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Scientific research, externalities and economic growth

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1. Introduction

Study of the causes of economic growth since the industrial revolution has highlighted the importance of technological development. This interpretation of long-period growth has come to the fore in the applied literature, and recently also in the theoretical literature which reprises Schumpeter's theories of the first half of the 1900s. On closer inspection, however, this interpretation is incomplete because it fails to consider the origin of technological advancement, namely the progress of science. Historians and scholars of science, in fact, stress the concomitance between the appearance of important scientific discoveries and the transition from a period of slow productivity growth to that of exponential expansion which led up to the contemporary age.

The alliance between basic research, technology and growth has been particularly close and fruitful since the nineteenth century. Rosenberg and Birdzell (1986; 1990) argue that economic miracle of the Western world can be explained by the marked increase in science's ability to investigate the secrets of nature since the end of the 1800s. This greater efficiency of basic research was initially due to

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important changes in its organization and closer interaction with the rest of society and with the economy.

In this chapter we put forward an analytical approach to economic growth which tries to capture the essential features of the interaction between the work of the scientific community and long-period economic activity.

The traditional theory of growth, which originated with Solow (1956), considers the academic world to be exogenous with respect to the economy. As in the case of other public goods, the production of knowledge is the task of the state. Exceptions in this theoretical tradition are the works of Karl Shell (1969, 1970), in which the production of knowledge is endogenous. In this model, the state collects resources from the activities of private agents in order to finance basic research, which is the public input to the private sector. The economic problem analysed by Shell is essentially that of the dynamic allocation of resources between the production of goods and the production of knowledge.

Still largely unexplored in the economics literature is the scientific research sector in relation to its forms of organization and the incentives - economic and otherwise - which motivate those who work in it.

With the advent of ‘endogenous growth theory’ - the new scientific paradigm for the analysis of growth - innovation has become a central topic of inquiry. The works of Romer (1990), Aghion and Howitt (1992), and Grossman and Helpman (1991) have generated a rich Schumpeterian strand in growth theory which draws heavily on the microeconomic literature on industrial innovation in which innovative firms get a patent that prevents others from profiting from new knowledge. These models, too, relegate the production of new opportunities for technological progress to a residual domain exogenous to the economy. The case of growth models with general purpose technology is emblematic of the limitations of this approach. GPTs, in fact, are radical changes in technologies which improve production possibilities in a wide range of sectors. These changes should certainly be associated with scientific advances which alter the constraints to which technologies are subject, but there is no trace of this phenomenon in these models.

The model analysed in this paper represents the working of an economy which consists of agents who may choose to work either in the goods production sector or in scientific research. These two economic activities are organized according to different objectives and rules. Research is financed by the state out of taxes, and its output is a public good that benefits all firms and improves their productivity. Researchers are engaged in competitions with other researchers for a new discovery. The probability of a new finding by a researcher is a function of his/her effort, and his/her interactions with other researchers.

(Bisogna continuare con la descrizione)

Among the main results of the model are ….(Si dovrebbero inserire I risultati, ma quali sono?)

The paper is organized as follows. The second section surveys the theoretical literature on the relationship between science and economic growth. The third section sets out the basic theoretical model. The fourth section analyses the model's equilibrium solution.

2. The related literature

Arrow's 1962 essay laid the basis for the economic analysis of the production

of and the demand for knowledge; analysis that was subsequently developed with reference to technological innovation. In the very general terms of Arrow's analysis, the various forms that knowledge can assume are likened to information. According to Arrow, on the supply side, once knowledge has been produced it can be transmitted at a cost considerably lower than that necessary for its production. On the demand side, information has the characteristic of non-rivalry in its consumption, because its use by one individual does not reduce the quantity available for consumption by another individual. These two features of knowledge make it similar to a public good - all the more so the greater the degree of excludability in consumption, which cannot be perfect.

Arrow's article prompted Dasgupta and David (1987) to investigate the fundamental differences between the production of knowledge in the institutions of science and technology. This important essay laid the basis for the modern economic theory of science. The main differences between the worlds of science and technological innovation reside in their organization and the goals pursued. The fundamental difference between science and technology concerns the dissemination of results, which is immediate and complete in scientific research\ as academic researchers seek to publish their discoveries as soon as possible and obtain, through peer evaluation, recognition by the scientific community of the validity of their results. This is contrary to what happens in technological research where new knowledge is kept secret.

The scientific community on the one hand enjoys the advantage of complete information; on the other, it is concerned to ensure the researcher's property right on the item of new knowledge that s/he has produced. Because full disclosure is the optimal solution from the point of view of society's well-being, this social norm adopted in the scientific community serves that purpose. Obviously, full disclosure conflicts strongly with the secrecy necessary to be able to profit from technological innovation. Firms, in fact, obtain a return on investments in R\&D in relation to the degree of market power that a patent or the restricted circulation of an innovation may generate for them.

Radically different from this objective is the ‘quest for priority’ in attribution of the paternity of a discovery that motivates academic researchers. The latter immediately submit the results of their work for publication which will certify their priority in the discovery. From this derives recognition in monetary terms (career advancement, awards, etc.) and in terms of reputation and prestige in the scientific community.

