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This paper discusses issues on the estimation of consumer demand equations subject to binding non-negative constraints. We propose computationally feasible specifications and a simulated maximum likelihood (SML) method for demand systems. Our study shows that the econometric implementation of the SML estimates can avoid high-dimensional integration problems. As contrary to the simulation method of moments and simulated pseudo-likelihood methods that require the simulation of demand quantities subject to non-negativity constraints for consumers in the sample, the SML approach requires only simulation of the likelihood function. The SML approach avoids solving for simulated demand quantities because the likelihood function is conditional on observed demand quantities.

We have applied SML approach for the linear expenditure system (LES) with non-negativity constraints. The results of a seven-goods demand system are presented. The results provide empirical evidence on the importance of taking into account possible cross equation correlations in disturbances.

Key Words: Simulated likelihood; Linear expenditure system; Non-negativity constraints; Multivariate censored variables; Nonlinear simultaneous equations.

JEL Classification Numbers: C15, C34, D12.

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1. INTRODUCTION

In examining consumer demand behavior, household or individual microeconomic data offer detailed information for empirical analysis. Important indicators of behavior and heterogeneous preferences associated with the age, sex, or level of education of consumers, can be treated explicitly with micro data but cannot easily be incorporated into aggregate demand analysis. However, household budget data, which contain information on the consumption of disaggregate commodities, often demonstrate a significant proportion of observations for which expenditures on some goods are zero. Since large data sets of consumer surveys are usually on short run purchases or consumption by consumers, zero expenditures are the result of short run consumption behavior or consumers' sensitivity to commodity prices.[1] Standard approaches to specifying and estimating demand systems that ignore the non-negativity constraints are not appropriate to study the short-run price response of consumer behavior. Hence, Deaton (1986, p. 1809) claimed that the problem of dealing with zero expenditures is one of the most pressing in applied demand analysis.

Papers by Wales and Woodland (1983), Lee and Pitt (1986, 1987), and Lee (1993) have proposed methods for estimating demand systems with binding non-negativity constraints. Wales-Woodland's approach is based on the Kuhn-Tucker conditions associated with a stochastic direct utility function. Lee and Pitt (1986), taking the dual approach, begin with the indirect utility function and show how virtual price relationships can take the place of Kuhn-Tucker conditions. Such microeconometric models are formulated within the classical utility maximization framework for consumer demand and the profit-maximization or cost-minimization framework for producers. The Kuhn-Tucker conditions of concave programming provide the basic equations for formulating the likelihood function for estimation. The models can easily be generalized to cases with quantity rationing and convex budget constraints (Lee and Pitt 1987). However, the implied statistical models are complex nonlinear multivariate or simultaneous equation Tobit models, limiting the empirical estimation to a small number of goods. This is so because without restrictive distributional assumptions, likelihood functions become rather complicated and involve multiple integrals in a complicated manner. Such difficulties may have hindered the progress of empirical application in the past.

The development of methods of simulated moments in a discrete choice model (McFadden, 1989; Pakes and Pollard, 1989) renews the possibility of overcoming some of the complexity for estimation. The consumer demand

[1]Consumer purchasing data for marketing research often have such a feature. Short run demand analysis is of particular interest in marketing science because the interest on producers' price promotions and issues of coupons (Chiang 1991, 1995).

systems with non-negativity constraints in econometrics are multivariate, nonlinear, simultaneous limited dependent variable systems. Contrary to discrete choice models, simulated method of moments for consumer demands with non-negativity constraints require more than a simple generation of random numbers and the direct simulation of model outcomes. In a discrete choice model, the direct simulation of choices can provide a simple frequency simulator of choice probabilities (Hajivassiliou and McFadden 1998, Laroque and Salanie 1993). For the consumer demand model, simulating the probabilities of demand goods binding in constraints by frequency requires solving nonlinear programming problems or equations of Kuhn-Tucker conditions. Since repeatedly solving such equations is rather impractical, direct simulation of demand quantities and their corresponding frequency counts for binding inequality constraints are not attractive. Therefore, it may be more practical to consider simulation estimation methods rather than the method of simulated moments. Alternative methods, such as importance sampling, may use the likelihood function. In this paper, we extend the SML methods in Börsch-Supan and Hajivassiliou (1993) and Lee (1992, 1995). This simulation estimation method can be implemented with Monte Carlo simulation techniques and conventional optimization methods. It avoids the technical difficulty of deriving the virtual prices in the previous approaches of Lee and Pitt (1986, 1987). Applications of simulation methods in various economic subjects can be found in Mariano et al. (2000).

