Department of Economic Studies
University of Naples "Parthenope"

DES - dp

Discussion Paper

No.4/2007

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2007

State Aid Policies and Underground Activities*

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December 12, 2006

Abstract

The main goal of this paper is to examine the implications of firm-oriented fiscal policies, such as capital subsidies and tax allowances, in an economy with an underground sector. In addition, we investigate whether the technology structure of "hidden" production may facilitate or counteract the effects of fiscal policies on firm behavior. Among our results we stress the following: **first,** capital subsidies promote tax evasion; these subsidies induce firms to increase actual capital accumulation (a level effect), but also produce a reduction in the regular share of aggregate capital stock (a composition effect). **Second**, tax relief reduces underground activities and fosters capital accumulation, as well as aggregate production. **Third**, the technology structure matters for determining how to allocate resources between formal and informal production, hence the amount of reported revenues.

Keywords: State aid, tax exemptions, capital subsidies, tax evasion, underground production, physical capital accumulation.

JEL classification: E26, E22, H25, H26.

*Different versions of this paper have been presented at the University of Aarhus (DK), at the University of Bath (UK), at the International Workshop *The Economic Impact of Financial and Labour Market Institutions* held at the University of Urbino, and at the Annual Meeting of the *Italian Economist Society (SIE).* We thank all the participants for suggestions. Comments from John Hudson and Philip Jones are gratefully acknowledged. We also thank participants at the II Workshop *Le cifre dell'economia sommersa e il loro utilizzo*, and participants at the Workshop *Mercato del lavoro e Mezzogiorno: nuove prospettive e vecchi paradigmi*, both held at the University of Naples "Parthenope".

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1 Introduction

This paper focuses on the nexus among selected firm-oriented fiscal policies (tax relief and capital subsidies), physical capital accumulation and underground activities. In particular, we examine the implications of fiscal policies aimed to support firms, such as capital subsidies and tax allowances, in the presence of an underground economy. In addition, we investigate whether the technology structure of "hidden" production may facilitate or counteract the effects (the effectiveness) of the fiscal policies on firm behavior.

Subsidies programmes and tax advantages for "infant" industries or depressed areas are often justified because the industry is not competitive enough and prices do not show full flexibility. However, granting capital subsidies and tax allowances to firms has also important implications for underground activities and tax evasion.

Underground activities occur in many countries, and there are significant indications that this phenomenon is large and increasing.[1] The estimated average size of the underground sector, as a percentage of total GDP, in the late 1990s was about 17 percent in OECD countries (Schneider and Enste, 2002).[2]

The paper presents an optimal-investment model in which a representative firm maximizes the expected cash flow, choosing simultaneously the optimal combination of capital stock (i.e. firm size) and its allocation between two possible technologies -regular and irregular-, conditional on a set of fiscal policy and technological parameters.

We are not aware of any contribution investigating how the optimal investment choice is related to underground economy and tax evasion. This is a major issue, because underground activities represent an additional financing source for investing, which is not subject to distortionary taxation. This means that fiscal policy results might differ from what we expect in a model that explicitly incorporates tax evasion. It is important to understand from a theoretical perspective the economic mechanisms operating in such a context.

We focus on the *moonlighting firm*, which operates simultaneously in the regular and irregular sectors, using the same stock of capital and evading taxation in the irregular sector. Such a firm is able to evade, like firms that operate only in the underground economy, but in addition, it can exploit a technological advantage. The paper explicitly considers two fiscal

[1] There is no universal agreement on what defines the underground economy. Moreover, the difficulty in defining the sector extends to the estimation of its size. We are concerned with the size of the underground economy as encompassing activities which are otherwise legal but go unreported or unrecorded.

[2] Estimates of the underground economy are particularly difficult, as the phenomenon is, by definition, not directly measurable. Several methods have been used for this purpose, some based on theoretical models, some based on econometrics and others on micro analysis of agent responses in particular surveys. See, among others, The Economic Journal (1999) symposium on the Hidden Economy, and Busato, Chiarini and Di Maro (2006).

policy experiments: we study the impact of permanent changes in tax rates and investment subsidies both on capital accumulation, and on its allocation across regular and underground production.

Our analysis focuses on the Italian economy due to the sizeable underground sector in Italy, and the high percentage of "moonlighters" which operate in the informal sector.[3] Furthermore, Italian governments have repeatedly supported firms with capital subsidies and tax allowances. We think that the theoretical scheme and its predictions may be applied, without loss of generality, to other countries.

Three main results can be drawn from our analysis. First, capital subsidies promote tax evasion; subsidies induce firms to increase actual capital accumulation (a level effect), but also produce a reduction in the regular share of aggregate capital stock (a composition effect). The investment subsidy is a non excludable public good that opens room for free-riding (tax evasion in other words). In this context the Government is not capable, because of (un-modeled) monitoring costs, to distinguish between regular and moonlighting firms. Firms therefore, have an incentive to declare a sufficiently small amount of revenues to be eligible for the subsidy, while investing relatively more in the underground economy and "pocketing the tax wedge". Second, a tax reduction reduces underground activities and foster capital accumulation and aggregate production. This is not a novelty, but we draw interesting insights which may not follow directly from intuition when tax reliefs are related to firms' technology. Indeed, the third main result of our analysis asserts that technology matters (the labelled *moonlighting effect* and *scale effect*, discussed below) for determining how to allocate resources between the formal and informal production, hence the amount of reported revenues.

The structure of the paper is as follows. Section 2 provides some stylized facts and defines the motivations of the paper. Section 3 explains the firm maximizing problem and characterizes the long run equilibrium. In Sections 4 and 5 the main results of technology and policy analysis are reported and commented upon. Finally, Section 6 presents some concluding remarks.

2 Fiscal Policies and Underground Activities in Europe: Selected Stylized Facts

The question of the relationship between taxation and the underground economy has received considerable attention, but we are not aware of economists or politicians who have considered

[3] In Italy, the National Institute of Statistics (ISTAT) produces several time series estimates of the underground economy and employment, disaggregated at regional level since 1995.

Table 1: Selected key indicators of State aid and tax rates, selected EU countries, 2004

Aid	*ES*	*DE*	*FR*	*IT*	*BE*	*UK*	*GR*	*SE*
% of the overall EU-25 amount	6.5	28.0	14.5	11.4	1.6	8.8	0.8	4.5
% of GDP	0.4	0.7	0.4	0.4	0.2	0.2	-	0.8
Effective corporate tax burden	36.1	36.0	34.8	32.0	29.7	28.9	28.0	24.8

Spain:ES; Germany:DE; France:FR; Italy:IT; Belgium:BE; United Kingdom:UK; Greece:GR; Sweden:SE.
Source: European Commission (2006); ZEW (2006).

the effects of capital subsidies on tax evasion and underground production.

A sustained expansion of public sector expenditures on welfare provision has led European firms to endure high corporate tax rates (see the last row of **Table 1** and Joumard, 2002). Many European governments, to stimulate demand for labor and promote capital accumulation in particular areas of the country or productive sector, have recently shifted the tax burden on capital away from such target ares, introducing generous tax relief policies for investment expenditure or raising tax allowances to offset corporate income tax. The rationale of these interventions, and in general of so-called *State aid*, is to address various market failures (externalities, merit or public goods and so on) or they may be justified using equity arguments, to improve social and regional cohesion, in order, for instance, to promote growth. European Union (EU) State aid policy is strictly regulated since it may harm competition. To this end, the European Commission Treaty (Art. 87) obliges EU governments to negotiate their allowances with the European Commission.

According to the European Commission (2006), in 2004 the most of the EU-25 member States' aid was earmarked for the manufacturing sector (59%) while a further 23% was directed towards agriculture. Overall, EU-25 State aid accounted, in 2004, for more than 61.4 billion euros. **Table 1** presents some figures on State aid and tax rates; in particular, the size of total aid in each country is shown as a percentage of its own GDP, ans as a percentage of the overall EU-25 amount of State aid.

There are several instruments of State aid (see Nitsche and Heidhues, 2006 and European Commission, 2006) . *Grants* and *Tax Exemptions* and *Equity Participation* comprise aid that is transferred in full to the recipient and accounts for the vast majority of aid in all Member States. *Soft Loans* and *Tax Deferrals* cover transfers in which the aid element is the interest saved by the recipient during the period in which the capital transferred is at his/her disposal. *Guarantees*, expressed in nominal amounts guaranteed, incorporate aid elements corresponding to the benefit which the recipient receives free of charge or at lower than the market rate. The share of each aid instrument in total aid to manufacturing and

Table 2: Distribution of aid by instruments, selected EU countries, 2002-2004

	ES	*DE*	*FR*	*IT*	*BE*	*UK*	*GR*	*SE*
Grants	59.9	40.3	51.6	66.9	88.6	55.6	66.4	33.7
Tax exemption	29.2	39.3	30.6	9.7	3.4	32.4	33.6	63.4
Equity participation	0.3	1.9	0.1	0.7	0.1	1.2	0.0	0.6
Soft Loans	10.6	1.2	15.3	3.9	6.1	10.8	0.0	2.2
Tax deferrals	0.0	0.0	0.1	18.7	0.6	0.0	0.0	0.0
Guarantees	0.0	17.4	2.3	0.1	1.2	0.0	0.0	0.1

Spain:ES; Germany:DE; France:FR; Italy:IT; Belgium:BE; United Kingdom:UK; Greece:GR; Sweden:SE.
Source: European Commission (2006).

services sectors for some selected countries is reported in **Table 2**.[4]

State aid accounts for a considerable share of the GDP in EU countries; moreover, countries with high corporate taxation, such as Germany, France and Italy, also provide large amounts of total EU State aid. This evidence suggests that there could be as yet unexplored connections among State aid policies, tax policy and the underground economy.

