only generates costs without affecting the exchange rate. However, any announced precommitment by the central bank to avoid this futile part of the intervention volume will be time-inconsistent since the central bank will always be better off when it reneges on its promise ex post. We will define the *intervention bias* as the absolute value of this futile component of the equilibrium intervention volume:

$$B_t \equiv \left| - \left[ \frac{\varphi\delta + \zeta D_4(k^2 + \varphi\delta^2)}{k^2} \right] \varepsilon_t \right| \tag{20}$$

This equation allows us to examine the effect of various *political* and *economic* parameters on the intervention bias which can be summarized by the following proposition:

**Proposition 2:**     The intervention bias ($B_t$) at any given time will be *lower*,
1. the *longer* the planning horizon of the central bank ($\beta$),
2. the *higher* the variance of the innovation in the target ($\sigma_v^2$),
3. the *lower* the variance of the misperception error ($\sigma_\eta^2$),
4. the *higher* the degree of persistence in the exchange rate shock ($\zeta$),

A proof of this proposition is given in Appendix F. The economic intuition can be obtained by noting that a reduction of the intervention bias can arise for two reasons. On the one hand, it could be the consequence of an increase in the central bank's *concern for the future* (which is measured by the absolute value of $D_4$) for a given value of $E_t(s_{t+1}-T_{t+1})$. Alternatively, a decrease in $B_t$ could be caused by a reduction in the (absolute value of the) expected future gap itself for a given value of $D_4$.

Apart from the degree of persistence in the exchange rate shock ($\zeta$), the changes in the parameters indicated in Proposition 2 will all cause an increase in the central bank's concern for the future. As for the effect of the length of the policy horizon ($\beta$) this is rather trivial in the light of the previous discussion. An important implication of this result is that a longer policy horizon will entail an improved ability to counter the time-inconsistency problem because it decreases intervention costs without sacrificing effectiveness. In other words, a higher value of $\beta$ will allow a larger relative share of any *given* intervention volume to influence the exchange rate. This is because the relative influence of the constituent shocks of $p_t$ on the intervention volume will increase at the expense of a diminished relative influence of $\varepsilon_t$. It is well-known from the literature that the length of the policy horizon is positively related to the degree of *central bank independence* (see Cukierman (1992), Chapter 18 and Eijffinger and De Haan (1996)). In this respect, we can conclude that a very independent central bank, such as the Deutsche Bundesbank, will trade less foreign exchange reserves in vain than more dependent (and more myopic) central banks.

Furthermore, the effects on the variance of the innovation to the target ($\sigma_v^2$) and the variance of the market's misperception error ($\sigma_\eta^2$) can be understood from the way in which they affect the speed of learning ($\theta$) as shown in Proposition 3 below. An increase in the relative variance $\sigma_v^2/\sigma_\eta^2$ will increase the informativeness of last period's intervention residual and will, therefore, cause the market to be better informed about the current stance of the target ($p_t$). Since this implies a stronger link between periods, the central bank will henceforth display a greater concern for the future which will lower the intervention bias.

Contrary to the other shocks, a larger degree of persistence in the exchange rate shock ($\zeta$) will not alter the central bank's concern for the future but will increase the future expected marginal loss instead. As can be seen from equation (13), any given realization of $\varepsilon_t$ will now to a larger extent feed through into the expected future gap between the spot rate and the target. This will mitigate the central bank's response to the exchange rate shock. This result is intuitively plausible since the degree of exchange rate persistence can be seen as an indicator of the strength of the market sentiment underlying bandwagon effects. Hence, irrespective of the central bank's response to its own subjective preferences, its degree of activism towards the objectively verifiable exchange rate shock will be weaker if this underlying sentiment becomes stronger (i.e. if $\zeta$ increases).

**The determinants of intervention credibility**

From the preceding discussion it will be clear that the market's speed of learning ($\theta$) plays a crucial role in this model. From the central bank's point of view this parameter can be seen as a measure of the degree of *intervention credibility*. A higher speed of learning simply means that the central bank will, on average, transmit more information about its target to the market *ex post*. Consequently, interventions will become more predictable *ex ante* since an increase in the speed of learning enables the market to calculate a more accurate (ex ante) expectation of the current state of the target. The effect of various institutional parameters on the degree of intervention credibility is summarized by the following proposition:

**Proposition 3:**  The market's speed of learning ($\theta$) will be *higher*,
1. the *shorter* the central bank's planning horizon ($\beta$),
2. the *higher* the variance of the innovation to the target ($\sigma_v^2$),
3. the *lower* the variance of the misperception error ($\sigma_\eta^2$),

A proof of this proposition is given in Appendix F. The fact that a longer planning horizon causes the market to be more slow in recognizing innovations to the central bank's target may

seem counterintuitive at first sight. However, it should be kept in mind that the concomitant smaller degree of policy activism inevitably implies that the central bank will also reveal its preferences (in particular last period's innovation ($v_{t-1}$)) to a smaller extent. In other words, since it holds that $\partial D_4/\partial \beta$ is strictly smaller than zero (see Appendix F) it turns out that an increase in $\beta$ will cause innovations to the central bank's target to become relatively less relevant in explaining the average intervention residual (the denominator in equation (12)). The latter will convey less information as a result. Hence, it can be concluded that a more independent central bank will also be more eager to preserve its information advantage. Put differently, an independent central bank will be characterized by a relatively large degree of impredictability in its intervention *tactics* [23].

