A new perspective to rational expectations: maximin rational expectations equilibrium*

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Abstract: We introduce a new notion of rational expectations equilibrium (REE) called maximin rational expectations equilibrium (MREE), which is based on the maximin expected utility (MEU) formulation. In particular, agents maximize maximin expected utility conditioned on their own private information and the information that the equilibrium prices generate. Maximin equilibrium allocations need not to be measurable with respect to the private information of each individual and with respect to the information that the equilibrium prices generate, as it is in the case of the Bayesian REE. We prove that a maximin REE exists universally (and not generically as in Radner (1979) and Allen (1981)), it is efficient and incentive compatible. These results are false for the Bayesian REE.

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1 Introduction

In seminal papers, Radner (1979) and Allen (1981) prove the *generic* existence of a rational expectations equilibrium (REE). Indeed, Kreps (1977) provides an example that shows that REE may not exist universally. However, a careful examination of Krep's example of the nonexistence of the REE indicates that there is nothing wrong with the REE concept other than the fact that we impose on agents the Bayesian (subjective expected utility) decision doctrine. We show that replacing the Bayesian expected utility by the maximin expected utility (MEU) leads to a REE which turns out to be efficient and incentive compatible in Kreps's example. This poses the following questions that we will address in this paper:

Why should one dictate a priori a Bayesian expected utility maximization? Does the replacement of the Bayesian doctrine with the MEU provide better outcomes? Does the REE exist universally under the MEU decision making? Is it efficient and incentive compatible? The Bayesian and the MEU formulations provide two different expected utility functional forms; is the MEU formulation superior to the Bayesian?

We introduce a new notion of REE which abandons the Bayesian decision making adopted in the papers of Radner (1979) and Allen (1981). Under the Bayesian decision making, agents maximize their subjective expected utilities conditioned on their own private information and also on the information that the equilibrium prices generate. The resulting equilibrium allocations are measurable with respect to the private information of each individual and also with respect to the information the equilibrium prices generate and clear the market for every state of nature.

Our non-expected utility reformulation of the rational expectations equilibrium of Radner (1979) and Allen (1981) is based on the adoption of the MEU (see Gilboa and Schmeidler (1989)). Specifically, in our new setup agents maximize their MEU conditioned on their own private information and also on the information the equilibrium prices have generated. In this setting the resulting maximin REE may not be measurable with respect to the private information of each individual and also with respect to the information that the equilibrium prices generate (contrary to the Bayesian REE). Nonetheless, market clearing occurs for every state of nature.

An attempt to introduce non-expected utility into general equilibrium theory was previously made by de Castro-Yannelis (2008). Specifically, de Castro-Yannelis (2008) showed that by replacing the Bayesian (subjective expected utility) by the maximin expected utility, the conflict between efficiency and incentive compatibility ceases to exist. In this paper, we continue this line of research by introducing non-expected utility into the rational expectations equilibrium.

The introduction of the MEU into the general equilibrium modeling, enables us to prove that the maximin REE exists universally under the standard continuity and concavity assumptions of the utility function. Furthermore, we show that the maximin REE is incentive compatible and efficient. These results are false for the Bayesian REE (see Kreps (1977) and Glycopantis-Yannelis (2005), p.31 and also Example 9.1.1, p.43).

The paper is organized as follows: in Section 2 we introduce the notion of maximin REE. In Section 3 we compare the maximin REE with the Bayesian REE. In particular, we show that in the Kreps's example 3.4, where a REE does not exist, a maximin REE does exist. Section 4 states the assumptions which guarantee the existence of a maximin REE. Sections 5 and 6 prove the efficiency and the incentive compatibility of the maximin REE. The related literature is discussed in Section 7. Some concluding remarks and open questions are collected in Section 8. The appendix contains the proof of the existence of the maximin REE.

2 Differential information economy and maximin REE

2.1 Differential information economy

We define the notion of a finite-agent economy with differential information. Let Ω be the finite set of states of nature and $\mathcal{F}$ be an algebra on Ω. Let $\mathbb{R}^{\ell}_{+}$ be the commodity space and I be a set of n agents. **A differential information exchange economy** $\mathcal{E}$ is a set

$$\mathcal{E} = \{(\Omega, \mathcal{F}); (X_i, \mathcal{F}_i, u_i, e_i, \pi_i) : i \in I = \{1, \ldots, n\}\},$$

where for all $i \in I$

- $X_i : \Omega \to 2^{\mathbb{R}^{\ell}_{+}}$ is agent i'**random consumption set** of each agent[4].

[4]Throughout the paper, we consider for each $i \in I$ and $\omega \in \Omega$, $X_i(\omega)$ to be the commodity space, i.e., $X_i(\omega) = \mathbb{R}^{\ell}_{+}$, expect when additional assumptions on X_i are required.

- $\mathcal{F}_i$ is a measurable partition[5] of $(\Omega, \mathcal{F})$ denoting **the private information** of agent i. The interpretation is as usual: if $\omega \in \Omega$ is the state of nature that is going to be realized, agent i observes $E^{\mathcal{F}_i}(\omega)$ the element of $\mathcal{F}_i$ which contains ω.

- a **random utility function** representing her (ex post) preferences:

$$\begin{array}{rrcl} u_i: & \Omega \times I\!\!R^\ell_+ & \to & I\!\!R \\ & (\omega, x) & \to & u_i(\omega, x). \end{array}$$

- a **random initial endowment** of physical resources represented by the function

$$e_i : \ \Omega \to I\!\!R^\ell_+.$$

We assume that e_i is $\mathcal{F}_i$-measurable and $e_i(\omega) \in X_i(\omega)$ for all $\omega \in \Omega$.

- π_i is a **probability** on Ω, whose role will be clarified below. It is assumed that $\pi_i(\omega) > 0$ for all $\omega \in \Omega$.

The structure above does not describe yet the preference of each agent. In fact, we will consider two types of preferences: the Bayesian or Expected Utility (EU) preferences (described in section 2.2 below) and the Maximin Expected Utility preference (described in section 2.3). The above structure, including each agent's preference, is common knowledge for all agents.

As usual, we can interpret the above economy as a two time period ($t = 1, 2$) model, since the ex ante stage ($t = 0$) does not play a role in rational expectation theory. At the interim stage, $t = 1$, agent i only knows that the realized state belongs to the event $E^{\mathcal{F}_i}(\omega^*)$, where ω^* is the true state at $t = 2$. With this information (or with the information acquired through prices, as we discuss below), agents trade. At the ex post stage ($t = 2$), agents execute the trades according to the contract agreed in period $t = 1$, and consumption takes place.

We define a **price vector** p as a function from Ω to the simplex of $I\!\!R^\ell_+$, denoted by Δ, such that $p(\cdot)$ is $\mathcal{F}$-measurable. Notice that since for each ω, $p(\omega) \in \Delta$, then $p(\omega) \neq 0$. This guarantees that $p : \Omega \to \Delta$ is a non-zero function.

Define for each price vector p, the **budget set** of agent i in state ω as follows:

$$B_i(\omega, p(\omega)) = \{y_i \in X_i(\omega) \, : \, p(\omega) \cdot y_i \leq p(\omega) \cdot e_i(\omega)\}.$$

[5]By an abuse of notation we will still denote by $\mathcal{F}_i$ the algebra that the partition $\mathcal{F}_i$ generates.

In order to introduce the rational expectation notions in section 3, we need the following notation. Let $\sigma(p)$ be the smallest sub-algebra of $\mathcal{F}$ for which p is measurable and let $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$ denote the smallest algebra containing both $\mathcal{F}_i$ and $\sigma(p)$.

A function $x : I \times \Omega \to I\!R^{\ell}_{+}$ is said to be a **random consumption vector or allocation** if for each $i \in I$ and $\omega \in \Omega$, $x_i(\omega) \in X_i(\omega)$. Define for all $i \in I$, the sets

$$\begin{aligned} L_{X_i} &= \{x_i : \Omega \to I\!R^{\ell}_{+} \ : \ x_i(\omega) \in X_i(\omega) \text{ for all } \omega \in \Omega\}, \\ \bar{L}_{X_i} &= \{x_i \in L_{X_i} : \ x_i(\cdot) \text{ is } \mathcal{F}_i\text{-measurable}\}. \\ \bar{L}^{REE}_{X_i} &= \{x_i \in L_{X_i} : \ x_i(\cdot) \text{ is } \mathcal{G}_i\text{-measurable}\}. \end{aligned}$$

Let $L_X = \prod_{i \in I} L_{X_i}$, $\bar{L}_X = \prod_{i \in I} \bar{L}_{X_i}$ and $\bar{L}^{REE}_X = \prod_{i \in I} \bar{L}^{REE}_{X_i}$.

An allocation (i.e., $x \in L_X$) is said to be **feasible** if

$$\sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i(\omega) \quad \text{for all } \omega \in \Omega.$$

2.2 Expected utility (EU)

We define now the (Bayesian or subjective expected utility) interim expected utility. For each i, let $(\Omega, \mathcal{F}, \pi_i)$ be a probability space and $\Pi_i \subset \mathcal{F}$ be any partition of Ω. For any assignment $x_i : \Omega \to I\!R^{\ell}_{+}$, agent i's **interim expected utility** function with respect to Π_i at x_i in state ω is given by

$$v_i(x_i|\Pi_i)(\omega) = \sum_{\omega' \in \Omega} u_i(\omega', x_i(\omega'))\pi_i(\omega'|\omega),$$

where

$$\pi_i(\omega'|\omega) = \begin{cases} 0 & \text{for} \quad \omega' \notin E^{\Pi_i}(\omega) \\ \frac{\pi_i(\omega')}{\pi_i\left(E^{\Pi_i}(\omega)\right)} & \text{for} \quad \omega' \in E^{\Pi_i}(\omega). \end{cases}$$

We can also express the interim expected utility using conditional probability as

$$v_i(x_i|\Pi_i)(\omega) = \sum_{\omega' \in E^{\Pi_i}(\omega)} u_i(\omega', x_i(\omega')) \frac{\pi_i(\omega')}{\pi_i\left(E^{\Pi_i}(\omega)\right)}.$$

In the applications below, the partition Π_i will be the original partition $\mathcal{F}_i$ or, more frequently, the partition generated by the prices, $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$.

