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Inference on Income Inequality and Tax Progressivity Indices: U-Statistics and Bootstrap Methods

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Abstract

This paper discusses asymptotic and bootstrap inference methods for a set of inequality and progressivity indices. The application of non-degenerate U-statistics theory is described, particularly through the derivation of the Suits-progressivity index distribution. We have also provided formulae for the "plug-in" estimator of the index variances, which are less onerous than the U-statistic version (this is especially relevant for those indices whose asymptotic variances contain kernels of degree 3). As far as inference issues are concerned, there are arguments in favour of applying bootstrap methods. By using an accurate database on income and taxes of the Spanish households (statistical matching EPF90-IRPF90), our results show that bootstrap methods perform better (considering their sample precision), particularly those methods yielding asymmetric CI. We also show that the bootstrap method is a useful technique for Lorenz dominance analysis. An illustration of such application has been made for the Spanish tax and welfare system. We distinguish clear dominance of cash-benefits on income redistribution. Public health and state school education also have significant redistributive effects.

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1. INTRODUCTION

Numerous empirical studies in applied welfare economic analysis are based on a microeconomic framework where the input information consists in a sample of individuals or households. This information generally comes from household surveys or administrative records – or, in some cases, from a statistical fusion of both types of datasets. The range of these studies is quite wide, covering from income inequality and redistribution to tax and transfer incidence analysis or microsimulation of public policies.

An important output from such analysis is the estimation of statistics, namely, vertical and horizontal inequality indices, Lorenz curves, redistributive effects and progressivity measures of a tax or a set of taxes. These indices are of primary interest, not only for academic purposes related to this field but also for policy makers to acquire knowledge of the welfare policies therein embedded.

As well as the estimation of these indices, statistical inference is another quantitative aspect which needs to be considered. Although theoretical properties of global inequality and tax progressivity indices have been studied by many authors, statistical inference analysis has not been completely adopted in applied research. In the case of Gini-related inequality indices, we can find results in Fraser (1957), Gastwirth (1974), Cowell (1989). Results for the generalized entropy class of inequality measures are provided in Cowell (1989) and Thistle (1990). Bishop *et al.* (1998) derived the sampling distribution of Kakwani and Reynolds-Smolensky indices. These are only a few examples taken from the huge amount of important research in this field. The first part of this paper is related to this area. The U-statistics theory can be used as a unifying framework to derive the large-sample distribution of inequality and progressivity indices, particularly those which admit representation in terms of non-degenerate U-statistics. This property can be used to estimate the indices of interest and their corresponding sampling variances.

Once the analytical expressions of the index variances are obtained, there are two alternative ways to estimate the elements of the variance formulae, namely estimators based on U-statistics and those known as 'plug-in' estimators. The calculation *via* U-statistics mainly involves dealing with expressions of computational cost $\mathrm{O}(n^3)$. Estimators based on the principle of substitution or 'plug in' estimators are of lower order, reducing the computational burden of the variance expressions. The 'plug-in' estimators of sampling variances have been obtained in the paper for all the indices. We wish to highlight the derivation of the sampling variance of horizontal-inequality Atkinson-Plotnick index and particularly of the Suits progressivity-index, which has a very complicated variance formula.

The next step is to set confidence intervals or hypothesis testing. Both the classical hypothesis testing based on the asymptotic approach and the bootstrap procedure can be used for inference. Focusing on bootstrap confidence intervals (CI), for example, we have an easy way to evaluate the index values regarding their sampling variability -see Mills and Zandvakili (1997) who compared bootstrap intervals of Gini and Theil indices with those obtained from the normal approximation. Another more recent study is from Biewen (2002) who extended the analysis to poverty and mobility measures and who also carried out a simulation study to analyse the finite sample behaviour of the two approaches.

We have complemented these two studies by applying the bootstrap procedure to the inequality and progressivity indices as well as to the Lorenz curve ordinates corresponding to income deciles. The estimation and inference on Lorenz curves deserves special attention. Beach and Davidson (1983) derived the sampling distribution of Lorenz ordinates and proposed dominance tests. Davidson and Duclos (1997) and Duclos (1997) extended these results for correlated samples and formalized estimation and inference on these indices by deducing the asymptotic distribution of a set of quantiles in the Lorenz curves.

Bootstrapping indices or Lorenz curves present an important advantage with respect to the asymptotic formulae: bootstrapping incorporates the correlation structures existing in the microdata, such as dependency which occurs between pre-tax and post-tax income in a cross-sectional study or between variables coming from consecutive panel-data waves. The asymptotic approach can also deal with these data dependencies but with a complex process of deriving the covariance structures.

We have performed two empirical exercises.

First, we compare the sampling performance of bootstrap inference results with those obtained from the normal approximation through a simulation analysis. This experiment uses a microdata set coming from the statistical matching EPF90-IRPF90 as the parent population. The simulation is carried out considering different sample sizes and for a set of selected indices. The performance of different bootstrap methods for the construction of confidence intervals together with the standard asymptotic interval is evaluated in terms of their coverage probability. Results reveal that bootstrap methods work better even for large samples, particularly those bootstrap methods yielding asymmetric CI.

Second, we describe the progressivity and redistributive profile of the Spanish tax-benefit system as a whole by using data from EPF90-IRPF90. This empirical exercise is based on the estimation and bootstrapping of Lorenz and concentration curves. Within a multiple hypothesis-testing framework, dominance analysis has been made between different taxes and benefits to analyze their individual effect on income redistribution. These tests enable us to establish a statistical ordering of the components of the tax and benefit systems in terms of their progressivity and redistributive effects.

This paper is organized as follows: Section 1 is this introduction. Section 2 offers a general description of the approaches mentioned therein. First, the estimates of the respective indices based on U-statistics are presented along with the "plug-in" variance estimations. In addition, the detailed process of deriving the asymptotic distribution of a particular index is explained through the demonstration of the Suits progressivity-index (see Appendix A). Second, we describe bootstrap methods for the setting up of inference procedures and we also present multiple tests to be used in later sections. Section 3 provides results of the Monte Carlo exercise performed for the comparison of the asymptotic and bootstrap confidence intervals. In Section 4, bootstrap multiple tests are applied in order to test welfare dominance of taxes and benefits in Spain. A clear ordering of the different components of the Spanish tax and welfare systems has been established. Section 5 provides brief concluding remarks.

2. INEQUALITY AND PROGRESSIVITY INDICES: U-STATISTICS AND BOOTSTRAP METHODS

As mentioned in the introduction, U-statistics theory gives us a general setting for deducing the index asymptotic distributions and especially for assessing the asymptotic variance formulas to be applied in confidence intervals or hypothesis testing (non-parametric setting). A simple illustration is given by the well known Gini index, which can be expressed in terms of statistical functionals as follows: $G_X = \frac{\eta_1}{2\mu_x}$, where $\eta_1 = \iint |x_1 - x_2| dF(x_1) dF(x_2)$, and x represents the pre-tax income variable with distribution function $F(x)$ and μ_x is the population mean of x. In an earlier application, Hoeffding showed that the usual estimator of the functional η_1:

$$\hat{\eta}_1 = \frac{1}{n(n-1)} \sum_{i \neq j} \left| x_i - x_j \right| , \qquad (1)$$

which is known as the Gini mean difference statistic is itself a U-statistic, specifically the U-statistic related to the kernel $h(x_1, x_2) = \left| x_1 - x_2 \right|$. Furthermore, any inequality or progressivity measure expressed in terms of functionals is consistently estimated by their corresponding U-statistics.

Let *X* represent the pre-tax income variable and *Y* denote a general variable that in some cases will represent tax or benefit (*T*), and in others, the post-tax income, defined as *Y=X-T* (benefits can be treated as a negative tax and then should be added to X to get final income). Consider that *X* and *Y* are jointly distributed under *H(x,y)* with *F(x)* and *G(y)* the respective marginal functions of *X* and *Y.* Table I summarizes the index expressions in terms of statistical functionals.

{ insert Table I}

In addition, the U-statistics that estimate the respective functionals are shown in the last section of table I. Replacing every functional by its corresponding U-statistic in the index expression and by the application of Slutsky's Theorem, we obtain consistent estimators of the indices. The following step- which is the central point- is to derive the asymptotic variances of the indices estimators.

Some indices are U-statistic of degree 1, such as Theil and Atkinson inequality measures. In fact, they are functions of a sum of identical and independent random variables. Thus, they could be dealt with by simply applying the Central Limit Theorem (CLT), which is a particular case of Hoeffding's theorem. The other indices are either functionals of degree 2 or emerge as combinations of functionals of degree 2. In these cases, the application of Hoeffding's theorem 7.1 on the asymptotic multivariate normality of a vector of U-statistics together with the Delta method leads to the index asymptotic variances. Consistent estimates of the variances can be obtained from estimators based on U-statistics.

However, despite their very interesting properties (see Lee (1990) for example), the computation of the U-statistics which estimates the elements of the variance formulae implies averaging $\binom{n}{3}$ or $\binom{n}{2}$ terms. If the sample size n is large –which is the case for common household surveys- this computation can be costly. Let us show, as an example, equation A11 in appendix A: the U-statistic which estimate the element (4,3) of the covariance matrix of the Suits index is O(n^3) [1], whereas its plug-in estimator -see equation A16 in Appendix A- is O(n^2). The plug-in method also leads to consistent estimators (assuming continuity of the functionals) and they are easier to program. Finally, the plug-in estimators corresponding to the asymptotic variances of the indices presented in table I are described in Table II.

{insert Table II}

After the respective standard errors are calculated, confidence intervals and hypothesis tests can be carried out to verify changes in income inequality or tax progressivity, issues that policy makers are frequently interested in. An example of asymptotic tests for assessing changes in income inequality and tax- progressivity in the context of tax reform is given by Bishop *et al*. (1998). This kind of test applies not only for independent samples, but also to correlated data: for example, comparing two indices from a single sample or from completely dependent samples (two waves from a balanced panel).