The incentives system that operates in research is characterized by great uncertainty and by the principal’s difficulty of monitoring effort. The evolution of state-organized academic research seems to have struck a balance between the private motivations of researchers and the needs of society. Individual scientists take part in contests in which those who obtain an innovative result first receive recognition from the scientific community and the advantages that ensure therefrom. Because the work of those who do not win is valueless, the contest belongs to the category of tournaments in which the winner takes all (Dasgupta, 1989; Lazear, 1997).

Comparison with reality shows that this system efficiently incentivizes academic researchers, in that they are generally highly motivated and committed to their research. In effect, this result also derives from the assurance of an income, often from teaching duties, which mitigates the effects of the risk in research.

The rules of the academic world favour the spread of forms of collaboration and information-sharing which have important externalities. Work in academic departments is characterized by forms of knowledge sharing and ideas' discussion as seminars and mimeo circulation and also by several informal ways of externalities in everyday life interactions. The transmission of tacit knowledge takes place in academic departments whose composition is an important factor in the work of individual researchers. This relationship may also hold among researchers belonging to different institutions but who work in the same field and interact with each other to form 'invisible colleges' (David, 1998). Furthermore, scientific work is often carried out by teams of researchers, in that the advantages deriving from obtaining priority are generally indivisible, while the pooling of kindred and specialized skills considerably increases the chances of success (Stephan and Levin, 1992). Data on publications show that collaborations have increased over time.

Externalities in knowledge production have been analyzed by Carraro and Siniscalco (2001) in a model that concerns a race between academic researchers and researchers in private firms to a specific discovery with possible commercial use. This paper shows under what conditions the coexistence of Science and Technology institutions can be welfare maximizing.

Finally there are a number of papers that have tried to measure empirically the influence of scientific advances on technological innovation and on productivity of economic systems. The influence of scientific advances on technological innovation, and on the productivity of economic systems, has been the subject of applied inquiry for a number of years. The studies by Mansfield (1991, 1995) are based on surveys of firms' opinions on the importance of scientific advances for innovation in products and processes. The first study was based on a sample of 76 of the largest USA firms and found that in the period 1975-1985 around 11\% of new products and 9\% of new processes could not have been developed without the results of academic research conducted in the previous fifteen years.

An equally direct approach has been used by Adams (1990), who estimates the contribution of scientific knowledge to productivity growth in 18 manufacturing sectors. The main feature of this study is its meticulous construction of an indicator of the stock of scientific knowledge obtained by considering both the number of publications in scientific fields closest to the sector’s technology since the 1930s, and the scientific personnel employed in the sector.

Another strand of studies consider the spatial effects of research spillover on the innovative activities of firms. Among the most important of these studies is Jaffe (1989), which considers data on corporate patents in each state of the USA. The estimation of a model of simultaneous equations shows that there are important spillover values for academic research, especially in the cases of pharmaceuticals and chemicals industries.

3. The Economy

A class of growth models that can be used to represent the salient features of the science sector described in the previous sections comprises so-called neo-Schumpeterian models. Here we follow the framework of Aghion-Howitt (1992) in which there is no capital accumulation.

In our economy there is a continuum of individuals, of measure 1, who can find employment in one of two different sectors: one is a competitive consumption good sector, the other is the basic research sector which produces the body of knowledge used in the production process of the final good. Manufacturing firms are owned by all agents in the economy, and labour and capital markets are perfecly competitive. The state owns and organizes the science sector. Time is indexed by *t*, while the state of knowledge is indexed by *k*.

The consumption good, which acts as numeraire, is produced using the following technology:

$$Y_{kt} = R_k l_{kt}^{\alpha} Z^{1-\alpha} \qquad (1)$$

with $0 < \alpha < 1$, where l_{kt} denotes the number of specialised workers used at time *t*, R_k is a technological parameter which measures the productivity of the basic knowledge freely disposable in the technological era *k*, and *Z* is an input available with fixed supply, that in the following we normalize to 1.

In this economy, innnovation consists in the birth of a new body of knowledge, *k+1*, produced in the science sector, able to increase the productivity of final good workers by a constant parameter $\gamma > 1$. That is to say, as common in Shumpeterian growth models, we assume that:

$$R_k = \gamma^k \qquad (2)$$

Consequently *k* denotes the type of basic knowledge and the technological era that comes to an end with a scientific discovery and the introduction in manufacturing of an innovation. Because the parameters that define the economy, and therefore the choices made by the agents, remain constant during each technological era, henceforth we can simplify the notation by omitting the time index *t* when it is not indispensable.

Each individual has an infinite life-span and is characterised by one (identical for all agents) intertemporal utility function of consumption and effort required by the job performed. We assume the following instantaneous utility function for scientists:

$$u_{kt}^{R} = c_{kt} - dx_{kt}^{1+\sigma} R_{kt} \qquad (3)$$

with $0 < \sigma < 1$, where c is consumption and x is effort on the job. As a matter of fact, scientists make research essentially applying cognitive resources and effort whose disutility of effort depends also on the extent of knowledge that must be mastered on the job.