2.3 Maximin Expected Utility (MEU)

As before, let $\Pi_i \subset \mathcal{F}$ be a partition of Ω. The maximin utility of each agent i with respect to Π_i of Ω is:

$$\underline{u}_i^{\Pi_i}(\omega, x_i) = \min_{\omega' \in E^{\Pi_i}(\omega)} u_i(\omega', x_i(\omega')).$$

Whenever for each agent i the measurable partition Π_i is his private information $\mathcal{F}_i$, then we do not use the superscript, i.e.,

$$\underline{u}_i(\omega, x_i) = \min_{\omega' \in E^{\mathcal{F}_i}(\omega)} u_i(\omega', x_i(\omega')).$$

On the other hand, when we deal with the notion of rational expectations equilibrium (according to which agents take into account also the information that the equilibrium prices generate), then for each agent i the measurable partition Π_i is $\mathcal{G}_i$ and the maximin utility is defined as

$$\underline{u}_i^{REE}(\omega, x_i) = \min_{\omega' \in E^{\mathcal{G}_i}(\omega)} u_i(\omega', x_i(\omega')), \text{ where } \mathcal{G}_i = \mathcal{F}_i \vee \sigma(p).$$

As the reader can see, this corresponds to an interim notion and does not require any expected utility representation. Therefore, we could have named it only *maximin* preference instead of MEU. However, this interim preference may come from an ex ante MEU, as defined by Gilboa and Schmeidler (1989), as we clarify below.

2.4 Motivating MEU preferences

We can introduce the MEU preferences defined above in a more intuitive way. For this, let us consider again the standard Bayesian (subjective expected utility) preference defined above. Recall that we have assumed that the prior π_i is defined on $\mathcal{F}$. What does justify this assumption? This is exactly the central tenet of the Bayesian paradigm: the agent has a prior about everything that he ignores, that is, a prior for all ω's. However, the Bayesian paradigm has been the target of many criticisms and it seems desirable to consider other paradigms. For this, let us make the weaker assumption that the agent has a prior not over all events, but only about those events that he can observe, that is, events measurable with respect to the private information partition $\mathcal{F}_i$. Although this assumption may yet be subject to criticism, it is a weaker assumption and may be justified on the grounds that each agent, by observing the occurrence of the events $E^{\mathcal{F}_i}(\omega)$, could learn their likelihood.

Therefore, it seems reasonable to assume that π_i is the prior of each agent $i \in I$, restricted to $\mathcal{F}_i$, the private information of agent i. The *ex ante* preference $\succeq_i^o$ of individual i can then be described as:[6,7]

$$f \succeq_i^o g \;\Leftrightarrow\; \int_\Omega u_i(\omega, f(\omega)) d\pi_i \geq \int_\Omega u_i(\omega, g(\omega)) d\pi_i \quad \text{for all } f, g \in \bar{L}_{X_i}.$$

This ex ante preference seems a completely standard EU preference. However, as de Castro-Yannelis (2008) have noticed, these preferences are incomplete. To see this, it is sufficient to observe that the preferences are capable of comparing *only* $\mathcal{F}_i$-measurable allocations. If the allocation h is not $\mathcal{F}_i$-measurable, its integral $\int h \, d\pi_i$ is not defined and, therefore, it is not possible for individual i to compare h with any other allocations. In other words: neither $f \succeq_i^o h$ nor $h \succeq_i^o f$ hold for any allocation f, which is the same as saying that the preference $\succeq_i^o$ is *incomplete*. As it was discussed in de Castro-Yannelis (2008), individuals can complete their preferences by adopting the maximin expected utility (MEU) of Gilboa and Schmeidler (1989).[8] We now recall the formal definition of preferences due to de Castro-Yannelis (2008).

Let $\mathbb{P}$ denote the set of measures $\mu : \mathcal{F} \to [0, 1]$. Define for each i, the following set

$$\mathcal{P}_i = \{\mu \in \mathbb{P} : \mu(A) = \pi_i(A), \text{ for all } A \in \mathcal{F}_i\}. \tag{1}$$

Thus, $\mathcal{P}_i$ is the set of all extensions of π_i from $\mathcal{F}_i$ to $\mathcal{F}$, that is the set of all probability measures defined in $\mathcal{F}$ that agree with π_i in the event that individual i is informed about. We now consider the preference $\succeq_i$ which extends $\succeq_i^o$ from $\bar{L}_{X_i}$ to the set of all allocations, L_{X_i}, i.e.,

$$f \succeq_i g \;\Leftrightarrow\; \min_{\mu \in \mathcal{P}_i} \int_\Omega u_i(\omega, f(\omega)) \, d\mu \geq \min_{\mu \in \mathcal{P}_i} \int_\Omega u_i(\omega, g(\omega)) \, d\mu \quad \text{for all } f, g \in L_{X_i}. \tag{2}$$

The preferences $\succeq_i$ are *complete*. De Castro and Yannelis (2008) have proved

[6] We use the notation $\succeq_i^o$ instead of the more standard $\succeq_i$ for a reason that will become clear in a moment.

[7] In this section, we will consider only *ex ante* preferences. The natural interim counterpart of the ex ante maximin expected utility preference defined here is the maximin utility defined on section 2.3.

[8] We will actually consider a special case of Gilboa-Schmeidler's preference: the one with the richest set of possible priors. De Castro-Yannelis (2008) show that this specialization is important: other preferences do not have the same incentive-compatibility property. See details in that paper.

that the preferences $\succeq_i$ given by (2) can be equivalently characterized by:

$$f \succeq_i g \;\Leftrightarrow\; \int_\Omega \min_{\omega' \in E^{\mathcal{F}_i}(\omega)} u_i(\omega', f(\omega'))\, d\pi_i$$
$$\geq \int_\Omega \min_{\omega' \in E^{\mathcal{F}_i}(\omega)} u_i(\omega', g(\omega'))\, d\pi_i \quad \text{for all } f, g \in L_{X_i}.$$

When we are interested in the notion of rational expectation equilibrium, we need to change expression (1) since agents take into account also the information that the equilibrium prices generate, $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. Therefore, $\mathcal{P}_i$ defined in (1) must be replaced by[9]

$$\mathcal{P}_i^{REE} = \{\mu \in \mathbb{P} : \mu(A) = \pi_i(A), \text{ for all } A \in \mathcal{G}_i\}. \tag{3}$$

Thus, $\mathbb{P}_i^{REE}$ is the set of all extensions of π_i from $\mathcal{G}_i$ to $\mathcal{F}$, that is the set of all probability measure defined in $\mathcal{F}$ that agree with π_i in the events that individual i is informed about. Then, we consider the preference $\succeq_i^{REE}$ which extends $\succeq_i^o$ from $\bar{L}_{X_i}^{REE}$ to the set of all allocations, L_{X_i}, i.e.,

$$f \succeq_i^{REE} g \;\Leftrightarrow\; \min_{\mu \in \mathcal{P}_i^{REE}} \int_\Omega u_i(\omega, f(\omega))\, d\mu \geq \min_{\mu \in \mathcal{P}_i^{REE}} \int_\Omega u_i(\omega, g(\omega))\, d\mu \quad \text{for all } f, g \in L_{X_i}.$$

Similarly to (2) observe that the preferences $\succeq_i^{REE}$ given by the above expression can be equivalently characterized by:

$$f \succeq_i^{REE} g \;\Leftrightarrow\; \int_\Omega \min_{\omega' \in E^{\mathcal{G}_i}(\omega)} u_i(\omega', f(\omega'))\, d\pi_i$$
$$\geq \int_\Omega \min_{\omega' \in E^{\mathcal{G}_i}(\omega)} u_i(\omega', g(\omega'))\, d\pi_i \quad \text{for all } f, g \in L_{X_i}.$$

Now, as we said before, the interim specialization of this preference corresponds to the maximin utility defined on section 2.3.

3 Maximin REE vs the Bayesian REE

3.1 Rational expectations equilibrium (REE)

Recall that $\sigma(p)$ is the smallest sub-algebra of $\mathcal{F}$ for which p is measurable and $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$ denotes the smallest algebra containing both $\mathcal{F}_i$ and $\sigma(p)$. We shall

[9]Note that this definition requires π_i to be defined on $\mathcal{G}_i$ instead of just $\mathcal{F}_i$. However, the justification (based on learning) given above for π_i being defined on $\mathcal{F}_i$ also works for $\mathcal{G}_i$.

also condition the expected utility of the agents on $\mathcal{G}_i$ which produces a random variable. The notion below is due to Radner (1979) and Allen (1981).

Definition 3.1 *A price vector p and a feasible allocation x are said to be a* **rational expectations equilibrium** *(REE) for the economy $\mathcal{E}$ if*

(i) *for all i the allocation $x_i(\cdot)$ is $\mathcal{G}_i$-measurable;*

(ii) *for all i and for all ω, $x_i(\omega) \in B_i(\omega, p(\omega))$;*

(iii) *for all i and for all ω, x_i maximizes the interim expected utility function $v_i(x_i|\mathcal{G}_i)(\omega)$ subject to $B_i(\omega, p(\omega))$.*

Remark 3.2 Since, the REE is an interim solution concept, one should expect that in condition (iii) above the budget set is interim, i.e.,

$$\sum_{\omega' \in E^{\mathcal{G}_i}(\omega)} p(\omega') \cdot x_i(\omega') \frac{\pi_i(\omega')}{\pi_i\left(E^{\mathcal{G}_i}(\omega)\right)} \leq \sum_{\omega' \in E^{\mathcal{G}_i}(\omega)} p(\omega') \cdot e_i(\omega') \frac{\pi_i(\omega')}{\pi_i\left(E^{\mathcal{G}_i}(\omega)\right)} \tag{4}$$

instead of

$$p(\omega) \cdot x_i(\omega) \leq p(\omega) \cdot e_i(\omega). \tag{5}$$

However, we show that (4) and (5) are equivalent. Indeed, since $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$, then $p(\cdot)$ is $\mathcal{G}_i$-measurable for all $i \in I$, as well as $x_i(\cdot)$ (see condition (i)). Furthermore, since for all $i \in I$, $e_i(\cdot)$ is $\mathcal{F}_i$-measurable and $\mathcal{F}_i \subseteq \mathcal{G}_i$, then $e_i(\cdot)$ is $\mathcal{G}_i$-measurable. Therefore, for all $\omega \in \Omega$

$$\begin{aligned} \sum_{\omega' \in E^{\mathcal{G}_i}(\omega)} p(\omega') \cdot x_i(\omega') \frac{\pi_i(\omega')}{\pi_i\left(E^{\mathcal{G}_i}(\omega)\right)} &= p(\omega) \cdot x_i(\omega) \quad \text{and} \\ \sum_{\omega' \in E^{\mathcal{G}_i}(\omega)} p(\omega') \cdot e_i(\omega') \frac{\pi_i(\omega')}{\pi_i\left(E^{\mathcal{G}_i}(\omega)\right)} &= p(\omega) \cdot e_i(\omega). \end{aligned}$$

This means that (4) is equivalent to (5).

