A Stochastic Dynamic Programming Model of Direct Subsidy Payments and Agricultural Investment

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Abstract

A stochastic dynamic programming model is used to compare the farmland investment impact of a fully decoupled direct payment and a standard price subsidy. The direct payment induces the farmer to invest because it lowers the farm's debt to asset ratio, which in turn reduces the probability of bankruptcy. The value of the real option to defer the investment decision is lower with a lower risk of bankruptcy, and thus the direct payment results in a higher probability of immediate investment. Simulation results demonstrate that for a farm facing moderate revenue and land price variability, the impact of a decoupled direct payment on farm investment is nearly as large as the investment impact of an equal-sized price subsidy. These results suggest that direct payments, such as those associated with U.S. production flexibility contracts, should be carefully scrutinized in on-going multilateral trade negotiations.

This paper is based on a larger research report, which was prepared for the OECD and presented at the OECD office in Paris in May, 2003. Many helpful comments from participants of the OECD workshop have been incorporated into this draft.

I. Introduction

In recent years, several papers have examined the link between decoupled farm subsidies (hereafter referred to as direct payments) and farm output. Wealth enhancing payments that do not depend on farm output or farm size may increase agricultural production and investment by freeing up financial resources for a credit-constrained farmer, by providing a farmer with better terms of credit and by reducing a farmer's aversion toward engaging in risky production and investment activities (Chavas and Holt; Hennessy; Tielu and Roberts; Young and Westcott; Burfisher, Robinson and Thierfelder; OECD 2002; USDA). A farmer may also increase production in anticipation that future direct payments will be based on historic levels of production (OECD 2000). Benjamin shows how labour market imperfections can eliminate the separation between farm household consumption and production decisions. Without separation, direct payments can potentially raise farm production. Vercammen identifies other economic linkages between direct payments and farm output including binary labour market decisions by the farm household, intergenerational transfer of farm assets, a rising marginal rate of taxation on farm income and a wedge between the cost of borrowing and the cost of saving.

Understanding the link between a direct payment and farm output is important, especially from an international trade perspective. The international community is increasingly scrutinizing the potential trade impacts of various types of farm support programs. Direct payments have received little attention in previous World Trade Organization (WTO) negotiations, but this has now changed given the recent emergence of large-scale direct payment programs such as production flexibility contracts in the

U.S., which are typically reported as "green".[1] The international community's initial enthusiasm for direct payments will erode if it is discovered that these payments can raise farm output substantially through indirect mechanisms.

An area that has received comparatively little attention is the dynamic link between direct payments and investment in farm assets such as land and machinery. These dynamic investment impacts are important from a trade perspective because a higher level of farm investment can result in higher levels of farm output for many subsequent years. It is well known that in the absence of market failures, there is no link between a direct payment and farm investment because the optimal level of investment depends only on the internal rate of return and the market rate of interest, and neither of these variables are impacted by a direct payment. The various types of market failure that dynamically link farm investment to a direct payment are similar to those previously discussed (e.g., lack of risk sharing and credit constraints). Credit market constraints are particularly important for the case of agricultural investment because agriculture tends to be highly capitalized and highly dependent on debt capital.

The purpose of this paper is to use a dynamic programming model of farm investment and credit market failure to demonstrate that there may exist a strong and positive link between a direct payment and farm investment in a typical owner-operator farm operation. The model is quite simple (i.e., most of the usual complexities of farm decision making under risk have been stripped away) and only a single credit market imperfection is introduced. Consequently, all of the investment impacts that result from the direct payment can be attributed to this single market imperfection. The credit market

[1] The USDA acknowledges the theoretical links between a direct payment and farm output, but finds that these links are empirically insignificant in a survey of U.S. farmers.

fails because farm lenders are assumed to employ a rules-based approach to farm foreclosure (i.e., bankruptcy) decisions. The lenders' rule is to declare the farm bankrupt and seize the farm's assets if the farm becomes insolvent (i.e., farm equity erodes to zero). Assuming a rules-based approach is reasonable because of the comparatively high degree of asymmetric information within a typical agricultural lending relationship and because insolvency is a natural bankruptcy trigger for lenders in more general lending situations.[2]

In this model, the rules-based approach to bankruptcy is a market failure because at the point of insolvency, the expected long-run financial viability of the farmer is still positive. This notion of "premature bankruptcy" has previously been examined by Vercammen (2000). Vercammen showed that premature bankruptcy results in a positive option value associated with the deferral of the investment decision. The larger the farm's debt to asset ratio, the larger the associated option value and the greater the probability the farmer will defer the investment decision. In this paper, Vercammen's finding is used to establish a link between a direct payment and farm investment.[3] A lower debt to asset ratio, which results from a direct payment, reduces the investment option value, which in turn increases the probability that a farmer will make an

[2] A comparatively small number of North American farmers actually exit agriculture each year because of full-blown bankruptcy. One reason for this outcome is that various forms of mediation and settlement mechanisms have been imposed on (or are made available to) the agricultural community. Nevertheless, as long as the farmer suffers some level of financial distress when the business becomes insolvent, the qualitative nature of the results presented here will remain intact.

[3] Unlike Vercammen (2000), option values are not explicitly isolated and examined in this paper. Only part of the incentive to forego an investment with a positive rate of return is due to the option value. Even in a single-period model with no dynamic connections and thus with no option value, a risk-neutral farmer who faces the risk of premature bankruptcy may choose to not make a seemingly profitable investment.

immediate investment. These indirect investment impacts are shown to be surprisingly strong.

The results of this research are consistent with the findings of several related studies. Using a static framwork, Mahul shows that a farmer who faces an external liquidation cost when debt servicing obligations cannot be met, will forego making some risky investments that have a positive short-run rate of return. In a more general setting, Holt and Milne and Robertson use stochastic continuous time optimal control to theoretically examine the dividend and investment decisions of a firm that faces liquidation if cash balances fall to zero. The solution to this type of problem involves "barrier control". Specifically, no investment takes place if the firm's cash balance is below a threshold level that depends on the current stock of capital. If the cash balance rises above this threshold, then investment is increased until the cash balance falls to exactly the threshold level. Certainly a direct payment would increase the expected level of investment in these types of barrier control models.

The dynamic programming model used for this analysis can be described as follows. A risk-neutral farmer chooses whether or not to invest in a single unit of homogenous land for each of T periods (the investment decision is fully irreversible).[4] The farmer's objective is to maximize the expected net worth of the farm as of date T. Farm debt is random over time because farm revenues are random. The value of the farm is also random over time because the random price of farmland is positively correlated

[4] The model is described in terms of investment in land, but the results are applicable to a wide range of agricultural investments including machinery, buildings, livestock inventory and various production technologies. The extent that investment in agricultural land raises farm output at a macro level depends on how the land was being utilized prior to the investment.

with farm revenues. If farm debt rises to the level of farm value at any point prior to date T, then farm bankruptcy occurs and the farmer receives zero net worth at date T. Expected cumulative investment as of date 0 is compared for three separate scenarios: (1) no subsidy; (2) a standard price subsidy for each of the T periods; and (3) a direct payment each period with present value equal to the expected present value of the price subsidy.

