UNIVERZA V MARIBORU
FAKULTETA ZA LOGISTIKO

Sabrina Trafela

MODELIRANJE MESTNE LOGISTIKE

MODELING OF CITY LOGISTICS

magistrsko delo

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Univerza v Mariboru

Fakulteta za logistiko

IZJAVA O AVTORSTVU

diplomskega dela

Spodaj podpisana SABRINA TRAFELA, študentka magistrskega študijskega programa Logistika sistemov, z vpisno številko 21003808, sem avtorica magistrskega dela:

Modeliranje mestne logistike

S svojim podpisom zagotavljam, da:

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- se zavedam posledic, ki jih dokazano plagiatorstvo lahko predstavlja za predloženo delo in za moj status na Fakulteti za logistiko Univerze v Mariboru;
- je diplomsko delo jezikovno korektno in da je delo lektoriral Aleš Zorko.

V Celju, dne ______________ Podpis avtorice:___________________

ZAHVALA

"The highest reward for man's labor is not what you will get for it, but what will you become." John Ruskin

Hvala vsem, ki so verjeli vame.

Prof. dr. Maks Oblak ter mentor prof. dr. Rudolf Pušenjak, brez vaju bi bil nastanek tega dela nemogoč.

Dear Judit and Szabo-Szoba members, thank you for the support and for pushing me forward when it was necessary.

Modeling of city logistics

Master thesis has it's fundamentals in the use of theoretical methods for modeling purposes on the case study of city logistics and modeling of traffic flow and transportation problem on the city of Győr, Hungary.

In practical part, we made a measurements of traffic flow on street sections in the city of Győr. With Android application, we gathered necessary data. We also prepared maps of the route sections of the city from various entry points to the final, destination point. Purpose of our thesis was finding the optimal solution for the delivery to the city centre with the use of strategic games.

Final step was a preparation of mathematical model in Mathematica programming environment, which gave us results, but most important is the development of the model itself. Model is constructed in a way that can be use in various possible situations and on different city centres or case studies.

Keywords: city logistics, strategic games, Nash Equilibrium, modeling of traffic flow.

Modeliranje mestne logistike

Magistrsko delo temelji na uporabi teoretičnih metod za modeliranje na primeru praktične študije primera mestne logistike oziroma modeliranje prometnih tokov in izbiro najboljše poti v pogledu transportnega problema mesta Győr na Madžarskem.

V praktičnem delu so bile opravljene meritve pretoka prometa na ulicah Győra. Z Android aplikacijo smo pridobili potrebne podatke ter pripravili zemljevide odsekov cest iz različnih začetnih točk do končne točke. Namen magistrskega dela je najti optimalno rešitev dostave v center mesta s strateškimi igrami.

Zadnji korak je bila izdelava matematičnega modela v programskem okolju Mathematica, ki nam je dal rezultate, a bolj pomemben je bil razvoj modela kot takšnega. Model je zgrajen tako, da je lahko uporabljen v raznih situacijah in na različnih mestnih odsekih ali študijah primerov.

Ključne besede: mestna logistika, strateške igre, Nashevo ravnotežje, modeliranje prometnih tokov.

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INTRODUCTION

Problem description

City logistics is one of the current problems in today's world. Except of the large cities like London, there is still a lot of traffic in the city centers. With time, companies all around the world started to realize, that big warehouses are not necessary or even »green«, so they look forward to organize their business in the »lean« way and the principle of »just-in-time« is becoming every day practice. Still, with the just-in-time principle, companies need a certain amount of stocks in their warehouses or in the places, where they storage their goods. The problem arises, if the order comes in late and the company doesn't have enough resources to continue the work without disturbances.

Those are the reasons that before implementing "just-in-time" principle in a business, everyone should make a research about how much resources do they need to implement it. From the research view, knowing the traffic flow around and in the city is very important because of the rush hours and traffic jams. If there is a rush hour, companies can be aware of that and can make the arrangements to deliver the goods to them before or after the rush hour.

Objectives, purpose and path

Main objectives of our master thesis are based on the real and useful value of gathering together the theoretical knowledge in the frame of solving real-life problems. While making a decision to write the thesis on certain area of expertise we decided, that we would like to use theoretical knowledge in the case study of city of Győr, Hungary, which was our home for 4 months. Real-life experience of living in the city itself gave us the chance and the possibility to discover the traffic flow and the problems. Because of the variety of small shops in the city center, we became interested, how is it possible to deliver goods to the city center, if there are many pedestrian-only streets.

This brings us to the purpose of the master thesis - first purpose was to extend and use the theoretical knowledge gathered during formal education in the frame of the subject Modeling dynamical processes in logistics. Second purpose was the use of the knowledge on the alive system, such as city center or cities in general. Third and last purpose was upgrade of our own knowledge and abilities to finalize the project such as research in this area of expertise.

We decided to work on the thesis in steps. First step was the table of contents of the thesis, which was the backbone of the whole project. After that we began with the searching for the appropriate theoretical fundamentals and scientific papers. During that time, we became a part of Erasmus experience in the city of Győr, Hungary, for which we decided to be our subject of interest for the purposes of case study. Next step was the preparation of materials and data for the practical part and the preparation of model in the frame of Wolfram Mathematica software. Last step was the analysis of results and the brainstorming about further possibilities and research.

Assumptions and limitations

Based on the chosen city, city of Győr in Hungary we can assume that the working methods for city logistics analysis and optimization are different but the guidelines remain the same as planning the city logistics in big urban areas.

Our limitation is based on the city itself. In our research, we will take a part of the city of Győr and model it for the purpose of city logistics in the frame of dynamical modeling of city supply.

Work methods

We will use following methods:

- the descriptive method, with which we will make the state of the art of the theory;
- the classification method, with which we will define the concepts;
- the mathematical modeling, with which we will gather the results.

1 LITERATURE REVIEW

1.1 City logistics

Nowadays, we can hear a lot about city logistics. But what city logistics really is? Based on our research, we cannot define this term exactly. Only familiar thing that authors have in common while researching city logistics are the topics that it covers topics of logistics, which is efficient supply chain, improved infrastructure and reduced logistics costs, topic of economy which means growth in service, sector attract, investments and new jobs, topic of urbanization covers efficient and reliable supply and sustainability, the topic of traffic means less trucks, lower congestion, intelligent routing and higher truck utilization and final topic, environment, that covers improvement of life quality, noise reduction and air quality improvement. There was only one author that defined city logistics. As Ehmke (2012, p. 9) summarized after Taniguchi, City Logistics as "the process for totally optimizing the logistics and transport activities by private companies in urban areas while considering the traffic environment, the traffic congestion and energy consumption within the framework of a market economy." Benjelloun and Crainic (2009, p. 46) said that City Logistics aims to reduce the nuisances associated to freight transportation while supporting the sustainable development of urban areas. It proceeds generally through the coordination of shippers, carriers, and movements, and the consolidation of loads of different customers and carriers into the same environment-friendly vehicles.

Most authors describe city logistics in the frame of transportation or freight delivery. Diziain, Ripert and Dablanc (2012, p. 268) state that logistics is vital to the life of cities and their residents. It is a major provider of wealth and a source of employment. Large logistics facilities, serving increasingly national and international markets have become a crucial element of dynamic metropolitan economies.

The next definition that comes really close to what we are researching was stated by Awasthi and Chauchan (2012, p. 574). It says that 'Logistics is that part of the supply chain process that plans, implements, and controls the flow and storage of goods, services, and related information from the point of origin to the point of consumption in order to meet customer's requirements". The logistics associated with consolidation, transportation, and distribution of goods in cities is called city logistics. From a systems point of view, city logistics consists of many subsystems involving different stakeholders namely shippers, receivers, end consumers, transport operators and public administrators. The end-consumers are residents or the people that live and work in the metropolitan areas. Shippers (whole-salers) supply good to the receivers (retailers, shopkeepers) through transport operators (or carriers). Administrators repre- sent the government or transport authorities whose objective is to resolve conflict between city logistics actors, while facilitating sustainable development of urban areas.

The existing studies on city logistics planning can be mainly classified into (a) survey based approaches, (b) simulation based approaches, (c) multi-criteria decision making based approaches, (d) heuristics based approaches and (e) cost-benefit analysis based approaches (Awasthi & Chauchan, 2012, p. 575).

Studies on simulation based approaches involve simulation of urban freight over the road networks to assess their impacts. Barceló and Grzybowska combine vehicle routing models and microscopic traffic simulation to model and evaluate city logistics applications. Taniguchi and Tamagawa proposed a simulation based approach for evaluating city logistics measures considering the behavior of several stakeholders. Taniguchi & Yamada use bi-level optimization model for representing the behavior of stakeholders associated with urban goods delivery. Taniguchi and Van Der Heijden present a methodology for evaluating city logistics initiatives using a dynamic traffic simulation with optimal routing and scheduling. Awasthi and Proth propose a system dynamics based approach for city logistics decision making. Segalou et al. propose a traffic simulation model for environmental impact assessment of urban goods movement. Marquez and Salim use scenario analysis to investigate the sensitivity of urban freight patterns to various greenhouse gas abatement policy measures (in Awasthi & Chauchan, 2012, p. 574-584).

DHL company divided city logistics effects in 5 different pillars. Those are (Pušenjak, personal communication, 22. december 2012):

- logistics;
  - efficient supply chain;
  - improved infrastructure;
  - reduced logistics cost;
- economy;
  - growth in service;
  - sector attract;
  - investments;
  - jobs;
- urbanization;
  - efficient and reliable supply;

  - sustainability;

- traffic;
  - less trucks;
  - lower congestion;
  - intelligent routing;
  - higher truck utilization;

- environment;
  - improve life quality;
  - noise reduction;
  - air quality improvement.

In our thesis, we are going to pay attention on traffic point of view and research. As Ehmke, Meisel and Mattfeld (2012, p. 339) stated, city logistics is about routing and scheduling logistics operations in urban areas. Concerning transportation, it seeks for approaches allowing for fast, accurate and reliable pickup and delivery operations as conducted by parcel services or waste disposal services. Nowadays, city logistics service providers have to consider dynamics within logistics processes, e.g., shorter delivery times, higher schedule reliability and delivery flexibility. Furthermore, service providers compete against other road users for the scarce traffic space of inner cities. In conurbanisations, traffic infrastructure is regularly used to capacity.

Realistic travel time estimations for the links of the traffic network are one of the most crucial factors for the quality of routing, since travel times in road networks heavily depend on network load, Network loads in urban areas are highly fluctuant with respect to different network links and times of the day, resulting in traffic jams. Hence, city logistics routing can not rely on mere travel distances. For the most part, a single travel time value per link, as provided by today's digital road maps, only insufficiently represents the traffic situation. City logistics routing requires time-dependent travel times capturing load fluctuations for each network link (Ehmke, Meisel & Mattfeld, 2012, p. 341).

Travel time determination is a long established field of research. Traditionally, travel times have primarily been of interest in the context of modeling traffic flows and quality. The process of travel time determination consists of two basic steps. Firs, traffic flow data is collected empirically. Then, the collected data samples are analyzed and extrapolated in terms of traffic flow models providing travel times. The collection of traffic flow data is usually carried out by stationary sensors or by manual short-time census. Traffic flows in urban road networks are highly fluctuant with respect to different network links, times of the day and day of the week. In order to derive travel times for city logistics, area-wide data collection is necessary (Ehmke, Meisel & Mattfeld, 2012, p. 341).

Data analysis is usually carried out by parameterizing data flow models by collected data samples, resulting in sped-flow diagrams or daily curves of traffic flows. However, traffic flows on urban main street are subject to a large variety of influences leading to modeling obstacles. A detailed reconstruction of travel times from traffic flow samples is complex. In sum, the provision of reliable travel times for city logistics routing is a challenging task and valid approaches are rare (Ehmke, Meisel & Mattfeld, 2012, p. 342).

1.2 Game theory and mathematical modeling

The operation of modern societies (which in nowadays we can also call economies) is based on networking, which are shown in different shapes as for example transportation networks, logistics network, telecommunication networks and also economic networks etc. Interconnection is more and more valuable. All of those networks are actually huge systems and this was the reason for the development of new research that brought to life a new theoretic basics with which it is possible to address those complex network systems. Analysis of supply chain is research area, which is interdisciplinary by its nature and it is the subject of intensive studies. Some authors look on the supply chain problem as optimization problem and they model the supply chain problems statically. Here, the fact that supply chains are dynamic, is neglected (Pušenjak, Oblak & Lipičnik, 2012, p. 75). In this chapter of our thesis, we will follow the path of Pušenjak, Oblak and Lipičnik (2012, p. 75), that described the supply chain as the system of equilibrium and non-equilibrium state of the competition, which is the part of strategic games, that we will mention in the future.

Dynamical approach in the frame of multilevel structure of supply chain gives us possibility to control different types of streams, where the most important are logistics, information and financial stream in the frame of system network (Pušenjak, Oblak & Lipičnik, 2012, p. 76). The dynamic approach that will be used in our thesis will be based on the queuing theory and the theory of strategic games.

Basic assumption for the strategic games (in accordance with game theory) is, that the players, which are involved in the play, choose their actions rationally, which means that they make choices based on the goal that they win the game. Game theory assumes that the player of the strategic games play for the win, better position, success in negotiations, satisfaction, etc. Effort for winning is described in the field of sociology. In the field of logistics, strategic games can represent achievement on the technical areas, such as optimization of route, minimization of fuel consumption, environment protection, etc. This field makes strategic games very important part of logistics field of expertise (Pušenjak, Oblak & Fošner, 2013, p. 100).

A strategic game consists of (Pušenjak, Oblak & Fošner, 2013, p. 100):

- *a set of players*;
- for each player, a *set of actions*: in certain strategic games a set of actions also represents the set of available moves, which is a typical case of chess game;
- for each player, *set of action profiles*: individual action profile is a set of actions, in which each player can choose only one action in the set of actions;
- *priority scale:* the set of action profiles, which defines this type of strategic game as an ordinal priority scale.

A very wide range of situations may be modeled as strategic games. For example, the players may be organizations, the actions prices and the preferences a reflection of the organization's profits. Time is absent from the model. The idea is that each player chooses her action once and for all, and the players choose their actions "simultaneously" in the sense that no player is informed, when she closes her action, of the action chosen by any other player (Osborne, 2002, p. 12).

A very famous author, winner of the Nobel prize was dealing with the theory of strategic games. His name was John F. Nash, Jr. His solution theory has two components. First, each player in strategic game chooses her action according to the model of rational choice, given her belief abouh other players' action. Second, every player's belief abouh the other players' action is correct. These two components are embodied in the following definition. He developed so called Nash equilibrium which is described as an action profile a^* with the property that no player i can do better by choosing an action different from a_i^*, given that every other player j adheres to a_j^* (Osborne, 2002, p. 21).

1.2.1 Modeling of transport systems with strategic games

Now we have the basic idea of strategic games, so we can continue to the use of game theory in the field of logistics. Game theory in general and especially strategic games can be very valuable tool if we have to model transport systems, prepare certain models to make effective decision making or optimization of transport systems.

Transport systems are diverse and are the subject of intensive studying in the frame of logistics. In basis, transport systems are dynamical, but the evolution of those systems in the dependence of time is too complex to make the calculations in the "pen and paper" way. While using evolutionary way there are a lots of obstacles such as time barriers, lack of knowledge, experience, etc., that we have to take into consideration. If we want to do the planning and managing of transport systems for decision making and it's optimization, strategic games are one of the most appropriate tools for it. We will present the usage of strategic game in a transport system modeling (Pušenjak, Oblak & Fošner, 2013, p. 100).

In the next subsection we will describe the strategic game, that is the key element in our practical part of the thesis, called "Choosing a route". First, we will present a strategic game with the table approach and in further subsections, we will describe the theoretical meaning of the used terminology together with Nash Equilibrium Theory.

1.3 Strategic games

1.3.1 Description of situation of strategic game "Choosing a route" - modeling for 2 travelers with table approach

Let's take a look to strategic game "Choosing a route", if there are only two possible ways between points A and B. Transport problem can be shown with the use of table only in the case if there are 2 vehicles driving throught roads, connecting points A and B. This means that the players in this strategic game are 2 travelers, where in the table, rows are corresponding for first travelers and columns for the second traveler. Let's make an agreement that traveler chooses route section X if he is driving on first road and route section Y if he is driving on second road. Choice of route sections X means first strategy and the choice of route section Y second strategy for independent traveler.

Set of possible strategies consists of two elements X and Y, and it's writen in a form $S = \{X, Y\}$. As we can see, set of strategies is exactly the same as in extended transport problem (chapter 1.2.1.2). If there are only two route sections between points A and B, corresponding table has only 2 rows and 2 columns.

As said, number of players is consistent with number of travelers (or vehicles), that are labeled with n and in our case $n = 2$. Number of strategies is labeled with m and in our case is $m = 2$. Number of action profiles is $n^m = 2^2 = 4$ and those profiles are gathered in the set $AP = \{XX, XY, YX, YY\}$.

Figure 1.1: *Strategic game "Choosing a route" - 2 travelers*

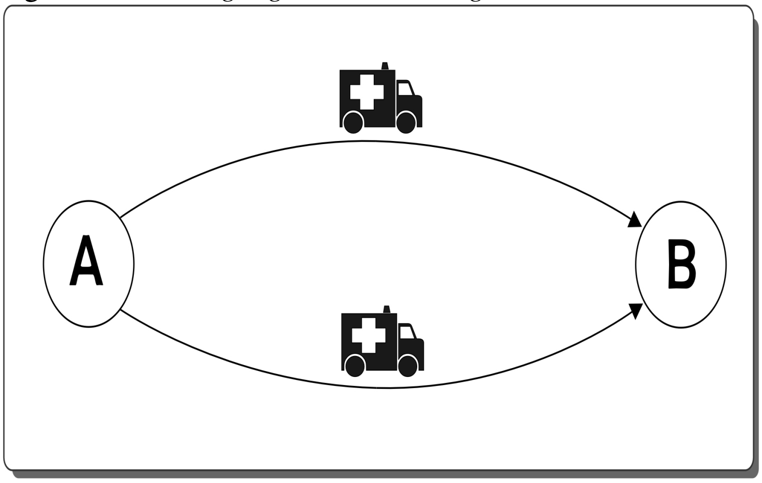


Table 1: *Strategic game "Choosing a route" between 2 route sections*

		2nd player	
		X	Y
1st player	X	$-22,-22$	$-20,-21$
	Y	$-21,-20$	$-21.5,-21.5$

Source: Pušenjak, personal communication, 14. january 2013

Action profile $a=(XX)$ coresponds to situation, where both vehicles are driving through first route section, action profiles (XY) and (YX) correspond to situation, where each vehicle drives through each route section, where the difference between the profiles is that in (XY) first traveler chooses route section X and the second traveler chooses route section Y, while in second action profile (YX) first traveler chooses route section Y and second traveler chooses route section X. If both travelers choose route section Y, we get action profile (YY).

Let's assume that the vehicle needs 20 minutes of driving time from point A to point B if it's driving through route section X. For each additional vehicle, driving time increases for 2 minutes, if there are two vehicles on route section X.

Let's also assume that if the vehicle is driving through route section Y, it needs 21 minutes to come from point A to point B. Driving time for each additional vehicle is increased for 0.5 minutes.

For the outcome of strategic game we choose similar labeling as in extended approach, so we label the outcome with negative value of travel time. Strategic game with table approach can be represented with the table 1.

1.3.2 Nash Equilibrium in strategic game "Choosing a route" with 2 travelers

We figured, that there are four possible action profiles for which we want to figure it out if they represent Nash Equilibrium or not.

Definition of Nash Equilibrium: action profile a^* of a strategic game with ordinal priority scale is Nash Equilibrium, if action profile a^* for each player $i = 1, 2, ...n)$ and for each action a_i of i-th player is at least as good as action profile (a_i, a^*_{-i}), in which i-th player changes his action a^*_i to action a_i when all the others, let's say j-th player preserves his action a^*_j.

We can express the definition mathematicaly with the use of inequality

$$u_i(a^*) \geq u_i(a_i, a^*_{-i}) \tag{1.1}$$

First, we have to examine action profile $a^* = (X, X)$. For determining Nash Equilibrium we will use the inequality (1.1) that defines Nash Equilibrium if it satisfies all actions a_i for i-th player. We use the table 1 to read the outcomes of strategic game in addressed action profile for both players (travelers) and we get $u_1(a^*) = -22$ and $u_2(a^*) = -22$. To discover, if action profile a^* represents Nash Equilibrium, we have to validate the inequality $u_i(a^*) \geq u_i(a_i, a^*_{-i})$ for both players (Pušenjak, personal communication, 3. june 2013).

For first player index i is $i = 1$ and for index $-i$ in this case corresponds the value $-i = 2$, which gives us changed action profile $(a_1, a^*_2) = (Y, X)$. We read the value of the outcome of strategic game u_i from the table 1, which is $u_1(a_1, a^*_2) = u_1(Y, X) = -21$. With this the inequality $u_i(a^*) \geq u_i(a_i, a^*_{-i})$ does not apply because the reverse statement is true $u_1(a^*) = -22 < u_1(a_1, a^*_2) = -21$, which is sufficient to establish that action profile $a^* = (X, X)$ is not Nash Equilibrium no matter if the inequality $u_i(a^*) \geq u_i(a_i, a^*_{-i})$ is true for the second player or not, because both players chose the strategy of route section X.