The REE is an interim concept since agents maximize conditional expected utility based on their own private information and also on the information that equilibrium prices have generated. The resulting allocation clears the market for every state of nature.

It is by now well known that a rational expectations equilibrium (REE), as introduced in Radner (1979) may not exist. It only exists in a generic sense and not universal. Moreover, it fails to be fully Pareto optimal and incentive compatible and it is not implementable as a perfect Bayesian equilibrium of an extensive form game (Glycopantis-Muir-Yannelis (2005)).

3.2 Maximin REE

We now define the notion of a maximin REE.

Definition 3.3 *A price vector* p *and a feasible allocation* x *are said to be a* **maximin rational expectations equilibrium** *(MREE) for the economy* $\mathcal{E}$ *if:*

(i) *for all* i *and for all* ω *the allocation* $x_i(\omega) \in B_i(\omega, p(\omega))$*;*

(ii) *for all* $i \in I$ *and for all* $\omega \in \Omega$, $\underline{u}_i^{REE}(\omega, x_i) = \max_{y_i \in B_i^*(\omega,p)} \underline{u}_i^{REE}(\omega, y_i)$, *where*

$$B_i^*(\omega, p) = \left\{ y_i \in L_{X_i} \ : \ p(\omega') \cdot y_i(\omega') \leq p(\omega') \cdot e_i(\omega') \ for\ all\ \omega' \in E^{\mathcal{G}_i}(\omega) \right\}.$$

Conditions (i) and (ii) indicate that each individual maximizes her maximin expected utility conditioned on her private information and the information the equilibrium prices have generated, subject to the budget constraint.

A *free disposal* REE or maximin REE is defined as before, except that the feasibility of the allocations is defined with an inequality, i.e.,

$$\sum_{i \in I} x_i(\omega) \leq \sum_{i \in I} e_i(\omega) \quad \text{for all } \omega \in \Omega.$$

Either a REE or a maximin REE are said to be (i) *fully revealing* if the price function reveals to each agent all states of nature, (ii) *partially revealing* if the price function reveals some but not all states of nature.

3.3 Relationship between the maximin REE and the Bayesian REE

We now show that the notions of maximin REE and of REE seem to be not comparable as the following example (due to Kreps (1977)) indicates. In particular, in the Kreps's example, which proves that the REE does not exist, we will show that a maximin REE does exist. From this we can conclude that maximin REE and REE are two different solution concepts.

Example 3.4 There[10] are two agents, two commodities and two equally probable states of nature $\Omega = \{\omega_1, \omega_2\}$. The primitives of the economy are:

$$\begin{aligned} e_1 &= \left(\left(\frac{3}{2}, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{3}{2}\right)\right) \quad \mathcal{F}_1 = \{\{\omega_1\}, \{\omega_2\}\}; \\ e_2 &= \left(\left(\frac{3}{2}, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{3}{2}\right)\right) \quad \mathcal{F}_2 = \{\{\omega_1, \omega_2\}\}. \end{aligned}$$

The utility functions of agents 1 and 2 in states ω_1 and ω_2 are given as follows

$$\begin{aligned} u_1(\omega_1, x_1, y_1) &= \log x_1 + y_1 \qquad u_1(\omega_2, x_1, y_1) = 2\log x_1 + y_1 \\ u_2(\omega_1, x_2, y_2) &= 2\log x_2 + y_2 \qquad u_2(\omega_2, x_2, y_2) = \log x_2 + y_2. \end{aligned}$$

It is well known that for the above economy a rational expectations equilibrium does not exist (see Kreps (1977)), however we will show below that a maximin REE does exist.

The information generated by the equilibrium price can be either $\{\{\omega_1\}, \{\omega_2\}\}$ or $\{\{\omega_1, \omega_2\}\}$. In the first case the maximin REE coincides with the Bayesian REE, therefore it does not exist. Thus, let us consider the case $\sigma(p) = \Omega$, i.e., $p(\omega_1) = p(\omega_2) = p$ and $q(\omega_1) = q(\omega_2) = q$.

Since for each ω, $E^{\mathcal{G}_1}(\omega) = \{\omega\}$, agent 1 solves the following constraint maximization problems:

Agent 1 in state ω_1:

$$\max_{x_1(\omega_1), y_1(\omega_1)} \log x_1(\omega_1) + y_1(\omega_1) \quad \text{subject to}$$

$$p x_1(\omega_1) + q y_1(\omega_1) \leq \frac{3}{2}(p+q).$$

[10]We are grateful to T. Liu and L. Sun for having checked the computations of Example 3.4.

Thus,

$$x_1(\omega_1) = \frac{q}{p} \quad y_1(\omega_1) = \frac{3}{2}\frac{p}{q} + \frac{1}{2}.$$

Agent 1 in state ω_2:

$$\max_{x_1(\omega_2), y_1(\omega_2)} 2\log x_1(\omega_2) + y_1(\omega_2) \quad \text{subject to}$$

$$px_1(\omega_2) + qy_1(\omega_2) \leq \frac{3}{2}(p+q).$$

Thus,

$$x_1(\omega_2) = \frac{2q}{p} \quad y_1(\omega_2) = \frac{3}{2}\frac{p}{q} - \frac{1}{2}.$$

Agent 2 in the event $\{\omega_1, \omega_2\}$ maximizes

$$\min\{2logx_2(\omega_1) + y_2(\omega_1); logx_2(\omega_2) + y_2(\omega_2)\}.$$

Therefore, we can distinguish three cases:

I Case: $2logx_2(\omega_1) + y_2(\omega_1) > logx_2(\omega_2) + y_2(\omega_2)$. In this case, agent 2 solves the following constraint maximization problem:

max $logx_2(\omega_2) + y_2(\omega_2)$ subject to $px_2(\omega_1) + qy_2(\omega_1) \leq \frac{3}{2}(p+q)$ and $px_2(\omega_2) + qy_2(\omega_2) \leq \frac{3}{2}(p+q)$. Thus,

$$x_2(\omega_2) = \frac{q}{p} \quad y_2(\omega_2) = \frac{3}{2}\frac{p}{q} + \frac{1}{2}.$$

From feasibility it follows that $p = q$, and

$$(x_1(\omega_1), y_1(\omega_1)) = (1, 2) \quad (x_1(\omega_2), y_1(\omega_2)) = (2, 1)$$

$$(x_2(\omega_1), y_2(\omega_1)) = (2, 1) \quad (x_2(\omega_2), y_2(\omega_2)) = (1, 2).$$

Notice that $2logx_2(\omega_1) + y_2(\omega_1) = 2log2 + 1 > log1 + 2 = logx_2(\omega_2) + y_2(\omega_2)$.

II Case: $2logx_2(\omega_1) + y_2(\omega_1) < logx_2(\omega_2) + y_2(\omega_2)$. In this case, agent 2 solves the following constraint maximization problem:

max $2logx_2(\omega_1) + y_2(\omega_1)$ subject to $px_2(\omega_1) + qy_2(\omega_1) \leq \frac{3}{2}(p+q)$ and $px_2(\omega_2) + qy_2(\omega_2) \leq \frac{3}{2}(p+q)$ Thus,

$$x_2(\omega_1) = \frac{2q}{p} \quad y_2(\omega_1) = \frac{3}{2}\frac{p}{q} - \frac{1}{2}.$$

From feasibility it follows that $p = q$, and

$$(x_1(\omega_1), y_1(\omega_1)) = (1, 2) \quad (x_1(\omega_2), y_1(\omega_2)) = (2, 1)$$

$$(x_2(\omega_1), y_2(\omega_1)) = (2, 1) \quad (x_2(\omega_2), y_2(\omega_2)) = (1, 2).$$

Clearly, as noticed above, $2log2 + 1 > log1 + 2$. Therefore, in the second case there is no maximin REE.

III Case: $2logx_2(\omega_1) + y_2(\omega_1) = logx_2(\omega_2) + y_2(\omega_2)$. In this case, agent 2 solves one of the following two constraint maximization problems:

max $logx_2(\omega_2)+y_2(\omega_2)$ or max $2logx_2(\omega_1)+y_2(\omega_1)$ subject to $px_2(\omega_1)+qy_2(\omega_1) \leq \frac{3}{2}(p+q)$ and $px_2(\omega_2) + qy_2(\omega_2) \leq \frac{3}{2}(p+q)$. In both cases, from feasibility it follows that $p = q$, and

$$(x_1(\omega_1), y_1(\omega_1)) = (1, 2) \quad (x_1(\omega_2), y_1(\omega_2)) = (2, 1)$$

$$(x_2(\omega_1), y_2(\omega_1)) = (2, 1) \quad (x_2(\omega_2), y_2(\omega_2)) = (1, 2).$$

Hence, since $2logx_2(\omega_1) + y_2(\omega_1) = 2log2 + 1 > log1 + 2 = logx_2(\omega_2) + y_2(\omega_2)$, there is no maximin REE in the third case.

Therefore, we can conclude that the unique maximin REE allocation is given by

$$(x_1(\omega_1), y_1(\omega_1)) = (1, 2) \quad (x_1(\omega_2), y_1(\omega_2)) = (2, 1)$$

$$(x_2(\omega_1), y_2(\omega_1)) = (2, 1) \quad (x_2(\omega_2), y_2(\omega_2)) = (1, 2).$$

Observe that the maximin REE bundles are not $\mathcal{F}_i$-measurable.

Remark 3.5 It should be noted that in the above example whenever agents maximize a Bayesian (subjective) expected utility as Kreps showed, the REE either revealing or non revealing does not exist. However, allowing agents to maximize a non expected utility, i.e., the maximin expected utility, we showed that a maximin REE exists. The example makes clear that the Bayesian choice of optimization seems to impose a functional restriction on the utility functions which does not allow agents to achieve the desired outcome. The functional form of the maximin expected utility seems to be achieving what we want agents to accomplish, i.e., to reach an equilibrium outcome. As we will see in the next section, this outcome is incentive compatible and efficient.

Remark 3.6 As we have already observed the maximin REE allocations may not be $\mathcal{G}_i$-measurable. However, if we assume strict concavity and $\mathcal{F}_i$-measurability of the random utility function of each agent, then the resulting maximin REE allocations will be $\mathcal{G}_i$- measurable, as the following proposition indicates.