Some insights into the size of the underground economy in EU countries are given in the Flynn Report (European Commission, 1998), which estimated that unofficial production of goods and services in European countries ranged between 7 and 16% of total GDP, with considerable differences among States. Although underground economy estimates strongly depend upon the estimation method used, the Flynn report suggested the existence of three different groups of countries. First, the Scandinavian countries, which, along with Ireland and Austria, had the lowest share of the underground economy, about 5% of GDP. By contrast, the largest shares of informal activities were recorded in southern countries, in particular Italy and Greece, with a size of about 20% of GDP, followed by Belgium and Spain, with slightly lower figures . Finally, a third group of countries, UK, Germany and France displayed intermediate size of shadow activities compared to the first two groups. More recent figures seem to confirm this ranking of countries (Schneider and Enste, 2002).

While the justification for State aid policies have often been discussed, and their ex-ante effects have (albeit much less) been investigated, their implications in contexts characterized by the presence and the persistent nature of the informal economy are entirely neglected.

The paper discusses possible interactions between State aid, and in particular tax ad-

[4]The aid schemes directed to the manufacturing/services sectors have been broken down according to their objective into two main categories: i) Horizontal objectives (R&D; Environment; Small and Medium-sized Enterprises -SMEs; Commerce; Employment; Regional aid); ii) Sector Aid (Manufacturing; Services; Coal mining; Other non-manufacturing; Transport). State aid for horizontal objectives amounted to 76% of total aid in 2004; moreover, among the above listed categories of horizontal objectives, regional aid and SMEs aid played a major role.

vantages and capital subsidies, in the presence of tax evasion and an underground economy, focusing on the productive activity of firms operating simultaneously in both the official and unofficial sector.

3 The Model

We assume a simple model with homogeneous good that can be produced using two different technologies, the regular technology and the underground one; regular production is taxed while underground production is not declared to the fiscal authorities.

The definition of irregular production needs some specification. Denote with K the firm's capital stock, and with μ and $(1-\mu)$ respectively, the share of capital allocated to the regular and irregular sector. Following an imaginative but incisive classification (e.g. Cowell, 1990), irregular production can be undertaken either by a completely irregular firm (hereinafter defined as a *ghost firm*), or by a firm which acts only partially in the underground sector (hereinafter defined as a *moonlighting firm*). Capital accumulation and allocating decisions would be different in the two cases: the share of capital invested in the regular sector would be $\mu = 0$ for a ghost firm, while it belongs in $\mu \in (0,1)$ for a moonlighting firm. Dealing with a ghost firm means considering that all the production is hidden. In this paper we consider a representative firm which operates "above" as well as "under" ground, producing an identical homogeneous good and using a unique stock of capital, but declaring to the fiscal authorities only a share of its production.

The literature usually assumes that underground firms are ghost firms and therefore less productive than regular firms. Typical explanations include lower entrepreneurial ability, difficulty in getting financial support and high transaction costs due to the necessity to locate "trustworthy" trading partners.[5] "Moonlighting technology" may relax the limits usually assessed for underground firms, generating a specific externality, which cannot be exploited by ghost firms.[6]

To have an idea of the significance of moonlighter behavior in underground activities, we refer to a recent work by Hibbs and Piculescu (2006) and a survey by Censis (2005). Hibbs and Piculescu, using data from the World Bank, point out that more than 60% of 3,818 interviewed enterprises, distributed over 54 countries, are used to operating both in the

[5] See Anderberg et al. (2003); Loayza (1994).

[6] The moonlighter may undertake irregular activities to obtain one of the following alternatives: (a) "extra-profit ": this is the situation of medium size productive units, largely regular, with their own brand, which exploit underground production to gain extra profits (but also the partial decentralization by a regular firm toward smaller and irregular productive units referred to as "local underground districts"; (b) "surviving ": this situation applies to small firms producing largely underground, which use regular production as a convenient screen to avoid fiscal controls. For greater details for Italy see Lucifora (2003) and Roma (2001).

Table 3: Sales amount reported by a typical firm for tax purposes (percent), main World regions, 1999-2000

Regions	*tax compliance*	*tax evasion*
East Asia & Pacific	70.54	29.46
Europe & Central Asia	89.35	20.65
Latin America & Caribbean	75.11	24.89
Middle East & North Africa	75.13	24.87
OECD	93.55	6.45
South Asia	93.7	6.3
Sub-Saharan Africa	78.62	21.38

Source: World Bank.

Table 4: Irregular firms as a percentage of total firms, main Italian regions, 2005

Irregular Firms	North-West	North-East	Center	South	Italy
Ghost Firms (1)	5.8	4.9	6.8	16.8	9.7
Moonlighting Firms (2)	29.7	31.3	41.7	59.9	43.4
Total Irregular Firms (1+2)	35.5	36.2	48.5	76.7	53.1

Source: Authors' elaboration upon Censis (2005).

official and unofficial sector. The phenomenon is indirectly estimated by the World Bank's World Business Environment Surveys (WBES) through the average percentage of total sales that firms report for tax purposes. This evidence is shown in **Table 3** for the main World regions. Since all respondents are legally registered firms, the high percentage of tax evasion, defined as the full sales minus the percentage of tax compliance with fiscal law, suggests that the representative firm is often a moonlighter.

More detailed data are available for Italy. A recent survey by Censis (2005), whose main evidence is shown in **Table 4**, highlights that most of the Italian firms which operate in the underground economy are moonlighters, while ghost firms are a residual share of the total firms.[7]

These remarks suggest that partial tax evasion (moonlighting technology) may offer some additional convenience to irregular entrepreneurs compared to total tax evasion (ghost technology).

[7] The survey conducted by the World Bank (2000) estimates the informal economy by asking managers the question "Recognizing the difficulties many enterprises face in fully complying with taxes and regulations, what percentage of total sales you estimate the typical firm in your are of activity reports for tax purposes?".

The survey carried out by Censis (Centre for Social Studies and Policies) is based on qualitative methodology, namely interviewing selected witnesses (managers, union representatives, public officers and so on).

3.1 Technology under Tax Evasion

Each firm can decide to implement regular production (no tax evasion), underground production (ghost firm) or both (moonlighting firm). Let μ or $(1-\mu)$ denote the share of capital (K) allocated to the regular or underground sector. Hence the output of the two specialized firms, i.e. regular and ghost is respectively:[8]

$$\begin{aligned} Y_R &= A(\mu K)^a, \mu = 1, 0 < a < 1 \\ Y_U &= B\left((1-\mu)K)\right)^a, \mu = 0, 0 < a < 1, \end{aligned}$$

where a represents the elasticity of capital stocks in the two sectors. The two sectors use identical technologies with the exception of the two scaling factors A and B. As occurs in a two-sector model with sector specific externalities, we assume that from the perspective of a firm operating in a single sector, the two parameters are taken as positive constants, while for a firm operating simultaneously in the two sectors (*moonlighting*) B is a function of the total use of capital:[9]

$$B = K^{a\sigma}.$$

The parameter σ measures the size of the external effect internalized by the simultaneous implementation of the regular and underground technology. Indeed, firms which have a regularly registered/declared activity can exploit, during their "irregular production", a broad series of indirect benefits, stemming from regular production, and not available for ghost firms, such as: a more extensive, high-qualified network of suppliers and/or customers; the possibility of using several fiscal policy benefits, such as the fiscal allowances for investment; the possibility of accessing the market of bank loans and so on.[10] In the rest of the paper we refer to this as the *moonlighting effect.* Condition 1 below suggests that the size of the externality should be sufficiently low as to ensure that returns to scale are not increasing at the firm level:

[8] To simplify the analysis we are considering a single factor technology which employs only capital; this is tantamount to a constant returns of scale technology, with capital and labor inputs: in this case, output as well as capital would be measured per unit of employee. Moreover, since both sectors produce the same commodity, the capital elasticities are assumed to be identical.

[9] In a different context (real business cycle model with indeterminacy) Benhabib and Farmer (1996) used a similar formulation to represent aggregate capital and labour external effects in a two-sector model. Of course, we rule out any kind of indeterminacy, as Condition 1 and Proposition 2 below demonstrate.

[10] Thus, we are emphasizing the technology of this partial tax evader against a total tax evader firm. The latter is less efficient given that its technology wastes the advantages of the external effect σ. Since we are only interested in examining tax evasion, we do not consider regular firms in our analysis.

Condition 1 $0 < \sigma < \frac{1-a}{a}$.