An alternative method to estimate the sampling distribution and perform hypothesis tests based on these sampling results is given by bootstrap techniques. Basically, bootstrap is a resampling method for simulating the empirical distribution of an estimator, in our case, an

[1] The estimators corresponding to the functionals which are multiplied by 4 in matrix A5 are O(n^3).

inequality or progressivity index or the Lorenz curve ordinates, which we denote by *I*. This procedure mainly consists in the extraction of *R* independent samples of size *n* drawn with replacement from the original sample data (bootstrap resample). The statistic is computed for each resample yielding $\hat{I}^*$, the so-called *bootstrap replication* of the statistic $\hat{I}$. An estimation of the sampling variability of $\hat{I}$ is obtained by applying the expression of the standard deviation to the *R*-length vector of bootstrap replications. Let us now focus on estimating confidence intervals. We consider three procedures. First is the percentile method to estimate confidence intervals. This procedure is described as follows: from *R* bootstrap samples, an estimation of the empirical function $\hat{F}_R$ of the statistic of interest *I* is obtained:

$$\hat{F}_R(x) = \frac{1}{R}\sum_{i=1}^{R} I_{-\infty,x}\left(\hat{I}_i^*\right) \tag{2}$$

For a significance level of α, the percentile method consists in computing the (α/2) and (1-α/2) empirical percentiles, denoted by $\hat{F}_R^{-1}\left(\frac{\alpha}{2}\right)$ and $\hat{F}_R^{-1}\left(1-\frac{\alpha}{2}\right)$ respectively and finally computing the confidence interval as follows [2]

$$\left[\hat{I}_{L1}^*, \hat{I}_{L2}^*\right] = \left[\hat{F}_R^{-1}\left(\frac{\alpha}{2}\right), \hat{F}_R^{-1}\left(1-\frac{\alpha}{2}\right)\right] \tag{3}$$

Second is an improved version of the percentile method is BC*a*, “bias-corrected and accelerated”, based on a correction of the formulas corresponding to the empirical percentiles (see Efron and Tibshirani (1993), chapter 14). Third comes the bootstrap approach for the construction of confidence intervals which is bootstrap-*t*, based on the same logic that underlies the construction of the Student’s *t* interval. This method consists in, first computing

$$t^*(r) = \frac{\hat{I}^*(r) - I}{\hat{es}^*(r)}, \; r = 1,\ldots,R \tag{4}$$

[2] For the construction of confidence intervals, drawing at least R=1000 replications is suggested.

where $\hat{I}^{*}(r)$ and $\hat{es}^{*}(r)$ are the bootstrap replications of the statistic of interest and its standard error, respectively. Subsequently, a number of *R* replications of $t^{*}(r)$ are generated in order to replicate the "bootstrap-*t* table", by using the percentile method previously explained. Finally, the α-percentile is $\hat{t}^{(\alpha)}$ so that $\sum_{r=1}^{R} I\left\{t^{*}(r) \leq \hat{t}^{(\alpha)}\right\}/R = \alpha$. Hence, the bootstrap-*t* interval is defined as:

$$\left[\hat{I} - \hat{t}^{(1-\alpha)} \cdot \hat{se}, \hat{I} - \hat{t}^{(\alpha)} \cdot \hat{se}\right] \qquad (5)$$

Both BC_a and bootstrap-*t* are second-order accurate, i.e. the corresponding approximated coverage converges to the nominal coverage at a rate of 1/*n*, as compared to other bootstrap methods, the standard and percentile among them, which are first-order accurate (rate of $1/\sqrt{n}$). An important drawback of the bootstrap-*t* concerns the estimation of the denominator in equation (4). When the analytic expression of the standard error of the index is unknown, it is necessary to compute its bootstrap estimate, which means two nested levels of resampling, which incurs resampling every resample. This notably increases the computational cost of the process as well as adding a new approximation error.

From the idea that underlies bootstrap methods and the relationship between confidence intervals and hypothesis tests, inference is easy to carry out by computing an empirical approximation to the *p*-value of a test. We will consider the application of this procedure under two different scenarios that are quite common in income and tax related studies. The first scenario concerns the situation in which two separate samples are drawn from independent distributions: $(X_1, X_2, ..., X_n)$, $(Y_1, Y_2, ..., Y_m)$. This would cover the case of inequality or progressivity comparisons between different years or countries by using independent samples. The second scenario refers to having correlated data. The observed data in this case, $\{(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)\}$, is assumed to come from a joint distribution and each observation of the sample consists of a vector of two components, for instance pre-tax income and post-tax income measured over the same individuals/households in a particular year or two consecutive observations extracted from panel waves.

Now, suppose we are interested in testing the null $H_0 : I_1 = I_2$ against $H_1 : I_1 > I_2$ $(H_1 : I_1 < I_2)$, which is redefined as $H_0 : D = 0$ against $H_1 : D > 0$ ($H_1 : D < 0$). The testing procedure based on bootstrap involves to first computing the difference statistic $\hat{D} = \hat{I}_1 - \hat{I}_2$. Then, in a second step and under the first scenario, the resampling is performed separately on each sample, while under the second scenario, the joint sample is resampled as a whole, where each pair of observations that belong to a same individual/household is treated as a block or unit. Finally, the p-value of the test is computed from the bootstrap distribution of the statistic $\hat{D}$ that is obtained after a fixed number *R* of iterations.

The testing procedure is somewhat more complicated in the case of testing Lorenz dominance between two distributions, since the test statistic is no longer a single value but a vector of curve ordinates. This means we are in a multiple testing context. In this case, the bootstrap procedure is applied for computing the individual p-values corresponding to each individual hypothesis. The test statistic is defined as $\hat{D}(p_i), i = 1, \ldots, k$, that is, the estimated difference between the two curves evaluated in a set of ordinates, which are commonly established to be the deciles, that is, *k*=9 and, $p_1 = 0.1, \ldots, p_9 = 0.9$.

At this stage we distinguish two types of multiple hypothesis testing. The first, and also the largest group, comprises those tests that aim to assess dominance between curves. For example, the assessment of Lorenz dominance can be used to test the progressivity and redistributive effect of a tax or benefit through income scale. The tests are set up in a way that rejection of the null hypothesis implies dominance between the selected curves. In other words, the *research hypothesis* here is the alternative hypothesis since the researcher designs the test expecting the data to give evidence in favour of H_1. The null hypothesis tests are expressed as a union of individual hypotheses, $H_0 : \bigcup_{i=1}^{9} \{D(p_i) = 0\}$, whilst the alternative hypothesis is defined as the intersection of individual alternative hypotheses, H_1: $H_1 : \bigcap_{i=1}^{9} \{D(p_i) > 0\}$ or else $H_1 : \bigcap_{i=1}^{9} \{D(p_i) < 0\}$. The decision rule consists in rejecting $H_0 \Leftrightarrow p\hat{-}value_j < \alpha_j \quad \forall j = 1,...,9$[3]. For this group of tests, the Intersection-Union Tests (IUT) Theory (see Casella and Berger (1990) theorem 8.3.23) offers an upper limit for the global type I error ($\alpha*$), which is set as the highest of the individual significance levels, i.e. $\alpha^* = \sup_{j\in(1,..,M)} \alpha_j$, where α_j is the significance level corresponding to the *j*-th individual hypothesis, j=1,…,k. For example, by setting the individual level at 0.05, the global type I error is bound to be lesser or equal to 0.05.

Now, assume we are interested in testing proportionality of a tax or benefit. We consider that a tax/benefit is proportional when the difference between the Lorenz income curve and the concentration tax curve equals zero for all the ordinates in the curve. Note that according to the way in which the null hypothesis in the tests already discussed (tests type I) is set up, its acceptance does not imply proportionality but ‘non-dominance’. Therefore, the assessment of proportionality claims for a different type of test. This second type of testing procedure is based on the Union-Intersection Test (UIT) Theory, which uses the Bonferroni inequality as a solution to the problem of multiplicity. The null hypothesis for this second type of test is

[3] In the particular case of comparing the Lorenz curves of two distributions A and B, the non rejection of the H0 will indicate that the differences between both curves are not statistically significant, but this statement neither implies the equality nor proportionality between distributions.

expressed as an intersection of individual hypotheses, $H_0 : \bigcap_{i=1}^{9} \left\{ D(p_i) = 0 \right\}$, whilst the alternative is expressed as a union of individual hypotheses: $H_1 : \bigcup_{i=1}^{9} \left\{ D(p_i) \neq 0 \right\}$. The decision rule is set as follows[4]: the null hypothesis is rejected if and only if $\min_j \left\{ \widehat{p - value_j} \right\} < \alpha / 9$. Observe that to the contrary of the previous tests, the research hypothesis is now the null hypothesis.

Finally, the relationship between bootstrapping and U-statistics is of interest. Helmers (1991), Arcones and Giné (1992), prove that the bootstrap approximation to the finite distribution of U-statistics implies an improvement of size of $n^{-1/2}$ with respect to the asymptotic approach.

3. COMPARING ASYMPTOTIC AND BOOTSTRAP CONFIDENCE INTERVALS: EVIDENCE FROM A SIMULATION ANALYSIS

This section aims to study the finite sampling behaviour of inequality and progressivity indices through the asymptotic and bootstrap approaches. In particular, we compare the performance of bootstrap methods with those obtained from the normal approximation through a simulation study. The proposed confidence intervals in section 2 are then estimated on these samples.

In our analysis, a large sample of approximately 21,500 observations (the Spanish EPF90-IRPF90 statistical matching, Calonge and Manresa (2001), is regarded as the parent

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[4] The acceptance of H_0 implies that both curves or distributions are proportional. However, note that the rejection of H_0 is not conclusive about the statistically significant difference between the two curves, namely the 'Lorenz dominance of A over B' or vice versa within an income inequality context.

population from which, small samples of different fixed sizes are randomly drawn[5]. An important feature in this experiment is the fact that it uses real data on household income and taxes, in contrast to other studies, such a the one of Biewen (2002), where data is simulated from a parametric distribution. We have the opportunity to evaluate the behaviour of the proposed inference techniques regarding common problems, such as the presence of outliers in income and tax variables and censoring for the income-tax variable. These data characteristics, which may have an important effect on the estimated indices are, however, more difficult to reproduce in a classical simulation study where data is drawn from a pre-fixed parametric distribution. Furthermore, the whole complexity of large household databases can barely be replicated in a simulated environment.

This exercise is carried out for a proposed set of income inequality indices - Gini, Theil, deciles of income Lorenz curve - and Kakwani and Suits tax progressivity indices. Table III shows the population parameters of the mentioned indices computed on the reference population.

{Insert Table III}

We consider four types of confidence intervals. The first one is the classical standard interval based on the normal approximation. The second type of interval, named modified standard, is obtained from the first one by replacing the asymptotic standard error by its bootstrap estimation. The third and fourth are respectively the BCa and bootstrap-*t* intervals already mentioned. To evaluate the sampling accuracy of the proposed methods we focus on the estimated coverage level computed over each type of confidence interval. The coverage level

[5] Briefly, this database links two sources of information: EPF90 Household Spanish Survey and a representative sample of 1990 income-tax payers. The database contains then information on income, direct and indirect taxes and socio-economic characteristics of the members of the household. Household pre-tax income and tax/benefit payments have been equivalised by using the following scale: $s=(A+0,5*C)^{0,5}$, where A and C represent the number of adults and children in a household.

is defined as the percentage of times an interval includes the real parameter. The simulation runs 2000 replications of the following algorithm for three different sample sizes n (=200, 1000, 2000).

Repeat for $i=1,\ldots,S(=2000)$ the following steps: First, draw a random sample without replacement of size n from the simulation universe and compute the statistic of interest. Second, compute the standard interval for each index. Third, generate R=1000 bootstrap resamples (extractions with replacement the same size n as the original sample) from the sample obtained in the first step. In this procedure, the bootstrap distribution of the index is obtained and used for computing the bootstrap confidence intervals: modified standard, BC*a* and bootstrap-*t*. For each interval, accumulate the coverage indicator value (Ic=1 if the interval includes the parameter, 0 otherwise), compute the shape and length measurements. Finally, the approximate real coverage and the average values for shape, length and interval extremes are computed as the final results of the simulation.