Proposition 3.7 *Let (p, x) be a maximin REE and $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$ for all $i \in I$. Assume that for all i, (i) $u_i(\cdot, y)$ is $\mathcal{G}_i$-measurable for all $y \in I\!R^\ell_+$ and (ii) $u_i(\omega, \cdot)$ is strictly concave for all $\omega \in \Omega$. Then $x_i(\cdot)$ is $\mathcal{G}_i$-measurable for all $i \in I$.*

Proof: Assume on the contrary that there exist $i \in I$ and $a, b \in \Omega$ such that $a \in E^{\mathcal{G}_i}(b)$ and $x_i(a) \neq x_i(b)$. Consider $z_i(\omega) = \alpha x_i(a) + (1-\alpha)x_i(b)$ for all $\omega \in E^{\mathcal{G}_i}(b)$, where $\alpha \in (0,1)$, and notice that $z_i(\cdot)$ is $\mathcal{G}_i$-measurable. Moreover,

$$\underline{u}_i^{REE}(b, z_i) = \min_{\omega \in E^{\mathcal{G}_i}(b)} u_i(\omega, z_i(\omega)) = \min_{\omega \in E^{\mathcal{G}_i}(b)} u_i(\omega, \alpha x_i(a) + (1-\alpha)x_i(b))$$

Since $u_i(\cdot, y)$ is $\mathcal{G}_i$-measurable for all $y \in I\!R^\ell_+$, from strict concavity of u_i it follows that

$$\begin{aligned} \underline{u}_i^{REE}(b, z_i) = u_i(b, \alpha x_i(a) + (1-\alpha)x_i(b)) &> \alpha u_i(b, x_i(a)) + (1-\alpha)u_i(b, x_i(b)) \\ = \alpha u_i(a, x_i(a)) + (1-\alpha)u_i(b, x_i(b)) &\geq \underline{u}_i^{REE}(b, x_i). \end{aligned}$$

Since (p, x) is a maximin REE it follows that $z_i \notin B_i^*(b, p)$, that is there exists a state $\omega_i \in E^{\mathcal{G}_i}(b)$ such that

$$p(\omega_i) \cdot z_i(\omega_i) > p(\omega_i) \cdot e_i(\omega_i) \;\Rightarrow\; \alpha p(\omega_i) \cdot x_i(a) + (1-\alpha)p(\omega_i) \cdot x_i(b) > p(\omega_i) \cdot e_i(\omega_i).$$

Moreover, since $p(\cdot)$ and $e_i(\cdot)$ are $\mathcal{G}_i$-measurable and $x_i(\omega) \in B_i(\omega, p(\omega))$ for all ω (see condition (i) in Definition 3.3), it follows that $p(\cdot) \cdot e_i(\cdot) > p(\cdot) \cdot e_i(\cdot)$, which is a contradiction. □

It was shown in Example 3.4 that the maximin and the Bayesian REE are not comparable. We will show below that whenever the utility functions are $\mathcal{F}_i$-measurable, then any maximin REE is also a REE and vice versa. Note that in Example 3.4, utility functions are not $\mathcal{F}_i$-measurable and therefore Example 3.4 does not fulfill the assumptions of Lemma 3.8 below.

Lemma 3.8 *Assume that for all $i \in I$ and for all $y \in I\!R^\ell_+$, $u_i(\cdot, y)$ is $\mathcal{F}_i$-measurable. If (p, x) is a REE, then (p, x) is a maximin REE. The converse is also true if $x_i(\cdot)$ is $\mathcal{G}_i$-measurable for all $i \in I$.*

Proof: All we need to show is that the maximin expected utility and the interim expected utility coincide. Since for all $i \in I$ and for all $y \in I\!R_+^\ell$, $u_i(\cdot, y)$ is $\mathcal{F}_i$-measurable and $\mathcal{F}_i \subseteq \mathcal{G}_i$, then $u_i(\cdot, y)$ is $\mathcal{G}_i$-measurable.

Moreover, since for each $i \in I$, $x_i(\cdot)$ is $\mathcal{G}_i$-measurable it follows that for all $i \in I$ and $\omega \in \Omega$, both maximin and interim utility function are equal to the ex-post utility function. That is,

$$\underline{u}_i(\omega, x_i) = \min_{\omega' \in E^{\mathcal{G}_i}(\omega)} u_i(\omega', x_i(\omega')) = u_i(\omega, x_i(\omega)) \tag{6}$$

and

$$v_i(x_i|\mathcal{G}_i)(\omega) = \sum_{\omega' \in E^{\mathcal{G}_i}(\omega)} u_i(\omega', x_i(\omega')) \frac{\pi_i(\omega')}{\pi_i\left(E^{\mathcal{G}_i}(\omega)\right)} = u_i(\omega, x_i(\omega)). \tag{7}$$

From (6) and (7) it follows that for all i and ω, $\underline{u}_i(\omega, x_i) = v_i(x_i|\mathcal{G}_i)(\omega)$. Therefore, we can conclude that (p, x) is a maximin REE if and only if (p, x) is a Bayesian REE. □

Remark 3.9 The above lemma remains true if we replace the $\mathcal{F}_i$-measurability of the allocations by the strict concavity of the random utility functions. This follows by combining Proposition 3.7 and Lemma 3.8.

4 Existence of a Maximin REE

In this section we prove the existence of a maximin REE. It should be noted that under the assumptions, which guarantee that a maximin REE exists, the Bayesian REE need not exist. The following assumptions are needed:

$(A.1)$ For every $i \in I$ and $\omega \in \Omega$, $X_i(\omega)$ is a non-empty, convex and closed subset of $I\!R_+^\ell$;

$(A.2)$ For every $i \in I$ and $\omega \in \Omega$, the initial endowment $e_i(\omega)$ belongs to the interior of $X_i(\omega)$;

$(A.3)$ For every $i \in I$ and $\omega \in \Omega$, the function $u_i(\omega, \cdot)$ is continuous, concave and strongly monotone.

Theorem 4.1 (Main Existence Theorem) *: Assume that assumptions* $(A.1), (A.2)$ *and* $(A.3)$ *hold, then there exists a maximin REE in* $\mathcal{E}$.

The following auxiliary theorem plays an important role in the proof of Theorem 4.1. The proofs of both theorems are in the appendix.

Theorem 4.2 *Suppose that assumptions* $(A.1), (A.2)$ *and* $(A.3)$ *hold and that for every* $i \in I$ *and* $\omega \in \Omega$, $X_i(\omega)$ *is a compact set. Then, a free disposal maximin REE exists.*

4.1 Proof of the main existence theorem

For $k \in \mathbb{N}$, $i \in I$ and $\omega \in \Omega$, let $X_i^k(\omega)$ be the set of all $x \in X_i(\omega)$ such that

$$\sum_{h=1}^{\ell} x^h \leq k \sum_{\omega \in \Omega} \sum_{h=1}^{\ell} e_i^h(\omega).$$

Assumption (A.1) implies that $X_i^k(\omega)$ is compact and convex; while assumption (A.2) guarantees that it is also non empty for all k. Therefore, by the Auxiliary Theorem 4.2 it follows that for every k, there exists a free disposal maximin REE, i.e., there exists a sequence $(p^k, x^k) \in L_{X^k} \times L_{\triangle}$, where for all $i \in I$,

$$\begin{aligned} L_{X_i^k} &= \{x_i : \Omega \to \mathbb{R}_+^{\ell} : \; x_i(\omega) \in X_i^k(\omega) \text{ for all } \omega \in \Omega\} \\ L_{X^k} &= \prod_{i \in I} L_{X_i^k} \\ L_{\Delta} &= \{p : \Omega \to \Delta \text{ such that } p(\cdot) \text{ is } \; \mathcal{F}-\text{measurable}\}, \end{aligned}$$

such that for all $k \in I\!N$,

(i^k) for all i and $\omega, x_i^k(\omega) \in B_i(\omega, p^k(\omega))$.

(ii^k) for all $i \in I$ and $\omega \in \Omega$, $\underline{u}_i^{REE}(\omega, x_i^k) = \max_{y_i \in B_i^*(\omega, p^k)} \underline{u}_i^{REE}(\omega, y_i)$, where

$$B_i^*(\omega, p^k) = \left\{ y_i \in L_{X_i^k} \ : \ p^k(\omega') \cdot y_i(\omega') \leq p^k(\omega') \cdot e_i(\omega') \text{ for all } \omega' \in E^{\mathcal{G}_i}(\omega) \right\}.$$

(iii^k) $$\sum_{i \in I} x_i^k(\omega) \ \leq \ \sum_{i \in I} e_i(\omega) \qquad \text{for all } \omega \in \Omega.$$

Since for any i and $k \in I\!N$, the sequence (p^k, x^k) belongs to the compact set $L_\Delta \times L_{X^k}$, there exists a subsequence, still denoted by (p^k, x^k), which converges[11] to (p^*, x^*). We need to prove that (p^*, x^*) is a maximin REE. First of all, notice that $p^* \in L_\Delta$ and $x^* \in L_X$; moreover from (i^k) and (iii^k) it follows that for every $i \in I$ and $\omega \in \Omega$, $x_i^*(\omega) \in B_i(\omega, p^*(\omega))$ and $\sum_{i \in I} x_i^*(\omega) \leq \sum_{i \in I} e_i(\omega)$, which actually becomes an equality because of the monotonicity assumption, i.e., $\sum_{i \in I} x_i^*(\omega) = \sum_{i \in I} e_i(\omega)$. To complete the proof, we must show that for all $i \in I$ and $\omega \in \Omega$, $\underline{u}_i^{REE}(\omega, x_i^*) = \max_{y_i \in B_i^*(\omega, p^*)} \underline{u}_i^{REE}(\omega, y_i)$ where

$$B_i^*(\omega, p^*) = \left\{ y_i \in L_{X_i} \ : \ p^*(\omega') \cdot y_i(\omega') \leq p^*(\omega') \cdot e_i(\omega') \text{ for all } \omega' \in E^{\mathcal{G}_i}(\omega) \right\}.$$

Assume, on the contrary, that there exist $i \in I$, $\bar{\omega} \in \Omega$ and $y_i \in L_{X_i}$ such that $\underline{u}_i^{REE}(\bar{\omega}, y_i) > \underline{u}_i^{REE}(\bar{\omega}, x_i^*)$ and $p^*(\omega) \cdot y_i(\omega) \leq p^*(\omega) \cdot e_i(\omega)$ for all $\omega \in E^{\mathcal{G}_i}(\bar{\omega})$. By the continuity of the maximin expected utility function and assumption $(A.2)$, without loss of generality we may consider an allocation y_i such that

$$p^*(\omega) \cdot y_i(\omega) < p^*(\omega) \cdot e_i(\omega) \quad \text{for all } \omega \in E^{\mathcal{G}_i}(\bar{\omega}). \tag{8}$$