The restriction on the size of the moonlighting effect σ and, consequently, the exclusion of any sort of increasing returns of scale, is a necessary assumption to allow the moonlighting firm to choose a finite optimal size of capital dimension, capturing the positive interaction between regular and underground production.[11]

Given these assumptions, total production value is computed by linearly aggregating regular and underground produced outputs. Incorporating the external effect B, total production reads:

$$Y = Y_R + Y_U = A(\mu K)^a + (1-\mu)^a K^{a(1+\sigma)}. \tag{1}$$

The firm behaves as a partial tax-evader, because it complies with fiscal law only for regular production.[12]

The institutional side of the model is defined by the triple $\{\rho, \tau, s\}$, where τ defines a proportional tax rate levied on output, s represents a surcharge factor ($s > 1$) levied on the tax rate if a firm is detected evading; finally, ρ is the probability that a firm is detected and convicted of tax evasion.

The firm's revenues may be expressed as follows:

REV	$\rightarrow$	**D**etected $\underset{(\sim\rho)}{}$	$REV_D = (1-\tau)Y_R + (1-\tau s)Y_U$
	$\searrow$		
		Not **D**etected $\underset{\sim(1-\rho)}{}$	$REV_{ND} = (1-\tau)Y_R + Y_U$

and the expected revenues, under risk neutrality, are:

$$\begin{aligned} E(R) &= E(\rho)REV_D + E(1-\rho)\,REV_{ND} = \\ &= (1-\tau)Y_R + (1-\rho\tau s)Y_U. \end{aligned} \tag{2}$$

It can be shown that the following condition ensures the existence of both productions:

Condition 2 $s > 1$ *and* $\rho s < (1-\rho)$.

[11] In the appendix it is shown that a sufficient condition to allow saddle path stability is: $\sigma < (a-1)^2/a < (1-a)/a$.

[12] The pre-tax technology implies that the regular/underground production ratio is a decreasing function of the total amount of capital, reflecting endogenous total factor productivity in the evaded production. After considering tax enforcement parameters, this negative relationship no longer applies, as the results shown in Sections 5 and 6 demonstrate.

Condition 2 states that the surcharge must be higher than unity, and that the expected surcharge must be lower than the threshold $(1-\rho)$. Otherwise the expected return to a unit of evaded production, $(1-\rho)\tau - \rho\tau s$, would be negative, such that it would not be worthwhile for the firm to operate in the underground sector.[13]

3.2 Value of the Firm

At time zero the firm is endowed with a given positive amount of capital $(\bar{K}_0)$, and with an intertemporally fixed flow of a non-capital resource (labor, land), which are normalized to unity.

Each instant a firm maximizes the intertemporal cash-flow function, choosing how many resources to allocate to the regular production, μ, and how much revenue to invest, I. Investing is a costly process for firms; the assumption here adopted is that the adjustment costs are a convex function of the rate of change of the capital stock (no learning by doing):

$$C(I) = I^b; b > 1.$$

In addition, we assume that investments are encouraged by the government, which provides a capital contribution proportional to total investment, α, to firms which are willing to increase their capital stock. We assume that government is neither able to know whether new capital will be employed in regular or irregular production, nor has accountability tools at its disposal enforcing the firm to declare only the capital regularly employed.[14]

The value of the firm is the expected present value of its revenues net of expenditures on capital input. The representative firm maximizes expected cash flow $\mathcal{V}$ subject to a constraint set:

[13] In this paper we use a simple tax evasion model. There are many issues, concerning the penalty rate, the possibility of detection and audit, that we cannot discuss here. See among others, the survey of Andreoni, Erald and Feinstein (1998), Slemrod and Yitzhaki (2002), Bayer (2006) and Sandmo (2006). Both empirical and theoretical literature usually considers taxation and regulation as the main causes of the existence of the underground sector (see Thomas, 1992; Tanzi, 1980; Dallago,1990). Analysis of tax evasion, starting from Allingham and Sandmo (1972) and Yitzhaki (1974), focuses on the structure of marginal taxation, and/or the consequences for private/social welfare, without investigating the link between tax evasion and technology (see Cowell, 1990; Trandel and Snow, 1999, for surveys on tax evasion, and Alm, 1985 for the welfare effects of evasion). On the other hand, when focusing on the technology of underground activities, the literature very often concentrates on the labor input, neglecting capital utilization (see Portes, Castells and Benton, 1989; Boeri and Garibaldi, 2001; Busato and Chiarini, 2004; Busato, Chiarini and Rey, 2005).

[14] This assumption along with Condition 2 in the main text is a strong incentive toward underground production. A different situation would occur if the fiscal authorities were more effective in allowing incentives to capital than in detecting tax evasion. In this case the rational agent would choose to produce irregularly, $Y_U > 0$, but seek incentives only on the regular share of its investment, $\alpha\mu I$. This hypothesis complicates the analysis considerably, generating unstable and oscillating equilibria.

$$\max_{\{I,\mu\}} \mathcal{V} = \int_{t=0}^{\infty} e^{-rt}\Pi dt \tag{3}$$

$$s.to : \Pi = (1-\tau)A(\mu K)^a + (1-\rho\tau s)(1-\mu)^a K^{a(1+\sigma)} - I - I^b + \alpha I$$

$$: \dot{K} = I - \delta K \tag{4}$$

$$: 0 \leq \mu \leq 1$$

$$: \bar{K}_0 > 0 \tag{5}$$

$$: \lim_{t\to\infty} e^{-rt}\phi_0 K = 0 \tag{6}$$

$$: \alpha \in (0,1)\,; s > 1; \rho s < (1-\rho)\,; 0 \leq \tau \leq 1; 0 < \sigma < \frac{1-a}{a}.$$

The quantity $(1-\tau)A(\mu K)^a + (1-\rho\tau s)(1-\mu)^a K^{a(1+\sigma)}$ represents firm's expected revenues, net of taxation, I is the amount of gross investment, and δ is the physical depreciation rate of capital. The amount αI denotes an investment allowance, where α belongs to the $(0,1)$ interval. This amount could account for several different types of State aid described in **Section 2**, such as grants to firms investing in less developed areas (regional aid), financial facilities to Small and Medium-sized Enterprisers (SME aid), financial facilities for specific sectors (sector aid).

Defining ϕ_0, ϕ_1 and ϕ_2 Lagrange multipliers, the current value Hamiltonian $\mathcal{H}$ reads:

$$\mathcal{H} = \int_{t=0}^{\infty} e^{-rt} \left\{ \begin{array}{c} (1-\tau)A(\mu K)^a + (1-\rho\tau s)(1-\mu)^a K^{a(1+\sigma)} + \alpha I + \\ -I - I^b + \phi_0 (I - \delta K) + \phi_1 \mu - \phi_2(\mu - 1) \end{array} \right\} dt,$$

where r is the exogenous discount rate.

The first order conditions obtain:[15]

[15] The optimization problem is well defined,e.g. the objective function is concave. Actually: $\partial^2\Pi/\partial^2\mu, \partial^2\Pi/\partial^2 K, \partial^2\Pi/\partial^2 I < 0$.

$$\frac{\partial H}{\partial I} = 0 : \alpha - 1 - bI^{b-1} + \phi_0 = 0 \tag{7}$$

$$\frac{\partial H}{\partial \mu} = 0 : (1-\tau)aA\mu^{a-1}K^a - (1-\rho\tau s)a(1-\mu)^{a-1}K^{a(1+\sigma)} + \phi_1 - \phi_2 = 0 \tag{8}$$

$$-\frac{\partial H}{\partial K} = \dot{\phi}_0 - r\phi_0 : \tag{9}$$

$$\dot{\phi}_0 = r\phi_0 - \left[(1-\tau)aA\mu^a K^{a-1} + (1-\rho\tau s)(1-\mu)^a a(1+\sigma)K^{a(1+\sigma)-1} - \phi_0\delta\right]$$

$$\mu \geq 0; \phi_1 \geq 0 \tag{10}$$

$$-\mu \geq 1; \phi_2 \geq 0 \tag{11}$$

$$\dot{K} = I - \delta K. \tag{12}$$

Proposition 1 below proves that the model has an interior solution.

Proposition 1 *Fof firms with moonlighting technology it is not worth either becoming completely regular ($\mu = 1$) nor turning into a ghost firm ($\mu = 0$): i.e. the model does not admit corner solutions.*

Proof. μ as well as $(1-\mu)$ are the basis of a negative power in **Eq. 8**, such that to have a finite solution they must necessarily lie in the open interval $(0, 1)$. ■

Manipulation of the first order conditions leads to the following conditions characterizing optimal capital accumulation and tax evasion:

$$[(\phi_0 - 1 + \alpha)/b]^{1/(b-1)} = I \tag{13}$$

$$(1-\tau)aA\mu^{a-1}K^a - (1-\rho\tau s)a(1-\mu)^{a-1}K^{a(1+\sigma)} = 0 \tag{14}$$

$$\dot{\phi}_0 = (r+\delta)\phi_0 - (1-\tau)aA\mu^a K^{a-1} - (1-\rho\tau s)(1-\mu)^a a(1+\sigma)K^{a(1+\sigma)-1} \tag{15}$$

$$\dot{K} = I - \delta K. \tag{16}$$

The investment function (**Eq. 13**) has standard characteristics: for a given level of fiscal allowances, α, investment is increasing in ϕ_0, and gross investment is zero when the marginal value of capital is just equal to the market price of capital, normalized to 1, net of fiscal allowance. The allowance of fiscal incentives to capital accumulation clearly increases investment.

Eq. 14 ensures the optimal allocation of capital between regular and underground production: the marginal effect of a capital reallocation on the net-of-tax revenues in the two sectors must be equal.

