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Optimal Dynamic Management of Agricultural Land-Uses: An Application of Regime Switching

Graeme J. Doole and Greg L. Hertzler

The capacity of global agricultural production to meet increased demand for food from population growth and wealth accumulation is threatened by extensive land degradation. Nonetheless, previous research has focused primarily on the dynamic implications of input management and ignored land-use choice. This paper extends this theory through an examination of the intertemporal management of agricultural land through the use of non-crop inputs, such as fertilizer, and land uses that either degrade or restore productivity. The need to consider the relative total asset value of alternative crops over time is demonstrated. Moreover, higher output prices for degrading crops are shown to increase their relative value, motivating the later adoption of substitutes. An inability of land markets to reflect differences in resource quality and low capital malleability promote greater degradation. However, substitution of complementary effects through input use may help to sustain productivity. These factors are discussed in the context of crop sequence management in Western Australian cropping systems.

Key Words: crop sequences, land degradation, regime switching

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Global agricultural production must increase by around 40 per cent over the next 20 years if increased demand due to population growth and wealth accumulation is to be satisfied (Organization for Economic Co-operation and Development and the Food and Agriculture Organization of the UN (OECD-FAO), 2009). However, this is seriously constrained by extensive land degradation, particularly in developing countries, that directly decreases primary productivity (Bruinsma, 2009). An improved understanding of the efficient management of land-use sequences can help to offset these constraints, as rotation with crops and pastures that have a beneficial impact on the inherent productivity of a soil has been used for centuries to sustain or increase the yields of agricultural plants (Doole and Pannell, 2009).

The beneficial impacts of land-use sequences can be classified as either direct or indirect. Direct benefits are the lowering of risk and the smoothing of input demand. Indirect benefits are those that influence profit by increasing the production of subsequent crops (Hennessy, 2006). Examples are the interruption of pest and disease

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cycles, the reduction of soil erosion, nitrogen fixation by legumes, enhanced soil structure, management of crop residues, and weed management (Doole and Pannell, 2009). Aggregation of indirect benefits provides an indication of the productivity of the land resource; this is analogous to a form of capital stock that partly determines crop production.

McConnell (1983) formalized the relationship between capital theory and the base productivity of agricultural land. In this model, conservation reduced current production and degradation could not be offset through the addition of non-soil inputs, such as fertilizer (Barrett, 1991; McConnell, 1983). Barbier (1990) extended this framework to incorporate "productive" and "ameliorative" inputs. The first increased both output and soil loss, for example deep cultivation. The second decreased erosion but did not affect crop production directly, for example the construction of terraces. Determining the optimal usage of the former is similar to renewable resource exploitation, while the latter resembles traditional investment theory (Clarke, 1992; LaFrance, 1992). Links to capital theory are even stronger when ameliorative practices enter as stock variables and investment therefore has a lasting impact on conservation (Grepperud, 1997).

The single crop approach adopted in these papers disregards complementary effects between agricultural practices. However, these can often be important in reality (Orazem and Miranowski, 1994). Indirect effects may be incorporated by identifying the optimal allocation of a given area of land among crop and non-crop land uses, such as pasture, at each point in time (Goetz, 1997). However, aggregation of the impacts of land use on productivity across an entire farm provides a coarse approximation of their value. A more precise examination requires analysis at the field level, particularly since spatial heterogeneity in land quality is a characteristic of many agricultural systems. In this case, it is appropriate to analyze crops as discrete choices rather than proportions.

Conceptual analyses of the optimal rotation between land uses that degrade or restore land quality have been formulated. Hertzler (1990) analyzed a multiple crop system through the inclusion of time as a state variable. However, non-crop inputs and costs incurred at the transition between individual crops (switching costs) were not incorporated, and both are shown here to be an important component of the switching decision. Willassen (2004), in comparison, included switching costs in an analysis of the fallow-cultivation cycle of traditional agriculture. Nonetheless, non-crop inputs, such as fertilizer or herbicide, were omitted. This paper extends this literature through the inclusion of both multiple land uses and non-crop inputs in a model incorporating transition costs. Key findings are discussed in relation to single crop models and the management of crop sequences in Western Australian agricultural systems.

The switching problem and necessary conditions for an optimal solution are presented in the next section. The model is based on the general framework of Doole (2009). Implications for optimal land management under single and multiple crops are outlined in the third and fourth sections, respectively. Conclusions are presented in the final section.

A Regime Switching Model of Agricultural Land Management

This section contains a description of an optimal switching model for the analysis of multiple crops and presents necessary conditions for its solution. Assume that a producer must determine the most profitable use of a field between t_0 and t_2. A given enterprise, $I = 1$, is active at the outset. However, the farmer may decide to switch to a successive regime, $I = 2$, at any time, t_1, between these endpoints. The moment before the switch occurs is denoted t_1^- and the moment after the switch has occurred is t_1^+. Regime i is therefore active over the closed interval $t = \left[t_i^+, t_{i+1}\right]$ where $t_0 \leq t_1 \leq t_2$. The framework could incorporate n switches but only one is incorporated here for clarity of exposition. Although an abstraction, the number of switching moments must be determined *ex ante* since the endogenous determination of total switching moments in multiple-phase control problems incorporating switching costs has proven intractable.

It is assumed that the quality of a fixed area of land in terms of agricultural production may be described by a composite index denoted by a single state variable, $x(t)$. Investment in land capital is represented by an increase in this index, while disinvestment causes a decline.

Productivity may be manipulated through the intensity of management inputs, the control variable denoted as $u_i(t)$. Controls are defined as continuous functions for generality but in most contexts are likely to be discrete practices. Subscription by regime index permits the set of management inputs (U_i, where $u_i(t) \in U_i$) to differ for each land use. For example, a herbicide may control weeds effectively in crop 1 but harm crop 2 significantly. In this case, U_1 may contain this herbicide but U_2 would not.

Productivity is also influenced through crop choice. Rates of degradation and renewal for each land use i are described through motion functions, $f_i(x(t),u_i(t))$. The units of measurement for the motion functions will depend on the definition of the state variable. A regime may be either degrading ($f_{\deg}(\cdot) < 0$) or restoring ($f_{res}(\cdot) > 0$) of land quality. Examples are wheat crops that degrade soil structure and pasture legumes that restore soil nitrogen and organic matter. The case where land is not affected through crop choice is ignored to focus on situations of practical relevance.