While, to simplify algebra, we assume that workers in manufacturing derive utility from consumption only:

$$u_{kt}^{y} = c_{kt}$$

The intertemporal preference rate, *r*, is constant and in equilibrium coincides with the rate of interest at which firms collect savings.

3.1. Science sector

Science sector in this economy produces the new basic knowledge which is a public good freely disposible for the production of the final good. As is well known, public good production usually involves strong problems with workers' incentives and effort. In our model, this issue is crucial since new knowledge production - hence economic growth - depends on effort of scientists.

The main characteristic of the academia is the high value attached to the priority of discovery. As a consequence of the norm of "priority" in scientific discoveries researchers compete in contests to be the first who introduces an innovation, and be rewarded by the scientific community. In this "winner take all" contest the prize consists in a monetary reward, m_{k+1} that is funded by the State and will last untill a new innovation and a new tchnological era arrive.

In this model innovation is uncertain and, following the literature on patent races, we assume that the probability that a single researcher obtains an innovation depends on the effort that he devotes to the research actitivity, and follows a Poisson stocastic process with arrival rate given by:

$$\theta(x_k) = \theta x_k + \theta h \bar{x}_k \qquad (4)$$

where x_k is the effort employed by the scientist in the research activity, $\bar{x}_k$ is the average effort of the research sector, and θ and h are two positive parameters. While, as concerns the probability that an innovation occurs in the economy, we have:

$$\theta n_k (x_k + h\bar{x}_k) \qquad (4.a)$$

The role of colleagues is of a paramount importance in doing science. In fact, good science is done in communities of scientists where cooperation between colleagues is very strong. Scientists talk with other scientists, share ideas, discuss one another work. This occur in informal way and in formal presentations of seminars and papers. The interchanges that result from such discussions can make spectacular differences in science.

The importance of the group is enhanced also by the rules which govern the academia, in fact the rule of priority induces to exchange ideas in order to obtain as early as possible the recognition of others, in other words, it works the rule of "full disclosure" (Dasgupta and David, 1994) that increases the interconnections among researchers and the externalities effects.

To capture such important aspects we have assumed that the productivity of a single researcher depends not only upon his own effort but also upon the effort put in the research activity by his colleagues, rapresented by the average effort of the scientists group. We use the average effort since it better represents the intellectual and psychological resources of others to which scientist may have access, which are relevant not only for their quantity but rather for their quality.

Given the above hypotheses the total expected benefits deriving from being an innovator are:

$$V_{k+1} = \theta\left(x_k + h\bar{x}_k\right)\int_{t_0}^{\infty} e^{-\left[r+\theta n_{k+1}\left(x_{k+1}+h\bar{x}_{k+1}\right)\right]\left(t-t_0\right)} m_{k+1} dt \tag{5}$$

Substituting this expression in equations (3), (4) and (4.a), and solving the integral, we obtain:

$$V_{k+1} = \frac{\theta\left(x_k + h\bar{x}_k\right)m_{k+1}}{r + \theta n_{k+1}\left(x_{k+1} + h\bar{x}_{k+1}\right)} \tag{6}$$

Hence, the total expected benefits deriving from partecipate in the research sector is given by the following:

$$U^R{}_k = \int_{t_0}^{\infty} e^{-\left[r+\theta n_k\left(x_k+h\bar{x}_k\right)\right]\left(t-t_0\right)}\left[V_{k+1} - R_k dx_k{}^{1+\sigma}\right]dt \tag{7}$$

3.2. The consumption good sector

In the consumption good sector each worker can supply inelastically one unit of labour factor, and there is no disutility connected with work. The expected utility obtainable by workers in this sector is:

$$U^y{}_k = \int_{t_0}^{\infty} e^{-\left[r+\theta n_k\left(x_k+h\bar{x}_k\right)\right]\left(t-t_0\right)} w_k dt \tag{8}$$

where w_k is the wage obtainable in that sector.

Consumption sector receives technology from the research sector at no cost, but it pays taxes that the state uses to fund the research sector. Considering the production function (1) and bearing in mind that this sector operates in perfect competition, profits net of taxes are defined as follows:

$$\pi = (1-\tau)Y_k - w_k l_k - r$$

where τ denotes the tax rate. Maximization of this function yields the wages in the consumption good sector, as given by:

$$w_k = (1-\tau)\alpha R_k l_k^{\alpha-1} \quad (9)$$

3.3. The public sector

The state levies taxes, τY on the consumption good sector in order to finance production of knowledge by the research sector.

$$m_k = \tau Y_k . \quad (10)$$

4. Equilibrium

Equilibrium in this economy is defined by both the optimal level of effort that each scientist puts in the research acitvity and in the optimal number of scientists that are allocated to the science sector.