As second action profile, let's check the action profile $a^* = (X, Y)$. From the table 1 for this case we read the outcome of strategic game for mentioned action profile for both players (travelers) and we get $u_1(a^*) = -20$ and $u_2(a^*) = -21$. To check if the action profile a^* is Nash Equilibrium, we have to re-validate the inequality $u_i(a^*) \geq u_i(a_i, a^*_{-i})$ for both players. For first player $i = 1$, and for this case $-i = 2$, which gives us changed action profile $(a_1, a^*_2) = (Y, Y)$. We read value of outcome of strategic game $u_1(a_1, a^*_2) = u_1(X, Y) = -21.5$ from the table 1. With this, the inequality $u_i(a^*) \geq u_i(a_i, a^*_{-i})$ does apply because we can write $u_1(a^*) = -20 \geq u_1(a_1, a^*_2) = -21.5$. Now we have to validate the inequality also for the second player. Here $i = 2$ and index $-i$ has a value of $-i =$

1. With this, we get the changed action profile $(a_1^*, a_2) = (X, X)$. Inequality is in this case writen as $u_2(a^*) \geq u_2(a_1^*, a_2)$ and it obviously applies, because $u_2(a^*) = -21 \geq u_2(a_1^*, a_2) = -22$. Inequality $u_i(a^*) \geq u_i(a_i, a_{-i}^*)$ is true for both players, which means that the action profile $a^* = (X, Y)$ is Nash Equilibrium.

Third action profile is $a^* = (Y, X)$. Outcomes of the strategic game for both players in this action profile are $u_1(a^*) = -20$ and $u_2(a^*) = -21$. We validate the inequality $u_i(a^*) \geq u_i(a_i, a_{-i}^*)$ for both players in a way, that for first player, $i = 1$ and $-i = 2$, with which we get changed action profile $(a_1, a_2^*) = (X, X)$. We read the outcomes of the strategic game $u_1(a_1, a_2^*) = u_1(X, X) = -22$ from the table 1. With this, the inequality $u_1(a^*) \geq u_1(a_1, a_2^*)$ does apply because we can write $u_1(a^*) = -20 \geq u_1(a_1, a_2^*) = -22$. For the second player, $i = 2$ and $-i = 1$. With this we get changed action profile $(a_1^*, a_2) = (Y, Y)$ with it's outcome $u_2(a_1^*, a_2) = u_2(Y, Y) = -21.5$. Inequality is in this case writen as $u_2(a^*) \geq u_2(a_1^*, a_2)$ and it obviously applies, because $u_2(a^*) = -21 \geq u_2(a_1^*, a_2) = -21.5$. Inequality $u_i(a^*) \geq u_i(a_i, a_{-i}^*)$ is also true, which means that the action profile $a^* = (X, Y)$ is also Nash Equilibrium (Pušenjak, personal communication, 3. june 2013).

Let's continue with the last action profile $a^* = (Y, Y)$. We can see from the table 1 in this case the outcomes of strategic games in mentioned action profile for both players (travelers) and we get $u_1(a^*) = -21.5$ and $u_2(a^*) = -21.5$. To define if the action profile a^* represents the Nash Equilibrium, we have to re-validate the inequality $u_i(a^*) \geq u_i(a_i, a_{-i}^*)$ for both players. For the first player, $i = 1$ and for this case $-i = 2$, which gives us changed action profile $(a_1, a_2^*) = (X, Y)$. We read value of outcome of strategic game $u_1(a_1, a_2^*) = u_1(X, Y) = -20$ from the table 1. With this, the inequality $u_i(a^*) \geq u_i(a_i, a_{-i}^*)$ does not apply because the reverse statement is true ($u_1(a^*) = -21.5 < u_1(a_1, a_2^*) = -20$), which is sufficient to establish that action profile $a^* = (Y, Y)$ is not Nash Equilibrium no matter if the inequality $u_i(a^*) \geq u_i(a_i, a_{-i}^*)$ is true for the second player or not, because both players chose the strategy of route section Y (Pušenjak, personal communication, 3. june 2013).

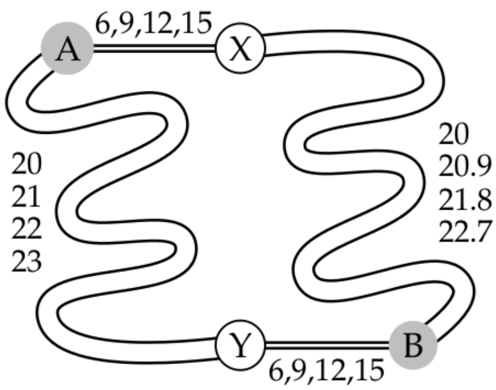
The outcome of strategic game "Choosing a route" for two players is interesting, if we compare it to extended transport problem with four passanger (next subsection). As a reader can see, we figured that each action profile, where two travelers choose strategy X and two travelers choose strategy Y is Nash Equilibrium, while in the case of two travelers, Nash Equilibrium is when one traveler chooses strategy X and the other chooses strategy Y.

Both Nash Equilibriums belong to action profiles, where none of the players has the reason to change his strategy, because there is no possible way to improve the outcome of strategic game. With other words, both players should preserve his decision, while it belongs to action profile (X, Y) or (Y, X) because those two represent Nash Equilibrium.

1.3.3 Description of situation of strategic game "Choosing a route" - modeling of complex transport system

Four travelers have to go from the point A to point B at the same time. Each of the travelers has to decide which route, that is available, to choose. Between point A and point B are two different ways, where one goes through point X and the other one through point Y. Routes, that connect the points A and B are shown in Figure 1.2. Road sections between point A and point X or between point Y and point B are local roads that are short but really narrow. For overcoming the distance on the road section $A-X$ or $Y-B$, each vehicle needs 6 minutes of drive, but the travel time extends for further 3 minutes for each additional vehicle. If, for example, there are two vehicles driving from A to X, each vehicle needs 9 minutes. If there are three vehicles, each of them needs 12 minutes etc. Roads between A and Y and between X and B are wide and long intercity roads. For overcoming the distance between point A and point Y, vehicle needs 20 minutes, but each further vehicle increases time travel for 1 minute per vehicle. From X to B, vehicle also needs 20 minutes and each further vehicle increases time travel for 0.9 minute per vehicle. On Figure 1.2 we can see the travel times for each vehicle, if on each road are driving 1, 2, 3 or 4 vehicles. Described situation should be modeled as strategic game and Nash Equilibrium should be found. With Nash Equilibrium we have to define if in the case that all four passangers choose the same route, if there would be better that one of them would choose the other one (independent of the traveler). We also need to make a judgement about if the better decision is if 3 of the travelers use one road section and one of them uses the other one. Based on this, we have to study the case of 2 travelers choosing one road and the other two the other road (Pušenjak, Oblak & Fošner, 2013, p. 100).

Figure 1.2: *Original network*



Source: *Osborne, 2002*

1.3.4 The structure of strategic game "Choosing the route"

Strategic game with which we can model the situation of choosing a route is defined with following mathematical structures (Pušenjak, Oblak & Fošner, 2013. p. 101):

- the set of players: In strategic games, there are four travelers ($i = 1, 2, ..., 4$). Because the number of players ($n = 4$)is higher than 2, strategic game cannot be illustrated with the table. Modeling will be made in the software environment Mathematica or Matlab;

- set of strategies or actions: Set of strategies includes all possible road sections, that connects the starting point A with the destination point B. Between A and B there are two road sections, where the first one goes through point X and the other one through point Y. Because X and Y point define certain road section, we can define the set of strategies in following form: $S = \{X, Y\}$, where X and Y are two typical elements of the set. For $m = 2$ we define the number of elements of the set S or number of strategies;

- set of action profiles: Because there are four players in the strategic game and there are only two possible strategies, is the number of action profiles $p = n^m = 4^2 = 16$. In Mathematica, we generate all 16 action profiles with the comands `n=4;Tuples[{X,Y},n]`. Command `Tuples` generates the set of all possible set (all possible p) strategies or actions of n players. In accordance to the syntax of the programming environment Mathematica, each set of actions is defined as a list, where elements of the list are divided between each other with comma or closed with the pair of braces. Because of the use of braces and not parentheses is the display of each action profile in Mathematica different from the display of action profile in our thesis. Generated action profiles belong to each of four travelers of strategic game;

- priority scale: priority scale is defined by the outputs of strategic games for each player. For the output of certain player in individual action profile is the most appropriate if we take negative value of his travel time. With this type of adaptation we reach the best possible outcome for each player that corresponds to the shortest travel time. Action profiles are classified in accordance to priority scale from the best to the worst action profile.

1.3.5 Calculating the strategic game "Choosing the route"

We already mentioned that we cannot model the strategic game "choosing the route" with table, but we have to calculate the strategic game with mathematical approaches such as theory of sets, combinatorics and mathematical analysis (Pušenjak, Oblak & Fošner, 2013, p. 102).

We will begin with mathematical description of strategic game "choosing the route" in a way, that we provide the number of players and number of strategies. Number of players in our case is defined with parameter n, which means $n = 4$ and number of strategies or actions with parameter m, which is $m = 2$. Based on our wishes, we can change the parameters. Set of strategies in our case has two elements X and Y and we define it as $S = \{X,Y\}$. First strategy X means, that the traveler chooses the route section $A-X-B$ and the second strategy Y means, that the traveler chooses a route section $A-Y-B$. We should know that all four travelers are starting the travel from point A and the destination point is point B, where each traveler can choose only one strategy X or Y. That means that the travel is defined by set of four strategies. Random set of four strategies can be written as $a = (a_1, a_2, a_3, a_4)$ and it's called action profile, where action a_i, $(i = 1,2,...,4)$ can take state $a_i = X$ or $a_i = Y$. In general, the set of all action profiles is labeled with AP and it's made from all possible combination of n actions. The set of action profiles for the strategic game "choosing the route" is defined by following equation (Pušenjak, Oblak & Fošner, 2013, p. 102):

$$\begin{aligned} AP = \{&(X,X,X,X),(X,X,X,Y),(X,X,Y,X),(X,X,Y,Y),\\ &(X,Y,X,X,),(X,Y,X,Y),(X,Y,Y,X),(X,Y,Y,Y,),\\ &(Y,X,X,X),(Y,X,X,Y),(Y,X,Y,X),(Y,X,Y,Y,),\\ &(Y,Y,X,X),(Y,Y,X,Y),(Y,Y,Y,X),(Y,Y,Y,Y)\}. \end{aligned} \tag{1.2}$$

For each action profile from the set AP there are four possible outcomes of the game, which means one outcome for each traveler. Our task is to construct that type of general function of outcomes of strategic game $u(i,k)$, with which we can calculate the outcome of the strategic game for arbitrary i-th traveler $(i = 1,2,...,n)$ in arbitrary action profile k $(k = 1,2,...,n^2)$. This function is defined as negative value of travel time, which means that we take $u(i,k) = -t_p(i,k)$, where $t_p(i,k)$ represents travel time of i-traveler in k-action profile. Travel time of each traveler depends on the choice of strategy (if he chooses strategy X and travels through route section $A-X-B$ or if he chooses strategy Y and travels through route section $A-Y-B$) and also on the number of vehicles on the route (in our case there can be a group of two, three or four vehicles) (Pušenjak, Oblak & Fošner, 2013, p. 102-103).

If we introduce a parameter $p(i,k)$ and $q(i,k) = 1-p(i,k)$ and we define it in a way that $p(i,k) = 1$ when the traveler chooses strategy X and $p(i,k) = 0$ when traveler chooses strategy Y, then we can express the travel time $t_p(i,k)$ as a sum $t_p(i,k) = p(i,k)t_x(i,k) + q(i,x)t_y(i,k)$, where $t_x(i,k)$ represents travel time on the route section $A-X-B$ and $t_y(i,k)$ represents travel time on route section $A-Y-B$. From this sum formulation and taking the link $q(i,k) = 1 - p(i,k)$ into consideration we can conclude that travel time $t_p(i,k) = t_x(i,k)$ when traveler chooses the strategy X is the same to $t_p(i,k) = t_y(i,k)$, when traveler chooses strategy Y. While defining travel times $t_x(i,k)$ and $t_y(i,k)$ we have to take into consideration the number of vehicles driving on the route section $A-X-B$ or $A-Y-B$ (Pušenjak, Oblak & Fošner, 2013, p. 103).

Let's label the number of vehicles on the route $A-X-B$ (which means with the choice of strategy X) in k action profile with $s_X(k)$ and the number of vehicles on the route $A-Y-B$ (with the choice of strategy Y) in k action profile with $s_Y(k)$. In the case of several vehicles on the route $A-X-B$ we have to add additional time $t_X^{add} = [s_X(k)-1]\cdot[3+0.9]$ to the sum $6+20=26$ (On route section $A-X$ we have to add 3 minutes to each additional vehicles and on route section $X-B$ additional 0.9 minutes per vehicle. It is obvious that in the case of only one vehicle ($s_X(k)=1$) on the route section $A-X-B$, we don't add any additional time, which means $t_X^{add}=0$). We also have to add additional time $t_Y^{add} = [s_Y(k)-1]\cdot(1+3)$ with several vehicles on the route section $A-Y-B$ to the sum $20+6=26$ (on the route section $A-Y$ we have to add 1 minute per each additional vehicle and on route section $Y-B$ additional 3 minutes per vehicle. Also in this case, we don't add additional time t_Y^{add}, which means that $t_Y^{add}=0$ or in case that on route section $A-Y-B$ is driving only one vehicle $s_Y(k)=1$) (Pušenjak, Oblak & Fošner, 2013, p. 103).

General function of outcomes of strategic game "Choosing a route" is therefore composed as follows (Pušenjak, Oblak & Fošner, 2013, p. 103):

$$\begin{aligned} t_p(i,k) &= p(i,k)\{26+[s_X(k)-1]\cdot(3+0.9)\}+q(i,k)\{26+[s_Y(k)-1]\cdot(1+3)\} \\ &= p(i,k)\{26+[s_X(k)-1]\cdot 3.9\}+q(i,k)\{26+[s_Y(k)-1]\cdot 4 \end{aligned} \tag{1.3}$$

$$u(i,k) = -t_p(i,k) \tag{1.4}$$

Example. Let's calculate travel time of all four travelers with action profile (X,X,X,X). We can conclude from equation (1.2) that we are dealing with first action profile, which means that $k=1$. Number of vehicles on route sections $A-X-B$ and $A-Y-B$ are $s_X(1)=4$ and $s_Y(1)=0$. For first traveler, $i=1$ we also have to calculate $p(1,1)=1$, $q(1,1)=1-p(1,1)=0$ and we set the travel time (Pušenjak, Oblak & Fošner, 2013, p. 103):

$$t_p(1,1) = 1 \cdot [26 + (4-1) \cdot 3,9] + 0 \cdot [26 + (0-1) \cdot 4] = 37.7$$

for second traveler ($i = 2$) we set $p(2,1) = 1$, $q(2,1) = 1 - p(2,1) = 0$ and we set the travel time

$$t_p(2,1) = 1 \cdot [26 + (4-1) \cdot 3,9] + 0 \cdot [26 + (0-1) \cdot 4] = 37.7$$

and we can make similar calculation for third and fourth traveler. Because we are dealing with action profile (X,X,X,X), where all four passangers choose route section $A - X - B$, travel times are the same for all four of them (37.7 minutes). Outcomes of strategic game with action profile (X,X,X,X) are also the same for all four travelers (-37.7 units).

Example. Let's calculate travel times and the outcomes of strategic game of all four travelers with action profile (X,X,Y,Y). We can conclude from equation (1.2) that we are dealing with fourth action profile, which means that $k = 4$. Number of vehicles on route sections $A - X - B$ and $A - Y - B$ are $s_X(4) = 2$ and $s_Y(4) = 2$. For first traveler, $i = 1$ we also have to calculate $p(1,4) = 1$, $q(1,4) = 1 - p(1,4) = 0$ and we set the travel time (Pušenjak, Oblak & Fošner, 2013, p. 104)

$$t_p(1,4) = 1 \cdot \{26 + [2-1] \cdot 3,9\} + 0 \cdot \{26 + [2-1] \cdot 4\} = 29.9$$

for second traveler ($i = 2$) we set $p(2,4) = 1$, $q(2,4) = 1 - p(2,4) = 0$ and we set the travel time

$$t_p(2,4) = 1 \cdot \{26 + [2-1] \cdot 3,9\} + 0 \cdot \{26 + [2-1] \cdot 4\} = 29.9$$

for third traveler ($i = 3$) we set $p(3,4) = 0$, $q(3,4) = 1 - p(3,4) = 1$ and we set the travel time

$$t_p(3,4) = 0 \cdot \{26 + [2-1] \cdot 3,9\} + 1 \cdot \{26 + [2-1] \cdot 4\} = 30$$

Calculation for the forth traveler ($i = 4$) is the same as for the third, the only difference is concerning index i. In this case, we get $p(4,4) = 0$, $q(4,4) = 1 - p(4,4) = 1$ and we calculate the travel time as follows:

$$t_p(4,4) = 0 \cdot \{26 + [2-1] \cdot 3,9\} + 1 \cdot \{26 + [2-1] \cdot 4\} = 30$$

We can see that the change with index i does not influence on the final outcome. This is the consequence of the fact, that third and forth traveler chose the same strategy Y, which means that they both travel on route section $A - Y - B$ (Pušenjak, Oblak & Fošner, 2013, p. 104).

1.3.6 Nash Equilibrium in strategic game "Choosing a route"

In the problem "Choosing a route" label $u(i,k)$ for the function of outcomes of strategic games showed certain advantages. This label is useful especially for the reason, that we can use it in software environment Mathematica or Matlab. If we want to calculate Nash Equilibrium with the software program, we have to reconcile the definition of Nash Equilibrium with this label. In standard definition of Nash Equilibrium we labeled equlibrium action profile with a^* and with $u_i(a^*)$ the outcome of strategic game of i-player with this profile. We can see, that the label is not useful for programming purposes. If we assume, that we will label Nash Equilibrium with k-action profile, than we can say that $u(i,k) = u_i(a^*)$ (Pušenjak, Oblak & Fošner, 2013, p. 104).

By definition, a^* is Nash Equilibrium, if considered $u_i(a^*) \geq u_i(a_i, a^*_{-i})$ for each $i = 1, 2, ..., n$, where (a_i, a^*_{-i}) represents action profile, where i-player changes strategy (action) a^*_i to action a_i and every other player preserves chosen strategy, what can be labeled symbolicaly with the use of symbol a^*_{-i}. Let's come to agreement, that changed action profile (a_i, a^*_{-i}) will be labeled as $k'(i)$-profile. We will present the way of calculating number $k'(i)$ below (Pušenjak, Oblak & Fošner, 2013, p. 104).

The proces of determining the number $k'(i)$

1. We minimize number k for 1. This means that action profiles are re-numbered into sequence of profiles, numbered with $0, 1, ..., n^2 - 1$.

2. We assign a BCD code to number $k - 1$. This means, that we decode decimal number to binary number. The outcoming binary number is a number with n-digits.

3. On i-th spot of BCD code of number $k - 1$ we perform the complementary operation. This means that we change number 0 to 1 and reversed. The complementary operation consists to change of strategy of i-player.

4. We decode given BCD code to decimal number, with which we get a number $k'(i) - 1$.

5. We get the desired number $k'(i)$, if we maximize number $k'(i)-1$ for 1.

Example. Let's test the proces described abowe with first traveler ($i=1$) and first action profile $k=1$ of strategic game "Choosing a route". We can presume from equation (1.2), that the mentioned action profile is $a=(X,X,X,X)$. Binary code of number $k-1=0$ is $(k-1)_{BCD}=0000$, complement of first number ($i=1$) of binary number is 1, with which we get binary code $(k'(1)-1)_{BCD}=1000$. With conversion into decimal number we get $k'(1)-1=1\times 2^3+0\times 2^2+0\times 2^1+0\times 2^0=8$. Desired number $k'(1)$ is $k'(1)=8+1=9$. Based on equation (1.2) we can check, that 9th action profile is $a'=(Y,X,X,X)$ which means that only first traveler changed strategy X to strategy Y while all the others maintained chosen strategy X. This outcome is exactly, what we wanted to achieve (Pušenjak, Oblak & Fošner, 2013, p. 105).

By determining the number $k'(i)$ adaptation $u[i,k'(i)]=u_i(a_i,a^*_{-i})$ is true and we can transform the definition of Nash Equilibrium as follows.

Action profile of strategic game with ordinal priority scale is Nash Equilibrium with k-th element of set AP, if k-th profile for each player ($i=1,2,...,n$) and every action a_i of i-th player is at least as good as $k'(i)$-th action profile, in which i-th player changes action a^*_i to action a_i, while every other, let's say j-th player, preserves his action a^*_j. We can define this mathematicaly using inequality (Pušenjak, Oblak & Fošner, 2013, p. 104)

$$u(i,k)\geq u[i,k'(i)] \tag{1.5}$$

that defines Nash Equilibrium, if it's fullfilled for all actions a_i of i-th player ($i=1,2,...,n$).

Let's check if the action profile $a=(X,X,X,X)$ is Nash Equilibrium with the help of definition with inequality (1.5).

In this case $k=1$, number $k'(i)$ is already set for first player, $i=1$ and we get $k'(1)=9$. With the use of equation (1.3) we calculate travel times $t_p(1,1)$ and $t_p(1,9)$ for first traveler ter for action profiles at $k=1$ and $k'(1)=9$ and then with the use of equation (1.5) also corresponding outcome (outcome) of strategic game $u(1,1)$ and $u(1,9)$. We already calculated travel time in the example 1, but we will do it again here just for the purpose of greater transparency.

