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Title: Consumption and Income Smoothing

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Consumption and income smoothing[i]

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Abstract

This paper presents a two sector dynamic general equilibrium model in which **income smoothing** takes place within the households (intra-temporally), and **consumption smoothing** takes place among the households (inter-temporally). Idiosyncratic risk sharing within the family is based on an *income smoothing contract*. There are two sectors in the model, the regular sector and the underground sector, and the smoothing comes from the underground sector, which is countercyclical with respect aggregate GDP. The paper shows that the simulated disaggregated consumption and income series (that are the regular and underground consumption flows) are more sensitive to exogenous changes in sector-specific productivity and tax rates than regular and underground income flows, and that this picture is reversed when the aggregate series are considered.

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1 Introduction.

This paper presents a two sector dynamic general equilibrium model in which **income smoothing** takes place within the households (intra-temporally), and **consumption smoothing** takes place among the households (inter-temporally). There are two sectors in the model, the regular sector and the underground sector.[i.] Idiosyncratic risk sharing within the family is based on an income-smoothing contract, and the smoothing comes from the underground sector, which is countercyclical with respect to aggregate GDP.

The model joins together three elements to understand dynamic consumption and production decision making: consumption smoothing, income smoothing, and risk sharing using a contract based upon a countercyclical underground sector[ii]. The contract provides the insurance opportunity for smoothing income and consumption. Intra-sector production smoothing, as well as the underground activities, are generated by distortionary taxation.

There are three main results:

First, the model shows that the simulated *disaggregated consumption series* (that are the regular and underground consumption flows in this context) are more sensitive to exogenous changes in sector-specific productivity and tax rates than regular and underground income flows. Selected volatility measures for disaggregated consumption series are greater than or equal to those of the corresponding income series.

Second, the picture is reversed when the *aggregate series* (regular plus underground) are considered. The impulse response function of aggregate consumption to innovations in productivity and tax rates is below the response of aggregate income. In addition, aggregate consumption is less volatile than aggregate income (over 1000 simulations), which is one of the most robust empirical stylized facts matched by equilibrium growth models.

Third, we show that in our model agents are able to disentangle consumption and income by relying on the countercyclical behavior of the underground sector. In this sense, the underground sector offers risk sharing opportunities to household's members, which is exploited by entering the income-smoothing contract.

The model is calibrated for the Italian economy because it presents a large underground sector, which allows appreciating better the impact of underground activities on the overall

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[i] There is no universal agreement on what defines the underground economy. Most recent studies use one of more of the following definitions: (a) unrecorded economy (failing to fully or properly record economic activity, such as hiring workers off-the-book); (b) unreported economy (legal activity meant to evade the tax code); (c) illegal economy (trading in illegal goods and services). We are interested about the size of the underground economy as encompassing those activities which are otherwise legal but go unreported or unrecorded, i.e. bits (a) and (b).

[ii] Underground activities are especially significant in many European countries where the underground sector represents from 15 to 35 percent of the GDP. In the U.S. the underground sector is about 8 or 9 percent of the GDP. Even though it cannot be considered a small fraction, it is much lower than the European counterparts (Busato and

economy. Needless to say, this analysis is addressed to European countries like Belgium, Denmark, Greece, Portugal and Spain (see Busato and Chiarini, 2004).[iii]

The paper is organized as follows. In Section 2 we summarize the empirical evidence about underground activities, consumption smoothing and income smoothing. Section 3 describes the structure of the model, and Section 4 discusses our choice of functional forms and parameters values. Section 5 outlines simulation results, while Section 6 presents our conclusions.

2. Selected stylized facts

In all industrialized countries the underground sector is large both in terms of the labor input and in terms of the output produced. Schneider and Enste (2000) show that the underground economic activity represents on average 16.9% of the GNP for the OECD countries. Different measurement techniques provide similar approximate magnitudes of the size and development of the underground economy.[iv] In addition, what is most interesting from the short run perspective, is the countercyclical behavior of the underground component. Busato and Chiarini (2004) present evidence of this phenomenon for Italy, New Zealand, the United States and the United Kingdom.

Income smoothing is a relevant phenomenon, as well. Mordoch (1995) suggests that a great deal of risk is averted by households in inputs allocation.[v] This means that the members of a family can pool together their resources, in particular their labor supplies, and allocate them more efficiently over time, and across sectors (see also Gallie and Paugam, 2000).[vi]

Between consumption data and the majority of theoretical models, there exist two critical discrepancies, frequently neglected. First, consumption data usually refers to households, while majority of theoretical models is cast into the representative agent framework. Second, since a countercyclical underground sector may offer risk sharing opportunities to workers, the

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Chiarini, 2004 or Schneider and Enste, 2000).

[iii] Our model is calibrated to match selected moments for the Italian economy within the sample 1970-1996. We choose Italy because of the availability of a complete data set on the underground sector (Bovi, 1999). Note however, that Italy presents a share of underground economy equally to approximately 20 or 30 percent of the GDP, which according to Schneider and Enste (2000), corresponds to several European countries like Norway, Belgium, Portugal, Spain, Greece and Denmark.

[iv] There exist several methods of estimating the size of the underground economy. A detailed survey of the three most widely used methods to measure the hidden activity (the direct approaches, the indicator approach and the model approaches) are discussed in Schneider and Enste (2000). See also Feige (1989), Thomas (1992; 1999) and Chiarini and Marzano (2004) among others.

[v] Note, however, that altruism within extended family is still controversial, at least for the United States. Altonji, Hayashi and Kotlikoff (1992) for instance reject the hypothesis of altruism within the extended family. But, in the conclusions, the authors underline how they do believe that significant altruistically motivated transfers occur in United States, especially among wealthy, who are underrepresented in the Panel Study on Income Dynamics (PSID).

[vi] For example we may think to the fact that in a large percentage of married and/or cohabiting persons between 25 and 54 years old, there is a single job-holder (i.e. Gallie and Paugam, 2000).

conclusions of models that do not explicitly incorporate this sector, may be seriously compromised for economies in which underground economy represents a significant share of actual GDP.

The model presented in the following Section specifically addresses these issues. We tell a simple story of a two persons household, and we compare the individual consumption and income profiles, with those of the family.

3 Structure of the Model

The model draws from Busato and Chiarini (2004), while differing in two important aspects. First, we generalize the household structure by introducing a simple form of heterogeneity;[vii] second, we model income smoothing and within-family consumption reallocation by a specific contract.

There are three agents in the model: the firm, the extended family, and the government. In addition there are the regular and the underground sectors.

The members of the extended family choose consumption, investment, and hours worked at each date and in each sector to maximize the expected discounted value of the family's utility subject to an income smoothing contract, a budget constraint, a proportional tax rate on the regular wage, and the law of motion for the capital stock.

In each period, the firm rents capital only in the regular sector, but hires labor in both sectors.[viii] It produces output in both the sectors by solving a period-by-period profit maximization problem. The firm solves this series of one-period problems subject to a probability that it may be discovered producing in the underground economy, convicted of tax evasion and subject to a penalty surcharge.

Finally, a government levies proportional taxes on output and labor income, and balances its budget at each point in time. We assume that government spending on goods and services does not contribute to either production or to extended family utility.

3.1 The Extended Families.

Consider a production economy populated by many consumers. Each consumer works in only one of the two sectors. They receive incomes that are functions of idiosyncratic shocks. Within the economy there exist extended families, exogenously determined and of fixed size. Within each family, the members have perfect information concerning each other's idiosyncratic shocks

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[vii] We will see that, given the structure of our income smoothing contract, we may rewrite the heterogeneous agent model as a representative agent model. This claim is proved in Proposition 1.

[viii] This assumption reflects a basic stylized fact of many underground economies.

to each sector.[ix] For simplicity we assume that there exists one family, which is composed by two working individuals, Mr. κ and Miss. *l*. [x] Without loss of generality, we assume that Mr κ works in the regular sector, while Miss. *l* works in the underground sector.

Since Mr. κ, and Miss. *l* are good friends until many years, we may argue that their preferences are not too far. Hence, assuming that they have the same utility function for consumption should not be seen as a forcing. They have, however, a different preference structure for labor supply, which is consistent with the fact they work in different sectors. We can use a variant of the Cho and Rogerson's (1988) extended family labor supply model. Specifically, extended family composed of Mr. and Mrs. κ is characterized by the following instantaneous utility function:

$$U\left(c_t^{\kappa}, c_t^{l}, l_t^{\kappa}, l_t^{l}\right) = \phi_{\kappa} u\left(c_t^{\kappa}\right) + \phi_l u\left(c_t^{l}\right) - v\left(l_t^{\kappa}\right) l_t^{l} - \mu\left(l_t^{l}\right), \qquad (1)$$

where $u(c^{\kappa}_t)$ and $u(c^{l}_t)$ represent utility from Mr. κ and Miss. *l* consumption, and $v(l^{\kappa}_t)l^{l}_t$ describes the disutility of working in both sectors. We interpret the last term, $\mu(l^{l}_t)$, as reflecting the idiosyncratic cost of working in the underground sector. This cost may be associated in particular with the lack of any social and health insurance in the underground sector.[xi] Given their long standing friendship, Mr. κ shares part of Miss. *l* concerns of working in that sector. For the same reason, we assume: φ_k= φ_l= ½.[xii]

An aspect of primary interest in our labor market is workers' labor supply in the two sectors of the economy. Mr. κ , which works in the regular sector, supplies l^k_t, and receive a wage $w^k_t = w_t(1-\tau_t)$, where τ_t is the stochastic tax rate on wage income. Miss *l,* who works in the other sector, offers l^l_t, and earns a wage $w^l_t= w_t$. The family budget constraint is:

$$w_t(1-\tau_t)l_t^k + w_t l_t^l + R_t K_t^{tot} = C_t^{tot} + X_t^{tot}, \qquad (2)$$

where $C_t^{tot} = c_t^k + c_t^l$ and X^{tot}_t represents total consumption and total investment by the family, respectively. Eventually they pool their savings together, and rent the grand total, X^{tot}_t, to the firms, whose capital stock evolves according to the following state equation:

$$K_{t+1}^{tot} = (1-\delta)K_t^{tot} + X_t^{tot}, \qquad (3)$$

where δ denotes the exogenous and constant depreciation rate. We refer to equations (1) to (3) as to the Heterogeneous Agent Model (HAM).