The optimal level of effort undertaken by scientists, x_k, maximizes the present net value of the total expected benefits deriving from doing research. We assume that a scientist does not have a strategic behaviour so that she does not consider the effect of her effort on the arrival rate of discoveries in the economy. In this case, the maximization of the total benefits gives rise to the following first order equilibrium condition:

$$\frac{\theta m_{k+1}}{r + \theta n_{k+1}\left(x_{k+1} + h\bar{x}_{k+1}\right)} - d(1+\sigma)R_k x_k^{\sigma} = 0 \quad (11)$$

According to equation (11), each researcher chooses the optimal amount of effort by equating the expected discounted marginal benefit of one more unit of effort to the marginal disutility that derives from effort. The optimal choice of effort depends positively on the prize for a discovery and negatively on the likelihood that a new finding will sign the end of the period during which he is the winner of the race, hence, it depends on how many resources (scientists and effort) the economy will put on research in the next knowledge era *k+1*.

Since individuals can choose, without sustaining costs, to participate in the labour market either as workers in the consumption sector or as researchers in the science sector, in equilibrium the maximum utility yielded by the two types of activity should be the same. By equations (9) and (10) we have the following equilibrium condition for the labour market:

$$V_{k+1} - dR_k x_k^{1+\sigma} = w_k \qquad (12)$$

where effort in research is the optimal value that derives from equation (11).

Given that individuals are homogeneous, equilibrium will be simmetric, which implies that $\bar{x}_k = x_k$. Finally, in equilibrium all individuals find employment, then we have:

$$n_k + l_k = 1 \qquad (13)$$

4.1. Dynamics

The analysis of general equilibrium dynamics can be summarized by the last three equilibrium conditions. After substitution of equations (11) and (13) in equation (12) we obtain the following difference equation in the variable n_k in the domain (0, 1):

$$\Psi(n_{k+1}) = \Omega(n_k) \qquad (14)$$

where

$$\Psi(n_{k+1}) \equiv \frac{\theta(1+h)\tau\gamma(1-n_{k+1})^{\alpha}}{r + \theta(1+h)n_{k+1}D^{\frac{1}{1+\sigma}}(1-n_{k+1})^{\frac{\alpha-1}{1+\sigma}}}, \qquad (15)$$

with $D \equiv \dfrac{(1-\tau)\alpha}{d(1+\sigma)(1+h)-d}$;

and

$$\Omega(n_k) \equiv (1-n_k)^{\frac{(\alpha-1)\sigma}{1+\sigma}}\left[dD^{\frac{\sigma}{1+\sigma}} + (1-\tau)\alpha D^{-\frac{1}{1+\sigma}}\right] \qquad (16)$$

As in Aghion and Howitt's model, equation (14) enables us to determine the amount of labour employed in the science sector in era k as function of the labour employed in the successive technological era. In fact, we are able to characterize the functions $\Psi(n_{k+1})$ and $\Omega(n_k)$.

Given that:

$$\frac{\partial\Omega(n_k)}{n_k} = \frac{(1-\alpha)\sigma}{1+\sigma}\left[dD^{\frac{\sigma}{1+\sigma}} + (1-\tau)\alpha D^{-\frac{1}{1+\sigma}}\right](1-n_k)^{\frac{(\alpha-1)\sigma-1-\sigma}{1+\sigma}} > 0;$$

and

$$\frac{\partial\Psi(n_{k+1})}{\partial n_{k+1}} = -\Psi(n_{k+1})^2(1-n_{k+1})^{-\alpha-1}\frac{1}{\tau\gamma} *$$

$$\left[\frac{r\alpha}{\theta(1+h)} + D^{\frac{1}{1+\sigma}}(1-n_{k+1})^{\frac{\sigma+\alpha}{1+\sigma}} + D^{\frac{1}{1+\sigma}}\frac{1-\alpha}{1+\sigma}n_{k+1}(1-n_{k+1})^{\frac{\alpha-1}{1+\sigma}} + \alpha D^{\frac{1}{1+\sigma}}n_{k+1}(1-n_{k+1})^{\frac{\alpha-1}{1+\sigma}}\right] < 0$$

we know that $\Omega(n_k)$ is monotone increasing and $\Psi(n_{k+1})$ is monotone decreasing. Furthermore,

$$\lim_{n_{k+1}\to 1}\Psi(n_{k+1})=0\,; \qquad \Psi(0)=\frac{\theta(1+h)\tau\gamma}{r}\,;$$

$$\lim_{n_1\to 1}\Omega(n_k)=\infty\,; \qquad \Omega(0)=dD^{\frac{\sigma}{1+\sigma}}+D^{-\frac{1}{1+\sigma}}(1-\tau)\alpha\,.$$

Hence, an intersection between the functions exists if $\Psi(0)>\Omega(0)$. In this case, we can apply the implicit function theorem and define the difference equation that describes the economy dynamics:

$$n_k = \Gamma(n_{k+1}), \tag{17}$$

with $\dfrac{\partial\Gamma(n_{k+1})}{\partial n_{k+1}}<0$.

The study of equilibrium dynamics can proceed as in Aghion and Howitt, (1992). We assume agents have perfect foresight, hence the economy employment of scientists at each era k is determined according to the forecast of the future employment n_{k+1}. Equilibrium dynamics are defined by sequences of scientists employment that start from the value n_0 and go into the future, and satisfy the difference function $n_k = \Gamma(n_{k+1})$.

