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Joscha Beckmann[1]

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Abstract

Although the literature on purchasing power parity (PPP) is rich in controversy, the relative contribution of prices and nominal exchange rates to real exchange rate movements which restore PPP disequilibria has rarely been put under any close scrutiny. Using monthly data from 1973:01 to 2009:12 from the USA, UK, Germany, France and Japan, this paper as a first step applies a cointegrated VAR framework to test for stationary real exchange rates and linear adjustments in prices and nominal exchange rates. As a second step, ESTR error correction models are fitted to test whether nonlinear error correctional behaviour characterizes the data. The results clearly indicate that the nominal exchange rate is responsible for the nonlinear mean reverting behaviour in real exchange rates and also mainly drives overall adjustment. Applying dynamic stochastic simulations based on the estimated models, this study also confirms recent results that the half-life times of real exchange rate shocks are significantly smaller than the consensus benchmark of three to five years.

JEL Classification: E44, F31, G15

Keywords: Purchasing power parity; cointegration; nonlinear vector error correction

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1 University of Duisburg-Essen. *– This paper was written partly while the author was a visiting scholar at Cass Business School, City University London. The author is grateful for valuable comments and helpful suggestions from Lucio Sarno.**– All correspondence to Joscha Beckmann, Department of Economics, Chair for Macroeconomics, Universitätsstr. 12, 45117 Essen, Germany, E-Mail: joscha.beckmann@uni-due.de.*

1. Introduction

While purchasing power parity has been the subject of numerous studies, it nevertheless remains a controversial research area in economics.[1] Although the overall evidence suggests that purchasing power parity is relevant to some extent in the long run, many researchers have rejected it because they find evidence of a unit root in the real exchange rate. In particular, the high degree of persistence in real exchange rates and the dynamics of real exchange rate adjustments mean that some puzzles remain to be solved. For a long time, the mixed empirical evidence on PPP was attributed mainly to the argument that, owing to the very low adjustment to PPP, the sample for the recent floating period was too short to detect a statistically significant mean reversion (Juselius and MacDonald, 2004; Froot and Rogoff, 1995). From a theoretical point of view, slow adjustment to PPP can be explained, for example, by inter-temporal smoothing or cross-country wealth distribution (Cheung and Lai, 1998; Rogoff, 1996). However, the recent findings by some authors (Taylor et al., 2001; Kilian and Taylor 2003) that major real exchange rates can well be characterized by nonlinear mean reverting processes offer another explanation for the failure to reject the unit root hypothesis for real exchange rates based on linear models: adjustment increases with the degree of deviation from PPP, so standard univariate unit root tests have very low statistical power in rejecting a false null hypothesis (Taylor et al., 2001).[2]

In fact, most studies that have reported a very low degree of adjustment base their analysis on a linear framework (Sarno et al., 2004). However, although the empirical evidence suggests that real exchange rates show nonlinear mean reverting behaviour, the question of whether exchange rates or prices are mainly responsible for such adjustment during the present floating period has not yet been closely examined in this context. Even in a linear framework, only a small number of studies deal with the question of whether nominal exchange rates or prices are mainly responsible for real exchange rate adjustment to PPP. This is surprising, considering that sluggish price adjustment is also highlighted in the literature as an explanation for PPP deviations (Rogoff, 1996; Cheung et al., 2004). In addition, traditional international macroeconomic models of the nominal exchange rate, such as the monetary approach, as well as “new” open economy models, rely on the assumption that purchasing power parity is at least valid in the long run (Taylor et al., 2001).

[1] For an overview of the literature on purchasing power parity see Sarno and Taylor (2003) and Sarno (2005).
[2] In a recent study, Frömmel et al. (2010) showed that unit root tests against nonlinear alternatives exhibited a limited ability to identify the precise form of nonlinearity.

This study is the first to fit a nonlinear error correction model which links exchange rates and prices to deviations from PPP for the post-Bretton Woods period on a monthly basis. The main aim is to shed some light on the speed and pattern of adjustment by dissecting the role of nominal exchange rates and prices. To tackle this issue, the remainder of the paper is organized as follows: Section 2 summarizes the empirical literature on price and nominal exchange rate adjustment as well as that on nonlinear mean reverting behaviour in real exchange rates, and points out the contribution this study makes to the literature. Section 3 describes the empirical framework used, and presents the results. As the initial stage of the analysis, a cointegration analysis is applied to test for PPP and mean reverting behaviour in a linear framework. The ESTR models applied thereafter also allow for nonlinear error correction behaviour to PPP deviations both for nominal exchange rates and for prices. Finally, the half-life times of shocks to real exchange rates is calculated based on the previous results by applying dynamic stochastic simulations. Section 4 concludes.

2. Literature review

Although the empirical record on PPP is enormous, the contribution of nominal exchange rates and prices to the adjustment process has been considered in only a minor number of studies. In an early paper, Wei and Parsley (1995) study the deviation from purchasing power parity in 12 tradable sectors of 91 OECD countries pairs, based on annual data for the recent float period. They find evidence that nominal exchange rate stability in terms of low volatility produces a faster rate of convergence towards purchasing power parity. Another interesting result pointed out by the authors is that countries with large deviations from PPP show a faster convergence, which indicates nonlinearity in the rate of mean reversion.[3] Analyzing multilateral real exchange rates from 93 countries, Goldfajn and Valdes (1996) find evidence that prices as well as exchange rates adjust to disequilibria in terms of probability, with the latter case occurring more frequently. They also point out that fixed exchange rate regimes are more likely to suffer from overvaluations of the real exchange rate. However, the study only refers to appreciations in the real exchange rates in terms of positive deviations from PPP.[4]

Engel and Morley (2001) apply a state pace model to exchange rates and prices and allow for a transitory dynamic in equilibrium prices and exchange rates as well as in disequilibrium components. The results suggest the speed of adjustment is higher for prices than for nominal

[3] The authors formally test for nonlinearity by adding a term of the initial deviation squared to the regression.
[4] Disequilibria are defined as short and medium deviations from positive PPP. The authors neglect periods of undervaluation, as one main focus of their study relates to exchange rate crises and their policy implications.

exchange rates, although the latter adjusts with a larger size.[5] The authors point out that their results do not contradict the view that nominal exchange rates adjust more. Cheung et al. (2004) analyze the role of nominal exchange rate and price adjustment using impulse-response analysis for five major exchange rates. Their results show that nominal exchange rate adjustment is mainly responsible for the speed of adjustment towards PPP. However, they also point out that nominal exchange rates converge more slowly than prices and point to possible nonlinearity in PPP convergence as a caveat of their study and an important area for further research.

Nonlinear real exchange rate dynamics can be formally derived in the context of international arbitrage costs (Taylor et al., 2001). For the law of one price, bands of transactions and iceberg costs are related to TAR models, where the transition from one regime to another is discrete once the threshold is reached. See Sarno et al. (2004) for a survey of empirical studies related to this issue. However, recognizing that PPP applies to a basket of goods and the transaction or iceberg costs of different goods are not necessarily the same, it is relatively straightforward to apply smooth transition regressive models, which generate switching regression in a way that the transition from one regime to another is not discrete, but smooth to PPP (Taylor et al., 2001; Teräsvirta, 1998).