[ix] This hypothesis will be important since it simplifies agents' interaction after the contract introduction.

[x] We choose to restrict the analysis to one family to keep notation simple. The size and the number of the extended family can easily be enlarged.

[xi] Note that since working in the underground sector is costly, as we can see from the last term of the instantaneous utility function, we rule out equilibria were all labor supply goes to the underground sector. In other words, if $l^l_t \rightarrow \infty$, then $\mu(l^l_t) \rightarrow \infty$, making this decision too costly for the family.

[xii] The choice of ½ is without loss of generality, it just simplifies the algebra.

But one day, unexpectedly, a spark ignites between the two. Mr. κ and Miss. *l* fall in love, and are offered an *"consumption and income smoothing contract (CISC)"*. Readers unfamiliar with Contract theory would call it a "marriage" contract. The contract, defined below, says the consumers should pool together income (and thus labor supply) and consumption.

Definition 1 (Consumption and Income Smoothing Contract) *The contract has three features:*

1. $l^k_t = \theta_t L_t$ *and* $l^l_t = (1-\theta_t) L_t$. *This means, that Mr. κ and Miss. l pool together their labor supplies,* L_t*, then they allocate a share* θ_t *to regular sector, and the remaining* $(1-\theta_t)$ *to underground sector.*
2. *The family will choose total consumption* C^{tot}_t*. Then Mr. κ and Miss. L consumption will be* $c^k_t = \omega^* C^{tot}_t$ *and* $c^l_t = (1-\omega^*) C^{tot}_t$.[xiii] *When agents have the same utility function for consumption,* $\omega^*=1/2$.[xiv]
3. *We assume that agents accept the contract, that it holds for each period in time, and that it is incentive compatible and perfectly enforceable.*[xv]

In this paper we do not consider strategic interaction among agents. The contract has the simple goal to pool together labor supply, and consumption, insuring the family against idiosyncratic shocks. In addition, its structure serves as foundation of Proposition 1.

Proposition 1 (Representative Agent Model and Heterogenous Agents Model) *Under the income smoothing contract, as in Definition* 1, *the Heterogeneous Agent Model (Equations (1) to* (3)) *is equivalent to a Representative Agent Model characterized by instantaneous utility function* $U(c^j_t, \theta^j_t) = u(c^j_t) - v(\theta^j_t)(1-\theta^j_t) - \mu(1-\theta^j_t)$, *budget constraint* $w_t(1-\theta^j_t \tau_t) + R_t K^j_t = c^j_t + X^j_t$ *and capital accumulation constrain (3). Equivalence is in the sense of having the same First Order Conditions.*

Proof. *See Appendix A .*

Remark 1 ("A Transparent Representative Agent Model") *By Proposition* 1, *we transform the HAM into a Representative Household model of a special kind.*[xvi] *The novelty of our approach consists in inspecting the composition mechanism of income and consumption flows*

[xiii] In this way individual consumption is disentangled from individual income. It may be interesting to note that this is the argument behind the risk sharing and consumption literature (see Deaton, 1992 for a survey).

[xiv] This claim can be showed quite easily, but for completeness we precise the argument in Appendix B.

[xv] By definition, an implicit contract will need to be sustained as an equilibrium in the interaction between the parties (Salanie', 1997). The contract we present in this model has the very simple goal to provide insurance against production idiosyncratic risk. For this reasons we assume that agents accept the contract.

[xvi] Notice that this an application of an already known result. Indeed Rubinstein (1974) shows that when agents have homogenous beliefs, and time additive utility function with linear risk tolerance, and same exponent, these

occurring within the family, and across sectors.

Specifically, our model generates, for both income and consumption, three series: two "pre-contract" or disaggregated series (regular and underground), and one "after-contract" or aggregated series. The former series refer to individual consumers, and we interpret them as consumption and income profile which arise without contract. The latter ones belong to the household. Then we can explicitly compare the stochastic properties of the different series. We may think to ours, as to a

"Transparent Representative Agent Model".[xvii]

Relying on Proposition 1, and assuming that there exists a continuum of households, which are uniformly distributed over a unit interval, we specify a following functional form for the j-*th* household momentary utility function.[xviii] Specifically (1) becomes:[xix]

$$u\left(c_t^j;\theta_t^j\right)=\left\{\frac{\left(c_t^j\right)^{1-q}-1}{1-q}-h\frac{\left(\theta_t^j\right)^{1+\gamma}}{1+\gamma}\left(1-\theta_t^j\right)-f\frac{\left(1-\theta_t^j\right)^{1+\eta}}{1+\eta}\right\} \qquad (4)$$

To have a well behaved utility function, we assume that $h, f \geq 0; \gamma, \eta > -1$, that all the parts of the momentary utility function are well behaved.[xx] The first quantity denotes the utility from aggregate consumption stream, while the second term represents the overall disutility of working;[xxi] the last term reflects the idiosyncratic cost of working in the underground sector. In particular, this cost may be associated with the lack of any social and health insurance in the underground sector.

3.1.1 Productivity shocks and tax tates.

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assumptions imply demand aggregation.

[xvii] We choose to end up with a representative agent model since the collected data on income and consumption refers, more or less implicitly, to a representative household. In other words, we harmonize the theoretical scheme with the data. If, for instance, we had chosen to calibrate directly the heterogeneous agent model, we could not be sure anymore of equivalence between theory and data.

[xviii] The generalization to continuum of households is not necessary, but is consistent with the traditional set up of equilibrium growth models (see Mehra and Prescott, 1985 or Danthine and Donaldson, 1996).

[xix] This specification is adapted from Cho and Rogerson (1988) and Cho and Cooley (1994). Unlike the extreme cases of indivisible labor, where all the fluctuations occur on the extensive margin, and the divisible labor in which the fluctuations take place on the intensive margin, in this formulation of preferences households may allocate their time along both margins (intensive-hours and extensive-employment margin). Cho and Rogerson achieve this feature by introducing heterogeneity into the opportunity sets of household decision makers, and Cho and Cooley introduce some fixed costs of going to work that are not explicitly modelled. This allows us to capture changes in labor in both the market and the underground sector simultaneously, and it is consistent with the data, where we observe substantial variations in both the markets.

[xx] Restriction on the utility function to make the inter-temporal optimization problem well defined are derived in Busato and Chiarini (2004).

[xxi] Notice that there exist perfect substitutability across sector, in the sense that there are no adjustment costs while transferring labor supply (demand) from a sector to an other. Each sector, however, has its own peculiar characteristics that the instantaneous utility function tries to capture with regard to consumer's behavior.

Finally, we formalize productivity and tax rates as a stochastic vector of variables that follow univariate AR(1) processes in log:

$$A_{t+1} = \Omega A_t + \varepsilon_t$$

where A_t is a vector $[M_t, Z_t, t_t, \tau_t]$ containing the productivity shocks, M_t, Z_t, the stochastic corporate tax rate, t_t, and the stochastic personal income tax rate τ_t. $\Omega = diag\ (\rho_i)$, where $i = m, z, t, \tau$, is a 4 x 4 matrix describing the autoregressive components of the disturbances relative to each of the four shocks. The innovation, $\varepsilon'_t = [\varepsilon_m, \varepsilon_z, \varepsilon_t, \varepsilon_\tau,]$ is a vector of *i.i.d.* random variables.

3.1.2 The Stochastic Dynamic Programming Problem for Households.

Let $Vt(K^j_t, A_t)$ be the value function for the household problem:

$$V_t\left(K^j_t; A_t\right) = \max_{K^j_{t+1}, \theta^j_t} \left\{u\left(c^j_t, \theta^j_t\right) + E\left[\beta V_{t+1}\left(K^j_{t+1}; A_{t+1}\right)\right]\right\}, \tag{5}$$

subject to momentary utility function (4), to budget constraint (A.5 in Appendix A), and the low of motion for the household capital stock (3). The optimality conditions for the problem are the Euler equation (6.1) and the intra-temporal consumption-Labor allocation condition (6.2):[xxii]

$$1 = \beta E\left(\left(\frac{c^j_{t+1}}{c^j_t}\right)^{-q} R_{t+1} / \Im_t\right)$$
$$\text{(6.1; 6.2)}$$
$$0 = -w_t \tau_t \left(c^j_t\right)^{-q} - \left(\theta^j_t\right)^{\gamma} + h\left(\frac{2+\gamma}{1+\gamma}\right)\left(\theta^j_t\right)^{1+\gamma} + f\left(1-\theta^j_t\right)^{\eta}$$

ho messo c^j minuscolo al numeratore, un cambiamento analogo lo faccio nelle appendici quando si parla di consumi della famiglia

where $\left(1-\delta+\left(1-t_{t+1}\right)M_{t+1}\alpha\left(K^i_{t+1}\right)^{\alpha-1}\left(\theta^i_{mt+1}\right)^{1-\alpha}\right) = \left(1-\delta+r_{t+1}\right) \equiv R_{t+1}$ from firm profit maximization (see below).