First, we have to calculate $s_X(1)=4$, $s_Y(1)=0$ and $p(1,1)=1$, $q(1,1)=1-p(1,1)=0$. After that we use formula (1.3) and we get

$$t_p(1,9)=1\cdot[26+(4-1)\cdot 3.9]+0\cdot[26+(0-1)\cdot 4]=37.7$$

and by the formula (1.4) we get $u(1,1) = -37.7$. For the profile with $k'(1) = 9$ we first define $s_X(9) = 3$, $s_Y(9) = 1$ and $p(1,9) = 0$, $q(1,9) = 1 - p(1,9) = 1$. With the use of formula (1.3) we calculate

$$\begin{aligned} t_p(1,9) &= p(1,9)\{26 + [s_X(9) - 1] \cdot 3.9\} + q(1,9)\{26 + [s_Y(9) - 1] \cdot 4\} \\ &= 0 \cdot \{26 + [3 - 1] \cdot 3.9\} + 1 \cdot \{26 + [1 - 1] \cdot 4\} \\ &= 26 \end{aligned}$$

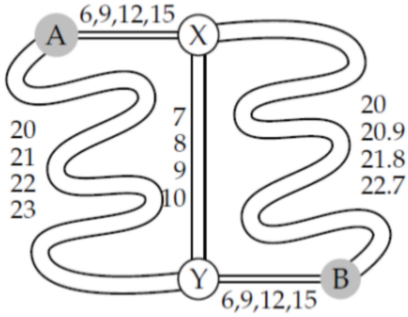
Corresponding outcome of strategic game is $u(1,9) = -26$. Because of that, action profile $a = (X,X,X,X)$ is not Nash Equilibrium no matter what were the outcomes of other outcomes of inequality (1.1) for other travelers.

1.3.7 Strategic game "Choosing a route" in reconstructed road network

This subsection describes the upgrade of the original strategic game "Choosing a route" with reconstructed road network. Let's say that with the traffic development, original road network was reconstructed with the construction of new, relatively short but wide road link between the points X and Y. With this, all four travelers from previous case get for possible choices to travel from point A to point B. Those are: $A-X-B$, $A-Y-B$, $A-X-Y-B$ and $A-Y-X-B$. Let's assume that the traveler, who chooses the route section $A-X-Y-B$ travels on the route section $A-X$ at the same time as any traveler, who chooses the route section $A-X-B$, and if he travels on route section $A-Y$, he drives at the same time as any traveler who drives on the route section $A-Y-B$ (Pušenjak, Oblak & Fošner, 2013, p. 101).

On the new constructed road between points X and Y one vehicle needs 7 minutes, while each additional vehicle increases the time travel for 1 minute per vehicle. We have to find Nash Equilibrium in the new situation and make a comparison of travel time for each additional traveler with his time before the new road between X and Y was constructed. Changed situation of strategic game is shown on the figure 1.3, where numbers next to route sections represent the travel times for each vehicle, if there are 1, 2, 3 or 4 vehicles on the road (Pušenjak, Oblak & Fošner, 2013, p. 101).

Figure 1.3: *Road network with new road section between points X and Y*



Source: Osborne, 2002

1.3.8 Structure of a strategic game "Choosing a route" in reconstructed road network

Strategic game with which we can model the situation of choosing a route in reconstructed road network is defined with following mathematical structures (Pušenjak, Oblak & Fošner, 2013, p. 101):

- the set of players: In strategic games, there are four travelers $(i = 1, 2, ..., 4, n = 4)$;
- set of strategies or actions: Set of strategies includes all possible road sections, that connects the starting point A with the destination point B. Between A in B there are four strategies, where first two strategies lead through the point X, while third and fourth strategy lead through point Y. We define the set of strategies as $S = \{X, XY, Y, YX\}$, where we labeled strategies X and Y for the route choice between A and B that were there before reconstruction and with the strategy XY and YX we labeled two new route choices, that were enabled by the new road section between points X and Y. In this case, strategy XY defines the possible route choice $A - X - Y - B$ and strategy YX defines route choice $A - Y - X - B$. With $m = 4$ we label the number of elements of set S or number of strategies;
- set of action profiles: Because there are four players in the strategic game and there are four possible strategies, respectively, is the number of action profiles $p = n^m = 4^4 = 256$ and index k of action profile may value the values on interval $k = 1, 2, ..., 256$. In Mathematica, we generate all 256 action profiles with the comands `n=4;Tuples[{X, XY, Y, YX},n]`. Command `Tuples` generates the set of all possible set (all possible m) strategies or actions of n players. Generated action profiles belong to each of four travelers of strategic game;

- priority scale: priority scale is defined by the outputs of strategic games for each player. For the output of certain player in individual action profile is the most appropriate if we take negative value of his travel time. With this type of adaptation we reach the best possible outcome for each player that corresponds to the shortest travel time. Action profiles are classified in accordance to priority scale from the best to the worst action profile.

1.3.9 Function of outcomes of strategic game "Choosing a route" in reconstructed road network

If the travelers are driving through original network, that consists of the routes $A-X-B$ and $A-Y-B$, travel times $t_p(i,k)$ and outcomes of strategic game $u(i,k) = -t_p(i,k)$ for i-th traveler are the same in the original and in the reconstructed road network. Despite of that on the route section $A-X$ we have to take into consideration changed number of travelers $s_X(k)+s_{XY}(k)$, where with $s_{XY}(k)$ we label number of travelers, that choose the route section $X-Y$ in the point X. If we take the number of parameter $p(i,k)$ as $p(i,k)=1$, than any of the vehicle can choose one of possible route sections, that goes through point X. Which route section the travelers chooses depends on the value of additional parameter $xy(i,k)$. If we have $p(i,k)=1$ and $xy(i,k)=1$, then the traveler will choose the route section $A-X-Y-B$. If the $xy(i,k)=0$, traveler will choose $A-X-B$. Travel time for any i-th traveler that drives throug point X can be calculated with the sum (Pušenjak, Oblak & Fošner, 2013, p. 101)

$$p(i,k)\left\{\overbrace{t_p(i,k)}^{A-X} + \overbrace{xy(i,k)t_p(i,k)}^{X-Y-B} + \overbrace{[1-xy(i,k)]t_p(i,k)}^{X-B}\right\} \tag{1.6}$$

If parameter $q(i,k)$ has the value of $q(i,k)=1$, we determine the travel time of i-th traveler in k-th action profile, that drives through the route section $A-Y$. We also need to consider $s_Y(k)+s_{YX}(k)$ of travelers on this route section, because in the point Y, $s_{YX}(k)$ travelers can choose the route section $Y-X-B$ if the parameter $yx(i,k)$ has the value $yx(i,k)=1$. If $yx(i,k)=0$, then there are only $s_Y(k)$ travelers on the route section $A-Y$, but there can be additional travelers $s_{XY}(k)$ who join the route section in the point Y from the route section $X-Y$. Travel time for any i-th traveler that goes through point Y can be calculated with the sum (Pušenjak, Oblak & Fošner, 2013, p. 101)

$$q(i,k)\left\{\overbrace{t_p(i,k)}^{A-Y} + \overbrace{yx(i,k)t_p(i,k)}^{Y-X-B} + \overbrace{[1-yx(i,k)]t_p(i,k)}^{Y-B}\right\} \tag{1.7}$$

Formula, that includes all the possibilities, has can be written in following form:

$$
\begin{aligned}
t_p(i,k) &= p(i,k)\Bigg\{ \overbrace{6+3\cdot[s_X(k)+s_{XY}(k)-1}^{A-X} \\
&+ \overbrace{xy(i,k)\{7+6+1\cdot[s_{XY}(k)-1]+3\cdot(s_Y(k)+s_{XY}(k)-1)\}}^{X-Y-B} \\
&+ \overbrace{[1-xy(i,k)]\{20+0.9\cdot[s_X(k)+s_{YX}(k)-1]\}}^{X-B} \Bigg\} \\
&+ q(i,k)\Bigg\{ \overbrace{20+1\cdot[s_Y(k)+s_{YX}(k)-1]}^{A-Y} \\
&+ \overbrace{yx(i,k)\{7+20+1\cdot[s_{YX}(k)-1]+0.9\cdot[s_X(k)+s_{YX}(k)-1]\}}^{Y-X-B} \\
&+ \overbrace{[1-yx(i,k)]\{6+3\cdot[s_Y(k)+s_{XY}(k)-1]\}}^{Y-B} \Bigg\} \qquad (1.8) \\
u(i,k) &= -t_p(i,k) \qquad (1.9)
\end{aligned}
$$

Let's explain the meaning of the formula on the following example.

Example. Let's take a look at travel times and their outcomes of strategic game for action profile $a=(X,XY,Y,YX)$ (Pušenjak, Oblak & Fošner, 2013, p.102).

In the set of strategies $S=\{X,XY,Y,YX\}$, $a=(X,XY,Y,YX)$ represents the 28th action profile ($k=28$). For the first travelers, the value of paramteters are as follows: $p(1,28)=1$, $q(1,28)=1-p(1,28)=0$, $s_X=1$, $s_Y=1$, $s_{XY}=1$, $s_{YX}=1$, $xy(1,28)=0$, $yx(1,28)=0$. With the use of formulas (1.8) and (1.9) we get (Pušenjak, Oblak & Fošner, 2013, p. 102)

$$
\begin{aligned}
t_p(1,28) &= 1\cdot\{6+3\cdot(1+1-1)+0\cdot[7+6+1\cdot(1-1)+3\cdot(1+1-1)] \\
&+ 1\cdot[20+0.9\cdot(1+1-1]\} \\
t_p(1,28) &= 9+0+20.9 \\
t_p(1,28) &= 29.9 \\
u(1,28) &= -t_p(1,28) = -29.9
\end{aligned}
$$

For the second traveler, the values of the parameters are as follows: $p(2,28) = 1$, $q(2,28) = 1 - p(1,28) = 0$, $s_X = 1$, $s_Y = 1$, $s_{XY} = 1$, $s_{YX} = 1$, $xy(2,28) = 1$, $yx(2,28) = 0$. With the use of formulas (1.8) and (1.9) we get (Pušenjak, Oblak & Fošner, 2013, p. 102)

$$\begin{aligned}
t_p(2,28) &= 1 \cdot \{6 + 3 \cdot (1 + 1 - 1) + 1 \cdot [7 + 6 + 1 \cdot (1 - 1) + 3 \cdot (1 + 1 - 1)] \\
&+ 0 \cdot [20 + 0.9 \cdot (1 + 1 - 1]\} \\
t_p(2,28) &= 9 + 0 + 16 \\
t_p(2,28) &= 25 \\
u(2,28) &= -t_p(2,28) = -25
\end{aligned}$$

Values of the parameters for the third traveler are following: $p(3,28) = 0$, $q(3,28) = 1 - p(1,28) = 1$, $s_X = 1$, $s_Y = 1$, $s_{XY} = 1$, $s_{YX} = 1$, $xy(3,28) = 0$, $yx(3,28) = 0$.

$$\begin{aligned}
t_p(3,28) &= 1 \cdot \{20 + 1 \cdot (1 + 1 - 1) + 0 \cdot [7 + 20 + 1 \cdot (1 - 1) + 0.9 \cdot (1 + 1 - 1)] \\
&+ 1 \cdot [6 + 3 \cdot (1 + 1 - 1]\} \\
t_p(3,28) &= 21 + 0 + 9 \\
t_p(3,28) &= 30 \\
u(3,28) &= -t_p(3,28) = -30
\end{aligned}$$

For the fourth traveler, values of parameters are $p(4,28) = 0$, $q(4,28) = 1 - p(4,28) = 1$, $s_X = 1$, $s_Y = 1$, $s_{XY} = 1$, $s_{YX} = 1$, $xy(4,28) = 0$, $yx(4,28) = 1$. This means (Pušenjak, Oblak & Fošner, 2013, p. 102):

$$\begin{aligned}
t_p(4,28) &= 1 \cdot \{20 + 1 \cdot (1 + 1 - 1) + 1 \cdot [7 + 20 + 1 \cdot (1 - 1) + 0.9 \cdot (1 + 1 - 1)] \\
&+ 0 \cdot [6 + 3 \cdot (1 + 1 - 1]\} \\
t_p(4,28) &= 21 + 27.9 + 0 \\
t_p(4,28) &= 48.9 \\
u(4,28) &= -t_p(4,28) = -48.9
\end{aligned}$$

With this, we explained the meaning and the use of reconstructed road network function in strategic game “Choosing a route” .

1.3.10 Nash Equilibrium and it's meaning in strategic game "Choosing a route" in reconstructed road network

We can calculate Nash Equilibrium in strategic games with reconstructed road network on very similar way as in the road network before reconstruction. That means, that the action profile $a^* = (a_1, a_2, a_3, a_4)$, $a_i \in S = \{X, XY, Y, YX\}$ is Nash Equilibrium if the inequality (1.5) applies for all players of strategic game ($i = 1, 2, ..., 4$). We will present the meaning on the example below (Pušenjak, Oblak & Fošner, 2013, p. 103).

Let's test, if the action profile $a = (X, X, Y, XY)$ represents Nash Equilibrium.

With the help of function `profil=Tuples[{X,XY,Y,YX},4]` we first see, that $a = (X, X, Y, XY)$ is 10th action profile ($k = 10$). At first, we validate the inequality (1.5) for the first traveler ($i = 1$). We start the procedure with the change of strategy of first traveler. Important difference in comparison with the original road network is, that now each traveler can change his strategy on 3 different ways, while in original network, there was only one change possible. First traveler can now change his strategy X in a way, that he chooses strategy Y, XY, or YX. For each of those changes we have to validate the inequality (1.5). Only when inequality applies for all the possible changes and for all travelers, it represents Nash Equilibrium (Pušenjak, Oblak & Fošner, 2013, p. 103).

If first traveler changes strategy X for strategy Y, we get a changed action profile (Y, X, Y, XY) which is in correlation with number $k'(1) = 138$. Travel times for this profile are following (Pušenjak, Oblak & Fošner, 2013, p. 103):

$$
\begin{aligned}
t_p(1,138) &= 33 \\
t_p(2,138) &= 20 \\
t_p(3,138) &= 33 \\
t_p(4,138) &= 28
\end{aligned}
$$

and the outcomes are negative values of travel times. Because the outcome of strategic game for 10th profile $a = (X, X, Y, XY)$ equals $u(1,10) = -t_p(1,10) = -32.9$, inequality (1.1) applies, because $u(1,10) = -32.9 > -33 = u(1,138)$ is true (Pušenjak, Oblak & Fošner, 2013, p. 103).

If the first traveler changes his strategy to XY, we get changed action profile (XY, X, Y, XY), that corresponds to value $k'(1) = 73$. Travel times for this profile are (Pušenjak, Oblak & Fošner, 2013, p. 104):

$$
\begin{aligned}
t_p(1,73) &= 28 \\
t_p(2,73) &= 32.9 \\
t_p(3,73) &= 29 \\
t_p(4,73) &= 32.9
\end{aligned}
$$

It's the same here, that the outcomes are negative values of travel times. Because the outcome of $u(1,73) = -28$ for the first traveler, we get the inequality $u(1,10) = -32.9 < -28 = u(1,73)$, which is in negation of inequality (1.5), we can assume, that the action profile $a = (X,X,Y,XY)$ is not Nash Equilibrium. Further validation of changes of strategies in unnecessary for other players (Pušenjak, Oblak & Fošner, 2013, p. 104).

Calculating Nash Equilibrium is an extensive task, that is mostly done by computer programmes. From the programing view point there are no difficulties there, because the definition of Nash Equilibrium, that we stated with inequality (1.5), can be used in any programing language (i.e. FORTRAN, C, or C++) and we are not limited by the use of Mathematica or Matlab. With the use of computer programes we can discover all Nash Equlibriums (the previous example of 10th action profile is only one of them). In addition, we can calculate the sum of travel times of all four travelers for each action profiles, which serves us for defining Pareto optimum (Pušenjak, Oblak & Fošner, 2013, p. 105).

Pareto optimum corresponds to action profiles (there can be various) for which the sum of travel times of all four travelers is the lowest. Similar analysis as is shown above, reveals that 23rd action profile $a = (X,XY,XY,Y)$ is Nash Equilibrium. However, we can express our surprise, because Nash Equilibrium of the action profile $a = (X,XY,XY,Y)$ is not Pareto optimum. To confirm this, we first compute the sum of travel times for the action profile $a = (X,XY,XY,Y)$, which results in $\sum_{i=1}^{i=4} t_p(i,23) = 4 \times 32 = 128$. Next, the Nash Equilibrium profile $a = (X,XY,XY,Y)$ is compared with previously analized 10th action profile $a = (X,X,Y,XY)$, where the total sum of travel times is equal to $\sum_{i=1}^{i=4} t_p(i,10) = 32.9 + 32.9 + 29 + 28 = 122.8$ and therefore has lower value than the total sum of a Nash Equilibrium for profile $a = (X,XY,XY,Y)$ (Pušenjak, Oblak & Fošner, 2013, p. 105).

The meaning of Nash Equilibrium in reconstructed road network is, that none of the travelers can changes his route (change his strategy) for the purpose of reducing travel time. Said diferently, all travelers in Nash Equilibrium insist with their choice, because there is no reason to change the strategy. We have to add, that Nash Equilibrium are also all permutations of action profile $a = (X,XY,XY,Y)$, that are all together 12 of them and can be generated in the Mathematica environment with the use of function $Permutations[\{X,XY,XY,Y\}]$. As an outcome we get following action profiles together with $a = (X, XY, XY, Y)$ (Pušenjak, Oblak & Fošner, 2013, p. 105):

$$
\begin{aligned}
a &= (X, XY, Y, XY) \\
a &= (X, Y, XY, XY) \\
a &= (XY, X, XY, Y) \\
a &= (XY, X, Y, XY) \\
a &= (XY, XY, X, Y) \\
a &= (XY, XY, Y, X) \\
a &= (XY, Y, X, XY) \\
a &= (XY, Y, XY, X) \\
a &= (Y, X, XY, XY) \\
a &= (Y, XY, X, XY) \\
a &= (Y, XY, XY, X)
\end{aligned}
$$

Based on this we can say, that Nash Equilibrium is each action profile, where two travelers choose the strategy XY (or route section $A-X-Y-B$), one chooses strategy X (or route section $A-X-B$) and one chooses strategy Y (or route section $A-Y-B$). Travel times of random traveler in those action profile are 32 minutes and the sum of all four travelers is 128 minutes (Pušenjak, Oblak & Fošner, 2013, p. 105).

1.3.11 Strategic game "Two players with different strategy sets"

We can present strategic game "Two players with different strategy sets" with the table. Rows in the table correspond to the first player, while columns correspond to the second player. Since we assume a different number of strategies of both players, the number of rows differs from the number of columns. This type of table represents the prototipe of strategic game in the transport problem "Choosing a route", where each traveler (player) starts the journey from different entry point (starting location) and all the routes (strategies) that are available for first traveler, can be (but it is not necessary) different from the routes (strategies) that are available for second traveler. Each cell in the table corresponds to individual action profile. Inputs in the cells are the outcomes of strategic games of both players. Outcomes can be travel times or even better, negative values of travel times, because based on this, the best possible outcome corresponds to the highest negative value (Pušenjak, personal communication, 21. june 2013).

1.3.12 Structure of strategic game "Two players with different strategy sets"

We will present only one game for the presentational purposes. Set of players of this strategic games consists of two travelers, that are begining the journey from two different starting points. First player (traveler) has the choice of two different strategies or actions with two elements $S_1 = \{C, D\}$. Second player can reach the same destination as first player through four different ways and his set of strategies is $S_2 = \{EG, EF, FG, FH\}$. We can present the game with the table with 2 rows and 4 columns. Number of action profiles is therefore $2 \times 4 = 8$ and the set of action profiles is (Pušenjak, personal communication, 21. june 2013):

$$AP = \{CEG, CEF, CFG, CFH, DEG, DEF, DFG, DFH\} \tag{1.10}$$

that can be generated in the software environment Mathematica with the use of the command `AP=Tuples[S1,S2]`. A set of action profiles, that is written above is organized, which means that the action profile CEG equals to $k = 1$, action profile CEF equals to $k = 2$ and action profile DFH equals to $k = 8$ (Pušenjak, personal communication, 21. june 2013).

Table 2: *Strategic game "Two players with different strategy sets"*

		2nd player			
		EG	EF	FG	FH
1st player	C	1,0	1,0	3,2	3,2
	D	2,3	0,1	2,3	0,1

Source: Osborne, 2002

In the future, we will need numbers of changed action profiles. If first player changes his decision in first action profile CEG, we get changed action profile DEG, which equals to the number $k' = 5$. If second player changes his strategy in first action profile CEG and he goes to i.e. fourth possible strategy, we get changed action profile CFH, which equals to the number $k' = 4$, etc (Pušenjak, personal communication, 21. june 2013).

1.3.13 Defining Nash Equilibrium in a strategic game "Two players with different strategy sets"

Definition of Nash Equilibrium for this type of strategic game is the same, as in previous cases, which means that action profile a^* of a strategic game with ordinal priority scale is Nash Equilibrium, if action profile a^* for each player $i = 1, 2, ...n)$ and for each action a_i of i-th player is at least as good as action profile (a_i, a^*_{-i}), in which i-th player changes his action a^*_i to action a_i when all the others, let's say j-th player preserves his action a^*_j (Pušenjak, personal communication, 21. june 2013).

