Firm corruption in the presence of an auditor

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Abstract

This paper develops a framework to explore firm corruption taking account of interaction with an auditor. The basic idea is that an auditor can provide auditing and other (consultancy) services. The extent of the other services depends on firm profitability. Hence auditor profitability can increase with firm corruption that may provide an incentive to collude in corrupt practices. This basic idea is developed using a game theoretic framework. It is shown that a multiplicity of equilibria exist from stable corruption, through auditor controlled corruption, via multiple equilibria to honesty on behalf of both actors. Following the development of the model various policy options are highlighted that show the difficulty of completely removing corrupt practices.

Keywords: firm corruption, auditor corruption, perfect equilibrium
JEL codes: C70, D21, K42, L21

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1 Introduction

A characteristic feature of the economic literature on corruption is that this is viewed as a state oriented problem to which firms respond; see, for example, Rose-Ackerman (1999). Other studies relate corruption to growth, poverty or governance aspects, especially for the case of developing countries; see, for instance, Bhagwati (1982). In a context similar to that analysed by Rose-Ackerman (1999), Lambert-Mogiliansky and Sonin (2006) analyse corruption and collusion in procurement. They argue that a corrupt agent would be willing to "sell" his decision in return for a bribe. They conclude that collusion is more likely to take place within auctions in cases where firms are small relative to the market. They also argue that the risks of collusion and of corruption need to be addressed simultaneously. Lambert-Mogiliansky and Sonin (2006, p. 884) argue that:

> Besides empirical evidence, there are theoretical issues motivating the study of links between collusion and corruption. First, any cartel must solve a series of problems including agreeing how to share the spoils, securing enforcement, and deterring entry[...] A corrupt auctioneer can contribute to solving some of these problems, for example by providing means of retaliation to secure enforcement or creating barriers to entry. Second, corrupt auctioneers might seek to extract rents.

This idea that an external agent is necessary for the stability of corruption and extracts rents in generating this stability is used in this paper. But the emphasis is shifted from the external agent being an auctioneer to being an auditor. On the same theme Svensson (2005) recognises that corruption can also involve, for example, collusion between firms or misuse of corporate assets. This latter approach is adopted here, and concentrates on corrupt practices within the private firm sector. In particular a game theoretic framework is developed that examines incentives for firms to be corrupt given market based monitoring by auditors. This general idea of a game theoretic approach to corruption is, of course, not original. Carilllo (2000) constructs a dynamic model of corruption within which agents are aware of their "propensity for corruption" and their clients choose an optimal level of bribe to be offered. Such a framework provides an explanation for different implicit prices for illegal services (bribes or kick-backs) for similar countries (or organisations within similar countries), based on an analysis of reaction of clients. Two of these ideas are carried forward into the current discussion: that there is a propensity for corruption and that the reactions of other agents (here auditors) are important for the equilibria that can be generated. Specifically firm level corruption has been a significant problem, and so is worthy of further analysis. Between 1997 and 2002 nearly 10 per cent of US listed companies restated their earnings at least once due to accounting irregularities (cited in Aglietta and Reberioux, 2005). In addition, earnings restatements because of financial irregularities in the USA reached 414 cases in 2004 (cited in Coffee, 2005). These irregularities provide a context to the current discussion in a manner that is perhaps more informative than famous headline cases of firm corruption. They provide background for the opinion expressed by Jensen (2006, p. 14) while discussing the overvaluation of equity in the late 1990s and early 2000s

> In part the massive overvaluation of equity that occurred was an understandable market mistake. Society seems to overvalue what is new. But this catastrophic overvaluation was also the result of misleading data from

> managers, large numbers of naive investors, and breakdowns in the agency relationship within companies, in investment banks, and in Audit and law firms many of whom knowingly contributed to the misinformation that fed the overvaluation.

A similar view is presented by Stiglitz (2003). While discussing Enron (Stiglitz, 2003, p.244) he suggests that

> Enron used accounting tricks that were increasingly becoming standard. It appears that its chief financial officer made the same discovery that so many other corporate executives made during the nineties: the same accounting tricks that could be used to distort information to boost stock market prices could be used to enrich themselves at the expense of other shareholders.

Key issues emerge from the Jensen-Stiglitz opinions cited here. Firm corruption involves (a) a breakdown in agency relationships; (b) misleading activity not only by managers but also by (for example) audit firms; and (c) "accounting tricks" that were increasingly becoming standard. But given (a), (b) and (c) it is apparent that only some firms were corrupt, even though (following Stiglitz) the "tricks" were becoming standard. The 414, or 10 per cent, cases of earnings restatements is a significant number but a small proportion of the total number of US firms. Hence many firms decided not to do what was apparently "standard" practice. A preliminary conclusion might therefore suggest itself: that a breakdown in agency relationships is a necessary but not sufficient condition for firm corruption. Sufficiency would appear to require (a) an agency breakdown in the relationship between firms and their owners; (b) a willingness on the part of firms to exploit this and engage in "tricks"; and (c) collusion by supporting actors (e.g. auditing firms) in the "tricks". In this paper a framework is developed that assumes agency breakdown has occurred and explores the possibility of firms exploiting this and collusion by auditing firms. The basic idea in this framework is that firm corruption involves collusion between firms and auditors. But a central problem exists in this relationship: auditors provide auditing and other consultancy services. The extent of these 'other' services depends on firm profitability i.e. the ability to buy them. In turn, firm and auditor profitability increases with corruption. This can provide an incentive for not only firm corruption but also auditor collusion in this corruption. The core problem analysed in this paper has, of course been recognised by other authors. For example Posner (2006, p. 11) gives a characteristically pithy summary of the core idea for the current discussion:

> Corporate executives, moreover, hire and pay the auditors who certify the correctness of the corporation's financial statements, dangle consulting contracts in front of auditors who also offer consulting services

In brief, a game theoretic framework is developed here that is used to examine whether, and in what circumstances, stable, equilibrium corruption is possible. The structure of the paper is as follows. In the next section key assumptions involved with a corruption game are developed. In section three explicit payoffs are specified. Following this, in section four, the equilibria in the corruption game are explored. Section five explores possible policy implications. Finally, brief conclusions are drawn. The final substantive section of the discussion highlights a number of key policy conclusions that follow from the framework developed here.

2 The corruption game

In this paper we consider the scenario of a firm that has the option of pursuing a profit-making corrupt prospect in the knowledge that such corrupt activity would be detected by its auditor. However, the firm also purchases consultancy services from the auditor. We consider whether there are conditions under which corruption may be an equilibrium and examine the effectiveness of various regulatory policy interventions in dealing with corruption. We now set out the precise framework of the corruption game.

A 1. *The game has two players: a Monopolist (M) and an Auditor (A).*

A 2. *Each player's action set has two elements: Corrupt (C) or Honest (H).*

Corollary 1. *Given the action set $\{C, H\}$ we rule out the possibility that the Auditor could mislead the regulatory authorities by indicating that the Monopolist has been corrupt when it has not been corrupt.*

A 3. *The players choose their actions sequentially over two periods: the Monopolist is assumed to be the leader and the Auditor the follower. Hence, with subscripts denoting the period $\{1, 2\}$, we have:*

$$\begin{array}{ll} \textit{Period 1: M chooses} & \{C_1, H_1\}; \\ \textit{Period 2: A chooses} & [\ldots, \{C_2, H_2\}]. \end{array}$$

Corollary 2. *It follows from A1 and A2 that retaliation by the Monopolist to $\{C_1, H_2\}$ is ruled out in this game.*

This restriction of the model seems reasonable because if the Monopolist sacks the Auditor for failing to support a corrupt strategy this would involve public disclosure of the corruption. The implication of this assumption is that we can restrict analysis to a two-stage, rather than three-stage, game.

A 4. *The game is one of complete and symmetric information.*

When the Auditor selects its action, $\{C_1, H_1\}$ is known. In addition, this assumption allows the use of backward induction to solve the game.

A 5. *There is an exogenous non-corrupt gross profit for the Monopolist:*

$$\Pi_M^H > 0.$$

Thus corruption does not affect the profit attributable to the firm's underlying activity.

A 6. *The players are risk neutral and expected profit maximisers.*

A 7. *(i) The firm buys (compulsory) Auditing services and additional consultancy services from the Auditor. (ii) Ex-ante the returns to the consultancy services are uncertain. As both agents are risk neutral they share this risk with a contract that has payment based upon expected returns. (iii) A constant proportion $\alpha \in [0, 1)$ of the Monopolist's profit is allocated to purchase Auditor consultancy services. (iv) The Monopolist's gross non-corrupt profit reflects productive and market characteristics and benefits from these Auditor services. The Monopolist's net non-corrupt profit is therefore:*

$$\Pi_M\{H_1, \ldots\} \equiv \Pi_M^H(1 - \alpha).$$

A 8. *(i) The Monopolist has an opportunity to undertake corrupt activities to the value $\gamma(g)\Pi_M^H$. The parameter g measures the extent of the corrupt activity relative to the exogenous non-corrupt gross Monopoly profit. (ii) The return to the corrupt activity $\gamma(g)$ is continuous and concave on g, reflecting diminishing returns: $\gamma(g) > 0$, $\gamma''(g) \leq 0$, $\forall g \in (0, \infty)$. (iii) Corruption produces an additional gross (before taking into account consultancy fees and any penalties for detected corruption) profit gain to the Monopolist over the non-corrupt gross Monopoly profit:*

$$\Pi_M^C \equiv (1 + \gamma(g))\Pi_M^H.$$

It is important to note that the level of corruption g in *A*8 is not a (continuous) choice variable of the Monopolist, rather, the Monopolist faces a discrete choice between not being corrupt $\{H_1, \ldots\}$ and pursuing a corrupt prospect, under $\{C_1, \ldots\}$, of value $\sigma(g)\Pi_M^H$. This reflects the fact that in many cases a firm may have a limited set of opportunities for corrupt activities making g discrete rather than continuous.