It is assumed that the use of an input has a net increasing impact on crop yield. Application increases base productivity, therefore $[f_i(\cdot)]_u > 0$, where $[\cdot]_u$ denotes the derivative of the function in square brackets with respect to (w.r.t) the subscripted term. An increase in productivity following input use will augment crop yield, $y_i(x(t))$, through the intuitive assumption, $[y_i(\cdot)]_x > 0$. An example is nitrogen fertilizer that increases crop yield through increasing soil nitrogen. Other types of input may be more applicable to certain problems. For example, "productive" inputs may be used to investigate the management of systems where practices, such as deep cultivation, increase output but degrade base productivity. These may be easily incorporated in this framework with adjustment of the relevant relationships. The critical difference between productive inputs and those analyzed in this paper is that the latter allow intensification to occur without degradation.

A state vector could represent base productivity in place of a composite index. This vector would contain a number of individual determinants of production, each with its own motion equation and associated control set. For example, state-transition equations representing a weed population and total soil nitrogen could be included. The control set for the former could incorporate different intensities of herbicide, while that for the latter could involve alternative levels of nitrogen fertilizer. The use of a composite index is retained for broader relevance and to avoid problems associated with dimensionality.

The initial level of land quality is denoted $x(t_0) = x_0$. The state trajectory is determined by:

$$\dot{x}(t) = f_i(x(t),u_i(t)), \tag{1}$$

for $I = \{1,2\}$. This is continuous but non-differentiable at the switching time, t_1.

A continuous profit function $\pi_i(x(t),u_i(t),t)$ is defined for each regime i:

$$\pi_i(x(t),u_i(t),t) = \int_{t_{i-1}^{+}}^{t_i^{-}} e^{-\delta t}(p_i y_i(x(t)) - c_i(x(t),u_i(t)))dt, \tag{2}$$

where $e^{-\delta t}$ is a discount factor with δ as the discount rate, p_i is the price per unit of output, $y_i(x(t))$ is output, and $c_i(x(t)), u_i(t))$ is the cost of inputs. As defined earlier, $[y_i(\cdot)]_x > 0$. Costs are assumed to increase as land quality declines due to decreases in the effectiveness of inputs. For example, more expensive cultural treatments are required when a herbicide-resistant weed population develops (Doole and Weetman, 2009). Therefore, $[c_i(\cdot)]_x < 0$. However, this assumption may be relaxed with little effect on the following discussion. In addition, inputs are costly, so $[c_i(\cdot)]_u > 0$.

Land has a salvage value defined through the function $e^{-\delta t_2^-}h(x(t_2^-))$. This is assumed to increase with the productivity of land, so $[e^{-\delta t_2^-}h(x(t_2^-))]_x > 0$. Moving from one land use to another incurs a switching cost, $e^{-\delta t_1^-}s(x(t_1^-))$. This increases with declining land quality, therefore $[e^{-\delta t_1^-}s(x(t_1^-))]_x < 0$. An example is pasture establishment for which costs increase as weed populations burgeon (Doole, 2009). The latter assumption may not be relevant for certain problems, in which case it may be disregarded with little implication for the main argument.

The producer's problem for $I = \{1,2\}$ is:

$$(3)\quad \max_{u_i,t_i} J = \int_{t_0^+}^{t_1^-} \pi_1(x(t),u_1(t),t)dt - e^{-\delta t_1^-} s(x(t_1^-)) + \int_{t_1^+}^{t_2^-} \pi_2(x(t),u_2(t),t)dt + e^{-\delta t_2^-} h(x(t_2^-)),$$

subject to,

$$(4)\quad \dot{x}(t) = f_i(x(t),u_i(t)), \text{ and,}$$

$$(5)\quad x(t_0) = x_0.$$

This problem incorporates two standard free-time optimal control problems with terminal value functions. Solution is complicated because the management of the first regime influences the second through the state variable and the optimal switching time must be endogenously determined. Necessary conditions for the solution of a similar model have been derived (Amit, 1986). However, that formulation does not include a terminal value function. Solution therefore requires Theorem 1.

Theorem 1 (Doole, 2009). Let $(x^*(t),u_i^*(t),t_i^*)$ for $I = \{1,2\}$ denote the trajectory that maximizes J in Equation (3) subject to the constraints in Equation (4) and Equation (5). This is the *optimal trajectory*. A Hamiltonian function for each regime i is defined as:

$$(6)\quad H_i(x(t),u_i(t),\lambda_i(t),t) = \pi_i(x(t),u_i(t),t) + \lambda_i(t)f_i(x(t),u_i(t),t).$$

Under the optimal trajectory there exists a vector of piecewise continuous adjoint functions, $\lambda = [\lambda_1(t), \lambda_2(t)]$, that each satisfies, over the relevant closed interval $t = \left[t_{i-1}^+, t_i^-\right]$:

$$(7)\quad \dot{\lambda}_i(t) = -\frac{\partial H_i(x(t),u_i(t),\lambda_i(t),t)}{\partial x(t)}.$$

The optimal control function within each land use i must obey:

$$(8)\quad \frac{\partial H_i(x(t),u_i(t),\lambda_i(t),t)}{\partial u_i(t)} = 0.$$

The following conditions must be satisfied at the final time:

$$(9)\quad H_2(x(t),u_2(t),\lambda_2(t),t)\Big|_{t_2^-} + \frac{\partial e^{-\delta t_2^-} h(x(t_2^-))}{\partial t_2^-} = 0, \text{ and,}$$

$$(10)\quad \lambda_2(t_2^-) = \frac{\partial e^{-\delta t_2^-} h(x(t_2^-))}{\partial x(t_2^-)}.$$

The adjoint variables must satisfy, at the switching time t_1:

$$(11)\quad \lambda_1(t_1^-) + \frac{\partial e^{-\delta t_1^-} s(x(t_1^-))}{\partial x(t_1^-)} = \lambda_2(t_1^+).$$

The Hamiltonian functions for each regime, at switching time t_1, must obey:

$$(12)\quad H_1(x(t),u_1(t),\lambda_1(t),t)\Big|_{t_1^-} - \frac{\partial s(x(t_1^-),t_1^-)}{\partial t_1} = H_2(x(t),u_2(t),\lambda_2(t),t)\Big|_{t_2^+},$$

for $t_0^- < t_1^- < t_2^-$.