3.2 The Firms.

There are I firms. Each firm $i \in I$ produces both in the regular and in the underground sector using two different production functions:

$$y^i_{mt} = M_t \left(k^i_t\right)^{\alpha} \left(l^i_{mt}\right)^{1-\alpha}; y^i_{ut} = Z_t l^i_{ut} \tag{7}$$

The regular output, y^i_{mt}, is the result of capital, k^i_t and regular labor, l^i_{mt}, applied to a Cobb

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[xxii] Appendix A characterizes in details the model, and states precisely the solution procedure.

Douglas production function. The underground output, y^i_{ut}, is produced with a production function which uses only underground labor, l^i_{ut},. Finally, M_t and Z_t are the idiosyncratic stochastic productivity shocks. [xxiii]This formulation is consistent with the behavior underling the existence of a underground sector. Indeed the firms have no incentive to invest capital in the underground sector.

In equilibrium each firm allocates a share, θ^i_t of the total labor, L_t, to regular production (therefore $l^i_{mt} = \theta^i_t L_t$) and the remainder, $1 - \theta^i_t$ to the other sector (therefore $l^i_{ut} = (1-\theta^i_t)L_t$).[xxiv] Normalizing L_t to unity, we can rewrite (7) as:

$$y^i_{mt} = M_t\left(k^i_t\right)^\alpha\left(\theta^i_t\right)^{1-\alpha}; y^i_{ut} = Z_t\left(1-\theta^i_t\right) \qquad (8)$$

When the firm produces in the regular sector, its output is taxed with certainty at the stochastic rate t_t. When producing in the underground sector, the firm may be discovered, with probability p, and forced to pay the stochastic tax rate, t_t, increased by a surcharge factor, $s > 1$, applied to the standard tax rate.

Assuming that the firm produces in both sectors, we can describe its revenues as follow:

$$y^i_{D,t} = (1-t_t)y^i_{mt} + \left(1-st_t\right)y^i_{ut} \text{ with prob. } p$$

$$y^i_{ND,t} = (1-t_t)y^i_{mt} + y^i_{ut} \qquad \text{with prob. } (1-p),$$

where $y^i_{D,t}$ is the output when the firm's underground activity is detected and $y^i_{ND,t}$ is the output produced when the firm's underground activities go undetected. The expected value of the output is then given by $E\left(y^i_t / \Im_t\right) = py^i_{D,t} + (1-p)y_{ND,t}i$.

The production costs come from the labor hired in both sectors, and from rented capital. The cost of regular labor is represented by wage paid for hours worked; a wage that is augmented by social security taxes at a rate that we will assume is equal to the corporate income tax rate, t_t. In

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[xxiii] Notice that this structure is equivalent to a more general set up with two production functions which use both the inputs, like for example $y^i_{mt} = M_t (k^i_t)^\alpha (l^i_{mt})^{1-\alpha}$ and $y^i_{ut} = Z_t (k^i_{ut})^\beta (l^i_{ut})^{1-\beta}$. According to Uzawa (1965) and Lucas (1988) if $\beta<\alpha$ we can set the smaller elasticity to zero without loss of any generality. It follows that the share of capital in the labor intensive production function is null and therefore an optimizing firm would choose $k_{ut} = 0$ for each t. Because we assume that the underground sector is labor intensive, we rely on this argument.

[xxiv] The use of the share is also consistent both with the fact that labor supply per person is approximately stationary in many economies although the real wage grows, and with the utility function, homogenous in consumption, that we adopt to model the household preferences. The aim is, therefore, to analyze the movement of resources between the two sectors, to understand how agents want to move inputs out of the market and into the underground. The reallocation of hours from market to informal sector rather than exclusively from leisure to labor, increases the volatility of the official labor input for a given technology shock.

accordance with the rationale behind underground activities, we assume that the firm does not pay social contributions for labor input employed in the underground sector.[xxv] Formally, firm costs are defined by:[xxvi]

$$CO\left(\theta_t^i, K_t^i\right) = w_t + w_t t_t \theta_t^i + r_t K_t^i \qquad (9)$$

At each date *t*, firm *i* maximizes period expected profits:

$$\max_{\left(\theta_t^i, k_t^i\right) \geq 0} E\left(y_t^i / \Im_t\right) - CO\left(\theta_t^i, k_t^i\right),$$

to derive the regular share of aggregate labor demand $\left(\theta_t^i\right)^* = \left(k_t^i\right)^* \left[\frac{(1-t_t)(1-\alpha)M_t}{(1-pst_t)Z_t + w_t t_t}\right]^{\frac{1}{\alpha}}$, the underground component $1-\left(\theta_t^i\right)^* = 1-\left(k_t^i\right)^* \left[\frac{(1-t_t)(1-\alpha)M_t}{(1-pst_t)Z_t + w_t t_t}\right]^{\frac{1}{\alpha}}$ and the capital demand $\left(k_t^i\right)^* = \left(\theta_t^i\right)^* \left[\frac{(1-t_t)\alpha M_t}{R_t}\right]^{\frac{1}{1-\alpha}}$ [xxvii]

3.3 The Government.

Under Proposition 1 the flow government budget constraint is:[xxviii]

$$w_t \tau_t \theta_t + \left(pst_t\right) y_{ut} + t_t y_{mt} = G_t, \qquad (10)$$

where $G_t = \overline{G}$.[xxix]

3.4 Equilibrium

Equilibrium for our model is described as a variant on a Recursive Competitive Equilibrium (RCE) of Prescott and Mehra (1980) notion.. Specifically, aggregate and individual quantities

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[xxv] Note that the tax structure is critical for the existence of underground activities, and therefore for the source of risk sharing.

[xxvi] Here we have already implemented the features of consumption and income smoothing contract into i-th firm objective. For details, see Appendix A, Proposition 1, Lemma 2.

[xxvii] Notice that in this context the regular share of aggregate labor demand equals the regular labor demand itself, because we normalize total labor to unity. The same holds for the underground labor market segment.

[xxviii] See Appendix A, Proposition 1, Lemma 3.

[xxix] Notice that the Government balances its budget only in expectation, since with probability 1 - *p* some firms and workers are evading. Hence equation (10) will not be satisfied on a state by state basis.

coincide, and equilibrium can be characterized as the F.O.C. of the Representative Household on which market clearing conditions have been imposed.

4 Calibration.

The system of equations we use to compute the dynamic equilibria of the model depends on a set of 12 parameters. Six pertain to household preferences, (q, h, f, η, γ, β), four to the structural-institutional context (the probability of a firm being detected p, the surcharge factor s, the equilibrium income and corporate tax rates t and τ), and the remaining two parameters to technology (the capital elasticity α, and the capital depreciation rate δ). The fact that the data on the underground economy is difficult to obtain substantially complicates the calibration. Because we are not aware of other studies which calibrate the parameters of a general equilibrium model augmented with an underground sector, we precisely detail our calibration procedure below.

1. **The probability of being detected, *p*.** We calibrate this parameter by estimating the unconditional mean of the ratio of number of inspected firms to their total number.[xxx] For Italy, as well as for the majority of countries, only a portion of these data are publicly available. For the Italian economy, the Ministry of Labor reports that the number of inspected firms has been 118,119 in 2000, 106,307 in 1999 and 95,676 in 1998. The overall number of firms in the Italian Economy has been 4,639,393 in 2000, 4,472,375 in 1999 and 4,311,369 in 1998. As suggested above, we first compute the probability of being detected in each year, p^*_t, and then we estimate the aggregate probability as $p = \frac{1}{T}\sum_{t=1}^{T} p_t^*$. For the Italian economy $p = 0.03$.[xxxi] Even though this is not an efficient estimate, it represents the best possible calibration for this parameter, given the available data.
2. **The surcharge factor s, the income tax rate *t*, and the corporate tax rate τ**. The parameter s represents the surcharge on the standard tax rate that a firm, detected employing workers in underground sector, must pay. According to the Italian Tax Law (Legislative Decree 471/97, Section 13, paragraph l) the surcharge equals 30 percent of the statutory tax rate if the firm pays the fine when detected, or 200 percent when the firm refuses to pay.[xxxii] We present results for both the values, s = 1.3 and s = 2.00.

[xxx] Note that an inspected firm is not necessarily convicted of evasion and therefore fined. Since inspections are based either on private information of Institutions, or randomly, it may happen that behavior of a perfectly honest firm will be inspected.

[xxxi] These data are available on line at the web site of the Italian Ministry of Labor, at the URL *http://www.minlavoro.it/Personale/div7-conferenzastampa..Ol032001.htm.*

[xxxii] In this case the firm will prosecuted under Criminal Law perspective, and if condemned pay 200 percent.