Section 2 discusses the consumer demand systems with non-negativity constraints. Section 3 discusses a linear expenditure system (LES). Section 4 first introduces a specification of stochastic disturbances with relatively restricted correlation across equations, which does not require high demand integration. Then, it relaxes the restricted correlation structure and introduces simulation method for its estimation. We proposes the SML method with a smooth recursive conditioning (SRC) simulator, which is also known as the GHK simulator (Geweke, 1991; Börsch-Supan and Hajivassiliou, 1993; Keane, 1994) for estimating the LES. The applications of these methods for estimating a seven-goods demand system are presented in Section 5. The implications of the results are discussed. Section 6 concludes the paper with a summary.

2. CONSUMER DEMAND WITH NON-NEGATIVITY CONSTRAINTS

Before deriving our method for estimating demand systems, we must first define consumer demand with non-negativity constraints. To begin, zero expenditures as a result of rational consumer behavior can be derived from the classical utility maximization framework. Let $U(x; \varepsilon)$ be a utility function with m commodities $x_1, ..., x_m$, where $x = (x_1, \ldots, x_m)$, and ε is a vector of stochastic terms which are known by the individual consumer but unknown by the econometrician. The vector ε represents unobserved preferences in consumers which affect their demand. In short-run demand models which are relevant for (short-time) consumer survey data, ε may capture unmeasured consumption in previous periods in a myopic dynamic setting and may be functions of demographic characteristics of the consumers. The utility maximization model of the consumer is

$$\max_x \{U(x;\varepsilon) : v'x = 1, x \geq 0\}, \tag{2.1}$$

where $v = p/M$ is a m-dimensional vector of goods prices normalized by income M. Note that U is strictly increasing and strictly quasi-concave so as to guarantee a unique solution for the demand vector, x^*. Furthermore, assuming that U is continuously differentiable, the demand, x^*, can be characterized by the Kuhn-Tucker conditions.

Let $x^* = (0, ..., 0, x^*_{l+1}, ..., x^*_m)'$ be a demand vector where the first l goods, with $l \geq 0$, are not consumed and all remaining goods (indexed $l+1$ through m) are consumed. The Kuhn-Tucker conditions for x^* are

$$\frac{\partial U(x^*;\varepsilon)}{\partial x_i} - \lambda v_i \leq 0, \quad i = 1, ..., l,$$

$$\frac{\partial U(x^*;\varepsilon)}{\partial x_i} - \lambda v_i = 0, \quad i = l+1, ..., m, \tag{2.2}$$

and $v'x^* = 1$ where λ is the Lagrange multiplier corresponding to the budget constraints. The Kuhn-Tucker conditions can equivalently be expressed in terms of virtual prices (Neary and Roberts, 1980). Virtual prices at x^* are

$$\xi_i = \frac{\partial U(x^*;\varepsilon)}{\partial x_i} \Big/ \lambda, \quad i = 1, ..., m. \tag{2.3}$$

Using virtual prices, the Kuhn-Tucker conditions can be rewritten as

$$\xi_i \leq v_i, \quad i = 1, ..., l; \qquad \xi_i = v_i, \quad i = l+1, ..., m, \tag{2.4}$$

and $v'x^* = 1$. Econometric models can be derived with the specification of either the direct utility function (Wales and Woodland, 1983) or the indirect utility function (Lee and Pitt, 1986). With the direct utility function specification, one specifies a direct utility function U. The Kuhn-Tucker conditions imply a system of equations:

$$v_m \frac{\partial U(x^*;\varepsilon)}{\partial x_i} - \xi_i \frac{\partial U(x^*;\varepsilon)}{\partial x_m} = 0, \quad i = 1, ..., l,$$

$$v_m \frac{\partial U(x^*;\varepsilon)}{\partial x_i} - v_i \frac{\partial U(x^*;\varepsilon)}{\partial x_m} = 0, \quad i = l+1, ..., m-1, \tag{2.5}$$

and $v'x^* = 1$. With the indirect utility function specification, one specifies either a system demand equations or an indirect utility function. With an indirect utility function, a system of demand equations can be derived by Roy's identity:

$$0 = D_i\left(\xi_1, ..., \xi_l, v_{l+1}, ..., v_m; \varepsilon\right), \quad i = 1, ..., l$$

and

$$x_i^* = D_i\left(\xi_1, ..., \xi_l, v_{l+1}, ..., v_m; \varepsilon\right), \quad i = l+1, ..., m-1. \tag{2.6}$$

In both systems, the implied endogenous variables are $x_{l+1}^*, ..., x_m^*$ and $\xi_1, ..., \xi_l$, where the ξ's are latent variables with $\xi_i \le v_i$, $i = 1, ..., l$.