In addition:

$$(13)\quad H_1(x(t),u_1(t),\lambda_1(t),t)\Big|_{t_1^-}. - \frac{\partial s(x(t_1^-),t_1^-)}{\partial t_1} \le H_2(x(t),u_2(t),\lambda_2(t),t)\Big|_{t_2^+},$$

for $t_0^- = t_1^- < t_2^-$, and,

$$(14)\quad H_1(x(t),u_1(t),\lambda_1(t),t)\Big|_{t_1^-} - \frac{\partial s(x(t_1^-),t_1^-)}{\partial t_1} \ge H_2(x(t),u_2(t),\lambda_2(t),t)\Big|_{t_2^+},$$

for $t_0^- < t_1^- = t_2^-$.

Conditions in Equation (6) to Equation (10) are consistent with the solution of a standard free-time optimal control problem with a salvage value. Equations (11)–(14) are not. These collectively specify the relationships that must hold at the switching time. Equation (11) states that it is optimal to switch from one agricultural practice to another when the marginal value of renewal or degradation matches that within the next regime. (The level of base productivity at which this occurs in this paper is referred to as the "switching state" throughout.) Similarly, Equation (12) outlines that it is beneficial to switch to the successive regime when it is more profitable to do so and incur switching costs than remain in the active land use. The expression in Equation (13) states that the first regime should never be active if its dynamic value is dominated by the successive enterprise at all potential switching moments. In a similar vein, Equation (14) describes that the second enterprise should not be utilized if its capital value never matches that within the first regime.

Optimal Management of Individual Crops

This section examines the optimal management of agricultural land within individual regimes. This outlines the implications of salvage value for effective stewardship and provides a foundation for the discussion of switching dynamics that follows.

The Hamiltonian function for each regime i is:

$$H_i(x(t),u_i(t),\lambda_i(t),t) = e^{-\delta t}(p_i y_i(x(t)) - c_i(x(t),u_i(t))) + \lambda_i(t) f_i(x(t),u_i(t)). \quad (15)$$

The Hamiltonian function ($H_i(\cdot)$) represents the total capital value of regime i and consists of two terms. The first is discounted profit. The second is the user benefit or user cost associated with current management, $\lambda_i(t)\, f_i(\cdot)$. This is the total gain or loss in future profit from time t to the end of the regime following an increase or decrease in base productivity. User benefit/cost involves two terms. The shadow price of renewal or degradation ($\lambda_i(t)$) reflects the value of a unit change in base productivity at time t in terms of profit earned over the remainder of the regime's duration. The second term is the unit change in land quality, as defined by the motion function.

Together with the state Equation (4) and the initial condition Equation (5), optimal trajectories within a given land use must satisfy:

$$\frac{\partial H_i(\cdot)}{\partial u_i(t)} = -e^{-\delta t}[c(x(t),u_i(t))]_u + \lambda_i(t)[f_i(x(t),u_i(t))]_u = 0, \text{ and,} \quad (16)$$

$$\dot{\lambda}_i(t) = -\frac{\partial H_i(\cdot)}{\partial x(t)} = -e^{-\delta t}\left(p_i[y_i(x(t))]_x - [c_i(x(t),u_i(t))]_x\right) - \lambda_i(t)[f_i(\cdot)]_x. \quad (17)$$

The first equation, Equation (16), identifies that inputs will be used up to the point where their marginal cost ($c(\cdot)]_u$) is equal to their marginal benefit ($\lambda_i(t)[f_i(\cdot)]_u$). Marginal benefit consists of the physical relationship between input application and the rate of degradation/renewal ($[f_i(\cdot)]_u$) multiplied by the marginal value of this change in base productivity ($\lambda_i(t)$). This specification contrasts that presented with the inclusion of "productive" inputs that enter the production function of crops directly (Barbier, 1990; Clarke, 1992; LaFrance, 1992). In that case, the marginal benefit of input use is marginal value product ($p_i[y_i(x(t), u_i(t))]_u$, where $[y_i(\cdot)]_u > 0$), the value of marginal output accruing to input application.

The second equation, Equation 17, identifies that the rate of depreciation/appreciation of land capital is the total of its marginal contribution to direct profits and capital investment under optimal management. Greater insight can be gained through focusing on the dynamics of the second regime. Integration of Equation (15) for this land-use yields:

$$\lambda_2(t) = e^{-\delta s}\int_{\varsigma}^{t_2} e^{-\alpha}\left(p_2(\cdot)[y_2(\cdot)]_x - [c_2(\cdot)]_x\right)dt + e^{-\alpha}[h(x(t_2))]_x, \quad (18)$$

where $\alpha = (\delta + [f_2(\cdot)]_x)(t - \varsigma)$. Equation (18) identifies that the shadow price of a change in base productivity at time ς ($\varsigma > t_1$) is the present value of the marginal profit earned between the present and the terminal time. Marginal profit is discounted by δ. In addition, it is discounted or compounded by the rate, $[f_i(\cdot)]_x$, at which degradation or renewal change with land quality.

The rate of degradation under a given crop may increase as land quality declines. For instance, soil loss accelerates at increased depth because low organic matter reduces the binding of aggregates (Goetz, 1997). This implies $[f_i(\cdot)]_x < 0$ and is analogous to a greater discount rate. Moreover, the rate of renewal is likely to decrease as land quality improves because of diminishing marginal returns. Counter-examples do exist. For example, it is generally easier for crop or pasture plants to compete with lower weed populations. However, diminishing marginal returns to renewal are likely to effectively bound improvements in yield (and consequently profit) at some threshold. Therefore, $[f_i(\cdot)]_x < 0$ is a more realistic assumption.

Degradation ($\dot{x}(t)<0$) reduces profit in the second regime through decreasing yield ($[y_2(\cdot)]_x > 0$), increasing input costs ($[c_2(\cdot)]_x > 0$), and imposing user costs ($\lambda_2(t)f_2(\cdot) < 0$). A lower shadow price implies a decrease in the future profitability of this regime. This reduces user cost and thus encourages more intensive resource use. The causes of such a reduction are apparent from Equation (18). The discount rate represents the opportunity cost of capital. Higher returns elsewhere in the economy therefore motivate degradation (McConnell, 1983). As noted earlier, degradation rates may increase as land quality declines. This will promote exploitation so that discounting has a lesser effect on the profits accruing to degradation. Declines in marginal profit, such as those brought about by lower prices, will also decrease the shadow price.