Smooth transition autoregressive models have been successfully applied to exchange rates by various researchers. Taylor and Peel (2000) estimate ESTAR models for deviations of the sterling-dollar and mark-dollar nominal exchange rates from monetary fundamentals proxied by money velocity. They find that the adjustment increases with large deviations. In the context of real exchange rates Taylor et al. (2001) fit ESTAR models to four major real exchange rates against the dollar. Their results suggest that the exchange rates under observation are well characterized by nonlinear mean reverting processes. Running a Monte Carlo simulation, the authors also show that standard univariate unit root tests have low power compared to multivariate unit root tests.[6] Kilian and Taylor (2003) apply the same model to quarterly data for Canada, France, Germany, Italy, Switzerland, Japan and the UK and report similar results. Wu and Hu (2009) provide further evidence of nonlinear real exchange rate dynamics by

[5] To be more precise, the speed of adjustment is measured as the extent to which the gap to equilibrium in the current period $(q_{t+1} - \overline{q_{t+1}})$ adjusts to the gap in the previous period $(q_t - \overline{q_{t)}})$. The authors point out that a VECM estimation does not measure the speed of adjustment, as it refers to a constant equilibrium $(\overline{q})$ when analyzing how much of the gap in the current period $(q_{t+1} - \overline{q})$ has been carried through from the previous period $(q_t - \overline{q})$. They conclude that the exchange rate has to adjust more than prices, owing to a larger gap.

[6] Taylor et al. (2001) point out that rejecting the unit root in a linear model might even indicate that the real exchange rate is close to its equilibrium in which the real exchange rate will be closer to a unit root, according to the results of their ESTAR model.

extending the analysis of Killian and Taylor (2003) to include the Harrod/Balassa/Samuelson effect by allowing for deviations from purchasing power parity to return to an equilibrium trend.

The work closest to this contribution is the analysis carried out by Sarno and Valente (2006), who apply a nonlinear Markov-switching error correction model to real exchange rates in the G5 countries using a dataset of more than one hundred annual observations. Their results show that long-run purchasing power parity is valid, and also indicate that relative prices are responsible for an adjustment during regimes of fixed exchange rates, while nominal exchange rates mainly restore disequilibria when exchange rates are allowed to flow freely. As a consequence, the persistence of shocks to the real exchange rate during fixed regimes is much higher, as nominal exchange rates cannot adjust. However, Sarno and Valente (2006) use annual data in their analysis. Furthermore, the transition parameters of the exponential function are exogenous in their approach and the study does not focus exclusively on the present floating area.

Altogether, a nonlinear analysis based on monthly data for the post-Bretton Woods period is important if we want to shed some light on the relative contribution of nominal exchange rates and prices to the adjustment dynamics which restore disequilibria from purchasing power parity. This allows some interesting conclusions with respect to the poor empirical record of PPP established in the literature. As already pointed out, it has been recognized that such results should not be attributed only to low adjustment, but also to nonlinear mean reverting behaviour. The framework considered in this study allows for nonlinear adjustment and does not impose any restrictions on the dynamics of nominal exchange rates and prices. Hence, it should be able to deliver further insight into the adjustment process. An interesting question, for example, is whether the half-life time of shocks based on this model is compatible with previous findings. Another issue is whether stickiness in nominal prices can be considered as an explanation for deviations from PPP.

In a more general context, such an analysis is important not only in relation to the validity of purchasing power parity but also for the general modelling of nominal exchange rates. It is now widely recognized that the poor empirical record of monetary exchange rate models and PPP, which in terms of forecasting was first highlighted by Meese and Rogoff (1983), can be put down to the fact that a linear modelling of the nominal exchange rate is inadequate. Stock and Watson (1988) have shown that univariate and bivariate macroeconomic time series are subject to substantial instabilities. This is compatible with the concept of regime-sensitive cointegration introduced by Siklos and Granger (1997). One way to proceed is therefore to apply regime-switching models or to test for structural breaks, so as to obtain time-varying estimates for the coefficients in the long-run exchange rate equation.[7] Another possibility, however, would be to rely on a linear model for the coefficients in the exchange rate equation and allow for nonlinear mean reverting adjustment behaviour. This correlates with the case where instabilities occur in the error adjustment but not in the determination coefficient itself. The latter is strongly related to the smooth transition models already described, which allow a distinction between states with large and small deviations from fundamental values.

3. Empirical analysis

3.1 Data and methodological issues

The sample used in this paper contains monthly data for nominal exchange rates and consumer price indexes running from January 1973 to December 2009 for the USA, UK, Germany, France and Japan. All series are expressed in logarithms and are taken from the International Financial Statistics of the International Monetary Fund. Prices have been seasonally adjusted.

Sarno and Valente (2006) point out that the univariate unit root tests that are often applied in the literature suffer from a loss of power because they imply common factor restrictions and rely on error rather than structural dynamics. This implies that they do not allow for a different adjustment of prices and exchange rates to deviations from purchasing power parity. The same argument is true with respect to panel unit root tests. Considering the aim of the present study, a bivariate system with price differentials and exchange rates is used for the remaining analysis and multivariate unit root tests are considered. The real exchange rate, X_t, is then defined, in logarithmic form, as

$$X_t = s_t - (p_t - p_t^*) \tag{1}$$

with s_t as the logarithm of the nominal exchange rate expressed as the domestic price of the foreign currency and p_t and p_t^* as logarithms of domestic and foreign price levels. If purchasing power parity holds, nominal exchange rates and prices are cointegrated with the cointegration vector $\beta^{'} = (1, -1)$ and X_t is a realization of a stationary process. A weak version of PPP requires only that a linear combination of exchange rates and prices is found to be stationary (Cheung and Lai, 1998).

[7] Models of this kind have been applied, for example, by Schinasi and Swamy (1989), Wolff (1987), Sarno et al. (2004), De Grauwe and Vansteenkiste (2007), Frömmel et al. (2005a,b), Goldberg and Frydman (1996, 2001) and Beckmann et al. (2011).

Sarno and Valente (2006) also base their analysis on a bivariate system. Engel and Morley (2001) provide evidence that domestic and foreign prices share similar convergence speeds, while Cheung et al. (2004) report similar adjustment dynamics for both. Furthermore, Juselius and MacDonald (2004) point out that using price differentials might remove the non-stationarity in the real exchange rate that is often observed in the literature.[8] For the present analysis, previous unit root tests suggest that nominal exchange rates and the price differential are integrated of order one in all cases, which implies that a cointegration analysis is adequate.[9]

3.2 Cointegration analyses

We start our analysis by applying the multivariate cointegration test of Johansen (1988), which draws upon the following vector autoregression representation (VAR):

$$\Delta Y_t = \Pi Y_{t-1} + \Gamma(\mathrm{L})\Delta Y_{t-l} + \Phi D_t + \epsilon_t, \quad t = 1, \dots, T. \tag{2}$$

The non-stationary behaviour is accounted for by a reduced rank $(r < p)$ restriction of the long-run level matrices Π, which can be fragmented into two $r \times p$ matrices α and β' ($\Pi = \alpha\beta'$). β' gives the coefficients of the variables for the r long-run relation, while α contains the adjustment coefficients describing the reaction of each variable to disequilibria from the r long-run relations given by the $r \times 1$ vector $\beta' Y_{t-1}$. The deterministic components are given by the $(p \times 1)$ vector ΦD_t, while ϵ_t describes an independent and identically distributed error term. The term $\Gamma(\mathrm{L})\Delta Y_{t-l}$ describes the short-run dynamics of the model using p equations between current variables, L lagged variables and equilibrium errors (Juselius, 2006). However, instead of modelling this term explicitly, this paper focuses on the nonlinear error correction mechanism to be described in detail in Section 3.3.

