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# ADDITIVE STABLE SOLUTIONS ON PERFECT CONES OF COOPERATIVE GAMES

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# Additive stable solutions on perfect cones of cooperative games.*

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**Abstract**

Closed kernel systems of the coalition matrix turn out to correspond to cones of games on which the core correspondence is additive and on which the related canonical barycentric solution is additive, stable and continuous. Different perfect cones corresponding to closed kernel systems are described.

**Key words:** Cooperative game, core, barycenter of the core, perfect cone of games.

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# 1 Introduction

An $n$-person cooperative game with player set $N = \{1, 2, \dots, n\}$ is a map $v : 2^N \to \mathbb{R}$ with $v(\phi) = 0$. Here $2^N$ is the collection of subsets of $N$. The family $G^N$ of $n$-person games is a linear space of dimension $2^n - 1$. The core $C(v)$ of a game $v \in G^N$ is the bounded polyhedral set

$$C(v) = \{x \in \mathbb{R}^n | \sum_{i=1}^{n} x_i = v(N), \sum_{i \in S} x_i \geqslant v(S) \text{ for each } S \in 2^N\}.$$

Games with a non-empty core are called balanced games and the cone of such games is denoted by $BA^N$. In this paper we concentrate on subcones of $BA^N$. For a $v \in BA^N$ the non-empty core $C(v)$ is equal to the convex hull of its finite set of extreme points: $C(v) = conv(ext(C(v))$. At first sight, an appealing payoff distribution of the worth $v(N)$ of the grand coalition for $v \in BA^N$ is the average of the extreme points of the core, which we will call the *straight barycenter* and denote it by $\beta(v)$.
Example 1.1 shows that this straight barycenter $\beta$ is discontinuous on $BA^N$ for $n > 2$, and that is not very attractive. As an escape from this inconvenience we will introduce special cones of games, called perfect cones, where taking into account in an implicit way the natural multiplicities of extreme points, the resulting barycentric solution is not only continuous but also additive. Furthermore, on these cones the core is an additive correspondence.

**Example 1.1**
Let $N = \{1, 2, 3\}$ and let $v : 2^N \to \mathbb{R}$ be the game with $v(N) = 2$, $v(\{1, 3\}) = v(\{2, 3\}) = 1$, $v(\{1, 2\}) = 0$, $v(\{1\}) = v(\{2\}) = v(\{3\}) = v(\emptyset) = 0$; and for each $k \in \mathbb{N}$, let $v_k : 2^N \to \mathbb{R}$ be the game with $v_k(\{3\}) = k^{-1}$ and $v_k(S) = v(S)$ for $S \neq \{3\}$. Then $ext(C(v)) = \{(0, 0, 2), (0, 1, 1), (1, 0, 1), (1, 1, 0)\}$ consists of four elements and the straight barycenter $\beta(v)$ is equal to $(\frac{1}{2}, \frac{1}{2}, 1)$. For each $k \in \mathbb{N}$ the set

$$ext(C(v^k)) = \{(0, 0, 2), (0, 1, 1), (1, 0, 1), (1 - k^{-1}, 1, k^{-1}), (1, 1 - k^{-1}, k^{-1})\}$$

consists of five elements, such that

$$\beta(v^k) = \frac{1}{5}(3 - k^{-1}, 3 - k^{-1}, 4 + 2k^{-1}).$$

Note that

$$\lim_{k \to \infty} \beta(v^k) = \frac{1}{5}(3, 3, 4) \neq (\frac{1}{2}, \frac{1}{2}, 1) = \beta(v),$$

while $\lim_{n\to\infty} \parallel v_n - v \parallel = \lim_{n\to\infty} \max\{|v_n(S) - v(S)| \, |S \in 2^N\} = 0$. Hence $\beta : BA^N \to \mathbb{R}^n$ is not continuous for 3-person games.