We can express the definition mathematicaly with the use of inequality

$$u_i(a^*) \geq u_i(a_i, a^*_{-i}) \tag{1.11}$$

Let's start with defining Nash Equilibrium for first action profile $AP(1) = CEG$ (Pušenjak, personal communication, 21. june 2013). We will present only this case for the purpose of presentation, while all the other cases can be calculated individually by the reader.

First, we read both outcomes of strategic game for action profile a^*, where we get $u_1(a^*) = 1$ and $u_2(a^*) = 0$. To check if action profile a^* represents Nash Equilibrium, we validate the inequality (1.11) for both players and for all strategies that are available for individual player. For first player $i = 1$, and for index $-i$ in this case, value $-i = 2$ coressponds. This gives us changed action profile $(a_1, a^*_2) = (D, EG)$ with outcome $u_1(a_1, a^*_2) = u_1(D, EG) = 2$. With this, the inequality (1.11) is not valid $(1 < 2)$ and we know, that this is not Nash Equilibrium, independently of other results for second player (Pušenjak, personal communication, 21. june 2013).

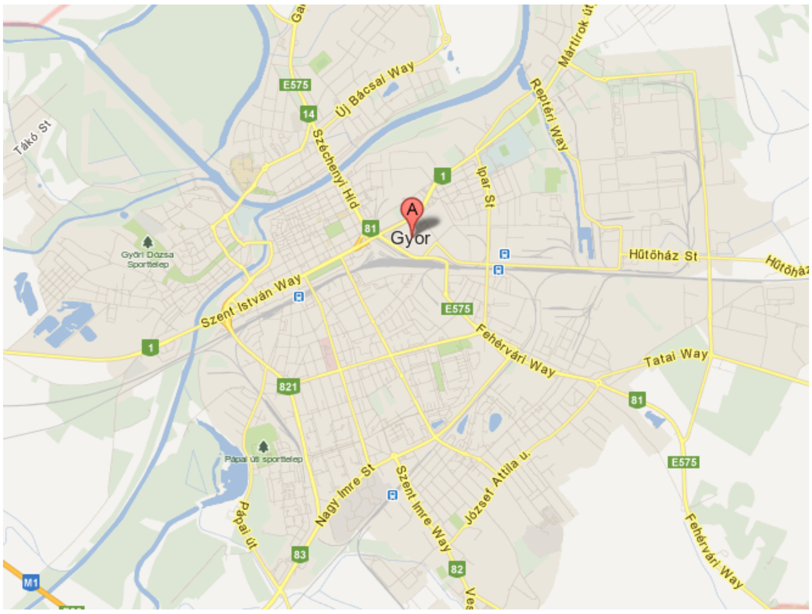
Because this type of strategic game is different from all the others, we will still check the validy of the inequation also for second player. For first player $i = 2$, and for index $-i$ in this case, value $-i = 1$ coressponds. Second player can change his strategy EG in three different ways. First possible changed action profile is $(a^*_1, a_2) = (C, EF)$, which corresponds to number $k' = 2$ for which we read the value of outcome of strategic game $u_2(a^*_1, a_2) = u_2(C, EF) = 0$. In this case, inequality (1.11) can be written as $u_2(a^*) \geq u_2(a^*_1, a_2)$ and it's obviously valid because $u_2(a^*_1, a^*_2) = 0 = 0 = u_2(a^*_1, a_2)$. Independent of the validity of inequation (1.11), action profile (C, EG) is not Nash Equilibrium. Validation of inequality (1.11) for second player is not finished yet. He can change his strategy EG for strategy FG, for which action profile (C, FG) corresponds and we get changed serial number $k' = 3$. The outcome of second player for changed action profile is $u_2(a^*_1, a_2) = u_2(C, FG) = 2$, which means that second player can improve his outcome so inequality (1.11) is not valid. Instead of inequality (1.11) in this case, inequality $u_2(a^*_1, a^*_2) = 0 < 2 = u_2(a^*_1, a_2)$ is valid. With changed action profile we can find out, that original action profile (C, EG) cannot be Nash Equilibrium not even for second player. The only thing that remains is the test, where second player changes his strategy EG for third option, which is strategy FH. This strategy corresponds to action profile

(C,FH) and changed serial number $k' = 4$. Outcome of second player for this action profile is $u_2(a_1^*, a_2) = u_2(C,FH) = 2$, therefore it's also true $u_2(a_1^*, a_2^*) = 0 < 2 = u_2(a_1^*, a_2)$ as in previous test. This means that the player can improve his outcome and action profile (C,FH) is not action profile (Pušenjak, personal communication, 21. june 2013).

2 CITY LOGISTICS: CITY OF GYŐR

City of Győr is one of the most important city of northwest Hungary and it is also the capital of Győr-Moson-Sopron county. City lies in the halfway between Budapest and Vienna and the road there is one of the most important roads of Central Europe. Győr is the sixth largest city in Hungary (Győr, b. d.).

Figure 2.1: City of Győr



Source: "Győr" [Google Maps], b. d.

The area along the Danube River has been inhabited by varying cultures since ancient times. The first large settlement dates back to the 5th century BCE; the inhabitants were Celts. They called the town Arrabona, a name that was used for eight centuries; its shortened form is still used as the German (Raab) and Slovak (Ráb) names of the city. Roman merchants moved to Arrabona during the 1st century BCE. Around 10 CE, the Roman army occupied the northern part of Western Hungary, which they called Pannonia. Although the Roman Empire abandoned the area in the 4th century due to constant attacks by the tribes living to the east, the town remained inhabited. Around 500 the territory was settled by Slavs, in 547 by the Lombards, and in 568-c.800 by the Avars, at that time under Frankish and Slavic influence. Between 880 and 894, it was part of Great Moravia, and then briefly under East Frankish dominance (Győr, b. d.).

The Magyars occupied the town around 900 and fortified the abandoned Roman fortress. Stephen I, the first king of Hungary, founded an episcopate there. The town received its Hungarian name Győr. The Hungarians lived in tents, later in cottages, in what is now the southeastern part of the city centre. The town was affected by all the trials and tribulations of the history of Hungary: it was occupied by Mongols during the Mongol invasion of Hungary (1241–1242) and then was destroyed by the Czech army in 1271 (Győr, b. d.).

After the disastrous battle of Mohács, Baron Tamás Nádasdy and Count György Csesznekу occupied the town for King Ferdinand I while John Zápolya also was attempting to annex it. During the Ottoman occupation of present-day central and eastern Hungary[1] (1541 - late 17th century), Győr's commander Kristóf Lamberg thought it would be futile to try to defend the town from the Turkish army. He burned down the town and the Turkish forces found nothing but blackened ruins, hence the Turkish name for Győr, Yanık kale ("burnt city") (Győr, b. d.)

During rebuilding, the town was surrounded with a castle and a city wall, designed by the leading Italian builders of the era. The town changed in character during these years, with many new buildings built in Renaissance style, but the main square and the grid of streets remained. In 1594, after the death of Count János Csesznekу, captain of Hungarian foot-soldiers, the Ottoman army occupied the castle and the town (Győr, b. d.).

In 1598 the Hungarian and Austrian army took control of it again and occupied it. During the Turkish occupation the city was called Yanık (Győr, b. d.).

In 1683, the Turks returned briefly, only to leave after being defeated in the Battle of Vienna (Győr, b. d.).

During the following centuries, the town became prosperous. In 1743 Győr was elevated to free royal town status by Maria Theresa. The religious orders of Jesuits and Carmelites) settled there, building schools, churches, a hospital and a monastery (Győr, b. d.).

In June 14, 1809, this was the site of the Battle of Győr (Battle of Raab), where the Grand Armee of Napoleon defeated the Hungarian noble rebels and Austrian corps. Napoleon occupied the castle and had some of its walls blown up. The leaders of the town soon realized that the old ramparts were not useful any more. Most of the ramparts were destroyed, allowing the town to expand (Győr, b. d.).

In the mid-19th century, Győr's role in trade grew as steamship traffic on the River Danube began. The town lost its importance in trade when the railway line between Budapest and Kanizsa superseded river traffic after 1861. The town leaders compensated for this loss with industrialisation. The town prospered till World War II but, during the war, several buildings were destroyed (Győr, b. d.).

During World War II, after the Nazis took control in Hungary, they organized the deportation of the Jews. They comprised 12.6% of the Győr population according to the 1910 census. On May 13, 1944, the Nazis ordered them into a ghetto. Two trains were loaded to transport the Jews of the town and the neighboring villages to Auschwitz on June 11 and on June 14. Altogether, 5,635 people were deported on these two days. Nearly all were killed in the extermination camp or along the way. Some exempted Jews were left behind, but they were massacred on March 26, 1945, just hours before liberation by the Allies (Győr, b. d.).

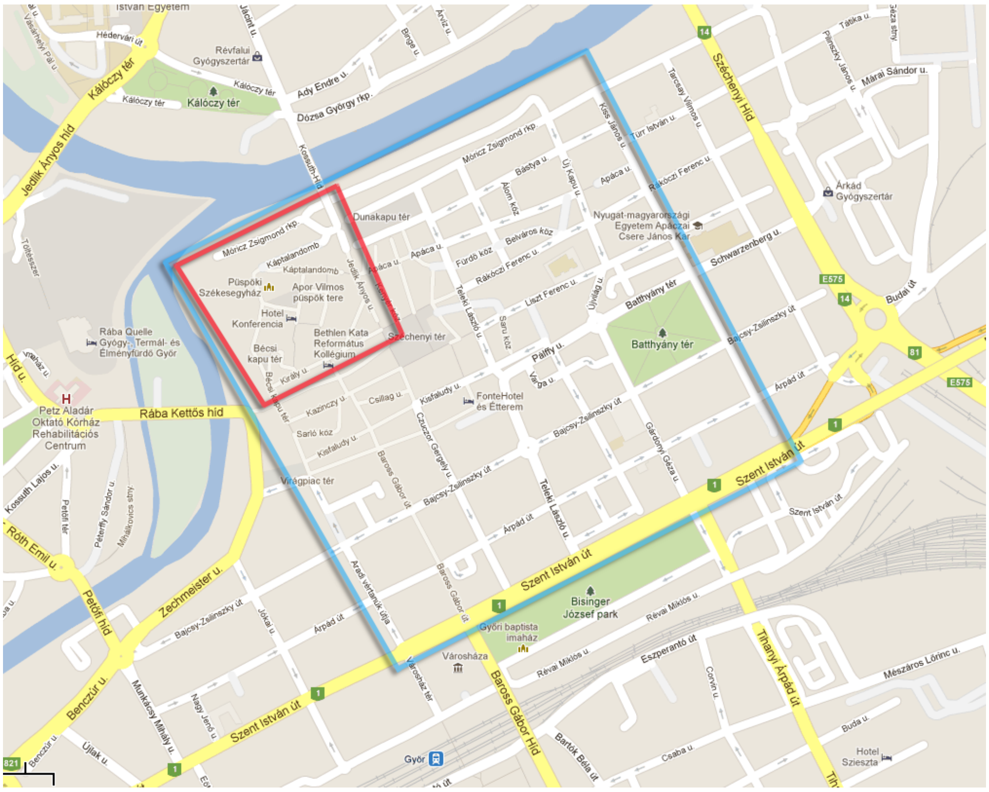
The 1950s and 60s brought more change: only big blocks of flats were built, and the old historical buildings were not given care or attention. In the 1970s the reconstruction of the city centre began; old buildings were restored and reconstructed. In 1989 Győr won the European award for the protection of monuments (Győr, b. d.).

3 DATA AND MODELS

3.1 Description of gathering data

Data we used for the purpose of preparing the dynamical model were gathered in city of Győr itself. First, we had to decide on the part of the city that we are going to use for this purpose. We made a tour of the city to get the personal experience about the borders of the city center and it's structure. With the personal comunication with the inhabitants of the city, we discovered, that the city center is divided into 2 different centers; the old city center and the new city center, as it's shown on the picture 3.1.

Figure 3.1: *Old and new city center of Győr*

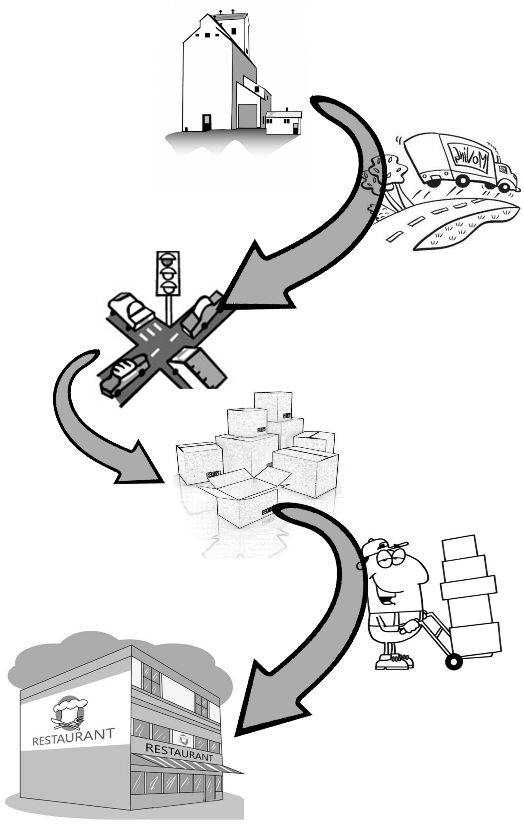


Source: "Győr" [Google Maps], b. d.

Historical city center has a historical church, basilika, hotel and some other monuments, while in the new city center mostly small shops and restaurants are situated. Historical city center is completely closed for traffic, while the modern city center has certain streets, that can bring you close to the pedestrian area.

In the modern part of the city center, we chose one of the restaurants, called Patio, for the destination point for our modeling purposes. This restaurant was chosen on purpose because it is situated in the pedestrian-only area and it's imposible to get there by car. This means, that we had to find the nearest possible point, where the car or other vehicle which delivers goods. Based on the personal comunication with the people of Győr (Makkos-Kaldi, personal communication, 16. february 2013), we discovered, that delivery of goods is organized in a way that the vehicle with goods comes to the nearest street and then people (drivers) carry the goods to the restaurants or shops or they put the goods on the hand trolley and transport them to the end destination.

Figure 3.2: *Delivery method of city of Győr*



We also found out, that most of the restaurants is buying supplies in the shop Metro near the city of Győr. Also, by personal experience, we discovered, that supplying the close city center is not neccesary with the truck, because most of the shops are small and the delivery of goods can be made with the car. Of course, delivery between shops and restaurants is a bit different, we assume that for the restaurants, because they have daily demand, while shops have mostly seasonal demand, but also we have to take into consideration that material has to be available for the customers at all times.

For the purpose of modeling the traffic flow for defining the best possible route we decided to gather data about traffic flow from the entrances to the city center to the destination point, which is the closest possible street to the restaurant.

3.2 Gathered data

First step of gathering data was seting up the entrances into the city center based on the map of Győr and defining the destination point. After that, we used Google Maps to prepare possible routes from all entrances to destination point. We were limited in a way, that a lot of streets inside of city center itself are one-way streets, so the set of possibilities was multilated. We present the possibilities in the table 3.

Table 3: *Set of possible route choices*

No.	*Entry point*	*Destination point*	*Distance*	*Driving time*
1	Raba Kettos Hid	Czuczor Gergely U.	2,1 km	6 min
2	Raba Kettos Hid	Czuczor Gergely U.	1,4 km	4 min
3	Raba Kettos Hid	Czuczor Gergely U.	1,8 km	5 min
4	Petofi Hid	Czuczor Gergely U.	1,6 km	4 min
5	Petofi Hid	Czuczor Gergely U.	1,4 km	4 min
6	Beke Hid	Czuczor Gergely U.	2,4 km	5 min
7	Beke Hid	Czuczor Gergely U.	2,0 km	4 min
8	Beke Hid	Czuczor Gergely U.	2,0 km	5 min
9	Kossuth Hid	Czuczor Gergely U.	1,6 km	4 min
10	Kossuth Hid	Czuczor Gergely U.	1,6 km	4 min
11	Kossuth Hid	Czuczor Gergely U.	1,8 km	5 min
12	Szechenyi Hid	Czuczor Gergely U.	1,4 km	3 min
13	Szechenyi Hid	Czuczor Gergely U.	1,8 km	4 min
14	Bisinger Joszef Hid	Czuczor Gergely U.	1,5 km	3 min
15	Fehervari Ut	Czuczor Gergely U.	1,6 km	3 min
16	Fehervari Ut	Czuczor Gergely U.	1,6 km	4 min
17	Tihanyi Arpad Ut	Czuczor Gergely U.	850 m	3 min
18	Barros Gabor Hid	Czuczor Gergely U.	850 m	2 min

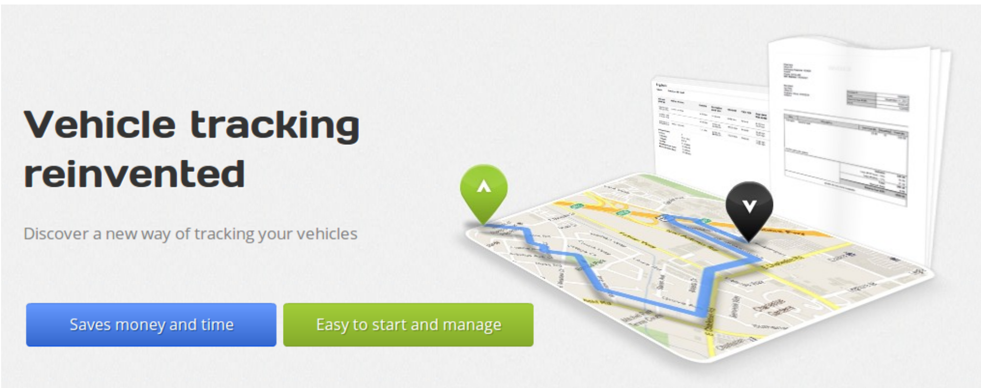
We decided to analyze all possible entrances to the city center because vehicles (cars, vans and trucks) can come to the city from all possible dirrections. From the entrance Szent

Istvan ut, vehicles mostly come from Vienna, while from the entrance Bisinger Joszef Hid, vehicles come from Budapest (as we mentioned in previous chapter, Győr is a a city with the cargo flow from Budapest to Vienna and reversed.

We gathered data about the driving time, distances and speed with the mobile phone (android) application called MyCarTracks. On their official website, we can see the description of the application, as follows: "MyCarTracks is new revolutionary way how to trace tracks of your vehicles, fleet and your employees and their tracks. This is a new trend which is constantly noticed and used by more and more fleet and vehicle owners." ("Tour - what is MyCarTracks" [MyCarTracks.com], b. d.).

"MyCarTracls is breaking traditional approach on vehicle/fleet tracking by removing the need to install GPS unit (blackbox, mileage logger) to your vehicle. As replacement it uses smartphone that you or your employees are carrying everytime driving a vehicle or working outside the office." ("Tour - what is MyCarTracks" [MyCarTracks.com], b. d.).

Figure 3.3: *My Car Tracks login screen - web service*

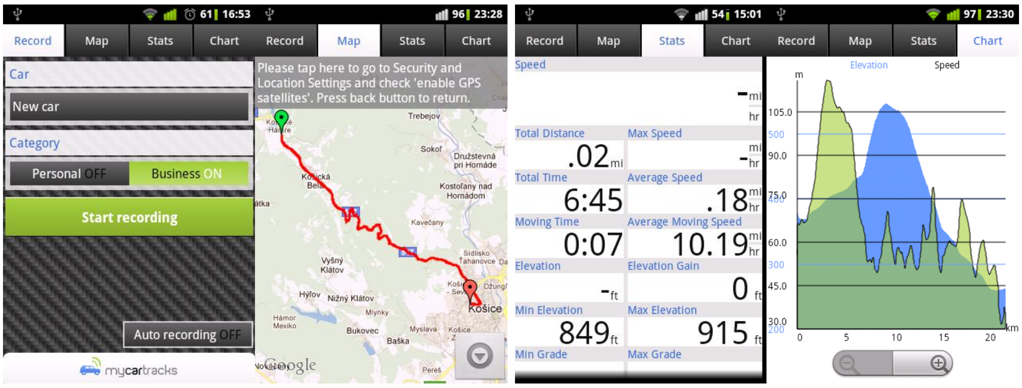


Source: "MyCarTracks" [MyCarTracks.com], b. d.

It's user friendly android app - the app records all our tracks. Developers used special auto recording feature - you start once and forget about the app, everything else it's recorded automaticaly. It can also be customized by the needs of the users and it has possibility of online and offline tracking. You can also generate monthly or daily mileage reports into pdf or excel file ("Tour - what is MyCarTracks" [MyCarTracks.com], b. d.).

With this application, we gathered necessary data for next step. We gathered data for different routes from entry points into city of Győr to the destination point with the use of Google Maps. After that, we made a measurements of set of routes and gathered data, that are shown in the next chapter.