In addition to the assumptions detailed above (i.e., risk neutrality, irreversible investment and exogenous consumption), supply response and investment adjustment costs are assumed away. The farm operates with constant returns to scale such that each unit of homogenous land receives the same revenue in a given period, and this revenue is drawn from a distribution that is independent of farm size. Because there is no supply response, the price subsidy (recall scenario 2 above) is really an area payment, whereby each unit of land receives a fixed payment regardless of the level of production. In a more general model, the price of farmland will react to changes in the demand for farmland and will therefore be endogenous. This level of complexity has been assumed away by making the price of farmland an exogenous stochastic process. The price of farmland is assumed positively correlated with revenues and the subsidy payment is assumed subject to different exogenous rates of capitalization, so there is at least some linkage between the price of farmland and the demand for farmland. Although the above assumptions limit the generality of the results, the fact that farm investment is strongly linked to a direct payment in a simple model suggests that this linkage is not likely to readily break down as the various assumptions are relaxed.

The stochastic dynamic programming investment model is constructed and discussed in the next section. Values for the parameters of the simulation model are selected in Section III. The simulation results are presented and discussed in Section IV. Conclusions and a discussion about the direction for future research are contained in Section V.

II. Stochastic Dynamic Programming Model

Beginning at date 0 a risk-neutral farmer chooses whether or not to invest in a unit of homogenous farmland for each of T periods in order to maximize the farm's expected net worth at date T. Off-farm employment income, capital and non-capital farm expense, personal consumption expense and savings external to the farm are the same for each unit of land and are implicitly netted out of farm revenue. Residual farm revenue (positive or negative), which is random over time, is fully used to pay down farm debt (negative debt implies savings). The outstanding principle component of farm debt falls or rises each period, depending on whether residual farm income is greater than or less than the interest owing on current outstanding farm debt. Net worth is equal to the market value of the farmland minus the outstanding principle component of farm debt.

The farmer's cost of borrowing (i.e., the rate of interest) is assumed constant over time. There is no risk premium built into the interest rate because the lender can instantly and costlessly seize and sell the farmer's land if the level of farm debt rises above the value of the land.[5] If the lender does seize the farm's assets prior to time T, the farm's net

[5] This assumption is valid in a continuous time model but is somewhat less suitable for the current discrete time model. With discrete time, farm debt might strictly exceed the value of the farmland if the lender chooses to wait until the farmer is deemed insolvent before seizing and reselling the assets.

worth equals zero at time T. As discussed above, this solvency-based foreclosure rule impacts the farmer's investment decision because, from the perspective of the farmer, foreclosure at the point of insolvency is generally premature.[6] Parameter values are assumed to be such that in the absence of premature foreclosure (or in the absence of risk in general), the farmer would choose to invest during each of the T periods. It is therefore only the risk of premature foreclosure that causes expected cumulative investment to drop below the maximum of T units.

The farmer faces two sources of risk that jointly determine the probability of bankruptcy: variable farm returns and a variable price of farmland. The former affects the temporal variation in debt and the latter affects the temporal variation in the bankruptcy trigger. An ideal specification of the model would have farm returns and the price of farmland cointegrated over time in a standard time series framework. There is a large literature which supports the notion that farm returns and the price of farmland are positively correlated over time, but not to the extent that the classic present value relationship between these two series can be statistically detected (e.g., Featherstone and Baker; Falk; Clark, Fulton and Scott; Falk and Lee). Unfortunately, it is not possible to solve the dynamic programming model if farm returns and the price of farmland are assumed cointegrated without restrictions. As an alternative, assume net farm revenues are repeatedly drawn from a stationary distribution, and the price of farmland evolves over time as a mean reverting stochastic process with the random component of this

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[6] Mean revenues are denoted $\overline{R}$. Farm equity is expected to grow as long as $\overline{R} > rd_t$ where d_t denotes outstanding debt at time t per unit of land and r is the rate of interest. The long run price of land, $\overline{P}$, is equal to $(\overline{R} - \delta)/r$ where δ is a measure of farm profitability. After substituting it can be seen that farm equity is expected to grow as long as $d_t < \overline{P} + \delta/r$. If the current price of land, P_t, is less than $\overline{P} + \delta/r$, then the $d_t = P_t$ foreclosure rule is premature because when $d_t = P_t$, farm equity is expected to grow.

process positively correlated with revenues. The long-run expected price of farmland is set equal to the capitalized value of mean farm returns after allowing for a predetermined profit margin.

Let the price of farmland measured at the beginning of the period evolve according to

(1) $$P_{t+1} = \theta\overline{P} + (1-\theta)P_t + \sigma_P Z_t^P$$

where Z_t^P is a standard normal random variable generated at the end of the period, σ_P is a parameter that governs the variability of the unanticipated change in the price of farmland, $\overline{P}$ is the long-term expected price of farmland and $\theta \in [0, 1]$ is a mean reversion weighting parameter.[7] The initial price of farmland is restricted equal to the long-term expected price by assuming $P_0 = \overline{P}$.

In the absence of any investment in period t, the equation of motion for debt, D_t, measured at the beginning of the period, can be written as

(2) $$D_{t+1}^{no} = (1+r)D_t - (\widetilde{R}_t + s)n_t - w$$

where r is the fixed rate of interest on outstanding debt, $\widetilde{R}_t$ is net revenues, which are drawn at the end of period t from a normal distribution with mean $\overline{R}$ and standard deviation σ_R, n_t is the number of units of land owned by the farmer at the beginning of period t, s is the area payment per unit of land and w is the whole farm direct payment. Both s and w are constant from period to period. The correlation coefficient between Z_t^P from equation (1) and $\widetilde{R}_t$ from equation (2) is denoted $\rho \in (0, 1)$.

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[7] If this model is viewed as an approximation of a continuous time model within which variables such as the price of land evolve over time as Brownian motion, then assuming that the price of land is normally distributed is not unreasonable.

If the farmer chooses to invest at the beginning of period t, then equation (2) becomes

$$D_{t+1}^{yes} = (1+r)(D_t + P_t) - (\widetilde{R}_t + s)(n_t + 1) - w\,. \tag{3}$$

Because it is useful to work with debt per unit of land (denoted d_t), rewrite equations (2) and (3) as

$$d_{t+1}^{no} = (1+r)d_t - (\widetilde{R}_t + s) - w / n_t \tag{4}$$

and

$$d_{t+1}^{yes} = \left(\frac{1+r}{n_t + 1}\right)\left(n_t d_t + P_t\right) - (\widetilde{R}_t + s) - \frac{w}{(n_t + 1)}\,. \tag{5}$$

The problem facing the farmer as of date 0 is to choose whether or not to invest in a unit of farmland for each of the subsequent T periods in order to maximize the expected value of terminal net worth, W(T), where W(T) = n(T)[P(T) – d(T)] if d(t) < P(t) for t ∈ {1, 2, ..., T} and W(T) = 0 otherwise. Assuming $D_0 < P_0$, the state equations describing the evolution of P(t) and d(t) are given by equations (1), (4) and (5). This specification of the problem requires discrete stochastic dynamic programming with numerical procedures to solve. The single control variable is the T period binary investment decision and the three state equations correspond to the price of land, unit debt and farm size.