Secondly, the intuition behind the effect of an increase in the relative variance $\sigma_v^2/\sigma_\eta^2$ is relatively straightforward. The average informativeness of the intervention residual will rise since this residual is now more likely to have been caused by innovations to the target rather than by misperception errors. Put differently, if preferences are relatively unstable over time and if the average distortion in intervention reports is relatively small, it will generally be easier for speculators to deduce these preferences from the observed actions of the central bank.

## 5. The determinants of intervention surprises and the importance of ambiguity

In this section we will identify those factors in our model which explain the effectiveness of sterilized foreign exchange interventions. On the basis of equations (3), (14) and (15) the current *intervention effectiveness* can be expressed as follows:

$$INV_t^R - INV_t^e = (\frac{\varphi\delta}{(k^2+\varphi\delta^2)}+\rho D_4)(1-\theta)\rho v_{t-1} - \rho\theta\eta_{t-1} +$$

$$+ (\frac{\varphi\delta}{(k^2+\varphi\delta^2)}+\rho D_4)(1-\delta\rho D_4(1-\theta))v_t + \eta_t \qquad (21)$$

This equation clearly shows that sterilized interventions are effective precisely because they provide the market with new (albeit contaminated) information about the central bank's short-term target. Moreover, it allows us to examine the determinants of the central bank's ability to

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[23] In this respect Dominguez and Frankel (1993a, p. 85) have noted that, '...consistently only about one-quarter of the variation in Bundesbank intervention is predictable..' .

generate intervention surprises. As for the first three shocks on the RHS, this ability depends on the extent to which speculators will underestimate the effect of a given shock to the state of the target when they form their expectations about the intervention volume.

First of all, the market cannot predict the full extent to which last period's innovation ($v_{t-1}$) will feed through into the current state of the target ($p_t$) *ex ante*. An increase in the realization of this shock will, therefore, induce a depreciation  (i.e. $\partial s_t / \partial v_{t-1} > 0$) since the concomitant increase in the reported intervention volume will *implicitly*[24] cause the market to revise its forecast of $p_t$ upwards *ex post*. On the other hand, an increase in last period's misperception error will have the opposite effect on the exchange rate ($\partial s_t / \partial \eta_{t-1} < 0$). This is because the market will erroneously regard part of the increase in $\eta_{t-1}$ as an increase in $v_{t-1}$ and will raise its ex ante expectation of $p_t$ accordingly. As argued before, the central bank will not respond directly to  the increase in $\eta_{t-1}$. However, the concomitant increase in $INV_t^e$ will be accommodated. The implicit ex post downward readjustment of the market's forecast of $p_t$ then results from the fact that this accommodation is less then perfect (i.e. $\partial INV_t / \partial INV_t^e = \varphi \delta^2 / (k^2 + \varphi \delta^2) < 1$).

Next, it can be seen that an increase in the current innovation to the target ($v_t$) will be particularly powerful in inducing a depreciation ($\partial s_t / \partial v_t > 0$). The basic reason for this is that this shock will implicitly release much information about $p_t$ to the market ex post. First of all because an increase in $v_t$ will not affect the market's ex ante forecast of the $p_t$ ($E(v_t | I_t) = 0$) and, secondly, because it will influence the intervention volume through *two* channels. On top of the *direct* rise in $p_t$, an increase in $v_t$ will also increase the central bank's future expected ability to induce a depreciation (i.e. $\partial E_t(INV_{t+1} - INV_{t+1}^e) / \partial v_t > 0$, as shown in Appendix D). This means that the central bank's incentive to suppress next period's expected intervention volume will be diminished. The resulting ceteris paribus increase in $INV_t$ will implicitly cause the market's ex post update of its forecast for $p_t$ to be over and above the upward adjustment justified by the afore-mentioned direct effect.

Finally, since both the central bank and the market decide on their actions before the current misperception error is realized, an increase in $\eta_t$ will neither affect ex ante expectations nor the actual intervention volume. Even so, the resulting increase in the reported intervention volume will still implicitly cause the market to readjust its forecast of the state of the target upwards ex

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[24] Of course, the market will only make one single *explicit* adjustment to its forecast of $p_t$ upon observing the reported intervention volume. However, conceptually one can think of this as the combined result of different implicit ex post updates which result from the various shocks on the RHS of equation (24).

post (i.e. $\partial s_t / \partial \eta_t > 0$). In this respect it should be noticed that the central bank can never be sure about the effectiveness of a given intervention operation ex ante. While all the other shocks on the RHS of equation (21) will be perfectly known by the central bank when it sets the intervention volume there always exists the possibility that its objectives will be partly frustrated by the market's current misperception.