The outline of this paper is as follows. In section 2 we introduce for $n$-person games the coalition matrix of size $(2^n - 1) \times n$, then define kernels and closed kernel systems leading to perfect cones of games on which the core and the barycenter are additive. In section 3 different perfect cones are discussed. Some of them are already well-known from the literature and our approach casts new light on them and on related solution concepts.

## 2 Closed kernel systems and perfect cones of games

Let $N = \{1, 2, \ldots , n\}$ and $S \in 2^N$. The characteristic vector $e^S$ of coalition $S$ is the vector in $\mathbb{R}^n$ with $(e^S)_i = 1$ if $i \in S$, and $(e^S)_i = 0$ otherwise. Let us arrange the non-empty coalitions of $N$ in a sequence $S_1, S_2, \ldots , S_{2^n-1}$ such that $e^{S_1} \geqslant_L e^{S_2} \geqslant_L \ldots \geqslant_L e^{S_{2^n-1}}$, where $\geqslant_L$ *is* the lexicographic order on $\mathbb{R}^n$. So, for $n = 3$ we obtain the sequence of coalitions $N, \{1, 2\}, \{1, 3\}, \{1\}, \{2, 3\}, \{2\}, \{3\}$ corresponding to the sequence of characteristic vectors $(1, 1, 1), (1, 1, 0), (1, 0, 1), (1, 0, 0), (0, 1, 1), (0, 1, 0), (0, 0, 1)$, decreasing according to the lexicographic order on $\mathbb{R}^3$. We can introduce now the $n$-person *coalition matrix* $A$, where the $n$ columns correspond to the players in the order $1, 2, \ldots , n$ and the $2^n - 1$ rows to the coalitions $S_1, S_2, \ldots , S_{2^n-1}$. Row $k$ in $A$ equals the characteristic vector of $S_k$. More precisely, the element $a_{pq}$ in cell $(p, q)$ of $A$ is equal to 1 if $q \in S_p$, and equals 0 otherwise.

The importance of the coalition matrix is revealed by the formula

$$C(v) = \{x \in \mathbb{R}^n | Ax \geqslant \underline{v}, \ \sum_{i=1}^{n} x_i \leqslant v(N)\}$$

where $\underline{v} = (v(S_1), v(S_2), \ldots , v(S_{2^n-1}))$.

A non-singular $n \times n$ - submatrix $B$ of $A$ with $e^N$ as first row will be called a *kernel*. The set of kernels will be denoted by $\mathcal{K}^N$. Given $v \in BA^N$, each kernel $B \in \mathcal{K}^N$ corresponds to a point $p(B, v)$ in the pre-imputation set

$$I^p(v) = \left\{ x \in \mathbb{R}^n \middle| \sum_{i=1}^{n} x_i = v(N) \right\}$$

of the game $v$, where $p(B, v)$ is the unique solution of the equation $Bx = \underline{v}_B$. Here $\underline{v}_B$ is the restriction of the vector $\underline{v}$ to those coordinates which correspond to rows of $B$.

If $p(B,v)$ is also a solution of the system of inequalities $Ax \geqslant \underline{v}$, then $p(B,v)$ is an extreme point of the core $C(v)$. Conversely, it follows from the theory of linear inequalities, that for each $x \in C(v)$, there is at least one kernel $B$ such that $x = p(B,v)$.

For further use we develop some notation for special kernels and then give an example. Let $A$ be the coalition matrix for $n$-person games. For each $\sigma \in \Pi(N)$, where $\Pi(N)$ is the set of $n!$ ordering of $N$, the kernel with rows $e^{\{\sigma(1)\}}, e^{\{\sigma(1),\sigma(2)\}}, \dots, e^N$ is denoted by $M^\sigma$. Then $p(M^\sigma, v)$ equals the marginal vector $m^\sigma(v)$ for which the $k$-th coordinate $(k \in N)$ is equal to $v(\{\sigma(1), \sigma(2), \dots, \sigma(k)\}) - v(\{\sigma(1), \dots, \sigma(k-1)\})$. Recall that the Shapley value (Shapley, 1953) of a game is the average of the $n!$ marginal vectors.