In Italy, corporations are subject to a progressive tax rate. A tax rate of 19 percent is applied to the share of profits that represents 7 percent of the firm's capitalization; the remaining portion is then subjected to an increased tax rate of 36 percent. We calibrate the steady state value of the corporate tax rate as the average of these two numbers, i.e. $t = 0.275$.

The personal income tax system is more complex, since we have five tax rates, spanning from 18.5 percent to 45.5 percent. The calibration of the income tax rate may be undertaken in two ways.[xxxiii] It may be estimated as the average tax rate, weighted by the relative share of population in each income class. It may also be estimated as the tax rate associated with the average income of the working population (Adults 15-64 years old). We rely on the second procedure and since the average income equals 18,246 Euros we estimate the income tax rate at 33.5 percent.

3. **The share of underground sector**, $1 - \theta$. To calibrate this parameter we refer to Schneider and Enste (2000) who estimate the share of the underground sector for a panel of OECD countries. The value for the Italian Economy, $1 - \theta = 0.30$, is also consistent with Mare's (1996) estimates.
4. **The preference parameters, q and β, the capital share, α, and the capital depreciation rate δ**. These parameters are set to values commonplace in this literature (e.g. Fiorito and Kollintzas, 1994, or Busato and Chiarini 2004). More precisely, we set $q = 1$, $\beta = 0.98$ and $\delta = 0.025$.
5. **Stochastic Shocks autocorrelation coefficients, $\rho_m, \rho_u, \rho_t, \rho_\tau$ and innovation amplitudes, $\sigma_m, \sigma_u, \sigma_t, \sigma_\tau$.** The ρ ' s are set to .90 and the σ's to 0.003. As we stress in Busato and Chiarini (2004) these values are much lower than the classical ones (see King and Rebelo, 1999). This means that the model has a particularly efficient amplification mechanism which allows us to employ very small shocks.
6. **The utility function parameters h, f, η and γ.** The calibration of these parameters is a not easy (see Cho and Cooley, 1994). We select them to match four moments: the ratio between standard deviation of total output $\sigma (Y_t^{tot})$, and the standard deviation of total consumption, $\sigma (C_t^{tot})$, the correlation between total output and total consumption $\rho(C_t^{tot}, Y_t^{tot})$, the correlation between underground income and total consumption $\rho(C_t^{tot}, y^l_t)$ and the correlation between regular income and total consumption $\rho(C_t^{tot},y^k_t)$. The

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[xxxiii] More precisely, the structure of the tax rates is the following as of 2001. For incomes less than 10,331 Euros tax rate is 18.5 percent, for incomes between 10,331 Euros and 15,496 Euros tax rate is 25.5 percent, for incomes between 15,496 Euros and 30,992 Euros tax rate is 33.5 percent, for incomes between 30,992 Euros and 63,283 Euros tax rate is 39.5 percent and, eventually, for incomes above 63,283 Euros tax rate is 45.5 percent.

calibrated values are $h = 0.55$, $f = 1.99$, $\eta = 1.40$, $\gamma = 3.00$.[xxxiv]

5 Simulation Results.

This section shows that an equilibrium growth model, augmented with an underground sector and an income smoothing contract operating within the households, generates consumption and income series consistent both with smoothness properties of actual data. Specifically, it is here shown that the "*individual*", "*pre-contract*", or *disaggregated series* (c^k_t, y^k_t, c^l_t and y^l_t) are sufficiently volatile as in the actual data, and that the "household", "after-contract" or aggregated consumption and income profiles (C^{tot}_t and Y_t^{tot}) satisfy smoothness properties presented by actual data.[xxxv] Note, however, that "pre-contract" series do not arise as actual income and consumption profiles, because household smooth pre-contract income and consumption series on a period by period basis.[xxxvi] For this reason, these series have been generated by numerically simulating the model.

Comparing the stochastic properties of these two sets of variables, we can explicitly capture the reallocation mechanism of consumption and income between agents and between sectors, which is usually implicit in data collection, and in previous consumption studies. We argue that this is the driving force that ties our results to the countercyclicality of underground activities.

5.1 Numerical Results.

We present our results from four different perspectives. First, we show that c^k_t, and c^l_t exhibits a volatility greater than or equal to y^k_t and y^l_t respectively.[xxxvii] Moreover, the former variables exhibit larger impact-response after innovations in productivity and tax rates. Second, the previous relationship is reversed when looking at aggregated series: C^{tot}_t is smoother and less sensitive to innovations than Y^{tot}_t. Third, we compare volatility, sensitivity to innovations, and correlation among the three consumption definitions, and among the three income components generated in our model.[xxxviii] Fourth, we analyze correlations between consumption and income series to draw additional evidence on the volatility of disaggregated components, and on the

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[xxxiv] Cho and Cooley (1994) calibrate these parameters for the United States, and choose $h = 6.0$, $f = 0.87$, $\eta = 0.62$, $\gamma = 2.00$. Note, however, that their formulation of the model addresses issues different from matching market and underground moments. Specifically, they study the implications of this kind of utility function for the volatility of hours, employment and productivity in the United States.

[xxxv] To precisely describe the series, we denote y^k_t, as "pre-contract" or "market income" earned from by Mr. *k,* from his job in the market sector, before pooling it together with Miss *l* "after-contract" or "underground income", y^l_t. Then c^k_t and c^l_t represent Mr. k and Mrs, *l,* pre-contract individual consumption profiles, respectively. Finally, Y^{tot}_t and C^{tot}_t define total income and total, after contract, consumption.

[xxxvi] Indeed, from consumption and incombe smoothing, Definition 1, point 2, it turns out that $c^k_t = c^l_t = 1/2\ C_t^{tot}$. See also Appendix B.

[xxxvii] Where volatility is measured as the standard deviation of the Hodrick-Prescott filtered series.

[xxxviii] What we do here differs from the first point. Here we compare time series properties among c^k_t, c^l_t, C^{tot}_t, and among y^k_t, y^l_t and Y^{tot}_t. In the first point, instead, we compare c^k_t with y^k_t and c^l_t with y^l_t.

smoothness of their aggregate counterparts.

Table 1 (see section 6.1.1) and Table 2 (see section 6.1.3), and Figures 1 to 5 present the main results.

5.1.1 Regular consumption is more volatile than regular income, and...

Figure 1 presents impulse response functions, and Table 1 presents selected time-series properties

(Figure 1 about here)

Figure 1 shows the first 32 quarter response of y^k_t, c^k_t and X^{tot}_t to a one standard deviation innovation in regular-sector productivity, underground sector productivity, corporate and income tax rates. The curves are the quarterly percentage deviations from a baseline scenario where all innovations are set to zero. As the four figures show, the response of regular consumption series is larger or equal than that of regular income component.

Also Table l suggests that "individual" series (c^k_t, y^k_t, c^l_t and y^l_t) respect the empirical evidence. The table shows that $\sigma_{c^k_t} > \sigma_{y^k_t}$ and that $\sigma_{c^l_t} = \sigma_{y^l_t}$. Precisely, $\sigma_{c^k_t} = 2.96$, $\sigma_{y^k_t} = 2.07$, and $\sigma_{c^l_t} = \sigma_{y^l_t} = 2.22$. It should be also noted that income components are quite volatile, consistently with the data.[xxxix] Finally, Table 3 (see Appendix C) shows that this result is robust to sensitivity analysis.

TABLE 1: STANDARD DEVIATIONS

	$\sigma (y^k_t)$	$\sigma (y^l_t)$	$\sigma (Y^{tot}_t)$	$\sigma (c^k_t)$	$\sigma (c^l_t)$	$\sigma (C^{tot}_t)$
Actual Data	2.27	1.11	**1.44**	-	-	**1.25**
Simul. Data (s=1.3)	2.07 (0.22)	2.22 (0.23)	**1.45** (0.15)	2.96 (0.28)	2.22 (0.23)	**1.17** (0.14)
Sim. Data (s=2.0)	1.99 (0.24)	1.94 (0.25)	**1.40** (0.14)	2.71 (0.29)	1.94 (0.25)	**1.14** (0.17)

Notes: The model is calibrated for Italian economy within the sample 1970-1996. C^{tot}_t represent the consumption of non durable goods and services, c^k_t and c^l_t represent the regular and underground component of consumption, respectively. Y^{tot}_t is the aggregate GDP, y^l_t is its underground component. Since regular and underground consumption data are not available, no statistics are available. The statistics are the means of 1000 simulations, of length 150 time periods. Each simulated series is detrended using Hodrick-Prescott filter before the statistics are calculated. The numbers in brackets are the small sample standard deviations. *Sources:* C^{tot}_t and Y^{tot}_t are withdrawn from Istat database, y^k_t and y^l_t

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[xxxix] As stressed in many contributions (e.g. Deaton 1992, Attanasio 1999, Attanasio and Rios-Rull 2000) both income and consumption are quite volatile, even though consumption smoothing is strong evidence across countries and data-sets.

are from Bovi (1999), while c^k_t and c^l_t are generated with our model.

5.1.2 .Aggregate consumption is less volatile than aggregate income

The analysis of aggregate variables presents a completely reversed picture.

(Figure 2 about here)

Figure 2 shows the first 32 quarter response of Y_t^{tot}, C^{tot}_t and X^{tot}_t to a one standard deviation innovation in regular-sector productivity, underground sector productivity, corporate and income tax rates. Notice that impulse response of aggregate consumption is smaller than or equal to that of aggregate income.