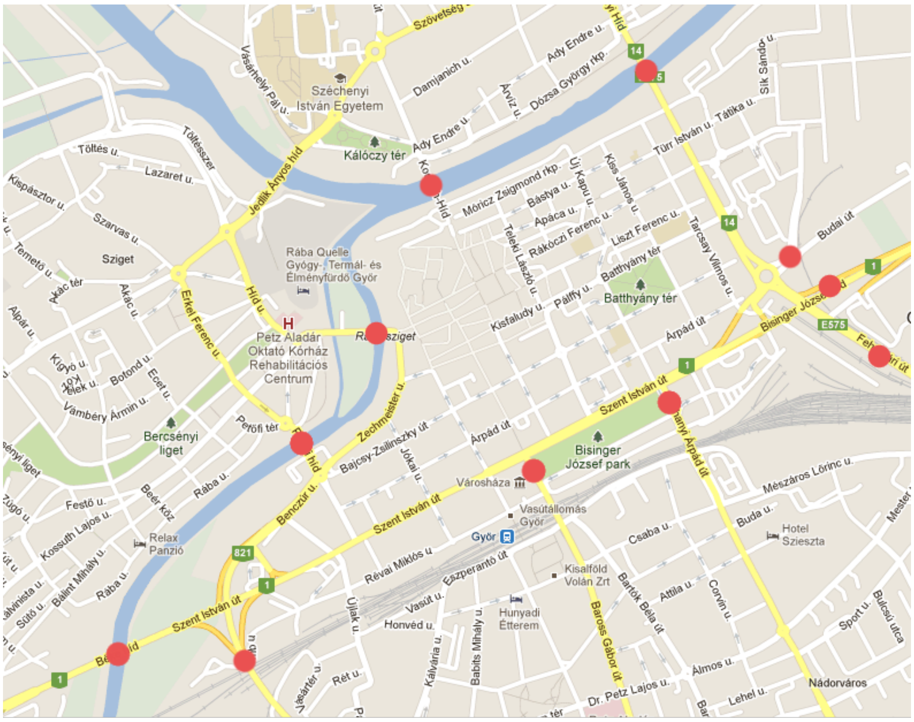
Figure 3.4: *My Car Tracks screenshots*



Source: "Mileage vehicle GPS tracker" [Google Play], b. d.

3.3 Spatial model of the selected area

Figure 3.5: *Entry points into city centre of Győr*



Source: "Győr" [Google Maps], b. d.

As it is seen on the map, theoretically, we have 11 entry points. With the use of Google Maps, we made a set of possible routes from each entry point to the destination point.

After that, measurements were made based on the set of routes. outcomes are shown below.

We have to take into consideration that the measurements were made by people, so we have to consider the human factor while comparing the outcomes. We get the most accurate distance in time if we look at the Google Maps data, while the measurements cannot start on the exact point that was set as an entry point.

We also have to explain that the average data that is shown in the following tables is just the average of the measurements for the purpose of comparing the Google Maps data[1] and the measurement's data. For this reason, data from Google Maps are displayed in italic. We present gathered data on following pages.

Entry point: Rabba Kettos hid

Destination point: Czuczor Gergely utca.

Route selection 1

Table 4: *Data for route selection 1*

	Distance	**Time**	**Average speed**
Google Maps	*2,1 km*	*6:00 min*	*21 km/h*
Measurement (night time)	2,09 km	7:34 min	30,05 km/h
Measurement (rush hour)	1,99 km	15:32 min	15,42 km/h
Average	**2,04 km**	**11:33 min**	**22,74 km/h**

Route selection 2

Table 5: *Data for route selection 2*

	Distance	**Time**	**Average speed**
Google Maps	*1,4 km*	*4:00 min*	*21 km/h*
Measurement (night time)	1,32 km	6:50 min	25,03 km/h
Measurement (rush hour)	1,26 km	8:40 min	16,53 km/h
Average	**1,29 km**	**7:45 min**	**20,78 km/h**

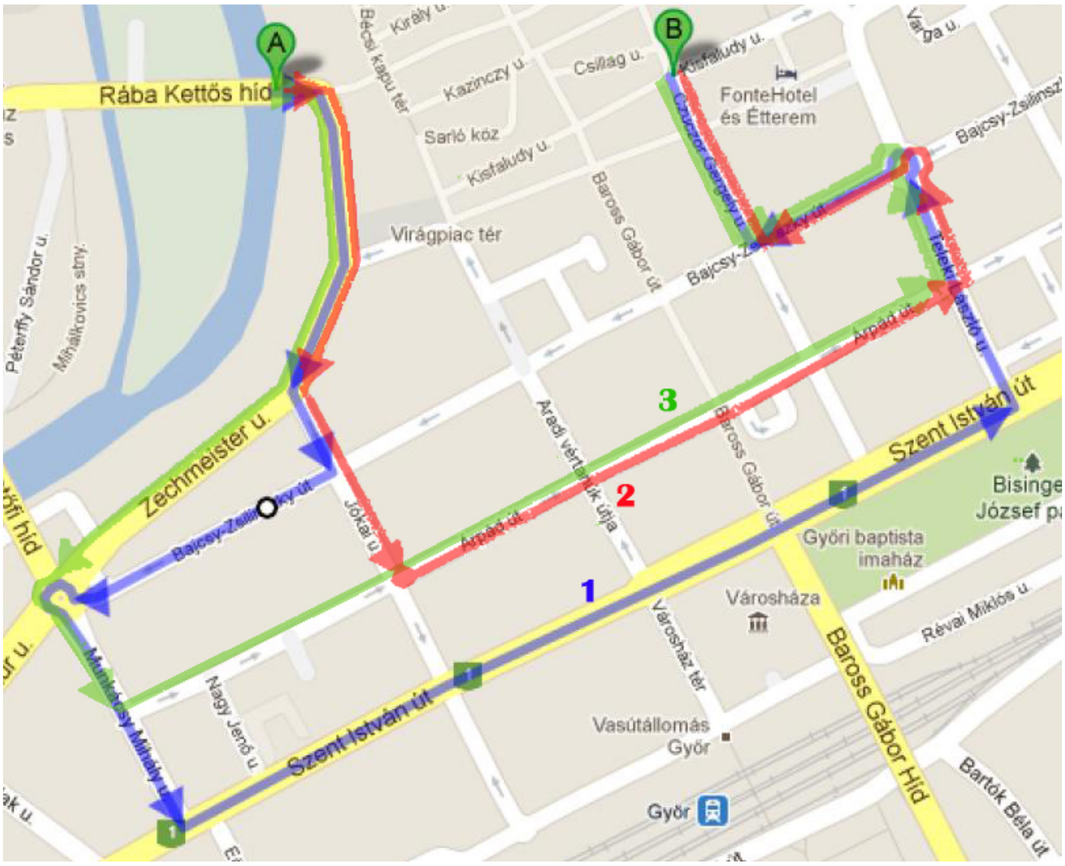
[1] *Average speed is calculated based on the basic mechanic's formula* $s = v \cdot t (distance = speed \cdot time)$

Route selection 3

Table 6: *Data for route selection 3*

	Distance	Time	Average speed
Google Maps	*1,8 km*	*5:00 min*	*21,6 km/h*
Measurement (night time)	1,74 km	4:03 min	25,69 km/h
Measurement (rush hour)	1,76 km	4:57 min	21,27 km/h
Average	**1,75 km**	**4:30 min**	**23,48 km/h**

Figure 3.6: *Map for route selection with entry point Raba Kettos Hid*



Source: "Győr" [Google Maps], b. d.

Entry point: Petofi Hid

Destination point: Czuczor Gergely utca.

Route selection 4

Table 7: *Data for route selection 4*

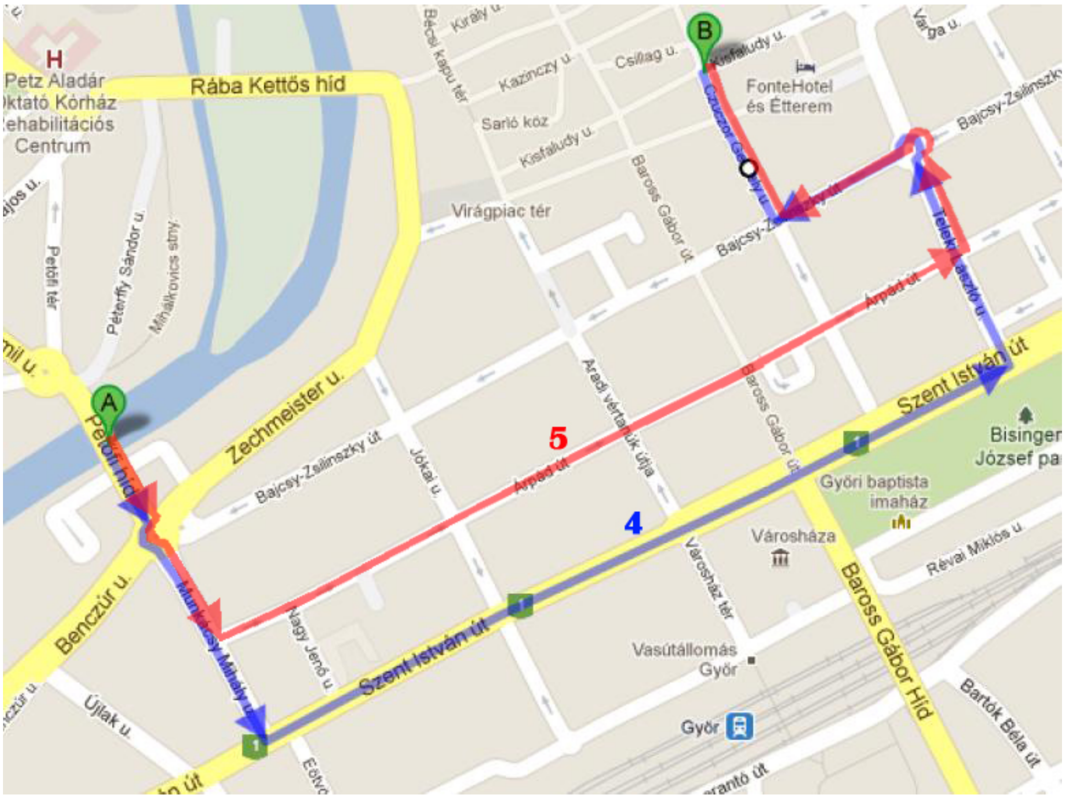
	Distance	**Time**	**Average speed**
Google Maps	*1,6 km*	*4:00 min*	*24 km/h*
Measurement (night time)	1,41 km	4:50 min	29,61 km/h
Measurement (rush hour)	1,34 km	15:56 min	13,75 km/h
Average	**1,37 km**	**10:23 min**	**21,68 km/h**

Route selection 5

Table 8: *Data for route selection 5*

	Distance	**Time**	**Average speed**
Google Maps	*1,4 km*	*4:00 min*	*21 km/h*
Measurement (night time)	1,15 km	6:51 min	20,57 km/h
Measurement (rush hour)	1,51 km	10:33 min	20,19 km/h
Average	**1,33 km**	**8:42 min**	**20,38 km/h**

Figure 3.7: *Map for route selection with entry point Petofi Hid*



Source: "Győr" [Google Maps], b. d.

Entry point: Beke Hid

Destination point: Czuczor Gergely utca.

Route selection 6

Table 9: *Data for route selection 6*

	Distance	**Time**	**Average speed**
Google Maps	*2,4 km*	*5:00 min*	*28,8 km/h*
Measurement (night time)	3,46 km	9:36 min	35,44 km/h
Measurement (rush hour)	3,44 km	14:59 min	26,61 km/h
Average	**3,45 km**	**12:17 min**	**31,02 km/h**

Route selection 7

Table 10: *Data for route selection 7*

	Distance	Time	Average speed
Google Maps	*2,0 km*	*4:00 min*	*30 km/h*
Measurement (night time)	1,86 km	2:43 min	41,16 km/h
Measurement (rush hour)	1,9 km	11:12 min	25,23 km/h
Average	**1,88 km**	**6:57 min**	**33,19 km/h**

Route selection 8

Table 11: *Data for route selection 8*

	Distance	Time	Average speed
Google Maps	*2,0 km*	*5 min*	*24 km/h*
Measurement (night time)	2,09 km	7:47 min	32,63 km/h
Measurement (rush hour)	1,99 km	10:34 min	22,69 km/h
Average	**2,04 km**	**9:10 min**	**27,66 km/h**

Figure 3.8: *Map for route selection with entry point Beke Hid*

Source: "Győr" [Google Maps], b. d.

Entry point: Kossuth Hid

Destination point: Czuczor Gergely utca.

Route selection 9

Table 12: *Data for route selection 9*

	Distance	**Time**	**Average speed**
Google Maps	*1,6 km*	*4:00 min*	*24 km/h*
Measurement (night time)	1,62 km	7:23 min	27,34 km/h
Measurement (rush hour)	1,57 km	8:37 min	23,36 km/h
Average	**1,60 km**	**8:00 min**	**25,35 km/h**

Route selection 10

Table 13: *Data for route selection 10*

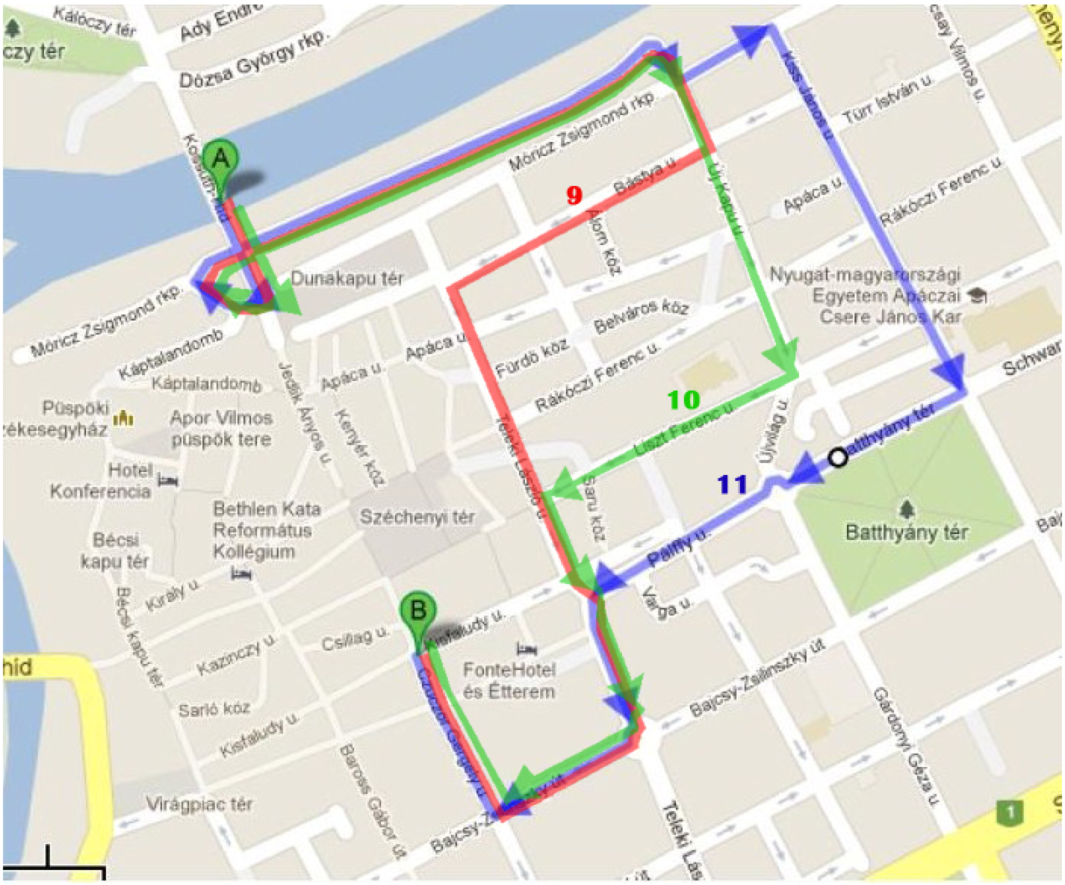
	Distance	**Time**	**Average speed**
Google Maps	*1,6 km*	*4:00 min*	*24 km/h*
Measurement (night time)	1,62 km	7:00 min	27,91 km/h
Measurement (rush hour)	1,52 km	8:20 min	24,36 km/h
Average	**1,57 km**	**7:40 min**	**26,14 km/h**

Route selection 11

Table 14: *Data for route selection 11*

	Distance	**Time**	**Average speed**
Google Maps	*1,8 km*	*5:00 min*	*21,6 km/h*
Measurement (night time)	1,99 km	7:28 min	32,07 km/h
Measurement (rush hour)	1,78 km	8:39 min	25,84 km/h
Average	**1,89 km**	**8:03 min**	**28,95 km/h**

Figure 3.9: *Map for route selection with entry point Kossuth Hid*



Source: "Győr" [Google Maps], b. d.

Entry point: Szechenyi Hid

Destination point: Czuczor Gergely utca.

Route selection 12

Table 15: *Data for route selection 12*

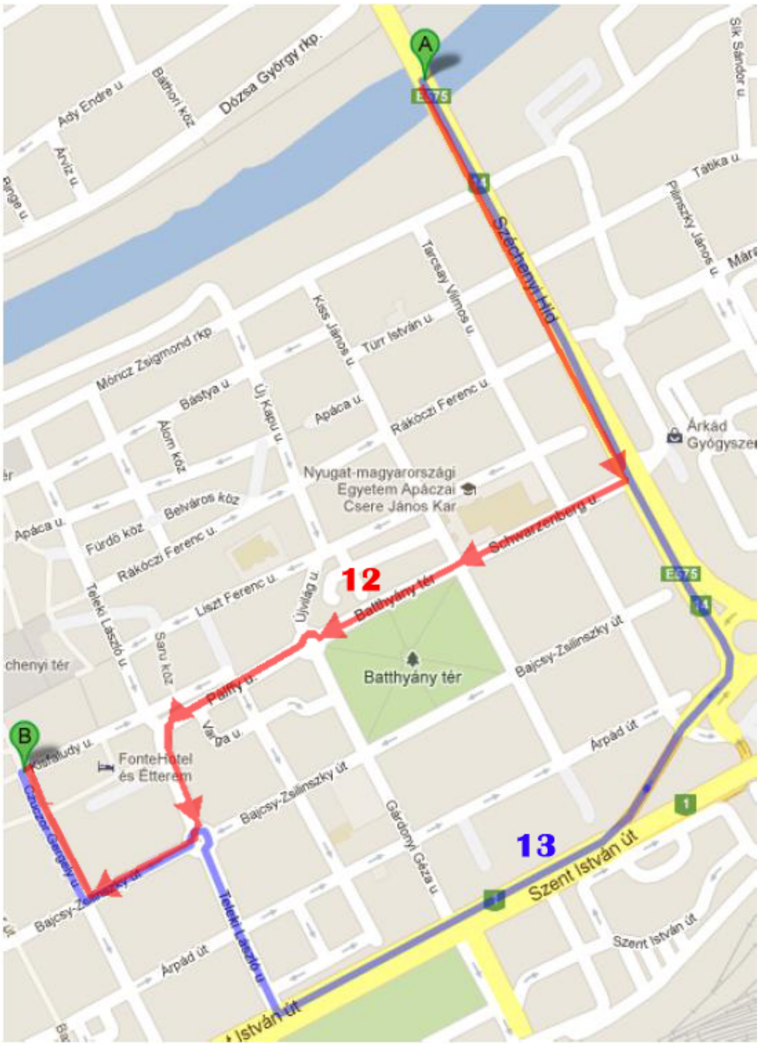
	Distance	Time	Average speed
Google Maps	*1,4 km*	*3:00 min*	*28 km/h*
Measurement (night time)	1,58 km	5:28 min	41,36 km/h
Measurement (rush hour)	1,72 km	9:01 min	34,06 km/h
Average	**1,65 km**	**7:14 min**	**37,71 km/h**

Route selection 13

Table 16: *Data for route selection 13*

	Distance	Time	Average speed
Google Maps	*1,8 km*	*4:00 min*	*27 km/h*
Measurement (night time)	1,96 km	5:48 min	46,67 km/h
Measurement (rush hour)	2,13 km	9:23 min	32,70 km/h
Average	**2,05 km**	**7:35 min**	**39,68 km/h**

Figure 3.10: *Map for route selection with entry point Szechenyi Hid*



Source: "Győr" [Google Maps], b. d.

Entry point: Bisinger Joszef Hid

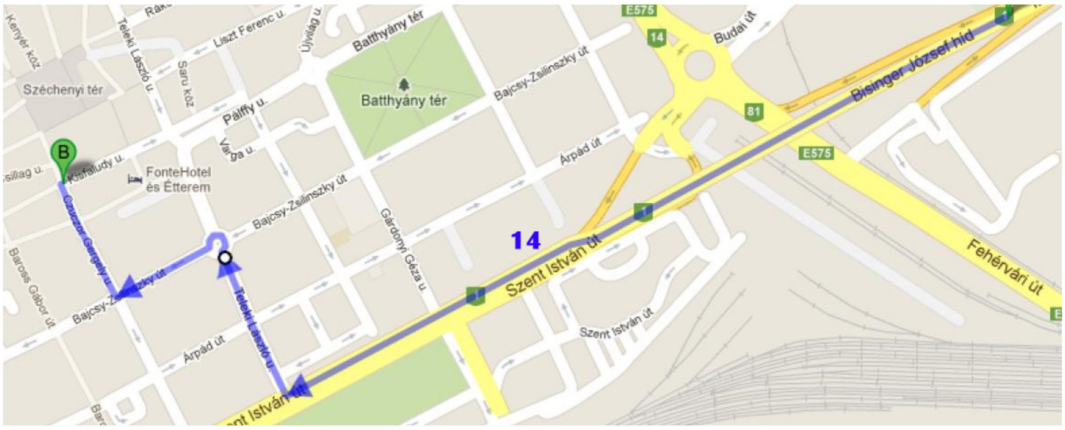
Destination point: Czuczor Gergely utca.