By continuity of the maximin expected utility function, there exists $\bar{k}$ such that for all $k > \bar{k}$, $\underline{u}_i^{REE}(\bar{\omega}, y_i) > \underline{u}_i^{REE}(\bar{\omega}, x_i^k)$. Since for all k, (p^k, x^k) is a free disposal maximin REE, from (ii^k) it follows that for all $k > \bar{k}$, there exists $\omega_k \in E^{\mathcal{G}_i}(\bar{\omega})$ such that

$$p^k(\omega_k) \cdot y_i(\omega_k) > p^k(\omega_k) \cdot e_i(\omega_k).$$

[11] We mean that the sequence (p^k, x^k) converges pointwise to (p^*, x^*), that is for all i and ω,

$$p^*(\omega) = \lim_{k \to \infty} p^k(\omega) \quad \text{and} \quad x_i^*(\omega) = \lim_{k \to \infty} x_i^k(\omega).$$

Since Ω is finite, we may conclude that there exists $\omega \in E^{\mathcal{G}_i}(\bar{\omega})$ such that for infinitely many $k \in I\!N$,

$$p^k(\omega) \cdot y_i(\omega) > p^k(\omega) \cdot e_i(\omega).$$

Therefore, by limit arguments,

$$p^*(\omega) \cdot y_i(\omega) \geq p^*(\omega) \cdot e_i(\omega),$$

which contradicts (8). Thus, (p^*, x^*) is a maximin REE and this completes the proof. □

Remark 4.3 Assume that assumptions $(A.1), (A.2)$ and $(A.3)$ hold. If for all $y \in I\!R^\ell_+$ and for all $i \in I$, $u_i(\cdot, y)$ is $\mathcal{F}_i$-measurable and $u_i(\omega, \cdot)$ is strict concave, then from Remark 3.9 and Theorem 4.1 it follows that there exists a REE in $\mathcal{E}$. .

Remark 4.4 Notice that in Example 3.4, where the REE does not exist, not all the above assumptions of Remark 4.3 are satisfied. In particular, the random utility functions are not $\mathcal{F}_i$-measurable. Hence, the Kreps's example of the nonexistence of a REE does not contradict Remark 4.3.

5 Efficiency of the maximin REE

We now define the notion of maximin Pareto optimality and we will prove that any maximin REE is maximin Pareto optimal.

Definition 5.1 *A feasible allocation x is said to be maximin efficient (or maximin Pareto optimal) with respect to information structure Π, if there do not exist a state $\bar{\omega}$ and an allocation $y \in L_X$ such that*

$$\begin{array}{ll} (i) & \underline{u}_i^{\Pi_i}(\bar{\omega}, y_i) > \underline{u}_i^{\Pi_i}(\bar{\omega}, x_i) \quad for\ all\ i \in I\ and \\ (ii) & \sum_{i \in I} y_i(\omega) = \sum_{i \in I} e_i(\omega) \quad for\ all\ \omega \in \Omega. \end{array}$$

Proposition 5.2 *If for any $i \in I$, and $t \in R^\ell_+$, $u_i(\cdot, t)$ is $\mathcal{F}_i$-measurable[12], then any maximin REE allocation is maximin efficient.*

[12]Notice that the measurability assumption of the utility does not imply that the maximin utility coincides with the ex post one, since the allocation may not be measurable.

Proof: Let (p, x) be a maximin REE and notice that since agents take into account the information that the equilibrium price have generated, then the private information of each agent is $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. Thus, for each $i \in I$, $\Pi_i = \mathcal{G}_i$ and $\underline{u}_i^{\Pi} = \underline{u}_i^{REE}$. Assume to the contrary that x is not maximin efficient, that is, there exist a state $\bar{\omega}$ and an allocation $y \in L_X$ such that

$$\begin{array}{ll} (i) & \underline{u}_i^{REE}(\bar{\omega}, y_i) > \underline{u}_i^{REE}(\bar{\omega}, x_i) \quad \text{for all } i \in I \ \text{ and} \\ (ii) & \sum_{i \in I} y_i(\omega) = \sum_{i \in I} e_i(\omega) \quad \text{for all } \omega \in \Omega. \end{array}$$

From condition (i) it follows that for all $i \in I$, $y_i \notin B_i^*(\bar{\omega}, p)$, that is there exists a state $\omega_i \in E^{\mathcal{G}_i}(\bar{\omega})$ such that $p(\omega_i) \cdot y_i(\omega_i) > p(\omega_i) \cdot e_i(\omega_i)$. Consider, the coalition S defined as follows:

$$S = \{i \in I \ : \ p(\bar{\omega}) \cdot y_i(\bar{\omega}) \leq p(\bar{\omega}) \cdot e_i(\bar{\omega})\}.$$

If S is empty, then $p(\bar{\omega}) \cdot y_i(\bar{\omega}) > p(\bar{\omega}) \cdot e_i(\bar{\omega})$ for all $i \in I$ and hence

$$p(\bar{\omega}) \sum_{i \in I} y_i(\bar{\omega}) > p(\bar{\omega}) \sum_{i \in I} e_i(\bar{\omega}),$$

which contradicts condition (ii). On the other hand, if $S \neq \emptyset$, then for all $i \in S$, consider the constant allocation h_i such that $h_i(\omega) = y_i(\bar{\omega})$ for all $\omega \in E^{\mathcal{G}_i}(\bar{\omega})$. Since $p(\cdot)$ and $e_i(\cdot)$ are $\mathcal{G}_i$-measurable, it follows that $h_i(\omega) \in B_i(\omega, p(\omega))$ for all $\omega \in E^{\mathcal{G}_i}(\bar{\omega})$, that is $h_i \in B_i^*(\bar{\omega}, p)$, and hence

$$\underline{u}_i^{REE}(\bar{\omega}, h_i) \leq \underline{u}_i^{REE}(\bar{\omega}, x_i) < \underline{u}_i^{REE}(\bar{\omega}, y_i),$$

because (p, x) is a maximin REE. Moreover, since $u_i(\cdot, y)$ is $\mathcal{G}_i$-measurable, it follows that

$$u_i(\bar{\omega}, y_i(\bar{\omega})) = u_i(\omega, y_i(\bar{\omega})) = \underline{u}_i^{REE}(\bar{\omega}, h_i) < \underline{u}_i^{REE}(\bar{\omega}, y_i) \leq u_i(\bar{\omega}, y_i(\bar{\omega})),$$

which is clearly a contradiction. Thus, x is maximin efficient. □

6 Incentive Compatibility of the maximin REE

We now recall the notion of coalitional incentive compatibility of Krasa-Yannelis (1994).

Definition 6.1 *An allocation x is said to be coalitional incentive compatible (CIC) if the following does not hold: there exist a coalition S and two states a and b such that*

$$\begin{array}{lll} (i) & E^{\mathcal{F}_i}(a) = E^{\mathcal{F}_i}(b) & \text{for all } i \notin S, \\ (ii) & e_i(a) + x_i(b) - e_i(b) \in I\!R^{\ell}_{+} & \text{for all } i \in S, \text{ and} \\ (iii) & u_i(a, e_i(a) + x_i(b) - e_i(b)) > u_i(a, x_i(a)) & \text{for all } i \in S. \end{array}$$

In order to explain what incentive compatibility means in an asymmetric information economy, let us consider the following two examples[13].

Example 6.2 Consider an economy with two agents, three equally probable states of nature, denoted by a, b and c, and one good per state denoted by x. The primitives of the economy are given as follows:

$$u_1(\cdot, x_1) = \sqrt{x_1};\quad e_1(a,b,c) = (20, 20, 0);\quad \mathcal{F}_1 = \{\{a,b\};\{c\}\}.$$
$$u_2(\cdot, x_2) = \sqrt{x_2};\quad e_2(a,b,c) = (20, 0, 20);\quad \mathcal{F}_2 = \{\{a,c\};\{b\}\}.$$

Consider the following risk sharing (Pareto optimal) redistribution of initial endowment:

$$\begin{aligned} x_1(a,b,c) &= (20, 10, 10) \\ x_2(a,b,c) &= (20, 10, 10). \end{aligned}$$

Notice that the above allocation is not incentive compatible. Indeed, suppose that the realized state of nature is a, agent 1 is in the event $\{a, b\}$ and she reports c, (observe that agent 2 cannot distinguish between a and c). If agent 2 believes that c is the realized state of nature as agent 1 has claimed, then she gives her ten units. Therefore, the utility of agent 1, when she misreports, is $u_1(a, e_1(a) + x_1(c) - e_1(c)) = u_1(a, 20 + 10 - 0) = \sqrt{30}$ which is greater than $u_1(a, x_1(a)) = \sqrt{20}$, the utility of agent 1 when she does not misreport. Hence, the allocation $x_1(a,b,c) = (20, 10, 10)$ and $x_2(a,b,c) = (20, 10, 10)$ is not incentive compatible. Similarly, one can easily check that when a is the realized state of nature, agent 2 has an incentive to report state b and benefit.

[13] The reader is also referred to Krasa-Yannelis (1994), Koutsougeras-Yannelis (1993) and Podczeck-Yannelis (2008) for an extensive discussion of the Bayesian incentive compatibility in asymmetric information economies.

In order to make sure that the equilibrium contracts are stable, we must insist on a coalitional definition of incentive compatibility and not an individual one. As the following example shows, a contract which is individual incentive compatible may not be coalitional incentive compatible and therefore may not be viable.