For the parametric estimation, the function form for the direct or indirect utility function and a distribution for ε need to be specified. In an empirical application of this model, it is important to select a system that satisfies globally the theoretical concavity property. This is so, because the structural equations (2.5) or (2.6) imply that the statistical model is a simultaneous nonlinear equations model with multivariate limited dependent variables. Amemiya (1974) and Gourieroux et al. (1980) have demonstrated that, for similar models, certain coherency conditions are required to guarantee that the implied distribution functions for the observable endogenous variables are proper distributions. In general, the coherency conditions fulfill the requirement that the mapping from the probability space of the disturbances to the sample space must be a well-defined onto single value function. For the consumer demand model, the coherency conditions are, first, that for each possible value of ε in the probability space a unique vector of endogenous variables, x^*, is generated by the structural equations and, second, that for every possible vector x^* there exists an ε vector that will generate it from the structural equations (see, e.g., Lee and Pitt 1987, Ransom 1987, and Soest et al., 1990 and 1993). If the specified utility $U(x;\varepsilon)$ is monotonic and strictly quasi-concave in x or the indirect utility function is derived from a utility with such properties, a unique demand vector x^* will exist for every ε. Satisfaction of the second coherency condition crucially depends on the manner in which the stochastic elements are introduced into the consumer's problem. In microeconometric models, random elements are unified components of a behavioral structure. If a sample observation cannot be realized by a specified structural model, the model is deemed to be too restrictive. If the observation is contaminated by measurement errors, measurement errors should then be introduced explicitly. For consumer demands for all goods to be nonessential, a stochastic utility specification must be able to generate demand quantities which can cover the entire simplex $\{x|v'x = 1, x \geq 0\}$ for each given price vector in a sample.

Demand systems derived from some popular flexible functional forms, e.g., the translog demand system (Christensen et al. 1975), may not satisfy the monotonicity or the concavity property of the utility function at any possible value of the parameter space. For such an approximated system, one has to be careful about the coherency conditions. Ransom (1987) showed that the quadratic utility model of Wales and Woodland (1983) does satisfy the coherency conditions even though the monotonicity condition does not satisfy globally. With proper restrictions on the parameter space of the translog demand system function introduced by Lee and Pitt (1986), Soest and Kooreman (1990) showed that the translog demand system will satisfy coherency conditions. Soest et al. (1993) further showed that not imposing coherency may yield inconsistent estimators. However, for any coherent model, a likelihood function can be derived using the relations (2.5) or (2.6), and the model can, in principle, be estimated after coherency conditions are properly imposed.

In this paper, we will focus our attention on empirical estimation of the linear expenditure demand system because of its simplicity and its global concavity property.

3. LINEAR EXPENDITURE DEMAND SYSTEM

The LES is derived from the Stone-Geary direct utility function of the form

$$U(x) = \sum_{i=1}^{m} \alpha_i \ln (x_i - \beta_i), \alpha_i > 0, (x_i - \beta_i) > 0, \tag{3.1}$$

where x_i is the quantity of good i and m is the number of goods. Maximizing the utility function (3.1) subject to the budget constraint, $\sum_{i=1}^{m} v_i x_i \leq 1$, yields the ordinary (i.e., Marshallian) demand function,

$$x_i = \beta_i - \frac{\theta_i}{v_i} \sum_{j=1}^{m} v_j \beta_j + \frac{\theta_i}{v_i}, \tag{3.2}$$

where $\theta_i = \frac{\alpha_i}{\sum_{j=1}^{m} \alpha_j}$. This system is attractive because of its linear structures in expenditures (i.e., the expenditure on each good is a linear function of all prices), even though it is restrictive in that the implied Engel curve is linear (Stone 1954, Pollak and Wales 1969). The implied notional expenditure share equation is

$$v_i x_i = v_i \beta_i + \theta_i \left(1 - \sum_{j=1}^{m} v_j \beta_j \right), \quad i = 1, ..., m. \tag{3.3}$$