A further interesting result concerns volatility measures for both aggregate series, $\sigma(C^{tot}_t)$ and $\sigma(Y^{tot}_t)$ (see Table l in Section 6.1.1). Note how aggregate consumption and aggregate income are less volatile than disaggregated counterparts, and, more importantly, that the former is smoother than the latter. Precisely, $\sigma(Y^{tot}_t)$ = 1.45 and $\sigma(C^{tot}_t) = 1.17$. Moreover, it is important to stress that we generate all these results with a low risk aversion coefficient, $q = 1$, consistent with empirical micro-studies (e.g. Dreze 19XX or Attanasio, 1999.).

These results are consistent with the widespread empirical evidence that aggregate consumption is smoother than aggregate income, which is one of the most robust empirical evidences matched by equilibrium growth models.

5.1.3 Smoothing and correlations.

Table 2 presents correlations for consumption and income series, at a disaggregated and aggregated level.

It is worth to notice that the correlation between aggregate consumption and output, $\rho(C^{tot}_t, Y^{tot}_t) = 0.69$, decomposes into correlations between total output and the two disaggregated consumption components, $\rho(c^k_t, Y^{tot}_t) = 0.95$ and $\rho(c^l_t, Y^{tot}_t) = -0.96$, respectively. Total and regular consumption are both pro-cyclical, but the former presents a weaker (positive) correlation with aggregate income.[xl] In the logic of our model, this comes from the fact that consumers allocate a share of income to underground consumption, which is countercyclical. Since we calibrate the weight of regular sector (0.725) to be larger than that of underground sector (0.275), total consumption ends up being pro-cyclical.

Second, Proposition 2 shows that a sufficient condition for aggregate consumption smoothing, is that correlation between regular and underground consumption, $\rho(c^k_t, c^l_t)$ should be smaller, in absolute value, than correlation between regular and underground output, $\rho(y^k_t, y^l_t)$.[xli]

[xl] Pro-cyclicality and Counter-cyclicality are defined in this contest with respect to total income, Y^{tot}_t.

[xli] The proof is trivial, but for completeness we present it in Appendix A.

TABLE 2: CORRELATIONS

	y^k_t	y^l_t	Y^{tot}_t	c^k_t	c^l_t	C^{tot}_t
y^k_t	1.00	-0.81	0.89	-	-	**0.77**
y^l_t		1.00	-0.45	-	-	**-0.54**
Y^{tot}_t			1.00	-	-	**0.80**
y^k_t	1.00	-0.98	0.95	0.95	-0.97	**0.70**
		(0.01)	(0.01)	(0.02)	(0.01)	(0.11)
y^l_t		1.00	-0.96	-0.94	1.00	**-0.51**
			(0.01)	(0.02)	-	(0.12)
Y^{tot}_t			1.00	0.95	-0.96	**0.69**
				(0.02)	(0.01)	(0.11)
c^k_t				1.00	-0.91	0.75
					(0.02)	(0.10)
c^l_t					1.00	0.42
						(0.13)
C^{tot}_t						
y^k_t	1.00	-0.97	0.99	0.96	-0.97	**0.68**
		(0.01)	(0.01)	(0.02)	(0.01)	(0.11)
y^l_t		1.00	-0.97	-0.95	1.00	**-0.61**
			(0.01)	(0.02)	-	(0.12)
Y^{tot}_t			1.00	0.96	-0.97	**0.69**
				(0.02)	(0.01)	(0.11)
c^k_t				1.00	-0.91	0.72
					(0.02)	(0.10)
c^l_t					1.00	-0.39
						(0.13)
C^{tot}_t						1.00

Notes: The first block of the table contains the correlations estimated for the actual data; the second blocks presents the correlations estimated on the simulated data. The first one refers to the case $s = 1.3$, the second to the case $s = 2.0$. The moments matched in the calibration of utility function parameters are presented in boldface. See Table 1 notes.

Proposition 2 (Smoothing and Correlation) *Aggregate consumption smoothing requires the correlation between regular and underground consumption,* $\left|\rho\left(c^k_t, c^l_t\right)\right|$ *to be smaller, in absolute value, than correlation between regular and underground production,* $\left|\rho\left(y^k_t, y^l_t\right)\right|$.

Proof. See Appendix A .

Table 2 shows how the model matches this restriction, since $\left|\rho\left(c^k_t, c^l_t\right)\right| = 0.91$ and $\left|\rho\left(y^k_t, y^l_t\right)\right| = 0.98$. In words, this means that regular consumption reacts less to innovations, than regular income does. Note that this argument parallels the key concept of the of risk sharing arguments discussed by Mace (1991) or Abel and Kotlikoff (1989). The difference is that in this case the insurance comes from income smoothing contract (i.e. a "real side" of the market) while in the works quoted above insurance originates from investing in financial securities.

5.1.4 Consumption and Income Smoothing: Inspecting the Composition

Here we complete the characterization of consumption and income smoothing, inspecting the composition mechanism operating before aggregation of consumption and income series. Specifically, we compare impulse response functions and volatility measures for all income definitions, Y^{tot}_t, y^k, y^l, and for all consumption components C_t, c^k, c^l.

(Figure 3 about here)

In Figure 3 the impact on the endogenous variables of a one-standard-deviation shock to regular sector productivity, directly increases capital investment, regular output and regular consumption. Note that total consumption rises only gradually. In particular, the household composed of Mr. and Mrs. *k* does not choose to adjust consumption completely after an innovation. This is due to inter-temporal substitution and wealth effects, as in a traditional Robinson Crusoe economy, but in addition we see the redistribution effect within the family, previously defined. Indeed, regular and underground consumption components move always in opposite directions, and the former is much more responsive than aggregate variables.[xlii]

Productivity shocks in the underground sector, and increases in tax rates, reverse the picture, yielding opposite effects. Now aggregate consumption and production are reduced, together with investment. In spite of the remarkable jumps in underground output and consumption, a rise in income and corporate taxes reduces regular consumption, total consumption and investment, thereby impoverishing the economy and causing a recession.

Notice that these patters are consistent with a traditional equilibrium growth model (e.g. Christiano, 1989 or King and Rebelo, 1999). The new insight of our approach consists in the opportunity to understand the composition of aggregate in terms of disaggregated variables. Concluding, the most interesting results we observe from the four panels of Figure 3 is that c^k_t responds to innovations more than C^{tot}_t does, and, in absolute terms, also more than c^l_t. In addition, total consumption presents a highly persistent response after the shocks.

(Figure 4 about here)

Impulse response functions of production series (Figure 4) display an analogous picture,

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[xlii] We are suggesting that market and underground consumption profiles, which are defined both as a precontract series, are highly negatively correlated. Table 2 (see section 6.1.3) shows that correlation between market and underground consumption equals -0.91. To see more clearly forces' interaction, consider the following example. Suppose we have a positive productivity innovation in market sector: market income and labor demand increase, and, since we know the two sectors have negative correlation, underground income and labor demand fall. Then the family reallocates its labor supply to the more productive sector, subtracting it from the less productive. Since labor supply cannot be traded (e.g. we cannot short sell H hours worked in one the sector), consumption would follow approximately income dynamic. Chiarini and Marzano (2006) examine the relationship between market and underground consumption profiles in Italy using econometric techniques in a partial equilibrium framework. Their empirical estimates suggest that the two forms of consumption are complements, so implying that when underground consumption rises, marginal utility of the market consumption rises too. This finding do not contrast with the evidence shown in this paper. In fact, given the negative correlation reported in Table 2, a rise in underground consumption reduces market consumption, rising its marginal utility.

where y^k_t and y^l_t are always negative correlated, and y^k_t is more sensitive to innovation than Y^{tot}_t and y^l_t. These results are robust, consistent with time series behavior of income and consumption, and support volatility and correlation measures already presented.

These results are confirmed by the graphical inspection of Hodrick-Prescott filtered series for consumption and income (Figure 5). The model generates pro-cyclical regular consumption movements, which are positively correlated with regular output. Underground consumption, instead, is countercyclical with respect to total consumption, and to regular consumption. Note that total consumption is always in between its regular and underground component. Analogous comments hold for production series.

(Figure 5 Panel A and Panel B about here)

6 Conclusions.

The purpose of this paper is to provide an original interpretation for consumption and income smoothing. The underground economy produces a *second cycle* which gives to households the opportunity to ensure themselves against bad times, by entering an *income smoothing contract*. Specifically, workers belonging to same extended family can insure themselves against fluctuations in regular and underground income, by entering the income smoothing contract. Specifically, each consumer-worker-investor can smooth aggregate income, even though disaggregated income and consumption components are more volatile by relying on this risk sharing mechanism. The introduction of the underground sector makes expansions less bright, and recessions less dark.

For a given institutional and productive structure, firms smooth production across sectors, and households smooth consumption inter-temporally and across sector. By this end, agents diversify both economic activities and labor input between sectors and (partially) protect the consumption flow from income volatility.

Our income smoothing contract represents, however, one out of many different ways to model income smoothing. We think that the economic literature would benefit from a more thorough investigation of these composition issues, because they are often ignored in the conventional consumption studies as well as in the dynamic equilibrium model literature.

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Appendix A: Proofs of Propositions in the Text

Proof of Proposition 1 (Representative Agent and Extended Family).

