Comment on 'The Spirit of Capitalism and Stock-Market Prices' by G.S. Bakshi and Z. Chen (AER, 1996) *

by

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1 Introduction

In a recent paper Bakshi and Chen (BC) (1996) argue that augmenting standard preferences by allowing wealth to directly enter the utility function helps to explain risk premia in capital markets. The motivation for this generalization comes from the observation that utility might not only be generated by consumption but also by the social status implied by high wealth. The most straightforward way to include this notion into neoclassical economics is to modify the utility function. BC propose various functional forms of preferences which have consumption, wealth and a social-wealth index as arguments. They focus on the implications of these preferences on risk premia by studying the volatility bounds of the intertemporal marginal rate of substitution (IMRS) as suggested by Hansen and Jagannathan (1991). They conclude that their preference specification enter the volatility bounds for more reasonable parameter values than the standard CRRA utility function. In particular, a relative risk aversion coefficient of about 10 is sufficient compared to about 100 for CRRA preferences.

This comment argues that this conclusion can be misleading because it does not take into account all restrictions among variables implied by the model. In particular, the authors treat consumption and wealth as independent variables when they compute the Hansen-Jagannathan bounds of the IMRS. However, one of the implications of the model is that the consumption-wealth ratio is constant and innovations in consumption and wealth are perfectly correlated. Once these restrictions are used to substitute out wealth from the IMRS, the same IMRS is obtained as in the standard CRRA specification (apart from a different discount factor) in which only consumption enters the IMRS. This implies that the preferences studied in BC have the same implications for risk premia as the standard CRRA preferences. For example, a high risk aversion coefficient is needed to enter the Hansen-Jagannathan bounds. As shown by Kocherlakota (1990), Svensson (1989), Campbell (1993) and Lettau and Uhlig (1997) the same issue arises in the related preference specification of Epstein and Zin (1989). This argument shows that it is very important to substitute out as many variables as possible from the IMRS in order to avoid misleading conclusions concerning the variability of the IMRS implied by the model.

The rest of this comment is organized as follows. Section 2 reproduces the important aspects of the model studied by BC and summarizes the implications for the Hansen-Jagannathan bounds. Section 3 substitutes out wealth from the IMRS using restrictions among consumption and wealth implied by the model and shows that the model reduces basically to the standard CRRA model. Section 4 shows that an extended version of the preferences has potentially richer implications for risk premia and Section 5 concludes.

2 The Model

2.1 The Setup

This section reproduces the most important parts of BC. Let C_t denote consumption of some investor, W_t her wealth and V_t some social-wealth index. Let the period utility function depend on these three arguments: $U(C, W, V)$. I impose the same restrictions on the derivatives of U as BC. They propose two different functional forms for U. Their Model 1 is a "absolute wealth" model:

$$U(C, W, V) = \frac{C^{1-\gamma} - 1}{1 - \gamma} W^{-\lambda}, \tag{1}$$

while their Model 2 includes the social-wealth index:

$$U(C, W, V) = \frac{C^{1-\gamma} - 1}{1 - \gamma} \left(\frac{W}{V} \right)^{-\lambda}. \tag{2}$$

See BC for motivations for these utility functions. I omit their Model 3 because it is essentially a variant of Model 2. To avoid excessive notation I present the model which is solved in section II in BC. Investors can invest in two assets. First, there is a riskless asset with return r_0 which is assumed to be constant over time. The price of the risky second asset is assumed to follow a Brownian motion:

$$\frac{dP_t}{P_t} = \mu dt + \sigma d\omega_t, \tag{3}$$

where ω_t is a standard Wiener process. The investor solves the following problem:

$$J(W_t, V_t) = \max_{C_s, \alpha_s} E_t \left[\int_t^\infty e^{-\rho(s-t)} U(C_s, W_s, V_s) ds \right] \tag{4}$$

subject to

$$dW_t = W_t(r_0 + \alpha_t(\mu - r_0)) - C_t + \alpha_t \sigma W_t d\omega_t, \tag{5}$$

where ρ denotes the discount rate.

2.2 The Solution

BC give the complete solution of the model. Here I present only the equations which are relevant for my argument. First, the consumption-wealth ratio is constant:

$$C_t^\star = \eta W_t^\star, \tag{6}$$

where variables with a '$\star$' denote the optimal choices in the above problem. The parameter η is a complicated function of the parameters of the model and is given in Proposition 2 of BC. Furthermore, the growth rates of consumption equals the growth rate of wealth:

$$\frac{dC_t^\star}{C_t^\star} = \frac{dW_t^\star}{W_t^\star} = \mu_w dt + \nu_w d\omega_t, \tag{7}$$

where μ_w and ν_w are again given in BC (p. 143). It is also worth noting that the coefficient of relative risk aversion (RRA) as measured by the curvature of the value function with respect to wealth is given by $\gamma + \lambda$ in this model.