{ insert tables IV and V}

Tables IV and V show the approximate coverage levels obtained from the Monte Carlo simulation already described. Results regarding shape, length and interval extremes are found in Appendix B (Tables B.I and B.II) In general traits, if we focus on coverage accuracy, results suggest that bootstrap confidence intervals are superior to those computed from the normal distribution. Particularly, the improved percentile method BC*a* and bootstrap-*t* achieve higher coverage accuracy compared to the modified standard. For the Gini index and the Lorenz curve deciles, the discrepancies between real and nominal coverage probabilities range from half to two percentage points when considering large samples (n=2000). Both methods BC*a* and bootstrap-*t* yield approximate coverage probabilities close to the nominal value of 0.95. However, some of the simulated-*p* are out of the acceptance interval $\hat{p} \in \{0.94, 0.96\}$ corresponding to the test $H_0 : \hat{p} = 0.95$ for *S=2000.*

Regarding the Theil index, results suggest a poorer performance for all the methods, denoted by coverage levels that are substantially distant from the nominal one. In any case, the coverage figures for bootstrap intervals still remain higher (for instance, in the case of *n*=2000 the standard method estimated coverage is 0.718 whereas the bootstrap-*t* presents a coverage value of 0.867, more than 14 percentage points higher).

On the other hand, Kakwani and Suits indices show a very similar behavioural pattern, that is, bootstrap intervals are more efficient in terms of coverage accuracy. BC*a* and bootstrap-*t* intervals show the highest 'real' coverage probabilities, with discrepancies between them not higher than two percentage points. Results regarding the progressivity indices show higher coverage when the index is evaluated on indirect taxes as opposed to direct taxes. Coverage levels for direct tax indices show a poor performance and do not present a clear increasing pattern with the sample size. This could be partially due to the higher presence of extreme values or zero observations in the distribution corresponding to direct taxation, though further evidence is needed for a firm conclusion.

Worthy of note are the differences between the intervals based on the normal approximation, namely the standard and the modified standard. Global results show a slightly better performance of the latter. More exactly, differences between the nominal and 'real' coverage probabilities reach values up to 23 and 18 percentage points in favour of the modified standard. These percentages correspond to the Theil index for *n*=2000. This indicates that even the simple alteration that the modified standard interval incorporates with respect to the standard one still implies an improvement.

Finally, based on these results, we can conclude that bootstrap intervals are more accurate than the ones based on the normal approximation, yielding wider and asymmetric intervals that can better capture the characteristics of the index finite sampling distribution[6]. An important point to highlight is that BCa and bootstrap-t performances are very much alike, which gives a strong argument in favour of the use of the BCa, whose computational cost is lower.

4. INFERENCE ON REDISTRIBUTION AND PROGRESSIVITY OF TAXES AND BENEFITS: AN APPLICATION TO THE SPANISH CASE.

In this section we estimate the redistributive effect and progressivity of taxes and benefits in Spain. We illustrate this using micro-data from the 1990 statistical matching *EPF-IRPF*. To evaluate the effects of taxes and benefits on income distribution we need first to identify who bears the tax burden and who receives the public benefits (economic incidence). The allocation of taxes and benefits to the households has been made according to the one proposed by Calonge and Manresa (1997) who followed the annual approach pioneered by Pechman and Okner (1974)[7].

On the tax-side, the burden has been imputed for each household according to different assumptions. The income tax, employee and self-employed social security contributions have been allocated to the households whose members pay these particular taxes. One of the most crucial shifting assumptions is that related to the social security contributions paid by employers

[6] Tables in Appendix B show the shape and length of the intervals. Bootstrap-t method shows more degrees of positive/negative asymmetry due to larger upper/lower limit estimations.

[7] In some sense, incidence shifting-assumptions are due to the degree of uncertainty about how the tax-shifting operates. However, the assumed hypotheses are based mainly on evidence found in the Spanish literature. See Argimón and González Páramo (1987) for a discussion of the incidence of social security contributions in Spain. See Quirmbach *et al.* (1996) for a discussion of the incidence of corporate tax.

and the corporation income tax. We assume that two-thirds of this social security contribution is shifted to employees and one-third to consumption (by increasing the goods and services price). Regarding corporation income tax we explore two different incidence hypotheses. In the first one, one-third of the tax is allocated in proportion to capital income, one-third to the property income in general and one-third is shifted to consumption. In the second hypothesis, which is more progressive, two-thirds are allocated in proportion to the capital owners and one-third to consumption. Taxation on consumption (value added tax, excise taxes, car tax, and property tax on houses) has been computed for every household in the sample considering the expenditure its members declared on levied commodities and services. Import taxes are allocated proportionally to the expenditure made by each household with respect to the total expenditure.

On the benefit-side, pension and unemployment benefits have been obviously allocated to the members of the households who received such benefits. The *per-capita* cost of benefits-in-kind provided by the public sector (education and health services) has been assigned to the members of the household who are considered recipients of those benefits . The overall taxes allocated to households represent 92% of the global revenue for the fiscal year 1990, which is a highly significant value of the total burden.

A standard approach to measuring the effect of a tax on the income distribution is to calculate the difference between the pre-tax and post-tax concentration indices. Denote by X the pre-tax income of a particular household, T the tax liability and $Y=X-T$ the post-tax income variable. The redistributive effect of a tax on income distribution RE is defined by the expression:

$$RE = G_X - C_Y = \frac{t}{1-t}K \tag{6}$$

where C_Y is the post-tax concentration index, G_X is the Gini coefficient of pre-tax income, K is the Kakwani progressivity-index and t is the tax average rate on X. It is evident from this formula that redistribution depends on two quantities: the liability progression measured by the Kakwani index and the average tax rate t. Thus, we can test if the *RE* of a particular tax/benefit is significant in statistical terms (H_0: $RE=0$). In this case, we know that if $n \rightarrow \infty$, the ratio $\frac{\widehat{RE}}{\sqrt{VAR(\widehat{RE})}}$ tends to a standard normal distribution , where the variance can be computed according to formula 4 in Table II .

Instead of focusing on merely 'single' indices, we have considered the analysis throughout the income distribution by using the Lorenz dominance criterion. We estimate the Lorenz and concentration curves underlying these indices and we perform tests of dominance between curves. We get a more detailed description of the redistributive patterns of tax and benefits by evaluating the following expression below (7), which is the counterpart of the one defined in (6) for a set of p-ordinates of the Lorenz curve, for example, the income deciles:

$$RE(p)=C_X(p) - L_X(p) \quad 0<p<1 \tag{7}$$

where $C_X(p)$ is the post-tax income concentration curve, $L_X(p)$ is the pre-tax income Lorenz curve and p are the pre-tax income deciles. The $RE(p)$ distances and pre-tax income deciles are represented on the y and x axes respectively. Each estimated ordinate is represented together with its corresponding confidence interval obtained from the application of the BC*a* method to a sample of size n=2000, randomly extracted from the sample. The number of bootstrap replicates has been set to R=1000. We should notice first that the multiple test with null hypothesis H_0: $RE(p)=0$ examines the dominance of the post-tax income concentration curve on the pre-tax income Lorenz curve. In other words; rejecting the null implies dominance of one curve over the other. Then, we test the existence of a "pro-poor" redistributive effect if $RE(p) > 0$ (which is the case when the post-tax income concentration curve lies above the pre-tax income Lorenz curve) and it is statistically significant. According to the way this particular multiple test is defined, this is equivalent to the case when the *RE* confidence intervals computed on *p*-ordinates do not cross the x-axis. On the contrary, a negative value for *RE(p)* indicates that we are testing a “pro-rich” redistributive effect: in this case, the post-tax concentration curve lies below the pre-tax income Lorenz curve. In both cases, the global redistribution achieved is measured by the area between the *D(p)*-curve and the abscissa-axis *p*, which, when doubled, coincides with the Reynolds-Smolensky-index.

Figure 1 shows the *RE(p)* curves evaluated on income deciles for the following variables: personal income tax (PIT), the corporate tax under the most progressive incidence assumption (CT(2)), direct taxes (DT) (which include PIT, CT(2) and the total amount of social security contributions allocated to employees), indirect taxes (IT) and benefits (BE).

{Insert figure 1}

At first view, figure 1 suggests that personal income tax, corporate tax, direct taxes and total benefits have a positive and statistically significant redistributive effect on income distribution since their confidence intervals calculated at $\alpha = 0.05$ do not include the abscissa axis for any of the mentioned curves, except for some deciles of CT(2). Figure 1 also shows that the impact of indirect taxation on income distribution is also statistically significant but with a negative redistributive effect. In fact, when we perform the corresponding multiple tests of Lorenz dominance, the null H_0: $RE(p)=0$ is rejected for all of them except for CT(2). In this last case, a more detailed look at the *p*-values of the individual tests show that the non-rejection is due to the first estimated decile, with an individual p-value of 0.096. Another conclusion on the tax-side is obtained by looking at the slope of the curves. For example, the *RE*(*p)* for personal income tax achieves its maximum, 0.031, in the eighth decile. Hence, the income-tax redistributes 3,14% of the total post-tax income amount from the two top deciles to the rest of the distribution.

Another interesting issue other than the testing of whether a tax or benefit has a statistically significant redistributive effect is to establish a statistical ordering of the different taxes and benefits in terms of their redistributive capacity. This is achieved by means of the *RE* dominance test described in section 2. This test has been applied to a set of pairs of tax(es)/benefit(s) selected according to the ordering given in figure 1. Results are presented in table VII.

{ insert table VII}

All pairwise comparisons except for the pair DT-PIT present global p-values equal to zero, which means that the left-handside tax/benefit RE-dominates the right-handside one. Corporate tax under the most progressive assumption exhibits a similar redistributive profile to the income tax, although its redistributive effect is lower.

The curve corresponding to direct taxes and the personal-income-tax curve cross, so a full test of dominance does not apply. However, individual p-values indicate that the direct taxes curve dominates the personal income tax curve up to the seventh decile. In other words, a multiple test of “partial” dominance on this part of the curve (deciles one to seven, both included) would clearly lead to the rejection of the null. So it would appear that income tax plays a prominent role in the highest part of the income distribution.

The most impressive result is the considerable dominance of total benefits on direct taxes, which emphasizes the redistributive power that benefits play on income redistribution. This curve reaches its maximum in the sixth decile, indicating a redistribution of 7% of the total amount of final income from deciles higher than the sixth towards the rest of the population.

A more detailed picture about benefits would be of interest. Figure 2 displays the RE(p) curves for different categories of benefits and Table VIII shows the global p-values corresponding to the RE-dominance tests performed to the pairs of benefits chosen.