Combining the investment function with **Eq. 16**, we obtain a dynamic system such that:

$$\begin{cases} \dot{\phi}_0 = (r+\delta)\,\phi_0 - (1-\tau)aA\,(\mu)^a\,K^{a-1} - (1-\rho\tau s)\,[1-\mu]^a\,a(1+\sigma)K^{a(1+\sigma)-1} \\ \dot{K} = I\left(\phi_0\right) - \delta K \\ \mu(K) = \frac{K^{\frac{a\sigma}{a-1}}\left(\frac{(1-\rho\tau s)a}{(1-\tau)aA}\right)^{\frac{1}{a-1}}}{1+K^{\frac{a\sigma}{a-1}}\left(\frac{(1-\rho\tau s)a}{(1-\tau)aA}\right)^{\frac{1}{a-1}}} \end{cases} \tag{17}$$

The first condition states that the marginal revenue of capital equals its user cost,$(r+\delta)\,\phi_0 - \dot{\phi}_0$; the second condition implies that K is increasing when ϕ_0 is so much higher than the marginal cost of capital, $1-\alpha$, as to achieve a level of net investment larger than physical depreciation of capital, δK. Finally, the third equation defines the equilibrium level of regular capital as a negative function of the total capital.[16]

3.3 The Steady State

3.3.1 Qualitative Analysis

The Steady state ($\dot{\phi}_0 = \dot{K} = 0$) is characterized by the three equations:

$$\begin{cases} \phi_0 = \frac{1}{(r+\delta)}\left[(1-\tau)aA\,(\mu)^a\,K^{a-1} + (1-\rho\tau s)(1-\mu)^a a(1+\sigma)K^{a(1+\sigma)-1}\right] \\ I\left(\phi_0\right) = \delta K \\ \mu(K) = \frac{K^{\frac{a\sigma}{a-1}}\left(\frac{(1-\rho\tau s)a}{(1-\tau)aA}\right)^{\frac{1}{a-1}}}{1+K^{\frac{a\sigma}{a-1}}\left(\frac{(1-\rho\tau s)a}{(1-\tau)aA}\right)^{\frac{1}{a-1}}} \end{cases} \tag{18}$$

The first equation suggests that in equilibrium (long run) the shadow price of capital is the discounted value of the net-of-tax marginal productivity of capital; the second condition states that the stock of capital is stable when investment is just equal to physical depreciation of capital; finally, the last relation expresses the optimal allocation of the capital stock between regular and underground production. It should be stressed that the long run equilibrium can only be described in a three-dimension space, and given the non-linearity of the involved relationships, we are compelled to use calibration to describe the nature of the steady state.

Proposition 2 *In the long run, the dynamic system of Eq. 17 admits a unique steady state.*

Proof. *APPENDIX* ■

[16] The negative relation expressed by the $\mu(K)$ equation in System 17 is a consequence of the endogenous TFP in underground technology.

Proposition 3 *The steady state of the dynamic system of Eq. 17 is always a saddle path.*
Proof. *APPENDIX* ■

3.3.2 Model Calibration

The model depends on five parameters. We calibrate these parameters for Italy, a country with a considerable size of hidden activities and high tax evasion. Moreover, as stressed earlyer in this paper, underground activities are characterized by high percentage of moonlighting firms, and the Governments in this economy have repeatedly supported firms with capital subsidies and tax allowances.[17]

The capital elasticity a, consistent with the standard literature, is set at the value 0.3; the exogenous discount rate, r, set at 0.025; the rate of physical depreciation of capital, δ, calibrated to 0.125. The technological parameters A and σ, are set, respectively tat 10 and 0.5. The latter, the *moonlighting effect*, must be consistent with Condition 1, defined above, and with the **Appendix** where we outline the model conditions to achieve a saddle path. With regard to parameter A, it represents the scale of production in regular technology and more details will be provided below.

Next, the tax rate, τ, is set at 0.4 to match the average high level of corporate taxation in Italy in recent years; the surcharge applied to tax evaders, s, following Italian civil law, is set at 1.3; the probability of being caught when cheating the government, ρ, is set at a very low value, 0.05, to give an idea of low enforcement, which can be assimilated to Italian actual conditions; finally, the size of incentives to capital accumulation, α, is set at 14% in the baseline calibration.[18]

Model Calibration: the benchmark

α	A	τ	σ	r	ρ	s	a	δ
0.14	10	0.40	0.5	0.025	0.05	1.3	0.3	0.125

Given this set of parameters, the solution of the dynamic system identifies a single long run equilibrium, given by the equilibrium vector:

$$\left(K^* = 15.5242; \phi_0^* = 2.0354; \mu^* = 0.8818\right).$$

[17] For the size of the underground economy, see, Baldassarini and Pascarella (2003), Schneider and Enste (2002), and Busato, Chiarini and Di Maro (2006). An outline of the State aid to firms in Italy may be found in Bosco (2002) and Ministero delle Attività Produttive (2005).

[18] Calibration of the fiscal parameters τ and α was chosen starting from the recent analysis of the Italian firm fiscal regimes addressed in Bontempi et al. (2001). In particular, incentives to investment identified as *Credito di Imposta* ranges from an average level of 0.14 for the Center-North regions, to 0.65 for the least developed region (Calabria). As for corporate taxation, the figure reported in KPMG (2004) for Italy is 37.25%. See also Busato and Chiarini (2004) for calibration of a macroeconomic model with tax evasion.

3.3.3 Steady State Relations (K, ϕ_0) and (μ, K)

The three steady state relations expressed by **System 18** can be geometrically represented in the space (K, ϕ_0, μ). In order to provide more insights into the local dynamics around the steady state, we prefer to represent them in two bi-dimensional graphs as in **Figure 1**.[19] The left panel of the Figure displays the two steady state relations $\dot{\phi}_0 = 0$ (the shadow price of capital) and $\dot{K} = 0$ (the stock of capital), while in the right panel the locus depicts the share of capital allocated in the regular sector, $\mu(K)$.

For our parametrization, the first equation of **System 18** describes a convex and negatively shaped curve: a larger amount of capital reduces its marginal productivity, so that in equilibrium a lower value for the shadow price for capital is commanded (see the $\dot{\phi}_0 = 0$ locus in the left panel of **Figure 1**). The locus $\dot{K} = 0$ (see the left panel of **Figure 1**) is displayed as an increasing relationship in the space (K, ϕ_0), consistently with the standard literature on the investment function; as investment must be equal to the depreciation in the capital stock, then, in order to maintain a higher stock of capital a higher shadow price is required.

The right-hand panel in **Figure 1** represents, in the space (μ, K), the relationship between the regular share of capital μ and the total stock of capital (K), defined by the last equation of **System 18**: for each level of K identified by the solution of **System 18**, a unique cash-flow maximizing value of μ is identified. The locus $\mu(K)$ is monotone and decreasing: given the nature of the *moonlighting effect*, the larger the amount of total capital, the more benefit is obtained in shifting it to underground production (e.g. μ drops).

The left-hand panel in **Figure 1** also displays the local dynamics: the stability arrows show that there is a single stable arm which leads the firm toward the long run equilibrium. The upper left side of the stable arm is characterized by a stock of capital lower than the equilibrium and a shadow price of capital higher than the equilibrium level, such that the rational firm increases the stock of capital (net investment is greater than capital depreciation) until the shadow price reaches its equilibrium level, at the steady state. When the capital stock dimension is lower than the optimal level, given **Eq. 14**, the regularity share, μ, is higher than optimal (see also the right-hand panel in **Figure 1**); during the process of capital accumulation, the firm also shifts capital into underground technology (i.e. μ drops). This allocating process lasts until the marginal productivity is equal across sectors (formal and informal sectors, see. **Eq. 14**). An analogous symmetric process applies when the capital dimension is higher than the optimal level and the firm operates on the lower and

[19] As stated above, we have a non-linear system described in a three-dimensional space. In order to find the steady state characteristics we must calibrate and simulate the system.

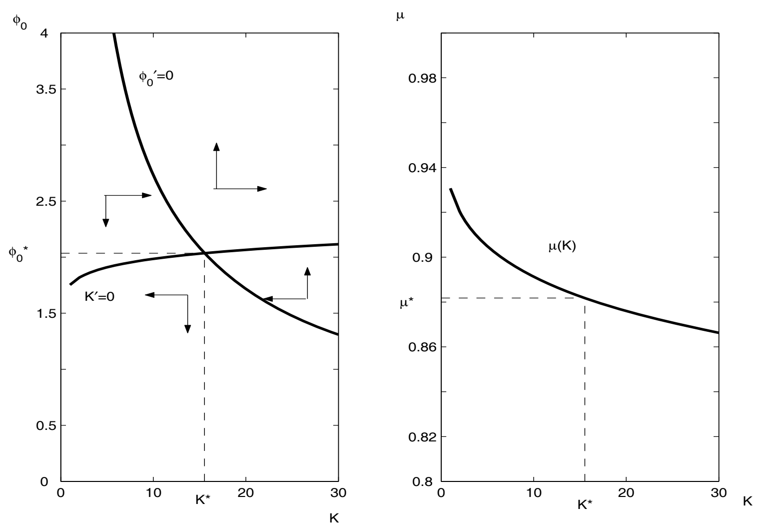

Figure 1: Points above the locus $\dot{\phi}_0 = 0$ are characterized, for each level of K, by a ϕ_0 higher than the equilibrium level; given the dynamic expressed in System 17 it implies a growth in the shadow price of capital (arrows pointing up). Similarly, when considering points above $\dot{K} = 0$, we register for each K a ϕ_0 higher than the equilibrium level; given the investment function, Eq. 13, and the dynamic expressed in System 17, it implies a growth of capital stock (arrows pointing right).

right-hand side of the stable arm.[20]

4 The Firm's Structure

In this section we discuss the role of the external effect "internalized" by the simultaneous implementation of regular and underground technology, σ, and the scale effect A. The latter is related to the dimensional effect of firms, whereas the former reflects, as stressed in the model, a broad series of indirect benefits, stemming from regular production, and not available for ghost firms.