Probability Transition Matrices

The next task is to construct a pair of probability transition matrices: one matrix corresponds to the case of no investment and the other matrix corresponds to the investment case. Let P^{min} and P^{max} denote the minimum and maximum price of farmland. Let h^p denote the number of discrete intervals between P^{Min} and P^{Max}. Let $P^0 = P^{min}$, $P^i =$

$P^{min} + (i - 0.5)(P^{max} - P^{min})/h^p$ for i$\in$ {1, …, h^p} and $P^{h^p+1} = P^{\max}$. For i$\in$ {1, …, h^p}, price interval i now refers to the price interval that is centered on P^i with width (P^{max} – P^{min})/h^p. Price interval 0 refers to the P^{min} reflecting barrier and price interval P^{h^p+1} refers to the P^{max} reflecting barrier. Intervals for debt are defined analogously.

It is necessary to calculate $PR^{i,j,k,l,n}$ where PR denotes probability, i is the index of the prevailing land price and j is the index of the prevailing unit debt at the beginning of the period, k refers to the land price interval and l refers to the unit debt interval at the end of the period, and n refers to the number of units of land in excess of n_0 held at the beginning of the period. The number of new units of land at the end of the period is equal to n if no investment takes place and n + 1 if investment does take place. $PR^{i,j,k,l,n}$ measures the probability that a farm with n units of land and with price centered in interval i and unit debt centered in interval j will finish the period with price in interval k and unit debt in interval l. This probability calculation must be made for the two alternative cases of with and without investment. The number of individual probability values in each of these two matrices will equal $(h^p + 2)^2(h^d + 2)^2T$.

Let $Z_t^R = (\widetilde{R}_t - \overline{R})/\sigma_R$ denote the standardized net returns variable. Now define

(6)

$$Z_{lower}^{d,l} \equiv \begin{cases} \dfrac{(1+r)d^j - s - \dfrac{w}{n_t} - d_{upper}^l - \overline{R}}{\sigma_R} & \textit{no investment} \\[2ex] \dfrac{\left(\dfrac{1+r}{n_t+1}\right)\left(n_t d^j + P^i\right) - s - \dfrac{w}{n_t+1} - d_{upper}^l - \overline{R}}{\sigma_R} & \textit{with investment} \end{cases}$$

The equation for $Z^{d,l}_{upper}$ is given by equation (6) with d^l_{lower} substituted for d^l_{upper}.[8] The subscripts "lower" and "upper" on the d^l variable indicates that the variable takes on a value equal to the respective endpoint of the d^l debt interval. It is easily shown using equations (4) through (6) that if land price is centered in interval i and unit debt is centered in interval j at the beginning of the period, then the probability that end-of-period debt will lie in interval l is equal to probability that the standardized normal random variable Z^R_t falls in interval $\left[Z^{d,l}_{lower}, Z^{d,l}_{upper}\right]$.

To calculate this latter probability, it is necessary to recognize that Z^P_t from equation (1) and Z^R_t are jointly normally distributed. It is therefore necessary to identify the analogous interval for Z^P_t such that a joint probability can be determined. Using equation (1), let

$$(7) \qquad Z^{P,k}_{lower} \equiv \frac{P^k_{lower} - \theta\overline{P} - (1-\theta)P^i}{\sigma_P} \quad and \quad Z^{P,k}_{upper} \equiv \frac{P^k_{upper} - \theta\overline{P} - (1-\theta)P^i}{\sigma_P}.$$

The probability that land price will fall in interval k and unit debt will fall in interval l given that price and unit debt at the beginning of the period are centered in intervals i and j, respectively, can now be expressed as

$$(8) \qquad PR^{i,j,k,l,n} = \int_{Z^{P,k}_{lower}}^{Z^{P,k}_{upper}} \int_{Z^{R,l}_{lower}}^{Z^{R,l}_{upper}} f(Z^R, Z^P; \rho)\, dZ^R dZ^P$$

where f(·, ·; ρ) is the probability density function for the standard bivariate normal distribution with correlation coefficient, ρ. For all possible values of n, equation (8) can be used to calculate the elements of the without-investment and with-investment

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[8] To deal with the reflecting barriers, set $d^0_{lower} = -\infty$, $d^0_{upper} = d^{\min}$, $d^{h_d+1}_{lower} = d^{\max}$ and $d^{h^d+1}_{upper} = \infty$. Make analogous adjustments for the Z^P variables in equation (7).

probability transition matrices. The procedure for numerically evaluating equation (8) is reported in the Appendix.

Recursive Solution Procedure

Let V_t^n denote a matrix with dimension ($h^P + 2$, $h^d + 2$). The i^{th} row of this matrix corresponds to price interval i and the j^{th} column corresponds to debt interval j. Element (i, j) of matrix V_t^n is a present value measure of expected date T net worth as of the beginning of period t given farm size n, assuming that land price is centered in price interval i and unit debt is centered in debt interval j. Element (i, j) of matrix V_T^n is equal to $(P_T - d_T)n_T$ if $P_T > d_T$ and zero otherwise. Element (i, j) of V_{T-1}^n is given by:

(9) $$V_{T-1}^n(i,j) = \begin{cases} (1+r)^{-1} \sum_{k=0}^{h^P+1} \sum_{l=0}^{h^d+1} PR_{no}^{i,j,k,l,n} V_T^n(k,l) & \textit{with no investment and } P^i \geq d^j \\ (1+r)^{-1} \sum_{k=0}^{h^P+1} \sum_{l=0}^{h^d+1} PR_{yes}^{i,j,k,l,n} V_T^{n+1}(k,l) & \textit{with investment and } P^i \geq d^j \\ 0 & \textit{with } P^i < d^j \end{cases}$$

The subscripts “no” and “yes” on the probability transition matrices within equation (9) identify whether the particular matrix has been derived with or without investment. The bottom component of equation (9) corresponds to farm bankruptcy. Equation (9) shows how recursion can be used to calculate V_{T-2}^n, V_{T-3}^n and so forth until period 0 is reached.

The dynamic programming problem can now be solved by determining the states and time periods for which investment takes place. Let I_t^n denote a matrix with the same dimension as V_t^n. Element (i, j) of this matrix takes on a value of 0 (no investment case) or 1 (investment case). In particular,

$$(10) \qquad I_t^n(i,j) = \begin{cases} 1 \;\; if \;\; V_{t,yes}^n(i,j) > V_{t,no}^n(i,j) \\ 0 \;\; if \;\; V_{t,yes}^n(i,j) \le V_{t,no}^n(i,j) \\ 0 \quad if \quad P^i < d^i \end{cases}$$

Equations (9) and (10) can be jointly used to solve the entire problem recursively, starting with time T – 1 and finishing with time 0.