A steady state equilibrium is defined as a value of n such that $n=\Gamma(n)$. Monotonicity properties of the functions $\Psi(n)$ and $\Omega(n)$ - see figure 1 - ensure that there exists a unique stationary solution to equation (17). This steady state $\bar{n}$ is asymptotically stable if :

$$\Psi_n(\bar{n})+\Omega_n(\bar{n})<0 \tag{18}$$

In this case, there are cycles of n_k that converge towards the steady state $\bar{n}$.

Otherwise, when inequality (18) is reversed, two different kinds of dynamic equilibrium may exist: one in which employment in basic research converges to a stable two-cycle where it assumes alternatively a low and a high value; another equilibrium in which a stable two-cycle is made by a nil value of employment. This interesting case defines a “no-growth trap” (see Aghion and Howitt, 1992) and happens when the economy converges to an equilibrium in which a high forecast of future employment in research determines a nil value today, that will remain the equilibrium outcome of the model dynamics in any time period. Inspection of figure 1 reveals that a no-growth trap may exist when both $\Psi(0)$ and $\Omega(0)$ are high because in this case the equilibrium cycle can be made by an high foreseen value of research employment to which corresponds a nil value of n_k.

Interestingly enough, such a dynamic equilibrium is more likely when the parameter h is high[1], hence static externalities in the scientific sector may cause nil investment in basic research. Actually, social interactions are an important feature of modern systems of scientific research in the industrialized world. Our result says that this phenomenon could be responsible for the evidence concerning many countries – not only the very poor – that choose to remain outside the international scientific community.

After convergence to a dynamic equilibrium, the economy evolves along a balanced growth path. Given the distribution of employment between science and good production, effort in basic research is determined by the equilibrium conditions equations (11) and (12). Then the rest of endogenous variables derives from general equilibrium of markets.

Even if our analysis of equilibrium dynamics focuses on employment, a similar study can be performed with reference to effort. In fact, equations (11), (12)

[1] It can be easily verified that $\frac{\partial \Psi(0)}{\partial h} > 0$; and $\frac{\partial \Omega(0)}{\partial h} > 0$.

and (13) can be manipulated to derive a difference equation in effort only. After these steps we are able to derive the equation:

$$\Psi^{x}\left(x_{k+1}\right)=\Omega^{x}\left(x_{k}\right), \tag{19}$$

where:

$$\Psi^{x}\left(x_{k+1}\right)\equiv\frac{\theta\tau\gamma D^{\frac{\alpha}{1-\alpha}}}{rx_{k+1}^{\frac{(1+\sigma)\alpha}{1-\alpha}}+\theta(1+h)\left(x_{k+1}^{\frac{1+\sigma\alpha}{1-\alpha}}-x_{k+1}^{\frac{(\alpha-1)\sigma}{1-\alpha}}D^{\frac{1}{1-\alpha}}\right)}; \tag{20}$$

and

$$\Omega^{x}\left(x_{k}\right)\equiv x_{k}^{\sigma}\left[D(1-\tau)\alpha+d\right]. \tag{21}$$

In figure 2 we represent the two functions. It is straightforward to verify that $\Omega^{x}\left(x_{k}\right)$ is concave increasing starting from the origin. We can prove that $\Psi^{x}\left(x_{k+1}\right)$ is monotone decreasing, and

$$\lim_{x_{k+1}\to\underline{x}^{+}}\Psi^{x}\left(x_{k+1}\right)=\infty\,;\qquad\lim_{x_{k+1}\to\infty}\Psi^{x}\left(x_{k+1}\right)=0$$

with $\underline{x}>0.$ Hence, equilibrium values of effort always exists, and we can define a difference equation that slopes downward and summarises the equilibrium dynamics of effort:

$$x_{k}=\Gamma^{x}\left(x_{k+1}\right) \tag{22}$$

Concerning this function some comments can be drawn similar to those that apply to the dynamics of employment n_{k}. In this case too, a stationary equilibrium exists that can be stable, and equilibrium cycles of effort should be associated with cycles of n_{k}. However, inspection of figure 2 allows us to rule out the possible existence of a no-growth trap in equilibrium effort dynamics. This result should mean that when foreseen employment in basic research in period $k+1$ deters any positive investment in research today, effort could assume any non-negative value. Hence, this feature of our model makes more realistic the definition of a no-growth trap.

In the following sections we will analyse the properties of equilibrium focusing on the stable steady state.

4.2. Comparative statics and balanced growth

In order to derive the effects of some relevant parameters on the steady state value of scientist employment, we concentrate on the case of a single steady state and we assume that it is stable. After some algebra we are able to state the following proposition:

Proposition 1.

a) Let us consider $\bar{n}$, the stable steady state of the equation $n_k = \Gamma(n_{k+1})$, then:

- *$\bar{n}$ increases when τ increases if $\tau < 1-\tau$;*
- *$\bar{n}$ increases when h or γ increase;*
- *$\bar{n}$ decreases with the interest rate r.*

b) Let us consider $\bar{x}$, the stable steady state of the equation $x_k = \Gamma(x_{k+1})$, then:

- *$\bar{x}$ increases with an increase of γ;*
- *$\bar{x}$ decreases with an increase of h or r;*
- *$\bar{x}$ has an ambiguous behaviour when τ increases.*

Proof in Appendix.