For each $k \in N$, the kernel of $A$ with row set $\{e^N\} \cup \{e^{\{i\}} | i \in N \backslash \{k\}\}$ is denoted by $E^k$. The kernels $E^1, E^2, \dots, E^n$ correspond to extreme points of the imputation set $I(v) = \{x \in \mathbb{R}^n | \sum_{i=1}^{n} x_i = v(N),\ x_i \geqslant v(\{i\}) \text{ for each } i \in N\}$ of a game $v$, if this imputation set is non-empty.

For each $k \in N$, the kernel of $A$ with row set $\{e^N\} \cup \{e^{N \backslash \{i\}} | i \in N \backslash \{k\}\}$ is denoted by $E^k_*$. The kernels $E^1_*, E^2_*, \dots, E^n_*$ correspond to the extreme points of the dual imputation set $I^*(v) = \{x \in \mathbb{R}^n | x_i \leqslant v^*(\{i\}) \text{ for each } i \in N, \sum_{i=1}^{n} x_i = v(N)\}$, if this set is non-empty. Here $v^*(\{i\}) = v(N) - v(N \backslash \{i\})$.

**Example 2.1**

Let $v$ be the 3-person game of example 1.1. Then $C(v) = \{x \in \mathbb{R}^3 | Ax \geqslant \underline{v},\ \sum_{i=1}^{n} x_i = v(N)\}$, where $A$ is the $7 \times 3$ - coalition matrix and $\underline{v} = (2,0,1,0,1,0,0)$. The extreme point $(1,0,1)$ of the core corresponds to the kernel $M^{(2,3,1)}$ and the extreme point $(0,1,1)$ to the kernel $M^{(1,3,2)}$. The extreme point $(0,0,2)$ corresponds to the three kernels $M^{(1,2,3)}, M^{(2,1,3)}$ and $E^3$. Further the extreme point $(1,1,0)$ corresponds to the kernels $M^{(3,2,1)}, M^{(3,1,2)}$ and $E^3_*$.

Take $\mathcal{L} \subset \mathcal{K}^N$, a non-empty set of kernels. Let $c_+(\mathcal{L})$ be the set of $n$-person balanced games, where the kernels in $\mathcal{L}$ are active i.e.

$$c_+(\mathcal{L}) = \{v \in G^N | p(B,v) \in C(v) \text{ for each } B \in \mathcal{L}\}.$$

Let $c_0(\mathcal{L})$ be the subset of games of $c_+(\mathcal{L})$, where each extreme point of $C(v)$ corresponds to at least one kernel in $\mathcal{L}$. Hence,

$$c_0(\mathcal{L}) = \{v \in c_+(\mathcal{L}) | ext(C(v)) = \{p(B,v) | B \in \mathcal{L}\}\}.$$

Note that for $v, w \in G^N$ and $B \in \mathcal{K}^N$ we have

$$p(B, v+w) = p(B,v) + p(B,w) \tag{2.1}$$

This implies that for each $\mathcal{L} \subset \mathcal{K}^N$, $\mathcal{L} \neq \emptyset$, the set $c_+(\mathcal{L})$ is a cone in the linear space $G^N$, that is $v + w \in c_+(\mathcal{L})$, if $v, w \in c_+(\mathcal{L})$. The set $c_0(\mathcal{L})$ is not necessarily a cone as we learn from example 2.3 below. This leads to the following

**Definition 2.1**
Let $\mathcal{L} \subset \mathcal{K}^N$, $\mathcal{L} \neq \emptyset$. Then $\mathcal{L}$ is called a *closed kernel system*, if $c_0(\mathcal{L})$ is a cone. For a closed kernel system $\mathcal{L}$, the cone $c_0(\mathcal{L})$ is called the *perfect cone* corrresponding to $\mathcal{L}$.