The variable of particular interest is expected cumulative investment from date 0 to date T as of date 0. Let CI_t^n be identical to I_t^n except the former denotes expected cumulative investment from time t to time T rather than current investment. The recursive formula for calculating element (i, j) of matrix CI_t^n is given by

$$(11) \qquad CI_t^n(i,j) = \begin{cases} \sum_{k=0}^{h^P+1} \sum_{l=0}^{h^d+1} PR_{no}^{i,j,k,l,n} CI_{t+1}^n(k,l) \quad with\, no\, investment \\ \sum_{k=0}^{h^P+1} \sum_{l=0}^{h^d+1} PR_{yes}^{i,j,k,l,n} CI_{t+1}^{n+1}(k,l)+1 \quad with\, investment \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad if\; P^i < d^i \end{cases}$$

To utilize equation (11), note that CI_T^n (i, j) = 0 for all combinations of i and j.

The total reduction in expected cumulative investment as of date 0 is given by T - CI_0^n (i, j) because T is equal to maximum feasible cumulative investment. It is useful to decompose this total reduction into two components: forced reduction due to bankruptcy and voluntary reduction due to implicit risk aversion. The forced reduction component is equal to T less the expected number of periods the farm expects to survive. This latter variable can be calculated by redefining the variables in equation (11) and adding a “1” to the top formula. The difference between the forced and total reduction in expected cumulative investment is equal to the voluntary component of reduced investment. This latter variable is of particular interest in this analysis, especially in the context of how it is impacted by the two types of subsidy schemes.

III. Simulation Model Calibration

The main purpose of this analysis is to determine how the investment impact of an area payment subsidy compares with the investment impact of an equivalent size direct payment subsidy. Equivalent size implies equal expected net present value of the subsidy as of date 0, assuming a constant rate of subsidization over time. To calculate the expected present value of the area payment, use equation (11) with the following modifications: substitute $(n_0 + n_t)s$ for the two expressions in period T, add $(n_0 + n_t)s$ to the first line and replace the "1" with $(n_0 + n_t + 1)s$ in the second line when $t < T$ and finally divide the double summation term by $1 + r$ to ensure discounting. Let $PV_{area}(j)$ denote the expected present value of the area payment as of date 0 given that the date 0 price of land is $\overline{P}$ and date 0 unit debt is centered in interval j.

The direct payment, w, is paid to the farmer regardless of farm size. If

$$(12) \qquad w = PV_{area}\ (j)\left(\frac{r}{1+r}\right)\left(1-(1+r)^{-T}\right)^{-1},$$

then a farmer with initial debt centered in interval j will expect to receive the same cumulative payment with the two different subsidy schemes. It is too complicated to compute equivalent direct payment values for all different values of j. Instead, an equivalent direct payment is calculated for the specific case of j^*. Investment impacts can therefore be meaningfully compared only for $j = j^*$.

Rather than attempting to calibrate the model to a particular real-world scenario, artificial (but realistic) parameter values will be utilized. The aim is to choose a simple set of parameter values for the base case and to then conduct sensitivity analysis to check the robustness of the results. Parameter selection for the base case begins by assuming a twenty year time horizon with a two year decision period (i.e., T = 10). Computer time required to solve the problem and the approximation error due to the discrete nature of the program place an upper limit on the chosen value of T. With a 24 month decision period, it is reasonable to set r = 0.1 (i.e., a 10 percent rate of interest).

With r = 0.1, the land price capitalization formula equals 1/0.1 = 10. Given this result, it is useful to normalize the model by setting average farm revenue, $\overline{R}$, equal to 1 and the long-run expected price of land, $\overline{P}$, equal to 10. However, with $\overline{R}$ = 1 and $\overline{P}$ = 10, the farmer would only just expect to break even on all land purchases. Through trial and error it was discovered that a 5 percent excess return provides a reasonably strong (but not overwhelmingly strong) incentive to purchase land for farmers with a moderate level of initial debt. Thus, $\overline{R}$ = 1.05 and $\overline{P}$ = 10 for the base case.

Now consider values for θ, P^{min}, P^{max} and σ_P. Recall that θ controls the rate of mean reversion in the state equation for the price of land. With $\theta = 0$, land price is a random walk (i.e., there is no mean reversion). With $\theta = 1$, the price of land is equal to $\overline{P}$ plus a random error term (i.e., both the short run and long run expected price do not change over time). The farmer faces a comparatively high risk of premature bankruptcy in the first case and a comparatively low risk in the second case. For the base case, an intermediate position was taken by setting $\theta = 0.5$. The remaining three parameters, P^{min}, P^{max} and σ_P, were chosen simultaneously to achieve moderate land price variability. With $\overline{P} = 10$, $P^{min} = 6$, $P^{max} = 14$ and $\sigma_P = 1$, moderate price risk is achieved and the probability that at least one of the reflecting barriers is reached over a 10 period horizon with a starting price of 10 is less than percent.

It is also necessary to choose appropriate values for d^{min} and d^{max}. Because bankruptcy occurs when $d_t \geq P_t$, it is reasonable to set $d^{max} = 15 \approx P^{max}$. Setting d^{min} involves a tradeoff. On the one hand, setting a large difference between d^{max} and d^{min} minimizes the probability that debt will hit the d^{min} reflecting barrier (in which case the results will be biased). However, a large difference corresponds to relatively wide debt intervals for the probability transition matrix, and the size of the approximation error is positively related to the size of the interval. Through trial and error it was discovered that setting $d^{min} = -15$ results in a reasonable balance between minimizing the degree of bias due to a binding lower reflecting barrier for debt and minimizing the approximation error due to an excessively large debt interval.

The variability in unit debt depends primarily on σ_R, which is the standard deviation of the normal distribution from which farm revenues are drawn. Because $\overline{R}$ = 1.05 in the base case, the chosen value of σ_R approximately represents the coefficient of variation of farm revenues. In order to compensate for the restrictive assumption that farm revenues are stationary over time (which limits the level of temporal variation in farm debt) σ_R was set at a relatively high level (0.75). The other important determinant of risk is the correlation coefficient between farm revenues and the price of land. Although the price of land should not theoretically change with revenue because the distribution from which revenue is drawn from each period is stationary, in reality it is common for farm returns and the price of land to be positively correlated over time. To capture this notion of risk, it was decided that setting $\rho = 0.5$ was reasonable.

Increasing the number of price and debt intervals in the probability transition matrices increases the accuracy of the model, but also rapidly increases the amount of computing time needed to solve the model. Having a larger number of intervals is more important for debt than for land price because debt is an endogenous variable with no natural reflecting barriers, whereas land price is exogenous with reflecting barriers that can be specified prior to solving the model. Through trial and error, it was discovered that a model with 3 intervals for price and 30 intervals for debt solves in about one hour on a standard home computer and results in a reasonably "stable" and "smooth" solution.[9]

For the remaining variables, it was decided that $n_0 = 5$ and $j^* = 21$ such that the farmer begins with 5 units of land and the analysis focuses on a farmer with an initial debt to asset ratio of 45 percent.[10] These values are reasonable for a farmer in his/her early years of expansion. The area payment parameter, s, was set to 0.2 for the base case and for all the different sensitivity scenarios. With $\overline{R} = 1.07$, setting s = 0.2 implies a subsidy rate of about 20 percent. The value of w that ensures an equivalent size direct payment is different for each different set of parameters because the expected cost of the area payment depends on the expected level of investment.