A particularly interesting feature of equation (21) is that positive and negative intervention surprises will cancel out on average (i.e. $E(INV_t^R - INV_t^e) = 0$). The reason for this is twofold. First of all, the central bank will not be able to fool the market *systematically* as far as the latter's perception of the actual intervention volume is concerned ($E(\eta_t) = 0$). Indeed, it can be argued that there may be certain periods in which (perhaps due to conscious manipulation by the central bank) the market will tend to over- or underpredict the actual intervention volume. Nevertheless, these periods cannot last *indefinitely* which is why our assertion about the inability to fool the market systematically will still hold even in the presence of such a temporary misperception bias[25]. Secondly, the central bank cannot *systematically* pursue an exchange rate target which is out of line with the underlying fundamentals ($E(v_t) = E(p_t) = 0$) since the exchange rate will be fully determined by them *on average.* Again, this does not exclude the possibility that a central bank may try pursue a target which deviates from the long run trend for a prolonged period of time[26]. Nevertheless, even such a central bank will not be able to resist fundamental forces indefinitely.

This provides an explanation for the fact that sterilized interventions sometimes have a substantial *short-run* effect on the exchange rate, on the one hand, but never seem to have a lasting effect, on the other. Consequently, sterilized interventions do not constitute a truly independent exchange rate policy tool for the policymaker who uses (other) monetary instruments solely for domestic purposes. In order to influence the exchange rate *systematically* policymakers will always have to alter the fundamentals which may be undesirable from the perspective of domestic monetary objectives.

**A rationale for ambiguity**
Many authors (e.g. Dominguez and Frankel (1993a)) have argued that the tendency of central banks tend to keep their intervention volumes secret is inconsistent with the signalling

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[25] This misperception bias could be modelled by introducing persistence in the realisation of the misperception error.

[26] Such a central bank could be characterized as having a relatively high degree of persistence in the innovation to its short-term target.

hypothesis. In this view interventions contain a signal about the central bank's planned monetary growth rates for the near future. Indeed, if the central bank wants to convey information about future monetary aggregates it will surely benefit more from giving a clear signal. However, as noted before, it is common practice for central bankers to send ambiguous rather than clear signals by means of sterilized interventions. This leaves us with two options. Either central bankers have consistently not been acting in a rational way for more than two decades or, alternatively, they have not been using sterilized interventions to convey information about money supply changes which are due in the near future. In our opinion, the latter is by far the most likely explanation. Our conviction in this respect is reinforced by the fact that a direct relationship between sterilized interventions and such tactical changes in monetary policy does not seem to be consistently supported empirically. In this respect it should be remembered that the whole point in conducting *sterilized* interventions is to have an *independent* policy tool. Clearly, this does not suggest that there should be any *systematic* relationship between sterilized interventions and subsequent changes in the money supply.

In our model the signalling channel reflects a subtle game between a central bank which retains private information about its short-term exchange rate target, on the one hand, and speculators who regard this target as an important anchor for exchange rate expectations, on the other. In this respect two questions need to be answered. First of all, we will discuss why speculators attach importance to a target about which is very likely to deviate from the rationally expected fundamental exchange rate (A). Secondly, we will address the question why central banks send ambiguous rather than clear signals about the short-term target. As for the first question we start by noting that it is common practice in the financial markets to spend time and money gathering information about the preferences of the monetary authorities. Again, the fact that the short-term target is likely to deviate from the fundamentally *expected* exchange rate is unimportant in a world where there is a lot of uncertainty about the *actual* fundamental exchange rate. Moreover, the mere fact that every trader knows that all other traders attach importance to the central bank's exchange rate target is in itself sufficient to establish a causal link between this target and exchange rate expectations. Nevertheless, one can easily think of some 'deeper' reasons why the central bank's short-term exchange rate target should matter even if it does not provide information about tactical changes in monetary policy. First of all, there could be a positive (psychological) spill-over effect from other areas of central bank decision making. As documented by Goodfriend (1986) central banks enjoy a certain degree of authority in the financial markets because they are perceived to have superior *'..wisdom, perception and relevant knowledge..'* (p.64). This degree of authority, in turn, is likely to be closely linked to the independence and reputation of the central bank in question. Although

central banks mainly derive this authority from their ability to influence domestic monetary policy, it is also bound to affect the foreign exchange markets' respect for the central bank in a positive way.

Next, the central bank might also use the short-term target to inform speculators about *its* opinion on the extent to which the present speculative bubble is a serious deviation from the (uncertain) fundamental exchange rate value. Furthermore, in the case of a presumably overvalued exchange rate, the short-term target may provide information about attempts to withstand protectionist pressures. Finally, our assumption about the distribution of the future fundamentals implies that both the central bank and speculators are faced with a vast array of possible future paths of various fundamentals. Although the central bank does not know which paths will be the relevant ones, it does have private information about how it will *react* to all these different paths. This provides a source of information for the markets which is quite different from the 'objective' actual distribution of the future fundamentals itself. It may very well be that the central bank uses the short-term target as a crude summary indicator of how it will react to all kinds of possible future fundamental developments.