4.1 The Moonlighting Effect

The new element in the technology that characterizes our firms is explicit specification of the advantages of operating in the different sectors (formal and informal). The moonlighting effect proposed in this paper shows how the evader and the regular entrepreneur are

[20] Of course, every path other than the saddle path taakes the firm far from the long run equilibrium to areas in which the transversality condition (Eq. 6) no longer applies.

"intertwined" and how great is the advantage of operating together through the economy as a result of transparent support of regular activity for irregular production. This insight follows directly from the model and, as stressed above, it is supported by empirical evidence, emerging from several surveys, on the characteristics of firms that operate in the underground sector.

Thus, unlike models which suggest that underground firms are less efficient and less productive than regular firms, the key assumption we support here is the existence of a different category of the shadow production. This firm is able to exploit profitable opportunities not available to ghost firms. In this section we perform a comparative dynamics analysis, considering how different values for the technological parameter σ affect the structure of the moonlighting firm, and the allocation of the total capital stock between the two technologies. Intuitively, a larger value for σ implies that the moonlighting firm strongly benefits from the simultaneity of its two productions.

Increasing the Moonlighting Effect

α	A	τ	$\sigma^{\bigstar}$	r	ρ	s	a	δ
0.14	10	0.40	1.0 (0.5)	0.025	0.05	1.3	0.3	0.125

For instance, doubling the size of the moonlighting effect from the benchmark value 0.5 (in brackets in the table) to 1 causes a considerable increase in the equilibrium level of the capital stock as well as a strong reduction in the share of regularity; the new equilibrium vector is:

$$\left(K^{*} = 20.6855; \phi_0^{*} = 2.0696; \mu^{*} = 0.7857\right).$$

These figures account for a 33% increase in the size of the capital stock, and quite a large increase in its irregular use, as μ is downsized by 10% compared to the benchmark calibration (see **Section 3.3.2**).

Figure 2 presents the graphical analysis of this shock. The change in the "internal externalities", σ, identifies a new steady state at a higher level of both capital and its shadow price.[21] There is initial overshooting of the shadow price of capital, generating an investment process, which lasts until the new long run equilibrium is reached. A larger value of σ triggers an investment process as well as a sudden drop in the regular share of capital, which persistently decreases until the stock of capital reaches its long run level. To better appreciate the intuition, it is useful to reconsider **Condition 1** (the restriction $\sigma < \frac{1-a}{a}$ aims

[21] The parameter reflects the dimension of an external effect internalized by the simultaneous implementation of regular and "hidden" technology.

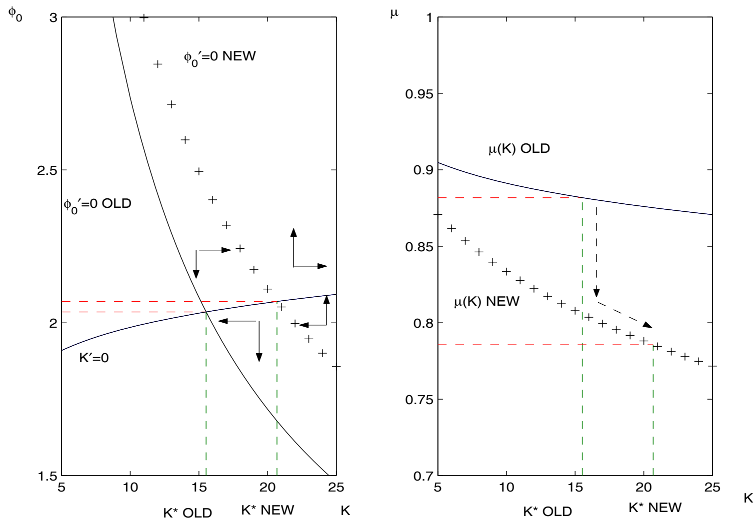

Figure 2: the rise in σ immediately rises the marginal productivity of capital in the underground technology, so that the shadow price of capital rises, and the curve $\dot{\phi}_0 = 0$ moves upward; simultaneously, the firm reallocates capital into the underground production, and the curve $\mu(K)$, in the right panel, moves downward.

to avoide the occurrence of increasing returns to scale). Without this assumption, there would be no equilibrium, and the concavity of the objective function would be no longer ensured. Even more interestingly, under increasing returns of scale due to the moonlighting effect, it is not worth continuing moonlighting, and the solution converges toward a ghost firm, i.e. $\mu \to 0$, while dimension is no longer determinate.[22]

The marked effect on firm structure arising from the different size of the moonlighting effect suggests how important the latter may be in order to determine the impact of policies. This topic will be investigated in **Section 5.3**.

4.2 The Scale of Regular Technology

Parameter A is the scale of production in regular technology, and it produces important implications for the relationship between firm size and underground activity:

$$Y_R = A\left(K_{REG}\right)^a \equiv A(\mu K)^a$$

Changes in this technological parameter generates great differences with the baseline

[22] Graphically, the locus $\dot{K} = 0$ would have the usual increasing shape, but we would also observe an increasing locus $\dot{\phi}_0 = 0$ situated above the $\dot{K} = 0$ such that no equilibrium could be found.

calibration, both in terms of optimal capital dimension and share of regularity. For instance, using a unitary productive scale, and leaving all the other parameters unchanged, we obtain a new long run equilibrium for optimal capital dimension and its allocation between the two sectors:

$$(K^* = 2.92; \mu^* = 0.28).$$

By contrast, a larger productive scale, say $A = 20$, generates the equilibrium:

$$(K^* = 34.94; \mu^* = 0.94).$$

This result is essentially derived from **Eq. 14**, which guarantees the optimal allocation of capital between the two productions.

These experiments suggest a strong and direct relationship between the scale of regular production, which can be considered a proxy for the firm size, and the choice to operate regularly. The literature is almost unanimous on the importance of firm size in affecting the propensity to operate in the underground economy. For instance, a recent survey (Di Nicola and Santoro, 2000), based on tax audits on a representative sample of Italian companies, points out the main characteristics of Italian firms which evade levied taxation. In particular, tax evasion is more widespread among small firms and new firms, especially when located in the south of Italy. Moreover, tax audits show that evasion is more common in firms with a weak property structure.

5 Fiscal Policy Experiments

Although a large body of literature already exists on the nature and effects of state aid, no general agreement is achieved about its final effects on the economy. Moreover, as far as we know, studies which deal with the effect of fiscal aid policies upon underground production are rare. This section presents two selected fiscal policy experiments, a tax reduction and a rise in capital subsidies, to evaluate their impact on the long run equilibrium as well as on the investment policy of the moonlighting firm.

5.1 Case # 1: Tax Advantages

There is widespread agreement concerning the depressing effect on investment of taxation (see Summers, 1981; Abel, 1982). However, here the tax policy provides useful insights for the comparative analysis we carry out below, with subsidies to capital stock in the presence of different firms' technology structures (A and σ).

In this section, we consider how tax relief on business may affect firm choice under moonlighting technology. Therefore we start our analysis by considering the effects of a cut in the corporate tax rate, τ, which we use as a tax-favoured treatment to investment, from the benchmark value 0.4 (in brackets in the table) to 0.2:

Reducing Corporate Tax Rate

α	A	$\tau^{\bigstar}$	σ	r	ρ	s	a	δ
0.14	10	0.20 (0.40)	0.5	0.025	0.05	1.3	0.3	0.125

Cutting corporate taxation by 50% causes an increase in the long run level of capital stock as well as the share of regularity: the new steady state equilibrium is given by the equilibrium vector:

$$(K^* = 21.6026; \phi_0^* = 2.0749; \mu^* = 0.9114).$$

The policy causes a 40% increase in capital stock, and a 3.4% increase in the size of the regular use of capital. The tax reduction increases the net-of-tax marginal revenue of capital. This occurs in more marked fashion in regular technology due to Condition 2.

Figure 3 presents the graphical analysis of this shock; the change in the taxation rate identifies a new steady state at higher level of both capital and its shadow price. There is initial overshooting of the shadow price of capital, generating an investment process, which lasts until the new long run equilibrium is reached.

It is then interesting to investigate the dynamics of the capital allocation, μ. The fall in the taxation ratio, $(1 - \rho\tau s)/(1 - \tau)$, due to the fall in the tax rate, alters the equilibrium relationship between K and μ. The relationship expressed in Eq. 14 balances the marginal productivity of capital in the two productions. Hence a variation in teh taxation ratio alters the relative profitability in the two productions.[23] A tax cut, *ceteris paribus*, induces the moonlighting firm to be more regular, experiencing initial overshooting in its regular size, for each level of the total capital stock.