For the base case, it is assumed there is no capitalization of the subsidy into the price of land. That is, values for $\overline{P}$, P^{min} and P^{max} do not change when either the area payment or direct payment is provided to the farmer. With r = 0.1 and s = 0.2, the three price variables will increase by 0.2/0.1 = 2 under the assumption of full capitalization.

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[9] With these values and with T = 10, each of the two probability transition matrices contain 32*32*5*5*10 = 256,000 cells. The program was written and solved using version 6.1 of Matlab. The code for running this program is available upon request.

[10] Unit debt intervals are of size 1 and $d^{min} = d^1 = -15$, so the 20^{th} debt interval ranges from 4.0 to 5.0. With $P_0 = \overline{P} = 10$, the midpoint of the 20^{th} interval corresponds to a debt to asset ratio of 45 percent.

The full capitalization case is not very interesting to consider because in this case the additional incentive for the farmer to invest after receiving the subsidy is very small. In the sensitivity analysis, results are presented for the case of 50 percent capitalization of the area payment and both 50 and 25 percent capitalization of the direct payment.[11] Table 1 summarizes the parameter values for the base case and for the four sensitivity analysis scenarios. Notice that in addition to examining how the results are impacted the capitalization assumption, sensitivity is also determined for revenue variability, the correlation between farm revenues and land price, and the length of the time horizon.

IV. Simulation Results

Figure 1 is a graph of the results for the base case. On the horizontal axis is different levels of the farm's debt to asset ratio at date 0. Ratios in excess of 0.75 are not considered because the bias in the results becomes significant at high levels of debt.[12] The results are plotted as if the initial debt to asset ratio is a continuous variable (this ratio actually increases in discrete jumps of size one).

[11] The capitalization rate for the direct payment should be such that the difference in the level of capitalization for the two types of subsidy schemes is equal to the difference in the increase in demand for land that results from the two types of subsidy schemes. This level of complexity is ignored in this analysis by assuming that the rate of capitalization for the direct payment is equal to either 100 percent or 50 percent of the rate of capitalization of the direct payment.

[12] Recalling the discussion in note #5, r should contain a risk premium that grows with the farm's level of debt. Assuming away this risk premium biases upward the farmer's incentive to invest, especially at high levels of debt.

The vertical axis is a measure of expected cumulative investment (measured in units of land) and expected survival of the farm (measured in years).[13] There are two lines in Figure 1 for the case of no subsidy. The solid line reflects the total reduction in investment that results from premature foreclosure and the dashed line reflects the forced reduction in investment that is due to bankruptcy. The vertical difference between the solid and dashed no-subsidy lines, which is quite large for this base case, is a measure of the voluntary reduction in investment due to the farmer's implicit aversion toward premature foreclosure.

The other two lines in Figure 1 correspond to expected cumulative investment with a 20 percent area payment and with a calibrated direct payment. The two subsidy schemes have equal expected present value when the initial debt to equity ratio is equal to 0.45 (i.e., the location of the vertical dotted line).[14] The vertical difference between each of these lines and the solid no-subsidy line is a measure of the extent that the respective subsidy has raised investment. Notice that with an initial debt to equity ratio of 0.45, the area subsidy has raised cumulative investment from about 6.2 to 8.4 whereas the direct payment has raised investment from about 6.2 to 8. The ratio of these differences is equal to (8.0 – 6.2)/(8.4 – 6.2) = 0.818. In other words, the increase in investment stemming from the direct payment is about 82 percent as large as the increase in investment stemming from the area payment. At lower levels of debt, the difference in the investment impact for the two types of subsidies is smaller, despite the fact that the

[13] Land units and years can be plotted on the same axis because the farmer is restricted to purchasing either no land or one unit of land per year.

[14] The value of the direct payment is smaller than the value of the area payment to the left of the vertical dotted line because investment (and thus the expected size of the area payment) is comparatively large at low levels of debt. The opposite is true to the right of the vertical dotted line.

expected value of the direct payment is less than the expected value of the area payment (see note 14).

The reason for these rather strong results is not immediately obvious because there are several interdependent forces at work. First, the area payment raises the direct marginal incentive to invest when $P_t < \overline{R}/r$ whereas there is no direct marginal incentive effect attached to the direct payment. Second, both types of subsidies lower the farm's debt to asset ratio, which in turn lowers the probability of future bankruptcy. A lower probability of bankruptcy implies a lower option value associated with a deferral of the investment decision and thus a higher probability of immediate investment. It is uncertain if the two types of subsidies have a differential impact on this option value effect. Third, as in the standard Dixit and Pindyck model of investment with uncertainty and irreversibility, the farmer may also have an incentive to defer the investment decision because of randomness in the price of land. The extent to which this option value is impacted differently by the two types of subsidies is unknown.

Figures 2 through 5 illustrate the results for the four alternative sensitivity scenarios. In Figure 2, the issue of subsidy capitalization is considered. As is expected, 50 percent capitalization of the area payment implies an overall lower level of cumulative investment when the area payment is provided to a farmer (i.e., the vertical difference between the two dark shaded schedules is less than in the base case). Figure 2 shows that the investment impact of a 50 percent capitalized direct payment is quite small. Given this result, it might be more reasonable to assume a 25 percent capitalization rate for the direct payment when the capitalization rate for the area payment is 50 percent. In this case (also illustrated in Figure 2), the investment impacts of the direct payment and the area payment are nearly indistinguishable. These results demonstrate that the comparative impact of a direct payment on investment appears to be highly dependent on the degree of subsidy capitalization.

The sensitivity of the results with respect to risk is illustrated in Figures 3 and 4. In Figure 3, the standard deviation of revenue is higher and in Figure 4 there is a stronger positive correlation between revenue and land price. Figure 3 looks quite similar to the base case in Figure 1, which implies that the investment results are not very sensitive to the standard deviation of revenue variable. This result stems from the assumption that revenues are stationary over time versus a more risky stochastic process such as mean reversion. In Figure 4, the higher level of risk has shifted all of the schedules down by a modest amount. The most interesting aspect of Figure 4 is that the stronger correlation between revenue and land price has made the investment impacts of the two types of subsidies virtually indistinguishable.

The last sensitivity result to be explored is with respect to the farmer's time horizon (Figure 5). Increasing the farmer's horizon from 20 to 30 years (T = 15 versus T = 10) narrows the difference in the level of cumulative investment for the two alternative investment schemes to a very small level. More research is necessary to provide a explanation for this result.

V. Conclusions

The dynamic programming model used to generate these results is very simplistic in that many important features of farm investment have been assumed away. Moreover, the knife-edge foreclosure rule that underlies this model does not accurately reflect the process of farm bankruptcy in the real world. Nevertheless, given the strength of the results presented here, it is likely that even in a more realistic investment environment, the link between a direct payment and farm investment is likely to be significant. Suggesting that a farm manager, who is cautiously investing in agricultural land because of concerns about bankruptcy, will invest more aggressively upon receipt of a direct payment is also likely to receive considerable anecdotal support.