If the terminal regime restores agricultural productivity ($\dot{x}(t) > 0$) then $\lambda_2(t)f_2(\cdot)$ instead represents a user benefit. A lower shadow price ($\lambda_2(t)$) will decrease the magnitude of user benefit and therefore decrease the incentive to retain this land use. In line with the results for a phase that degrades the resource base, this also occurs with a higher discount rate, a rate of renewal that declines with increased productivity, and lower marginal profit. However, there is greater incentive to retain such an enterprise if rates of renewal are augmented with increasing land quality, i.e., $[f_i(\cdot)]_x > 0$.

The producer will sell the land at the point where continuing farming of the last regime is unprofitable. Here the following relationship, consistent with Equation (9), holds:

$$H_2(\cdot)\Big|_{t_2^-} - \delta e^{-\delta t_2^-} h(x(t_2^-)) + e^{-\delta t_2^-}\left[h(x(t_2^-))\right]_x \dot{x}(t) = 0. \tag{19}$$

The first term (the Hamiltonian function for the second regime evaluated at t_2) represents the marginal value of extending the length of the final regime. The second and third terms are the rate at which the discounted salvage value of the farm changes with adjustment of the terminal time. The sum of these three terms must be zero at the optimal time of sale; otherwise it is profitable to continue management.

Prolonging the planning horizon will have two effects on the resale value of the farm. These are reflected in the second and third terms in Equation (19). First, salvage value will decrease through discounting. Second, it will decrease (increase) with retention of the last regime if this regime degrades (restores) land quality. However, the last factor declines in importance with decreases in the degree to which land markets reflect differences in productivity.

The price of agricultural land should reflect its expected long-term profitability under perfect information (Just and Miranowski, 1993). Optimal management consequently requires explicit consideration of current actions on the resale value of the farm (see Equation (18) and Equation (19)). However, this may not occur if there are information failures, notions of bequest are weak, or capital markets do not clear (McConnell, 1983). Incentives for conservation will be reduced if land markets do not properly account for differences in productivity (Clarke, 1992; Goetz, 1997). Suboptimal levels of exploitation will consequently be utilized, the degree of disinvestment in land capital depending on the extent to which the salvage value term is sensitive to degradation of the base resource. This will be highest in the extreme case where $h(x(t_2))$ is independent of land quality as producers will have no incentive to conserve the land resource in order to obtain a higher terminal value.

Optimal Management of Multiple Crops

This section focuses on the analysis of crop sequences utilizing the framework described in the second section. Determining the optimal rate of exploitation or investment across the relevant planning horizon is the sole consideration if a single crop exists. However, intertemporal planning requires simultaneous consideration of the relative (potential) profitability of the successive regime if a planting alternative is available.

Standard Switching Behavior under Degradation and Renewal

Suppose that the optimal switching time is freely variable $\left(t_{i-1}^- < t_i^- < t_{i+1}^-\right)$ and although it is presently more profitable to remain in the first enterprise, it will be optimal to switch to the successive regime at some stage. Prior to switching, the Left Hand Sides (LHSs) of the switching conditions in Equation 11 and Equation 12 will be greater than the Right Hand Sides.

Over time, any change in the level of the state variable under the active regime will modify both its own marginal value and that of the next regime. Adjustment will continue until the switching conditions hold with equality, beyond which it is more profitable to switch than remain in the first regime.

Detailed analysis is possible from manipulation of system equations and boundary conditions from Theorem 1. Current values are used here in order to simplify discussion. The necessary conditions for the first stage are:

$$\dot{x} = f_1(x,u_i) \text{ and } x(t_0) = x_0, \tag{20}$$

$$\frac{\partial H_i(\cdot)}{\partial u_1} = -[c_1(x,u_1)]_u + \phi_1[f_1(x,u_1)]_u = 0, \tag{21}$$

$$\begin{aligned}\dot{\phi}_1(t) = \delta\phi - \frac{\partial H_1(\cdot)}{\partial x} &= \phi_1(\delta - [f_1(x,u_1)]_x) \\ &- p_1[y_1(x)]_x + [c_1(x,u_1)]_x,\end{aligned} \tag{22}$$

where $H_i(\cdot) = p_i y_i(x) - c_i(x, u) + \varphi_i f_i(x, u_i)$ and $\varphi_i(t)$ is the current value costate where $\varphi_i(t) = e^{\delta t}\lambda_i(t)$. The switching conditions that hold prior to the switching time, assuming that switching does occur, are:

$$\lambda_1(\cdot) + \frac{\partial e^{-\delta t}s(\cdot)}{\partial x} \neq \lambda_2(\cdot), \text{ and} \tag{23}$$

$$H_1(\cdot) - \frac{\partial s(\cdot)}{\partial t_1} > H_2(\cdot). \tag{24}$$

The first switching condition, stated in Equation (23), specifies that the rate at which optimal profit changes with a change in the state variable is not equivalent between stages outside of the switching time. The LHS of this equation may approach the switching condition ($\lambda_1(\cdot) + [e^{-\delta t}s(\cdot)]_x = \lambda_2(\cdot)$) from above ($\lambda_1(\cdot) + [e^{-\delta t}s(\cdot)]_x > \lambda_2(\cdot)$ in Equation (23)) or from below ($\lambda_1(\cdot) + [e^{-\delta t}s(\cdot)]_x < \lambda_2(\cdot)$ in Equation (23)), depending on the parameters and functional forms in the problem.

The second switching condition, stated in Equation (24), specifies that the value of the first regime is greater than that of the second regime. Otherwise, it is profitable to switch into the next phase. Insights into optimal switching behavior can be gained through manipulation of Equation (24). Substitute the definition of the Hamiltonian function into this function to obtain:

$$\begin{aligned}&p_1y_1(x) - c_1(x,u) + \phi_1 f_1(x,u_i) - [s(\cdot)]_t \\ &\quad > p_2y_2(x) - c_2(x,u) + \phi_2 f_2(x,u_i).\end{aligned} \tag{25}$$

The Hamiltonian function for each stage represents its marginal value at time t. The first two terms on the LHS of Equation (25) ($p_1y_1(x) - c_1(x, u)$) represent current profit: total revenue minus total cost. The next term ($\varphi_1 f_1(x,u_i)$) represents the change in future profit associated with a change in the resource stock. The key term here is φ_1, which represents the marginal value of a change in the state variable (land quality) at time t. The state variable may decrease or increase through $f_1(x,u_i)$, depending on whether a land use degrades or restores base productivity and how this is modified by input management. The last term on the LHS of (Equation 25) ($[s(\cdot)]_t$) represents how time impacts the switching cost.