As long as the capital stock increases toward its steady state level, the firm will also reduce the share of regularity, adjusting toward its consistent equilibrium level of μ.

[23] Given Condition 2 in the main text, a fall in the tax rate necessarily causes, in the steady state solution (18), a fall in the taxation ratio $(1 - \rho\tau s)/(1 - \tau)$.

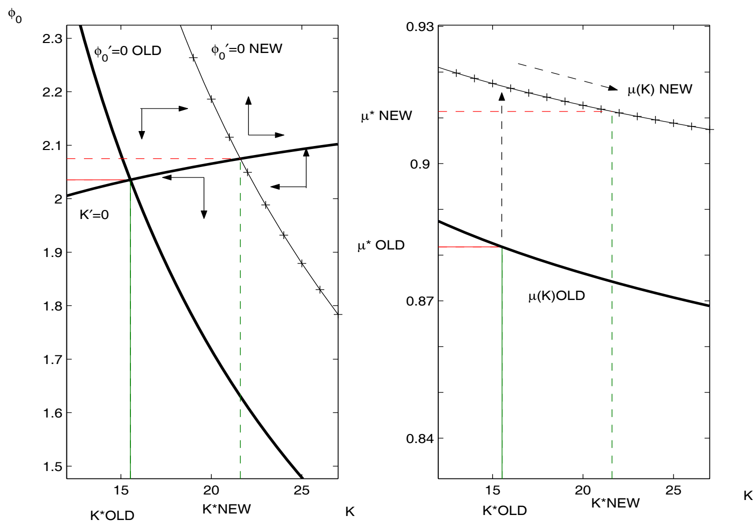

Figure 3: The new tax rate causes an upward shift, due to the increase in the marginal revenue of capital, in the curve $\dot{\phi}_0 = 0$, while the $\dot{K} = 0$ is unchanged. In the right-hand panel there is an upward translation of the curve $\mu(K)$ due to the fall in the taxation ratio. Given the initial stock of total capital, the share of its regular use, μ, jumps on the new $\mu(K)$ curve (the cross one), so that an initial overshooting occurs (see the dashed arrows).

5.2 Case # 2: Capital Subsidies

The empirical literature on the effects of capital subsidies has produced contrasting evidence. Roller et al. (2001) evaluate the effectiveness of state aid, and find that subsidies are the most effective tool, while tax reliefs are as efficient as subsidies only for regional aid. Bergstrom (1998) analyses the effects on total factor productivity of public capital subsidies to firms in Sweden between 1987 and 1993; the results suggest that while subsidization can influence the growth of value added, it does not affect either productivity or competitiveness. When considering studies available for Italy, Cannari and Chiri (2001), using a macroeconomic framework, find no evidence for a significant influence of investment subsidies either on value added growth or on employment growth in the manufacturing sector. By contrast, Pellegrini and Carlucci (2003), using a different estimation strategy, find a positive effect of capital subsidies on employment growth in the examined firms.[24] However, dealing with capital subsidies to firms also has important implications for underground activities and tax

[24]Of course, bureaucrats and politicians grant subsidies as representative of the firms or people living in the area or working in the sector. Anyway, also when having a positive impact on the economic aggregates, the dynamic effects of capital incentives must be accounted for, since they may distort the market structure if they keep inefficient firms afloat while inducing the more efficient ones to exit the market.

evasion. In this case, we attempt to draw on insights from these circumstances using our model and allowing for an increase in subsidies. The policy experiment we carry out with the model is a change (a doubling up) in the size of fiscal allowances to capital accumulation, moving from the benchmark value 0.14 (in brackets in the table) to 0.28:

Dispensing Capital Subsidies

$\alpha^{\bigstar}$	A	τ	σ	r	ρ	s	a	δ
0.28	10	0.40	0.5	0.025	0.05	1.3	0.3	0.125
(0.14)								

Increasing the size of subsidies to capital accumulation pushes up the equilibrium level of the capital stock up, as we would intuitively expect, but also generates a marginal reduction in the share of regularity. The new equilibrium is given by the equilibrium vector:

$$(K^* = 17.1079; \phi_0^* = 1.9069; \mu^* = 0.8796) .$$

These figures represent a 10% increase in capital stock and a 0.2% reduction in the regular use of capital. The increase in the incentive to capital accumulation reduces the cost of capital, such that there is an immediate effect on investment. In the left-hand panel in **Figure 4** the locus $\dot{K} = 0$ moves downward, and the shadow price of capital jumps on the new saddle path to a level higher than the new equilibrium, inducing a flow of new investment. The rise of capital stock alters the equilibrium marginal productivity (Eq. 14), such that as long as net investment is positive, the firm also reallocates capital between sectors. Since the TFP in the irregular technology is endogenous ($\sigma > 0$), it is optimal to reduce μ until the new equilibrium of capital is reached (right-hand panel in **Figure 4**).

The important point is that, contrary to the presumption that subsidies may also be useful for pushing firms to operate "over ground", in the presence of moonlighting technology, the incentives to improve capital stock are actually counterproductive in this sense, and increase the informal economy overall. To provide some empirical macroeconomic evidence to support our results, **Figure 5** reports investment incentives (source: Ministero Attività Produttive) and numbers of irregular workers (source: ISTAT) for each of the 20 Italian regions. Casual inspection suggests that there is a positive correlation between the two measures, which is particularly marked when considering irregular workers in the industrial sector (the coefficient of correlation is 0.88). In spite of the State aid grant to business over the decades, the phenomenon of underground activity persists and has increased. Indeed subsidies to capital stock have fuelled informal production.

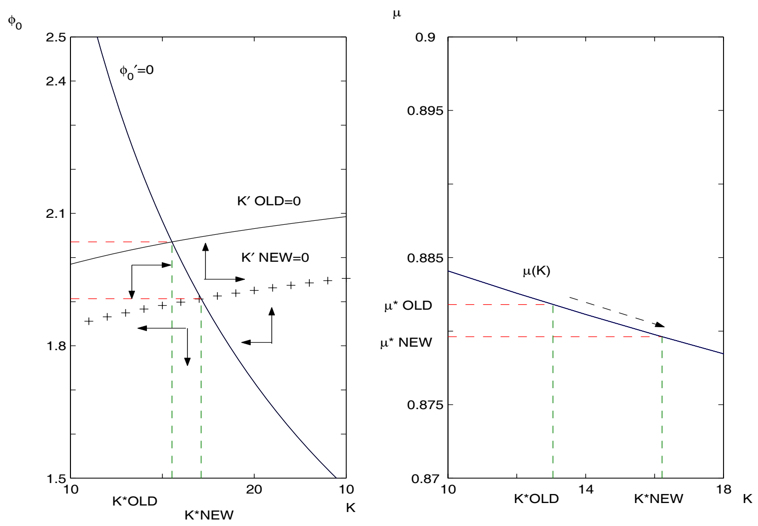

Figure 4: rising subsidies to investment generates capital accumulation, but it also spoors the allocation of capital into the underground production.

The most intuitive explanation of the positive correlation between the two indexes is typically traced back to the common factor “underdevelopment”. Indeed the underground economy and economic underdevelopment are often

analyzed together (Loayza, 1996; Johnson et al., 1997). The evidence shown by our model may introduce an additional argument, that is an opportunistic attitude of firms operating in areas with large fiscal benefits and a large underground economy. The investment subsidy is a non excludable public good that opens room for free-riding (tax evasion in this context). Here Government is not capable, because of un-modelled monitoring costs, to distinguish between regular and moonlighting firms. Firms therefore have an incentive to declare a sufficiently small amount of revenues as to be eligible for obtaining the subsidy, while investing relatively more in the underground economy and pocketing the tax wedge. Technology matters for determining the extent of the declared production.[25]

[25] The first order conditions suggest that the critical parameters for capital allocation in underground production are the size of the *moonlighting effect*, σ; the scale of regular production, A; and the taxation ratio $(1-\rho\tau s)/(1-\tau)$. In particular, if we calibrate the TFP in regular technology, A, to a value lower than the benchmark, the marginal effect of a rise in incentives on the irregular share of capital is sharper. For instance, setting A=5, the fall in the regular use of capital is about 0.5%; for A=2 the fall is about 1%. A similar effect occurs when calibrating higher values for the moonlighting effect σ.