Example 6.3 Consider an economy with three agents, two good and three equiprobable states of nature $\Omega = \{a, b, c\}$. The primitives of the economy are given as follows: for all $i = 1, 2, 3$, $u_i(\cdot, x_i, y_i) = \sqrt{x_i y_i}$ and

$$\begin{array}{rl} \mathcal{F}_1 = \{\{a,b,c\}\}; & e_1(a,b,c) = ((15,0);(15,0);(15,0)). \\ \mathcal{F}_2 = \{\{a,b\},\{c\}\}; & e_2(a,b,c) = ((0,15);(0,15);(0,15)). \\ \mathcal{F}_3 = \{\{a\},\{b\},\{c\}\}; & e_3(a,b,c) = ((15,0);(15,0);(15,0)). \end{array}$$

Consider the following redistribution of the initial endowments:

$$\begin{array}{rcl} \mathbf{x}_1(a,b,c) & = & ((8,5),(8,5),(8,13)) \\ \mathbf{x}_2(a,b,c) & = & ((7,4),(7,4),(12,1)) \\ \mathbf{x}_3(a,b,c) & = & ((15,6),(15,6),(10,1)). \end{array} \tag{9}$$

Notice that the only agent who can misreport either state a or b to agents 1 and 2 is agent 3. Clearly, agent 3 cannot misreport state c since agent 2 would know it. Thus, agent 3 can only lie if either state a or state b occurs. However, agent 3 has no incentive to misreport since she gets the same consumption in both states a and b. Hence, the allocation (9) is individual incentive compatible, but we will show that it is not coalitional incentive compatible. Indeed, if c is the realized state of nature, agents 2 and 3 have an incentive to cooperate against agent 1 and report b (notice that agent 1 cannot distinguish between b and c). The coalition $S = \{2, 3\}$ will now be better off, i.e.,

$$\begin{array}{rcl} u_2(c, e_2(c) + \mathbf{x}_2(b) - e_2(b)) & = & u_2(c,(0,15) + (7,4) - (0,15)) \\ = u_2(c,(7,4)) & = & \sqrt{28} > \sqrt{12} = u_2(c, \mathbf{x}_2(c)) \\ u_3(c, e_3(c) + \mathbf{x}_3(b) - e_3(b)) & = & u_3(c,(15,0) + (15,6) - (15,0)) \\ = u_3(c,(15,6)) & = & \sqrt{90} > \sqrt{10} = u_3(c, \mathbf{x}_3(c)). \end{array}$$

In Example 6.2 we have constructed an allocation which is Pareto optimal but it is not individual incentive compatible; while in Example 6.3 we have shown that an allocation, which is individual incentive compatible, need not be coalitional incentive compatible.

In view of Examples 6.2 and 6.3, it is easy to understand the meaning of Definition 6.1. An allocation is coalitional incentive compatible if no coalition of agents S can cheat the complementary coalition (i.e., $I \setminus S$) by misreporting the realized state of nature and make all its members better off. Notice that condition (i) indicates that coalition S can only cheat the agents not in S (i.e., $I \setminus S$) in the states that the agents in $I \setminus S$ cannot distinguish. If $S = \{i\}$ then the above definition reduces to individual incentive compatibility.

6.1 Maximin Incentive Compatibility

In this section we will prove that the maximin REE is incentive compatible. To this end we need the following definition of maximin coalitional incentive compatibility, which is an extension of the Krasa-Yannelis (1994) definition to incorporate maximin preferences (see also de Castro-Yannelis (2008)).

Definition 6.4 *A feasible allocation x is said to be maximin coalitional incentive compatible (MCIC) with respect to information structure Π, if the following does not hold: there exist a coalition S and two states a and b such that*

$$
\begin{array}{lll}
(i) & E^{\Pi_i}(a) = E^{\Pi_i}(b) & for\ all\ i \notin S, \\
(ii) & u_i(a, \cdot) = u_i(b, \cdot) & for\ all\ i \notin S, \\
(iii) & e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^{\ell}_{+} & for\ all\ i \in S,\ and \\
(iv) & \underline{u}_i^{\Pi_i}(a, y_i) > \underline{u}_i^{\Pi_i}(a, x_i) & for\ all\ i \in S,
\end{array}
$$

where for all $i \in S$,

$$
(*) \qquad y_i(\omega) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } \omega = a \\ x_i(\omega) & \text{otherwise.} \end{cases}
$$

According to the above definition, an allocation is said to be maximin coalitional incentive compatible if it is not possible for a coalition to misreport the realized state of nature and have a distinct possibility of making its members better off in terms of maximin expected utility. Notice that in addition to Definition 6.1 we require that agents in the complementary coalition to have the same utility in states a and b that they cannot distinguish. Obviously, if $S = \{i\}$ then the above definition reduces to individual incentive compatibility.

Remark 6.5 Example 6.2 shows that whenever agents use the Bayesian expected utility an allocation may not be incentive compatible. We now show that it is not the case when agents use the maximin expected utility. Precisely, if agents take into account the worse possible state that can occur, then the allocation $x_i(a, b, c) = (20, 10, 10)$ for $i = 1, 2$ in Example 6.2, is maximin incentive compatible. Indeed, if a is the realized state of nature, agent 1 does not have an incentive to report state c and benefit, because when she misreports she gets:

$$\underline{u}_1(a, y_1) = \min\{u_1(a, e_1(a) + x_1(c) - e_1(c)); u_1(b, x_1(b))\} = \min\{\sqrt{30}, \sqrt{10}\} = \sqrt{10}.$$

When agent 1 does not misreport, she gets:

$$\underline{u}_1(a, x_1) = \min\{u_1(a, x_1(a)); u_1(b, x_1(b))\} = \min\{\sqrt{20}, \sqrt{10}\} = \sqrt{10}.$$

Consequently agent 1 does not gain by misreporting contrary to the subjective expected utility, as we saw in Example 6.2. Similarly, one can easily check that agent 2, when a is the realized state of nature, does not have an incentive to report state b and benefit. Indeed, if the realized state of nature is a, agent 2 is in the event $\{a, c\}$. If agent 2 reports the false event $\{b\}$ then her maximin expected utility does not increase since

$$\begin{aligned}\underline{u}_2(a, y_1) &= \min\{u_2(a, e_2(a) + x_2(b)) - e_2(b); u_2(c, x_2(c))\} = \min\{\sqrt{20 + 10 - 0}, \sqrt{10}\} = \\ &= \sqrt{10} = \min\{\sqrt{20}, \sqrt{10}\} = \underline{u}_2(a, x_2).\end{aligned}$$

Remark 6.6 Condition (ii) of Definition 6.4 does not necessarily mean that for all $i \notin S$ and $y \in R^{\ell}_{+}$, $u_i(\cdot, y)$ is Π_i-measurable. Indeed it may be the case that there exists $\omega \in E^{\Pi_i}(a) \setminus \{a, b\}$ such that $u_i(\omega, \cdot) \neq u_i(a, \cdot) = u_i(b, \cdot)$. Moreover, condition (ii) is not required for each state, but only for the realized state of nature which the members of S may misreport. Observe that Definition 6.4 implicity requires that the members of the coalition S are able to distinguish between a and b; i.e., $a \notin E^{\Pi_i}(b)$ for all $i \in S$. One could replace condition (i) by $E^{\Pi_i}(a) = E^{\Pi_i}(b)$ if and only if $i \notin S$.

Lemma 6.7 *Condition (iv) and (∗) in the Definition 6.4, imply that for all $i \in S$,*

$$u_i(a, x_i(a)) = \min_{\omega \in E^{\Pi_i}(a)} u_i(\omega, x_i(\omega)) = \underline{u}_i^{\Pi_i}(a, x_i).$$

Proof: Assume, on the contrary, there exist an agent $i \in S$ and a state $\omega_1 \in E^{\Pi_i}(a) \setminus \{a\}$ such that $\underline{u}_i^{\Pi_i}(a, x_i) = u_i(\omega_1, x_i(\omega_1)) = \min_{\omega \in E^{\Pi_i}(a)} u_i(\omega, x_i(\omega))$.

Notice that

$$\underline{u}_i^{\Pi_i}(a, y_i) = \min_{\omega \in E^{\Pi_i}(a) \setminus \{a\}} \{u_i(a, e_i(a) + x_i(b) - e_i(b)); u_i(\omega, x_i(\omega))\}.$$

If, $u_i(a, e_i(a) + x_i(b) - e_i(b)) = u_i(a, y_i(a)) = \underline{u}_i^{\Pi_i}(a, y_i)$, then in particular $u_i(a, y_i(a)) \leq u_i(\omega_1, x_i(\omega_1)) = \underline{u}_i^{\Pi_i}(a, x_i)$. This contradicts (iv). On the other hand, if there exists $\omega_2 \in E^{\Pi_i}(a) \setminus \{a\}$ such that $u_i(\omega_2, x_i(\omega_2)) = \underline{u}_i^{\Pi_i}(a, y_i)$, then in particular $\underline{u}_i^{\Pi_i}(a, y_i) = u_i(\omega_2, x_i(\omega_2)) \leq u_i(\omega_1, x_i(\omega_1)) = \underline{u}_i^{\Pi_i}(a, x_i)$. This again contradicts (iv). □

6.2 Comparison between maximin CIC and CIC

Proposition 6.8 *If x is CIC, then it is also maximin CIC. The converse may not be true.*

Proof: Let x be a CIC and assume on the contrary that there exist a coalition S and two states a and b such that[14]

$$\begin{array}{lll} (i) & E^{\mathcal{F}_i}(a) = E^{\mathcal{F}_i}(b) & \text{for all } i \notin S, \\ (ii) & u_i(a, \cdot) = u_i(b, \cdot) & \text{for all } i \notin S, \\ (iii) & e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^{\ell}_{+} & \text{for all } i \in S, \text{ and} \\ (iv) & \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) & \text{for all } i \in S, \end{array}$$

where for all $i \in S$,

[14]Instead of $\mathcal{F}_i$, we can use any structure Π_i. This means that if in the Definition 6.1 we use Π_i instead of $\mathcal{F}_i$, then we would have maximin CIC with respect to Π_i. The proof is the same with the obvious adaptations.

$$y_i(\omega) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } \omega = a \\ x_i(\omega) & \text{otherwise.} \end{cases}$$

Notice that from (iv) and Lemma 6.7 it follows that for all $i \in S$,

$$u_i(a, e_i(a) + x_i(b) - e_i(b)) = u_i(a, y_i(a)) \geq \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) = u_i(a, x_i(a)).$$

Hence x is not CIC, which is a contradiction. For the converse, we construct the following counterexample. Consider the economy, described in Example 6.2, with two agents, three equally probable states of nature, denoted by a, b and c, and one good per state denoted by x. Assume that

$$\begin{array}{lll} u_1(\cdot, x_1) = \sqrt{x_1}; & e_1(a,b,c) = (20,20,0); & \mathcal{F}_1 = \{\{a,b\};\{c\}\}. \\ u_2(\cdot, x_2) = \sqrt{x_2}; & e_2(a,b,c) = (20,0,20); & \mathcal{F}_2 = \{\{a,c\};\{b\}\}. \end{array}$$