**Definition 2.2**
Let $\mathcal{K}$ be a perfect cone corresponding to the closed kernel system $\mathcal{L} \subset \mathcal{K}^N$. Then the map $\psi^{\mathcal{L}} : \mathcal{K} \to \mathbb{R}^n$, defined by

$$\psi^{\mathcal{L}}(v) = |\mathcal{L}|^{-1} \sum\nolimits_{B \in \mathcal{L}} p(B, v)$$

is called the *$\mathcal{L}$-barycentric solution* on $\mathcal{K}$, or shortly the *barycentric solution* on $\mathcal{K}$.

**Example 2.2**
For 2-person games there are three relevant kernel systems: $\mathcal{L}_1 = \{E_2\}$, $\mathcal{L}_2 = \{E_1\}$, $\mathcal{L}_3 = \{E_1, E_2\}$. Then $c_+(\mathcal{L}_1) = c_+(\mathcal{L}_2) = c_+(\mathcal{L}_3) = \{v \in G^{\{1,2\}} | v(\{1,2\}) \geqslant v(\{1\}) + v(\{2\})\}$, the cone $BA^{\{1,2\}}$ of balanced games. $\mathcal{L}_1$ and $\mathcal{L}_2$ are closed kernel systems corresponding to the cone of additive 2-person games, so

$$c_0(\mathcal{L}_1) = c_0(\mathcal{L}_2) = \{v \in G^{\{1,2\}} | v(\{1,2\}) = v(\{1\}) + v(\{2\})\}.$$

Further $c_0(\mathcal{L}_3) = BA^{\{1,2\}}$. Hence, for 2-person games all kernel systems are closed.

**Example 2.3**
Consider for 3-person games the kernel system $\mathcal{L} = \{M^{(1,2,3)}, M^{(2,1,3)}\}$. Then the (glove) game $v$ given by $\underline{v} = (1, 0, 1, 0, 1, 0, 0)$ with $C(v) = \{(0, 0, 1)\}$ is an element of $c_0(\mathcal{L})$ and also the ($\{1, 2\}$-unanimity) game $w$ with $\underline{w} = (1, 1, 0, 0, 0, 0, 0)$, for which the core is the line segment with extreme points $(1, 0, 0)$ and $(0, 1, 0)$. The core of $v + w$ is the triangle with vertices $(1, 0, 1), (0, 1, 1)$ and $(1, 1, 0)$, and $p(M^{(1,2,3)}, v + w) = (0, 1, 1)$, $p(M^{(2,1,3)}, v+w) = (1, 0, 1)$, which implies that $v+w \in c_+(\mathcal{L})$. But $v+w \notin c_0(\mathcal{L})$ because the extreme point $(1, 1, 0)$ of $C(v + w)$ does not correspond to a kernel in $\mathcal{L}$. This implies that $\mathcal{L}$ is not a closed kernel system.

Now we come to our main theorem.

**Theorem 2.1**
Let $\mathcal{K}$ be a perfect cone corresponding to the closed kernel system $\mathcal{L} \subset \mathcal{K}^N$. Then

(i) $C : \mathcal{K} \twoheadrightarrow \mathbb{R}^n$ is an additive correspondence,

(ii) $\psi^{\mathcal{L}} : \mathcal{K} \to \mathbb{R}^n$ is an additive solution,

(iii) $\psi^{\mathcal{L}} : \mathcal{K} \to \mathbb{R}^n$ is continuous.

**Proof**

(i) Take $v, w \in \mathcal{K} = c_0(\mathcal{L})$. Then $v+w \in c_0(\mathcal{L})$ and we have to prove that $C(v+w) = C(v) + C(w)$. Note that $C(v) + C(w) = conv(ext(C(v))) + conv(ext(C(w))) = conv\{p(B,v)|B \in \mathcal{L}\} + conv\{p(B,w)|B \in \mathcal{L}\} \supset conv\{p(B,v) + p(B,w)|B \in \mathcal{L}\} = conv\{p(B, v + w)|B \in \mathcal{L}\} = C(v + w)$, where the last equality follows from $v + w \in c_0(\mathcal{L})$ and the earlier equality from (2.1). Since, trivially, $C(v) + C(w) \subset C(v + w)$, we have shown that $C(v + w) = C(v) + C(w)$.