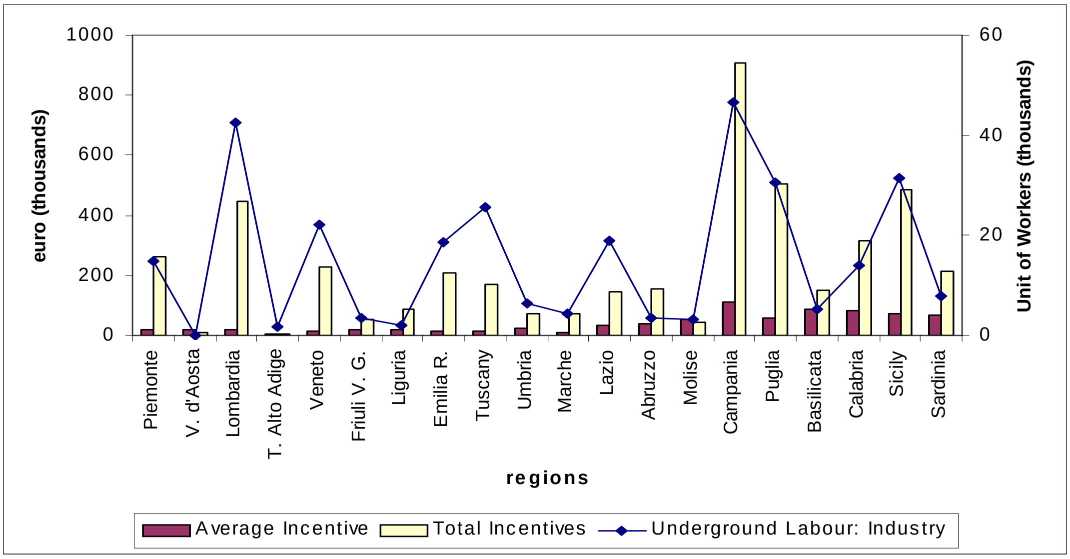

Figure 5: Undeground employment and capital subsidies in Italian regions, 1998.

5.3 Policy Implications

To discuss the implications of policy measures under underground economy, it is useful to provide a qualitative and quantitative syntheses of the experiments reported in the previous sections.

5.3.1 Output Effects

Table 5 reports the qualitative effects of the investigated policies on the steady state value of total capital K^* and its regular share μ^*. The table shows that a tax reduction and a more generous subsidies policy have a positive impact on capital accumulation, whereas their consequences on the underground capital share are conflicting. A tax reduction provides an incentive to shift capital into the regular economy, whereas increasing government subsidies - to raise investment- also encourages firms to engage in underground production. Moreover, the smaller the scale of production, the greater is the effect of encouraging underground production.

Output effects are presented in **Table 6**, where the effects of a tax reduction and of an increase in capital subsidy are investigated under three different scenarios (or also different moonlighting firms).

In the first simulation, Scenario 1, all the parameters are set at their benchmark cal-

Table 5: Qualitative effects of different fiscal policies on the long run equilibrium

Fiscal Policy	K^*	$\boldsymbol{\mu}^*$
Tax Relief	+	+
Increase in Capital subsidies	+	-

Table 6: Quantitative effects of fiscal policies on the long run equilibrium. Long run equiibrium without the policy is set at 100.

Fiscal Policy	*Scenario 1*	*Scenario 2*	*Scenario 3*
	$A = 10; \sigma = 0.5$	$A = 10; \sigma = 1$	$A = 1; \sigma = 0.5$
Tax Relief	110.7	109.6	103.6
Capital subsidies	102.9	103.2	103.7

Scenarios 1 and 2 assume two of the many possible values for the technological parameters σ and A. Given the non-linearity of the relations involved, we tested (through numerical calibration) for the monotone pattern of the relationship between total production and both σ and A, which is confirmed.

ibration (see **Section 3.3.2**); by contrast, in the other two simulations the technological parameters σ and A are modified.

In the third column of the Table, Scenario 2, the moonlighting effect is set at 1 (double that of Scenario 1), while in the last simulation (Scenario 3) the technological parameter A is set at 1 (sharply reduced against the benchmark calibration). Scenario 2 could well account for an economic environment with a higher opportunity for tax evasion. Notice that parameter σ can also be broadly considered a policy instrument, in the sense that the possibility for the moonlighting firm to exploit the external effect of the aggregate capital is supposed to be a function of the institutional and social framework in which firms operate. Finally, Scenario 3 could well represent small sized firms, which are among the main beneficiaries of the horizontal State aid policy.

The output effects confirm the importance of these parameters in determining the effectiveness of the policies. Two issues deserve attention. First, both corporate tax reductions and increases of capital subsidies exert a positive impact on total output, as a consequence of the policy effects already pointed out in **Section 5**. Second, the impact of the corporate tax reduction decreases, shifting from the benchmark scenario to the other two, whereas the impact on the total output of the larger amount of capital subsidies increases. This asymmetric performance is referable to the different qualitative effect of each of the examined policies on the share of regularity, μ, as summarized in **Table 5**.[26]

[26] The model allows us to consider also the effects of change in enforcement on the size and allocation of total capital, as well as the effects on total output. As demonstrated in Chiarini and Marzano (2005), improving the level of enforcement causes an increase in the equilibrium level of the share of regularity, but also a reduction in capital stock and in total output.

In order to explain the different output effects, one should keep in mind the qualitative effects of fiscal policies reported in **Table 5**, and to consider how total factor productivity (TFP) works in moonlighting technology, through **Eq. 1**, reported below for the reader's convenience:

$$Y = Y_R + Y_U = A(\mu K)^a + (1-\mu)^a K^{a(1+\sigma)}.$$

The larger the moonlighting effect, the higher is total factor productivity in underground technology, whereas the lower the scale effect, A, the lower is the TFP in the regular technology. Consequently, both Scenario 2 and 3 display a firm structure in which underground technology is relatively more profitable than in Scenario 1 (i.e. the benchmark calibration).

Tax cut policy and firm's technology structure. Under Scenario 2, characterized by a strong moonlight effect σ, after a tax cut, the capital re-allocation toward formal production (i.e. a relatively larger μ), would prove more costly compared to the benchmark calibration. An analogous effect operates under Scenario 3, characterized by a small firm's scale A: after a tax cut, the capital re-allocation toward the formal production (a larger μ) would be relatively less profitable than under Scenario 1. In these new technological structures, a tax cut policy produces a more costly shift towards regular sector production.

These arguments clearly explain why a corporate tax cut, under different advantages of operating in underground activities, provides an increase in total welfare of 9.6% (Table 6, Scenario 2) and 3.6% (Scenario 3) compared to a benchmark figure of 10.7% (Scenario 1). In other words, when considering a tax reduction, output gains are increasingly lower moving from Scenario 1 to Scenarios 2 and 3 since the expansion of regular production occurs under more productive underground technology.

Capital subsidies policy and firm's technology structure. This reasoning helps to interpret the consequences of more generous capital subsidies under the three different scenarios. In fact, given the nature of the TFP characterizing the moonlighting firm, an increase in investment allowances leads to a fall (albeit small) in the share of regularity μ. This outcome has been described in Section 5.2. The reduction in the share of capital allocated to the regular sector, activates a reverse process compared to the previously examined policy. After a rise in subsidies to moonlighting firm investment, the capital shift toward underground technology is more profitable both under Scenario 2 (σ high) and 3 (A small) than the benchmark calibration (Scenario 1). Compared to the benchmark case, now output gains are grater for larger moonlighting effects and lower productive scales: as the firm adds capital to its underground production, with much more favourable externalities and scale effects, this unit of capital becomes slightly more productive than under benchmark technology.

Doubling the size of the incentives to investment (from the benchmark 0.14 to 0.28) induces a 2.9% increase of total welfare under the benchmark calibration compared to an increase of 3.2% and 3.7% under, respectively, Scenarios 2 and 3.

5.3.2 Sensitivity Analysis

Further useful insights may be drawn from the analysis of the reaction function, which expresses the long run values of total capital and its regular share as a function of the various sizes of each single fiscal policy parameter. The right-hand panel in **Figure 6** shows the effects of the variation in a single parameter (tax rate, and moonlighting effect) on the size of regular capital, (μ), while in the left-hand panel the reaction functions for total capital stock are displayed.

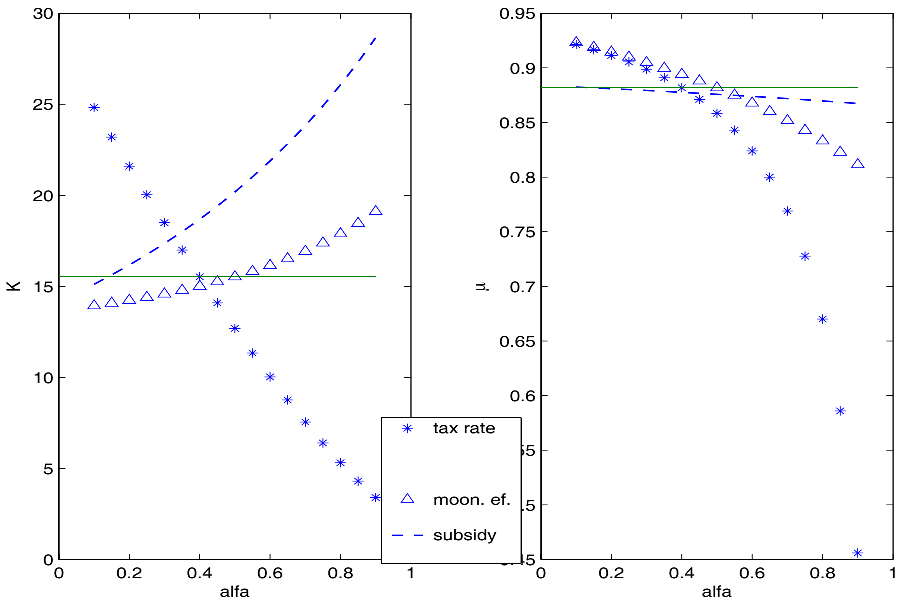

Figure 6: reaction to taxation (star); incentives (dash); moonlighting effect (triangle). The range of variation of each parameter is (0.1-0.9). The horizontal line represents the equilibrium level of total capital (left panel) and its regular use (right panel) in the benchmark calibration. Each reaction function crosses the horizontal line when the relevant fiscal policy parameter reach its benchmark value.