For the specification of all models, the choice of the lag length is based on tests for autocorrelation and ARCH-effects. According to Rahbek et al. (2002), the results we gain in the following are still robust under the ARCH-effects that remain in some cases. To identify the number of cointegrating relations r we rely on the trace test developed by Johansen (1988).[10]

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[8] This might be the case if both price levels are integrated of order two I(2) and cointegrated so that their linear combination is integrated of order one I(1). Juselius and Mac Donald (2004) treat the real exchange rate as an I(1) variable in their analysis. For an analysis of PPP in an I(2) framework see Frydman et al. (2010).

[9] The results are available upon request.

[10] The idea of the test is to separate the eigenvalues $\lambda_i, \mathrm{i} = 1 \dots \mathrm{r}$, which correspond to stationary relations, from those eigenvalues λ_i, $\mathrm{i} = \mathrm{r} + 1 \dots, \mathrm{p}$ which belong to non-stationary eigenvectors. The test statistic of the corresponding likelihood test, the so-called trace test, is given by $trace(r) = -T\sum_{i=r+1}^{p} \log(1 - \hat{\lambda}_i)$.

The results are presented in Table 1.

-Table 1 about here-

The hypothesis of a zero rank is clearly rejected in each case. On the other hand, the hypothesis of one cointegrating vector cannot be rejected at the one percent level. As a first result, we therefore conclude that exchange rates and prices are cointegrated. Table 2 shows that the hypothesis of no autocorrelation is not rejected in nearly each case.[11]

-Table 2 about here-

After determining the rank, the Johansen approach provides the maximum likelihood estimates of the unrestricted cointegrating relations $\beta' Y_{t-1}$, which are identified by the normalization on one variable.[12] This allows us to put the nature of the long-run relationship under closer scrutiny. Normalization has been carried out on the nominal exchange rates in order to achieve comparable results. Table 3 shows that nominal exchange rates and prices enter with a reversed sign for all currencies. Furthermore, both are considered significant according to their t-statistics, which can be shown for the nominal exchange rate by normalizing on prices.[13]

-Table 3 about here-

As a first result, it should be noted that a weak version of PPP is valid for each currency pair. However, nominal exchange rates do not contribute to mean reverting behaviour, in the sense that the adjustment coefficients are not significant. On the contrary, prices do adjust but the magnitude of adjustment is very small for all exchange rates, implying implausibly low rates of mean reversion.

A clear interpretation of weak PPP in general is rather difficult, so the next step is to test whether the strong version of PPP holds by imposing the restriction $\beta' = (1, -1)$. This restriction is rejected for each case except France, which implies that a strong version of PPP cannot be observed for the other countries. Sarno and Valente (2006) point out that this rejection might be due to structural breaks or instabilities. The same argument can be applied with respect to adjustment coefficients in the case of weak PPP as explained above. In the

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Under the null hypothesis of $(p-r)$ unit roots, λ_i, i = (r + 1 ...., p) should behave like random walks and the test statistic should be small. Starting with the hypothesis of full rank, the number of cointegrating relations is determined using a top-bottom procedure until the null cannot be rejected (Juselius, 2006).

[11] The results of the LM tests on ARCH effects are available upon request.

[12] In cases of a rank larger than one it is necessary to impose only identifying restrictions on β in order to achieve interpretable economic relationships for the long-run structure, otherwise the cointegration vector is unique.

[13] The results are available upon request.

present case, a closer inspection of the recursively calculated trace test, whose components should grow linearly over time in the case of constancy, confirms this view.[14] Hence, the application of more sophisticated methods seems necessary.[15]

To draw a preliminary conclusion, the strong form of purchasing power parity cannot be verified in a multivariate linear framework except for France. If the restriction $\beta' = (1 - 1)$ is not applied, very small adjustment only stems from prices. These results are in line with the findings of earlier cointegration studies which do not find significant mean reversion of real exchange rates for the post-Bretton Woods period (Mark, 1990; Sarno and Valente, 2006). Furthermore, Juselius and MacDonald (2000; 2004), also apply a similar cointegrated Var approach when analyzing international parity relations, and find that prices rather than exchange rates adjust towards long-run steady states.[16] For our next task, we put under closer scrutiny the question of whether the role of prices and nominal exchange rates for adjustment is different when a nonlinear framework is applied.

3.3 Framework for nonlinear error correction

To put the nonlinear mean reverting behaviour that has been observed by many researchers under consideration an ESTR error correction model is now applied in this section.[17] This allows a comparison of the adjustment process in periods of high and low deviation from purchasing power parity for both nominal exchange rates and prices. But in order to achieve clear interpretable and comparable results we impose the restriction $\beta' = (1, -1)$. Hence, we test for nonlinear error correction with respect to deviations from a strong version of purchasing power parity by considering the following nonlinear error correction model:

$$\Delta Y_t = c_1 + \alpha_1 \, (X_{t-1} - \mu) + (\tilde{c}_1 + \tilde{\alpha}_1 (X_{t-1} - \mu)) G(\gamma, X_{t-j} - \mu) \tag{3}$$

The term ΔY_t denominates a 2 x 1-vector which contains changes in the nominal exchange rate and the price differential, while X_t refers to the real exchange rate. The terms c_1 and α_1 correspond to the (linear) lower regime, while $(c_1 + \tilde{c}_1)$ and $(a_1 + \tilde{\alpha}_1)$ belong to the

[14] The results of the recursively calculated trace test for the different currencies are available upon request.
[15] As outlined in Section 2, one possibility would be the application of regime-switching models for the coefficients in the cointegration vector. However, considering the aim of this study, the ESTR models applied in the next section seem more appropriate.
[16] Both studies focus on a different research question as they also include interest rates and treat the real exchange rate as integrated of order one, as mentioned in 3.1.
[17] ESTR models of nominal exchange rates have, for example, been applied in the context of covered and uncovered interest rate parity and market efficiency by Sarno et al. (2006) and Baillie and Kilic (2006).