An interesting feature of this model is that providing the farmer with either type of subsidy is welfare enhancing. In the absence of a subsidy, the level of investment is below the first best level because of a market failure (i.e., premature bankruptcy). The subsidy raises the investment toward the first best level and it must therefore raise overall welfare, unless the transaction cost of distributing the subsidy to the farmer is excessive. This aspect of farm subsidies in a second best environment arises in many different situations, even though it is seldom acknowledged in multilateral discussions about the need to reduce farm subsidies.

There are many ways the model can be improved. It would be interesting to determine the extent that the link between a direct payment and farm investment would disappear if the irreversible investment assumption is relaxed. Similarly, if other determinants of net farm income (e.g., off-farm income and consumption expense) were made endogenous within the model, then the probability of bankruptcy would be reduced because the farmer would have an implicit mechanism for stabilizing income. The model is also lacking an appropriate specification of supply response. The interaction between supply response and investment may have important implications for the results presented here. Probably the easiest way to improve the model is to increase the accuracy of the results in exchange for longer computation time by increasing the number of price and debt intervals in the probability transition matrix.

Finally, the model could be calibrated more closely to a real world investment environment and the various results could be subjected to empirical verification. For example, the model predicts that in the absence of a subsidy, farm investment is lower the higher the level of revenue and price variability and the higher the debt to asset ratio. These are testable hypotheses that could be examined using secondary data. If farm investment can only be weakly linked to these two variables, then it is unlikely that a direct payment will have a significant impact on farm investment.

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Appendix

The purpose of this appendix is to explain the procedure for numerically evaluating equation (8) in the text. The procedure is based on Mee and Owen's (1983) approximation formula for the cumulative density function for a standardized bivariate normal distribution. The general problem is to evaluate

$$\text{(A.1)} \quad PR = \int_{-\infty}^{a} \int_{-\infty}^{b} f(x, y; \rho) dxdy$$

where f(x, y; ρ) is the standardized bivariate normal density function. There are two cases to consider. First, suppose the absolute value of a equals or exceeds the absolute value of b. In this case, if $a \leq 0$ then set $\mu = -\rho\phi(a)/\Phi(a)$ and $\sigma = (1 + \rho a\mu - \mu^2)^{0.5}$ where $\phi(\cdot)$ is the probability density function for a univariate standard normal random variable and $\Phi(\cdot)$ is the associated cumulative density function. Now, $PR \approx \Phi(a)\Phi((b - \mu)/\sigma)$. For the case where $a > 0$, then $\mu = \rho\phi(-a)/\Phi(-a)$, σ takes on the same expression as the previous case and $PR \approx \Phi(b)-\phi(-a)\Phi((b - \mu)/\sigma)$. Now suppose the absolute value of a is less than the absolute value of b. In this case, if $b \leq 0$ then $\mu = -\rho\phi(b)/\Phi(b)$ and $\sigma = (1 + \rho b\mu - \mu^2)^{0.5}$ and $PR \approx \Phi(b)\Phi((a - \mu)/\sigma)$. If $b > 0$ then $\mu = \rho\phi(-b)/\Phi(-b)$, σ takes on the same expression as in the previous case and $PR \approx \Phi(a) -\phi(-b)\Phi((a - \mu)/\sigma)$.

Equation (A.1) can be used to evaluate the more general expression

$$\text{(A.2)} \quad PR = \int_{a_L}^{a_H} \int_{b_L}^{b_H} f(x, y; \rho) dxdy$$

by recognizing that

$$
\text{(A.3)} \quad \begin{aligned}
\int_{a_L}^{a_H}\int_{b_L}^{b_H} f(x,y;\rho)dxdy &= \int_{-\infty}^{a_H}\int_{-\infty}^{b_H} f(x,y;\rho)dxdy - \int_{-\infty}^{a_L}\int_{-\infty}^{b_H} f(x,y;\rho)dxdy \\
&\quad - \int_{-\infty}^{a_H}\int_{-\infty}^{b_L} f(x,y;\rho)dxdy + \int_{-\infty}^{a_L}\int_{-\infty}^{b_L} f(x,y;\rho)dxdy
\end{aligned}.
$$

The approximation implied by equations (A.1) and (A.3) is remarkably accurate. The results are generally accurate to no less than two digits to the right of the decimal within the PR result. Accuracy is maximized when ρ falls in the interval [-0.5, 0.5].

Table 1: Parameters Settings for Base Case and Sensitivity Analysis

Base Case	$P^{min} = 6$, $P^{max} = 16$, $\overline{P} = 10$, $\theta = 0.5$, $d^{min} = -15$, $d^{max} = 15$, $h^P = 3$, $h^d = 30$, $\overline{R} = 1.05$, $\sigma_P = 1$, $\sigma_R = 0.75$, $\rho = 0.5$, $r = 0.1$, $T = 10$, $n_0 = 5$, $j^* = 20$, $(s, w)^1 = \{0, 0\}$, $(s, w)^2 = \{0.2, 0\}$, $(s, w)^3 = \{0, w(j^*) = 2.046\}$
Subsidy Capitalization	base case parameters except $P^{min} = 6.5$ (7), $P^{max} = 14.5$ (15), $\overline{P} = 10.5$ (11) and $w(j^*) = 2.0571$ (2.039) with 25% (50%) capitalization
High Revenue Variability	base case parameters except $\sigma_R = 1$, $w(j^*) = 1.9685$
High Correlation	base case parameters except $\rho = 0.75$, $w(j^*) = 2.039$
Long Time Frame	base case parameters except $T = 15$, $w(j^*) = 2.2827$

Note: $w(j^*=20)$ ensures that expected present value of area payment with $s = 0.2$ is equal to present value of direct payment for farmer with initial debt, $d_0(20) = 4.50$.