Corner solution (zero demand) for good i can occur only if the parameter β_i is negative. Goods for which the corresponding β_i is negative are referred to as 'inessential' because of the common interpretation of the β's as representing subsistence or committed expenditure. The LES requires that all goods that have zero demands have a corresponding $\beta_i \leq 0$; the non-negativity of the α_i''s rules out inferiority; and concavity requires that every good must be a substitute for every other good. Furthermore, through Pigou's law (e.g., Deaton, 1974), the additivity of preferences implies that, for large numbers of goods, income and price elasticities are approximately proportional.

Variation in tastes across consumers is introduced into the utility function (3.1) by treating the α_i parameters as stochastic;

$$\alpha_i = e^{\varepsilon_i}, \tag{3.4}$$

where the disturbances $(\varepsilon_1, ..., \varepsilon_m)$ are normally distributed with mean $(\gamma_1, ..., \gamma_m)$ and a constant variance-covariance matrix Σ.

Demographic variables, such as family size and age composition, have traditionally played a major role in the analysis of household budget data (Pollak and Wales 1981). Here, demographic variables are introduced into the demand system by treating γ_i, the mean of ε_i in (3.4), as a linear function of the demographic variables, z, i.e., $\gamma_i = z\delta_i$. Since the additive form of the utility function in (3.1) is invariant with respect to scaling, the normalization $\varepsilon_m = 0$ is made. At the sample observation $x^* = \left(0, ..., 0, x_{l+1}^*, ..., x_m^*\right)'$ with $x_i^* > 0, i = l+1, ..., m$, in the demand regime with the first l goods not being consumed, the virtual prices are

$$\xi_i = v_i \frac{s_m^* - v_m \beta_m}{s_i^* - v_i \beta_i} e^{\varepsilon_i}, \quad i = 1, ..., m-1, \tag{3.5}$$

where $s_i^* = v_i x_i^*$ is the expenditure share of the ith commodity. The optimality conditions for x^* are, after logarithmic transformation,

$$\varepsilon_i \leq \ln\left(-v_i \beta_i\right) - \ln\left(s_m^* - v_m \beta_m\right), \quad i = 1, ..., l \tag{3.6}$$

and

$$\varepsilon_i = \ln\left(s_i^* - v_i \beta_i\right) - \ln\left(s_m^* - v_m \beta_m\right), \quad i = l+1, ..., m-1. \tag{3.7}$$

Let $e_1 = (\varepsilon_1 - \gamma_1, ..., \varepsilon_l - \gamma_l)^{'}, e_2 = (\varepsilon_{l+1} - \gamma_{l+1}, ..., \varepsilon_{m-1} - \gamma_{m-1})^{'}$ and $s_2 = \left(s^*_{l+1}, ..., s^*_{m-1}\right)^{'}$. Then, (e_1, e_2) is $N(0, \Sigma)$-distributed, where

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \tag{3.8}$$

and

$$e_1 = \Sigma_{12}\Sigma_{22}^{-1}e_2 + \eta, \tag{3.9}$$

with $E(\eta) = 0$ and $var(\eta) = \Omega \equiv \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$. This implies that the regime switching conditions can be written as

$$\eta \leq \begin{pmatrix} \ln(-v_1\beta_1) - \ln(s^*_m - v_m\beta_m) - \gamma_1 \\ \vdots \\ \ln(-v_l\beta_l) - \ln(s^*_m - v_m\beta_m) - \gamma_l \end{pmatrix} - \Sigma_{12}\Sigma_{22}^{-1}e_2 \equiv r. \tag{3.10}$$

The above equation provides a one-to-one mapping of (e_1, e_2) to (η, s_2). Given the specified joint density function of (e_1, e_2), the joint density of (η, s_2) can be determined.