To summarize, we do not believe that there exists a strong and narrow *direct* relationship between sterilized interventions and tactical changes in monetary policy. Rather, we argue that the effectiveness of the signalling channel is related to the central bank's short-term target which in itself may reflect many aspects of the central bank's overall assessment of the current situation in the foreign exchange markets. It should be noted that arguing that this assessment includes the central bank's *views* on certain fundamental developments is clearly different from stating that the central bank will consciously *change* some of the fundamentals it controls following an intervention operation.

We now turn to the central bank's rationale for sending ambiguous signals. To assess the impact of ambiguity on the ability of sterilized interventions to affect the spot rate in general we take the variance of equation (21) as a measure of *average intervention effectiveness* (V)[27]:

$$V = (\frac{\varphi\delta}{k^2+\varphi\delta^2}+\rho D_4)^2 \ ((1-\theta)^2\rho^2+(1-\delta(1-\theta)\rho D_4)^2)\sigma_v^2 \ + \ (1+\rho^2\theta^2)\sigma_\eta^2 \tag{22}$$

A central bank that chooses to send completely unambiguous signals ($\sigma_\eta^2 = 0$), will grant speculators a maximal speed of learning ($\theta = 1$) as can be seen from equation (9). The measure for average intervention effectiveness then simplifies as follows:

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[27] We assume the following: $E[v_i\eta_j] = 0 \ \forall \ i,j$ , $E[v_t v_s] = 0 \ \forall \ t \neq s$ and $E[\eta_l\eta_m] = 0 \ \forall \ l \neq m.$

$$V = (\frac{\varphi \delta}{k^2 + \varphi \delta^2} + \rho D_4)^2 \sigma_v^2 \qquad\qquad (23)$$

Comparing equations (22) and (23), we arrive at the following proposition:

**Proposition 4:**   The average effectiveness of sterilized interventions will be *strictly larger* if the central bank retains a *strictly positive degree of ambiguity* concerning the intervention volume compared to the situation where the central bank always chooses to reveal this volume perfectly.

A proof of this proposition is given in Appendix G. The key point to note is that in the absence of ambiguity the intervention volume will always perfectly reveal the short-term target to the market *ex post*. Consequently, the central bank's ability to influence the spot rate through the signalling channel will be very limited since speculators only face uncertainty about the *current* innovation to the target ($v_t$). By introducing ambiguity the central bank can signal some of its private information without completely revealing the current state of its preferences at the same time.

As noted before, private information can be considered as a valuable asset because it endows the central bank with a tactical advantage[28] which can be used to mitigate the time-inconsistency problem. In this respect there seems to be an inherent trade-off involved. On the one hand the central bank will have to transmit some of its private information to the market in order to influence the spot rate. On the other hand, however, the central bank's tactical advantage vis-à-vis the market (which is at the heart of its ability to influence the spot rate in the first place) will be diminished by the very act of intervening. Clearly, the absence of ambiguity will not completely deprive the central bank of its tactical information advantage because it receives 'new' private information in every period through the current realization of the innovation to the target ($v_t$). Nevertheless, Proposition 4 indicates that the central bank will be able to derive more effectiveness from its private information on average if it does not immediately signal all of this newly arrived information to the markets but preserves some of it for later use instead.

To sum up, it turns out that the transmission of noisy rather than clear signals constitutes an important complement of private information since it significantly extends the information

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[28] Obviously, the extent of this tactical advantage is inversely related to the speed of learning ($\theta$).

advantage enjoyed by the central bank. In spite of this, it cannot be concluded a priori that the practice of sending ambiguous signals will be in the interest of the central bank from the perspective of minimizing its intertemporal loss function (5). More specifically, from Proposition 2 it can be seen that the intervention bias will be strictly lower in the absence of ambiguity. This means that the amount of futile foreign exchange transactions will increase as a consequence of noisy signalling. The fact that many central banks prefer not to reveal their intervention data indeed suggests that ambiguity will be conducive to the central bank's welfare on balance. A formal proof of this is, however, beyond the scope of this paper.

## 6. Conclusion

This paper examines the central bank's attempts to influence the spot rate by means of sterilized foreign exchange interventions. To this end we have examined a dynamic game in which the central bank retains private information about its short-term exchange rate target and in which speculators are subject to ambiguity concerning the true intervention volume.
By the very act of intervening the central bank will transmit some information about its short-term target to the market which will lead speculators to revise their expectations about the future spot rate. This is because speculators consider the central bank's preferences to be an important expectations anchor in a world where the actual fundamental exchange rate is highly uncertain. The dynamic game analyzed in this paper contains a learning process on the part of speculators which introduces a link between periods. This link will induce the central bank to take the future consequences of its current actions into account which will generally reduce the degree of activism. This result is the exchange rate policy equivalent of the well-known reputational story in the literature on time-inconsistency in monetary policy games.