Route selection 14

Table 17: *Data for route selection 14*

	Distance	**Time**	**Average speed**
Google Maps	*1,5 km*	*3:00 min*	*30 km/h*
Measurement (night time)	1,52 km	7:12 min	41,54 km/h
Measurement (rush hour)	1,57 km	11:48 min	33,50 km/h
Average	**1,54 km**	**9:30 min**	**37,52 km/h**

Figure 3.11: *Map for route selection with entry point Bisinger Joszef Hid*



Source: "Győr" [Google Maps], b. d.

Entry point: Fehervari ut

Destination point: Czuczor Gergely utca.

Route selection 15

Table 18: *Data for route selection 15*

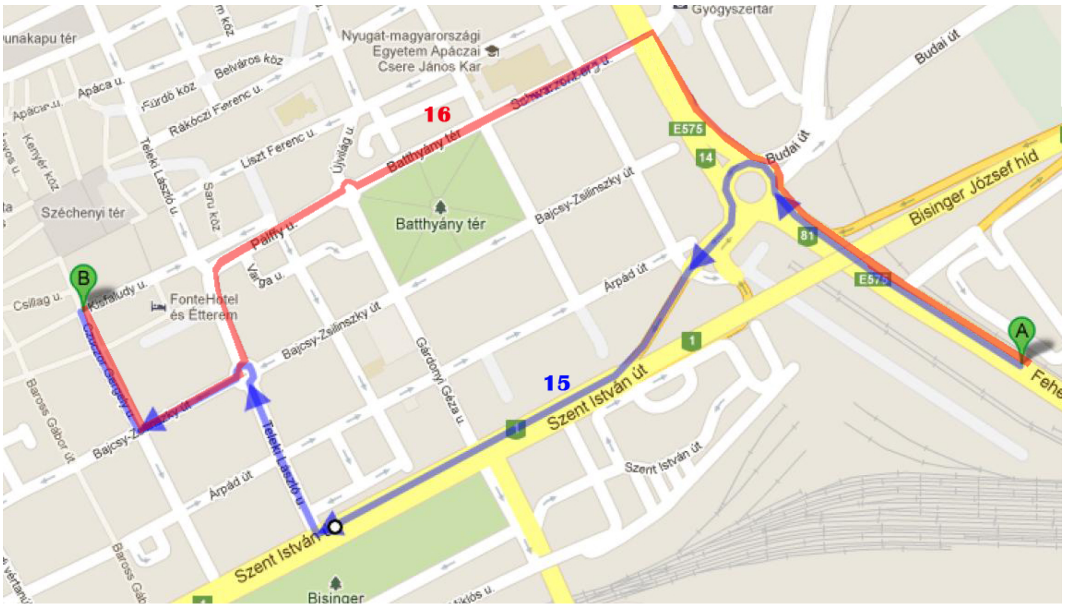
	Distance	**Time**	**Average speed**
Google Maps	*1,6 km*	*3:00 min*	*32 km/h*
Measurement (night time)	1,72 km	4:28 min	37,49 km/h
Measurement (rush hour)	1,88 km	10:36 min	23,31 km/h
Average	**1,8 km**	**7:32 min**	**30,4 km/h**

Route selection 16

Table 19: *Data for route selection 16*

	Distance	**Time**	**Average speed**
Google Maps	*1,6 km*	*4:00 min*	*24 km/h*
Measurement (night time)	1,73 km	3:47 min	44,59 km/h
Measurement (rush hour)	1,8 km	5:27 min	37,12 km/h
Average	1,76 km	4:37 min	37,85 km/h

Figure 3.12: *Map for route selection with entry point Fehervari ut*



Source: "Győr" [Google Maps], b. d.

Entry point: Tihanyi Arpad ut

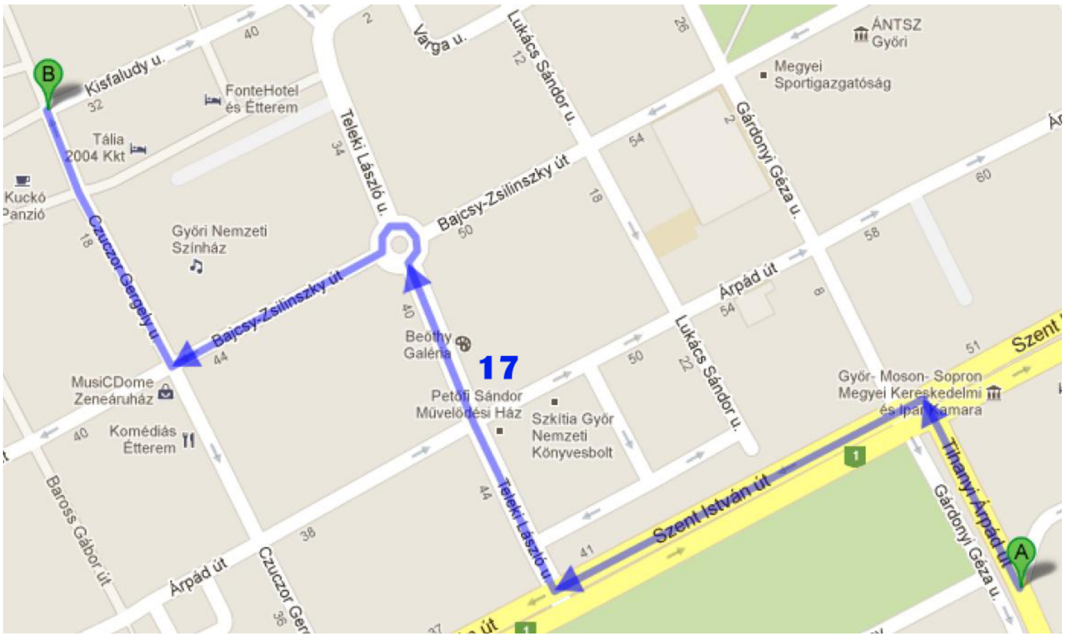
Destination point: Czuczor Gergely utca.

Route selection 17

Table 20: Data for route selection 17

	Distance	**Time**	**Average speed**
Google Maps	*850 m*	*3:00 min*	*17 km/h*
Measurement (night time)	1,01 km	4:57 min	25,99 km/h
Measurement (rush hour)	1,09 km	8:44 min	23,59 km/h
Average	**1,05 km**	**6:50 min**	**24,79 km/h**

Figure 3.13: *Map for route selection with entry point Tihanyi Arpad ut*



Source: "Győr" [Google Maps], b. d.

Entry point: Baross Gabor Hid

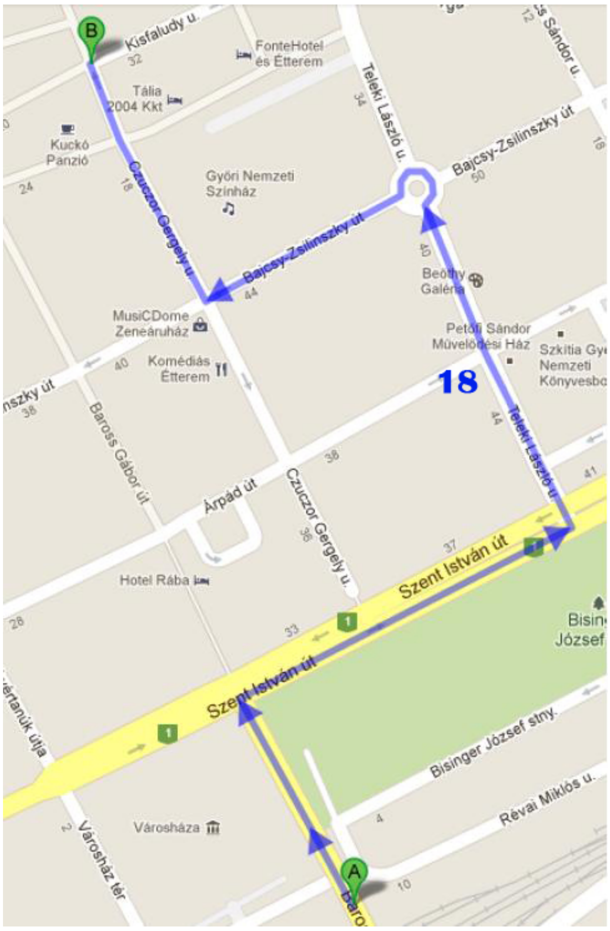
Destination point: Czuczor Gergely utca.

Route selection 18

Table 21: *Data for route selection 18*

	Distance	Time	Average speed
Google Maps	*850 m*	*2:00 min*	*25,5 km/h*
Measurement (night time)	923 m	5:37 min	36,46 km/h
Measurement (rush hour)	999 m	15:59 min	19,78 km/h
Average	**961 m**	**10:48 min**	**28,12 km/h**

Figure 3.14: *Map for route selection with entry point Barros Gabor Hid*



Source: "Győr" [Google Maps], b. d.

3.4 Model

The case that we will describe in this chapter is just a continuum of the example in the chapter 1.2.1.1. We decided to continue the case here for the purpose of easier imagination for the reader to know, what is the model of our master thesis, how is it structured and how to calculate it. Actual modeling was made based on gathered data that we presented in previous subsection and with the Mathematica programming language.

3.4.1 Wolfram Mathematica

We will describe just a bit about Mathematica here. Authors (developers) state: "Almost any workflow involves computing results, and that's what Mathematica does—from building a hedge-fund trading website or publishing interactive engineering textbooks, to developing embedded image-recognition algorithms or teaching calculus." ("Mathematica" [Wolfram.com], b. d.).

"Mathematica is renowned as the world's ultimate application for computations. But it's much more—it's the only development platform fully integrating computation into complete workflows, moving you seamlessly from initial ideas all the way to deployed individual or enterprise solutions." ("Mathematica" [Wolfram.com], b. d.).

For over 20 years, faculty and staff worldwide have used Mathematica for everything from teaching simple concepts in the classroom to doing serious research using some of the world's largest clusters. Mathematica continues to provide faculty with interactive lessons to engage students, deepening their understanding and preparing them for the future across a wealth of disciplines. Academic researchers can utilize Mathematica to quickly and accurately analyze data, test hypotheses, and document results. And because Mathematica delivers more capabilities, taking the place of several specialized kinds of software, schools can utilize Mathematica at a lower cost across campus ("Higher education" [Wolfram.com], b. d.).

3.4.2 Nash Equilibrium in Mathematica

A Nash equilibrium of a game is a strategy combination such that no party can improve its situation by changing its strategy, assuming the complementary strategies of the other players stay the same. The strategies are said to be "mutual best responses". This Demonstration lets you examine 10 continuous strategy games and see the payoff surfaces of each of the players as well as the sum of the payoffs. These surfaces can be visualized in three dimensions or in two dimensions. The Demonstration also calculates "best response" curves for each of the players, shown in orange and blue, respectively, and projects the Nash equilibrium point onto each best response curve. The total payoff surface is colored green for those strategy combinations that would improve total wealth relative to that achieved at the Nash equilibrium. The "green zone" thus represents the strategy combinations that, if wealth were transferrable between the players through enforceable contracts, would be Pareto superior to the Nash equilibrium ("Nash Equilibria with continuous strategies" [Demonstrations Wolfram.com], b. d.).

As we mentioned in the theoretical part, the point of our masther thesis is to find best possible choice of routes in road network from different entry points to the city center. Based on that we will present a basic Nash Equilibrium code, that was downloaded from the Wolfram Demonstration Projects website ("Nash Equilibria with continuous strategies" [Demonstrations Wolfram.com], b. d.).

3.4.3 Analytical determination of the travel time and outcomes of strategic game "Choosing a route" between two transport connections

As we said, let's continue the example about two transport connections here. Two outcomes of strategic game correspond to each action profile from the set AP. Now let's construct that type of function for the outcomes of strategic game $u(i,k)$, with which we can calculate outcome of strategic game for any i-th traveler $(i = 1,2)$ in arbitrary k-th action profile $(k = 1,2,3,4)$ in a way, that we will get the outcomes shown in a table. This function is defined as a negative value of travel time, which means that we take $u(i,k) = -t_p(i,k)$, where $t_p(i,k)$ is labeled as travel time of i-th traveler in k-th action profile (Pušenjak, Oblak & Fošner, 2013, p. 103).

Travel time of any player depends of the choice of strategy, which means if the strategy X is chosen, he travels on the route section $A-X-B$ or if the strategy Y is chosen, he travels on the route section $A-Y-B$. It's also important if the traveler is traveling alone on the route section or if there are two vehicles on the road (Pušenjak, Oblak & Fošner, 2013, p. 103).

If we introduce the parameter $p(i,k)$ and $q(i,k) = 1 - p(i,k)$ and we define them in a way that $p(i,k) = 1$, when traveler chooses strategy X or $p(i,k) = 0$ if the traveler chooses strategy Y, then we can express travel time $t_p(i,k)$ as a sum

$$t_p(i,k) = p(i,k)t_X(i,k) + q(i,k)t_Y(i,k) \tag{3.1}$$

where $t_X(i,k)$ represents travel time on the route section $A-X-B$ and $t_Y(i,k)$ represents travel time on route section $A-Y-B$. From the record of mentioned sum with the consideration of the equality $q(i,k) = 1 - p(i,k)$ we can see, that the travel time is $t_p(i,k) = t_X(i,k)$ when traveler chooses the strategy X and that it equals to $t_p(i,k) = t_Y(i,k)$ when traveler chooses strategy Y. While determining travel times $t_X(i,k)$ and $t_Y(i,k)$ we also have to take into consideration if there is one or more vehicle on the route section (Pušenjak, Oblak & Fošner, 2013, p. 103).

Let's label number of vehicles on the route section $A-X-B$ in k-th action profile with $s_X(k)$ and the number of vehicles on the route section $A-Y-B$ with k-th action profile with $s_Y(k)$. We should not forget that in the case of multiple vehicles on the same route section in strategy X we have to add additonal time to the travel time ($t_X^{add} = [s_X(k)-1] \cdot 2$)[2]. On the route section $A-X-B$ we have to add 2 minutes for each additonal vehicle. It is clear that if there is only one vehicle on the mentioned section and $s_X(k) = 1$, we don't add any additional time ($t_X^{add} = 0$). If there are two vehicles on the route section $A-Y-B$, we have to add $t_Y^{add} = [s_Y(k)-1] \cdot 0.5$[3] to travel time of 21 minutes (Pušenjak, Oblak & Fošner, 2013, p. 103).

General function for the strategic game of "Choosing a route" is therefore built in the following way (Pušenjak, personal communication, 17. march 2013):

$$\begin{aligned} t_p(i,k) &= p(i,k)\{20 + [s_X(k)-1] \cdot 2\} + q(i,k)\{21 + [s_Y(k)-1] \cdot 0.5\} \\ u(i,k) &= -t_p(i,k) \end{aligned} \tag{3.2}$$

Let's calculate the travel times of both travelers for action profile (X,X).

Because we know that (X,X) is the first action profile, it means that $k = 1$. Number of vehicles on the route sections $A-X-B$ and $A-Y-B$ are $s_X(1) = 2$ and $s_Y(1) = 0$. For the first traveler $i = 1$ we calculate also $p(1,1) = 1$ and $q(1,1) = 1 - p(1,1) = 0$ and we set the travel time (Pušenjak, Oblak & Fošner, 2013, p. 103):

$$t_p(1,1) = 1 \cdot [20 + (2-1) \cdot 2] + 0 \cdot [21 + (0-1) \cdot 0.5] = 22$$

For second traveler $i = 2$ we set $p(2,1) = 1$ and $q(2,1) = 1 - p(2,1) = 0$ and we get the same outcome than for the first traveler

[2]We have to have in mind, that the value 2 in the formula consists of 2 minutes for each vehicle, which is consistent with the example and it does not represent the general function/formula.

[3]We have to have in mind, that the value 0,5 in the formula consists of 0,5 minutes for each vehicle, which is consistent with the example and it does not represent the general function/formula.

$$t_p(2,1) = 1 \cdot [20 + (2-1) \cdot 2] + 0 \cdot [21 + (0-1) \cdot 0.5] = 22$$

Because in action profile (X,X) both travelers choose the strategy X, travel times are the same for both of them, which equals to

$$\begin{aligned} u(1,1) &= -t_p(1,1) = -22 \\ u(2,1) &= -t_p(2,1) = -22 \end{aligned}$$

units. Calculated outcome for action profile (X,X) is identical to the outcome, that was shown in the table 1 (Pušenjak, Oblak & Fošner, 2013, p. 104).

3.4.4 Modeling of strategic game on case study of city of Győr, Hungary

As we mentioned, added value of our master thesis reffers to development of mathematical model in software environment Mathematica, which can also be easily transfered to other modeling environments. With this, we wanted to prepare a frame and get a higher knowledge of solving transportation problems in the frame of mathematical modeling, which can be used also for the future research.

Mathematical model is presented below.

```
Print[Text[Style["Strategic game - "Choosing a route "", FontSize → 24, TextAlignment →Center]]];

Print[Text[Style["between two transport connections"", FontSize → 24, TextAlignment →Center]]];

Nstr = 2;

i = 1;

k = 1;

p[i, k] = 1;

q[i, k] = 1 - p[i, k];

s_x[k]=2;

s_y[k]=0;

t_p[i, k]=p[i, k]*(20+(s_x[k]-1)*2)+q[i, k]*(21+(s_y[k]-1)*0.5);

u_1[X, X]=-t_p[1, 1];

i = 1;

k = 2;

p[i, k] = 1;
```

q[i, k] = 1 - p[i, k];

s_x[k]=1;

s_y[k]=1;

t_p[i, k]=p[i, k]*(20+(s_x[k]-1)*2)+q[i, k]*(21+(s_y[k]-1)*0.5);

u_1[X, Y]=-t_p[1, 2];

i = 1;

k = 3;

p[i, k] = 0;

q[i, k] = 1 - p[i, k];

s_x[k]=1;

s_y[k]=1;

t_p[i, k]=p[i, k]*(20+(s_x[k]-1)*2)+q[i, k]*(21+(s_y[k]-1)*0.5);

u_1[Y, X]=-t_p[1, 3];

i = 1;

k = 4;

p[i, k] = 0;

q[i, k] = 1 - p[i, k];

s_x[k]=0;

s_y[k]=2;

t_p[i, k]=p[i, k]*(20+(s_x[k]-1)*2)+q[i, k]*(21+(s_y[k]-1)*0.5);

u_1[Y, Y]=-t_p[1, 4];

i = 2;

k = 1;

p[i, k] = 1;

q[i, k] = 1 - p[i, k];

s_x[k]=2;

s_y[k]=0;

t_p[i, k]=p[i, k]*(20+(s_x[k]-1)*2)+q[i, k]*(21+(s_y[k]-1)*0.5);

u_2[X, X]=-t_p[2, 1];

i = 2;

k = 2;

p[i, k] = 0;

q[i, k] = 1 - p[i, k];

s_x[k]=1;

s_y[k]=1;

t_p[i, k]=p[i, k]*(20+(s_x[k]-1)*2)+q[i, k]*(21+(s_y[k]-1)*0.5);

u_2[X, Y]=-t_p[2, 2];

i = 2;

k = 3;

p[i, k] = 1;

q[i, k] = 1 - p[i, k];

s_x[k]=1;

s_y[k]=1;

t_p[i, k]=p[i, k]*(20+(s_x[k]-1)*2)+q[i, k]*(21+(s_y[k]-1)*0.5);

u_2[Y, X]=-t_p[2, 3];

i = 2;

k = 4;

p[i, k] = 0;

q[i, k] = 1 - p[i, k];

s_x[k]=0;

s_y[k]=2;

t_p[i, k]=p[i, k]*(20+(s_x[k]-1)*2)+q[i, k]*(21+(s_y[k]-1)*0.5);

u_2[Y, Y]=-t_p[2, 4];

strategy[1]=X;

strategy[2]=Y;

i=1;

Do[profile[i]={{j, k}, strategy[j], strategy[k]}};

Print[“Action profile[“, i, “] = “, profile[i][[2]]];

Print[“Indeks of cell of action profile: j = “, profile[i][[1, 1]], “, k = “, profile[i][[1, 2]]];

i=i+1, {j, 1, 2}, {k, 1, 2}];

Print[“Number of action profiles = “, Nprofile=i-1];

Print[“Calculation of outcomes of strategic game for individual action profile”];

i=1;

Do[outcome[profile[i][[2]]]={$u_1$strategy[j], strategy[k]], u_2[strategy[j], strategy[k]]};

Tableoutcome[j, k]=outcome[profile[i][[2]]];

i=i+1, {j, 1, 2}, {k, 1, 2}]

Do[Print[“Outcome of strategic game(“, profile[i][[2]], “) = “];

Print[outcome[profile[i][[2]]]], {i, 1, Nprofile}];

```
Print["Table of outcomes of strategic game "];

TraditionalForm[Array[Tableoutcome, {2, 2}]]

Do[Nash=1;

Print["Analysis of action profile ", profile[i][[2]]];

j=profile[i][[1, 1]];

k=profile[i][[1, 2]];

Print["j = ", j, " k = ", k];

For[1_1=1, 1_1 ≤Nstr, 1_1++, If[1_1 ≠ j && u_1[strategy[1_1], strategy[k]]<=u_1[strategy[j], strategy[k]],

Print[u_1[strategy[1_1], strategy[k]], " <= ", u_1[strategy[j], strategy[k]], " Nash = ", Nash],

If[1_1 == j && u_1[strategy[1_1], strategy[k]] == u_1[strategy[j], strategy[k]],

Print[u_1[strategy[1_1], strategy[k]], " = ", u_1[strategy[j], strategy[k]], " Nash = ", Nash],

Print[u_1[strategy[1_1], strategy[k]], " > ", u_1[strategy[j], strategy[k]], " Nash = ", Nash=0]]]];

If[Nash == 1, Print["Action profile", profile[i], "is Nash Equilibrium"],

Print["Action profile", profile[i], "isn't Nash Equilibrium"]];

, {i, 1, Nprofile}];
```

3.5 Results

We are presenting final results, that were made using data in the table 22 and with a model for software environment Mathematica, below. Model is written in a way, that it calculates Nash Equilibrium and Pareto optimum as two possible choices for defining optimal solution in a road network. We focused on road network for the purpose of delivery of goods needed in the city center itself.