Under the assumption that (e_1, e_2) is jointly normal, the density of η is also normal and is independent of s_2 because η is independent of e_2 in (3.9) and s_2 is determined by e_2 from (3.7). Let $f_1(\eta)$ be the conditional density function of η and $f_2(e_2)$ be the density function of e_2, the likelihood function for an observation x^* is

$$L_c(x^*) = \left(\int_{\{\eta:\eta\leq r\}} f_1(\eta)d\eta\right) f_2(e_2) \left|\frac{\partial e_2}{\partial s_2}\right|. \tag{3.11}$$

Since $s^*_m = 1 - \sum_{j=l+1}^{m-1} s^*_j$, the Jacobian of the transformation is

$$\left|\frac{\partial e_2}{\partial s_2}\right| = \left|\begin{pmatrix} \frac{\partial\varepsilon_{l+1}}{\partial s^*_{l+1}} & \cdots & \frac{\partial\varepsilon_{l+1}}{\partial s^*_{m-1}} \\ \vdots & & \vdots \\ \frac{\partial\varepsilon_{m-1}}{\partial s^*_{l+1}} & \cdots & \frac{\partial\varepsilon_{m-1}}{\partial s^*_{m-1}} \end{pmatrix}\right|.$$

The matrix $\frac{\partial e_2}{\partial s_2}$ can be written as

$$\frac{\partial e_2}{\partial s_2} = H + cab^{'}, \tag{3.12}$$

where H is a diagonal matrix with diagonal elements $h_{ii} = \frac{1}{s^*_i - v_i\beta_i}$, $i = l+1, \cdots, m-1$;

$$c = \frac{1}{s^*_m - v_m\beta_m},$$

and $a = b = \mathbf{1}$, where $\mathbf{1}$ is a vector of ones.

Since $\frac{\partial e_2}{\partial s_2}$ has a certain matrix pattern in (3.12), it is possible to obtain an analytical formula for the determinant. Using Graybill (1983, Theorem 8.4.3, p.203), we note that

$$\begin{aligned}
\left|\frac{\partial e_2}{\partial s_2}\right| &= \left[1 + c\sum_{i=l+1}^{m-1}\frac{a_i b_i}{h_{ii}}\right]\prod_{i=l+1}^{m-1} h_{ii} \\
&= \left[1 + \frac{1}{s_m^* - v_m\beta_m}\sum_{i=l+1}^{m-1}(s_i^* - v_i\beta_i)\right]\prod_{i=l+1}^{m-1}\frac{1}{(s_i^* - v_i\beta_i)} \\
&= \left[\frac{s_m^* - v_m\beta_m + \sum_{i=l+1}^{m-1}(s_i^* - v_i\beta_i)}{s_m^* - v_m\beta_m}\right]\prod_{i=l+1}^{m-1}\frac{1}{(s_i^* - v_i\beta_i)} \\
&= \sum_{i=l+1}^{m}(s_i^* - v_i\beta_i) \Big/ \prod_{i=l+1}^{m}(s_i^* - v_i\beta_i)\,.
\end{aligned}$$

The likelihood function with n observed is

$$L = \prod_{i=1}^{n}\prod_{c}\left[L_c(x_i^*)\right]^{I_i(c)}, \qquad (3.13)$$

where $I_i(c)$ is an indicator such that $I_i(c) = 1$ if the observed consumption pattern x_i^* for individual i is in the demand regime c and $I_i(c) = 0$ otherwise.

4. SIMULATED MAXIMUM LIKELIHOOD ESTIMATION

The empirical implementation of (3.13), however, is troubled by the computational complexity of the likelihood function, i.e., the estimation would require numerical integration involving multiple probability distributions in (3.11). The problem is somewhat simpler for the case of production. With a translog cost function, the linearity of the derived demand equations allows for additive and normal errors. Estimation of a translog cost function with three inputs has been accomplished in Lee and Pitt (1987) by the Gaussian quadrature; however, evaluation of multiple integrals by numerical methods even in the normal case can effectively be accomplished only for small numbers of goods.

Before methods of simulation estimation have been invented, a strategy of empirical modeling is to specify relatively restrictive covariance structures across equations so as to reduce the presence of high dimensional integrals. A possible specification is to assume that disturbances have an error component structure (Hausman and Wise 1978; Butler and Moffitt

1982). An error component specification for ϵ's is

$$\epsilon_i = u + w_i, \quad i = 1, \cdots, m-1, \tag{4.1}$$

where $u, w_i, i = 1, \cdots, m-1$ are mutually independent. With the error component structure in (4.1), the optimality conditions for x^* in (3.6) and (3.7) become

$$w_i \leq \ln(-v_i\beta_i) - \ln(s_m^* - v_m\beta_m) - u, \quad i = 1, \cdots, l \tag{4.2}$$

and

$$w_j = \ln(s_j^* - v_j\beta_j) - \ln(s_m^* - v_m\beta_m) - u, \quad j = l+1, \cdots, m-1. \tag{4.3}$$