A 9. *(i) In the case in which both players choose to be corrupt, the payoff to each agent is uncertain as Nature assigns a probability $\sigma(g)$ to the corrupt activity being detected and a strictly positive penalty being imposed on both Monopolist ($F_M > 0$) and Auditor ($F_A > 0$). (ii) Under $\{C_1, H_2\}$, the monopolist incurs the penalty F_M with certainty.*

Remark 1. *A*9(*ii*) *follows logically given Corollary 1 and A*4.

Corollary 3. *It follows from A*6 *and A*9 *that the players' payoffs following Nature's actions under $\{C_1, C_2\}$ can be represented by an "expected" payoff with probability weights $\sigma(g)$ and $1 - \sigma(g)$.*

We now introduce the first of a number of critical values of g that will be helpful in developing the results of the game.

Definition 1. $\hat{g} \equiv \inf(g : \sigma(g) = 1)$.

A 10. *The probability of corruption being detected under $\{C_1, C_2\}$ depends upon the level of Monopolist corruption, g, with $\sigma(0) = 0$ and according to the **corruption detection profile** $\sigma_i(g)$ $(i = 1, 2)$ where either (i) $\sigma_1(g) \in [0, 1]$, where $\sigma_1'(g) > 0$ and $\sigma_1''(g) > 0$ for $\forall g \in [0, \hat{g})$ and $\sigma_1'(g) = 0$ for $\forall g \in [\hat{g}, \infty)$ and $\hat{g} > 0$ or (ii) $\sigma_2(g) \in [0, 1)$ where $\sigma_2'(g) > 0$, $\sigma_2''(g) < 0$ and $\lim_{g \to \infty} \sigma_2(g) = T$ where $T \in (0, 1)$.*

Thus, in either case, $\sigma(g)$ is positive monotonic for $\sigma(g) < 1$, which would appear to be reasonable as higher levels of g are likely to be more conspicuous and hence more likely to be detected. The conditions regarding the second derivatives in these definitions are necessary to ensure that the functions $\sigma(g)$ and $\omega(g)$ (defined later) cross only once on their upward sloping segments. This "well behaved" property helps to facilitate transparency in the model and keep the analysis manageable. If, with relatively simple functional forms and simple interactions between these functions, unusual results arise then this will be of greater interest than if the model were so complex that it could support any outcome however unusual.

Corollary 4. *$\hat{g}$ is not defined under corruption detection profile $\sigma_2(g)$.*

The following assumption is a logical extension of the Monopolist 'non-retaliation' and Auditor 'non-misleading' properties of the model (see Corollaries 2 and 1 respectively).

A 11. *The payoffs to each player under $\{H_1, C_2\}$ are the same as under $\{H_1, H_2\}$:*

$$\Pi_i\{H_1, C_2\} \equiv \Pi_i\{H_1, H_2\} \qquad i = \{M, A\}.$$

Costs are mostly not specified explicitly within the model (they play an unspecified role in Π_M^H and $\omega(g)$, defined later), however, the following assumption introduces a cost differential for the Auditor under corruption relative to honest behaviour.

A 12. *The Auditor incurs a cost c_A of supplying services to the Monopolist. These costs are higher under $\{C_1, C_2\}$ than under $\{\ldots, H_2\}$, respectively c_A^C and c_A^H. The cost differential is defined $\triangle c \equiv c_A^C - c_A^H$ and is assumed (i) to be positive and constant (not a function of the level of corruption) and, (ii) $\triangle c < \alpha F_m$.*

We argue that the positive differential is a sensible assumption given the higher transaction costs involved with hiding corrupt practices. The constancy of this differential is not as problematic as it may appear, the reason being that we are only interested in comparisons over no corruption and a given level of corruption - the level of corruption is not a continuous choice variable.

As we see later, $A12(ii)$ ensures that the set of values of g for which the Auditor would support corruption is non-empty.

The game is illustrated in extensive form in Figure 1. Nodes M and N relate to the Monopolist and Nature, respectively, and nodes A_1 and A_2 relate to the Auditor. Payoffs are reported in parentheses - the single payoff following N is explained in Corollary 3.

Insert Figure 1 here.

Some further useful characteristics of the game are outlined below, their purpose will become apparent later.

Definition 2. *Let $\varphi(\sigma) \equiv \frac{\sigma}{1-\sigma} F_A + \frac{\triangle c}{1-\sigma}$.*

Lemma 1. *$\varphi(\sigma)$ is: (i) positive monotonic, (ii) convex in σ, and (iii)* $\lim_{\sigma \to 1^-} \varphi(\sigma) = \infty$.

Proof. It follows from A9 and A12(i) that F_A and Δc are strictly positive, hence (i) $\varphi'(\sigma) = \frac{F_A}{1-\sigma} + \frac{\sigma F_A + \triangle c}{(1-\sigma)^2} > 0$ and (ii) $\varphi''(\sigma) = 2\frac{F_A}{(1-\sigma)^2} + 2\frac{\sigma F_A + \triangle c}{(1-\sigma)^3} > 0$. (iii) Since F_A and $\triangle c$ are exogenous and finite, $\lim_{\sigma \to 1^-} \frac{1}{1-\sigma} = 0$, and so $\lim_{\sigma \to 1^-} \varphi(\sigma) = \infty$. □

The L.H.S of Figure 2, which we will see later captures the relevant information relating to the Auditor's decision, illustrates $\varphi(\sigma)$. The R.H.S. of Figure 2 is concerned with parameters affecting the Monopolist's decision of which Definition 3 introduces a key aspect.

Insert Figure 2 here.

Definition 3. *Let $\omega(g) \equiv \frac{\Pi_M^H}{F_M} \gamma(g)$. We refer to $\omega(g)$ as the Monopolist's **corruption technology profile**.*

Lemma 2. *$\omega(g)$ is (i) continuous, and (ii) concave.*

Proof. Given Π_M^H and F_M are strictly positive and exogenous, the proof follows from the properties of $\gamma(g)$ in $A8(ii)$. □

We now define further key values of g, examples of which are illustrated in Figure 2.

Definition 4. *Let (i) g^* be the discrete level of corruption available to the Monopolist under $\{C_1, \ldots\}$, (ii) $g^{**} \equiv (g : \varphi(\sigma(g)) = \alpha F_M)$.*

We will see in the next Section that g^{**} defines the level of corruption which produces a detection probability under which the Auditor is indifferent between $\{C_1, H_2\}$ and $\{C_1, C_2\}$ and that for $g^* < (>) g^{**}$ the Auditor would support (not support) a corrupt Monopolist.

Remark 2. *There exist feasible profiles $\sigma(g)$ for which g^{**} is not defined. However, it follows from $A12(ii)$ that if g^{**} is defined it always yields $\sigma(g^{**}) \in (0,1)$.*

Definition 5. *Let (i) $\tilde{g} \equiv \inf(g : \omega(g) = 1)$; (ii) $\tilde{\tilde{g}} \equiv \sup(g : \omega(g) = 1)$; (iii) $\bar{g} \equiv \inf(g : \omega(g) = \sigma(g), g \in \mathbb{R}_{++})$; (iv) $\bar{\bar{g}} \equiv \sup(g : \omega(g) = \sigma(g), g \in \mathbb{R}_{++})$; (v) $g^{max} \equiv (g : \arg\max \omega(g))$.*

Having introduced various critical values of g in the model, we can now outline the relationships between the level of Monopolist corruption and the return to corruption.

Definition 6. *We define three categories of the Monopolist's corruption technology profile, $\omega_i(g)$ $(i = a, b, c)$. In addition to the conditions placed upon $\omega(g)$ from $A8(ii)$, we have that: (i) $\omega_a(g) \equiv (\omega : \omega'(g) > 0, \forall g \in [0, \infty); \lim_{g\to\infty} \omega(g) > 1)$; (ii) $\omega_b(g) \equiv (\omega : \omega'(g) > 0, \forall g \in [0, g^{max}); \omega'(g) < 0, \forall g \in (g^{max}, \infty); \omega(g^{max}) \geq 1)$; (iii) $\omega_c(g) \equiv (\omega : \omega'(g) > 0, \forall g \in [0, \infty);\ \lim_{g\to\infty} \omega(g) = S, S \in (0,1))$.*

Insert Figure 3 here.

Given the characterisations of $\omega(g)$ in Definition 6 and $\sigma(g)$ in $A10$, we now set out five cases describing different possible relationships between $\omega(g)$ and $\sigma(g)$ in the following Definition.

Definition 7. *(i) **Case1**: $\sigma_1'(0) > \omega'(0)$; (ii) **Case2**: $\sigma_1'(0) < \omega'(0)$ and $\exists \hat{g}$ and $\bar{g}$ s.t. $\hat{g} > \bar{g}$; (iii) **Case3**: $\sigma_1'(0) < \omega'(0)$ and $\exists \tilde{g}, \hat{g}$ s.t. $\tilde{g} \leq \hat{g}$; (iv) **Case4**: $\sigma_2'(0) < \omega'(0)$ and $\bar{g}$ may exist but not $\bar{\bar{g}} \neq \bar{g}$; (v) **Case5**: $\sigma_2'(0) > \omega'(0)$ and $\bar{g}$ may exist but not $\bar{\bar{g}} \neq \bar{g}$.*

The list of Cases outlined in Definition 7 is not intended to be exhaustive. There are many obvious, though uninteresting, ways of extending the Cases but we have tried to keep them to a minimum in order to allow us to explore the equilibria arising from the model with reasonably well-behaved functions. In particular, we have explicitly limited the number of times $\omega(g)$ and $\sigma(g)$ can cross. The more complicated the scenarios the more arbitrary the predictions. Examples of the Cases1-3 are illustrated in Figure 4.