A particularly interesting feature of this paper is that it shows that the effectiveness of the signalling channel need not be in conflict with the fact that central banks prefer to conceal their intervention data. On the contrary, interventions will be more effective on average if the central bank retains a certain degree of ambiguity. This result arises on the fact that interventions in this model are not meant to convey a signal about future monetary aggregates (in this case it would certainly be in the interest of the central bank to make the signal as clear as possible). Rather, they will transmit a signal about the central bank's short-term exchange rate target. Essentially, by contaminating the market's perception of the actual intervention volume the central bank will be able to use its information advantage to a greater extent.

Our model also has a number of additional features which are observed in reality. First of all, the central bank can never be sure about the effectiveness of its intervention policy ex ante since its attempts to influence the exchange rate may be frustrated (but also enhanced) by the market's current misperception. Next, we also find that the futile component of intervention operations (i.e. the intervention bias) will decrease if the central bank in question becomes more independent. This effect arises because the length of the policy horizon is positively related to the degree of central bank independence. Finally we find that sterilized interventions cannot systematically affect the exchange rate even though there may be a substantial short run effect. This result stems from the fact that central banks will not be able to fool the market systematically about the actual intervention volume, on the one hand, and the inability of the central bank to systematically pursue a target that differs from the underlying fundamental trend, on the other. This leads us to conclude that the use sterilized interventions will not provide policymakers with an independent tool for exchange rate policy.

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# Appendix A: Calculation of $\partial INV_{t+1}^e/\partial INV_t$

Leading equation (6) by one period and taking expectations conditional on the market's information set in period t+1 ($I_{t+1}$) we obtain:

$$E(INV_{t+1}|I_{t+1}) \equiv INV_{t+1}^e = D_1 E(\varepsilon_{t+1}|I_{t+1}) + D_2 E(p_{t+1}|I_{t+1}) + D_3 INV_{t+1}^e +$$
$$D_4 E(E_{t+1}(p_{t+2})|I_{t+1}) + D_5 E(E_{t+1}(INV_{t+2}-INV_{t+2}^e)|I_{t+1}) + D_6 E(E_{t+1}(\varepsilon_{t+2})|I_{t+1}) \tag{A.1}$$

With regard to this equation the following can be noted:

$$E(p_{t+1}|I_{t+1}) = \rho^2 p_{t-1} + \rho E(v_t|I_{t+1})$$

$$E(E_{t+1}(p_{t+2})|I_{t+1}) = E(p_{t+2}|I_{t+1}) = \rho^3 p_{t-1} + \rho^2 E(v_t|I_{t+1})$$

$$E(E_{t+1}(\varepsilon_{t+2})|I_{t+1}) = E(\varepsilon_{t+2}|I_{t+1}) = \zeta\varepsilon_{t+1} \tag{A.2}$$

$$E(E_{t+1}(INV_{t+2}-INV_{t+2}^e)|I_{t+1}) = E((INV_{t+2}-INV_{t+2}^e)|I_{t+1}) = 0$$

The last expression in (A.2) simply states that the expected value of surprise interventions in period t+2 based on the information available to speculators in period t+1 should be equal to zero. Otherwise, these interventions could not have been unexpected in the first place. Plugging equation (A.2) into (A.1) and rearranging we obtain:

$$INV_{t+1}^e = \frac{1}{1-D_3} [(D_1+\zeta D_6)\varepsilon_{t+1} + (D_2+\rho D_4)(\rho^2 p_{t-1} + \rho E(v_t|I_{t+1}))] \tag{A.3}$$

To get an expression for $E(v_t|I_{t+1})$ we note that, using (4) and (6), speculators will have observed the following in period t:

$$INV_t^R = (D_1+\zeta D_6)\varepsilon_t + D_2(\rho p_{t-1}+v_t) + D_3 INV_t^e +$$
$$D_4 E_t(\rho^2 p_{t-1}+\rho v_t+v_{t+1}) + D_5 E_t(INV_{t+1}-INV_{t+1}^e) + \eta_t \tag{A.4}$$

While they know the exact value of the LHS of this equation, speculators will form an expectation about all the terms appearing on the RHS based on $I_{t+1}$. To keep the calculations manageable we will introduce some bounded rationality on the part of the speculators by assuming the following:

$$E(E_t(INV_{t+1}-INV_{t+1}^e)|I_{t+1}) = 0 \tag{A.5}$$

Using (A.5) we can rewrite equation (A.4) as follows:

$$INV_t^R - g(t) = (D_2 + \rho D_4)\nu_t + \eta_t$$
$$where \quad g(t) \equiv (D_1 + \zeta D_6)e_t + (D_2 + \rho D_4)\rho p_{t-1} + D_3 INV_t^e \tag{A.6}$$

All the terms in g(t) are incorporated into $I_{t+1}$ but, by contrast, speculators cannot decompose the RHS of (A.6) into its constituent shocks. Hence, we will call the LHS of (A.6) the *intervention residual*. It will be shown later that $E_t(INV_{t+1}-INV_{t+1}^e)$ actually depends on $\nu_t$ (see Appendix D). So by introducing a limited degree of bounded rationality (equation (A.5)) we assume that speculators overlook the fact that intervention residual realized in the previous period has also been *indirectly* affected by $\nu_t$ through $E_t(INV_{t+1}-INV_{t+1}^e)$. The market will subsequently use the intervention residual to obtain an optimal forecast for $\nu_t$ which yields[29]:

$$E(\nu_t|I_{t+1}) = \frac{\theta}{(D_2 + \rho D_4)} [INV_t^R - g(t)] \quad where \quad \theta \equiv \frac{(D_2 + \rho D_4)^2 \sigma_\nu^2}{(D_2 + \rho D_4)^2 \sigma_\nu^2 + \sigma_\eta^2} \tag{A.7}$$

Equation (7) in the main text can then easily be obtained by plugging (A.7) into (A.3), using the fact that $INV_t^R = INV_t + \eta_t$ and taking the first order condition with respect to $INV_t$.