Figure 6 shows that reaction functions are always monotone, but they are also non linear. The figure highlights the deep impact of taxation (cross-line) both on the capital stock and on the size of its irregular use; none of the other parameters have a quantitatively similar impact. Tax policy is the only measure able to generate a co-movement in the two objective variables, total capital and regular share: starting from the baseline value of taxation, 0.4, a fall in tax rate generates a capital as well as a regular share increase, and a welfare gain.

Judging from **Figure 6**, we should also conclude that the fiscal authorities should be very careful when planning policies to support investment, especially in areas where the underground economy is sizeable. The figure shows that capital subsidies impact deeply on capital accumulation, but they also produce an increase in the irregular use of capital. In designing policy subsidies to the stock of capital, what should be taken into account is the "nature" of the firm, and, in particular, whether in the sector, and also in the area where the firm operates, a large part of output is unreported.

As incentives to investment in the presence of moonlighters always produce incentives to go underground, it may be argued there is the risk that this policy proves, via underground activities, to be a time-inconsistent policy. If government policies support moonlighting firms, the latter will have less incentive to increase their reported capital, since doing so could entail a reduction in the level of incentives they enjoy.[27] Therefore, the government would be forced to provide incentives to capital accumulation for longer than expected.

6 Conclusions

In this paper we have investigated the effects of two fiscal policies to support firms, which can be broadly included among State aid policies, in a context characterized by hidden activity.

As we have seen, State aid accounts for a considerable share of GDP in EU countries; moreover, among the different instruments, a very large amount of aid is provided directly through grants to small sized firms. Comparison of EU tax rates and state aid presented in **Section 2** reveals that countries with high corporate taxation, such as Germany, France and Italy, also provide large amounts of total EU State aid. In particular, in these countries high corporate rates are recorded along with high tax exemptions. Since tax burden has always been considered one of the main causes of growth in underground activities, there could be strong links between State aid policies, tax policy and the underground economy.

The innovation of the paper is twofold: first, we represent a specific technological advantage (the "internal externalities") of *moonlighting* firms over *ghost* firms. Second, we consider the implication of this framework for assessing selected aid policies under tax evasion.

In this regard, we set out an optimal investment model in which a representative firm maximizes the expected cash flow, selecting the optimal combination of capital stock and its

[27] See, for instance, Tornell (1991) analysis for trade policies.

allocation between regular and irregular technologies. The model provides several striking policy implications.

First of all, the troubling aspects of capital subsidies. In the context of moonlighting firms, which display a kind of technology that is certainly not purely theoretical, but is strongly supported by empirical evidence, government's incentives to capital stock prove an incentive not only to capital accumulation but also to its underground use. This policy is clearly counterproductive if a government aims also to reduce underground production and tax evasion.

The second striking result concerns the troubling aspects of the technology of moonlighters. Although tax allowances and capital subsidies both cause a positive effect on total output, their effectiveness is basically related to the "internal externalities" and to the size of the benefit recipients.

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7 Appendix

Proposition 2 *In the long run, the dynamic System of Equation 17 admits a unique steady state.*

Proof. *System 18 can be written as follows:*

$$\begin{cases} \phi_0 = \frac{1}{(r+\delta)}\left[(1-\tau)aA\left[\mu^*(K)\right]^a K^{a-1} + (1-\rho\tau s)\left[1-\mu^*(K)\right]^a a(1+\sigma)K^{a(1+\sigma)-1}\right] \\ \phi_0 = 1-\alpha+b\left(\delta K\right)^{b-1} \\ \mu^*(K) = \frac{CK^d}{1+CK^d} \end{cases}$$

First Step:

To show that the first equation expresses ϕ_0 as a monotone and decreasing function of the stock of capital K .

$$\frac{d\phi_0(K)}{dK} = \varphi\left[(a-1)\left[\mu^*(K)\right]^a K^{a-2} + a\mu^*(K)'\left[\mu^*(K)\right]^{a-1}K^{a-1}\right] +$$

$$+\chi\left[(a(1+\sigma)-1)(1-\mu^*(K))^a K^{a(1+\sigma)-2} - a\left[1-\mu^*(K)\right]^{a-1}K^{a(1+\sigma)-1}\mu^*(K)'\right]$$

$$\varphi = \frac{(1-\tau)aA}{(r+\delta)} > 0; \chi = \frac{(1-\rho\tau s)a(1+\sigma)}{r+\delta} > 0.$$

The expression derived from equation 14 in the main text:

$$\mu^*(K) = \frac{CK^d}{1+CK^d}; d = \frac{a\sigma}{a-1} < 0; C = \left(\frac{(1-\rho\tau s)a}{(1-\tau)aA}\right)^{\frac{1}{a-1}} > 0$$

is a strictly decreasing and monotone function of K:

$$\frac{d\mu^*(K)}{dK} = \frac{CdK^{d-1}\left[1+CK^d\right]-CdK^{d-1}\left(CK^d\right)}{\left[1+CK^d\right]^2} = \frac{CdK^{d-1}}{\left[1+CK^d\right]^2} < 0 \quad \forall a < 1$$

It implies that the first term of $\frac{d\phi_0(K)}{dK}$ is always negative, such that we need to demonstrate that the second one is negative too:

$$\left[a(1+\sigma)-1\right]\left[1-\mu^*(K)\right]^a K^{a(1+\sigma)-2} - a\left[1-\mu^*(K)\right]^{a-1}K^{a(1+\sigma)-1}\mu^*(K)' < 0$$

Using again the definition of $\mu^*(K)$ as well as $\frac{d\mu^*(K)}{dK}$ we get:

$$\left[a(1+\sigma)-1\right]\left[\frac{1}{1+CK^d}\right]^a K^{a(1+\sigma)-2} - a\left[\frac{1}{1+CK^d}\right]^{a-1}K^{a(1+\sigma)-1}\frac{CdK^{d-1}}{\left[1+CK^d\right]^2} < 0$$

$$\left[a(1+\sigma)-1\right]\left[\frac{1}{1+CK^d}\right]^a K^{a(1+\sigma)-2} - aK^{a(1+\sigma)-2}\frac{CdK^d}{\left[1+CK^d\right]^{a+1}} < 0$$

$$\left[a(1+\sigma)-1\right]\left[\frac{1}{1+CK^d}\right]^a - a\frac{CdK^d}{\left[1+CK^d\right]^{a+1}} < 0$$

$$\left[a(1+\sigma)-1\right] - a\frac{CdK^d}{\left[1+CK^d\right]} < 0$$

$$\left[a(1+\sigma)-1\right] - ad\mu < 0.$$

As μ is a majorant of this last expression, we consider the case $\mu = 1$ to get:

$$\sigma < (a-1)^2/a < (1-a)/a$$

This condition can be considered a sufficient condition to obtain the required monotony of the relation $\dot{\phi}_0 = 0$.

Second Step

The second equation expresses ϕ_0 a monotone and increasing function of the stock of capital K. Indeed, $\frac{d\phi_0}{dK} = b(b-1)(\delta K)^{b-2} > 0$ for each $b > 1$.

Given that the codomain of the first equation is $(0;+\infty)$ *while the second equation has codomain* $(1-\alpha;+\infty)$*, it follows that there exists a single value of* K *such that the two equations simultaneously apply.* ■

Proposition 3 *The steady state of the dynamic System of Equations 17 is always a saddle path.*

Proof. The first step is to combine the last Equation of System 17 into the first one:

$$\begin{cases} \dot{\phi}_0 = (r+\delta)\,\phi_0 - (1-\tau)aA\,(\mu^*)^a\,K^{a-1} - (1-\rho\tau s)\,[1-\mu^*]^a\,a(1+\sigma)K^{a(1+\sigma)-1} \\ \dot{K} = I\,(\phi_0) - \delta K \end{cases}$$

The Jacobian of this System of Equations, evaluated at the steady state is:

$$\begin{bmatrix} r+\delta & -\partial^2\Pi/\partial^2 K \\ \partial I/\partial\phi_0 & -\delta \end{bmatrix}$$

and it has a trace and a determinant given by:

$TR = r; DET = -\delta\,(r+\delta) + \partial I/\partial\phi_0\left(\partial^2\Pi/\partial^2 K\right);$

where

$\partial^2\Pi/\partial^2 K = d\left[(1-\tau)aA\,(\mu^*)^a\,K^{a-1} + (1-\rho\tau s)(1-\mu^*)^a a(1+\sigma)K^{a(1+\sigma)-1}\right]/dK$

Local stability, and in particular saddle path stability, requires that the trace should be positive, while the determinant should be negative, when evaluated at the steady state. Under our parametrization it implies that the condition $\partial^2\Pi/\partial^2 K < 0$, which is the necessary condition to get a concave objective function, is also a sufficient condition to get saddle path stability. Given the demonstration of the first step of Proposition 2, it follows that

$\partial^2\Pi/\partial^2 K < 0$.

This result implies that the Determinant of Jacobian matrix of linearized System 17 is negative, and it underlines that the equilibrium is a saddle path. ■