(nonlinear) upper regime. G is a bounded continuous exponential transition function which lies between 0 and 1 and has the following form:

$$G\big(\gamma, X_{t-j} - \mu\big) = 1 - \exp\left(-\gamma (X_{t-j} - \mu)^2\right) \tag{4}$$

This specification implies that the lower (upper) regime is associated with small (large) values of X relative to its mean μ, which is determined via the transition function (van Dijk et al., 2002). The exponential transition function is symmetrically inverse-bell-shaped, so that an adjustment for deviations above and below equilibrium is symmetric. The parameter γ determines the smoothness of the transition, with lower absolute values implying slower transition, while the delay parameter j determines the number of periods needed to generate an adjustment of the nominal exchange or prices after deviations from PPP equilibrium (Taylor et al, 2001). In order to create a scale-free smoothness parameter, γ is normalized by the standard deviation of the transitional variable X_{t-j}.

It is first necessary to formally test for nonlinearity, though it is also important to choose an adequate transition variable, which in the present study means the choice of a lag order for the real exchange rate. Both issues can be tackled by applying a test introduced by Luukonen et al. (1988). They suggest an LM test for nonlinearity based on the following third order Taylor approximation of the transition function (Teräsvirta, 1998; Franses and van Dijk, 2000)[18]:

$$\Delta Y_t = \varphi_0 + \varphi_1 (X_{t-1} - \mu) + \varphi_2 (X_{t-1} - \mu)\big(X_{t-j} - \mu\big) + \varphi_3 (X_{t-1} - \mu)\big(X_{t-j} - \mu\big)^2 + \varphi_4 (X_{t-1} - \mu)\big(X_{t-j} - \mu\big)^3 + \epsilon_t \tag{5}$$

The null hypothesis which refers to the linear model being adequate is tested as $Ho{:}\ \varphi_i = 0$ with $i = 2,3,4$ against the alternative $H1$ that at least one $\varphi_i \neq 0$, implying that the higher order terms are significant (Teräsvirta, 1998). The test statistic has a X^2 distribution with three degrees of freedom. If the hypothesis of linearity is rejected, one way to choose the transition variable is to compute the test statistic for several transition functions, i.e. different values of j, and select the configuration for which its value is maximized (van Dijk, Teräsvirta and Franses, 2002; Taylor et al., 2001). Teräsvirta (1998) has shown that this approach works well in most cases.

[18] In the case of small samples, F versions of the LM test statistics are preferable as they have better size properties. The test can also be applied in order to distinguish between a logistic and an exponential transition function (van Dijk et al., 2002; Teräsvirta, 1998). However, a logistic transition function does not seem to be attractive in the context of real exchange rates, as such a formulation would imply different behavior for positive and negative deviations from PPP (Taylor, et al., 2001).

It is important to note that the choice of j has significant implications for interpretation. Taylor et al. (2001) and Sarno and Valente (2006) point out that a small value of j is preferable, as a large value implies that it takes a long time for the real exchange rate to start showing mean reverting behaviour after a shock, i.e. before prices and nominal exchange rates adjust after disequilibria. On the other hand, Sarno et al. (2004) analyze the law of one price on a broad basis and show that the delay parameter for their self-exciting TAR model varies across sectors and countries, as the reactions of market participants to arbitrage opportunities vary.[19] For the present study, delays from one to twelve months are considered.[20] This allows us to take into account the possibility that the degree of mean reversion varies between currencies.

The results of the LM tests presented in Table 4 show that the hypothesis of linearity is clearly rejected at the one percent level for all currencies and for nearly every choice of the lag order j.

-Table 4 about here-

Hence, the conclusion is that a nonlinear framework is adequate. The results also suggest that a delay parameter of two is an adequate choice for all configurations, with the test statistic being maximized for each currency pair. This implies that the first regime corresponds to the case from two months previously where the real exchange rate rarely deviated from long-run equilibrium in terms of PPP, while the second regime corresponds to a situation two months previously where deviation from the equilibrium was large.

3.4 Estimation results

As a next step, the nonlinear error correction model described in equation (3) is estimated by nonlinear least squares. The analysis has been carried out in a 2-dimensional nonlinear system estimation for each currency, with the real exchange rate restricted to having the same equilibrium for both prices and the nominal exchange rate.[21] In order to test for autocorrelation we apply an LM test developed by Eitrheim and Teräsvirta (1996), which is a generalization of the test for serial correlation in an AR-model suggested by Breusch and Pagan (1979). The idea of this test is to perform a regression analysis on the residuals from the original model on its p-lagged values and the partial derivatives with respect to the parameters of the model. The test

[19] Many of the estimated delay parameters provided by the authors imply that markets react with a delay of one to one and a half years. A delay parameter of one is reported in only 6 out of 43 cases.

[20] Longer delays have turned out to be less suitable in previous estimations carried out by the author. The results are available upon request.

[21] Besides the economic intuition this is necessary, as a different equilibrium level would correspond to a different deterministic in the long-run relations, implying that the error correction would refer to different cointegrating relations.

statistic is then obtained as nR^2 from this regression and is X^2 distributed with p degrees of freedom asymptotically (Franses and van Dijk, 2001).

Previous tests for the basic model suggested positive autocorrelation of a low order. As this might result in a downward bias of the standard error estimation, changes in the nominal exchange rate or the price differential in the previous period have been included in both the linear and the nonlinear parts of the corresponding equation. Based on the following estimation, further step-by-step restrictions have been applied to achieve a reduced form of the equation. For the exchange rate, the adjustment coefficient for small deviations α_1 turned out not to be significant in each case, according to t-statistics and further Wald tests. The change in the nominal exchange rate in the previous period for each currency pair turned out to be important only for the linear part of the equation, according to the same procedure. For the price equation, the only hypothesis that could not be rejected in each case was that the nonlinear adjustment coefficient $\tilde{\alpha_1}$ is not significant.

Taking these findings into account, the exchange rate equation boils down to

$$\Delta s_t = c_1 + \vartheta \Delta s_{t-1} + (\tilde{c}_1 + \tilde{\alpha}_1(X_{t-1} - \mu))G(\gamma, X_{t-2} - \mu) \tag{6}$$

while the price equation can be written as

$$\Delta(p_t - p_t^*) =$$
$$c_1 + \alpha_1 \ (X_{t-1} - \mu) + \vartheta \Delta(p_{t-1} - p_{t-1}^*) + \left(\tilde{c}_1 + \tilde{\vartheta}\Delta(p_{t-1} - p_{t-1}^*)\right)G(\gamma, X_{t-2} - \mu) \tag{7}$$

Further restrictions on coefficients, which turned out to be insignificant in some cases but not in all, did not introduce any important changes. The final results, based on these findings, are given in Table 5.

-Table 5 about here-

It may be worthwhile mentioning first of all that the observed insignificance of the transition parameter γ in some cases cannot be interpreted as evidence against a smooth transition model, as the t-statistics should be interpreted with caution (van Dijk et al., 2002; Taylor et al., 2001). The first main result is that the nominal exchange rate always shows mean reverting behaviour when the absolute deviations from PPP were already large two months previously. As mentioned above, no exchange rate adjustment can be observed in the case of small deviations because α_1 turns out to be not significant. The finding that the change in the nominal exchange rate in the previous period is significant is not surprising, considering that nominal exchange rates can well be characterized as a random walk process.

On the other hand, price adjustment can be observed in the cases of France and Germany for the linear part of the equation, while additional adjustment in the nonlinear part always does not occur with $\alpha_1^{\sim}$ being insignificant. Hence, the price differential does not contribute to the nonlinear pattern of real exchange rate adjustment.