## Appendix B: Proof of the existence of $D_4$ and calculation of boundaries for this coefficient.

From equation (11) in the main text we can derive the following:

$$\frac{\partial F(D_4)}{\partial D_4} = -\frac{2\beta\varphi\delta\rho^2 k^2 \sigma_\nu^2 \sigma_\eta^2}{[k^2((D_2 + \rho D_4)^2 \sigma_\nu^2 + \sigma_\eta^2)]^2} (D_2 + \rho D_4) \tag{B.1}$$

Furthermore, from equation (11) it is clear that $D_4$ will be strictly negative if a solution exists. This allows us to draw the following conclusion from equation (B.1):

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[29] Equation (A.7) can be found by means of linear regression. If it holds that $INV_t^R = g(t)$ speculators will know for sure that there was no innovation to the central bank's objectives in period t. Therefore, the intercept in this regression will be equal to zero. As far as the slope of the regression line is concerned, the following holds: $\theta/(D_2+\rho D_4) = Cov([INV_t^R-g(t)],\nu_t)/Var([INV_t^R-g(t)])$.

$$< 0 \quad if \quad D_4 > - \frac{1}{\rho} \frac{\varphi \delta}{(k^2 + \varphi \delta^2)}$$

$$\frac{\partial F(D_4)}{\partial D_4} \qquad = 0 \quad if \quad D_4 = - \frac{1}{\rho} \frac{\varphi \delta}{(k^2 + \varphi \delta^2)} \qquad \text{(B.2)}$$

$$> 0 \quad if \quad D_4 < - \frac{1}{\rho} \frac{\varphi \delta}{(k^2 + \varphi \delta^2)}$$

An examination of the function $F(D_4)$ yields:

$$F(0) = - \frac{\beta \varphi \delta \rho}{k^2} \frac{D_2^2 \sigma_\nu^2}{D_2^2 \sigma_\nu^2 + \sigma_\eta^2} \equiv \gamma D \quad where \quad 0 \leq \gamma \leq 1$$

$$F\left( - \frac{1}{\rho} \frac{\varphi \delta}{(k^2 + \varphi \delta^2)} \right) = 0 \qquad \text{(B.3)}$$

$$\lim_{D_4 \to - \infty} F(D_4) = - \frac{\beta \varphi \delta \rho}{k^2} \equiv D$$

Equations (B.2) and (B.3) can be summarized by the following picture[30]:
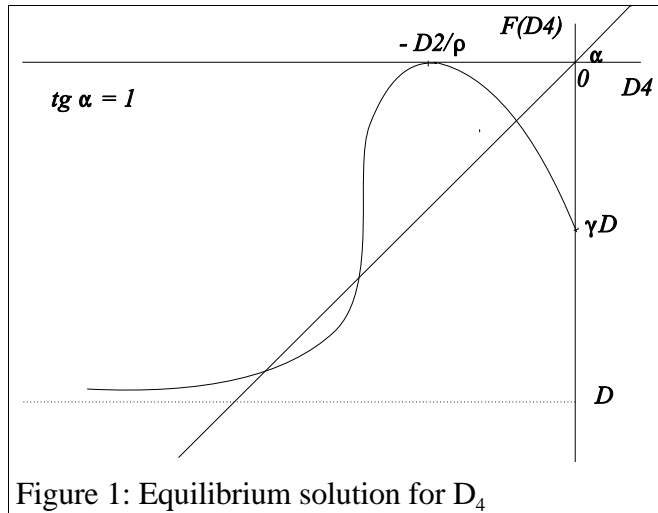


Figure 1: Equilibrium solution for $D_4$

Figure 1 reveals that the function $F(D_4)$ is monotonically increasing between minus infinity (where $F(D_4)$ approaches a minimum at $- \beta \varphi \delta \rho / k^2$ ) and $- D_2/\rho$ where the function reaches a maximum. Within this part of the domain of $F(D_4)$ (i.e. within the interval $(-\infty, - D_2/\rho)$ ) there

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[30] A similar picture appears in Cukierman (1992), chapter 15 appendix C.

are two possibilities. First of all, it could be that $F(D_4)$ never intersects with the 45°-line in which case there is no solution for $D_4$ in this range. Alternatively, there could also be two intersections with the 45°-line (as shown in Figure 1) yielding two solutions in the range under consideration[31].