(ii) The additivity of $\psi^{\mathcal{L}}$ follows from the fact that for all $B \in \mathcal{L}$ and $v, w \in \mathcal{K}$ : $p(B, v + w) = p(B, v) + p(B, w)$.

(iii) The continuity of $\psi^{\mathcal{L}}$ follows from the fact that for each $B \in \mathcal{L}$, the function $v \mapsto p(B, v) = B^{-1}\bar{v}_B$ is continuous on $\mathcal{K}$. □

In section 3 we describe some closed kernel systems and the corresponding barycentric solutions.

# 3 Examples of closed kernel systems and barycentric solutions

Let us start with the cone $CONV^N$ of convex games (see Shapley, 1971). A game $v \in CONV^N$ if $v(S \cup T) + v(S \cap T) \geqslant v(S) + v(T)$ for all $S, T \in 2^N$. It is well-known (see Ichiishi, 1983) that a game $v$ is convex if and only if each marginal vector $m^\sigma(v)$ is in the core. Hence, we have

**Theorem 3.1**
The kernel system $\mathcal{L}_c = \{M^\sigma|\sigma \in \Pi(N)\}$ is a closed kernel system corresponding to the perfect cone $CONV^N$ of convex games. The corresponding barycentric solution $\psi^{\mathcal{L}_c}$ coincides with the Shapley value on $CONV^N$. Further $C : CONV^N \twoheadrightarrow \mathbb{R}^n$ is additive.

**Remark 3.1**
In Dragan et al. (1989), in an alternative way it was also proved that the core is additive on the cone of convex games.

Let us consider now the kernel system $\mathcal{L}_i = \{E^1, E^2, \dots, E^n\}$.
Then for each $v \in c_+(\mathcal{L}_i)$, the core element $p(E^k, v)$, where $p_i(E^k, v) = v(\{i\})$ if $i \in N\backslash\{k\}$ and $p_k(E^k, v) = v(N) - \sum_{i \in N\backslash\{k\}} v(\{i\}$, is an extreme point of the imputation set $I(v)$.
Hence, $I(v) = conv\{p(E^k, v)|k \in N\} \subset C(v)$. Since, trivially, $C(v) \subset I(v)$, we conclude that $I(v) = C(v)$. So $ext(C(v)) = ext(I(v)) = \{p(B, v)|B \in \mathcal{L}_i\}, c_+(\mathcal{L}_i) = c_0(\mathcal{L}_i)$. Hence, we obtain

**Theorem 3.2**
The kernel system $\mathcal{L}_i = \{E^i|i \in N\}$ is a closed kernel system, corresponding to the perfect cone $K_i$ of balanced games for which the core is equal to the imputation set. Further $K_i = \{v \in G^N|v(S) \leqslant \sum_{i \in S} v(i)$ for all $S \in 2^N$, $\sum_{i \in N} v(i) \leqslant v(N)\}$, and the additive barycenter solution $\psi^{\mathcal{L}_i}$ coincides with CIS, the center of the imputation set solution.

Now, we consider the kernel system $\mathcal{L}_d = \{E^1_*, E^2_*, \dots, E^n_*\}$. Similarly as above we obtain

**Theorem 3.3**
The kernel system $\mathcal{L}_d = \{E^i_*|i \in N\}$ is a closed kernel system, corresponding to the perfect cone $K_d$ of balanced games $v$ for which $C(v) = I^*(v)$. Further

$$K_d = \left\{ v \in G^N|v^*(S) \geqslant \sum_{i \in S} v^*(i) \text{ for all } S \neq N, \sum_{i \in N} v^*(i) = v(N) \right\},$$

where

$$v^*(S) = v(N) - v(N\backslash S) \text{ for each } S \in 2^N.$$

The additive barycenter solution $\psi^{\mathcal{L}_d}$ coincides with $ENSR$, the rule which splits equally the non-separable rewards.