The results are also compatible with the findings of the cointegration analysis in section 3.2. Error correction behaviour of the nominal exchange rate cannot be observed in a linear framework, while the role of prices is not clear-cut. However, when comparing the results for the linear part of the equation with those for the adjustment coefficients reported in the previous section one should keep in mind that these two cannot be directly compared, as the strong version of PPP is under observation in this section. What we can say however, is that the adjustment coefficients for linear price adjustments for both weak and strong PPP in both sections are of very small size relative to the adjustment coefficient of the nominal exchange rate in the nonlinear part of equation (6). Overall, therefore, the model confirms that the adjustment increases with the size of the deviations from PPP. If deviations are small, only prices drive a very low adjustment in some cases. If real exchange rates further depart from PPP, the nominal exchange rate is solely responsible for the increasing adjustment. Hence, nominal exchange rates are also the main driver of the overall adjustment.

3.5 Half-life times of real exchange rate shocks

In the last stage of the analysis, half-life times of real exchange rate shocks are calculated, based on the estimated models from the previous section. This delivers further insights into the mean reverting properties and allows a comparison with previous studies. Prior to estimations based on nonlinear models, the consensus results of most studies suggested that it takes 3 to 5 years for real exchange rate shocks to have diminished by half (Rogoff, 1996; see Cheung et al., 2004, for an overview). Estimations of half-life times based on smooth transition models that have been provided, for example, by Taylor et al. (2001), Sarno and Valente (2006), Lothian and Taylor (2008) and Norman (2010) suggest that real exchange rates mean revert much faster, particularly if shocks are large.

To estimate the half-lives of shocks within our nonlinear framework, the approach of Gallant et al. (1993) for calculating generalized impulse response functions (GIRF) is applied in a similar manner as in Taylor et al. (2001). The calculation of nonlinear impulse response functions presents several difficulties because the shapes of the impulse response functions depend on the initial condition and the size of the shock. If the initial condition is chosen far from the mean, the impulse response function will tend towards zero more quickly. The degree of mean reversion also increases with the size of the shock (Norman, 2010; Taylor et al, 2001).

Accounting for all dynamics, and in a similar fashion to Sarno and Valente (2006), the focus is on the impulse response of the real exchange rate X_t to shocks based on the model presented in equations (6) and (7) before restrictions have been applied.[22] Speaking generally, the GIRF is calculated as the average difference between two different realizations of the stochastic process X_{t+l} which start at identical historical values. The first realization is hit by a shock of varying size at time t while the second displays a base run without additional shocks (Paya and Peel, 2006; Norman, 2010).

$$GIRF_l(l, u_t) = E(X_{t+l}|u_t = \phi, X_{t+l}, \omega_{t-2}) - E(X_{t+l}|u_t = 0, X_{t+l}, \omega_{t-2}) \qquad (8)$$

where $l = 1,2 \ldots$, denotes the time horizon, $u_t = \phi$ is a shock occurring at time t and ω_{t-2} corresponds to the history of the real exchange rate X_t. The half-life time of a shock then corresponds to the number of months until the shock has diminished by half. Since analytic expressions for the conditional expectations involved in Eq. (8) are not available for $l > 1$, stochastic Monte Carlo simulations are necessary to obtain a GIRF (Gallant et al., 1993; Taylor et al., 2001; Paya and Peel, 2006).[23]

Analogous to Taylor et al. (2001), half-life times are calculated conditional on the initial exchange rate equilibrium as well as on average historical value. In the first case, the transition variable X_{t-2} is set to the estimated equilibrium μ, as considered in Sections 3.3 and 3.4, and 5000 simulations with and without shock $u_t = \phi$ at time t for $l = 200$ are carried out for each of the four models. To obtain estimates based on the average historical value, 200 simulations for $l = 200$ with different starting dates are implemented. For each date, X_{t-2} is set equal to X_t if the real exchange rate is above the estimated equilibrium μ, while X_{t-2} is set to the absolute distance from the equilibrium level if X_t is below μ.[24]

[22] Preliminary simulations for the model including restrictions did not produce significantly different results.
[23] Owing to the large numbers of simulations and the Law of Large Numbers, this procedure should give results identical to calculations by multiple integration (Gallant et al., 1993; Taylor et al., 2001).
[24] This distinction is necessary considering the symmetric adjustment in ESTR models and the fact that only positive shocks are considered. For further details see Taylor et al. (2001).

Table 6 gives the results for four different sizes of shock expressed in percentages, where $u_t = \in [5,10,20,40]$.[25] The estimated half-lives show that the speed of mean reversion increases with the size of the shock. Similarly to the results provided by Taylor et al. (2001), the estimated half-lives are significantly lower for estimates based on initial history. As mentioned above, this pattern also mirrors nonlinearities because the initial condition is chosen farther from the mean when historical values are considered.

-Table 6 about here-

Altogether, the results suggest that the half-life time of real exchange rate shocks is significantly lower than five years. All estimates based on historical values are below three years, while half-life times based on equilibrium values never exceed four-and-a-half years. These results are in line with previous findings based on smooth transition models for real exchange rates. In a recent study, Norman (2010) found a large proportion of half-lives to be below five years, with nearly thirty percent below three years. Half-life times provided by Taylor et al. (2001) are slightly lower for calculations based on historical values and higher for initial equilibrium estimates, but display the same pattern. The only study which calculates half-lives based on a full VECM with prices and nominal exchange rates is provided by Sarno and Valente (2006). Their estimates for flexible regimes based on initial history and annual data suggest that on average shocks of ten and twenty percent are reduced by half after less than one year, while the half-life times of smaller shocks are below two years. The results of this study, based on initial history, suggest half-lives between eighteen months for the UK and thirty-three months for Japan. The differences might be explained by the focus in this work on the post-Bretton Woods period and the use of monthly data.[26] As pointed out by Paya and Peel (2006), the speed of adjustment in ESTAR models might still be upwardly biased, indicating even smaller half-lives, as provided by this study. Summing up this section and taking into account that the estimates based on initial history are more reasonable from an economic point of view, we provide further evidence that real exchange rates mean revert much faster than suggested by the consensus benchmark of three to five years.

[25] Sarno and Valente (2006) do not consider shocks of forty percent but calculate half-lifes for one percent shocks. However, owing to the fact that this study uses monthly data, shocks of forty percent are also considered, similarly to Taylor et al. (2001). Shocks of one percent did not seem to differ from five percent shocks so the author decided that their presentation is not necessary to demonstrate the nature of nonlinear adjustment.

[26] The sample by Sarno and Valente (2006) also includes time periods such as the inter-war floating period where previous studies have found evidence of a faster mean reversion of real exchange rates compared to the recent floating period (Sarno and Valente, 2006; Taylor and McMahon, 1988).