{Insert figure 2}

Results show first that the redistribution effect of cash benefits (CB) and benefits-in -kind (BiK) are both statistically significant. Figure 2 also indicates a clear dominance ordering between the two curves, illustrated by the non-overlapping between the ordinates' interval estimates. Cash benefits have the largest redistributive effect on income distribution - all pairwise comparisons involving CB on the left hand-side result in p-values equal to zero, as shown in table VIII. The redistributive effect of BiK is also remarkable. Breaking it down into three categories –'health services' (HB), 'primary and secondary education' (EdI) and 'university education' (EdII) – enable us to establish a dominance ordering. Health services have the largest *RE* followed by 'primary and secondary education'.

{Insert table VIII}

In fact, as the positive-slope fragment of the benefit curves shows, 60% of the population with lower resources is benefit receiver as compared to a situation in which benefits are evenly distributed. Finally, public expenditure on university education has null redistributive effect: the test of proportionality computed on its RE curve results in a global p-value of 0.810, which indicates that the null hypothesis of proportionality can not be rejected (see also figure II).

Once we have measured and evaluated the redistributive profile of a tax or benefit it is interesting to picture the contribution of each decile to the global redistribution (RE). The following difference

$$RE(p_i)\text{-}RE(p_{i-1}) \qquad (7)$$

measures the "redistributive load" carried by decile i. If we take the personal income tax shown in figure 3 as an example, the ninth and tenth deciles account for percentages of 13.8% and 86,2% of *RE*, respectively. The redistributive effect induced by direct taxation as a whole proves to be less progressive because in this case, taxpayers spread from the sixth to the tenth deciles. The contribution of the ninth and tenth deciles to the redistribution is of the order of 33% and 55% respectively. Each receiver decile captures around 20-22% of the redistributive effect, except for the fourth that receives 15% of the overall income that is redistributed.

{Insert figure 3}

Figure 3 also reveals that indirect taxation causes regressive income redistribution. Note that the poorest deciles up to the seventh decile are punished in the sense that a larger net income would be expected, had the indirect tax been replaced by a proportional one. As a result of the regressivity of indirect taxes, the two highest deciles become "winners". Particularly, the income redistributed towards the richest deciles due to indirect taxation amounts to 1,15% of the post-tax income, from which 87% is perceived by the top decile.

In the same way as the redistributive profile was defined from the concept of *RE(p)* distance, we can define the progressivity profile of a tax as

$$TR(p)=L_X(p) - C_T(p)\,. \qquad (9)$$

where TR(p) indicates the difference between the pre-tax income Lorenz curve and the tax concentration curve (see figure 4). Twice the region defined between the TR(p) curve and the abscissas axis coincides with the Kakwani index[8]. The results on TR-dominance multiple tests are shown in table VIII.

{Insert figure IV}

{ Insert table IX}

The following features can be drawn on the tax-side . First, the personal income tax has clearly the most progressive profile, as indicated by the p-values corresponding to the

[8] This distance is interpreted as the fraction of total tax that is transferred from households with income less than 100p percentile towards the upper side of the distribution as a result of the system's progressivity.

pairwise comparisons between this tax and each of the remaining taxes. Comparing the CT progressivity profiles under the two different incidence assumptions is of a special interest. The corporate tax variant (2) has been tested as a progressive one against CT(1) tax. In fact, the shape of the CT(1) curve rather suggests a proportional or regressive profile. However, the test of regressivity fails to reject the null hypothesis with a global p-value of 0.413 (observe the overlapping for some decile intervals with the *x* axis). Moreover, a second test of proportionality is not significant (the null hypothesis of proportionality is rejected). This illustrates that the acceptance of the null in a test of progressivity/regressivity (assessed by type I tests) does not imply the proportionality of the tax or benefit (assessed by type II tests). Hence, neither a regressive nor a proportional nature can be inferred for CT(1). It is obvious that the pattern of corporate tax is pretty sensitive to the incidence assumption adopted, and more research is needed about shifting assumptions.

The direct taxes and the CT(2) curves cross at the fourth decile. However, by examining the individual p-values, we can establish a partial dominance of DT over CT(2) for the first three deciles, and a partial dominance of CT(2) over DT for the part of the curve that comprises the fifth to the ninth deciles (both included). As for the dominance of direct taxes over corporate tax variant (1), the test of TR-dominance fails to reject the null hypothesis with a global p-value of 0.241. But once again, the partial test applied to deciles up to the seventh is rejected with a p-value of 0.022, which means that in this part of the curve, direct taxes are statistically more progressive than CT(1).

The reading from these curves is just the opposite to that of the tax curves. A benefit curve lying under the abscissas axis will indicate progressivity of the benefit. The most striking feature that arises from figure 5 is the fact that all benefit curves except for the one related to university education have similar shapes. Moreover, most of these curves cross each other, thus impeding a TR dominance ranking among them.

University education exhibits the lowest progressivity profile among all the benefits considered. The test with null *TR(p)*=0 fails to be rejected for this type of benefits (with a global p-value of 0.206), indicating a non-significant progressive effect. At the same time, the corresponding test of proportionality fails to be rejected, which implies the statistically significant proportionality of university education. All tests of progressivity performed over the remaining benefit curves are significant, that is, the null hypothesis *TR(p)*=0 is rejected for all of them.

{Insert Table IX}.

Note that for these tests, the alternative hypothesis is $TR(T_1) < TR(T_2)$, which means that benefit T_1 is more progressive than benefit T_2. Regarding the setting up of a ranking, an examination of the *p*-values provided in Table IX reveals that cash benefits, benefits in kind and health benefits TR dominate university education. The rest of the pairs do not cross except for CB-BiK and EdI-EdII. For the first pair, the global dominance test cannot be rejected, with a *p*-value=0.376. A closer look at the individual *p*-values shows an important overlapping for deciles above the third one. This is not the case for the second pair of curves, for which, although presenting a global *p*-value of 0.346, the overlapping only occurs at the first decile .

The main findings from the progressivity and redistributive effect analyses are summarized in the following lines. Although direct taxes play an important role in the redistribution made by the Spanish tax-benefit system, total benefits appear to be the component of the system that exerts the largest redistributive effect. A closer examination of the benefits shows that cash benefits followed by benefits in kind are those mainly responsible for the redistribution induced. Indirect taxes cause a significant and negative redistributive effect. It is surprising to observe that corporate tax, even under the most progressive assumption, does not produce a statistically significant redistribution.

In terms of progressivity, PIT is the most progressive tax and its *TR* curve statistically dominates the *TR* curves of the other the taxes. Opposite to PIT, indirect taxes present a significant regressive nature. Corporate tax variant (2) proves to be more progressive than under variant (1). The CT(2) and DT curves cross at the fourth decile. However, a partial TR dominance of CT(2) over DT for deciles higher than the fourth can be established.

5. CONCLUSIONS

Inference methods on income, tax and benefit distributions are crucial in applied economic welfare. It is well-known that the asymptotic distributions of a set of inequality and progressivity indices can be derived by using non-degenerate U-statistics theory. As an application of this theory, we have described its main guidelines and derived the Suits-progressivity index distribution. We have also provided a formula for the “plug-in” estimator of the variance indices, which are less onerous – in terms of computational cost – than the U-statistic version (this is specially relevant for those indices whose asymptotic variances contain kernels of degree 3).

As far as inference issues are concerned, there are arguments in favour of using bootstrapping as an alternative to the classic approach. By using the statistical matching EPF90-IRPF90 - which contains more accurate information on income and taxes than the merely EPF90 household survey - our results show that bootstrap methods perform better (considering their sample precision), particularly those bootstrap methods yielding asymmetric CI).

Another interesting application of the bootstrap technique is the formulation of multiple hypothesis tests for the assessment of Lorenz dominance and other related concepts. We have showed that the bootstrap method is a useful technique for Lorenz dominance analysis. Moreover, it can be applied even in the case when the independence-between-samples assumption does not hold.

An illustration of such application has been made for the Spanish tax and welfare system. According to this analysis, we distinguish clear dominance of cash-benefits on income redistribution. Public health and state school education also have significant redistributive effects.

APPENDIX A

DERIVATION OF THE SUITS INDEX ASYMPTOTIC DISTRIBUTION

Let X be the pre-tax income variable and Y the tax/benefit variable, G_X the pre-tax income Gini index and CR_Y the relative concentration index corresponding to the tax or benefit Y. The Suit index of tax progressivity, Suits (1977), is defined as:

$$S = CR_Y - G_X \tag{A1}$$

where

$$CR_Y = \frac{2}{\mu_Y}\operatorname{cov}(Y, F_1(X)) \tag{A2}$$

and $F_1(x) = \frac{1}{\mu_X}\int_0^x ydF(y)$ is the first moment distribution function of X, F is the distribution function of variable X, and μ_X and μ_Y are the population means of X and Y, respectively. Given that the covariance term in A.2 can be decomposed as $E[YF_1(X)] - E(Y)E[F_1(X)]$, the formula A.1 for the Suits index is reformulated in terms of functionals as:

$$S = \left[\frac{\eta_2}{\mu_X\mu_Y} - \frac{\eta_3}{\mu_X}\right] - \frac{\eta_1}{2\mu_X} \tag{A3}$$

where η_1, η_2 and η_3 are defined in table I. Assume that an i.i.d. random sample is drawn from the joint distribution $H(x,y)$, with $F(x)$ and $G(y)$ the respective marginal functions, then it follows from Slutsky's theorem that a consistent estimator, $\hat{S}$, for the Suits index is obtained from the U-statistics associated with the functionals η_1, η_2 and η_3 (see table I):

$$\hat{S} = \left[\frac{\hat{\eta}_2}{\overline{xy}} - \frac{\hat{\eta}_3}{\overline{x}}\right] - \frac{\hat{\eta}_1}{2\overline{x}} \tag{A4}$$

Asymptotic normality of S is established by applying Hoeffding's theorem 7.1 which is about the joint distribution of a vector of U-statistics, leading to the following lemma:

Lemma 1. Let *H(x,y)*, *F(x)* and *G(y)* be continuous functions with finite second central order moments for *F* and *G*, that is $E(|X|)<\infty$, $E(|Y|)<\infty$ and $E(|X|^2)<\infty$ $E(|Y|^2)<\infty$, then the random vector [$\sqrt{n}(\bar{x}-\mu_X)$, $\sqrt{n}(\bar{y}-\mu_Y)$, $\sqrt{n}(\hat{\eta}_2-\eta_2)$, $\sqrt{n}(\hat{\eta}_3-\eta_3)$, $\sqrt{n}(\hat{\eta}_1-\eta_1)$] follows an asymptotically multivariate normal distribution with zero mean and variance-covariance matrix:

$$\Sigma_{5\times5}=\begin{bmatrix} \sigma_X^2 & \sigma_{XY} & 2\upsilon(\mu_X,\eta_2) & 2\upsilon(\mu_X,\eta_3) & 2\upsilon(\mu_X,\eta_1) \\ & \sigma_Y^2 & 2\upsilon(\mu_Y,\eta_2) & 2\upsilon(\mu_Y,\eta_3) & 2\upsilon(\mu_Y,\eta_1) \\ & & 4\upsilon(\eta_2,\eta_2) & 4\upsilon(\eta_2,\eta_3) & 4\upsilon(\eta_2,\eta_1) \\ & & & 4\upsilon(\eta_3,\eta_3) & 4\upsilon(\eta_3,\eta_1) \\ & & & & 4\upsilon(\eta_1,\eta_1) \end{bmatrix}, \qquad \text{(A5)}$$

where σ_X^2, σ_Y^2, σ_{XY} are the variances of X and Y and covariance between them respectively. The terms $\upsilon_{ij}(.)$ represent the covariance between functionals. Next, the application of the Delta method leads us to the asymptotic distribution of Suits index:

$$\sqrt{n}(\hat{S}-S)\xrightarrow{d} N(0,J\Sigma J') \qquad \text{(A6)}$$

where J is the partial derivative of S with respect to $\eta=(\mu_X \quad \mu_Y \quad \eta_2 \quad \eta_3 \quad \eta_1)'$:

$$J=-\frac{1}{\mu_X}\begin{bmatrix}(CR_Y-G_X) & \frac{\eta_2}{\mu_Y^2} & -\frac{1}{\mu_Y} & 1 & \frac{1}{2}\end{bmatrix}$$

can also be expressed as,

$$J=-\frac{1}{\mu_X}\left\{\begin{bmatrix}CR_Y & \frac{\eta_2}{\mu_Y^2} & -\frac{1}{\mu_Y} & 1 & 0\end{bmatrix}-\begin{bmatrix}G_X & 0 & 0 & 0 & -\frac{1}{2}\end{bmatrix}\right\} \qquad \text{(A.7)}$$

The asymptotic variance of the Suits index is established in the following theorem[9]:

---

[9] The quadratic form defined in A.6 is equal to $J_1\Sigma J_1' + J_2\Sigma J_2' - 2J_2\Sigma J_1'$ where j_1 and j_2 are the sub vectors in expression A.7. Note that the two first terms are the asymptotic variances of the relative concentration index, CR_Y, and Gini index, *Gx,* respectively.

Theorem 1. Under the same conditions set for Lemma 1, $\hat{S}$ is a consistent estimate of S with asymptotic normal distribution, $\sqrt{n}(\hat{S}-S)\xrightarrow{\ \ell\ } N\left(0,\sigma_S^2\right)$, where the variance is defined as

$$
\begin{aligned}
&\sigma_S^2 = \sigma_{CR_Y}^2 + \sigma_{G_X}^2 \\
&-\frac{2}{\mu_X^2}\left[\sigma_X^2 CR_Y G_X + \frac{\eta_2}{\mu_Y^2}\sigma_{XY} G_X + 2\upsilon(\mu_X,\eta_3)G_X + 2\frac{\upsilon(\eta_2,\eta_1)}{\mu_Y} - 2\frac{\upsilon(\mu_X,\eta_2)}{\mu_Y}G_X\right. \\
&\left.-\upsilon(\mu_X,\eta_1)CR_Y - \frac{\eta_2}{\mu_Y^2}\upsilon(\mu_Y,\eta_1) - 2\upsilon(\eta_3,\eta_1)\right].
\end{aligned}
\tag{A.8}
$$

and σ_G^2 and $\sigma_{CR_Y}^2$ are the variances of $\sqrt{n}(\hat{G}_X - G_X)$ and $\sqrt{n}(\hat{CR}_Y - CR_Y)$:

$$
\sigma_G^2 = \frac{1}{\mu_X^2}\left[\sigma_X^2 G_X^2 + \upsilon(\eta_1,\eta_1) - 2\upsilon(\mu_X,\eta_1)G_X\right]
$$

$$
\begin{aligned}
\sigma_{CR_Y}^2 = &\frac{CR_Y}{\mu_X^2}\left[CR_Y\sigma_X^2 + \frac{2\eta_2}{\mu_Y^2}\sigma_{XY} - \frac{4\upsilon(\mu_X,\eta_2)}{\mu_Y} + 4\upsilon(\mu_X,\eta_1)\right] + \\
&\frac{\eta_2}{\mu_X^2\mu_Y^2}\left[\frac{\sigma_Y^2}{\mu_Y^2} + \frac{4\upsilon(\mu_Y,\eta_2)}{\mu_Y} + 4\upsilon(\mu_Y,\eta_3)\right] + \frac{4}{\mu_X^2}\left[\frac{\upsilon(\eta_2,\eta_2)}{\mu_Y^2} + \upsilon(\eta_3,\eta_3) - \frac{2\upsilon(\eta_2,\eta_3)}{\mu_Y}\right]
\end{aligned}
$$

A.1 Variance estimator of $\hat{S}$ based on U-statistics

We illustrate the estimation of the covariance terms $\upsilon_{ij}(.)$. Let us consider –as an example- the element $\upsilon(\mu_X,\eta_2)$. First, we obtain the symmetric kernels that estimate functionals μ_X and η_2. Given that $\bar{x}$ and $\hat{\eta}_2$ (see table I) are the U-statistics of μ_X and η_2 respectively, the corresponding kernels are $h(X_1)=X_1$ and $h((X_1,Y_1),(X_2,Y_2)) = X_1Y_2I_0\left(X_1<X_2\right)+X_2Y_1I_0\left(X_2<X_1\right)$ with respective degrees m_a =1 and m_b =2. Therefore, the coefficient related to this element is $m_am_b=2$ and the functional

$$\begin{aligned}\upsilon(\mu_X,\eta_2) &= E\left[X_1\left[X_1Y_2I_0\left(X_1<X_2\right)+X_2Y_1I_0\left(X_2<X_1\right)\right]\right]-\mu_X\eta_2\\ &= \iint\iint x_1\left[x_1y_2I_0\left(x_1<x_2\right)+x_2y_1I_0\left(x_2<x_1\right)\right]\\ &\quad \times dH(x_1,y_1)dH(x_2,y_2)-\mu_X\eta_2\end{aligned}, \tag{A.9}$$

is consistent estimated as follows:

$$\hat{\upsilon}(\mu_X,\eta_2)=\frac{1}{n(n-1)}\sum_{i<j}(X_i+X_j)[X_iY_jI_0(X_i<X_j)+X_jY_iI_0(X_j<X_i)]-\bar{X}\hat{\eta}_2 .$$

Let us derive now another example, $\upsilon(\eta_2,\eta_3)$, which leads to a kernel of degree three. Following the same procedure as the previous steps, the functional is derived as:

$$\begin{aligned}\upsilon(\eta_2,\eta_3) = \iint\iint\int&\left[x_1y_2I_0\left(x_1<x_2\right)+x_2y_1I_0\left(x_2<x_1\right)\right]\\ &\times\left[x_1I_0\left(x_1<x_3\right)+x_3I_0\left(x_3<x_1\right)\right]\\ &\times dH(x_1,y_1)dH(x_2,y_2)dF(x_3)-\eta_2\eta_3,\end{aligned} \tag{A.10}$$

Its corresponding estimator based on U-statistics is computed by the following expression,

$$\begin{aligned}\hat{\upsilon}(\eta_2,\eta_3) = &\frac{1}{n(n-1)(n-2)}\sum_{i<j<k}\\ &\times\left\{\sum_{\Pi(i,j,k)}h((X_{\pi_i},X_{\pi_i}),(X_{\pi_j},X_{\pi_j}),(X_{\pi_k},X_{\pi_k}))\right\}-\hat{\eta}_2\hat{\eta}_3,\end{aligned} \tag{A.11}$$

where the summation is over the set of all permutations of the n sample elements and $h(.)$ is the kernel defined as:

$$\begin{aligned}h((X_1,Y_1),(X_2,Y_2),(X_3,Y_3)) = &\left[X_1Y_2I_0\left(X_1<X_2\right)+X_2Y_1I_0\left(X_2<X_1\right)\right]\\ &\times\left[X_1I_0\left(X_1<X_3\right)+X_3I_0\left(X_3<X_1\right)\right]\end{aligned}$$

Other covariance elements of the matrix A.5 can be obtained following the same procedure.

A.2 Variance estimator of $\hat{S}$ based on plug-in estimators

Finally, we have calculated the plug-in estimators corresponding to the elements $\upsilon_{ij}(.)$. Let us focus again on the two functionals we have explained before. Consider, first, the functional $\upsilon(\mu_X, \eta_2)$. A slight re-ordering of the integration variables in A.9 leads to the following expression:

$$\begin{aligned}\upsilon(\mu_X,\eta_2) &= \int\int x_1 \left\{ \int\int [x_1 y_2 I_0(x_1 < x_2) + x_2 y_1 I_0(x_2 < x_1)] dH(x_2, y_2) \right\} \\ &\times dH(x_1, y_1) - \mu_X \eta_2 = \iint x_1 \Delta_2(x_1, y_1) dH(x_1, y_1) - \mu_X \eta_2\end{aligned} \tag{A.12}$$

where $\Delta_2(x_1, y_1)$ can be reformulated as

$$\Delta_2(x_1, y_1) = x_1 \mu_Y \left[1 - H_1(x_1, \cdot)\right] + y_1 \mu_X F_1(x_1) \tag{A.13}$$

and $H_1(x_1, \cdot) = \dfrac{1}{\mu_Y} \int_0^{x_1} \int_0^{+\infty} y_2 dH(x_2, y_2)$. An unbiased and consistent estimator for Δ_2 is:

$$d_{2i} = \frac{n}{n-1} \left[X_i \left[\overline{y} - \overline{g}_i p_i \right] + Y_i \overline{x}_i p_i - \frac{1}{n} Y_i X_i \right] \tag{A.14}$$

where $\overline{x}_i$ represents the mean income corresponding to individuals of the sample with income less than or equal to X_i; $\overline{g}_i$ is the mean tax for those individuals in the sample with income less than or equal to X_i and p_i is the percentage of individuals with income less than or equal to X_i. From expression (A.12) the corresponding plug-in estimator can be obtained. The application of the principle of substitution twice leads to:

$$\tilde{\upsilon}(\mu_X, \eta_2) = \frac{1}{n} \sum_{i=1}^{n} X_i d_{2i} - \overline{x} d_2 \tag{A.15}$$

In the same way, the plug-in estimator corresponding to functional $\upsilon(\eta_2, \eta_3)$ is defined as:

$$\tilde{\upsilon}(\eta_2, \eta_3) = \frac{1}{n}\sum_{i=1}^{n} d_{2i} d_{3i} - d_2 d_3 \tag{A.16}$$

This procedure applies to the elements in matrix A.5. By replacing the elements $\upsilon_{ij}(.)$ in the index variance formulae with their respective plug-in estimators, we obtain the consistent estimate for σ_S^2 defined in table II. The application of the principle of substitution twice does not necessarily imply consistency. Continuity of the functionals appears to ensure consistency of the plug-in estimators.

REFERENCES

Arcones M, Giné E. 1992 .On the bootstrap of U and V statistics. *Annals of Statistics* **20**: 655-674.