Lemma 3. *Under corruption technology $\omega_c(g)$ **Case3** is not defined.*

Proof. The proof follows directly from the the requirement in Case 3 of the existence of $\hat{g}$ in Definition 7(iii), which is ruled out under corruption technology $\omega_c(g)$ since, by Lemma 2(ii) $\omega_c(g)$ is concave and by Definition 6 (iii) $\lim_{g\to\infty} < 1$. □

Insert Figure 4 here.

Definition 7 (iv) and (v) ensure that there is a limit to the number of times the $\omega_i(g)$ and $\sigma_2(g)$ functions can cross. The following Remark makes it clear that the assumptions of the model also ensure similar crossing properties between the $\omega_i(g)$ and $\sigma_2(g)$ functions.

Remark 3. *Given $\omega(g)$ is strictly concave and $\sigma_1(g)$ is strictly convex for $g \in (0, \tilde{g})$, then: (i) $\sigma_1'(0) > \omega'(0)$ in **Case 1** implies $\sigma_1'(0) > \omega'(0)$ $\forall g \in (0, \tilde{g})$; (ii) $\omega(g)$ and $\sigma_1(g)$ in **Case 2** cross exactly once for $g \in (0, \tilde{g})$.*

For the analysis in Section 4 it is useful to make a distinction between *Perfect Nash Equilibria* (*PNE*) corruption profiles that are effectively unconstrained and those that are constrained. The following Definition makes explicit what is meant in each Case.

Definition 8. *(i) A PNE is said to support **unconstrained corruption** if corruption is PNE for $\forall g \in [a, \infty)$ where $a \in \mathbb{R}_{++}$; (ii) A PNE is said to support **constrained corruption** if $g \in [a, b]$ is PNE where a and b are finite, $a, b \in \mathbb{R}_{++}$ and $a \leq b$.*

Finally, it is also useful to classify Cases where corruption is guaranteed to be an equilibrium for sufficiently small levels of g^*.

3 Payoff Specification and equilibria

This section will specify an explicit payoff structure for the corruption game set out above. First, from $A5$, $A7$ and $A11$, the payoffs corresponding to z in Figure 1 are:

$$z_M \equiv \Pi_M\{H_1, H_2\} \equiv \Pi_M\{H_1, C_2\} \equiv (1-\alpha)\Pi_M^H, \tag{1a}$$

$$z_A \equiv \Pi_A\{H_1, H_2\} \equiv \Pi_A\{H_1, C_2\} \equiv \alpha(1-\alpha)\Pi_M^H - c_A^H. \tag{1b}$$

Given $A9$, if the Auditor does not collude in the corruption the Monopolist will face a punishment cost of F_M with certainty. Given $A7$ and $A8$ the payoffs corresponding to y in Figure 1 are:

$$y_M \equiv \Pi_M\{C_1, H_2\} \equiv (1-\alpha)[(1+\gamma(g))\Pi_M^H - F_M], \tag{2a}$$

$$y_A \equiv \Pi_A\{C_1, H_2\} \equiv \alpha[(1+\gamma(g))\Pi_M^H - F_M] - c_A^H. \tag{2b}$$

If the Auditor colludes in the corruption the probability of corruption being detected is $\sigma(g)$, by $A10$. By $A9$, if the Auditor is found to be corrupt there is a punishment cost of F_A. Hence, given $A6$ and $A12$, the payoffs corresponding to x in Figure 1 are:

$$x_M \equiv E(\Pi_M\{C_1, C_2\}) \equiv (1-\alpha)[(1+\gamma(g))\Pi_M^H - \sigma(g)F_M], \tag{3a}$$

$$x_A \equiv E(\Pi_A\{C_1, C_2\}) \equiv \alpha[(1+\gamma(g))\Pi_M^H - \sigma(g)F_M] - c_A^C - \sigma(g)F_A. \tag{3b}$$

The system of equations defined by [1], [2] and [3] in Section 3 can be used to derive the conditions under which each of the three scenarios in the game is a *PNE*. We begin by identifying the conditions under which each scenario is a *Unique Perfect Nash Equilibrium* (*UPNE*) and then consider the case of *Multiple Perfect Nash Equilibria* (*MPNE*). We will see later that separating the characterisations of *PNE* in this way facilitates the policy discussion in Section 6.

UPNE 1. $\{C_1, C_2\}$ *From Figure 1 this corruption equilibrium requires $x_A > y_A$ and $x_M > z_M$, hence, respectively, from (1a), (2b), (3a) and (3b):*

$$\varphi(\sigma) < \alpha F_M, \tag{4a}$$

$$\omega(g) > \sigma(g). \tag{4b}$$

Corollary 5. *Condition (4a) will be met and the Auditor will support Monopoly corruption iff $\sigma(g^{**}) > \sigma(g^*)$.*

UPNE 2. $\{C_1, H_2\}$ *This equilibrium involves attempted Monopoly corruption controlled by the Auditor. In terms of Figure 1 it requires* $x_A < y_A$ *and* $y_M > z_M$*, hence, respectively, from (1a), (2a), (2b) and (3b):*

$$\varphi(\sigma) > \alpha F_M, \tag{5a}$$
$$\omega(g) > 1. \tag{5b}$$

UPNE 3. $\{H_1, H_2\} \equiv \{H_1, C_2\}$ *This 'honesty' equilibrium arises under two different sets of circumstances, if: (i)* $x_A > y_A$*,* $z_M > x_M$*, requiring, respectively (4a) and:*

$$\omega(g) < \sigma(g), \tag{6a}$$

and (ii) $x_A < y_A$*,* $z_M > y_M$*, requiring, respectively (5a) and:*

$$\omega(g) < 1. \tag{6b}$$

We now consider the circumstances under which there are *MPNE.*

MPNE 1. $\{C_1, C_2\}$,$\{C_1, H_2\}$ *From Figure 1 these corruption equilibria require* $x_A = y_A$ *and* $x_M, y_M > z_M$*, hence, respectively, from (1a), (2a), (2b), (3a) and (3b):*

$$\varphi(\sigma) = \alpha F_M, \tag{7a}$$
$$\omega(g) > 1. \tag{7b}$$

MPNE 2. $\{C_1, C_2\}$,$\{H_1, \ldots\}$ *From Figure 1 these corruption equilibria require* $x_A > y_A$ *and* $x_M = z_M$*, hence, respectively, (4a), and from (1a) and (3a):*

$$\omega(g) = \sigma(g). \tag{8}$$

MPNE 3. $\{C_1, H_2\}$,$\{H_1, \ldots\}$ *From Figure 1 these corruption equilibria require* $x_A < y_A$ *and* $y_M = z_M$*, hence, respectively, (5a), and from (1a) and (2a):*

$$\omega(g) = 1. \tag{9}$$

MPNE 4. $\{C_1, C_2\}$,$\{C_1, H_2\}$,$\{H_1, \ldots\}$ *From Figure 1 these corruption equilibria require* $x_A = y_A$ *and* $x_M = y_M = z_M$*, hence, respectively, (7a), and from (8) and (9):*

$$\omega(g) = \sigma(g) = 1. \tag{10}$$

Lemma 4. *MPNE4 is not feasible.*

Proof. From Lemma 1(iii) $\lim_{\sigma \to 1^-} = \infty$ and hence from Definition 4 $\sigma(g^{**}) < 1$ which contradicts (10). □

Remark 4. *For completeness, note, there are no pure strategy PNE under* $x_A = y_A$ *where either (i)* $x_M > z_M > y_M$*, or (ii)* $y_M > z_M > x_M$*.*

Remembering that g^* is not a (continuous) choice variable, there is a clear way of ranking the three *UPNE* from a public policy point of view at a given level of g^*. *UPNE3* is the most desirable outcome as this involves the guarantee of no corrupt activity. *UPNE1* is clearly the least desirable outcome as corrupt activities may be going on undetected. *UPNE2* is an improvement upon *UPNE1* inasmuch as corruption, although it is not prevented, is detected through the functioning of the Auditor. Similarly, cases where *UPNE2* supports unconstrained corruption may involve very high levels of abuse which, though not avoided, are detected, whilst unconstrained corruption under *UPNE1* may be very high and go undetected.

Definition 9. *Labeling UPNEk* $(k = 1, 2, 3)$, *corruption equilibria are monotonically "worsening" ["improving"] in* g^* *if increasing* g^* *leads to smaller [larger]* k *for* $\forall g^* \in (0, \infty)$.

Definition 10. *Let* ***small-scale*** *(i)* ***corruption*** *be* g^* *that is supported by UPNE1 or UPNE2 for* $\forall g^* \in (0, a)$ *(ii)* ***honesty*** *be* g^* *that is supported by UPNE3 for* $\forall g^* \in (0, a)$, *where* a *is finite and* $a \in \mathbb{R}_{++}$.

4 Analysis

In this section we are interested in establishing the conditions under which corruption might be an equilibrium and even an unconstrained equilibrium. We are also interested in how these conclusions are affected by changes in the corruption technology and detection profiles. In particular we will seek to establish whether the equilibria of the model under a particular technology/detection profile combination is monotonically worsening, improving or non-monotonic in the level of g^*. However, it is important to be clear about what we are seeking to establish in this exercise. Given g^* is an exogenous variable we are not actually concerned with changes in the level of g^* as this is not in the gift of either of the players or the regulatory authority. Instead, "increasing" g^* is a simple way of representing local stretching or shrinking of the corruption technology or detection profile in such a way that the relative position of these functions to the right of g^* are shifted downwards to occur at a lower level of g.