Under the assumption that w_i, $i = 1, \cdots, m-1$, and u are normally distributed with means $\gamma_1, \ldots, \gamma_{m-1}, 0$ and variances σ_j, $j = 1, \cdots, m$, respectively, the likelihood function for an observation x^* satisfying (4.2) and (4.3) will be

$$\begin{aligned} L_c(x^*) = & \frac{\sum_{j=l+1}^{m}(s_j^* - v_j\beta_j)}{\prod_{j=l+1}^{m}(s_j^* - v_j\beta_j)} \int_{-\infty}^{\infty} \prod_{i=1}^{l} \Phi\left(\frac{\epsilon_i(x^*, u) - \gamma_i}{\sigma_i}\right) \\ & \cdot \prod_{j=l+1}^{m-1} \frac{1}{\sigma_j}\phi\left(\frac{\epsilon_j(x^*, u) - \gamma_j}{\sigma_j}\right) \cdot \frac{1}{\sigma_m}\phi\left(\frac{u}{\sigma_m}\right) du, \end{aligned} \tag{4.4}$$

where $\epsilon_i(x^*, u) = \ln(s_i^* - v_i\beta_i) - \ln(s_m^* - v_m\beta_m) - u$, $i = 1, \cdots, m-1$; Φ is the standard normal distribution function and ϕ is the standard normal density function. This likelihood function (4.4) involves only a single integration and can be numerically evaluated with the Gaussian quadrature formula (Stroud and Secrest 1966). The error component structure for ϵ's is restrictive. If u were to capture unobserved individual characteristics, these unobserved characteristics are assumed to have the same effect on all the virtual prices (or inverse demand functions). An alternative view of the error component specification for the LES corresponds to that $\alpha_i = e^{w_i}$ for $i = 1, \cdots, m-1$ and $\alpha_m = e^u$ with α's being mutually independent.

To relax the correlation structure on ϵ's, the resulted likelihood will involve multiple integral and an effective simulation method will be needed. As the method of simulated moments is not attractive, an alternative is the SML method. A computationally tractable simulator which does not have a large simulation variance is, however, important for the SML method. Statistical properties such as small biases and efficiency depend on the likelihood simulator (see, e.g., Lee 1992, 1995). For the simulation of choice probabilities of a multinomial probit model, the SRC simulator is shown to be an effective simulator (Geweke 1991, Börsch-Supan and Hajivassiliou 1993, and Keane 1994). It is feasible to generalize the SRC simulator for discrete choices to the consumer demand model with non-negativity constraints. This simulator is smooth and is an importance sampling simulator. Also, it has a useful restriction, namely, that the simulated probabilities are bounded between zero and one. The simulated likelihood for the model with a SRC simulator can be derived through a sequence of operators by changing variables. It is achieved by transforming η in (3.11) to a vector of random variables which make simulation easier. Such transformation is possible for this model due to the normality of η and the rectangle of the range $\{\eta : \eta \leq r\}$ for the LES in the integration (3.11). Since $var(\eta) = \Omega = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$ for the LES from (3.9) is positive definite, there exists a lower diagonal Choleski matrix with positive diagonal elements such that $DD' = \Omega$.

Define a transformation $\zeta = D^{-1}\eta$. The transformed random variable ζ becomes a standard multivariate normal variable. The likelihood function in (3.11) becomes

$$L_c(x^*) = \left(\int_{\{\zeta : D\zeta \leq r\}} \left[\prod_{i=1}^{l} \phi(\zeta_i) \right] d\zeta \right) f_2(e_2) \left| \frac{\partial e_2}{\partial s_2} \right|, \tag{4.5}$$

where $\phi(\zeta_i)$ is the standard normal density of ζ_i, $\zeta = (\zeta_1, ..., \zeta_l)$, and

$$D = \begin{pmatrix} d_{11} & 0 & \cdots & 0 \\ d_{21} & d_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ d_{l1} & d_{l2} & \cdots & d_{ll} \end{pmatrix}. \tag{4.6}$$

The range $\{\zeta : D\zeta \leq r\}$ can be rewritten recursively as

$$\frac{1}{d_{11}} r_1 \geq \zeta_1, \frac{1}{d_{22}} (r_2 - d_{21}\zeta_1) \geq \zeta_2, \cdots, \frac{1}{d_{ll}} (r_l - d_{l1}\zeta_1 - \cdots - d_{l,l-1}\zeta_{l-1}) \geq \zeta_l. \tag{4.7}$$