Argimón I. González-Páramo JM. 1987. Traslación e incidencia de las cotizaciones sociales por niveles de renta en España, 1980-1984", *Fundación Fondo para la Investigación Económica y Social*, *D.T*.

Beach C, Davidson R. 1983. Distribution free statistical inferences with Lorenz curves and income shares. *Review of Economics Studies* **50**: 723-735.

Biewen M. 2002 Bootstrap inference for inequality, mobility and poverty measurement. *Journal of Econometrics* **108**: 317-342.

Bishop J, Formby J, Zheng, B. 1998. Inference tests for Gini-based tax progressivity indexes *Journal of Business & Economic Statistics* **16:** 322-330.

Calonge S, Manresa A. 1997. Consecuencias redistributivas del estado del bienestar en España: un análisis empírico desagregado. *Moneda y Crédito* **204**: 13-65.

Calonge S, Manresa A. 2001. *Fusión estadística de bases de datos.* In Incidencia fiscal y del gasto público social sobre la distribución de la renta en España y sus CC.AA. Fundación BBVA. Madrid.
Online english version: Alegre *et al*. (2000) Statistical matching between diferent datasets: An application to the Spanish Household Survey (EPF90) and the income Tax File (IRPF90). http://selene.uab.es/mmercader/workshop/indexcast.html

Casella G, Berger RL (1990). *Statistical Inference.* Wadsworth & Brooks, Pacific Grove, California.

Cowell F. 1989. Sampling variance and decomposable inequality measures *Journal of Econometrics* **42**: 27-42

Gastwirth J. 1974. Large sample theory of some measures of inequality. *Econometrica* **42**: 191-196.

Davidson R, Duclos J-Y. 1997. Statistical inference for the measurement of incidence of taxes and transfers. *Econometrica* **65:** 1453-1465.

Duclos J-Y. 1997. The asymptotic distribution of linear indices of inequality, progressivity and redistribution. *Economic Letters* **54**: 51-57.

Efron B, Tibshirani R. 1993. *An Introduction to the Bootstrap.* Chapman & Hall. New York.

Fraser, D. 1957. *Nonparametric Methods in Statistics*, John Wiley. New York.

Helmers R. 1991. On the edgeworth expansion and the bootstrap approximation for a studentized U-statistic. *Annals of Statistics* **19**: 470-484.

Hoeffding W. 1948. A Class of Statistics with Asymptotically Normal Distribution *Annals of Mathematical Statistics* **19**: 293-325.

Kakwani N. 1976 Measurement of tax progressivity: an international comparison *Economic Record* **40**: 426-441.

Lee AJ. 1990. *U-statistics. Theory and Practice*. Owen DB (ed.). Marcel Dekker. New York.

Mills JA, Zandvakili S. 1997. Statistical inference via bootstrapping for measures of inequality. *Journal of Applied Econometrics* **12**: 133-150.

Pechman JA, Okner B. 1974. *Who Bears the Tax Burden?.* The Brookings Institution. Washington DC.

Quirmbach HC, Swenson CW, Vines CC. 1996. An experimental examination of general equilibrium tax incidence. *Journal of Public Economics* **61**: 337-358.

Thistle PD. 1990. Large sample properties of two inequality indices. *Econometrica* **58**: 725-728.

Table I. Formulas for indices, functionals and U-statistics.

Indices in terms of functional statistics

1. $G_X = \dfrac{\eta_1}{2\mu_X}$ **2.** $K = \dfrac{\eta_1^{'}}{2\mu_Y} - \dfrac{\eta_1}{2\mu_X}$ **3.** $S = \left[\dfrac{\eta_2}{\mu_X \mu_Y} - \dfrac{\eta_3}{\mu_X}\right] - \dfrac{\eta_1}{2\mu_X}$ **4.** $RE = \dfrac{\eta_{1Y}}{2\mu_Y} -$

5. $HI = \dfrac{1}{2\mu_Y}\left(\eta_{1Y} - \eta_1^{'}\right)$ **6.** $T_1 = \dfrac{1}{\mu_X}\gamma_1 - \log(\mu_X)$ **7.** $A_X^{\varepsilon} = 1 - \dfrac{\left(\mu_X^{(1-\varepsilon)}\right)^{1/1-\varepsilon}}{\mu_X}$ **.,** $\varepsilon \neq 1$

Functionals

$$\eta_1 = 4\operatorname{cov}(X, F(X)) = 4\left[\int_0^{+\infty} xF(x)dF(x) - \mu_X \frac{1}{2}\right] \qquad \eta_{1Y} = 4\operatorname{cov}(Y, G(Y)) = 4\left[\int_0^{+\infty} yG(y)dG(y) - \mu_Y \frac{1}{2}\right]$$

$$\eta_1^{'} = 4\operatorname{cov}(Y, F(X)) = 4\left[\int_0^{+\infty}\int_0^{+\infty} yF(x)dH(x,y) - \mu_Y \frac{1}{2}\right] \qquad \eta_2 = 2E\left[Y\mu_X F_1(X)\right] = 2\int_0^{+\infty}\int_0^{+\infty} y\left(\int_0^x udF(u)\right)dH(x,y)$$

$$\eta_3 = 2E\left(\mu_X F_1(X)\right) = 2\int_0^{+\infty}\left(\int_0^x udF(u)\right)dF(x) \qquad \gamma_1 = E(X\log(X)) = \left[\int_0^{+\infty} x\log(x)dF(x)\right]$$

$$\mu_X^{(k)} = E\left(X^k\right) = \int_0^{+\infty} x^k dF(x)$$

U-statistics

$$\hat{\eta}_1 = \frac{1}{n(n-1)}\sum_{i \neq j}\left|X_i - X_j\right| \quad \hat{\eta}_{1Y} = \frac{1}{n(n-1)}\sum_{i \neq j}\left|Y_i - Y_j\right| \qquad \hat{\eta}_{\mathbf{1}}^{'} = \frac{1}{n(n-1)}\sum_{i \neq j}(Y_i - Y_j)I(X_i > X_j)$$

$$\hat{\eta}_2 = \frac{2}{n(n-1)}\sum_{i<j} X_i Y_j I_0(X_i < X_j) + X_j Y_i I_0(X_j < X_i) \qquad \hat{\eta}_3 = \frac{2}{n(n-1)}\sum_{i<j} X_i I_0(X_i < X_j) + X_j I_0(X_j < X_i)$$

$$\hat{\gamma}_1 = \frac{1}{n}\sum_{i=1}^{n} X_i \log(X_i) \qquad \hat{\mu}_X^{(k)} = \frac{1}{n}\sum_{i=1}^{n} X_i^k = \overline{x}^{(k)}$$

where $I(X_i > X_j) = 1 \textit{ if } X_i > X_j \textit{ or } 0 \textit{ if } X_i = X_j$ $\textit{or } -1 \textit{ if } X_i < X_j$ $\qquad I_0(X_i < X_j) = 1 \textit{ if } X_i < X_j, 0 \textit{ otherwise}$

Notes:1.Gini, 2.Kakwani, 3.Suits, 4. Redistributive Effect, 5.Horizontal inequality, 6.Theil(l), , 7.Atkinson. The U-statistic that estimates $\mu_X^{(k)}$ equals the respective sample moment.

TABLE II. ASYMPTOTIC VARIANCE ESTIMATES OF INEQUALITY AND TAX PROGRESSIVITY INDICES.

Formulae

1 $$\hat{\sigma}^2_{G_x} = \frac{1}{\bar{x}^2}\left[\hat{G}^2_x \hat{\sigma}^2_x - 2\hat{G}_x \hat{\sigma}_{xd_1} + \hat{\sigma}^2_{d_1}\right]$$

2 $$\hat{\sigma}^2_K = \hat{\sigma}^2_{G_x} + \hat{\sigma}^2_{C_y} - \frac{2}{\overline{xy}}\left[\hat{\sigma}_{d_1 d'_1} + \hat{G}_x \hat{C}_y \hat{\sigma}_{xy} - \hat{\sigma}_{xd'_1}\hat{G}_x - \hat{\sigma}_{yd_1}\hat{C}_y\right]$$

3 $$\sigma^2_S = \hat{\sigma}^2_{CR_Y} + \hat{\sigma}^2_{G_x} - \frac{2}{\bar{x}^2}\left[\hat{G}_x\left\{\hat{\sigma}^2_x \hat{CR}_y + \frac{\hat{\eta}_2}{\bar{y}^2}\hat{\sigma}_{xy} + 2\hat{\sigma}_{xd_3} - \frac{2\hat{\sigma}_{xd_2}}{\bar{y}}\right\}\right.$$
$$\left. + \frac{2\hat{\sigma}_{d_2 d_1}}{\bar{y}} - \hat{CR}_Y\,\hat{\sigma}_{xd_1} - \frac{\hat{\eta}_2}{\bar{y}^2}\hat{\sigma}_{yd_1} - 2\hat{\sigma}_{d_3 d_1}\right]$$

4 $$\hat{\sigma}^2_{RE} = \hat{\sigma}^2_{G_x} + \hat{\sigma}^2_{G_y} - \frac{2}{\overline{xy}}\left[\hat{\sigma}_{d_1 d_{1Y}} + \hat{G}_x \hat{G}_y \hat{\sigma}_{xy} - \hat{G}_x \hat{\sigma}_{xd_{1Y}} - \hat{G}_y \hat{\sigma}_{yd_1}\right]$$

5 $$\hat{\sigma}^2_{HI} = \hat{\sigma}^2_{Gy} + \hat{\sigma}^2_{Cy} - \frac{2}{\bar{y}^2}\left[\hat{\sigma}^2_y \hat{G}_y \hat{C}_y + \hat{\sigma}_{d_{1Y} d'_1} - \hat{\sigma}_{yd'_1}\hat{G}_y - \hat{\sigma}_{yd_{1Y}}\hat{C}_y\right]$$

Expressions

Observation i of variable $d_1, d'_1, d_{1Y}, d_2, d_3$ is computed as:

$$d_{1i} = \frac{n}{n-1}\left[2(x_i - \bar{x}_i)p_i - (x_i - \bar{x})\right] \qquad d_{1Y_i} = \frac{n}{n-1}\left[2(y_i - \bar{y}_i)r_i - (y_i - \bar{y})\right]$$

$$d'_{1i} = \frac{n}{n-1}\left[2\left[y_i - \bar{g}_i\right]p_i - (y_i - \bar{y})\right] \qquad d_{2i} = \frac{n}{n-1}\left[x_i\left[\bar{y} - \bar{g}_i p_i\right] + y_i \bar{x}_i p_i - \frac{1}{n} y_i x_i\right]$$

$$d_{3i} = \frac{n}{n-1}\left[x_i(1 - p_i) + \bar{x}_i p_i - \frac{1}{n}x_i\right] \qquad \bar{x}_i = \frac{\sum_{j=1}^{n} X_j I(X_j \le X_i)}{\sum_{j=1}^{n} I(X_j \le X_i)}$$

$$\bar{g}_i = \frac{\sum_{j=1}^{n} y_j I(x_j \le x_i)}{\sum_{j=1}^{n} I(x_j \le x_i)} \quad \bar{y}_i = \frac{\sum_{j=1}^{n} y_j I(y_j \le y_i)}{\sum_{j=1}^{n} I(y_j \le y_i)} \quad r_i = \frac{1}{n}\sum_{j=1}^{n} I(y_j \le y_i)$$

$$p_i = \frac{1}{n}\sum_{j=1}^{n} I(X_j \le X_i)$$

Notes: The asymptotic variance of *Cy* is also represented by formula 1, after replacing x, η_1 and *d1* by y, η_2 and d_2, respectively. The asymptotic variance of *CRy* is derived in appendix A. I(A) represents the indicator function with value 1 if A is true, 0 otherwise. Theil and Atkinson variances are not included, they can be derived by conventional CLT methods. References for the variance of Gini and Kakwani/redistributive effect indices based on U-statistics formulae are Cowell (1989) and Bishop *et al.* (1998) respectively. We noted a typographical error in equations 23 and 28 of Bishop *et al.*(1998): G*y* should be replaced by *Cy.*

Table III. Parameters of the simulation: real values.