We begin the analysis by considering the first corruption technology $\omega_a(g)$ under each of the five Cases and then examine how the predictions of the model change by sequentially introducing technologies $\omega_b(g)$ and $\omega_c(g)$.

4.1 Corruption technology *a*

In this Section we begin to examine each of the Cases under the first corruption technology $\omega_a(g)$ and the *UPNE* that are supported under them. For convenience we refer to Caseji with Case$j$, $j \in \{1, 2, 3, 4, 5\}$ in accordance with Definition 7 and $i \in \{a, b, c\}$ refers to the corruption technology profile.

Proposition 1. ***Case1a*** *supports UPNE2, UPNE3 and MPNE3 depending upon* g^* *according to:*

$$\left\{ \begin{array}{c} \{H, H\} \\ \{H, H\}, \{C, H\} \\ \{C, H\} \end{array} \right\} if \left\{ \begin{array}{c} g^* \in (0, \bar{g}) \\ g* = \bar{g} \\ g* \in (\bar{g}, \infty) \end{array} \right\}.$$

Corollary 6. ***Case1a*** *(i) supports unconstrained* $UPNE2$ *corruption, and, (ii)* $UPNEk$ *are monotonically worsening in* g^*.

Note, that under ***Case1a***, there is never a possibility of complete regulatory failure: *UPNE1* is never feasible. For sufficiently low g^*, there is no corruption, whilst for sufficiently high g^* corruption is chosen by the Monopolist but detected by the Auditor.

We now consider what happens if the detection profile becomes less tough (σ_1 stretches to the right) and/or the the rewards to corruption becomes steeper in accordance with the scenario in ***Case2a***.

Proposition 2. ***Case2a*** *supports UPNE1-3 and MPNE2 and MPNE3 depending upon g^* according to:*

$$\left\{\begin{array}{c} \{C,C\} \\ \{C,C\},\{H,H\} \\ \{H,H\} \\ \{H,H\},\{C,H\} \\ \{C,H\} \end{array}\right\} if \left\{\begin{array}{c} g^* \in (0, min\{g^{**},\bar{g}\}) \\ g^* = min\{g^{**},\bar{g}\} \\ g^* \in (min\{g^{**},\bar{g}\},\tilde{g}) \\ g^* = \tilde{g} \\ g^* \in (\tilde{g},\infty) \end{array}\right\}.$$

One important thing to note is that the movement from ***Case1a*** to ***Case2a*** has introduced the equilibrium *UPNE1* under which the regulatory system fails and the Auditor colludes in the corrupt activity. However, the following Corollary has important implications for policymakers.

Corollary 7. ***Case2a*** *(i) supports unconstrained $UPNE2$ corruption, and, (ii) $UPNEk$ are non-monotonic in g^*: marginal adjustments in the detection or penalty regimes intended to move the equilibrium from UPNE1 (UPNE2) to UPNE3 may overshoot and result in UPNE2 (UPNE1).*

A further weakening (rightward-stretching) of the detection profile $\sigma_1(g)$ and/or improvement in the rate of return to corruption $\omega_a(g)$ results in a movement from ***Case2a*** to ***Case3a***.

Proposition 3. ***Case3a*** *supports UPNE1 and UPNE2 depending upon g^*, and may support MPNE1 or UPNE3, MPNE2, and MPNE3 depending upon g^{**} according to:*

$$\begin{array}{cc} g^{**} < \tilde{g} & g^{**} > \tilde{g} \end{array}$$
$$\left\{\begin{array}{cc} \{C,C\} & \{C,C\} \\ & \{C,C\},\{C,H\} \\ \{C,C\},\{H,H\} & \\ \{H,H\} & \\ \{H,H\},\{C,H\} & \\ \{C,H\} & \{C,H\} \end{array}\right\} if \left\{\begin{array}{c} g^* \in (0,g^{**}) \\ \tilde{g} < g* = g^{**} \\ \tilde{g} \geq g* = g^{**} \\ g^* \in (min\{g^{**},\tilde{g}\},\tilde{g}) \\ g^{**} \leq g^* = \tilde{g} \\ g^* \in (max\{g^{**},\tilde{g}\},\infty) \end{array}\right\}.$$

Corollary 8. ***Case3a*** *(i) for $g^{**} > \tilde{g}$ is monotonically improving in g; (ii) for $g^{**} \geq \tilde{g}$ is non-monotonic in g, following the same sequence of equilibria as* ***Case2a*** *but for different reasons; (iii) like* ***Case2a****, supports unconstrained UPNE2 corruption.*

Proposition 4. *(i) Under $\lim_{g\to\infty}\sigma_2(g) > \sigma^{**}$,* ***Case4a*** *and* ***Case3a*** *are equivalent (they support the same equilibria under the same conditions - see Proposition 3), and (ii) Under $\lim_{g\to\infty}\sigma_2(g) < \sigma^{**}$,* ***Case4a*** *supports only UPNE1:*

$$\left\{\ C,C\ \right\} for \left\{\ g^* \in (0,\infty)\ \right\}.$$

Corollary 9. ***Case4a*** *under $\lim_{g\to\infty}\sigma_2(g) > \sigma^{**}$ (i) is non-monotonic in g, and, (ii) supports unconstrained UPNE2 corruption. (iii)* ***Case4a*** *under $\lim_{g\to\infty}\sigma_2(g) < \sigma^{**}$, supports unconstrained and small-scale UPNE1 corruption.*

Therefore, even though in **Case 4** the detection profile $\sigma_2(g)$ lies everywhere below probability 1, so long as g^{**} exists, the outcomes of the model with corruption technology $\omega_a(g)$ are exactly the same as under Case 3. However, under ***Case4a*** with $\sigma_2(g) < \sigma^{**}$, $\{C_1, C_2\}$ is the only outcome.

Proposition 5. ***Case5a*** *under (i)* $\lim_{g\to\infty}\sigma_2(g) < \sigma^{**}$*, supports UPNE1, UPNE3 and MPNE2 depending upon* g^**, according to:*

$$\left\{\begin{array}{c}\{H,H\}\\ \{H,H\},\{C,C\}\\ \{C,C\}\end{array}\right\} \textit{if} \left\{\begin{array}{c} g^*\in(0,\bar{g})\\ g^*=\bar{g}\\ g^*\in(\bar{g},\infty)\end{array}\right\},$$

and, under (ii) $\lim_{g\to\infty}\sigma_2(g) > \sigma^{**}$ *and* $g^{**} < \bar{g}$*, supports UPNE2, UPNE3 and MPNE3 depending upon* g^**, and according to:*

$$\left\{\begin{array}{c}\{H,H\}\\ \{H,H\},\{C,H\}\\ \{C,H\}\end{array}\right\} \textit{if} \left\{\begin{array}{c} g^*\in(0,\tilde{g})\\ g^*=\tilde{g}\\ g^*\in(\tilde{g},\infty)\end{array}\right\},$$

and, under (iii) $\lim_{g\to\infty}\sigma_2(g) > \sigma^{**}$ *and* $\tilde{g} > g^{**} > \bar{g}$*, supports UPNE1-3 and MPNE1-3 depending upon* g^**, according to:*

$$\left\{\begin{array}{c}\{H,H\}\\ \{H,H\},\{C,C\}\\ \{C,C\}\\ \{C,C\},\{H,H\}\\ \{H,H\}\\ \{H,H\},\{C,H\}\\ \{C,H\}\end{array}\right\} \textit{if} \left\{\begin{array}{c} g^*\in(0,\bar{g})\\ g^*=\bar{g}\\ g^*\in(\bar{g},g^{**})\\ g^*=g^{**}\\ g^*\in(g^{**},\tilde{g})\\ g^*=\tilde{g}\\ g^*\in(\tilde{g},\infty)\end{array}\right\}.$$

and, under (iv) $\lim_{g\to\infty}\sigma_2(g) > \sigma^{**}$ *and* $g^{**} > \tilde{g}$*, supports UPNE1-3 and MPNE1 and MPNE2 depending upon* g^**, according to:*

$$\left\{\begin{array}{c}\{H,H\}\\ \{H,H\},\{C,C\}\\ \{C,C\}\\ \{C,C\},\{C,H\}\\ \{C,H\}\end{array}\right\} \textit{if} \left\{\begin{array}{c} g^*\in(0,\bar{g})\\ g^*=\bar{g}\\ g^*\in(\bar{g},g^{**})\\ g^*=g^{**}\\ g^*\in(g^{**},\infty)\end{array}\right\}.$$

Corollary 10. ***Case5a*** *(i) supports unconstrained UPNE1 corruption under* $\lim_{g\to\infty}\sigma_2(g) < \sigma^{**}$ *and unconstrained UPNE2 corruption otherwise, (ii) supports monotonically worsening UPNEk with* g^**, under both* $\lim_{g\to\infty}\sigma_2(g) < \sigma^{**}$ *and* $\lim_{g\to\infty}\sigma_2(g) > \sigma^{**}$ *given* $g^{**} < \bar{g}$ *and, (iii) is non-monotonic in* g^* *for* $\lim_{g\to\infty}\sigma_2(g) > \sigma^{**}$ *given* $g^{**} > \bar{g}$*.*

4.2 Corruption technology *b*

One of the main characteristics of corruption technology $\omega_a(g)$ is that, whilst it exhibits diminishing returns to the scale of corruption g ($\gamma(g)$ is strictly concave), the diminishing returns property is insufficiently pronounced to ever cause $\omega_a(g)$ to become decreasing in g. We now consider the case of corruption technology $\omega_b(g)$ under which, for sufficiently high levels of g^*, $\omega_a'(g) < 0$.