Then, the likelihood function in (4.5) is

$$\left[\int_{-\infty}^{d_{ll}^{-1} r_1} \int_{-\infty}^{d_{22}^{-1}(r_2 - d_{21}\zeta_1)} \cdot \int_{-\infty}^{d_{ll}^{-1}(r_l - d_{l1}\zeta_1 - \cdots - d_{l,l-1}\zeta_{l-1})} \left(\prod_{i=1}^{l} \phi(\zeta_i) \right) d\zeta_1 \cdots d\zeta_l \right] \cdot f_2(e_2) \left| \frac{\partial e_2}{\partial s_2} \right|. \tag{4.8}$$

The next step is to change the incomplete integral in the likelihood function in (4.5) to a complete integral by reforming some probability measures.

Define the truncated standard normal density as

$$\phi_{A_i}(\zeta) = \frac{\phi(\zeta)}{\Phi\left(\frac{1}{d_{ii}}\left(r_i - d_{i1}\zeta_1 - \cdots - d_{i,i-1}\zeta_{i-1}\right)\right)} \tag{4.9}$$

on the support $A_i = \left(-\infty, d_{ii}^{-1}\left(r_i - d_{i1}\zeta_1 - \cdots - d_{i,i-1}\zeta_{i-1}\right)\right]$ for $i = 1, ..., l$, where as a convention, $d_{10} = 0$ and $\zeta_0 = 0$. Then, the likelihood function for the LES in (4.8) can be written as a complete integral with respect to the truncated normal densities:

$$\left(\int_{A_1} \cdots \int_{A_l} \prod_{i=1}^{l} \Phi\left(\frac{1}{d_{ii}}\left(r_i - d_{i1}\zeta_1 - \cdots - d_{i,i-1}\zeta_{i-1}\right)\right) \cdot \prod_{i=1}^{l} \phi_{A_i}(\zeta_i) \cdot d\zeta_1 \cdots d\zeta_l\right)$$
$$\cdot f_2\left(e_2\right)\left|\frac{\partial e_2}{\partial s_2}\right|. \tag{4.10}$$

As any random variable with a given distribution can be generated from a uniform random number generator through its inverse distribution transformation, the truncated normal random variables can be transformed into uniform random variables. To do this, define the transformation as

$$u_i = \frac{\Phi(\zeta_i)}{\Phi\left(\frac{1}{d_{ii}}\left(r_i - d_{i1}\zeta_1 - \cdots - d_{i,i-1}\zeta_{i-1}\right)\right)},$$
$$\text{where} \quad \zeta_i \leq \frac{1}{d_{ii}}\left(r_i - d_{i1}\zeta_1 - \cdots - d_{i,i-1}\zeta_{i-1}\right) \tag{4.11}$$

for $i = 1, ..., l$. Conditional on $\zeta_1, ..., \zeta_{i-1}, u_i$ is a uniform random variable on $[0, 1]$. Such transformations define a sequence of conditional uniform random variables. Conversely, uniform random variables can be drawn from a uniform random generator and the ζ's can be solved recursively as functions of the u's:

$$\zeta_i = \Phi^{-1}\left[u_i \Phi\left(\frac{1}{d_{ii}}\left(r_i - d_{i1}\zeta_1 - \cdots - d_{i,i-1}\zeta_{i-1}\right)\right)\right], \tag{4.12}$$

starting with

$$\zeta_1 = \Phi^{-1}\left[u_1 \Phi\left(\frac{r_1}{d_{11}}\right)\right].$$

With the transformations in (4.11), (4.10) can be rewritten as

$$L_c(x^*) = \left[\int_0^1 \cdots \int_0^1 \prod_{i=1}^{l} \Phi\left(\frac{1}{d_{ii}}\left(r_i - d_{i1}\zeta_1 - \cdots - d_{i,i-1}\zeta_{i-1}\right)\right) du_1 \cdot \cdot du_l\right]$$
$$\cdot f_2\left(e_2\right)\left|\frac{\partial e_2}{\partial s_2}\right|. \tag{4.13}$$