Indices	Real value
Gini	0.38505
Theil(1)	0.27820
Lorenz deciles	
1st	0.02518
3th	0.11112
5th	0.24318
7th	0.42879
9th	0.70349
Kakwani	
Direct taxes	0.29947
Indirect taxes	-0.14368
Suits	
Direct taxes	0.34553
Indirect taxes	-0.15106

Table IV. Simulation Analysis: Real approximate coverage for inequality indices, Lorenz ordinates and progessivity measures.

Index	n = 200	n =1000	n = 2000	n = 200	n =1000	n = 2000
	Method Standard [1]				BC*a*	
Gini	0.783	0.795	0.788	0.901	0.929	0.931
Theil	0.948	0.779	0.718	0.83	0.852	0.858
Lorenz Deciles						
1	0.805	0.829	0.855	0.844	0.869	0.949
3	0.807	0.814	0.829	0.930	0.938	0.946
5	0.810	0.812	0.827	0.930	0.938	0.941
7	0.789	0.807	0.790	0.917	0.933	0.947
9	0.768	0.794	0.782	0.881	0.920	0.920
Kakwani (D)	0.839	0.826	0.841	0.892	0.881	0.901
Kakwani (I)	0.803	0.837	0.823	0.881	0.926	0.930
Suits (D)	0.680	0.685	0.716	0.878	0.855	0.855
Suits (I)	0.826	0.846	0.851	0.864	0.924	0.925

Notes: Nominal coverage: 0.95. [1]Asymptotic estimate of the standard error

Table V. Simulation Analysis: Real approximate coverage for inequality indices, Lorenz ordinates and progressivity measures.

Index	n = 200	n =1000	n = 2000	n = 200	n =1000	n = 2000
	Method Standard [2]				Bootstrap-*t*	
Gini	0.829	0.879	0.884	0.925	0.952	0.948
Theil	0.699	0.755	0.771	0.818	0.866	0.867
Lorenz Deciles						
1	0.865	0.886	0.911	0.823	0.894	0.961
3	0.868	0.899	0.910	0.929	0.950	0.963
5	0.871	0.893	0.906	0.934	0.949	0.957
7	0.851	0.881	0.896	0.933	0.948	0.958
9	0.783	0.849	0.870	0.903	0.940	0.938
Kakwani (D)	0.864	0.826	0.855	0.901	0.882	0.925
Kakwani (I)	0.857	0.897	0.913	0.918	0.935	0.948
Suits (D)	0.828	0.809	0.832	0.878	0.883	0.899
Suits (I)	0.848	0.891	0.906	0.906	0.929	0.943

Notes: Nominal coverage: 0.95. [2] Bootstrap estimate of the standard error

Table VI. RE Lorenz dominance : taxes and benefits.
H_0=D(p)=RE(T_1)-RE(T_2) = 0 H_1 = D(p)>0

Pairwise comparisons	Global-p-values
BE-DT, BE-PIT, BE-CT(2), BE-IT	0.000 0.000, 0.000 0.000
DT-PIT, DT-CT(2),DT-IT	0.039 0.000 0.000
PIT-CT(2), PIT-IT	0.000 0.000
CT(2)-IT	0.000

Table VII. RE Lorenz dominance: benefits.
H_0=D(p)=RE(T_1)-RE(T_2)=0 H_1=D(p)>0

Pairwise comparisons	Global-p-values
BE-BEiK, B-HB, BE-NUE, BE-UE	0.003, 0.000, 0.000, 0.000
BEiK-HB, BeiK-NUE, BeiK-UE	0.001, 0.000, 0.000
HB-NUE, HB-UE	0.000, 0.000
NUE-UE	0.003

Table VIII. TR Lorenz dominance: taxes.
H_0=D(p)=TR(T_1)-TR(T_2) = 0 H_1 = D(p)>0

Pairwise comparisons	Global-p-values
PIT-CT(2), PIT-DT, PIT-CT1, PIT-IT	0.004, 0.003, 0.000, 0.000
CT(2)-DT, CT(2)-CT(1), CT(2)-IT	0.550 0.000, 0.000
DT-CT(1), DT-IT	0.241 0.000
CT(1)-IT	0.052

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Table IX. TR Lorenz dominance: benefits.
H_0=D(p)=TR(T_1)-TR(T_2) = 0 H_1 = D(p)<0

Pairwise comparisons	Global-p-values
BE-BEiK, B-HB, BE-NUE, BE-UE	0.376 1.000 0.800 0.000
BEiK-HB, BeiK-NUE, BeiK-UE	0.846 0.555 0.015
HB-NUE, HB-UE	0.750 0.001
NUE-UE	0.3464

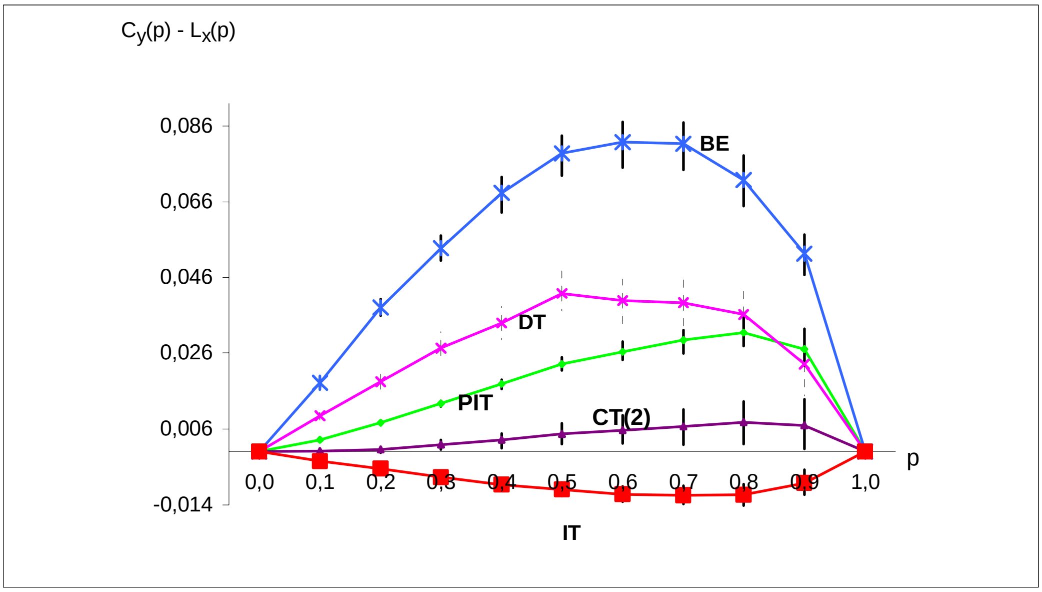

Figure 1. Redistributive effects

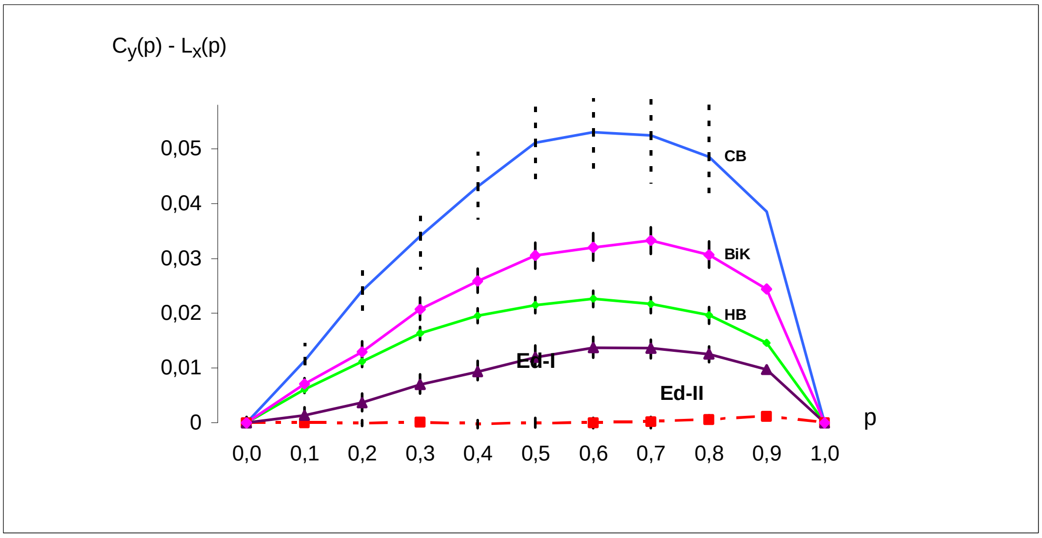

Figure 2. Benefit redistributive effects

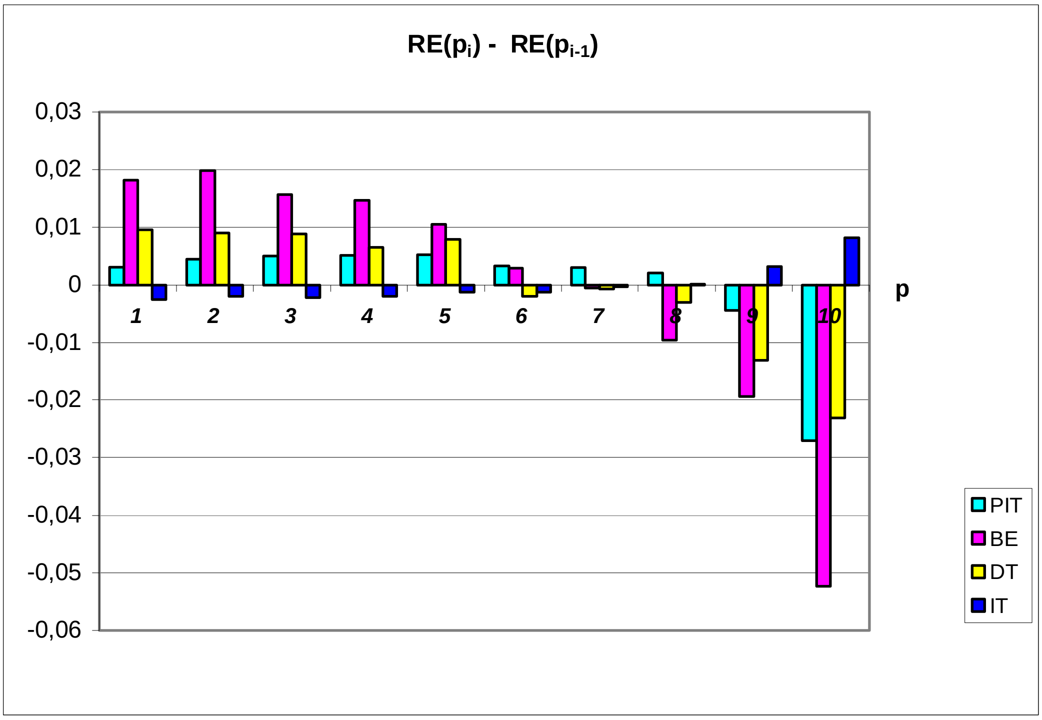

Figure 3. Contribution to the global redistribution per decile.