Proposition 6. *Moving from corruption technology* $\omega_a(g)$ *to* $\omega_b(g)$ *introduces MPNE3 and UPNE3 at the end of the sequence of equilibria in* g^* *under Cases 1 and 2 so the relevant sequence of equilibria under corruption technology* $\omega_b(g)$ *extends those in*

Propositions 1 and 2, and necessarily changes the interval over which UPNE2 exists, according to:

$$\left\{\begin{array}{c}\vdots\\ \{C,H\}\\ \{C,H\},\{H,H\}\\ \{H,H\}\end{array}\right\} if \left\{\begin{array}{c}\vdots\\ g^*\in(\tilde{g},\tilde{\tilde{g}}=\bar{\bar{g}})\\ g^*=\tilde{\tilde{g}}=\bar{\bar{g}}\\ g^*\in(\tilde{\tilde{g}}=\bar{\bar{g}},\infty)\end{array}\right\}.$$

Corollary 11. *Corruption technology $\omega_b(g)$ (i) rules out unconstrained corruption equilibria that prevailed in Cases 1 and 2 under corruption technology $\omega_a(g)$, (ii) makes Case 1 non-monotonic in g^*, whereas it was monotonically worsening under corruption technology $\omega_a(g)$.*

Proposition 7. *Moving from corruption technology $\omega_a(g)$ to $\omega_b(g)$ (i) introduces MPNE3 and UPNE3 at the end of the sequence of equilibria in g^* under **Case3** so the relevant sequence of equilibria under corruption technology $\omega_b(g)$ extends those in Proposition 3, and necessarily changes the interval over which UPNE2 exists, according to:*

$$\begin{array}{cc} g^{**}<\tilde{g} & \tilde{\tilde{g}}>g^{**}>\tilde{g} \end{array}$$
$$\left\{\begin{array}{cc}\vdots & \vdots\\ \{C,H\} & \\ & \{C,H\}\\ \{C,H\},\{H,H\} & \{C,H\},\{H,H\}\\ \{H,H\} & \{H,H\}\end{array}\right\} if \left\{\begin{array}{c}\vdots\\ g^*\in(\tilde{g},\tilde{\tilde{g}})\\ g^*\in(g^{**},\tilde{\tilde{g}})\\ g^*=\tilde{\tilde{g}}\\ g^*\in(\tilde{\tilde{g}},\infty)\end{array}\right\}.$$

and, (ii) for $g^{**}>\tilde{\tilde{g}}$:

$$\left\{\begin{array}{c}\{C,C\}\\ \{C,C\},\{H,H\}\\ \{H,H\}\end{array}\right\} if \left\{\begin{array}{c}g^*\in(0,min\{g^{**},\bar{\bar{g}}\})\\ g^*=min\{g^{**},\bar{\bar{g}}\}\\ g^*\in(min\{g^{**},\bar{\bar{g}}\},\infty)\end{array}\right\}.$$

Corollary 12. *Corruption technology $\omega_b(g)$: (i) rules out unconstrained corruption equilibria UPNE2 that prevailed in **Case3** under corruption technology $\omega_a(g)$, and, (ii) preserves the non-monotonicity of the UPNEk in g^* under $g^{**}<\tilde{g}$ and the monotonically improving UPNEk for $g^{**}>\tilde{g}$.*

Proposition 8. *(i) Under $\lim_{g\to\infty}\sigma_2(g)>\sigma^{**}$, Case 4 and Case 3, with corruption technology $\omega_b(g)$, are equivalent (they support the same equilibria under the same conditions - see Proposition 7), and (ii) under $\lim_{g\to\infty}\sigma_2(g)<\sigma^{**}$, moving from corruption technology $\omega_a(g)$ to $\omega_b(g)$ introduces MPNE2 and UPNE3 at the end of the sequence of equilibria in g^* under **Case4** so the relevant sequence of equilibria under corruption technology $\omega_b(g)$ extends those in Proposition 4, and necessarily changes the interval over which UPNE1 exists, according to:*

$$\left\{\begin{array}{c}\{C,C\}\\ \{C,C\},\{H,H\}\\ \{H,H\}\end{array}\right\} if \left\{\begin{array}{c}g^*\in(0,\bar{g}=\bar{\bar{g}})\\ g^*=\bar{g}=\bar{\bar{g}}\\ g^*\in(\bar{g}=\bar{\bar{g}},\infty)\end{array}\right\}.$$

Corollary 13. *Corruption technology $\omega_b(g)$: (i) rules out unconstrained corruption equilibria UPNE2 and UPNE1 that prevailed in **Case4** under corruption technology $\omega_a(g)$, and, (ii) preserves the non-monotonicity of the UPNEk in $g*$ under $g^{**}<\tilde{g}$ and the monotonically improving UPNEk for $g^{**}>\tilde{g}$.*

Proposition 9. *Moving from corruption technology $\omega_a(g)$ to $\omega_b(g)$ under* **Case5***, and (i) $\lim_{g\to\infty}\sigma_2(g) < \sigma^{**}$, introduces MPNE2 and UPNE3 at the end of the sequence of equilibria in g^* so the relevant sequence of equilibria under corruption technology $\omega_b(g)$ extends those in Proposition 5(i), and necessarily changes the interval over which UPNE3 exists, according to:*

$$\left\{\begin{array}{c}\vdots\\ \{C,C\}\\ \{C,C\},\{H,H\}\\ \{H,H\}\end{array}\right\} \textit{if} \left\{\begin{array}{c}\vdots\\ g^*\in(\bar{g},\bar{\bar{g}})\\ g^*=\bar{\bar{g}}\\ g^*\in(\bar{\bar{g}},\infty)\end{array}\right\},$$

and, under (ii) $\lim_{g\to\infty}\sigma_2(g) > \sigma^{**}$ and $g^{**} < \tilde{g}$, supports UPNE3 and MPNE3 at the end of the sequence of equilibria in g^* so the relevant sequence of equilibria under corruption technology $\omega_b(g)$ extends those in Proposition 5(ii) and (iii), and necessarily changes the interval over which UPNE3 exists, according to:

$$\left\{\begin{array}{c}\vdots\\ \{C,H\}\\ \{C,H\},\{H,H\}\\ \{H,H\}\end{array}\right\} \textit{if} \left\{\begin{array}{c}\vdots\\ g^*\in(\tilde{g},\tilde{\tilde{g}})\\ g^*=\tilde{\tilde{g}}\\ g^*\in(\tilde{\tilde{g}},\infty)\end{array}\right\},$$

and, under (iii) $\lim_{g\to\infty}\sigma_2(g) > \sigma^{**}$ and $\bar{\bar{g}} > g^{**} > \tilde{g}$, supports UPNE3 and MPNE3 at the end of the sequence of equilibria in g^* so the relevant sequence of equilibria under corruption technology $\omega_b(g)$ extends those in Proposition 5(iv), and necessarily changes the interval over which UPNE2 exists, according to:

$$\begin{array}{cc}\tilde{\tilde{g}} < g^{**} < \tilde{g} & \bar{\bar{g}} > g^{**} > \tilde{\tilde{g}}\end{array}$$
$$\left\{\begin{array}{cc}\vdots & \vdots\\ \{C,H\} & \\ & \{C,H\}\\ \{C,H\},\{H,H\} & \\ & \{C,H\},\{H,H\}\\ \{H,H\} & \\ & \{H,H\}\end{array}\right\} \textit{if} \left\{\begin{array}{c}\vdots\\ g^*\in(g^{**},\tilde{\tilde{g}})\\ g^*\in(g^{**},\bar{\bar{g}})\\ g^*=\tilde{\tilde{g}}\\ g^*=\bar{\bar{g}}\\ g^*\in(\tilde{\tilde{g}},\infty)\\ g^*\in(\bar{\bar{g}},\infty)\end{array}\right\}.$$

and, under (iv) $\lim_{g\to\infty}\sigma_2(g) > \sigma^{**}$ and $\bar{\bar{g}} < g^{**}$, supports UPNE3 and MPNE3 at the end of the sequence of equilibria in g^* so the relevant sequence of equilibria under corruption technology $\omega_b(g)$ is exactly in accordance with part (i) of this Proposition.

4.3 Corruption technology *c*

Finally, we consider corruption technology $\omega_c(g)$, which unlike technologies a b, has such strongly diminishing returns to corruption that $\omega_c(g)$ never reaches unity - which, of course, means that Auditor honesty will immediately rule out any corruption by the Monopolist.