Some interesting comments can be done on the statements of this proposition. Public policies aimed at the enlargement of the science sector can be realized by collecting more resources from the private sector that will be channelled to higher prizes for scientific discoveries. This kind of policy improves the reward of doing basic research. Strong effects on the size of science sector may derive from the strength of externalities in scientific environments. In fact, cooperative and collaborative departments may provide an incentive to joining the world of science. This effect accords with both common sense and a large part of sociological literature dealing with science.

Interesting results derive also from comparative statics of equilibrium effort of researchers. In fact, both the negative effects of externalities and the ambiguous effect of state resources seems to be counterintuitive. The ambiguous effect of state resources can be explained by considering that when state resources increase there are three effects: there is an increase in the resources devoted to the prize derived from an innovation, which rises the effort, but there is also an increase in the size of research sector which increases the probability that an innovation occurs in the economy, reducing in this way the duration of the prize; finally an increase in state resources reduces the wage obtainable in the alternative sector and, given the labour market equilibrium condition, this reduces also the reward of all the specialised workers and consequently their level of effort. Regard to the negative effect of externalities this can be explained by similar considerations. In this case we have two opposite effects, on one hand an increase in externalities increases the probability that a researcher obtains an innovation, but on the other there is an increase in the probability that an innovation occurs in the economy, which reduces the duration of the prize. This latter effect is greater since it is magnified by the size of research sector.

The opposite effects of externalities on size and effort in science sector provide us with an important motivation for the separate introduction of effort in our model. This effect reminds us that there are limits to collaborative behaviour that should consider the individual incentive to work in science.

In our economy, production of the final good increases only when an innovation occurs, and this is a probabilistic event. The expected average steady state rate of growth of per capita income depends on the number of researches employed in science, on the productivity of these workers, on the optimal level of effort and on the magnitude of the technological advance brought about by the innovation. In particular we consider the determinants of balanced growth characterized by the steady state values of scientists employment and effort $\bar{n}$, $\bar{x}$:

$$E(g) = E(\ln Y_t - \ln Y_{t-1}) = \theta\bar{n}(\gamma,\tau,h)\bar{x}(\gamma,\tau,h)(1+h)\ln\gamma \qquad (23)$$

Taking into account comparative statics results of Proposition 1 , we are able to write the following

Proposition 2. *In balanced growth equilibrium the expected rate of growth of per capita income increases with an increase of each of the parameters τ , γ . The sign of the effect of the importance of externalities in research - h - is undetermined.*

Proof: it is trivial from Proposition1.

This proposition gives us a picture that highlights the role that science sector may have in economic growth. It also summarises some important results of the paper.

In this model two forces drive the production of new knowledge and economic gowth. One is individual choice of scientists who take part of a complex organization in which incentives derive not only from money income but also from the community rules. The other is the collective choice made by all agents from wich the relative size of science sector derives. The monetary incentive to work in basic research has both an individual and an aggregate dimensions, since the second concerns the distribution of physical resources between the two sectors of the economy.

Externalities of average effort of scientists affect the individual probability of obtaining a new finding and represent the third context effect in science sector. As usual, externalities in production lower the individual incentive to effort, but cause increasing returns on aggregate activities. This seems to happen also in our model and the result of externalities on growth depends on the relative strenght of these two contrasting effects.

5. Conclusions

This paper represents the first attempt to the modelling of basic research and long run economic growth since work done by Karl Shell in the late sixties. As common in the framework of endogenous growth models, we provide a formalization of the interactions between the scientific sector and the rest of the economy which work both ways. Focus is on the role of externalities in basic research that affect the probability of success of each scientist racing for a new finding. The state organizes production of new knowledge - a public good - with resources taken from the private sector.

Scientists compete each other to get a priority over a discovery and these races are affected by several forms of social interactions. In fact, scientists informal interactions give rise to externalities that hasten discoveries. Given that science is financed by taxes taken from private firms, output growth and structure of basic research activity jointly determine the dynamics of the economy. This dynamics are not trivial as multiple steady state equilibria can derive from strong effects of social inteactions in science.

Here we set the main lines for the analysis of such an important issue for long run growth that in future work we will further develop in order to deal with welfare issues and public policy.

Appendix

A1. Properties of the function $\Psi^x(x_{k+1})$.