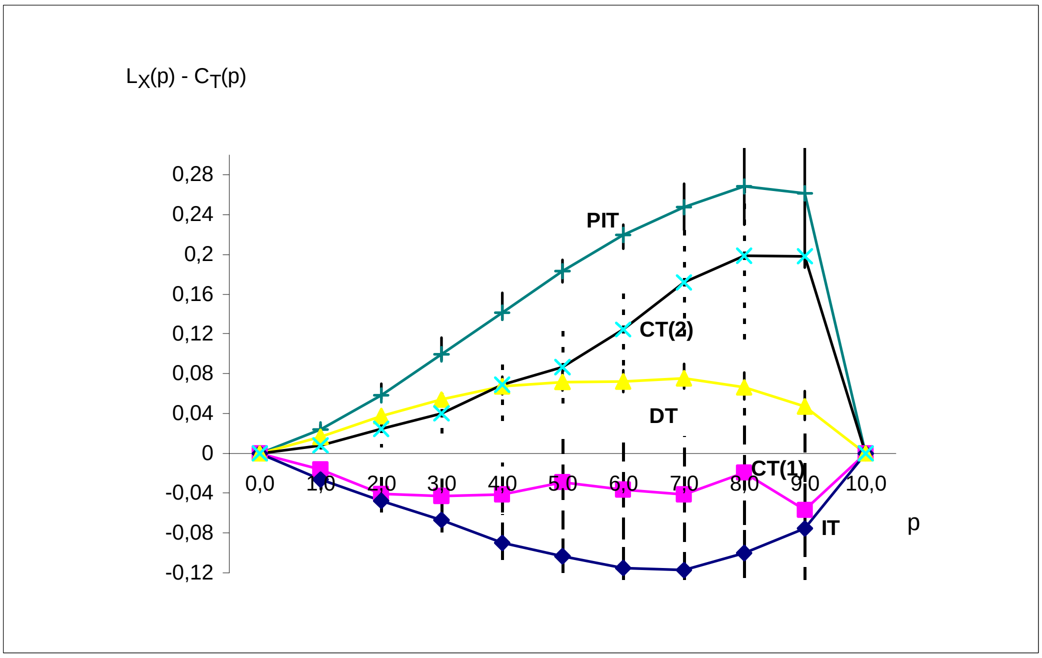

Figure 4. Progressivity profiles.

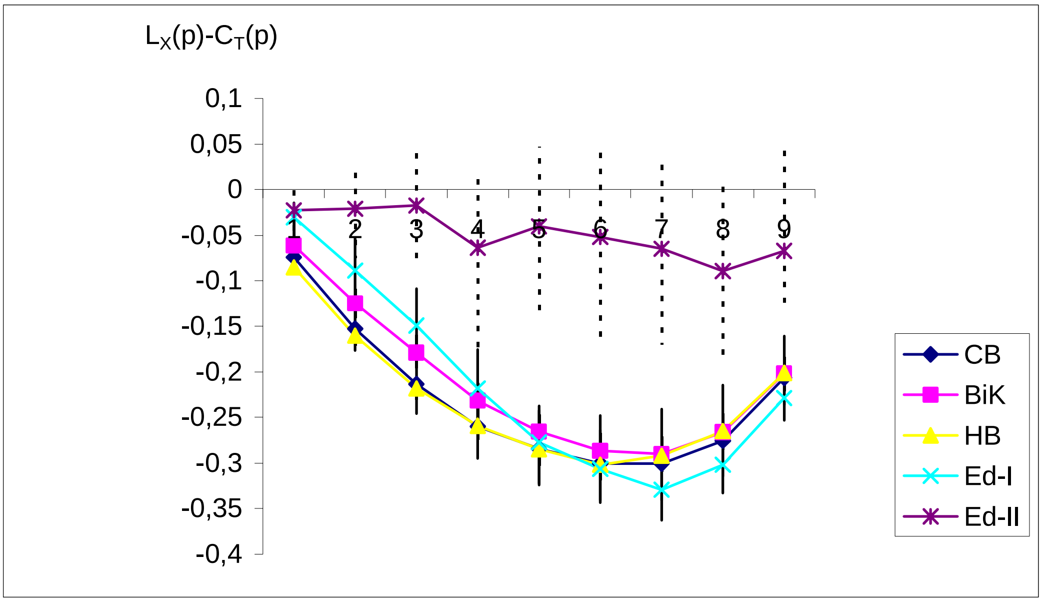

Figure 5. Progressivity profile: benefits.

APPENDIX B

Table B.1 Simulation Analysis: Interval length and shape(italics) for inequality indices, Lorenz coordinates and progressivity measures

Index	n = 200	n = 1000	n = 2000	n = 200	n = 1000	n = 2000
		Method Standard [1]			BC*a*	
Gini	0.08796 (*1.000*)	0.04371 (*1.000*)	0.03178 (*1.000*)	0.10862 (*1.3797*)	0.05980 *(1.3535)*	0.04752 (*1.3943*)
Theil	0.29020 (*1.000*)	0.10993 (*1.000*)	0.08039 (*1.000*)	0.20412 *(1.6633)*	0.13644 *(1.7072)*	0.11082 *(1.7047)*
Lorenz Deciles						
1	0.00736 *(1.000)*	0.00339 *(1.000)*	0.00247 *(1.000)*	0.01004 *(0.8437)*	0.00475 *(0.8695)*	0.00344 *(0.8753)*
3	0.02552 *(1.000)*	0.01182 *(1.000)*	0.00849 *(1.000)*	0.03489 *(0.8884)*	0.01717 *(0.8911)*	0.01274 *(0.8821)*
5	0.04698 *(1.000)*	0.02214 *(1.000)*	0.01616 *(1.000)*	0.06360 *(0.8434)*	0.03177 *(0.8597)*	0.02417 *(0.8529)*
7	0.06997 *(1.000)*	0.03411 *(1.000)*	0.02494 *(1.000)*	0.09405 *(0.7730)*	0.04830 *(0.8097)*	0.03764 *(0.8034)*
9	0.08834 *(1.000)*	0.04583 *(1.000)*	0.03374 *(1.000)*	0.11253 *(0.6489)*	0.06329 *(0.6991)*	0.05023 *(0.7161)*
Kakwani (D)	0.09269 *(1.000)*	0.04968 *(1.000)*	0.03847 *(1.000)*	0.10862 *(1.0833)*	0.05980 *(1.0721)*	0.04527 *(1.0834)*
Kakwani (I)	0.11048 *(1.000)*	0.05523 *(1.000)*	0.04057 *(1.000)*	0.14928 *(0.9758)*	0.07854 *(0.9493)*	0.06027 *(0.9242)*
Suits (D)	0.12343 *(1.000)*	0.07678 *(1.000)*	0.05998 *(1.000)*	0.18381 *(1.1158)*	0.10773 *(1.1238)*	0.08781 *(1.1949)*
Suits (I)	0.13922 *(1.000)*	0.06893 *(1.000)*	0.04940 *(1.000)*	0.17252 *(1.0770)*	0.09032 *(0.9781)*	0.06766 *(0.9406)*

Notes: Nominal coverage: 0.95. [1]Asymptotic estimate of the standard error

Table B.2. Simulation Analysis: Interval length and shape(italics) for inequality indices, Lorenz coordinates and progressivity measures

Index	n = 200	n = 1000	n = 2000	n = 200	n = 1000	n = 2000
		Method Standard [2]			Bootstrap-*t*	
Gini	0.09029 *(1.000)*	0.04864 *(1.000)*	0.03782 *(1.000)*	0.14255 *(1.6625)*	0.06502 *(1.4268)*	0.04824 *(1.3544)*
Theil	0.16421 *(1.000)*	0.10586 *(1.000)*	0.08435 *(1.000)*	0.27426 *(1.7748)*	0.17224 *(1.8674)*	0.13196 *(1.9061)*
Lorenz Deciles						
1	0.00851 *(1.000)*	0.00396 *(1.000)*	0.00285 *(1.000)*	0.01074 *(0.8229)*	0.00483 *(0.8944)*	0.00342 *(0.9173)*
3	0.02979 *(1.000)*	0.01426 *(1.000)*	0.01045 *(1.000)*	0.03763 *(0.8091)*	0.01744 *(0.8769)*	0.01263 *(0.9008)*
5	0.05350 *(1.000)*	0.02625 *(1.000)*	0.01971 *(1.000)*	0.07060 *(0.7705)*	0.03267 *(0.8433)*	0.02408 *(0.8644)*
7	0.07790 *(1.000)*	0.03961 *(1.000)*	0.03025 *(1.000)*	0.11046 *(0.6902)*	0.05095 *(0.7852)*	0.03773 *(0.8165)*
9	0.09069 *(1.000)*	0.05059 *(1.000)*	0.03940 *(1.000)*	0.16172 *(0.5252)*	0.07082 *(0.6735)*	0.05186 *(0.7148)*
Kakwani (D)	0.09291 *(1.000)*	0.04927 *(1.000)*	0.03866 *(1.000)*	0.12809 *(1.3513)*	0.06935 *(1.4074)*	0.05443 *(1.2305)*
Kakwani (I)	0.12494 *(1.000)*	0.06516 *(1.000)*	0.04923 *(1.000)*	0.19634 *(1.0196)*	0.08935 *(0.9951)*	0.06142 *(0.9457)*
Suits (D)	0.15775 *(1.000)*	0.09145 *(1.000)*	0.07402 *(1.000)*	0.24876 *(1.8209)*	0.15921 *(1.4793)*	0.117696 *(1.2621)*
Suits (I)	0.14369 *(1.000)*	0.07483 *(1.000)*	0.05543 *(1.000)*	0.22349 *(1.1719)*	0.10279 *(1.0413)*	0.069043 *(1.0761)*

Notes: Nominal coverage: 0.95. [2]Bootstrap estimate of the standard error