The LHS of Equation (25) will decline over time if this regime degrades the base resource and input management does not counteract this effect. This occurs since $f_{\deg}(\cdot) < 0$ in this case and the marginal value of the first regime falls through yield decreases and increasing costs. Alternatively, the LHS of Equation (25) will increase over time through $f_{res}(\cdot) > 0$ if the first regime restores land quality or the degrading impact of an enterprise is subsumed by the positive impacts of input application (e.g., soil infertility is overcome through use of fertilizer). In contrast to the degrading case, increases in productivity will increase yield and decrease costs.

A hypothetical trajectory for dynamic profit within a net degrading enterprise (i.e., one in which input application does not offset inherent degradation) is labelled H_1 in Figure 1. This trajectory is understood to be defined by all terms on the LHS of Equation (20), including those concerning switching costs. A rise in the dynamic value of a regime through investment in land quality is demonstrated for the second enterprise (curve H_2) in Figure 1.

Dynamic profit decreases along the curve H_1 in Figure 1. This is consistent, for example, with declines in profit with continuous cereal cropping because of decreases in soil organic matter. Dynamic profit will continue to decrease until the switching conditions in Equation (11)

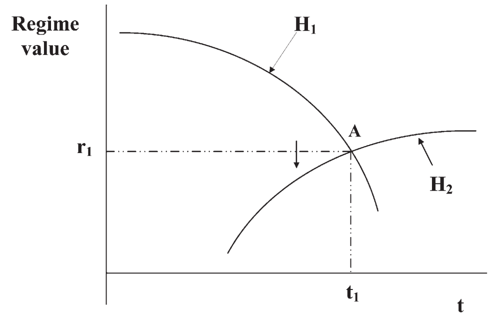

Figure 1. Dynamics of regime value following a price decrease for a degrading regime.

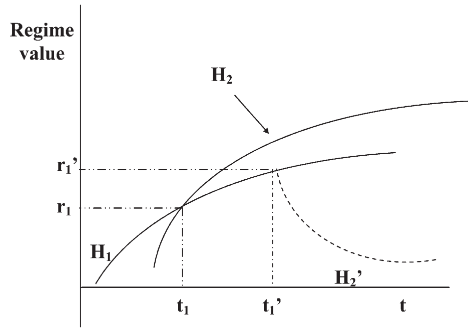

Figure 2. Dynamics of optimal switching when the active regime renews the quality of land.

and Equation (12) are satisfied with equality, at which point it is optimal to switch to the next regime. The optimal switching time occurs at point A at time t_1 in Figure 1. In the specific case illustrated here, it is optimal to switch given that remaining in the first enterprise will drive dynamic profitability below that which can be earned within the second enterprise. This is demonstrated in that remaining on H_1 past the point of intersection (point A) leads to a position where $H_2(\cdot) > H_1(\cdot) - \partial s(\cdot)/\partial t_1$.

The curve for the renewing enterprise (H_2) is concave in Figure 1 to reflect diminishing marginal returns to renewal. It is optimal to remain within enterprises that renew land quality across the entire planning horizon if there is only one state variable representing base productivity. For example, suppose that a producer has adopted a regime that renews land quality. Its dynamic profit could follow a path such as H_1 in Figure 2. It would be profitable to switch to another renewing enterprise (H_2) at t_1 in Figure 2 if dynamic profit were higher within this regime. However, it will never be profitable to switch to a degrading enterprise, such as H_2' at t_1' in Figure 2, if production functions are continuous—see later text for a discussion of the implications of discontinuous production functions—because a higher return is earned by not switching. This is evident in that dynamic profit along the trajectory H_1 is always higher than that on H_2' past switching time t_1'. This is a graphical illustration of the necessary condition defined in Equation 14 that describes that the successive regime should never be adopted if its value never matches that of the active enterprise.

This discussion has two important implications. First, it reinforces the importance of considering the relative profitability of the alternative crop. Second, it highlights the critical need to incorporate multidimensional relationships between enterprise use and base productivity if realistic optimal switching schedules are to be obtained. In reality, enterprises that renew base productivity are also likely to degrade one or more determinants of production. In this case, it may be optimal to switch to a degrading enterprise at some time. An example is where continued grazing of a legume pasture leads to soil compaction and eventually to declines in productivity.

Impacts of a Change in Output Price on Switching Time and State

Modification of optimal management following a change in output price for the active regime may be identified through taking the partial differential of the necessary conditions from Equations (20)–(25) with respect to p_1. Using Equation 22, the costate variable for the first phase is equal to:

$$\phi_1(t) = e^{\delta(t-\varsigma)} \int_{\varsigma}^{t_1} e^{-\alpha}\big(p_1(\cdot)[y_1(\cdot)]_x - [c_1(\cdot)]_x\big)dt, \tag{26}$$

where $\alpha = (\delta + [f_2(\cdot)]_x)(t - \varsigma)$. Substitution of this expression into Equation (21) and deriving the partial differential with respect to p_1 yields:

$$\Delta p_1 \frac{\partial [H_i(\cdot)]_u}{\partial p_1} = \Delta p_1 [f_1(\cdot)]_u e^{\delta(t-s)} \int_{\varsigma}^{t_1} e^{-\alpha} [y_1(\cdot)]_x dt. \tag{27}$$

The term Δp_1 represents the change in output price, whereas $[f_1(\cdot)]_u$ is the (positive) relationship between input application and land quality. These benefits of input application are expressed in farm profit through the relationship connecting base productivity and crop yield ($[y_1(\cdot)]_x$) between the time of application and the terminal time.

The differential of the necessary condition that determines the optimal switching time (Equation (25)) at time ς (where $\varsigma < t_1$) yields:

$$\Delta p_1 \frac{\partial E(\cdot)}{\partial p_1} = \Delta p_1 \left[y_1(x) + e^{\delta(t-s)} \int_{\varsigma}^{t_1} e^{-\alpha} [y_1(\cdot)]_x \, dt \right], \tag{28}$$

where $E(\cdot)$ denotes the LHS of Equation (25). The first term in the square brackets describes a change in the value of yield at time t. The second term in the square brackets represents the current value of changes in marginal product occurring across the remainder of the horizon. This expression is positive (negative) for a price increase (decrease). Nevertheless, its relationship with the optimal switching time is indeterminate, depending on the characteristics of the subsequent regime.