Figure 1: Base Case Results

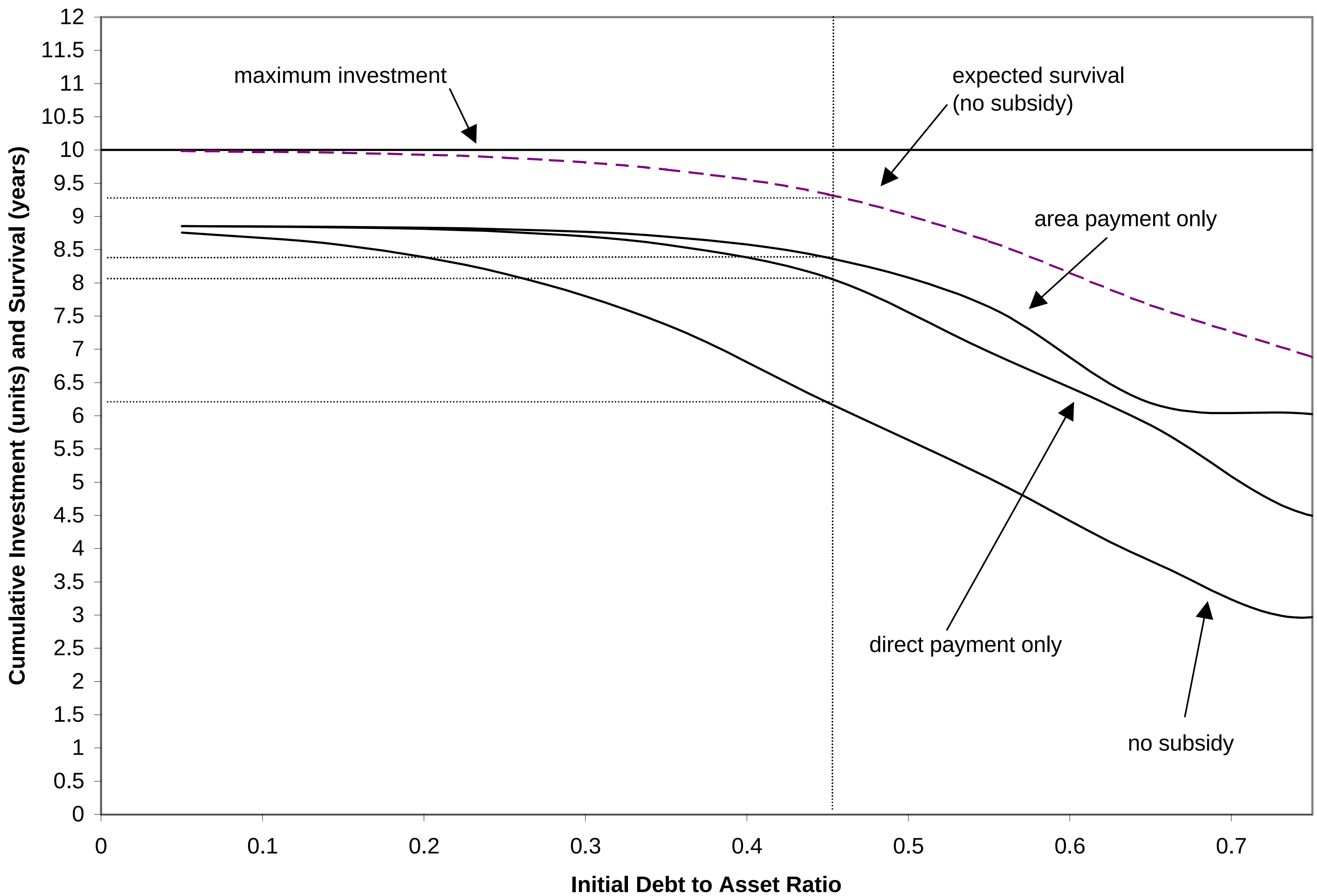

Figure 2: Capitalization Sensitivity Results

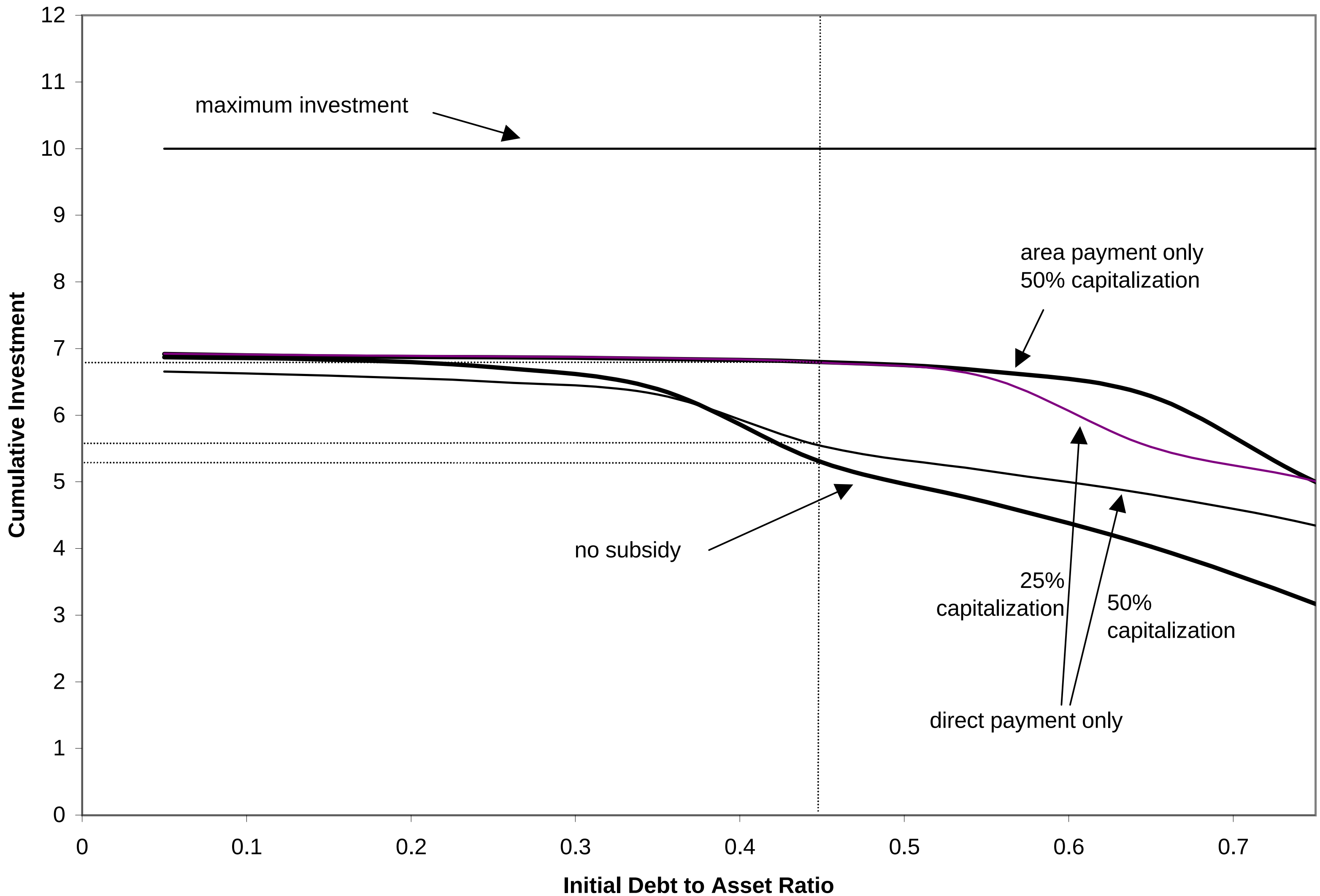

Figure 3: High Revenue Risk Sensitivity Results