Let us consider the function $\Psi^x(x_{k+1}) \equiv \dfrac{\theta\tau\gamma D^{\frac{\alpha}{1-\alpha}}}{r x_{k+1}^{\frac{(1+\sigma)\alpha}{1-\alpha}} + \theta(1+h)\left(x_{k+1}^{\frac{1+\sigma\alpha}{1-\alpha}} - x_{k+1}^{\frac{(\alpha-1)\sigma}{1-\alpha}} D^{\frac{1}{1-\alpha}} \right)}$, then

$$\frac{\partial \Psi^x(x_{k+1})}{\partial x_{k+1}} \equiv -\theta\tau\gamma D^{\frac{\alpha}{1-\alpha}} \frac{r\dfrac{(1+\sigma)\alpha}{1-\alpha} x_{k+1}^{\frac{(1+\sigma)\alpha}{1-\alpha}-1} + \theta(1+h)\left[x_{k+1}^{\frac{1+\sigma\alpha}{1-\alpha}-1} - x_{k+1}^{\frac{\sigma\alpha-\sigma}{1-\alpha}-1} D^{\frac{1}{1-\alpha}} \dfrac{\sigma\alpha-\sigma}{1-\alpha} \right]}{\left[r x_{k+1}^{\frac{(1+\sigma)\alpha}{1-\alpha}} + \theta(1+h)\left(x_{k+1}^{\frac{1+\sigma\alpha}{1-\alpha}} - x_{k+1}^{\frac{(\alpha-1)\sigma}{1-\alpha}} D^{\frac{1}{1-\alpha}} \right) \right]^2} < 0$$

We can write $\Psi^x(x_{k+1})$ as:

$$\Psi^x(x_{k+1}) \equiv \frac{\theta\tau\gamma D^{\frac{\alpha}{1-\alpha}}}{x_{k+1}^{\frac{(1+\sigma)\alpha}{1-\alpha}} \left[r x_{k+1}^{\frac{\alpha}{1-\alpha}} + \theta(1+h) x_{k+1}^{\frac{1}{1-\alpha}} - x_{k+1}^{\frac{-\alpha}{1-\alpha}} D^{\frac{1}{1-\alpha}} \theta(1+h) \right]}$$

This version of the function shows the denominator that is zero when the increasing function $r x_{k+1}^{\frac{\alpha}{1-\alpha}} + \theta(1+h) x_{k+1}^{\frac{1}{1-\alpha}}$ crosses the decreasing function $x_{k+1}^{\frac{-\alpha}{1-\alpha}} D^{\frac{1}{1-\alpha}} \theta(1+h)$. It can be easily seen that this intersection always happens for a positive value of x_{k+1}, hence we can state that:

$$\lim_{x_{k+1} \to \underline{x}^{+}} \Psi^{x}\left(x_{k+1}\right) = \infty\,; \qquad \lim_{x_{k+1} \to \infty} \Psi^{x}\left(x_{k+1}\right) = 0$$

A2. Proof of Proposition 1.

Part a). Starting from $\Psi(\overline{n}) = \Omega(\overline{n})$ we can write:

$$\frac{r(1-\overline{n})^{\frac{1-\alpha}{1+\sigma}}}{D^{\frac{1}{1+\sigma}}} + \theta(1+h)\overline{n} = \frac{\theta(1+h)\tau\gamma(1-\overline{n})}{dD + (1-\tau)\alpha} \tag{a1}$$

from which derives:

$$r(1-\overline{n})^{\frac{-(\alpha+\sigma)}{1+\sigma}} + \theta(1+h)D^{\frac{1}{1+\sigma}}\overline{n}(1-\overline{n})^{-1} - \frac{\theta(1+h)D^{\frac{1}{1+\sigma}}\gamma\tau}{dD + (1-\tau)\alpha} = 0\,.$$

Taking the differential of that equation with respect to $\overline{n}$ and τ we get:

$$\partial\overline{n}\left\{ r\frac{\alpha+\sigma}{1+\sigma}(1-\overline{n})^{\frac{-\alpha-\sigma}{1+\sigma}-1} + \theta(1+h)D^{\frac{1}{1+\sigma}}\left[(1-\overline{n})^{-1} + (1-\overline{n})^{-2}\overline{n}\right]\right\} +$$

$$+\,\partial\tau\left\{ -\theta(1+h)(1-\overline{n})^{-1}\overline{n}\frac{D}{(1+\sigma)(1-\tau)} - \theta(1+h)\gamma\frac{\left(D^{\frac{1}{1+\sigma}} + \frac{\tau}{1+\sigma}D^{\frac{1}{1+\sigma}-1}D_{\tau}\right)\left[dD + (1-\tau)\alpha\right] + \frac{\tau dD^{\frac{1}{1+\sigma}+1}}{1-\tau} + \alpha\tau D^{\frac{1}{1+\sigma}}}{\left[dD + (1-\tau)\alpha\right]^{2}}\right\} = 0$$

where $D_{\tau} = -\dfrac{\alpha}{d(1+\sigma)(1+h) - d}$, hence $\dfrac{\partial\overline{n}}{\partial\tau} > 0$ if $\tau < 1-\tau$.

From the above equations it can be easily verified that $\dfrac{\partial\overline{n}}{\partial\gamma} > 0$, and $\dfrac{\partial\overline{n}}{\partial r} < 0$.

As far as the effect of externalities on scientist employment is concerned, equation (a1) can be rearranged in the following way:

$$(1+h)=\frac{r(1-\overline{n})^{\frac{-\alpha-\sigma}{1+\sigma}}}{\theta D^{\frac{1}{1+\sigma}}\left[\frac{\tau\gamma}{dD+(1-\tau)}-\overline{n}(1-\overline{n})^{-1}\right]}$$

from which $\frac{\partial \overline{n}}{\partial h}>0$ derives.