Assume the first phase is a degrading regime, it is followed by one that restores land quality, and input application does not overcome the degradation caused by the first crop. In this case, dynamic profit declines over time with degradation according to the trajectory H_1 in Figure 3. The optimal path of degradation and renewal across the two enterprises is $\overline{ABD}$. However, regime value is promoted through the price increase according to Equation (28). Hence, the Hamiltonian trajectory shifts from H_1 to H_1' and the optimal path is $\overline{ACD}$. This causes an increase in profit and an increase in the switching time, *ceteris paribus*. By similar reasoning, it can be established that a price decrease for product from the first regime should encourage switching to occur sooner. Moreover, these relationships will hold regardless of whether the following enterprise degrades or renews land quality.

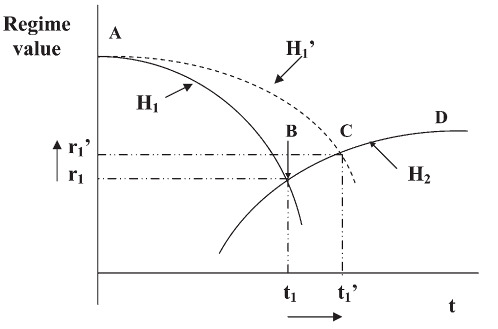

Figure 3. Dynamics of regime value following a price increase for a degrading regime.

Another instance is possible where the first regime renews soil quality. In this situation, the relationship between the optimal switching time and the output price depends on the nature of the successive enterprise. A price increase should delay switching if profit increases at a greater rate in the consecutive regime. (In contrast, switching will not occur if profit increases at a slower rate in the next stage, consistent with Equation (14).) For example, a price increase for the active regime in Figure 4 shifts the optimal switching time from t_1 to t_1' following a shift in trajectory from H_1 to H_1'. This occurs as the active enterprise is now more profitable and switching earlier has a higher opportunity cost. In line with above discussion, a producer will only switch to a degrading enterprise from a renewing regime if the latter starts to degrade land quality at some time. In this case,

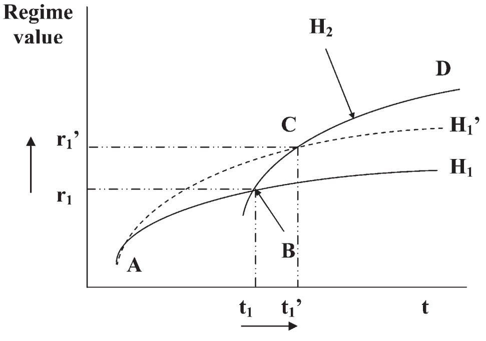

Figure 4. The effect of a price change for the active regime when the successive regime renews land quality at a greater rate.

the optimal response to a price change will mirror that within a degrading enterprise.

A number of previous studies highlight an inverse relationship between the optimal length of a degrading phase and a change in its output price (e.g., Goetz, 1997; Willassen, 2004). This result is dependent on problem structure and the nature of functions incorporated. It occurs since returns to degradation are higher following a price change and restorative enterprises are required to offset this damage. The positive association between output price and the length of a degrading regime found here is in accordance with the law of supply and typical agricultural practice. For instance, the higher profitability of cereal crops, relative to livestock enterprises, motivated extended cropping phases in Western Australia throughout the 1990s (Doole and Weetman, 2009). Increasing the area of land planted to degrading crops following an increase in output price is also consistent with recommendations from applications of equilibrium whole-farm optimization models, even those explicitly including degradation and renewal (e.g., Bathgate, Revell, and Kingwell, 2009; Doole et al., 2009).

The differential of the necessary condition that determines the optimal switching state (Equation (23)) at time ς (where $\varsigma < t_1$) yields:

$$(29) \qquad \Delta p_1 \frac{\partial F(\cdot)}{\partial p_1} = \Delta p_1 e^{\delta(t-s)} \int_{\varsigma}^{t_1} e^{-\alpha}[y_1(\cdot)]_x \, dt,$$

where $F(\cdot)$ denotes the LHS of Equation (23). Equation (23) determines the optimal level of the state variable at the switching time based on the relative value of the costate variables. A price increase ($\Delta p_1 > 0$) promotes the value of marginal product across the remainder of the horizon. This increases the magnitude of φ_1 for a given value of x.

This will have different implications on the optimal switching state, depending on the relative shapes of the costate trajectories. The curvature and orientation of the adjoint profiles will depend on the functions and parameters present within a problem. An adjoint trajectory (e.g., λ_1 in Figure 5a) is an increasing function of the state variable because investment in soil quality improves yield, decreases input costs, and reduces the need for inputs. However, the relative slopes of these functions will typically differ between stages, reflecting differences in marginal returns to investment in soil quality.

Two cases exist in the two-stage problem. First, assume that marginal returns to land investment are steeper in the first phase than the second (Figure 5a). An increase in output price for the first stage promotes the value of land quality, shifting the costate profile upwards from λ_1 to λ_1'. The optimal switching state therefore decreases from x_1 to x_2, as $\lambda_1' > \lambda_2$ at x_1 denoting that there is a marginal benefit accruing to reducing x in the first stage following a price increase. Second, alternatively, assume that marginal returns to land investment are steeper in the second phase than the first (Figure 5b). The condition $\lambda_1' > \lambda_2$ again holds at x_1; however, the optimal switching state is higher after the price increase due to the relative curvature of the costate trajectories.