4. Conclusion

Applying a nonlinear error correction model, this paper has dissected the contributions of nominal exchange rates and prices to PPP adjustment for the current floating period. The empirical results show firstly that strong PPP can only be verified for France, while a weak version holds for each country if a standard cointegration analysis is applied. The adjustment, which is of very small size, is then driven solely by prices. Applying nonlinear ESTR models to deviations from strong PPP then confirms that the nominal exchange rate starts to drive adjustment once deviations become large. The corresponding adjustment coefficients are considerably higher compared to the significant coefficients for price adjustment. The straightforward conclusion is that nominal exchange rates are responsible for the nonlinear mean reverting behaviour in real exchange rates. Furthermore, they are also the main driver of overall adjustment to PPP, considering the magnitude of the coefficients.

This implies that real exchange rate dynamics are far more complex than assumed by previous studies which have based their analysis on standard unit root tests and either reject PPP or report very low rates of mean reversion. As pointed out by Sarno and Valente (2006) and Taylor et al. (2001), these tests also have low power because they apply common factor restrictions to the dynamics of prices and nominal exchange rates and neglect nonlinear adjustment dynamics. Providing estimates for the half-life times of shocks based on a model which simultaneously allows for nonlinear adjustment without implying such restrictions, this study confirms previous results that the half-life times of real exchange rate shocks are significantly below the consensus benchmark of three to five years.

Taking these findings as a starting point, it seems doubtful whether sluggish prices offer an explanation for slow adjustment to PPP. The results from the multivariate linear model established in section 3.2 are compatible with results previously derived in the literature. This is in particular valid with respect to the argument raised by Rogoff (1996) who states that the convergence speed to PPP is too low to be fully explained by stickiness in nominal prices in the tradition of Dornbusch (1976). At first glance, the smaller half-lives of shocks established by this study therefore seem to be more consistent with standard macroeconomic models based on nominal rigidities. However, the overall results suggest that the nominal exchange rate carries most of the adjustment burden and the speed of price adjustment is not a main factor in the context of PPP. Hence, the argument of price stickiness might be invalidated not by low convergence of real exchange rates but by the minor contribution of prices to adjustment. A related argument has been raised by Cheung et al. (2004). A major task for theoretical models is to incorporate the nonlinear adjustment of real exchange rates while keeping an eye on the main role of nominal exchanges.

Altogether, the results are encouraging for the modelling of nominal exchange rates in the sense that the commonly applied concept of PPP is indeed important for the nominal exchange rate after one has distinguished between small and large deviations. An interesting area for further research would be a nonlinear error correction model for the nominal exchange rate based on a multivariate cointegration analysis, which besides prices also includes income, interest rates and money supply. Up to now, most studies, including this paper, have focused on one kind of fundamental factor when analyzing nonlinear error correction behaviour in nominal and real exchange rates.

Tables

Table 1: Rank test

p-r	R	Eig. Value	Trace	Trace*	Frac95	P-Value	P-Value*
a) UK/US							
2	0	0.121	62.644	61.613	20.164	0.000***	0.0000***
1	1	0.014	6.361	4.554	9.142	0.170	0.347
b) GE/US							
2	0	0.095	48.271	47.467	20.164	0000***	0000***
1	1	0.01	4.577	2.848	9.142	0.344	0.617
c) FR/US							
2	0	0.054	28.818	28.177	20.164	0.002***	0.003***
1	1	0.011	4.89	2.429	9.142	0.306	0.694
d) JP/US							
2	0	0.075	39.196	38.136	25.731	0.000***	0.001***
1	1	0.012	5.221	3.971	12.448	0.573	0.745

Note: Panel (a-d) reports the results of Johansen (1988, 1991) cointegration tests. Trace* and p-value* refer to Bartlett-corrected values. r denotes the cointegration rank. */**/*** implies rejection of the null hypothesis at the 10/5/1% significance level.

Table 2: Tests for autocorrelation

a)UK/US

LM(1):	ChiSqr(4)	=	7.658	[0.105]
LM(2):	ChiSqr(4)	=	6.098	[0.192]
LM(3):	ChiSqr(4)	=	3.977	[0.409]
LM(4):	ChiSqr(4)	=	8.110	[0.088]

b)GE/US

LM(1):	ChiSqr(4)	=	5.911	[0.206]
LM(2):	ChiSqr(4)	=	2.549	[0.636]
LM(3):	ChiSqr(4)	=	2.123	[0.713]
LM(4):	ChiSqr(4)	=	1.966	[0.742]

c)FR/US

LM(1):	ChiSqr(4)	=	4.176	[0.383]
LM(2):	ChiSqr(4)	=	7.110	[0.130]
LM(3):	ChiSqr(4)	=	9.001*	[0.061]
LM(4):	ChiSqr(4)	=	4.714	[0.318]

d)JP/US

LM(1):	ChiSqr(4)	=	8.009	[0.091]
LM(2):	ChiSqr(4)	=	5.798	[0.215]
LM(3):	ChiSqr(4)	=	7.963*	[0.093]
LM(4):	ChiSqr(4)	=	5.226	[0.265]

Note: Panel (a-d) reports LM tests on autocorrelation for the model described in section 3.2. The test is distributed as X^2 with degrees of freedom in parentheses. P-values are given in squared parentheses. */**/*** implies rejection of the null hypothesis at the 10/5/1% significance level.

Table 3: Cointegration vector and estimated adjustment coefficients

a)UK/US

s_t	$(p_t - p_t^*)$	Const.
1	-10.132***	1.043***
	(-5.474)	(-4.604)

Δs_t	$\Delta(p_t - p_t^*)$
0.000	0.001***
(0.218)	(5.466)

b)GE/US

s_t	$(p_t - p_t^*)$	Const.
1	-3.612***	-0.721***
	(-5.100)	(-4.312)

Δs_t	$\Delta(p_t - p_t^*)$
0.001	0.001***
(0.543)	(6.137)

c)FR/US

s_t	$(p_t - p_t^*)$	Const.
1	-0.895***	-5.051***
	(-3.218)	(-40.828)

Δs_t	$\Delta(p_t - p_t^*)$
0.003	0.004***
(0.869)	(4.885)

d)JP/US

s_t	$(p_t - p_t^*)$	Const.
1	-2.382***	-0.002***
	(-5.307)	(-5.211)

Δs_t	$\Delta(p_t - p_t^*)$
-0.004	0.004***
(-0.750)	(5.850)

Note: Panel (a-d) shows the estimates of the cointegration vector and reports the adjustment coefficients towards the long-run equilibrium for the exchange rate (Δs_t) and prices ($\Delta(p_t - p_t^*)$) with t-values in parentheses. As outlined in section 3.2, the nominal exchange rate is significant if the normalization is carried out on prices. */**/*** implies significance at the 10/5/1% significance level.