Proposition 10. *Moving from corruption technology $\omega_a(g)$ or $\omega_b(g)$ to $\omega_c(g)$ under (i)* ***Case1*** *results in universal UPNE3:*

$$\left\{\ H,H\ \right\} \textit{for} \left\{\ g^*\in(0,\infty)\ \right\},$$

(ii) ***Case2*** *results in the sequence of equilibria in* g^**:*

$$\left\{\begin{array}{c}\{C,C\}\\ \{C,C\},\{H,H\}\\ \{H,H\}\end{array}\right\} if \left\{\begin{array}{c} g^* \in (0, min\{\bar{g}, g^{**}\})\\ g^* = min\{\bar{g}, g^{**}\}\\ g^* \in (min\{\bar{g}, g^{**}\}, \infty)\end{array}\right\}.$$

Corollary 14. *Corruption technology* $\omega_c(g)$ *(i) rules out the non-monotonic sequence of equilibria in* g^* *under* ***Case1b****, and preserves the monotonically improving sequence of equilibria in* g^* *in* ***Case2a*** *and* ***Case2b****.*

Proposition 11. *Under* $\lim_{g\to\infty}\sigma_2(g) < \sigma^{**}$*,* ***Case4c (Case5c)*** *is equivalent to* ***Case4a (Case5a)*** *supporting universal UPNE1 (unconstrained UPNE1).*

Corollary 15. *Unconstrained UPNE1 is feasible with corruption technology* $\omega_c(g)$ *under Case4.*

Proposition 12. *Under* $\lim_{g\to\infty}\sigma_2(g) > \sigma^{**}$ *(i)* ***Case4c*** *supports the sequence of equilibria under* g^**, according to:*

$$\left\{\begin{array}{c}\{C,C\}\\ \{C,C\},\{H,H\}\\ \{H,H\}\end{array}\right\} \; if \; \left\{\begin{array}{c} g^* \in (0, g^{**})\\ g^* = g^{**}\\ g^* \in (g^{**}, \infty)\end{array}\right\}.$$

(ii) ***Case5c*** *supports the sequence of equilibria under* g^**, according to:*

$$\begin{array}{cc} g^{**} \leq \bar{g} & g^{**} > \bar{g} \end{array}$$
$$\left\{\begin{array}{lc}\{H,H\} & \{H,H\}\\ & \{H,H\},\{C,C\}\\ & \{C,C\}\\ & \{C,C\},\{H,H\}\\ \{H,H\} & \{H,H\}\end{array}\right\} \; if \; \left\{\begin{array}{c} g^* \in (0, min\{g^{**}, \bar{g}\})\\ g^* = \bar{g} < g^{**}\\ g^* \in (\bar{g}, g^{**})\\ g^* = g^{**} > \bar{g}\\ g^* \in (g^{**}, \infty)\end{array}\right\}.$$

Corollary 16. *For* $\lim_{g\to\infty}\sigma_2(g) > \sigma^{**}$ *Corruption technology* $\omega_c(g)$ *(i) rules out unconstrained corruption equilibrium UPNE2 under Cases 4 and 5 that prevailed under* $\omega_a(g)$ *(ii) eliminates non-monotonicity under Case4 that prevailed under* $\omega_a(g)$ *and* $\omega_b(g)$ *(iii) preserves non-monotonicity under Case5 that prevailed under* $\omega_a(g)$ *and* $\omega_b(g)$ *for* $g^{**} > \bar{g}$*.*

5 Policy Options

In this section we consider how the parameters of the model may be manipulated so as to change the outcome of the game for a given prospect g^*. We begin by asking whether the Monopolist can influence the outcome of the game. Given we are assuming that the Monopolist cannot determine the level of corruption, the only other candidate for an instrument that the Monopolist might exploit is α.[1]

Proposition 13. *If* $\omega(g^*) > \sigma(g^*)$ *and* $\sigma(g^*)$ *is greater than, but sufficiently close to,* $\sigma(g^{**})$ *then the monopolist can increase* α *(the share of profit devoted to Auditor services) strategically to move from UPNE2 to UPNE1.*

[1]It is conceivable that the Monopolist might be able to influence the profile $\gamma(g)$. However, in order to analyse this we would require a formal specification of the costs involved and this lies beyond the scope of the current work.

Proof. Let $\omega(g^*)$ at some initial level of α be $\omega(g^*, \alpha)$ where $\omega(g^*, \alpha) > \sigma(g^*)$. Accordingly, let $\sigma(g^*) > \sigma(g^{**}, \alpha)$ so that we have UPNE2 at α. Increasing α shifts αF_M (in Figure 2) to the left raising $\sigma(g^{**})$. However, given $\omega(g^*, \alpha) > \sigma(g^*)$, it follows there exists some $\Delta\alpha > 0$ such that $\omega(g^*, \alpha + \Delta\alpha) > \sigma(g^*)$. If $\sigma(g^{**}, \alpha)$ is sufficiently close to $\sigma(g^*)$, then $\sigma(g^{**}, \alpha + \Delta\alpha) < \sigma(g^*)$, hence yielding UPNE1. □

Definition 11. *If it exists, let $\underline{\Delta\alpha} > 0$ be the value of $\Delta\alpha$ which satisfies both $\sigma(g^{**}, \alpha + \Delta\alpha) < \sigma(g^*)$ and $\omega(g^*, \alpha + \Delta\alpha) > \sigma(g^*)$, where $\omega(g^*, \alpha) > \sigma(g^*)$ and $\sigma(g^*) > \sigma(g^{**}, \alpha)$.*

Hence, if $\underline{\Delta\alpha}$ exists then it is possible for the Monopolist to move the game from UPNE2 to UPNE1. It follows that the Monopolist may be able to exploit consultancy fees to 'bribe' the Auditor to be complicit in its corruption. However, although UPNE1 may be 'better' than UPNE2 for the Monopolist, inasmuch as it moves the Monopolist from a situation of incurring the fine F_M with certainty, to incurring it with some positive probability $\sigma(g^*) < 1$,[2] the above Proposition only establishes that there are circumstances under which it might bring about such manipulation of the Auditor. We now address the question regarding the conditions under which such manipulation would be in the interests of the Monopolist.

Proposition 14. *The Monopolist optimally selects to increase α by an amount $\Delta\alpha$ in order to bring about a move from UPNE2 to UPNE1 if:*

$$\Delta\alpha < \frac{(1-\alpha)(1-\sigma(g))F_M}{\{(1+\gamma(g))\pi_M^H - \sigma(g)F_M\}} \tag{11}$$

Proof. It is required to show that the (risk-neutral) Monopolist's expected profit under UPNE1 with $\alpha + \Delta\alpha$ is greater than the Monopolist's profit under UPNE2 with α. Replacing α in (3a) with $\alpha + \Delta\alpha$ and comparing with (2a) we have (11). □

Definition 12. *Let* $\overline{\Delta\alpha} \equiv \frac{(1-\alpha)(1-\sigma(g))F_M}{(1+\gamma(g))\pi_M^H - \sigma(g)F_M}$.

Lemma 5. *Although it is possible for the denominator of (11) to be non-positive, for $\{C_1, C_2\}$ to be a UPNE requires that $(1 + \gamma(g))\pi_M^H - \sigma(g)F_M > 0$, hence where the strategy of using α to move from UPNE2 to UPNE1 is feasible, then the denominator of (11) is positive.*

Proof. From (4b) UPNE1 requires that $\omega(g) > \sigma(g)$, hence $\frac{\pi_M^H}{F_M}\gamma(g) > \sigma(g)$. Multiplying by F_M and rearranging, we have $\gamma(g)\pi_M^H - \sigma(g)F_M > 0$, hence $1 + \gamma(g)\pi_M^H - \sigma(g)F_M > 0$. □

Remark 5. *If $\underline{\Delta\alpha} \in (0, \overline{\Delta\alpha})$ then the Monopolist can and will optimally raise α to move the game from UPNE2 to UPNE1.*

Proposition 15. *The range of values of $\Delta\alpha$ which are consistent with the Monopolist optimally choosing to stimulate a move from UPNE2 to UPNE1, $\Delta\alpha \in (0, \overline{\Delta\alpha})$, is (i) decreasing in α, π_M^H and $\gamma(g)$, (ii) increasing in F_M and (iii) may be increasing or decreasing in $\sigma(g)$.*

[2] By definition, under UPNE1, $\sigma(g^*) < \sigma(g^{**}) < 1$.

Proof. (i) This follows directly from the observation that $-\alpha$ appears only in the numerator of (11) whilst $\gamma(g)$ and π_M^H both appear only in the denominator of (11) with positive coefficients, hence the respective partial derivatives of $\overline{\Delta\alpha}$ in each Case are negative. (ii) This follows given, after some manipulation:

$$\frac{\partial\overline{\Delta\alpha}}{\partial F_M} = \frac{(1-\alpha)(1-\sigma(g))[(1+\gamma(g))\pi_M^H]}{\{.\}^2} > 0,$$

where $\{.\}$ is the denominator in (11), and given the assumptions of the model, the numerator of the derivative is positive. (iii) Given:

$$\frac{\partial\overline{\Delta\alpha}}{\partial\sigma(g)} = \frac{(1-\sigma(g)) - [(1+\gamma(g))\pi_M^H - \sigma(g)F_M]}{\{.\}^2},$$

the first term in the numerator $(1-\sigma(g))$ is non-negative by the assumptions of the model and [.] is also positive from Lemma 5. □

It follows that subject to UPNE1 and UPNE2 both being feasible following an increase in F_M, such an increase in the fine to the Monopolist will increase the range of values of $\Delta\alpha$ which would make a move from UPNE2 to UPNE1 attractive to the Monopolist. As we will see later, such an increase in F_M will also have a perverse effect on the Auditor which reinforces the likelihood of a move from UPNE2 to UPNE1 being feasible and optimal.

We now ask whether the regulatory body can influence the outcome of the game. The two obvious factors that the regulator can manipulate are the fines (to the Auditor (F_A) and the Monopolist (F_M) in the scenario where corruption is detected) and the probability of detection (by investing in the detection framework). We begin by examining the impact upon the game of raising the penalty to the Monopolist, F_M.