As far as the interval $(- D_2/\rho , 0)$ is concerned, one can observe that $F(D_4)$ is strictly decreasing in $D_4$ in this part of its domain. Furthermore, since both the maximum value of $F(D_4)$ (which is reached at $D_4 = -D_2/\rho$) and the value of $F(0)$ $(= \gamma D)$ are strictly smaller than zero, it must be that there exists one and only one solution for $D_4$ on the interval $(- D_2/\rho , 0)$.

These considerations lead us to make the following assumption concerning the boundary conditions for $D_4$:

$$- \frac{1}{\rho}D_2 < D_4 < 0 \qquad (B.4)$$

The reason for selecting this interval is twofold. First of all, we will ensure existence of an equilibrium value for $D_4$ by choosing the latter in the range described by equation (B.4). After all, there may or may not exist a solution in the interval $(-\infty , -D_2/\rho)$ depending on the exact parameter configuration of $F(D_4)$. Secondly, it is the only solution for $D_4$ that is consistent with the intuitively plausible notion that the central bank will ceteris paribus buy foreign exchange if it desires a depreciation. In other words, the following partial derivative (which can be obtained from equation (13) in the main text) should, in our view, be strictly positive:

$$\frac{\partial INV_t}{\partial p_t} = D_2 + \rho D_4 \qquad (B.5)$$

It can easily be seen that this derivative will only be positive if $D_4$ is in the range described by equation (B.4). Any other possible solution to $D_4$ would imply that the ceteris paribus deterring effect of higher future expectations of interventions (as measured by the term $\rho D_4$) outweighs the ceteris paribus effect of a stronger desired depreciation today (as measured by the term $D_2$). While the deterring effect mentioned is obviously highly relevant in explaining observed intervention behavior it cannot realistically be assumed to be this strong.

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[31] Of course it could also be the case that the 45°-line through the origin is at some point in the range specified exactly equal to the slope of the $F(D_4)$-curve, in which case there exists exactly one solution on the interval $(-\infty , - D_2/\rho)$.

## Appendix C: Derivation of the reaction function of the speculators

Equation (14) in the main text can easily be obtained as follows: First of all, we plug equation (A.6) into (A.7) and use the resulting expression in (A.3) to get an expression for $INV_{t+1}^e$ in terms of exogenous variables and undetermined coefficients only. Subsequently, we can replace the latter by using the expressions obtained in equation (10). Lagging the result by one period yields equation (14).

## Appendix D: Derivation of an expression for $E_t(INV_{t+1}\text{-}INV_{t+1}^e)$

Taking expectations conditional on the speculators' information set in period t across equation (13) and subtracting the resulting expression from (13) we obtain:

$$INV_t - INV_t^e = (\frac{\varphi\delta}{k^2+\varphi\delta^2}+\rho D_4)(p_t - E(p_t|I_t)) -$$
$$\delta D_4[E_t(INV_{t+1}-INV_{t+1}^e) - E(E_t(INV_{t+1}-INV_{t+1}^e)|I_t)]$$

(D.1)

Regarding this equation the following can be noted:

$$E(E_t(INV_{t+1}-INV_{t+1}^e)|I_t) = 0$$
$$p_t - E(p_t|I_t) = v_t + \rho(v_{t-1} - E(v_{t-1}|I_t))$$

(D.2)

Plugging the expressions obtained in this equation back into equation (D.1), using equation (A.7) and leading the result one period yields:

$$INV_{t+1} - INV_{t+1}^e = (\frac{\varphi\delta}{(k^2+\varphi\delta^2)}+\rho D_4)(v_{t+1} + \rho(1-\theta)v_t - \frac{\rho\theta(k^2+\varphi\delta^2)}{\varphi\delta+\rho D_4(k^2+\varphi\delta^2)}\eta_t)$$
$$- \delta D_4 E_{t+1}(INV_{t+2}-INV_{t+2}^e)$$

(D.3)

Finally, taking expectations across equation (D.3) conditional on the central bank's information set in period t we obtain:

$$E_t(INV_{t+1}-INV_{t+1}^e) = (\frac{\varphi\delta}{k^2+\varphi\delta^2}+\rho D_4)\rho(1-\theta)v_t$$

(D.4)

Here, we have used the fact that:

$$E_t(E_{t+1}(INV_{t+2} - INV^e_{t+2})) = E_t(INV_{t+2} - INV^e_{t+2}) = 0 \qquad (D.5)$$

Equation (D.5) is a direct result from the observation that the expression on the LHS of equation (D.1) is affected by shock realisations in periods t-1, t and perhaps later periods but most certainly not by shocks realized in periods t-i where i ≥ 2. Consequently, shocks that are realized in period t and that are therefore part of the central bank's information set in this period cannot influence surprise interventions in period t+2.

## Appendix E: Proof that, expect for the reaction coefficient for $\eta_{t-1}$, the central bank displays a less vigourous response to various shocks under asymmetric information.