Table 4: Teräsvirta test for nonlinearity and choice of the delay parameter

J	UK/US	GE/US	FR/US	JP/US
t-1	4.818	20.026***	4.996	21.456***
	(0.567)	(0.003)	(0.544)	(0.002)
t-2	39.129***	59.594***	57.911***	72.897***
	(0.000)	(0.000)	(0.000)	(0.000)
t-3	22.266***	29.244***	30.851***	41.459***
	(0.001)	(0.000)	(0.000)	(0.000)
t-4	23.907***	26.489***	33.653***	34.054***
	(0.001)	(0.000)	(0.000)	(0.000)
t-5	25.197***	22.407***	32.558***	26.392***
	(0.000)	(0.001)	(0.000)	(0.000)
t-6	20.654***	18.435***	34.273***	19.098***
	(0.002)	(0.005)	(0.000)	(0.004)
t-7	18.785***	17.148***	36.556***	13.557**
	(0.005)	(0.009)	(0.000)	(0.035)
t-8	18.443***	16.349**	31.653***	22.638***
	(0.005)	(0.012)	(0.000)	(0.001)
t-9	18.666***	20.926***	36.988***	12.485*
	(0.005)	(0.002)	(0.000)	(0.052)
t-10	15.447***	29.804***	41.084***	18.763***
	(0.017)	(0.000)	(0.000)	(0.005)
t-11	13.817***	33.876***	39.499***	34.123***
	(0.032)	(0.000)	(0.000)	(0.000)
t-12	15.200***	31.751***	38.639***	39.829***
	(0.019)	(0.000)	(0.000)	(0.000)

Note: The table displays the test statistic of the LM test for nonlinearity as described in Section 2.1 for different lag orders j, with P-values in parentheses. The test is distributed as X^2 with six degrees of freedom. For details, see Teräsvirta (1998). */**/*** implies rejection of the null hypothesis at the 10/5/1% significance level.

Table 5: Nonlinear VECM estimation for changes in the nominal exchange rate and price differentials

a)UK/US								
Δs_t	c_1	ϑ	$c_1^{\sim}$	$\alpha_1^{\sim}$	γ		μ	Test AC
	-0.001	0.366***	0.005	-0.040*	0.516		-0.545***	AR1=(0.219)
	(0.523)	(0.000)	(0.380)	(0.080)	(0.349)		(0.000)	AR2=(0.013)**
$\Delta(p_t - p_t^*)$	c_1	α_1	ϑ	$c_1^{\sim}$	$\vartheta^{\sim}$	γ	μ	
	-0.001*	0.006	0.633***	0.002**	-0.566***	17.644**	-0.545***	AR1=(0.997)
	(0.080)	(0.117)	(0.001)	(0.027)	(0.002)	(0.015)	(0.000)	AR2=(0.238)

b)GE/US								
Δs_t	c_1	ϑ	$c_1^{\sim}$	$\alpha_1^{\sim}$	γ		μ	
	-0.006	0.327***	0.008*	-0.020**	15.973		0.601***	AR1=(0.645)
	(0.115)	(0.000)	(0.099)	(0.032)	(0.380)		(0.000)	AR2=(0.183)
$\Delta(p_t - p_t^*)$	c_1	α_1	ϑ	$c_1^{\sim}$	$\vartheta^{\sim}$	γ	μ	
	-0.001***	0.009***	0.021	-0.007**	-0.415	0.165	0.601***	AR1=(0.997)
	(0.003)	(0.004)	(0.750)	(0.035)	(0.287)	(0.167)	(0.000)	AR2=(0.238)

c)FR/US								
Δs_t	c_1	ϑ	$c_1^{\sim}$	$\alpha_1^{\sim}$	γ		μ	
	0.003	0.315***	-0.017**	-0.048**	0.938		1.944***	AR1=(0.731)
	(0.587)	(0.000)	(0.034)	(0.034)	(0.509)		(0.000)	AR2=(0.211)
$\Delta(p_t - p_t^*)$	c_1	α_1	ϑ	$c_1^{\sim}$	$\vartheta^{\sim}$	γ	μ	
	0.000	0.004**	0.542***	0.001	-0.792***	0.201	1.944***	AR1=(0.222)
	(0.210)	(0.044)	(0.000)	(0.343)	(0.008)	(0.327)	(0.000)	AR2=(0.294)

d)JP/US								
Δs_t	c_1	ϑ	$c_1^{\sim}$	$\alpha_1^{\sim}$	γ		μ	
	-0.005*	0.316***	0.003	-0.010**	1.316		5.160***	AR1=(0.305)
	(0.067)	(0.000)	(0.354)	(0.049)	(0.422)		(0.000)	AR2=(0.262)
$\Delta(p_t - p_t^*)$	c_1	α_1	ϑ	$c_1^{\sim}$	$\vartheta^{\sim}$	γ	μ	
	-0.002	0.011	-0.077	0.006	0.315	0.515	5.160***	AR1=(0.348)
	(0.105)	(0.106)	(0.680)	(0.149)	(0.379)	(0.201)	(0.000)	AR2=(0.381)

Note: The table shows the estimation results for the reduced form of the nonlinear vector error correction models as described in section 3.4 with P-values in parentheses. */**/*** implies significance at the 10/5/1% significance level. AR in the right column refers to the p-values for the test on autocorrelation of order one and two as described in Section 3.4.

Table 6: Half-life times of real exchange rate shocks

a) Conditional on initial equilibrium

Shock (%):	40	20	10	5
UK	24	45	48	49
France	20	30	41	42
Germany	28	28	42	54
Japan	25	31	39	47

b) Conditional on average initial history

Shock (%):	40	20	10	5
UK	18	20	24	26
France	20	23	24	26
Germany	26	29	31	32
Japan	27	28	30	33

Note: The table shows the estimation results for the half-life times of real exchange rate shocks in terms of months, as described in Section 3.5.

References

Baillie, R.T. and R. Kilic (2006). Do asymmetric and nonlinear adjustments explain the forward premium anomaly? *Journal of International Money and Finance,* 25(1), 22-47.

Beckmann J., A. Belke and M. Kühl (2011). The stability of the dollar-euro exchange rate determination equation - a time-varying coefficient approach. *Review of World Economics*, 147(1), 11-40.

Breusch, T.S. and A.R. Pagan (1979). A simple test for heteroscedasticity and random coefficient variation. Econometrica, 47(5), 1287-1294.

Cheung, Y., K.S. Lai and M. Bergman (2004). Dissecting the PPP puzzle: The unconventional roles of nominal exchange rate and price adjustments. *Journal of International Economics*, 64(1), 135-150.

Cheung, Y. and K.S. Lai (1998). Parity reversion in real exchange rates during the post-Bretton Woods period. *Journal of International Money and Finance,* 17(4), 597-614.

De Grauwe, P. and I. Vansteenkiste (2007). Exchange rates and fundamentals: a nonlinear relationship? *International Journal of Finance and Economics*, 12(1), 37-54.

Dornbusch, R. (1976). Expectations and exchange rate dynamics. *The Journal of Political Economy*, 84(6), 1161-1176.

Dijk, D. van, T. Teräsvirta and P.H. Franses (2002), Smooth transition autoregressive models - A survey of recent developments. *Econometric Reviews*, 21(1), 1-47.

Eitrheim, Ø. and T. Teräsvirta (1996). Testing the adequacy of smooth transition autoregressive models. *Journal of Econometrics*, 74 (1), 59-75.