Part b).

At the steady state the equation $\Psi^{x}(x_{k+1})=\Omega^{x}(x_{k})$ can be written as:

$$\frac{\theta\tau\gamma D^{\frac{\alpha}{1-\alpha}}}{D(1-\tau)\alpha+d}+D^{\frac{1}{1-\alpha}}\theta(1+h)-r\overline{x}^{\frac{\alpha+\sigma}{1-\alpha}}-\theta(1+h)\overline{x}^{\frac{1+\sigma}{1-\alpha}}=0 \qquad \text{(a2)}$$

Differentiation of (a2) with respect to $\overline{x}$ and τ provides:

$$\partial\overline{x}\left\{-r\frac{\alpha+\sigma}{1-\alpha}\overline{x}^{\frac{\alpha+\sigma}{1-\alpha}-1}-\theta(1+h)\frac{1+\sigma}{1-\alpha}\overline{x}^{\frac{1+\sigma}{1-\alpha}-1}\right\}+$$

$$+\partial\tau\theta\gamma\left\{\frac{\left(D^{\frac{\alpha}{1-\alpha}}+\frac{\tau\alpha}{1-\alpha}D^{\frac{\alpha}{1-\alpha}-1}D_{\tau}\right)\left[D(1-\tau)\alpha+d\right]-\tau D^{\frac{\alpha}{1-\alpha}}\alpha\left[D_{\tau}(1-\tau)-D\right]}{\left[D(1-\tau)\alpha+d\right]^{2}}+\frac{1}{1-\alpha}\theta(1+h)D^{\frac{\alpha}{1-\alpha}-1}D_{\tau}\right\}=0$$

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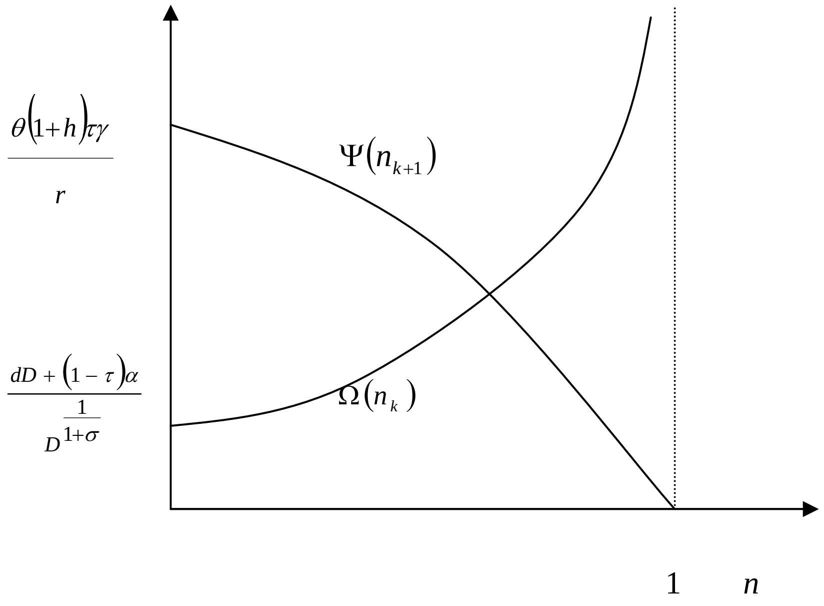

Figure 1. The existence of equilibrium dynamics of basic research employment.

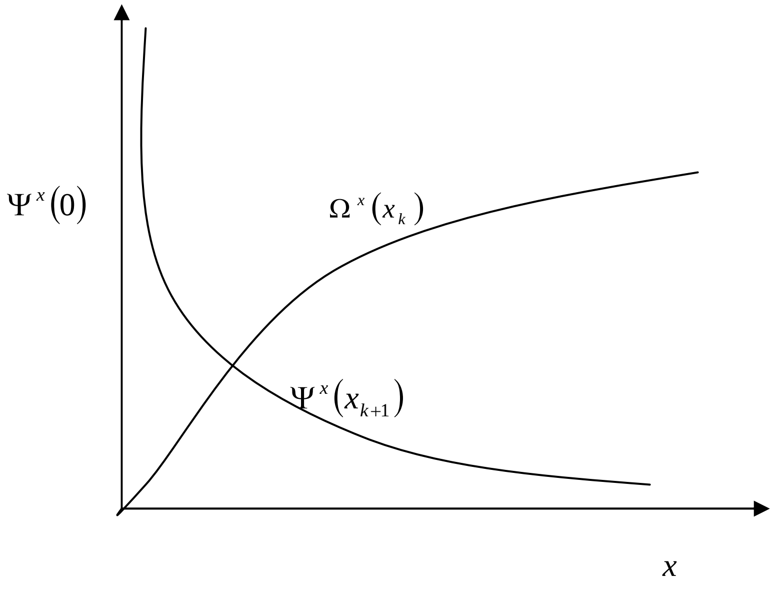

Figure 2. The existence of equilibrium dynamics of effort in science sector.