Proposition 16. *Increasing the Monopolist's fine under detected corruption, F_M (i) can eliminate all corruption with a sufficiently high fine, (ii) can, perversely, incentivise UPNE1 over UPNE2.*

Proof. (i) For $\{C_1, C_2\}$ to be UPNE requires, from (4a), that $\omega(g) \equiv \frac{\gamma(g)\pi_M^H}{F_M} > \sigma(g)$ and for $\{C_1, H_2\}$ to be UPNE requires, from (5b), that $\omega(g) \equiv \frac{\gamma(g)\pi_M^H}{F_M} > 1$. Hence, to rule out UPNE1 and UPNE2, respectively requires that $\sigma(g)F_M > \pi_M^H$ and $F_M > \pi_M^H$. (ii) Let $\omega(g)$ at some initial level of F_M be $\omega(g, F_M)$, where $\omega(g, F_M) > \sigma(g)$. Accordingly, let $\sigma(g^*) > \sigma(g^{**}, F_M)$ so that we have UPNE2. Increasing F_M shifts αF_M in Figure 2 to the left raising $\sigma(g^{**})$. However, given $\omega(g^*, F_M) > \sigma(g^*)$, it follows there exists some $\Delta F_M > 0$ such that $\omega(g^*, F_M + \Delta F_M) > \sigma(g^*)$. If $\sigma(g^{**}, F_M)$ is sufficiently close to $\sigma(g^*)$, then $\sigma(g^{**}, F_M + \Delta F_M) < \sigma(g^*)$, hence yielding UPNE1. □

Essentially, Proposition 16(i) refers to a case where $\omega(g)$ is lowered sufficiently that the corruption profile resembles **Case1c**: for $g \in (0, \infty)$, $\omega(g) < \sigma(g)$.

Corollary 17. *UPNEk can be non-monotonic in F_M.*

Corollary 18 is a warning that increasing the fine to the Monopolist on detection of corruption may have perverse effects if the fine is not set sufficiently high.

Corollary 18. *The regulatory authority can bring about a move from UPNE1 to UPNE2, causing the Auditor to be honest instead of supporting Monopoly corruption, by decreasing the monopoly penalty, F_M.*

We now examine the implications for the game of the regulator increasing the fine to the Auditor with corruption detected under UPNE1.

Proposition 17. *Increasing the fine to the Auditor, F_A, on detection of UPNE1 corruption (i) unambiguously reduces the range of g^* over which the Auditor will choose to be complicit in corrupt activities, promoting UPNE2 over UPNE1, (ii) cannot eliminate UPNE2.*

Proof. (i) This follows straightforwardly from Definition 2. Increasing F_A raises $\varphi(\sigma)$ for $\forall\sigma \in (0,1)$. Since, from (4a), UPNE1 requires that $\varphi(\sigma) < \alpha F_M$, increasing $\varphi(\sigma)$ reduces $\sigma(g^{**})$, the supremum of the set of $\sigma(g)$ for which the Auditor would support Monopoly corruption. (ii) This follows straightforwardly from the observation that F_A does not feature in the Monopolist's condition for UPNE2. □

Finally, we consider the possibility that the regulator could invest in improving the corruption detection framework, raising $\sigma(g)$.

A 13. *We assume, for simplicity, that investments in improving the corruption detection framework cause the profile $\sigma(g)$ to rise $\forall g \in (0,\hat{g})$ $[\forall g \in (0,\infty)]$in the case of σ_1 $[\sigma_2]$ so that the properties of the profile under A10 are preserved.*

Corollary 19. *(i) If $T > \sigma^{**}$ so that g^{**} does not exist, then a sufficiently large investment in improving detection will eventually yield $T < \sigma^{**}$ for which there will exist an associated g^{**}. (ii) Investment in improving detection cannot convert a $\sigma_2(g)$ detection profile into a $\sigma_1(g)$ profile.*

Lemma 6. *Under σ_1 and also σ_2 for $T > S$, g^{**} exists and any investment in improving the detection of corruption in accordance with A10 will lower the level of g^{**}.*

Proof. For this proof it is convenient to exploit the strict monotonicity of $\sigma(g)$ $(\sigma'(g) > 0)$ for $\sigma \in [0,1)$ $[\sigma \in (0,\infty)]$ under σ_1 $[\sigma_2]$. This allows us to invert the function giving $g(\sigma)$ for $\sigma_1 \in [0,1)$ and $\sigma_2 \in [0,T)$. Under σ_1, $\sigma(g^{**}) \in (0,1)$ exists and under σ_2 with $T > S$, $\sigma(g^{**}) \in (0,T)$ exists . Hence, inverting the function we can say in each case $g(\sigma^{**})$ exists. Given σ^{**} is determined by the interaction of $\varphi(\sigma)$ and αF_M, neither of which are affected by raising the $\sigma(g)$ profile, then σ^{**} is constant. However, an upward shift in $\sigma_1(g)$ for $\sigma \in (0,1)$ implies $g(\sigma^{**})$, and hence g^{**}, falls. A similar argument holds for an upward shift in σ_2 for $\sigma \in (0,T)$. □

We begin by considering the impact of investing in improved detection upon the Auditor.

Proposition 18. *Investment in corruption detection (i) under σ_1, and also σ_2 for $T > S$, unambiguously reduces the range of g^* for which the Auditor will choose to be complicit in corrupt activities, promoting UPNE2 over UPNE1, (ii) under σ_2 in the case of $T \leq S$ will reduce the range of g^* for which the Auditor will choose to be complicit if the shift in σ_2 is sufficiently large.*

Proof. (i) Given Lemma 6 Since the Auditor's complicity in corruption requires that $g^* \in (0, g^{**})$ the range of values of g^* consistent with Auditor complicity has fallen. (ii) It is sufficient to note that under σ_2 in the case of $T \leq S$, σ^{**} lies strictly above σ_2 $\forall g \in [0, \infty)$, hence g^{**} does not exist. However, since σ^{**} is fixed and lies in the open interval $(0, 1)$ there always exists a $\sigma > \sigma^{**}$ in the interval $(0, 1)$. Hence, a sufficiently large investment in improving detection will eventually shift σ_2 upwards raising T above S so that g^{**} exists. This reduces the interval of g^* under which the Auditor will be complicit in corruption from $(0, \infty)$ to $(0, g^{**})$. Further improvements in corruption detection will then have the effect described in (i). □

It follows that investments in improving corruption detection may have no effect upon the Auditor unless they are sufficiently large, hence local adjustments in the detection may not have any impact upon the the range of g^* supporting UPNE2. We now turn our attention to the impact of investments in corruption detection on the Monopolist.

Proposition 19. *Investments in improving corruption detection, however large, are completely ineffective at eliminating Monopoly corruption or even reducing the range of g^* for which the Monopolist is corrupt where corruption arises under technologies a and b in the region $\omega_a > 1$ or $\omega_b > 1$.*

Proof. This follows straight forwardly from the observation that UPNE2 requires $\omega > \sigma$ but since $\omega > 1$ and σ is constrained to lie at or below 1, any feasible increase in σ will not be enough to reverse the inequality between σ and ω. □

Corollary 20. *Investments in improving corruption detection are completely ineffective at addressing unconstrained UPNE2 under technology ω_a.*

Thus, whilst improving detection may deter the Auditor from being complicit in corrupt activities, on its own, this policy cannot eliminate all corruption, including possible unconstrained corruption. Also, we know from Proposition18 that such investments will eventually deter the Auditor from supporting corrupt activities, hence the most that could be achieved with this policy of improving corruption detection is to eliminate UPNE1. UPNE2 cannot be eliminated in this case, however much investment is undertaken.

Remark 6. *In line with A10, sufficiently large investments in improving corruption detection will eventually raise $\sigma_1(g)$ transforming Case3 into Case2 and Case2 into Case1. Similarly, investments will eventually raise $\sigma_2(g)$ transforming Case4 into Case5.*

Proposition 20. *If investments in improving corruption detection, required to bring about a change in Case as described in Remark6, are prohibitively expensive, improving corruption detection will not eliminate small-scale UPNE1 in Case2, Case3 and Case4.*

Proof. The proof follows from the definition of small-scale corruption and the observation that Cases2-4 support UPNE1 small-scale corruption since in each case $\sigma'(0) < \omega'(0)$. □

We now examine some of the characteristics of the 5 Cases in terms of the role that investments in detection improvement can have on deterring Monopoly corruption. We use the idea of arbitrarily small changes in corruption detection investment in order to emphasise that after the investment the local properties of the model are unchanged and we have not made a shift from one Case to another.

Proposition 21. *(i) Investments in improving corruption detection are completely ineffective in dealing with Monopolist corruption in Case1. (ii) Under Case2, arbitrarily small investments in improving corruption technology will always reduce the range of g^* for which the Monopolist chooses to be corrupt.*

Proof. (i) This follows directly from the fact that under Case1 the only corrupt equilibrium is UPNE2 where $\omega_a > 1$ or $\omega_b > 1$, which, from Proposition19, we know cannot be affected by detection investments. (ii) First, if $g^{**} > \bar{g}$ then Monopolist corruption occurs if $g^* \in (0, \bar{g})$. Investing in improving corruption detection will raise $\sigma_1(g)$ hence reducing $\bar{g}$ and with it the upper limit of g^* consistent with Monopolist corruption. Second, if $g^{**} \leq \bar{g}$ then Monopolist corruption occurs if $g^* \in (0, g^*)$. Investing in improving corruption detection raises $\sigma_1(g)$, which by Lemma6 reduces g^{**}, and with it the upper limit of g^* consistent with Monopolist corruption. □

Finally, we note that unlike Case1 where corruption detection was completely ineffective at deterring Monopoly corruption, and Case2 where, regardless of the corruption technology, such investments would always reduce the range of g^* under which the Monopolist would be corrupt, we have that Cases3-5 each have conditions under which improving corruption detection would and would not have benefits in terms of reducing the range of g^* consistent with Monopolist corruption. For brevity, the following Proposition identifies the Cases where local improvements in the corruption detection do not impact upon Monopolist corruption.