Engel, C. and J.C. Morley (2001). The adjustment of prices and the adjustment of the exchange rate. *NBER Working Papers, number 8550.*

Franses, P.H., D. van Dijk (2001). The forecasting performance of various models for seasonality and nonlinearity for quarterly industrial production. *Econometric Institute Report, numbers 2001-14.*

Franses, P.H., D. van Dijk (2000). Nonlinear time series models in empirical finance. *Cambridge University Press,* Cambridge.

Froot, K.A. and K. Rogoff (1995). Perspectives on PPP and long-run real exchange rates. Grossman, M. and K. Rogoff (eds.), *Handbook of International Economics*, 3-32.

Frömmel, M., Kruse, R., L. Menkhoff and P. Sibbertsen (2010). What do we know about real exchange rate nonlinearities? *CREATES Research Paper,* 2009-50.

Frömmel, M., R. MacDonald and L. Menkhoff (2005a). Markov switching regimes in a monetary exchange rate model. *Economic Modelling*, 22(3), 485-502.

Frömmel, M., R. MacDonald and L. Menkhoff (2005b). Do fundamentals matter for the D-Mark/Euro-Dollar? A regime switching approach. *Global Finance Journal*, 15(3), 321-335.

Frydman, R., M. Goldberg, S. Johansen and K. Juselius (2010). Testing hypotheses in an I(2) model with piecewise linear trends. An analysis of the persistent long swings in the Dmk/$ rate. *Journal of Econometrics*, 158(1), 117-129.

Gallant, A., P. Rossi and G. Tauchen (1993). Nonlinear dynamic structures. *Econometrica*, 61 (4), 871-907.

Goldberg, M.D. and R. Frydman (1996). Empirical exchange rate models and shifts in the co-integrating Vector. *Structural Change in Economic Dynamics*, 7(1), 55-78.

Goldberg, M.D. and R. Frydman (2001). Macroeconomic fundamentals and the DM/$ exchange rate: temporal instability and the monetary model. *International Journal of Finance and Economics*, 6(4), 421-435.

Goldfajn, I. and R.O. Valdes (1996). The aftermath of appreciations. *NBER Working Papers, number 5650.*

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control*, 12(2-3), 231-254.

Johansen, S. (1991). Estimation and hypothesis testing of cointegrated vectors in Gaussian vector autoregressive models. *Econometrica* 59(6), 1551-1580.

Juselius, K., (2006). *The cointegrated VAR model: Econometric methodology and macroeconomic applications*, Oxford University Press, Oxford.

Juselius, K. and R. MacDonald (2004). International parity relationships between the USA and Japan. *Japan and the World Economy*, 16(1), 17-34.

Juselius, K. and R. MacDonald (2000). Interest rate and price linkages between the USA and Japan: Evidence from the Post-Bretton Woods Period. *Discussion Papers 00-13, University of Copenhagen. Department of Economics.*

Kilian, L. and M. P. Taylor (2003). Why is it so difficult to beat the random walk forecast of exchange rates. *Journal of International Economics*, 60(1), 85-107.

Lothian, J.R. and M.P. Taylor (2008). Real exchange rates over the past two centuries: How important is the Harrod-Balassa-Samuelson effect? *The Economic Journal*, 118(532), 1742-1763.

Luukonen, R., P. Saikkonen, and T. Teräsvirta (1988). Testing linearity against smooth transition autoregressive models. *Biometrika* 75, 491-499.

Mark, Nelson C. (1990). Real and nominal exchange rates in the long run: An empirical investigation. *Journal of International Economics*, 28(1-2), 115-136.

Meese, R.A. and K. Rogoff (1983). Empirical exchange rate models of the seventies: Do they fit out of sample? *Journal of International Economics*, 14(1-2), 3-24.

Norman, S. (2010). How well does nonlinear mean reversion solve the PPP puzzle? *Journal of International Money and Finance,* 29 (5), 919-937.

Paya, I. and D. Peel (2006). On the speed of adjustment in ESTAR models when allowance is made for bias in estimation. *Economics Letters*, 90 (2), 272-277.

Rahbek, A., E. Hansen and J.G. Dennis (2002). "ARCH Innovations and their impact on cointegration rank testing, Department of Theoretical Statistics", *Centre for Analytical Finance,* Working Paper no.22, University of Copenhagen.

Rogoff, K. (1996). The purchasing power parity puzzle. *Journal of Economic Literature*, 34(2), 647-668.

Sarno, L. and G. Valente (2006). Deviations from purchasing power parity under different exchange rate regimes: Do they revert and, if so, how? *Journal of Banking & Finance*, 30, (11), 3147-3169.

Sarno, L., G. Valente and H.L. Leon (2006). Nonlinearity in deviations from uncovered interest parity: an explanation of the forward bias puzzle. *IMF Working Papers number 06/136.*

Sarno, L. (2005). Viewpoint: Towards a solution to the puzzles in exchange rate economics: where do we stand? *Canadian Journal of Economics*, 38, (3), 673-708.

Sarno, L., M.P. Taylor and I. Chowdhury (2004). Nonlinear dynamics in deviations from the law of one price: a broad-based empirical study. *Journal of International Money and Finance,* 23(1), 1-25.

Sarno, L., G. Valente and M.E. Wohar (2004). Monetary fundamentals and exchange rate dynamics under different nominal regimes. *Economic Inquiry*, 42(2), 179-193.

Sarno, L., and M.P. Taylor (2003). The economics of exchange rates. Cambridge, Cam-bridge University Press.

Schinasi, G.J. and P.A.V.B. Swamy (1989). The out-of-sample forecasting performance of exchange rate models when coefficients are allowed to change. *Journal of International Money and Finance*, 8(3), 375-390.

Stock, J.H. and M.W. Watson (1988). Testing for common trends. *Journal of the American Statistical Association*, 83(404), 1097-1107.

Siklos, P.L. and C.W.J. Granger (1997). Regime-sensitive cointegration with an application to interest-rate parity. *Macroeconomic Dynamics*, 1(3), 640-657.

Taylor, M.P. and P.C. McMahon (1988). Long-run purchasing power parity in the 1920s. *European Economic Review*, 32(1), S. 179-197.

Taylor, M.P. and D.A. Peel (2000). Nonlinear adjustment, long-run equilibrium and exchange rate fundamentals. *Journal of International Money and Finance*, 19(1), 33-53.

Taylor, M.P., D.A. Peel and L. Sarno (2001). Nonlinear mean-reversion in real exchange rates: Toward a solution to the purchasing power parity puzzles. *International Economic Review*, 42(4), 1015-42.

Teräsvirta, T. (1998). Modeling economic relationships with smooth transition regressions. *Giles, D.E.A., Ullah, A. (Eds), Handbook of applied economic statistics. Marcel Dekker: New York*, 507-552.

Wei, S. and D.C. Parsley (1995). Purchasing power disparity during the floating rate period: exchange rate volatility, trade barriers and other culprits. *NBER Working Papers number 5032.*

Wolff, C.C.P. (1987). Time-varying parameters and the out-of-sample forecasting performance of structural exchange rate models. *Journal of Business and Economic Statistics*, 5(1), 87-97.

Wu, J. and Y. Hu (2009). New evidence on nominal exchange rate predictability. *Journal of International Money and Finance*, 28(6), 1045-1063.