Lemma 1 (Households and Extended Families) *The consumer side of the heterogeneous agent model is represented as the following three equations:*

$$U\left(c_t^k, c_t^k, l_t^k, l_t^l\right) = \left(\tfrac{1}{2}\right)u\left(c_t^k\right) + \left(\tfrac{1}{2}\right)u\left(c_t^l\right) - v\left(l_t^k\right)l_t^l - \mu\left(l_t^l\right) \qquad (A.1)$$

$$w_t\left(1-\tau_t\right)l_t^k + w_t l_t^l + R_t K_t^{tot} = C_t^{tot} + X_t^{tot} \qquad (A.2)$$

$$K_{t+1}^{tot} = \left(1-\delta\right)K_t^{tot} + X_t^{tot} \qquad (A.3)$$

Then consumption and income smoothing contract (Definition 1) dictates following two conditions:

1. ***Income Pooling****:* $l_t^k = \theta L_t$ *and* $l_t^l = \left(1-\theta\right)L_t$.
2. ***Consumption Pooling****:* $c_t^l = c_t^k = \left(\tfrac{1}{2}\right)c_t^j$ *(see Appendix B).*

Now, normalizing L_t *to unity and implementing these features into (A.1)-(A.3), we rewrite them as:*

$$U\left(c_t^j, \theta_t^j\right) = u\left(\left(\tfrac{1}{2}\right)c_t^j\right) - v\left(\theta_t^j\right)\left(1-\theta_t^j\right) - \mu\left(1-\theta_t^j\right) \qquad (A.4)$$

$$w_t\left(1-\theta_t^j\tau_t\right) + R_t k_t^j = c_t^j + X_t^j \qquad (A.5)$$

$$K_{t+1}^{tot} = \left(1-\delta\right)K_t^{tot} + X_t^{tot} \qquad (A.6)$$

Specifying functional forms $v\left(\theta_t^j\right)\left(1-\theta_t^j\right) = h\dfrac{\left(\theta_t^j\right)^{1+\gamma}}{1+\gamma}\left(1-\theta_t^j\right)$ *and* $\mu\left(1-\theta_t^j\right) = f\dfrac{\left(\theta_t^j\right)^{1+\eta}}{1+\eta}$ *into Equation (A.4) we derive equation 4 in the text. Notice (A.3)* $\equiv$ *(A.6).*

$$u\left(c_t^j;\theta_t^j\right) = \left\{\frac{\left(c_t^j\right)^{1-q}-1}{1-q} - h\frac{\left(\theta_t^j\right)^{1+\gamma}}{1+\gamma}\left(1-\theta_t^j\right) - f\frac{\left(1-\theta_t^j\right)^{1+\eta}}{1+\eta}\right\} \qquad \text{(A.7)}$$

Concluding, after introduction of consumption and income smoothing contract, the consumers' side of the model is represented by equations (A.5), (A.6), (A.7).

Lemma 2 (Firms) *Firms are characterized by a production function, and a cost function.*

$$Y_{mt}^i = M_t\left(K_t^i\right)^{\alpha}\left(L_{mt}^i\right)^{1-\alpha} \text{ and } Y_{ut}^i = Z_t L_{ut}^i \qquad (A.8)$$

$$w_t\left(1+t_t\right)L_{mt}^i + w_t L_{ut}^i + r_t K_t^i \qquad (A.9)$$

Now, implementing consumption and income smoothing contract, we rewrite (A8) and (A9) as:

$$Y_{mt}^i = M_t\left(K_t^i\right)^{\alpha}\left(\theta_t^i\right)^{1-\alpha} \text{ and } Y_{ut}^i = Z_t\left(1-\theta_t^i\right) \qquad (A.10)$$

$$CO(\theta_t^i, K_t^i) = w_t + w_t t_t \theta_t^j + r_t K_t^i \qquad (A.11)$$

To derive (A.11), which equals eqation (9) in the text, just simplify the following: $w_t\left(1+t_t\right)\theta_t^j + w_t\left(1-\theta_t^j\right) + r_t K_t^i$. *Hence firms' problem is represented by equations (A.10) and (A.11).*

Lemma 3 (Government) *Government budget constraint is:*

$$w_t\tau_t l_t^k + \left(pst_t\right)Y_{ut} + t_t Y_{mt} = G_t$$

Implementing consumption and income smoothing, it becomes:

$$w_t\tau_t\theta_t + \left(pst_t\right)Y_{ut} + t_t Y_{mt} = G_t \qquad (A.12)$$

which is equation (10) in the text.

Finally, the decentralized model we study in this paper is represented by equations *(A.5), (A.6), (A.7)* for j-th household, *(A.lO), (A.lI)* for i-th firm, and *(A.12)* for government.

The solution method used to solve this artificial economy is that suggested by King, Plosser and Rebelo (1988a,b). To this end we transform the equilibrium characterization of the economy into an approximating first order autoregressive linear system, applying linear approximations (e.g. Campbell 1994; Uhlig 1999).

Proof of Proposition 2 (Smoothing and Correlations). Assume $\sigma^2(c^l_t) = \sigma^2(y^l_t)$ (Assumption 1), and $\sigma^2(c^k_t) > \sigma^2(y^k_t)$ (Assumption 2).[xliii] Let $C^{tot}_t = c^k_t + c^l_t$, and let $Y^{tot}_t = y^k_t + y^l_t$. Then $\sigma^2(C^{tot}_t) = \sigma^2(c^k_t) + \sigma^2(c^l_t) + 2\,\sigma(c^k_t, c^l_t)$ and $\sigma^2(Y^{tot}_t) = \sigma^2(y^k_t) + \sigma^2(y^l_t) + 2\,\sigma(y^k_t, y^l_t)$, where $\sigma^2(x)$ represents the variance of x, and $\sigma(x, y)$ represents the covariance between x and y. Aggregate consumption smoothing implies $\sigma^2(C^{tot}_t) < \sigma^2(Y^{tot}_t)$, or, equivalently $\sigma^2(c^k_t) + \sigma^2(c^l_t) + 2\,\sigma(c^k_t, c^l_t) < \sigma^2(y^k_t) + \sigma^2(y^l_t) + 2\,\sigma(y^k_t, y^l_t)$. Rewrite $\sigma(c^k_t, c^l_t)$ as $\rho(c^k_t, c^l_t)\sigma(c^k_t)\,\sigma(c^l_t)$ and $\sigma(y^k_t, y^l_t)$ as $\rho(y^k_t, y^l_t)\sigma(y^k_t)\,\sigma(y^l_t)$ where $\rho(x, y)$ stands for the correlation between x and y, and $\sigma(x)$ is the standard deviation of x. Therefore: $\sigma^2(c^k_t) + \sigma^2(c^l_t) + 2\,\rho(c^k_t, c^l_t)\sigma(c^k_t)\,\sigma(c^l_t) < \sigma^2(y^k_t) + \sigma^2(y^l_t) + 2\,\rho(y^k_t, y^l_t)\sigma(y^k_t)\,\sigma(y^l_t)$. By construction we have $\sigma^2(c^l_t) = \sigma^2(y^l_t)$ and obviously $\sigma(c^l_t) = \sigma(y^l_t)$. Since $\sigma^2(c^k_t) > \sigma^2(y^k_t)$ and obviously $\sigma(c^k_t) > \sigma(y^k_t)$, consumption smoothing now implies that $\rho(c^k_t, c^l_t)\sigma(c^k_t) < \rho(y^k_t, y^l_t)\sigma(y^k_t)$, or equivalently $\frac{\left|\rho\left(c^k_t; c^l_t\right)\right|}{\left|\rho\left(y^k_t; y^l_t\right)\right|} < \frac{\sigma\left(y^k_t\right)}{\sigma\left(c^k_t\right)} < 1$. Therefore $\frac{\left|\rho\left(c^k_t; c^l_t\right)\right|}{\left|\rho\left(y^k_t; y^l_t\right)\right|} < 1$ or $\left|\rho\left(c^k_t; c^l_t\right)\right| < \left|\rho\left(y^k_t; y^l_t\right)\right|$.

Remark 2 Notice that since $y^l_t = c^l_t$, we may rewrite Proposition 2 statement as follows $\left|\rho\left(c^k_t; y^l_t\right)\right| < \left|\rho\left(y^k_t; y^l_t\right)\right|$ or analogously as $\left|\rho\left(c^k_t; c^l_t\right)\right| < \left|\rho\left(y^k_t; c^l_t\right)\right|$.

Appendix B: Perfect Risk Sharing Scheme

QUESTA APPENDICE MI è POCO CHIARA, $C_{k,t}$ indica il consumo dell'agente k? Allora dovrebbe essere c^k_t.

Cosa rappresenta u'_k?

C^j per coerenza con il testo precedente dovrebbe essere minuscolo, vedi proposition 1 e eq. 6.1-2.

After entering the contract, consumers agree on a perfect risk sharing scheme, in the sense that they set ratio between marginal utilities equal to a constant, i.e.

$$\frac{u'_k\left(C_{k,t}\right)}{u'_l\left(C_{l,t}\right)} = \frac{\varphi_k}{\varphi_l}$$

Since $u'_k\left(c^k_t\right) = u'_l\left(c^k_t\right) = u'\left(C_t\right)$ we have

$$c^k_t = \frac{\varphi_k}{\varphi_l} c^l_t .$$

Assuming that both consumers have the same importance, we can set $\varphi_k = \varphi_l$ and therefore $c^k_t = c^l_t$. The two consumers will have an equal consumption profile. In terms of total consumption, we have $c^k_t = c^l_t = 1/2\, c^j_t$, where c^j_t represents consumption chosen by j-th household at time t.