Consider the allocation

$$\begin{aligned} x_1(a,b,c) &= (20,10,10) \\ x_2(a,b,c) &= (20,10,10). \end{aligned}$$

We have already noticed that such an allocation is not Bayesian incentive compatible (see Example 6.2), but it is maximin CIC (see Remark 6.5). □

6.3 The Maximin REE is maximin incentive compatible

Proposition 6.9 *Any maximin REE is maximin coalitional incentive compatible.*

Proof: Let (p, x) be a maximin REE. Since agents take into account the information generated by the equilibrium price p, the private information of each individual i is given by $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. Thus, for each agent $i \in I$, $\Pi_i = \mathcal{G}_i$ and $\underline{u}_i^{\Pi_i} = \underline{u}_i^{\mathcal{G}_i}$. Assume on the contrary that (p, x) is not maximin CIC. This means that there exist a coalition S and two states $a, b \in \Omega$ such that

$$\begin{array}{lll} (i) & E^{\mathcal{G}_i}(a) = E^{\mathcal{G}_i}(b) & \text{for all } i \notin S, \\ (ii) & u_i(a,\cdot) = u_i(b,\cdot) & \text{for all } i \notin S, \\ (iii) & e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^{\ell}_{+} & \text{for all } i \in S, \text{ and} \\ (iv) & \underline{u}_i^{REE}(a, y_i) > \underline{u}_i^{REE}(a, x_i) & \text{for all } i \in S, \end{array}$$

where for all $i \in S$,

$$y_i(\omega) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } \omega = a \\ x_i(\omega) & \text{otherwise.} \end{cases}$$

Notice that since (p, x) is a maximin REE, it follows from (iv) that for all $i \in S$ there exists a state $\omega_i \in E^{\mathcal{G}_i}(a)$ such that

$$p(\omega_i) \cdot y_i(\omega_i) > p(\omega_i) \cdot e_i(\omega_i) \geq p(\omega_i) \cdot x_i(\omega_i).$$

By the definition of y_i, it follows that for all $i \in S$, $\omega_i = a$, that is $p(a) \cdot y_i(a) > p(a) \cdot e_i(a)$, and hence $p(a) \cdot [x_i(b) - e_i(b)] > 0$. Furthermore[15], condition (i) implies that $p(a) = p(b)$ and hence $p(b) \cdot x_i(b) > p(b) \cdot e_i(b)$. This contradicts the fact that (p, x) is a maximin REE and therefore $x_i(b) \in B_i(b, p(b))$. □

Corollary 6.10 *Any maximin REE is maximin individual incentive compatible.*

Remark 6.11 It should be noted that the maximin REE in Example 3.4 is coalitional incentive compatible. Indeed if state ω_1 occurs and agent 1 announces ω_2, then

$$u_1(\omega_1, e_1^1(\omega_1)+x_1(\omega_2)-e_1^1(\omega_2), e_1^2(\omega_1)+y_1(\omega_2)-e_1^2(\omega_2)) = log2+1 < 2 = u_1(\omega_1, x_1(\omega_1), y_1(\omega_1)).$$

On the other hand, if state ω_2 occurs and agent 1 announces ω_1, then

$$u_1(\omega_2, e_1^1(\omega_2)+x_1(\omega_1)-e_1^1(\omega_1), e_1^2(\omega_2)+y_1(\omega_1)-e_1^2(\omega_1)) = 2 < 2log2+1 = u_1(\omega_2, x_1(\omega_2), y_1(\omega_2)).$$

Therefore, the unique maximin REE in Example 3.4 is maximin CIC.

7 Related literature

To the best of our knowledge no universal existence and incentive compatible results have been obtained for the maximin rational expectations equilibrium. It is well known by now that the Bayesian REE as formulated by Radner (1979) and

[15]Notice that for all i, $\sigma(p) \subseteq \mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. Thus, for all i, $p(\cdot)$ is $\mathcal{G}_i$-measurable. Therefore, condition (i) implies that $p(a) = p(b)$.

Allen (1981) it only exists generically and it may not be incentive compatible or efficient.

Condie and Ganguli (2010a, 2010b) obtained results on the generic existence of fully and partially revealing REE. In particular, Condie and Ganguli consider a financial market economy with asymmetric information, where some investors behave as Bayesian expected utility maximizers and some others as MEU maximizers. In modeling the asymmetric information, they do not adapt the partition approach of Radner and Allen and also they consider one good. Their REE notion has interesting features, i.e., allows simultaneously maximin expected utility and Bayesian expected utility decision making. However, on the Bayesian side of the decision making their notion of REE is not consistent with that of Radner (1979) and Allen (1981) as their allocations need not to be measurable with respect to the private information of each individual as well as the information that the equilibrium prices generate. Nonetheless, by allowing for mixed behavior (Bayesian and maximin) they are able to obtain generic non revealing and fully revealing rational expectations equilibrium. Although, our results are similar in spirit with the ones in Condie and Ganguli (2010a, 2010b), they are not directly related to theirs for several reasons. We follow the Radner and Allen partition approach to model the asymmetric information and we focus on the universal (not generic) existence of a maximin REE. Obviously, our notion is different than theirs as we do not allow for any Bayesian behavior. Furthermore, we examine the incentive compatibility and efficiency of our new maximin rational expectations equilibrium notion. It should be noted that with one good per state the $\mathcal{F}_i$-measurability of allocations is necessary and sufficient for the incentive compatibility (see for example Krasa-Yannelis (1994) and Glycopantis-Muir-Yannelis (2005)). In view of this result, one can conclude that the Condie-Ganguli's REE notion need not be incentive compatible, as the allocations in the Bayesian decision making of their model need not be measurable with respect to the private information of each individual investor.

Correia da Silva and Hervès Beloso (2009) provide an existence theorem for a Walrasian equilibrium for an economy with asymmetric information, where agents' preferences are represented by maximin expected utility functions. Their MEU formulation is in the ex-ante sense. This seems to be the first application of the MEU to the general equilibrium existence problem with asymmetric information. However, they do not consider the issue of incentive compatibility or the REE notion. Since, they work with the ex-ante maximin expected utility formulation, their results have no bearing on ours.

The recent work by Epstein and Schneider (2010) studies the usefulness of ambiguity aversion models in financial markets. Learning models under ambiguity have been studied by Epstein-Schneider (2007) and Kim-Pesce-Yannelis (2010). Welfare proprieties of ambiguity aversion are studied by Dana (2004). The above works are in the spirit of maximin expected utility decision making, but they are not directly related to ours.

8 Concluding remarks and open questions

We introduced a new rational expectations equilibrium notion which abandons the Bayesian (subjective expected utility) formulation. Our new rational expectations equilibrium notion is formulated in terms of the maximin expected utility. In particular, in our framework agents maximize maximin expected utility instead of Bayesian expected utility. Furthermore, the resulting equilibrium allocations need not to be measurable with respect to the private information and the information the equilibrium prices have generated as in the case of the Bayesian REE. Our new notion exists universally (and not generically), it is Pareto efficient and incentive compatible. These results are false for the Bayesian REE (see Kreps (1977) and Glycopantis-Yannelis (2005)). It seems to us that the adoption of the maximin expected utility solves the basic problems that the Bayesian REE notion faces as an equilibrium notion, i.e., universal existence, efficiency and incentive compatibility.

If the intent of the REE is to capture the idea of "good" contracts under asymmetric information, then clearly the results of this paper suggests that the maximin REE has appealing properties, i.e., it exists universally under the standard continuity and concavity assumptions, it is efficient and incentive compatible. But why the Bayesian doesn't have the same properties? Why the Bayesian REE doesn't exist in the Kreps example and the maximin does? The MEU seems to provide superior outcomes for two main reasons.

First, as it is argued in de Castro-Yannelis (2008) the Bayesian expected utility (EU) preference representation is incomplete, but the MEU is complete. This is obvious in the Ellsberg experiment where the EU makes the choices of the individuals contradictory, but the MEU forces people to make the right choices. There is a fundamental incompleteness in the EU which doesn't allow agents to make the correct choices. This incompleteness is not part of the MEU decision making and as a consequence the MEU allows agents to reach Pareto superior outcomes.

Second, in the MEU decision making incentive compatibility is in a way inherent in the definition because we take into account the worse possible state that can occur. In that sense agents cannot be cheated because the decision to take into account the worse possible state, has already prevented any potential cheating.

In a general equilibrium model with asymmetric information, we think that the MEU choice does not reflect pessimistic behavior, but rather incentive compatible behavior. If an agent plays against the nature (e.g., Milnor game), since, nature is not strategic, it makes sense to view the MEU decision making as reflecting pessimistic behavior. However, when you negotiate the terms of a contract under asymmetric information and the other agents have an incentive to misreport the state of nature and benefit, then the MEU provides a mechanism to prevent others from cheating you. This in not pessimism, but incentive compatibility. It is exactly for this reason that the MEU solves the conflict between efficiency and incentive compatibility (see for example de Castro-Yannelis (2008)). This conflict seems to be inherent in the Bayesian analysis (see Example 6.2 in Section 6).

We hope that our new formulation of the REE will find useful applications in many areas and especially in macroeconomic general equilibrium models.

We conclude this paper with some open questions:

Throughout we have used the assumption that there is a finite number of states. It is an open question if the main existence theorem can be extended to infinitely many states of nature of the world and even to an infinite dimensional commodity space. This is also the case for the theorems on incentive compatibility and efficiency.

In Glycopantis-Muir-Yannelis (2005) it was shown that the REE is not implementable as a perfect Bayesian equilibrium of an extensive form game. We conjecture that a new definition of perfect maximin equilibrium can be introduced, which will be compatible with the implementation of the maximin REE. What reinforces this conjecture is the fact that incentive compatible equilibrium notions, i.e., private core (Yannelis (1991)) and private value allocations (Krasa-Yannelis (1994)) are implementable as a perfect Bayesian equilibrium. Since, the maximin REE is also maximin incentive compatible, we believe that such a conjecture should be true.

It is also of interest to know if the results of this paper could be extended to a continuum of agents.

Based on the Bayesian expected utility formulation, Sun, Wu and Yannelis (2010) show that with a continuum of agents, whose private signals are independent conditioned on the macro states of nature, a REE universally exists, it is incentive compatible and efficient. These results are been obtained by means of the law large numbers. It is of interest to know if the theorems of this paper can be extended in such a framework which makes the law of large numbers applicable.

Furthermore, it is of interest to know under what conditions the core-value-Walras equivalence theorems hold for the maximin expected utility framework.