2.3 Implications for Asset Prices

BC use the Hansen-Jagannathan (HJ) (1991) bounds for Model 1 to show this preference specification is more successful in explaining risk premia than the standard CRRA specification. They do not compute HJ-bounds of Model 2 which includes the social-wealth index. They argue that it is hard to find a satisfying functional form for the social wealth index V and question whether it is appropriate to use aggregate data to test such as model.

The HJ-bounds are derived from the stochastic disount factor (SDF) implied by the model. The discrete time version of the SDF for for Model 1 is

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{W_{t+1}}{W_t}\right)^{-\lambda} \left(1 + \frac{\lambda}{\gamma - 1}\frac{C_{t+1}}{W_{t+1}}\right). \tag{8}$$

Using aggregate data, BC find that the model satisfies the HJ-bounds for parameter configurations which imply only moderately high risk aversion. For example, when γ and λ are both around 5, RRA is around 10 and the bounds are satisfied. The authors stress that this is a great improvement over the standard CRRA preferences which require a RRA of over 100 to satisfy the bounds.

3 Substituting out Wealth

In their empirical exercise, BC treat consumption and wealth as separate variables. However, the model has strong implications for their co-movement. In particular, (6) implies that the consumption-wealth ratio is constant and hence innovations in consumption growth and wealth growth are perfectly correlated (as can be seen also from (7)). These restrictions have very important implications for the HJ-bounds and asset prices in general. Substituting out wealth in the SDF (8) using (6) yields

$$M_{t+1} \quad = \quad \beta \left(\frac{C_{t+1}}{C_t}\right)^{-(\gamma+\lambda)} \left(1 + \frac{\lambda}{\gamma - 1}\eta\right)$$

$$= \quad \tilde{\beta} \left(\frac{C_{t+1}}{C_t} \right)^{-\text{RRA}},$$

where $\tilde{\beta} = \beta \left(1 + \frac{\lambda}{\gamma - 1}\eta\right)$. Hence the SDF is the same as in the standard CRRA preference case with a different discount rate. Recall the RRA is equal to $\gamma + \lambda$ here. This implies that the HJ-bounds are basically the same as in the CRRA case (only the discount factors differ). In particular, an extremely high risk aversion is needed to enter the bounds.

Note, that a very similar problem appears in the related preferences specification of Epstein and Zin (EZ) (1989). They propose a recursive utility function that allows to disentangle risk aversion and intertemporal substitution. In addition to consumption, the return of the market portfolio enters the SDF. As pointed out by Campbell (1993) this model implies a constant consumption-wealth ratio when returns are i.i.d.. He used that fact to solve out consumption. As shown in Kocherlakota (1990) and Lettau and Uhlig (1997), one can alternatively solve out the return on the market portfolio. They show that the SDF reduces to the standard CRRA case as long as returns are i.i.d.. The important implication in the EZ framework as well as in the BC model is that high risk aversion is needed to enter the HJ-bounds once all the model implications are used.

4 A More General Model

BC compute the HJ-bounds only for their Model 1. They argue that it would not be appropriate to test Model 2 with aggregate data. However, it is instructive to compute the SDF for Model 2:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{W_{t+1}}{W_t} \right)^{-\lambda} \left(\frac{V_{t+1}}{V_t} \right)^{-\lambda} \left(1 + \frac{\lambda}{\gamma - 1} \frac{C_{t+1}}{W_{t+1}} \right). \tag{9}$$

Using again the fact that the consumption-wealth ratio is constant to substitute out wealth, I obtain

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-(\gamma+\lambda)} \left(\frac{V_{t+1}}{V_t} \right)^{-\lambda} \left(1 + \frac{\lambda}{\gamma - 1} \eta \right). \tag{10}$$

Note that (10) does not reduce the the standard CRRA case as the growth of the social-wealth index variable V remains in the SDF. Hence if V is highly volatile the SDF becomes more volatile as well making it easier to enter th HJ-bounds. Whether the volatility of this variable on a high frequency is reasonable is of course open. But in principle a framework like Model 2 of BC has an additional channel to increase the volatility of the SDF compared to the standard CRRA model and Model 1 of BC.

5 Conclusion

This comment argues that it is important to take into account all restrictions among variables implied by the model when computing the SDF of a model. If this is not done, variables will be treated as independent time series often leading to an overestimation of the volatility of the SDF. The model studied by BC is one example of this effect, but the same argument holds for the Epstein-Zin (1989) preferences.

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