Many single-crop analyses identify an indeterminate relationship between price and land

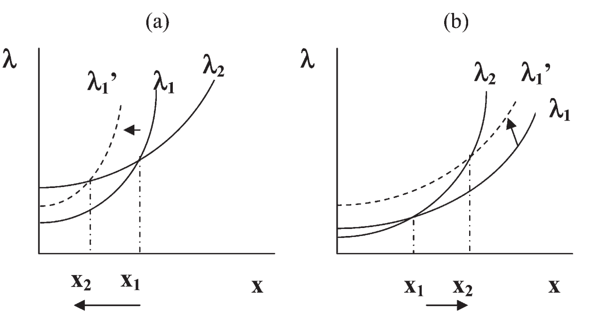

Figure 5. Changes in the switching state following a price increase in the first regime assuming no switching cost function is defined. Costate trajectories are denoted λ_1 and λ_2 for stage 1 and 2, respectively, with λ_1' indicating the adjoint trajectory for the first regime after the price increase.

degradation. A higher price may promote degradation through motivating greater use of inputs that improve yield and degrade or encourage conservation through increasing the marginal benefit of soil conservation (Clarke, 1992; Grepperud, 1997; LaFrance, 1992). The optimal response of management decisions to output price in this paper depends on relative returns to investment. A higher output price will promote greater investment in the soil resource through increasing the marginal benefit of input application (see Equation (27)). Moreover, it is worthwhile to invest in soil quality following a price increase in the first stage if the rate at which marginal returns to this investment change (i.e., the slope of the adjoint profile) is higher in the second stage.

Impacts of a Change in the Switching Cost

Switching costs impact optimal management through the necessary conditions in Equation (11) and Equation (12). Assume two stages exist, it is not yet optimal to switch, and use current-value terms for simplicity. This yields: $\phi_1(t) + [s(x)]_x \neq \phi_2(t)$ and $H_1 - [s(x)]_x\dot{x} > H_2$.

Improving land quality decreases transition costs (thus $[s(x)]_x < 0$). This is captured in the second term of $\phi_1(t) + [s(x)]_x \neq \phi_2(t)$. The net effect of transition costs on the switching state depends on the relative shapes of the costate profiles, as discussed in the previous section with reference to the output price. Increasing or including marginal transition costs increases the switching state if the slope of λ_1 dominates that of λ_2, as there are greater relative returns accruing to investment in stage 1 (Figure 6a). In contrast, increasing or including marginal transition costs lowers the switching state if the slope of λ_1 dominates that of λ_1, as there are lower relative returns accruing to investment in stage 1 (Figure 6b). Suppose nitrogen fertilizer is applied in both phases and the switching cost is dependent on soil nitrogen. Inclusion of the switching cost function reduces one component of the benefit of fertilizer application in the first phase. However, it may be optimal to invest further in soil quality if greater returns accrue to this activity in the first stage.

The temporal value of switching costs is captured in $H_1 - [s(x)]_x\dot{x} > H_2$ where $\dot{x} = f_i(x,u_i)$. The impact of increasing or including switching

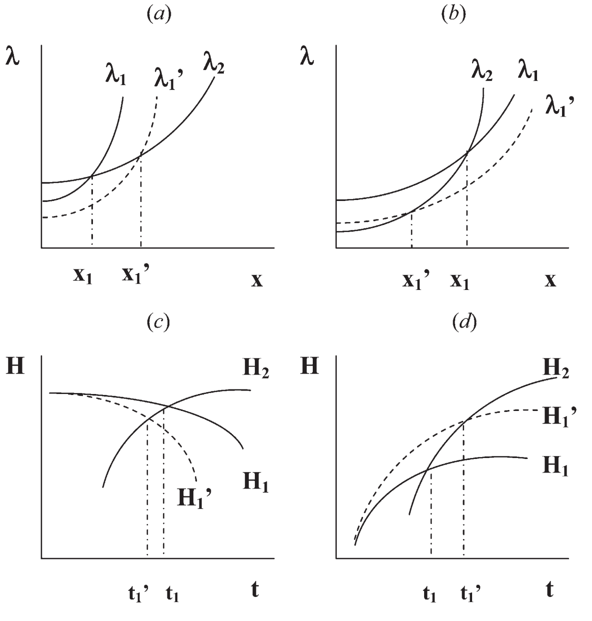

Figure 6. Influence of marginal switching costs on the adjoint trajectories (*a* and *b*) and Hamiltonian trajectories (*c* and *d*) in a two-stage problem. Trajectories without switching costs are denoted λ_1, λ_2, H_1, and H_2. Trajectories λ_1' and H_1' are the adjoint and Hamiltonian profiles with marginal switching costs.

costs depends on the types of regime involved. Increasing or including transition costs decreases the optimal switching time if a degrading regime is followed by a restorative stage (Figure 6c). This occurs because degradation ($f_i < 0$) increases switching costs through $[s(x)]_x < 0$. In contrast, increasing or including transition costs further promotes the value of a restorative enterprise. This shifts its Hamiltonian function upwards, promoting a later switching time (Figure 6d).

Switching Behavior with Discontinuous Production Functions

Production functions $y_i(x)$ may be discontinuous in some cases. Here, $y_i = 0$ for $x < \bar{x}$ and $y_i > 0$ for $x \geq \bar{x}$, where $\bar{x}$ is a threshold value indicating a certain level of soil quality required for crop growth. One example is that a saline water table should be maintained at least two metres below the soil surface to maintain growth of agricultural crops and pastures (Clarke et al., 2002).

This has interesting implications for switching behavior. The costate trajectory for the second stage is not defined for all levels of the state variable, just for $x \geq \bar{x}$. However, it may be shallower (Figure 7a) or steeper (Figure 7b) than the adjoint trajectory for the first stage. Thus, the impact of changes in the output price and switching cost on the optimal switching state remains indeterminate in the case of discontinuous production functions.

Suppose a discontinuous production function is defined for a degrading phase. Point *e* in Figure 7c denotes where it is profitable to begin producing in the (degrading) second regime that possesses the discontinuous yield function. The threshold of soil quality where this occurs may be defined $\hat{x}$, where $\hat{x} \geq \bar{x}$. In contrast, point *f* in Figure 7c indicates where production finishes under degradation as $x = \bar{x}$. (Although, converse to this illustrative example, it may be profitable to terminate production at $x > \bar{x}$.) Switching occurs at t_1 as $H_1 - [s(x)]_t < H_2$ holds over $[t_1^+, t_2]$, observable in Figure 7c as placement of the trajectory H_2 above H_1. Equation (13) describes that $H_1 - [s(x)]_t < H_2$ typically infers that $t_0^- = t_1^- < t_2^-$. However, its interpretation is different with a discontinuous $y_i(x)$ because H_2 is not defined over $[t_0, t_1^-]$.