**Remark 3.2**
The cone $K_d$ was studied in Driessen and Tijs (1983) and it was proved that the center of the core of a game in $K_d$ coincides with the nucleolus (Schmeidler, 1969) and with the $\tau$-value (Tijs, 1981).

In Brânzei and Tijs (2001) the cone $GBB^N$ of general big boss games (with player $n$ as big boss) is introduced, which contains the cone of big boss games (cf. Muto et al. (1988)). Here, $GBB^N = \{v \in G^N | f^n(v) \in C(v), g^n(v) \in C(v)\}$, where $f^n(v)$ is the extreme point $p(E^n, v)$ of the imputation set of $v$, and $g^n(v)$ is the extreme point $p(E^n_*, v)$ of the dual imputation set. They proved that

(i) $GBB^N$ is a cone,

(ii) $\emptyset = C(v) = \{x \in \mathbb{R}^n | v(i) \leqslant x_i \leqslant v^*(i)$ for each $i \in N\backslash\{n\}, \sum_{i=1}^{n} x_i = v(N)\}$

(iii) $B : GBB^N \to \mathbb{R}^N$ is additive, where $B(v) = \frac{1}{2}(f^n(v) + g^n(v))$

(iv) $B(v)$ is equal to $\tau(v)$ iff $v(N\backslash\{n\}) = \sum_{i=1}^{n-1} v(i)$.

In view of (ii) it is obvious that for a game $v \in GBB^N$ the extreme points are of the form $p(B^S, v)$, where for $S \subset N\backslash\{i\}$, $B^S$ is the kernel with row set $\{e^N\} \cup \{e^i | i \in S\} \cup \{e^{N\backslash\{i\}} | i \in N\backslash(S \cup \{n\})\}$. Note that $p_i(B^s, v) = v(\{i\})$ for each $i \in S$ and $p_i(B^S, v) = v^*(\{i\})$ for each $i \in N\backslash(S \cup \{n\})$. So we obtain

**Theorem 3.4**
The kernel system $\mathcal{L}_g = \{B^S | S \subset N\backslash\{n\}\}$ is a closed kernel system, corresponding to the perfect cone of general big boss games.
Further $C : GBB^N \twoheadrightarrow \mathbb{R}^n$, $\psi^{\mathcal{L}_g} : GBB^N \to \mathbb{R}^n$ are additive, and $\psi^{\mathcal{L}_g}(v) = B(v)$ for each $v \in GBB^N$.
In case $v(N\backslash\{n\}) = \sum_{i=1}^{n-1} v(i)$ it holds that $\psi^{\mathcal{L}_g}(v) = \tau(v)$.

**Example 3.1**
The game $v$, introduced in example 1.1, is a convex game as well as a general big boss game. Then $\psi^{\mathcal{L}_c}(v) = \frac{1}{6} \sum_{\sigma \in \Pi(N)} p(M^\sigma, v) = \frac{1}{6}(2(0,0,2) + 2(1,1,0) + (1,0,1) + (0,1,1)) = (\frac{1}{2}, \frac{1}{2}, 1) = \phi(v)$ and $\psi^{\mathcal{L}_g}(v) = \frac{1}{4}((0,0,2) + (1,1,0) + (1,0,1) + (0,1,1)) = (\frac{1}{2}, \frac{1}{2}, 1) = \tau(v)$. It is not difficult to find a game $w \in CONV^N \cap GBB^N$ with $\psi^{\mathcal{L}_c}(w) \neq \psi^{\mathcal{L}_g}(w)$.

# 4 Concluding remarks

Closed kernel systems give rise to interesting perfect cones of games, where the core and the related barycentric solution are additive. In the four discussed perfect cones of games the barycentric solutions are related to well-known one point solution concepts. A topic for further research is to develop a method to find all closed kernel systems in a systematic way. One can expect that interesting new cones of games will appear.

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