A Appendix: Proof of the Auxilary Theorem

A.1 Mathematical preliminaries

A **correspondence** $\varphi : X \to 2^Y$ is a function from X to the family of all subsets of Y. A correspondence $\varphi : X \to 2^Y$ is compact-valued (nonempty-valued, convex-valued) if $\varphi(x)$ is a compact (nonempty, convex) subset of Y for every $x \in X$. Let X and Y be sets. The **graph** G_Φ of a correspondence $\Phi : X \to 2^Y$ is the set $Gr_\Phi = \{(x, y) \in X \times Y : \quad y \in \Phi(x)\}$.

If X and Y are topological spaces, $\phi : X \to 2^Y$ is said to be **lower semi continuous (l.s.c.)** if the set $\{x \in X : \phi(x) \cap V \neq \emptyset\}$ is open in X for every open subset V of Y. The correspondence $\phi : X \to 2^Y$ is said to be **upper semi continuous (u.s.c.)** if the set $\{x \in X : \phi(x) \subset V\}$ is open in X for every open subset V of Y. It is known that a closed-valued correspondence with compact Hausdorff range space has a closed graph if and only if it is upper hemi continuous.

If (X, α) and (Y, β) are measurable spaces, Φ is said to have a **measurable graph** if Gr_Φ belongs to the product σ-algebra $\alpha \otimes \beta$. We are often interested in the situation where (X, α) is a measurable space, Y is a topological space and $\beta = \beta(Y)$ is the Borel σ-algebra of Y. A correspondence $\phi : X \to 2^Y$ from a measurable space (X, α) into a topological space Y is said to be **lower measurable** if $\{x \in X : \phi(x) \cap V \neq \emptyset\} \in \alpha$ for every V open in Y. It is known that if the correspondence ϕ is lower measurable and closed valued, then it has a measurable graph. Furthermore, if the measure space (X, α, μ) is complete and the correspondence ϕ from (X, α, μ) to 2^Y has a measurable graph, then it is lower measurable (see for example Castaing-Valadier (1977)).

Let $(\Omega, \mathcal{F}, \mu)$ be a measurable space and Y be a separable metric space. From **Kuratowski and Ryll-Nardzewski Measurable Selection Theorem** it follows that if $\phi : \Omega \to 2^Y$ is a lower measurable, closed and nonempty-valued correspondence, then there exists a measurable function $f : \Omega \to Y$ such that $f(\omega) \in \phi(\omega)$ for all $\omega \in \Omega$.

A.2 Proof of the auxiliary theorem

For all $i \in I$, let $B_i : \Omega \times \triangle \to 2^{I\!R^\ell_+}$ be the *random budget set* of agent i, defined by

$$B_i(\omega, q) = \{x_i(\omega) \in X_i(\omega) : \;\; q \cdot x_i(\omega) \leq q \cdot e_i(\omega)\}.$$

Let $D_i : \Omega \times \triangle \to 2^{I\!R^\ell_+}$ be the *maximin random demand set* of agent i, defined by

$$D_i(\omega, q) = \{x_i(\omega) \in B_i(\omega, q) : \;\; u_i(\omega, x_i(\omega)) = \max_{y_i \in B_i(\omega,q)} u_i(\omega, y_i)\}.$$

Lemma A.1 For each i, $D_i(\cdot, \cdot)$ has a jointly measurable graph, that is

$$Gr_{D_i(\cdot,\cdot)} \in \mathcal{F} \otimes \mathcal{B}(\triangle) \otimes \mathcal{B}(I\!R^\ell_+).$$

Proof: Since for all $i \in I$ and $\omega \in \Omega$, $e_i(\omega)$ is an interior point of $X_i(\omega)$, it follows that $B_i(\omega, \cdot)$ is lower hemi continuous. Moreover, it is easy to verify that $B_i(\omega, \cdot)$ has closed graph, with a compact range space, therefore it is upper hemi continuous. Consequently, for all $i \in I$ and $\omega \in \Omega$, $B_i(\omega, \cdot)$ is continuous. From Berge's maximum theorem, it follows that for each $\omega \in \Omega$, $D_i(\omega, \cdot)$ is upper semi continuous with non empty and compact values. Consequently, $D_i(\omega, \cdot)$ has closed graph and since Ω is finite, it follows that it has a jointly measurable graph. That is, for all $i \in I$,

$$\begin{aligned} Gr_{D_i(\cdot,\cdot)} &= \left\{(\omega, q, x_i) \in \Omega \times \triangle \times I\!R^\ell_+ \; : \; \omega \in \Omega \text{ and } (q, x_i) \in Gr_{D_i(\omega,\cdot)}\right\} \\ &= \bigcup_{\omega \in \Omega} \left[\{\omega\} \times Gr_{D_i(\omega,\cdot)}\right]. \end{aligned}$$

Since $\{\omega\}$ and $Gr_{D_i(\omega,\cdot)}$ are closed, it follow that $Gr_{D_i(\cdot,\cdot)} \in \mathcal{F} \otimes \mathcal{B}(\triangle) \otimes \mathcal{B}(I\!R^\ell_+)$. $\square$

Define the *aggregate maximin random excess demand* for the economy $\mathcal{E}$ by

$$\mathcal{Z}(\omega, q) = \sum_{i=1}^n D_i(\omega, q) - \sum_{i=1}^n e_i(\omega).$$

Finally let $\mathcal{W} : \Omega \to 2^\triangle$ be the (free-disposal) equilibrium correspondence defined by[16]

$$\mathcal{W}(\omega) = \{q \in \triangle : \quad \mathcal{Z}(\omega, q) \cap I\!R^\ell_- \neq \emptyset\}.$$

[16] $I\!R^\ell_-$ denotes the negative cone of $I\!R^\ell$.

Since each $D_i(\cdot,\cdot)$ has jointly measurable graph and Ω is finite[17], from Castaing-Valadier (1977, p.80), it follows that $D_i(\cdot,\cdot)$ is jointly lower measurable, and so is $\mathcal{Z}(\cdot,\cdot)$. Therefore the set $B = \{(\omega, q) \in \Omega \times \triangle : \ \mathcal{Z}(q,\omega) \cap \mathbb{R}^\ell_- \neq \emptyset\}$ belongs to $\mathcal{F} \otimes \mathcal{B}(\triangle)$. Notice that the set B coincides with the graph of the correspondence $\mathcal{W}$, i.e.,

$$B = G_{\mathcal{W}} = \{(\omega, q) \in \Omega \times \triangle : \ q \in \mathcal{W}(\omega)\}.$$

Hence, $G_{\mathcal{W}}$ belongs to $\mathcal{F} \otimes \mathcal{B}(\triangle)$, i.e., $\mathcal{W}(\cdot)$ has a measurable graph, and since Ω is complete, it follows from Castaing-Valadier (1977, p.80) that $\mathcal{W}(\cdot)$ is also lower measurable.

Since for each fixed $\omega \in \Omega$, $D_i(\omega,\cdot)$ is upper semi continuous, convex, nonempty and compact valued, so is $\mathcal{Z}(\omega,\cdot)$. Furthermore, for any fixed $\omega \in \Omega$, $Z(\omega,\cdot)$ satisfies Walras law, i.e., for all $q \in \triangle$ and for every $z \in Z(\omega, q)$, $q \cdot z \leq 0$.

It follows from the Gale-Nikaido-Debreu lemma that, for each fixed $\omega \in \Omega$, $\mathcal{W}(\omega)$ is nonempty, i.e., $\mathcal{W}(\cdot)$ is a nonempty valued correspondence.

Since for each fixed $\omega \in \Omega$, $\mathcal{Z}(\omega,\cdot)$ is upper semi continuous, it follows that the set $\{q \in \triangle : \mathcal{Z}(\omega, q) \cap V \neq \emptyset\}$ is closed for all V closed subset of $\mathbb{R}^\ell$. Hence for all $\omega \in \Omega$ the set $\{q \in \triangle : \mathcal{Z}(\omega, q) \cap \mathbb{R}^\ell_- \neq \emptyset\} = \mathcal{W}(\omega)$ is closed.

Consequently, $\mathcal{W}(\cdot)$ satisfies the conditions of the Kuratowski and Ryll-Nardzewski Measurable Selection Theorem and hence it admits a measurable selection. This means that there exists a $\mathcal{F}$-measurable function $p^* : \Omega \to \triangle$ such that $p^*(\omega) \in \mathcal{W}(\omega)$ for all $\omega \in \Omega$, i.e.,

$$\mathcal{Z}(\omega, p^*(\omega)) \cap \mathbb{R}^\ell_- \neq \emptyset \qquad \text{for all } \omega \in \Omega.$$

Notice that this means that:

(1′) for all i and ω, $x_i^*(\omega)$ maximizes $u_i(\omega,\cdot)$ subject to $p^*(\omega) \cdot x_i^*(\omega) \leq p^*(\omega) \cdot e_i(\omega)$

(2′) $\displaystyle\sum_{i \in I} x_i^*(\omega) \leq \sum_{i \in I} e_i(\omega)$ for all $\omega \in \Omega$.

[17] Since Ω is finite, it follows that $(\Omega, \mathcal{F}, \pi_i)$ is complete for each $i \in I$.

Consider for all $i \in I$, $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p^*)$. To complete the proof we must show that (p^*, x^*) is a free disposal maximin REE. Clearly, condition $(2')$ above holds and for each $i \in I$ and $\omega \in \Omega$, $x_i^*(\omega) \in B_i(\omega, p^*(\omega))$. Therefore, all it remains to be shown is that condition (ii) in Definition 3.3 is satisfied. Assume to the contrary that condition (ii) in Definition 3.3 does not hold, then there exist an agent $i \in I$, a state $\bar{\omega} \in \Omega$ and an allocation $y \in L_{X_i}$ such that $y \in B_i^*(\bar{\omega}, p^*)$ and

$$\underline{u}_i^{REE}(\bar{\omega}, y) > \underline{u}_i^{REE}(\bar{\omega}, x_i^*).$$

Since Ω is finite, there exists a state $\omega \in E^{\mathcal{G}_i}(\bar{\omega})$ such that $u_i(\omega, x_i^*(\omega)) = \underline{u}_i^{REE}(\bar{\omega}, x_i^*)$. This implies that

$$u_i(\omega, y(\omega)) \geq \underline{u}_i^{REE}(\bar{\omega}, y) > \underline{u}_i^{REE}(\bar{\omega}, x_i^*) = u_i(\omega, x_i^*(\omega)),$$

which contradicts condition $(1')$. Hence, (p^*, x^*) is a free disposal maximin REE. □

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