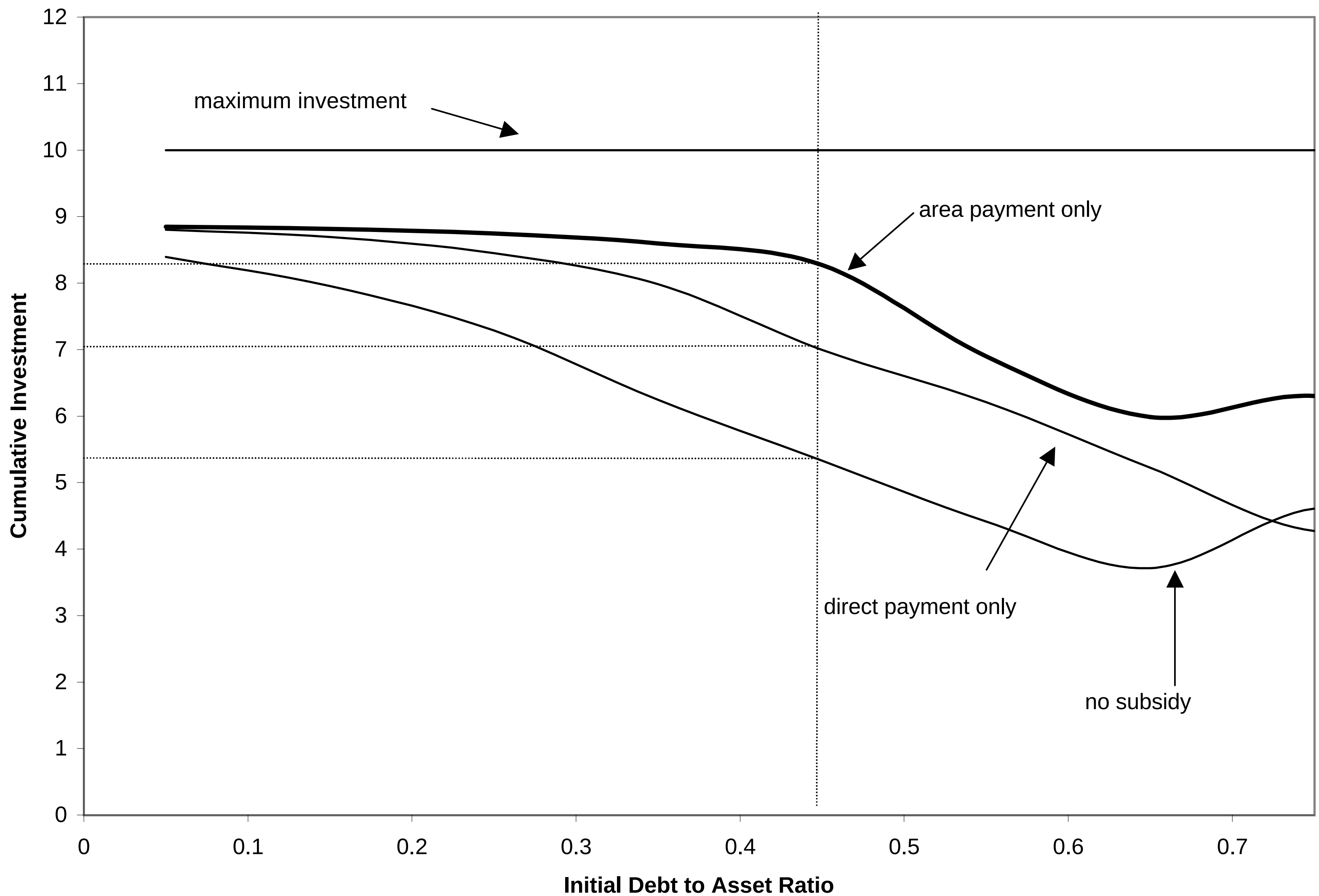

Figure 4: High Revenue-Price Correlation Sensitivity Results

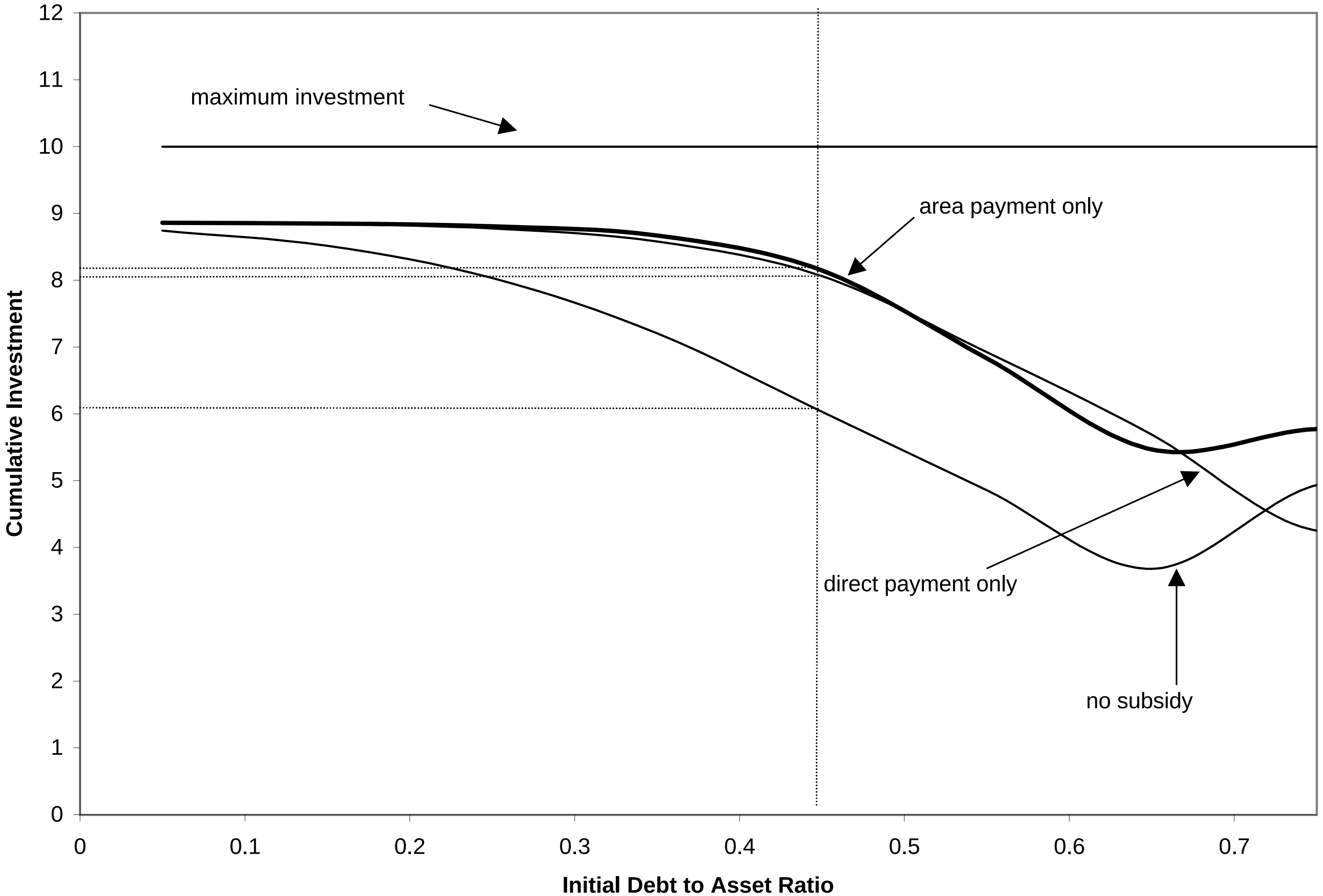

Figure 5: Long Time Horizon Sensitivity Results