Final results are shown below.

Table 22: *Set of times for different route choices*

No.	*Entry point*	*Night time*	*Rush hour*	*Average*
1	Raba Kettos Hid	454 sec	932 sec	693 sec
2	Raba Kettos Hid	410 sec	520 sec	465 sec
3	Raba Kettos Hid	243 sec	297 sec	270 sec
4	Petofi Hid	290 sec	956 sec	623 sec
5	Petofi Hid	411 sec	633 sec	522 sec
6	Beke Hid	576 sec	899 sec	737 sec
7	Beke Hid	163 sec	672 sec	417 sec
8	Beke Hid	467 sec	634 sec	550 sec
9	Kossuth Hid	443 sec	517 sec	480 sec
10	Kossuth Hid	420 sec	500 sec	460 sec
11	Kossuth Hid	448 sec	519 sec	483 sec
12	Szechenyi Hid	328 sec	541 sec	434 sec
13	Szechenyi Hid	348 sec	563 sec	455 sec
14	Bisinger Joszef Hid	432 sec	708 sec	570 sec
15	Fehervari Ut	268 sec	636 sec	452 sec
16	Fehervari Ut	227 sec	387 sec	277 sec
17	Tihanyi Arpad Ut	297 sec	524 sec	410 sec
18	Barros Gabor Hid	337 sec	959 sec	648 sec

We will present the outcome of the program, written in Mathematica, in following lines. Due to large amount of pages, we will present just the couple of lines from each set of outcome.

SETS OF STRATEGIES FOR NINE ENTRY POINTS

Rabba Kettos Hid {Route1, Route2, Route3}

3

Petoffi Hid {Route4, Route5},

2

Beke Hid {Route6, Route7, Route8}

3

Kossuth Hid {Route9, Route10, Route11}

3

Szechenyi Hid {Route12, Route13}

2

Bisinger Joszef Hid {Route14}

1

Fehervari ut {Route15, Route16}

2

Tihanyi Arpad ut {Route17}

1

Baross Gabor Hid {Route18}

1

THE SET OF ACTION PROFILES

{{Route1,Route4,Route6,Route9,Route12,Route14,Route15,Route17,Route18},

{Route1,Route4,Route6,Route9,Route13,Route14,Route15,Route17,Route18},

{Route1,Route4,Route6,Route10,Route12,Route14,Route15,Route17,Route18},

{Route1,Route4,Route6,Route10,Route12,Route14,Route16,Route17,Route18},

{Route1,Route4,Route6,Route11,Route12,Route14,Route15,Route17,Route18},

...

{Route3,Route5,Route8,Route11,Route13,Route14,Route15,Route17,Route18},

{Route3,Route5,Route8,Route11,Route13,Route14,Route16,Route17,Route18}}

216

216

LIST OF INDICES

{{1,4,6,9,12,14,15,17,18},{1,4,6,9,12,14,16,17,18},{1,4,6,9,13,14,15,17,18},

{1,4,6,9,13,14,16,17,18},{1,4,6,10,12,14,15,17,18},{1,4,6,10,12,14,16,17,18},

{1,4,6,10,13,14,15,17,18},{1,4,6,10,13,14,16,17,18},{1,4,6,11,12,14,15,17,18},

...

{3,5,8,10,13,14,15,17,18},{3,5,8,10,13,14,16,17,18},{3,5,8,11,12,14,15,17,18},

{3,5,8,11,12,14,16,17,18},{3,5,8,11,13,14,15,17,18},{3,5,8,11,13,14,16,17,18}}

Number of routes in transport network = 18

Until this point, results of every strategic game for the city of Győr are the same, because we have the same set of strategies for all measurements. This means, that all of the streets, where we were making measurements, are included in the final calculation. From this point on, results vary based on the input data, which means based on the travel times for certain route section of the city of Győr. We will present a section from each measurement section in following subsections.

3.5.1 Results for night time measurements

We inserted necessary data of the complete strategic game for the city of Győr into the model. Outcome of the program was following:

STRATEGIC GAME “CHOOSING A ROUTE” FROM DIFFERENT ENTRY POINTS OF CITY OF Győr

STRATEGIC GAME OF NINE PLAYERS WITH DIFFERENT SETS OF STRATEGIES

NIGHT TIME TRAVEL TIMES ANALYSIS

TRAVEL TIMES OF 9 PLAYERS IN 216 ACTION PROFILES

tp[1,1]=454

tp[1,2]=454

...

tp[9,214]=337

tp[9,215]=337

tp[9,216]=337

END OF TRAVEL TIME LIST FOR 9 PLAYERS IN 216 ACTION PROFILES

DETERMINATION OF THE 1. PLAYER’S OUTCOME IN 1. ACTION PROFILE

{Route1,Route4,Route6,Route9,Route12,Route14,Route15,Route17,Route18}

u[1,1]=-454

DETERMINATION OF THE 1. PLAYER’S OUTCOME IN 2. ACTION PROFILE

{Route1,Route4,Route6,Route9,Route12,Route14,Route16,Route17,Route18}

u[1,2]=-454

...

DETERMINATION OF THE 9. PLAYER’S OUTCOME IN 216. ACTION PROFILE

{Route3,Route5,Route8,Route11,Route13,Route14,Route16,Route17,Route18}

u[9,216]=-337

START OF SEARCHING PROCEDURE FOR NASH EQUILIBRIA

INDICES OF 1. ACTION PROFILE = {1,4,6,9,12,14,15,17,18}

Original AP[[1]][[1]] = Route1

Changed AP[[1]][[1]][[73]] = Route2

Original AP[1] = {Route1, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18}

1. Changed AP[1] OF 1. PLAYER = {Route2, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18}

Performing the 1. profile change CHAP[1][[1]] = Route2

DETERMINATION OF 1.- th CHANGED ACTION PROFILE NUMBER kch[i] FOR THE 1. PLAYER

kch[1] = 73

OUTCOME OF THE GAME AT THE ORIGINAL 1. ACTION PROFILE = {-454,-290,-576,-443,-328,-432,-268,-297,-337}

OUTCOME OF THE GAME AT THE 1. CHANGED ACTION PROFILE = {-410,-290,-576,-443,-328,-432,-268,-297,-337}

-454 < -410 Testing for Nash equilibrium for 1. player is negative, Nash = 0

...

Original AP[[216]][[7]] = Route16

Changed AP[[216]][[7]][[215]] = Route15

Original AP[216] = {Route3,Route5,Route8,Route11,Route13,Route14,Route16,Route17,Route18}

1. Changed AP[216] OF 7. PLAYER = {Route3,Route5,Route8,Route11,Route13,Route14,Route15,Route17, Route18}

Performing the 1.-th profile change CHAP[216][[7]] = Route15

DETERMINATION OF 1.- th CHANGED ACTION PROFILE NUMBER kch[i] FOR THE 7. PLAYER

kch[7] = 215

OUTCOME OF THE GAME AT THE ORIGINAL 216.-th ACTION PROFILE = {-243,-411,-467,-448,-348,-432,-227,-297,-337}

-227 ≥ -268 Testing for Nash equilibrium for 7. player is positive, Nash = 1

216. ACTION PROFILE {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18} ISN'T NASH EQUILIBRIUM

{-454,-290,-576,-443,-328,-432,-268,-297,-337}

{-454,-290,-576,-443,-328,-432,-227,-297,-337}

{-454,-290,-576,-443,-348,-432,-268,-297,-337}

...

{-243,-411,-467,-448,-348,-432,-268,-297,-337}

{-243,-411,-467,-448,-348,-432,-227,-297,-337}

DETERMINATION OF PARETO OPTIMAL ACTION PROFILE

ANALYSIS OF ACTION PROFILE: {Route1, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18}

-3425 < -3384 Pareto_optimum = 0

-3425 ≥ -3445 Pareto_optimum = 0

...

-3425 < -3210 Pareto_optimum = 0

216.-th Action profile {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18} isn't Pareto optimal

THE COMPLETE LIST OF NASH EQUILIBRIA

1. ACTION PROFILE {Route1, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18} ISN'T NASH EQUILIBRIUM

2. ACTION PROFILE {Route1, Route4, Route6, Route9, Route12, Route14, Route16, Route17, Route18} ISN'T NASH EQUILIBRIUM

...

216. ACTION PROFILE {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18} ISN'T NASH EQUILIBRIUM

SUMMARY OF PARETO OPTIMAL ACTION PROFILES

ANALYSIS OF 1.-th ACTION PROFILE: {Route1, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18}

1. - Action profile {Route1, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18} isn't Pareto optimal

ANALYSIS OF 2.-nd ACTION PROFILE: {Route1, Route4, Route6, Route9, Route12, Route14, Route16, Route17, Route18}

2. - Action profile {Route1, Route4, Route6, Route9, Route12, Route14, Route16, Route17, Route18} isn't Pareto optimal

...

ANALYSIS OF 216.-th ACTION PROFILE: {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18}

216. - Action profile {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18} isn't Pareto optimal

SUMMARIZED RESULT OF STRATEGIC GAME "CHOOSING A ROUTE" FROM DIFFERENT ENTRY POINTS OF CITY OF GYŐR FOR NIGHT TIME MEASUREMENT TIMES

162. ACTION PROFILE {Route3, Route4, Route7, Route10, Route12, Route14, Route16, Route17, Route18} IS NASH EQUILIBRIUM

162. - ACTION PROFILE {Route3, Route4, Route7, Route10, Route12, Route14, Route16, Route17, Route18} IS PARETO OPTIMAL WITH TOTAL SUM = -2737

If we interpret this result it means, that we have 9 travelers, that have all together a choice of 18 possible routes to come to the final destination that we set. This 9 travelers have also 216 different possibilities of choosing a route. We presented a few possibilities above within results to show the output of the whole program.

The result of 162th action profile for Nash Equilibrium means, that first traveler chooses route 3, second traveler chooses route 4, third traveler decides for route 7, fourth traveler chooses route 10, fifth traveler goes through route 12, sixth traveler drives by route 14, seventh traveler decides for route 16, eighth traveler chooses route 17 and the last one picks route 18.

As we can see, Pareto optimum represents the value of -2737, which means, that the absolute result for night time measurement is 2737 seconds (this is the sum of travel times of all roads, that are included in the solution of Nash Equilibrium or Pareto optimum). This is optimal solution for which way to use in night time.

3.5.2 Results for rush hour measurements

We inserted necessary data of the complete strategic game for the city of Győr into the model. Outcome of the program was following:

STRATEGIC GAME “CHOOSING A ROUTE” FROM DIFFERENT ENTRY POINTS OF CITY OF Győr

STRATEGIC GAME OF NINE PLAYERS WITH DIFFERENT SETS OF STRATEGIES

RUSH HOUR TRAVEL TIMES ANALYSIS

TRAVEL TIMES OF 9 PLAYERS IN 216 ACTION PROFILES

tp[1,1]=932

tp[1,2]=932

...

tp[9,216]=959

END OF TRAVEL TIME LIST FOR 9 PLAYERS IN 216 ACTION PROFILES

DETERMINATION OF THE 1. PLAYER’S OUTCOME IN 1. ACTION PROFILE

{Route1,Route4,Route6,Route9,Route12,Route14,Route15,Route17,Route18}

u[1,1]=-932

...

DETERMINATION OF THE 9. PLAYER’S OUTCOME IN 216. ACTION PROFILE

{Route3,Route5,Route8,Route11,Route13,Route14,Route16,Route17,Route18}

u[9,216]=-959

START OF SEARCHING PROCEDURE FOR NASH EQUILIBRIA

INDICES OF 1. ACTION PROFILE = {1,4,6,9,12,14,15,17,18}

Original AP[[1]][[1]] = Route1

Changed AP[[1]][[1]][[73]] = Route2

Original AP[1] = {Route1, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18}

1. Changed AP[1] OF 1. PLAYER = {Route2, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18}

Performing the 1. profile change CHAP[1][[1]] = Route2

DETERMINATION OF 1.- th CHANGED ACTION PROFILE NUMBER kch[i] FOR THE 1. PLAYER

kch[1] = 73

OUTCOME OF THE GAME AT THE ORIGINAL 1.-th ACTION PROFILE = {-932,-956,-899,-517,-541,-708,-636,-524,-959}

OUTCOME OF THE GAME AT THE 1.-th CHANGED ACTION PROFILE = {-520,-956,-899,-517,-541,-708,-636,-524,-959}

-932 < -520 Testing for Nash equilibrium for 1. player is negative, Nash = 0

...

Original AP[[216]][[7]] = Route16

Changed AP[[216]][[7]][[215]] = Route15

Original AP[216] = {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18}

1.-th Changed AP[216] OF 7. PLAYER = {Route3, Route5, Route8, Route11, Route13, Route14, Route15, Route17, Route18}

Performing the 1.-th profile change CHAP[216][[7]] = Route15

DETERMINATION OF 1.- th CHANGED ACTION PROFILE NUMBER kch[i] FOR THE 7. PLAYER

kch[7] = 215

OUTCOME OF THE GAME AT THE ORIGINAL 216.-th ACTION PROFILE = {-297,-633,-634,-519,-563,-708,-327,-524,-959}

OUTCOME OF THE GAME AT THE 1.-th CHANGED ACTION PROFILE = {-297,-633,-634,-519,-563,-708,-636,-524,-959}

-327 $\geq$ -636 Testing for Nash equilibrium for 7. player is positive, Nash = 1

216. ACTION PROFILE {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18} ISN'T NASH EQUILIBRIUM

{-932,-956,-899,-517,-541,-708,-636,-524,-959}

{-932,-956,-899,-517,-541,-708,-327,-524,-959}

...

{-297,-633,-634,-519,-563,-708,-327,-524,-959}

DETERMINATION OF PARETO OPTIMAL ACTION PROFILE

ANALYSIS OF ACTION PROFILE: {Route1, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18}

-6672 < -6363 Pareto_optimum = 0

-6672 $\geq$ -6694 Pareto_optimum = 0

...

-3425 < -3210 Pareto_optimum = 0

216.-th Action profile {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18} isn't Pareto optimal

THE COMPLETE LIST OF NASH EQUILIBRIA

1. ACTION PROFILE {Route1, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18} ISN'T NASH EQUILIBRIUM

2. ACTION PROFILE {Route1, Route4, Route6, Route9, Route12, Route14, Route16, Route17, Route18} ISN'T NASH EQUILIBRIUM

...

216. ACTION PROFILE {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18} ISN'T NASH EQUILIBRIUM

SUMMARY OF PARETO OPTIMAL ACTION PROFILES

ANALYSIS OF 1.-th ACTION PROFILE: {Route1, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18}

1. - Action profile {Route1, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18} isn't Pareto optimal

ANALYSIS OF 2.-nd ACTION PROFILE: {Route1, Route4, Route6, Route9, Route12, Route14, Route16, Route17, Route18}

2. - Action profile {Route1, Route4, Route6, Route9, Route12, Route14, Route16, Route17, Route18} isn't Pareto optimal

...

ANALYSIS OF 216.-th ACTION PROFILE: {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18}

216. - Action profile {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18} isn't Pareto optimal

SUMMARIZED RESULT OF STRATEGIC GAME "CHOOSING A ROUTE" FROM DIFFERENT ENTRY POINTS OF CITY OF GYŐR FOR RUSH HOUR TIMES

210. ACTION PROFILE {Route3, Route5, Route8, Route10, Route12, Route14, Route16, Route17, Route18} IS NASH EQUILIBRIUM

210. - ACTION PROFILE {Route3, Route5, Route8, Route10, Route12, Route14, Route16, Route17, Route18} IS PARETO OPTIMAL WITH TOTAL SUM = -5123

The result of 210th action profile for Nash Equilibrium means, that first traveler chooses route 3, second traveler chooses route 5, third traveler decides for route 8, fourth traveler chooses route 10, fifth traveler goes through route 12, sixth traveler drives by route 14, seventh traveler decides for route 16, eighth traveler chooses route 17 and the last one picks route 18.

Result is obvious for different reasons. First, cars can enter the city from all of the entry points to the city of Győr. Here the results could differ if we would set a limitation of delivery made by trucks, because in example, Kossuth Hid is not appropriate for the truck delivery, as we were told by the people, who live there.

Second, it the result makes sense from the point, that each delivery driver chooses his own route to come from entry point of the city of Győr to the destination point, which is Czuczor Gergely u. If all of the delivery drivers would choose one route there is a chance of traffic congestion.

In this case, Pareto optimum represents the value of -5123, which means, that the absolute result for night time measurement is 5123 seconds (this is the sum of travel times of all roads, that are included in the solution of Nash Equilibrium or Pareto optimum). This is optimal solution for which way to use in rush hour.

We can see, that the optimum in the case of rush hour measurements is almost 50 percent higher than the optimum that we got in the case of night time measurements. We can easily explain this result by the number of vehicles on the road in certain time frames. During the night, there was lower traffic density than in day time.

3.5.3 Results of average travel times

STRATEGIC GAME "CHOOSING A ROUTE" FROM DIFFERENT ENTRY POINTS OF CITY OF Győr

STRATEGIC GAME OF NINE PLAYERS WITH DIFFERENT SETS OF STRATEGIES

AVERAGE TRAVEL TIMES ANALYSIS

TRAVEL TIMES OF 9 PLAYERS IN 216 ACTION PROFILES tp[1,1]=693

tp[1,2]=693

...

tp[9,216]=648

END OF TRAVEL TIME LIST FOR 9 PLAYERS IN 216 ACTION PROFILES

DETERMINATION OF THE 1. PLAYER'S OUTCOME IN 1. ACTION PROFILE

{Route1,Route4,Route6,Route9,Route12,Route14,Route15,Route17,Route18}

u[1,1]=-693

...

DETERMINATION OF THE 9. PLAYER'S OUTCOME IN 216. ACTION PROFILE

{Route3,Route5,Route8,Route11,Route13,Route14,Route16,Route17,Route18}

u[9,216]=-648

START OF SEARCHING PROCEDURE FOR NASH EQUILIBRIA

INDICES OF 1. ACTION PROFILE = {1,4,6,9,12,14,15,17,18}

Original AP[[1]][[1]] = Route1

Changed AP[[1]][[1]][[73]] = Route2

Original AP[1] = {Route1, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18}

1. Changed AP[1] OF 1. PLAYER = {Route2, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18}

Performing the 1. profile change CHAP[1][[1]] = Route2

DETERMINATION OF 1.- th CHANGED ACTION PROFILE NUMBER kch[i] FOR THE 1. PLAYER

kch[1] = 73

OUTCOME OF THE GAME AT THE ORIGINAL 1.-th ACTION PROFILE = {-693,-623,-737,-480,-434,-570,-452,-410,-648}

OUTCOME OF THE GAME AT THE 1.-th CHANGED ACTION PROFILE = {-465,-623,-737,-480,-434,-570,-452,-410,-648}

-693 < -465 Testing for Nash equilibrium for 1. player is negative, Nash = 0

...

Original AP[[216]][[7]] = Route16

Changed AP[[216]][[7]][[215]] = Route15

Original AP[216] = {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18}

1.-th Changed AP[216] OF 7. PLAYER = {Route3, Route5, Route8, Route11, Route13, Route14, Route15, Route17, Route18}

Performing the 1.-th profile change CHAP[216][[7]] = Route15

DETERMINATION OF 1.- th CHANGED ACTION PROFILE NUMBER kch[i] FOR THE 7. PLAYER

kch[7] = 215

OUTCOME OF THE GAME AT THE ORIGINAL 216.-th ACTION PROFILE = {-270,-522,-550,-483,-455,-570,-277,-410,-648}

OUTCOME OF THE GAME AT THE 1.-th CHANGED ACTION PROFILE = {-270,-522,-550,-483,-455,-570,-452,-410,-648}

-277 $\geq$ -452 Testing for Nash equilibrium for 7. player is positive, Nash = 1

216. ACTION PROFILE {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18} ISN'T NASH EQUILIBRIUM

{-693,-623,-737,-480,-434,-570,-452,-410,-648}

{-693,-623,-737,-480,-434,-570,-277,-410,-648}

...

{-270,-522,-550,-483,-455,-570,-277,-410,-648}

DETERMINATION OF PARETO OPTIMAL ACTION PROFILE

ANALYSIS OF ACTION PROFILE: {Route1, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18}

-5047 < -4872 Pareto_optimum = 0

-5047 $\geq$ -5068 Pareto_optimum = 0

...

-4185 $\geq$ -4360 Pareto_optimum = 0

216.-th Action profile {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18} isn't Pareto optimal

THE COMPLETE LIST OF NASH EQUILIBRIA

1. ACTION PROFILE {Route1, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18} ISN'T NASH EQUILIBRIUM

2. ACTION PROFILE {Route1, Route4, Route6, Route9, Route12, Route14, Route16, Route17, Route18} ISN'T NASH EQUILIBRIUM

...