Proposition 22. *Arbitrarily small improvements in corruption detection do not reduce the range of g^* under which the Monopolist is corrupt (i) under Case3a and Case3b if $\tilde{g} < g^{**} \leq \tilde{\tilde{g}}$, (ii) under Case4a and Case 4c if $T < \sigma^{**}$ and $T < S < \sigma^{**}$, respectively, (iii) under Case4a and Case4b if $T > \sigma^{**}$ and respectively $g^{**} > \tilde{g}$, $g^{**} > \tilde{g}$, (iv) under Case5a and Case5b if $T > \sigma^{**}$ and $\bar{g} > g^{**}$, (v) under Case5c if $\sigma^{**} > T > S$, $T > \sigma^{**}$ or finally $S > T > \sigma^{**}$ and $\bar{g} \geq g^{**}$.*

Proof. □

6 Conclusions

This paper has had two broad objectives. First, to develop a model of firm corruption, taking account of auditor interaction, and to use this model to identify the possibility of stable corruption, where stability is viewed as an equilibrium in the game. The key driver to the relationship between the firm and the auditor is that increasing firm profitability, from corruption, indirectly increases the demand for consultancy services that the auditor provides in addition to auditing services. It has been shown here that a variety of equilibria are possible in the game, depending on particular parameterisations: corruption by both the firm and the auditor; firm corruption that is controlled by an honest auditor; multiple equilibria involving both corruption and honesty; honesty by both actors in the model. The multiplicity of possible equilibria in the model is interesting in its own right but is particularly useful in terms of the analysis of possible policy interventions that are considered in the final substantive section. This analysis of policy is the second broad objective of the discussion. In general terms some of the policy conclusions confirm what might be considered intuitively obvious anti-corruption policies. But some of the conclusions are less intuitively obvious and reflect firm-auditor interaction. The model has the property that it is possible to eliminate firm corruption by heavy monopoly penalties. This is a draconian policy that can imply closing firms down. This of course occurs in practice but is by no means universal. If penalties on firms are less than that necessary to close firms down, which is also the case in practice, a series of interesting policy conclusions can be derived from the model developed here. First, the monopolist can, in principle, 'bribe' the auditor by increasing consultancy payments. The result here is that the equilibrium of the game can, in principle, be shifted from "firm corruption that is controlled by an honest auditor" to "corruption by both the firm and the auditor". Secondly, even without the firm strategically buying consultancy services, increasing penalties on corrupt firms can be shown to undermine auditor honesty; a conclusion that follows from the interactions in the model. It follows that the efficiency of the auditing system may be improved by reducing penalties on corrupt firms.

It follows from these conclusions that unless anti-corruption polices are based on closing down corrupt firms, any penalties on corrupt firms are an inefficient policy option and should be used in combination with, or replaced by, other policy options. First there is the obvious option of punishing auditors. It has been shown here that this unambiguously promotes auditor honesty and does not have the perverse effects that can be identified when corrupt monopolists are punished. But auditor punishment does not remove firm corruption instead it results in a more effective auditing system. It follows that possible anti-social effects of corruption not considered here (for instance on consumers or other economic actors) still exist. A similar conclusion follows from investment in the detection of firm corruption. This can be shown to not eliminate corruption instead it promotes auditor honesty. It is appropriate to mention, here in the conclusion, various policy options that appear intuitively plausible but go beyond the confines of the model presented here. First, there is the possibility of making penalties endogenous and hence increasing with corrupt gains. This might eventually eliminate large scale corruption, but depending on the function used to define the penalties need not eliminate the perverse impacts of firm penalties in general. An interesting issue is when dual equilibria exist. This suggests that corruption might be understood as a focal point; the fact that it exists does not imply that non-corruption can also exist even with the same Monopoly and Auditor payoff structures. This possibility suggests anti-corruption policies that are beyond the framework developed here and might revolve round the expectations of the actors. One final issue that can be highlighted involves the non-monotonicity of the equilibria. It is clear from the analysis presented here that large-scale policy interventions, involving (for example) politically motivated policies aimed at 'getting tough' on corruption, may leap-frog the desired outcome and be counterproductive.

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Figure 1: **Extensive form representation of the corruption game**

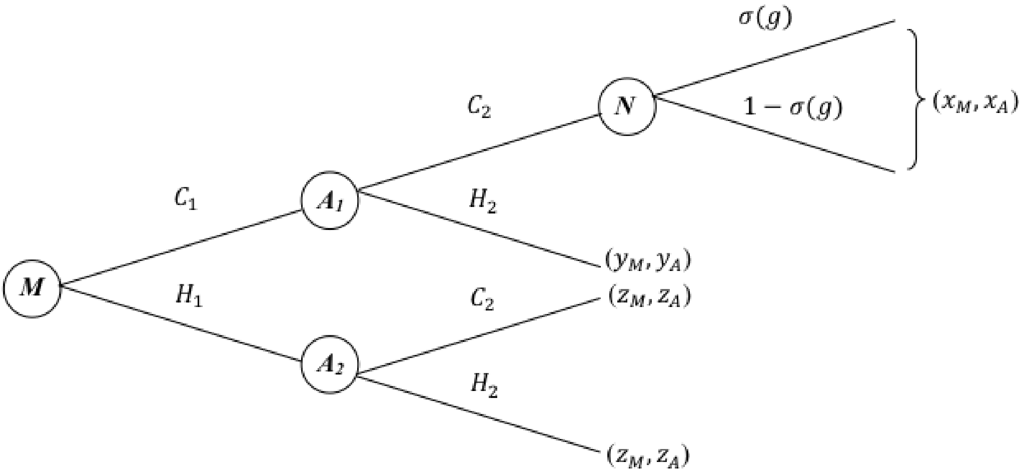

Figure 2: **Auditor's Decision**

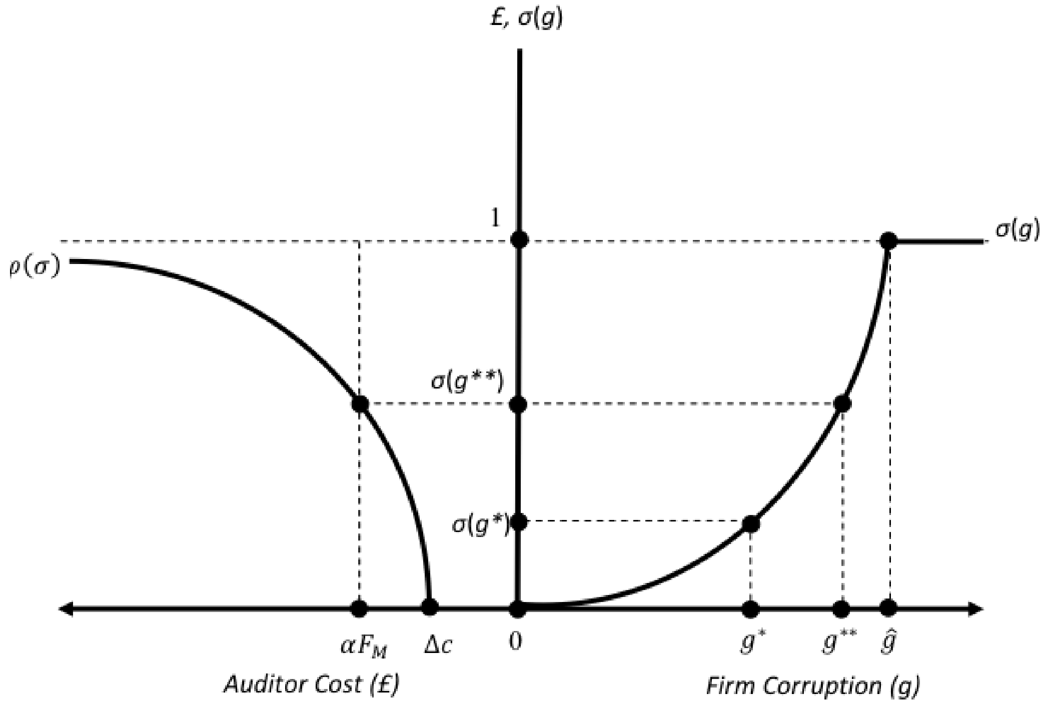

Figure 3: **Three Corruption Technologies** $\omega(g)$

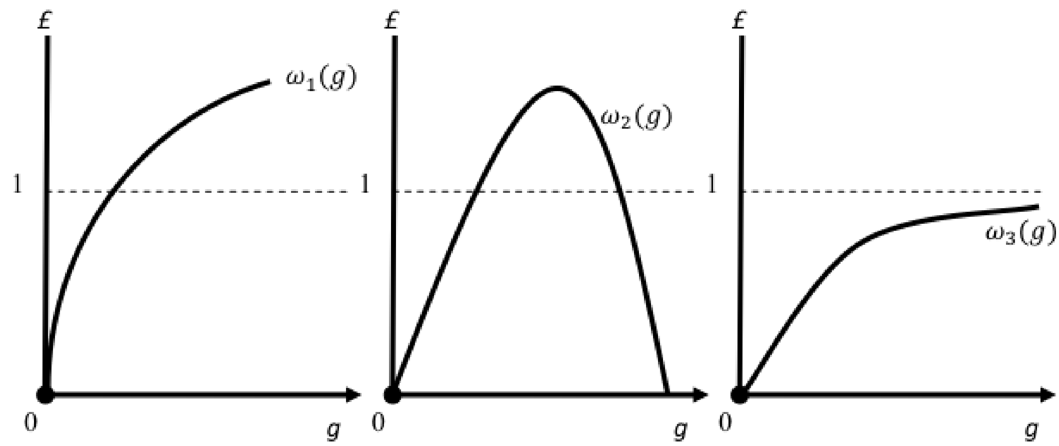

Figure 4: **Examples of the of Cases 1-3 the Relationship between Corruption Technology $\omega(g)$ and the Corruption Detection Profile $\sigma(g)$**