Suppose a discontinuous production function is defined for a restorative phase. Point *g* in Figure 7d denotes where it is profitable to begin producing in the (restorative) second regime that possesses the discontinuous yield function. This may occur at any $x \geq \bar{x}$. Regime value increases from here to its termination at point *h*

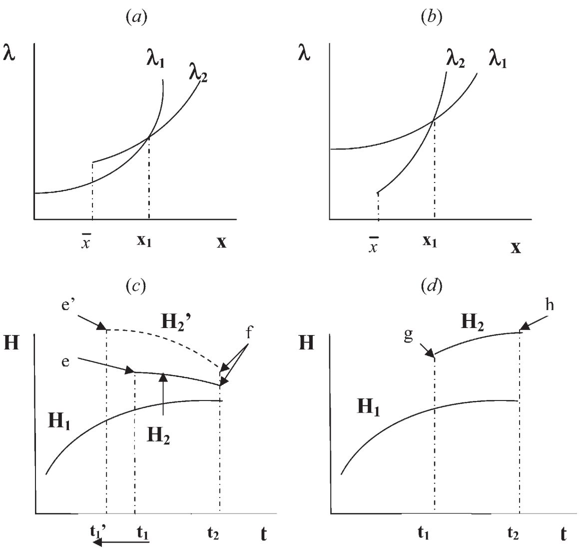

Figure 7. Influence of a discontinuous production function for the second regime on the adjoint trajectories (*a* and *b*) and Hamiltonian trajectories (*c* and *d*) in a two-stage problem. State $\bar{x}$ is a threshold value indicating a certain level of soil quality required for crop growth.

due to the renewal occurring under the phase. Switching occurs at t_1 as $H_1 - [s(x)]_t < H_2$ holds over $[t_1^+, t_2]$.

Parameter perturbation will alter the optimal switching time in many circumstances, as the shape and location of both H_1 and H_2 in (H, t) space will usually change in response. Suppose that price increases in the second stage. This decreases the threshold $\hat{x}$ at which the degrading phase becomes profitable; thus, less renewal is required in the first phase. This manifests itself as an extension of H_2 to H_2' in Figure 7c. Moreover, regime value across the phase is promoted through the price increase, resulting in an upward shift of H_2 to H_2' (Figure 7c). The net effect of these changes is that profit increases in the degrading phase and the switching time occurs earlier when a discontinuous production function is present.

Conclusions

Dual consideration of non-crop inputs and complementary effects between land uses in this analysis resolves a significant shortcoming in the analysis of land degradation. This addition brings such models closer to representing modern agricultural systems in which both non-crop inputs and indirect effects play important roles in maintaining farm profitability. A key result is that it is optimal to increase the use of a degrading crop with an increase in its output price, converse to the findings of several earlier studies. A second lesson is that an inability of land markets to reflect differences in resource quality reduces incentives for greater conservation. A third implication is that low capital malleability may promote degradation through increasing the cost of switching between alternative land uses. Overall, these factors identify an explicit need to consider dynamic factors in models of agricultural decision-making, particularly those involving land allocations among multiple land-uses.

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References

Amit, R. "Petroleum Reservoir Exploitation: Switching from Primary to Secondary Recovery." *Operations Research* 34(1986):534–49.

Barbier, E.B. "The Farm-Level Economics of Soil Conservation: The Uplands of Java." *Land Economics* 66(1990):199–211.

Barrett, S. "Optimal Soil Conservation and the Reform of Agricultural Pricing Policies." *Journal of Development Economics* 36(1991):167–87.

Bathgate, A.D., C. Revell, and R.S. Kingwell. "Identifying the Value of Pasture Improvement Using Whole Farm Modelling." *Agricultural Systems* 102(2009):48–57.

Bruinsma, J. *The Resource Outlook to 2050.* Paris: FAO, 2009.

Clarke, C.J., R.J. George, R.W. Bell, and T.J. Hatton. "Dryland Salinity in South-Western Australia: Its Origins, Remedies, and Future Research Directions." *Australian Journal of Soil Research* 40(2002):93–113.

Clarke, H.R. "The Supply of Non-degraded Agricultural Land." *Australian Journal of Agricultural Economics* 36(1992):31–56.

Doole, G.J. "A Practical Algorithm for Multiple-Phase Control Systems in Agricultural and Natural Resource Economics." *Journal of Agricultural and Resource Economics* 34(2009): 109–29.

Doole, G.J., A.D. Bathgate, and M.J. Robertson. "The Economic Value of Grazing Vegetative Wheat (*Triticum aestivum* L.) Crops in Mixed-Farming Systems of Western Australia." *Animal Production Science* 49(2009):807–15.

Doole, G.J., and D.J. Pannell. "On the Economic Analysis of Crop Rotations." *Crop Rotation: Economics, Impact, and Management*. Y.U. Berklian, ed., pp. 71–106. Hauppauge: Nova Science Publishers, 2009.

Doole, G.J., and E. Weetman. "Tactical Management of Pasture Fallows in Western Australian Cropping Systems." *Agricultural Systems* 104(2009): 24–32.

Goetz, R.U. "Diversification in Agricultural Production: A Dynamic Model of Optimal Cropping to Manage Soil Erosion." *American Journal of Agricultural Economics* 79(1997): 341–56.

Grepperud, S. "Soil Conservation as an Investment in Land." *Journal of Development Economics* 54(1997):455–67.

Hennessy, D.A. "On Monoculture and the Structure of Crop Rotations." *American Journal of Agricultural Economics* 88(2006):900–14.

Hertzler, G.L. "Dynamically Optimal Adoption of Farming Practices Which Degrade or Renew the Land. Agricultural Economics Discussion

Paper No. 5/90. University of Western Australia, Perth, 1990.

Just, R.E., and J.A. Miranowski. "Understanding Farmland Price Changes." *American Journal of Agricultural Economics* 75(1993):156–68.

LaFrance, J.T. "Do Increased Commodity Prices Lead to More or Less Soil Degradation?" *American Journal of Agricultural Economics* 36(1992):57–82.

McConnell, K.E. "An Economic Model of Soil Conservation." *American Journal of Agricultural Economics* 65(1983):83–89.

Orazem, P.F., and J.A. Miranowski. "A Dynamic Model of Acreage Allocation with General and Crop-Specific Soil Capital." *American Journal of Agricultural Economics* 76(1994):385–95.

Organization for Economic Co-operation and Development and the Food and Agriculture Organization of the UN (OECD-FAO). *Agricultural Outlook 2009–2018*. Paris: OECD, 2009.

Willassen, Y. "On the Economics of the Optimal Fallow-Cultivation Cycle." *Journal of Economic Dynamics & Control* 28(2004):1541–56.