216. ACTION PROFILE {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18} ISN'T NASH EQUILIBRIUM

SUMMARY OF PARETO OPTIMAL ACTION PROFILES

ANALYSIS OF 1.-th ACTION PROFILE: {Route1, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18}

1. - Action profile {Route1, Route4, Route6, Route9, Route12, Route14, Route15, Route17, Route18} isn't Pareto optimal

ANALYSIS OF 2.-nd ACTION PROFILE: {Route1, Route4, Route6, Route9, Route12, Route14, Route16, Route17, Route18}

2. - Action profile {Route1, Route4, Route6, Route9, Route12, Route14, Route16, Route17, Route18} isn't Pareto optimal

...

ANALYSIS OF 216.-th ACTION PROFILE: {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18}

216. - Action profile {Route3, Route5, Route8, Route11, Route13, Route14, Route16, Route17, Route18} isn't Pareto optimal

We already presented the data and final results for night time and rush hour measurements. We made the final calculation with a mentioned model, that represents average of all the measurements. Here are the results of average travel times, that are shown in Table 21. This average is calculated from actual measurement times and can be compared to Google Maps times.

SUMMARIZED RESULT OF STRATEGIC GAME “CHOOSING A ROUTE” FROM DIFFERENT ENTRY POINTS OF CITY OF GYŐR FOR AVERAGE TIMES

198. ACTION PROFILE {Route3, Route5, Route7, Route10, Route12, Route14, Route16, Route17, Route18} IS NASH EQUILIBRIUM

198. - ACTION PROFILE {Route3, Route5, Route7, Route10, Route12, Route14, Route16, Route17, Route18} IS PARETO OPTIMAL WITH TOTAL SUM = -4008

The result of 198th action profile for Nash Equilibrium means, that first traveler chooses route 3, second traveler chooses route 5, third traveler decides for route 7, fourth traveler chooses route 10, fifth traveler goes through route 12, sixth traveler drives by route 14, seventh traveler decides for route 16, eighth traveler chooses route 17 and the last one picks route 18.

Pareto optimum represents the value of -4008, which means, that the absolute result for night time measurement is 4008 seconds (this is the sum of travel times of all roads, that are included in the solution of Nash Equilibrium or Pareto optimum). This is optimal solution for which way to use based on average times.

We can see that the average solution is closer to a solution given for rush hour than the solution for night time. The difference is not significant, but the fact that it still exist can be explained that there is higher probability of getting to the store (or to the city center) “on time” in the case of delivery if we use the roads that were predicted by rush hour result.

CONCLUSION

Summary

Final thesis is based on theory of strategic games and Nash Equilibrium in connection with city logistics.

Theoretical part is divided into three basic parts. First part consists of short chapter about city logistics, where we made the literature review on the mentioned topic and we defined the concept of city logistics. Then follows the chapter about game theory and mathematical modeling, combined with modeling of transport systems and situations, that were basis for our model in software environment Mathematica, which is shown in practical part of final thesis.

As it was said, the chapter of strategic games is based on different cases. We presented the cases from simple strategic games, that can be modeled with table approach (by principle of the strategic game "The prisoner's dilemma"), to modeling of complex transport systems with the use of strategic games. Next step was a structure of strategic game "Choosing a route" in reconstructed road network, that is important from the viewpoint of long term projecting of road networks for the purpose of constructing new road sections or canceling certain road sections. This case can be used also if there is certain construction work being made on the road section, because we can project the conversion of traffic to other road sections. We finished theoretical part with the presentation of strategic game "Two players with different strategy sets". All of the mentioned cases are coherently linked with Nash Equililbrium theory and Pareto optimum. It's worth mentioning that Nash Equilibrium and Pareto optimum are not gathered on the same way, so there fore they do not necessary represent the same outcome of the strategic game.

Second chapter is about the city of Győr, Hungary, that was chosen for the purpose of our case study for the master thesis. We decided for this city based on the time spent there while doing internship at Szechenyi Istvan University. In this chapter, we described the city overall and it's history with the development through time.

With the third chapter comes the practical part. Here we described the way of gathering data and presented the way of delivering goods in the city center, based on the informations that we got with personal communication with people living in the city, working there or studying. We found out that city center is divided into two parts, old (or hystorical) city center and new (or modern/extended) city center. Our practical part consists the part of both city centers, because the purpose of it was that we calculate the optimal route for delivery into the city center, based on the entrances to it. Final point (destination point) was Czuczor Gergely u., which is the closest street before the pedestrian area. We had 9 entry points to the city center and one destination point.

For the need of modeling of strategic games we needed data about the lenght of the section from entry to destination point for each of possible routes, that were named "Route selection X", where X represents number of route, that is shown on the map in the practical part of the master thesis, and time, that was needed for the journey. We also gathered data about average speed of the journey, but those data were not cruical for our model, but are mentioned for the purpose of the further development of research work.

The basis of the actual measurement was the analysis of possible entry points to the city center to the destination point. We made the analysis with Google Maps, where we gathered approximate data about the lenght of the road section and time needed for the journey. Next step were the measurements. For the purpose of increased accuracy we didn't want to use only Google Maps data, but we made the measurements of all the possible ways in night time, when the density of the traffic is lower and in day time in the frame of rush hour periods, which means the times, when people migrate to work place or home, study, ...

Measurements were made with the android application, called My Car Tracks. Application allows vehicle tracking or better said, tracking of the mobile phone with GPS technology, with which it gathers data about location, speed, distance etc. Data are stored in the online database, where you can print the reports, define different vehicles and so on. We also presented a map of each possible route selection.

Modeling of strategic game based on the data for the city of Győr was made in the software environment Mathematica. Final product of our master thesis is a program, that covers the ful problem of city of Győr based on our input data. This means that it covers 9 entry points, 18 different roads and 216 action profiles. We presented a basic Mathematica model in the master thesis, that was used for calculating optimal transport ways, defining Nash Equilibrium and Pareto optimum. Final result, that was gathered from the last version of the program is that there is only one Nash Equilibrium and only one Pareto optimum between all 216 action profiles.

Result, that was achieved while writing master thesis is not only the result of strategic game based on the city of Győr; his added value is much bigger. From first idea about combining theory and practice, we made a huge step forward and we developed a tool (model), that can be used on any imaginable road network, independent of the starting point and road connections. We can se, that te basic idea was developed into a very powerful computer program, that can be used in complex system in different fields of logistics and economy and also in techical sciences (construction, traffic, etc.).

Evaluation of efficiency of the problem solution

Problem, that we defined at the begining of the writing of our final thesis, was concentrated mostly on the connection of theorethical knowledge with practical case study. While writing, we discovered, that there is a lot more behind this so together with our mentor, we made a big step forward. Based on the measurements we made in the city of Győr, which was our case study topic, we came from just combining theory with practice to preparing actual solution, that can be used in any traffic network around the world.

In our case, problem solution is connected to the companies, who deliver goods in the city center of Győr. With our solution (mathematical model), they can provide just-in-time delivery and logistics services. If we take a look at world wide usage of this, we made a progress in the field of new knowledge. Our approach can present a new model or way of thinking, which means a highly useful contribution to the world knowledge treasury.

Model can be used in various cases and by different stakeholdes. If we consider just latest happening in Győr, when Donau flodded almost the whole city, we could use the reconstructed road network materials to determine, which roads are still in use and how much volume of the cars can take. We could also determine, which additional roads to use and the municipaliy could inform people of the results.

Also, governments can use the model (with slight changes) in the way of reconstruction of road segments. Every country makes some changes every year and the model can be used in the same purpose as we explained for the city of Győr in the previous paragraph.

Conditions for the solution implementation

Our solution is not made in a way, that can be used by certain company in exactly this moment, it's more of a model, than companies or governments can use for their process optimization.

Solution, that we developed in the frame of Mathematica modeling software environment was used on the actual case in the city centre of Győr. Program itself was constructed in a way, that can be easily used in other cases or other city centres as well. For better understanding and easier presentation, we also developed a program that can be used for reconstructed road network and based on this, we can also get a preview of road usage and in the future, we can prepare a report on which roads are overfloded etc.

Possibility of further development

Model for calculating strategic games in the frame of transportation problem "Choosing a route" can be easily used for other cities as well. We tried to make a simple model, which can be transfered or used based on different data, while the programming part remains the same.

As mentined in the thesis, we gathered a lot of data with the My Car Track application. In our thesis, there was only time used, but others were mentioned exactly for the purpose of further development. We predict the next step of expanding our thesis in the research work and publishing the results with a model, that will contain extended approach, thinking also about other points of view, for example binding the DHL theory in practical approach, which means that the final result is based on formula, that includes time frame, distance, gasoline price or carbon footprint etc.

For further development, we can predict the usage of the program in other cities as well, to get the comparison to the city centre of Győr, which is used in our master thesis. While city centre of Győr is not a large area to cover, there was still a lot of options, a lot of road choice possibilities, so we can assume, that the research of bigger cities, even capitals, i.e. Paris or London, is very complex task.

We can say, that we actually achieved added value with our thesis, because with this program, further development can be made on other road networks (other case studies) and based on this, we can give our contribution to the world's knowledge treasury.

MODELIRANJE MESTNE LOGISTIKE

Magistrsko delo je nastalo pod mentorstvom prof. dr. Rudolfa Pušenjaka ter somentorice iz tujine, dr. Judit Makkos-Kaldi. Zaradi lažje komunikacije in časovne optimizacije pri pisanju smo se odločili, da je najbolje, če je zaključno delo napisano v angleškem jeziku.

Zaključno delo temelji na teoriji strateških iger, Nashevega ravnotežja ter Pareto optimuma v povezavi z mestno logistiko. V uvodnem delu smo predstavili problem, ki smo ga zaznali oziroma v začetku laično idejo o tem, kako poteka dostava dobrin v mestna jedra. Nadalje smo predstavili načrt, kako bomo dosegli zastavljen cilj ter možne omejitve, s katerimi bi se lahko srečali v času pisanja magistrskega dela.

Teoretični del je razdeljen na tri osnovne dele in sicer na krajše poglavje o mestni logistiki, kamor smo vključili trenutni pregled literature na to temo in poskušali predstaviti pojem mestne logistike kot takšen. Ugotovili smo, da večina avtorjev mestno logistiko opisuje kot proces popolne optimizacije logistike in transportnih aktivnosti, ki so vodeni s strani privatnih podjetij v urbanih naseljih, medtem ko je potrebno istočasno upoštevati še cestno omrežje, zasedenost cestnega omrežja in porabo energije v okviru zahtev na trgu. Sem smo za potrebe magistrskega dela dodali še klasifikacijo, ki sta jo razvila Awasthi in Chauchan (2012, str. 575), ki deli študije na temo planiranja mestne logistike v štiri sklope in sicer pristop, ki temelji na anketnih vprašalnikih, pristop, ki temelji na simulacijah, pristop ki temelji na teoriji multikriterijskih odločitvah, hevristični pristop ter stroškovna analiza. Ker smo se v našem delu osredotočili predvsem na simulacije, smo le-te tudi podrobneje analizirali.

Temu sledi poglavje o teoriji iger in matematičnem modeliranju ter modeliranju transportnih sistemov s strateškimi igrami. Osborne (2002, str. 12) pojasnjuje, da je s pomočjo strateških iger možno modelirati skorajda neomejeno množico problemov, kot na primer, igralci so lahko organizacije, akcije so cene in tako naprej. Čas v modelu manjka oziroma je namensko izvzet. Ideja je, da vsak izmed igralcev izbere svojo strategijo, ki je naknadno ne spreminja, hkrati pa vsi igralci izberejo strategijo istočasno. Sprememba strategije je nesmiselna, saj si igralec svojega rezultata ne more izboljšati. Nadalje smo pojasnili tudi modeliranje transportnih sistemov s pomočjo strateških iger v grobem. Transportni sistemi so v osnovi prekompleksni, da bi jih lahko računali po principu "papir in pero". V kolikor uporabimo evolucijsko teorijo, naletimo na mnogo ovir kot so časovne ovire, pomanjkanje znanja in izkušenj, itd., ki jih moramo pred tem proučiti. Zadnje podpoglavje je namenjenu prikazu različnih strateških iger in situacij, ki so bile osnova za izdelavo matematičnega modela v programskem orodju Mathematica in ki je prikazan v praktičnem delu zaključnega dela.

Kot rečeno, je poglavje o strateških igrah vezano tudi na različne primere. Predstavili smo primere od preprostih strateških iger, ki jih lahko modeliramo s pomočjo tabele (po principu strateške igre "Dilema dveh zapornikov"), kar smo opisali v poglavju "Modeliranje strateške igre "Izbira poti" za 2 potnika s tabelo", kamor smo nadalje dodali tudi izračun Nashevega ravnotežja. Nato smo prišli do modeliranja kompleksnih transportnih sistemov s pomočjo strateških iger, kamor spada modeliranje strateških iger s štirimi igralci ter določevanje Nashevega ravnotežja. Naslednji korak je bila struktura strateške igre "Izbira poti" v rekonstruiranem cestnem omrežju, ki je pomembna predvsem z vidika dolgoročnega projektiranja cestnih omrežij za namene izgradnje novih cestnih povezav ali prekinitev poti po določenih povezavah. Ta način je uporaben tudi v primeru, če se na določenem cestnem odseku opravljajo npr. gradbena dela, da vemo, kam preusmeriti promet, koliko ga je in ali bo na takšen način prišlo do zastojev etc. Na koncu smo zaključili teoretični del s predstavitvijo strateške igre dveh igralcev z različnima množicama strategij. Vsi našteti primeri so podprti z zgledi ter koherentno povezani še z Nashevim ravnotežjem in Pareto optimumom. Tukaj je potrebno omeniti, da Nashevo ravnotežje in Paretov optimum ne predstavljata nujno istega izzida strateške igre, predvsem pa sta pridobljena na različna načina.

Drugo poglavje se navezuje na mesto Győr na Madžarskem, ki je bilo izbrano za študijo primera za namene našega magistrskega dela. Za to mesto smo se odločili na podlagi opravljanja študijske prakse v Madžarskem mestu Győr. V tem poglavju smo opisali mesto kot takšno in njegovo zgodovino ter razvoj skozi čas. Smiselnost tega poglavja lahko pojasnimo predvsem z vidika razvoja mesta skozi čas. Glede na to, da je sedanje mestno jedro, ki predstavlja peš cono v mestu samem, v preteklosti predstavljalo glavno žilo in utrip mesta, je bilo nujno, da smo to vsaj okvirno predstavili. Konec koncev danes Győr predstavlja eno najpomembnejših mest v severozahodnem delu Madžarske ter hkrati prestolnico regije Győr-Moson-Sopron. Hkrati Győr predstavlja eno izmed najpomembnejših cestnih križišč Centralne Evrope, saj predstavlja vmesno postajo med Budimpešto in Dunajem.

S tretjim poglavjem se prične praktični del. Tukaj smo opisali način zbiranja podatkov in predstavili način dostave v samo mestno jedro na podlagi informacij, ki smo jih pridobili z osebno komunikacijo z ljudmi, ki prebivajo v mestu, so tam zaposleni ali tam študirajo. Izvedeli smo, da je samo mestno jedro razdeljeno na dva dela, staro oziroma zgodovinsko mestno jedro in novo oziroma moderno/razširjeno mestno jedro. Naš praktični del sicer obsega del obeh, saj je namen našega zaključnega dela, da na podlagi vhodov v mestno središče izračunamo, katera je optimalna pot za dostavo v mestno središče. Trenutno smo v programu, ki smo ga razvili za namene magistrskega dela, vključili le časovno komponento, saj bi v nasprotnem primeru zaključno delo presegalo določen obseg, hkrati pa lahko kot nadaljnje delo predvidimo vključitev še okoljske komponente, cene bencina in drugih dejavnikov. Končna točka, kjer lahko vozniki dostavnih vozil parkirajo svoja vozila je bila Czucor Gergely u., kar je tudi najbližje možno parkirno mesto pred vstopom v območje za pešce, ki signalizira zaprti del mestnega središča. Imeli smo torej 9 vstopnih točk (nekaj točk je bilo izločenih zaradi samo ene možne povezave od začetne do končne destinacije) in eno končno točko.

Za potrebe modeliranja strateških iger smo potrebovali podatke o razdalji od začetne do končne točke za vsako izmed možnih poti, ki smo jih poimenovali "Izbira poti X", kjer X označuje številko poti in je prikazana na zemljevidu v praktičnem delu magistrskega dela in čas, ki je bil porabljen za omenjeno pot. Pridobili smo tudi podatke o povprečni hitrosti in najvišji hitrosti vožnje, ki pa v našem modelu niso igrali ključne vloge, a smo jih zapisali, saj lahko predstavljajo možnost za nadaljnje raziskovalno delo.

Osnova za dejanske meritve je bila analiza možnih poti od vhodnih točk v mestno jedro do končne točke. To smo naredili s pomočjo Google zemljevidov, kjer smo pridobili tudi okvirne informacije o dolžini poti in času, ki je potreben, da se ta pot prevozi. Naslednji korak je bilo opravljanje meritev. Zaradi boljše natančnosti namreč nismo želeli uporabiti samo podatkov, pridobljenih z Google zemljevidi, temveč smo opravili meritve vseh možnih poti v nočnem času, ko je gostota prometa manjša ter v dnevnem času in sicer ob urah, ko se predvideva, da je konična ura (ang. rush hour), torej takrat, ko ljudje migrirajo na delovno mesto ali iz delovnega mesta, predavanj etc.

Meritve smo opravili s pomočjo aplikacije za mobilne telefone, ki temeljijo na operacijskem sistemu Android, imenovane My Car Tracks. Aplikacija omogoča sledenje vozilu oziroma natančneje mobilnemu telefonu in s pomočjo GPS tehnologije pridobi podatke o lokaciji, hitrosti, prevoženi razdalji etc. Podatki se zbirajo v bazi, ki jih je mogoče pregledati na spletnem portalu in s pomočjo katerega je mogoče tudi izpisati poročila etc. Nadalje smo tudi grafično prikazali za vsako izmed možnih poti.

Modeliranje strateške igre na podlagi podatkov za mesto Győr smo pripravili v programskem orodju Mathematica. Končni izdelek našega magistrskega dela je program, ki obsega popolni problem mesta Győr glede na naše vhodne podatke. To pomeni, da je v problem zajetih 9 izhodišč, 18 cest ter 216 akcijskih profilov. V magistrskem delu smo zapisali osnovni model, ki smo ga uporabili za izračunavanje optimalnih transportnih poti in določanje Nashevega ravnotežja ter Paretovega optimuma. Končni rezultat, ki smo ga pridobili iz zadnje verzije programa je, da med vsemi 216. akcijskimi profili obstaja samo eno Nashevo ravnotežje, ki hkrati predstavlja tudi Paretov optimum.

Rešitve smo razdelili na podpoglavja in sicer smo najprej pisali modeliranje strateške igre študije primera mesta Győr, kjer smo podali matemični zapis strateške igre v programskem okolju Mathematica. Nato smo razširili rezultate še na vse tri možnosti, ki smo jih pridobili s strani meritev. Prvo podpoglavje obsega računalniški izpis rezultatov za meritve, opravljene v nočnem času, kjer Nashevo ravnotežje predstavlja 162. akcijski profil. Paretov optimum predstavlja vrednost -2737, kar pomeni, da vsi igralci potrebujejo 2737 sekund, da pridejo iz začetne do končne točke. Drugo podpoglavje obsega računalniški izpis rezultatov za meritve, opravljene v času prometne konice, ko ljudje migrirajo iz točke A v točko B znotraj mesta. Nashevo ravnotežje v tem primeru predstavlja 210. akcijski profil, kar pomeni, da prvi potnik izbere tretjo cesto (t.i. "route 3"), drugi potnik izbere peto cesto, tretji potnik izbere osmo možnosti, četrti potnik se odloči za 10 opcijo, peti potnik za 12 opcijo, šesti potnik izbere cesto 14, sedmi potnik se odloči za 16 cesto, osmi potnik za cesto 17 in zadnji potnik izbere 18. cesto. Paretov optimum predstavlja vrednost -5123, kar pomeni, da vsi igralci potrebujejo 5123 sekund, da pridejo iz začetne do končne točke. Tretje podpoglavje obsega računalniški izpis rezultatov za povprečne meritve, kar pomeni, da smo v računalniški program vstavili povprečne vrednosti, ki predstavljajo srednjo vrednost med nočnimi meritvami in meritvami, narejenimi v času prometne konice. Nashevo ravnotežje predstavlja 198. akcijski profil. Paretov optimum predstavlja vrednost -4008, kar pomeni, da vsi igralci potrebujejo 4008 sekund, da pridejo iz začetne do končne točke.

Rezultat, ki smo ga dosegli pri pisanju magistrskega dela ni samo rezultat strateške igre na podlagi mesta Győr, temveč je dodana vrednost mnogo večja. Iz začetne ideje smo namreč razvili orodje oziroma model, ki je lahko uporabljen na katerem koli mestnem središču, na katerem koli cestnem omrežju in neodvisno od števila izhodišč in cestnih povezav. Hkrati je to močno orodje za nadaljnje načrtovanje kakršnega koli cestnega odseka. Kot lahko vidimo, se je osnovna ideja, ki je obsegala povezovanje teorije s praktičnim primerom razvila v zmogljiv računalniški program, ki ga je mogoče uporabljati v zelo kompleksnih sistemih na različnih področjih logistike in ekonomije, pa tudi v tehniki (gradbeništvo, promet, itd.).

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