The proof for the reaction coefficient for $\varepsilon_t$ and the one for $p_{t-2}$ can be seen quite easily by noting that the coefficient $D_4$ is strictly negative. Furthermore, the proof for the coefficient for $v_{t-1}$ follows from the fact that the inequality assumed initially[32] can be reduced to the following:

$$\frac{\varphi^2 \delta^3}{k^2}(1-\theta) > \rho D_4 \frac{(k^2 + \varphi \delta^2)^2}{k^2} \qquad (E.1)$$

This inequality always holds because the LHS of this equation is strictly positive while the RHS is strictly negative.

Finally, as far as the proof for the coefficient accompanying $v_t$ is concerned, we note that the assumed inequality can be rewritten as follows:

$$\frac{\varphi \delta}{(k^2 + \varphi \delta^2)} - \delta(1-\theta)(\rho D_4)^2 + \rho D_4 \frac{(k^2 + \theta \varphi \delta^2)}{(k^2 + \varphi \delta^2)} < \varphi \delta^2 \qquad (E.2)$$

Naturally, this inequality is always true since the first term on the LHS is strictly smaller than the term appearing on the RHS and both the second and the third term on the LHS are strictly negative.

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[32] By which we mean, for the sake of clarity, the inequality expressing that the coefficient under symmetric information is strictly greater than its counterpart under asymmetric information.

## Appendix F: Proof of Propositions 2 and 3

From equation (20) it can easily be seen that the absolute value of $B_t$ will be strictly increasing in $D_4$ since this coefficient is always negative. Consequently, to prove that, for example, the intervention bias will be reduced as a result of an increase in one of the parameters mentioned it is sufficient to prove that the coefficient $D_4$ is decreasing in this parameter. The latter proof can be obtained by calculating the partial derivative of $F(D_4)$ with respect to the parameter under consideration (see equation (11)). If it is found that $F(D_4)$ is decreasing in a certain parameter it can be concluded that $D_4$ itself will be a decreasing function of this parameter as well. This can be verified with the aid of Figure 1 from which it can be seen that a downward shift of the $F(D_4)$-curve implies a decrease in the equilibrium solution for $D_4$. The proof for $\beta$, $\sigma_v^2$ and $\sigma_\eta^2$ in Proposition 2 then simply follows from the following partial derivatives:

$$\frac{\partial F(D_4)}{\partial \beta} = -\frac{\varphi \delta \rho (D_2 + \rho D_4)^2 \sigma_v^2}{k^2((D_2 + \rho D_4)^2 \sigma_v^2 + \sigma_\eta^2)} \qquad < 0$$

$$\frac{\partial F(D_4)}{\partial \sigma_v^2} = -\frac{\beta \varphi \delta \rho (D_2 + \rho D_4)^2 \sigma_\eta^2}{k^2((D_2 + \rho D_4)^2 \sigma_v^2 + \sigma_\eta^2)^2} \qquad < 0 \qquad \text{(F.1)}$$

$$\frac{\partial F(D_4)}{\partial \sigma_\eta^2} = \frac{\beta \varphi \delta \rho (D_2 + \rho D_4)^2 \sigma_v^2}{k^2((D_2 + \rho D_4)^2 \sigma_v^2 + \sigma_\eta^2)^2} \qquad > 0$$

The proof for the parameter $\zeta$ can easily be obtained by noting that the partial derivative of the absolute value of the coefficient in equation (20) with respect to this parameter (which is equal to $(D_4(k^2 + \varphi \delta^2)) / k^2$) is negative since $D_4$ is strictly negative.

To prove Proposition 3 we start by noting that, using equation (9) the following will hold:

$$\frac{\partial \theta}{\partial \beta} = 2\rho (D_2 + \rho D_4)\sigma_v^2 \sigma_\eta^2 \frac{\partial D_4}{\partial \beta} \quad < 0 \qquad \text{(F.2)}$$

Furthermore, to prove the effect of an increase in $\sigma_v^2$ and the effect of a decrease in $\sigma_\eta^2$ on the speed of learning we can use the following part of equation (10):

$$D_4 = -\frac{\beta \varphi \delta \rho \theta}{k^2} \qquad \text{(F.3)}$$

From equation (F.1) it follows that a larger value of $\sigma_v^2$ and/or a smaller value of $\sigma_\eta^2$ will cause a decline in the coefficient $D_4$ (or, in other words, the absolute value of $D_4$ will increase

because this coefficient will always be non-positive). Since all the other parameters on the RHS of equation (F.3) are not affected by these variances, it must be true that the alterations in these variances mentioned will bring about an increase in the parameter $\theta$.

## Appendix G: Proof that the average intervention effectiveness is strictly larger in the presence of ambiguity

To prove that interventions will be more effective on average if the central bank retains some degree of ambiguity it is sufficient to prove that the coefficient accompanying $\sigma_v^2$ is strictly larger when it holds that $\sigma_\eta^2 > 0$. This amounts to proving the following inequality holds:

$$(1-\theta)^2\rho^2 + (1-\delta(1-\theta)\rho D_4)^2 > 1 \tag{G.1}$$

Since the coefficient $D_4$ is always strictly negative it must be that the second term on the LHS of this inequality is strictly greater than 1. Hence, this inequality will always hold since the first term on the LHS is strictly greater than 0.