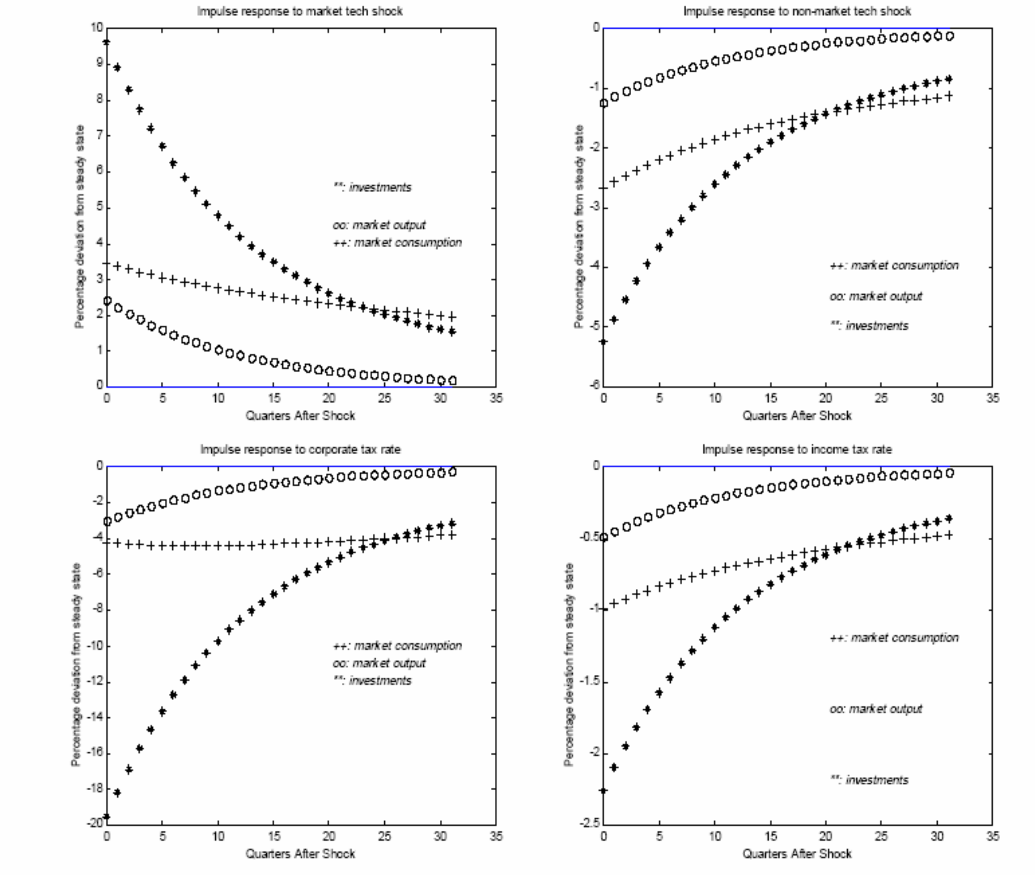
[xliii] Both assumptions are derived from empirical evidences, robust across countries and data sets, and matched by our model, see Deaton (1992), Attanasio (1999), Schneider and Enste (2000).

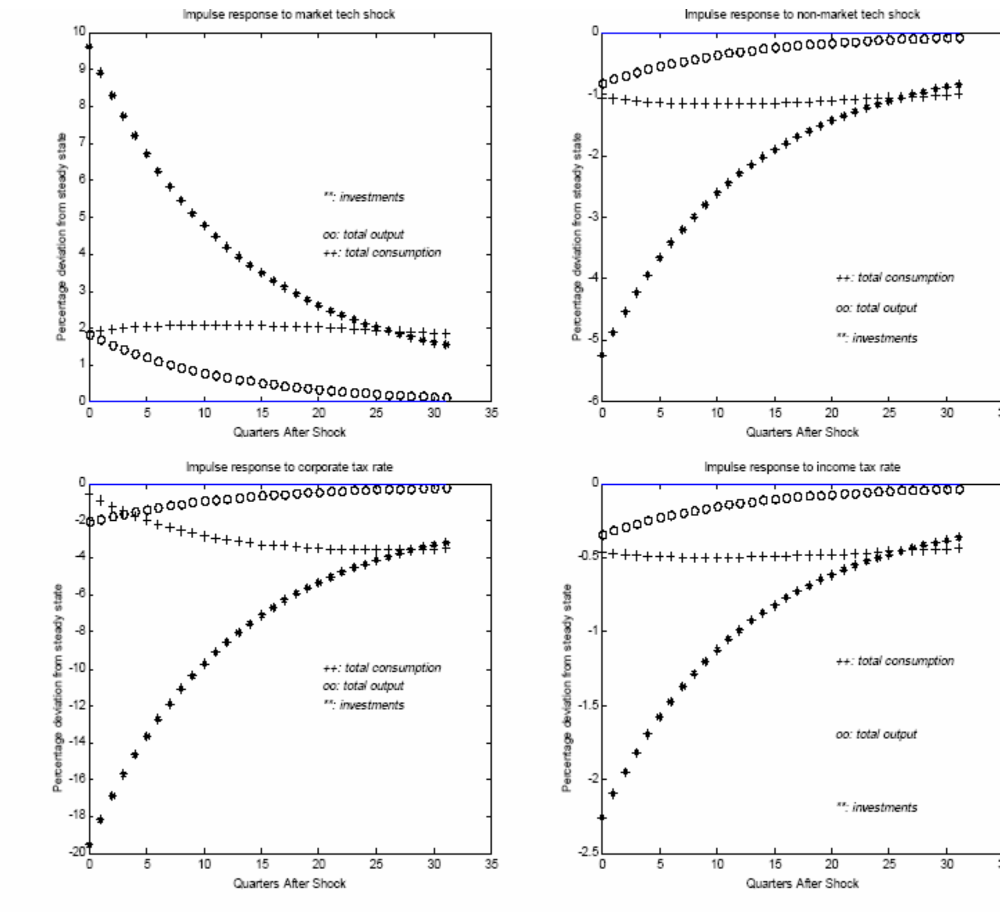
Appendix C: Sensitivity Analysis

	$\sigma_{Y_{tot}}$	$\sigma_{C_{tot}}$	$\sigma_{C_{tot}} / \sigma_{Y_{tot}}$	$\rho(Y_{tot}, C_u)$	$\rho(Y_{tot}, C_{tot})$
$h = 0.35$	1.05	0.96	0.91	-0.97	0.45
$h = 0.38$	1.02	0.91	0.90	-0.93	0.32
$h = 0.44$	1.27	0.97	0.76	-0.94	0.68
$h = 0.46$	1.32	1.20	0.91	-0.98	0.77
$h = 0.48$	2.03	1.86	0.92	-0.98	0.71
$\eta = 1.25$	1.67	1.30	0.78	-0.98	0.71
$\eta = 1.27$	1.34	1.21	0.90	-0.95	0.74
$\eta = 1.29$	1.1	0.90	0.82	-0.94	0.54
$\eta = 1.31$	1.13	1.11	0.98	-0.93	0.58
$\eta = 1.33$	1.35	1.25	0.92	-0.95	0.57
$\eta = 1.40$	1.26	1.10	0.87	-0.95	0.17
$f = 1.90$	1.36	1.18	0.86	-0.94	0.45
$f = 1.92$	1.41	1.19	0.84	-0.94	0.52
$f = 1.94$	1.25	1.13	0.90	-0.96	0.55
$f = 1.96$	1.25	1.04	0.83	-0.95	0.71
$f = 1.98$	1.49	1.36	0.90	-0.96	0.73
$f = 2.00$	1.10	1.05	0.95	-0.98	0.70

Notes: $\sigma_{Y_{tot}}$ represents the total production standard deviation, $\sigma_{C_{tot}}$ is total consumption standard deviation, $\rho(Y_{tot}, C_u)$ is the correlation coefficient between total production and underground consumption, and $\rho(Y_{tot}, C_{tot})$ is the correlation coefficient between total production and total consumption.

Appendix D: Figures

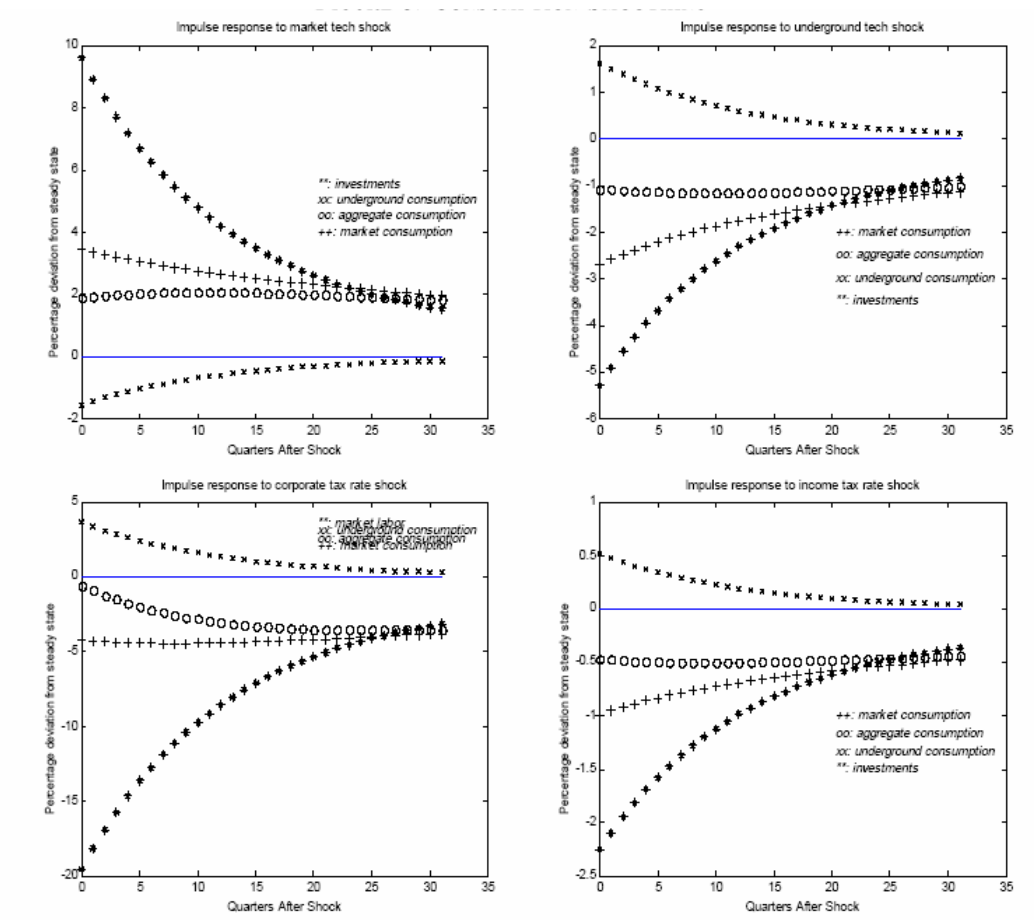



Impulse response to market tech shock

Impulse response to non-market tech shock

Impulse response to corporate tax rate

Impulse response to income tax rate


Impulse response to market tech shock
**: investments
xx: underground consumption
oo: aggregate consumption
++: market consumption
Percentage deviation from steady state
10
8
6
4
2
0
-2
0
5
10
15
20
25
30
35
Quarters After Shock
Impulse response to underground tech shock
++: market consumption
oo: aggregate consumption
xx: underground consumption
**: investments
Percentage deviation from steady state
2
1
0
-1
-2
-3
-4
-5
-6
0
5
10
15
20
25
30
35
Quarters After Shock
Impulse response to corporate tax rate shock
**: market labor
xx: underground consumption
oo: aggregate consumption
++: market consumption
Percentage deviation from steady state
5
0
-5
-10
-15
-20
0
5
10
15
20
25
30
35
Quarters After Shock
Impulse response to income tax rate shock
++: market consumption
oo: aggregate consumption
xx: underground consumption
**: investments
Percentage deviation from steady state
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
5
10
15
20
25
30
35
Quarters After Shock

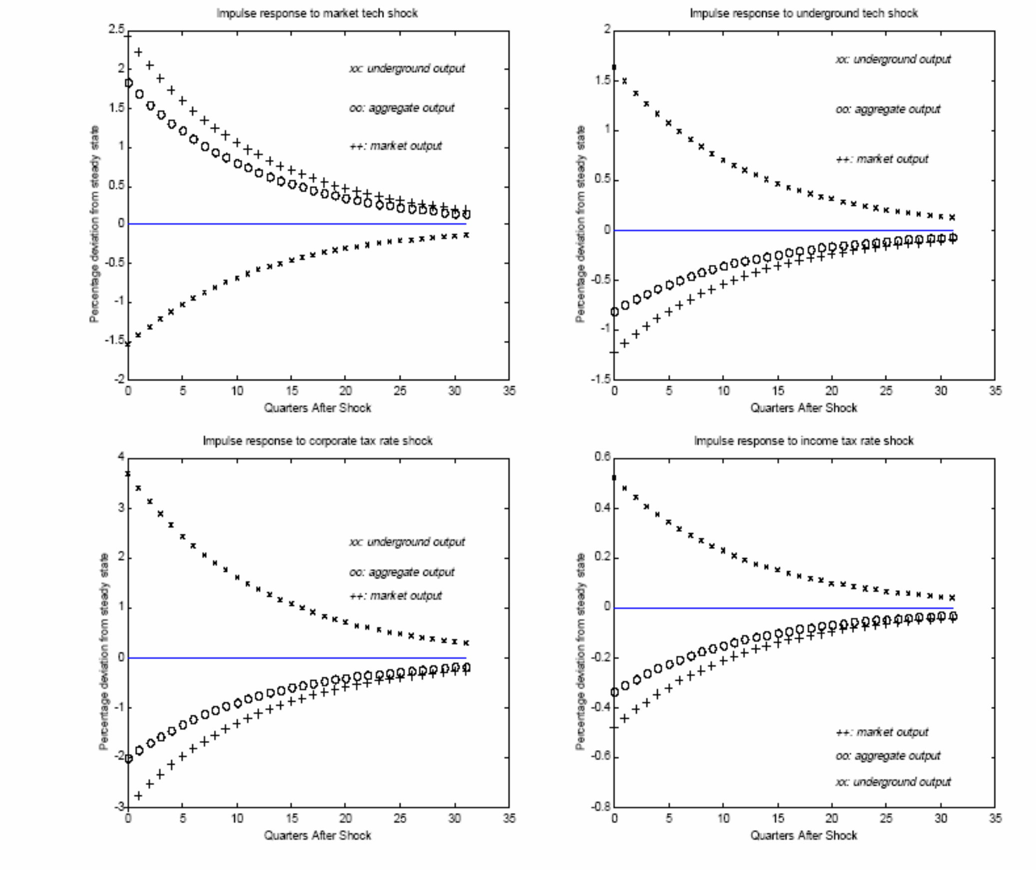
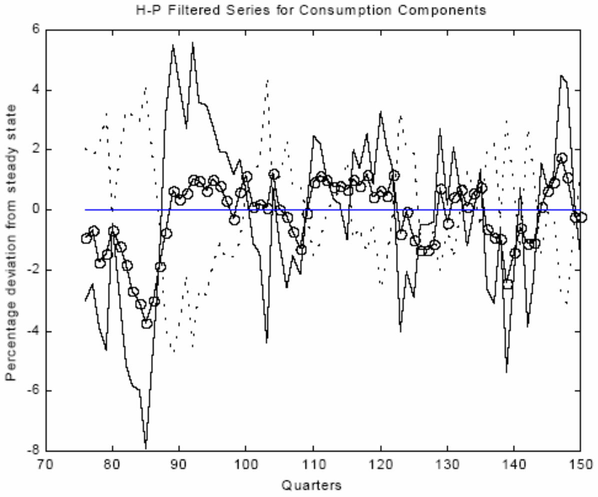
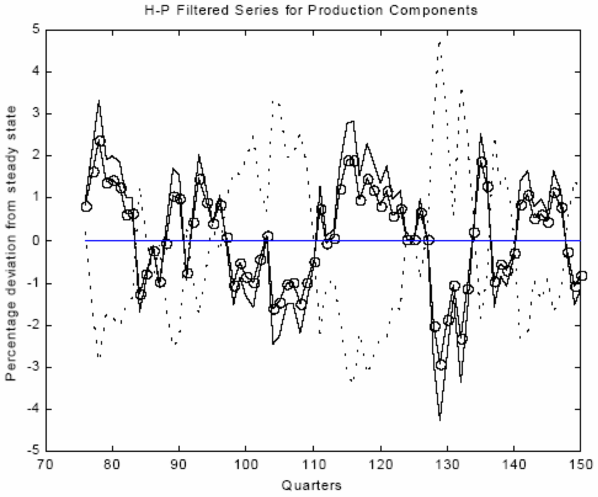

Impulse response to market tech shock
xx: underground output
oo: aggregate output
++: market output
Percentage deviation from steady state
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
5
10
15
20
25
30
35
Quarters After Shock
Impulse response to underground tech shock
xx: underground output
oo: aggregate output
++: market output
Percentage deviation from steady state
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
5
10
15
20
25
30
35
Quarters After Shock
Impulse response to corporate tax rate shock
xx: underground output
oo: aggregate output
++: market output
Percentage deviation from steady state
4
3
2
1
0
-1
-2
-3
0
5
10
15
20
25
30
35
Quarters After Shock
Impulse response to income tax rate shock
++: market output
oo: aggregate output
xx: underground output
Percentage deviation from steady state
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
5
10
15
20
25
30
35
Quarters After Shock

FIGURE 5: COMPOSITION MECHANISM FOR
CONSUMPTION AND PRODUCTION COMPONENTS



Panel A



Panel B

Figure 5: *Panel A*: solid line represents market production component y_t^{κ}, dashed line represent non-market production component, y_t^{l}, and the starred line the aggregate production component, Y_t^{tot}. *Panel B*: solid line represents market consumption component c_t^{κ}, dashed line represent non-market consumption component, c_t^{l}, and the starred line the aggregate consumption component, C_t^{tot}.