With r random draws, the likelihood at a sample observation x^* can be simulated as

$$L_{rc}(x^*) = \left[\frac{1}{r}\sum_{t=1}^{r}\prod_{i=1}^{l}\Phi\left(\frac{1}{d_{ii}}\left(r_i - d_{i1}\zeta_{1,t} - \cdots - d_{i,i-1}\zeta_{i-1,t}\right)\right)\right]\cdot f_2\left(e_2\right)\left|\frac{\partial e_2}{\partial s_2}\right|, \tag{4.14}$$

where $(\zeta_{1,t}, ..., \zeta_{l-1,t})$ is the corresponding vector of the ζ's from the tth random draw of $(u_{1,t}, ..., u_{l-1,t})$ in simulation. This simulator is smooth in parameters and generalizes the SRC to our model.

5. APPLICATION TO FOOD CONSUMPTION

A sample of 1,150 households was drawn from the 1978 Socioeconomic Survey of Indonesia (SUSENAS), a national probability sample of households. Food consumption (purchased and home-produced) of close to 100 separate items in the seven days prior to the date of enumeration is aggregated into seven categories: tubers, fruits, animal products (meat and dairy), fish, vegetables, grains, and others. A village is assumed to represent a distinct market, and the average price of every disaggregate item is calculated as the average price of the commodity consumed by the sampled households in the village. Price indices are computed by geometrically weighted component prices with the average budget shares of a larger administrative area, the kabupaten (regency).[2] There are 300 kabupatens in the sample. The absence of data on most non-food prices means that we must impose the assumption that foods and non-foods are separable in the utility function. Three demographic variables are identified: the number of household members 4 years of age and under (infants), the number aged 5 through 14 (children), and the number aged 15 and above (adults). Table 1 provides summary statistics on food consumption shares and normalized (by total food expenditure) prices, as well as demographic variables. As Table 1 indicates, six of the seven foods were not consumed by at least one household during the reference period. Half of the sampled households did not consume animal products, and one-third did not consume tubers or fruit. Only grain was consumed by all households. Even though the nonnegativity of the α's in a LES rules out inferiority, and concavity requires that every good must be a substitute for every other good. For this data, the goods are unlikely to be inferior and they can be substitutes for each other. The restrictions of the LES may be appropriate given the nature of these seven goods.

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[2] If a commodity's price was unavailable for a village, it was taken to be the average kabupaten price.

TABLE 1.

Summary Statistics

	Mean	**Standard Deviation**	**Frequency of Zero Consumption**
Tubers Share	.0313	.0521	473
Fruit Share	.0364	.0465	419
Animal Products Share	.0559	.0872	576
Fish Share	.1111	.0894	112
Vegetables Share	.1285	.0702	14
Others Share	.1806	.0792	1
Grain Share	.4562	.1630	0
Tuber Price	1.2763	1.0563	
Fruit Price	1.1868	1.0362	
Animal Products Price	1.0964	.8472	
Fish Price	1.1408	.8965	
Vegetable Price	1.1478	.7797	
Others Price	1.2136	1.0274	
Grain Price	1.1213	.8005	
Infants	.7757	.8541	
Children	1.6087	1.3969	
Adults	3.0330	1.3063	

Note:
(a) Sample size = 1150.
(b) Infants, children and adults are household members aged 0 to 4, 5 to 14, and 15 years of age and above, respectively.

The LES has been estimated with both the error component specification and the specification with generally correlated normal disturbances. We report both the maximum likelihood estimates (MLE) of the error component model and the SMLE of the general covariance model. Tables 2-5 report the SMLE results for the general covariance model and Tables 6-9 report the MLE for the error component model. One can then compare the two sets of results and their empirical implications for the Indonesia data.

In solving the β estimates of the LES, each estimated β_i must satisfy the condition $x_i - \beta_i > 0$. To satisfy these constraints, the parameterization of $\beta_i = \min(x_i) - \exp(\beta_i^*)$, where min is taken over the whole sample and β^*s are free parameters, are used during the estimation process. A second concern is that the estimated variance-covariance matrix Σ may not be positive definite; therefore, non-negative definiteness is imposed by replacing Σ with its Cholesky decomposition, i.e., $\Sigma = AA'$ where A is a lower triangular matrix.

Table 2 provides SMLEs of the LES with demographic effects. The coefficients β_i are all negative and are significantly negative for tubers, animal,

TABLE 2.

LES – SML Parameter Estimates

	Tubers	**Fruit**	**Animal**	**Fish**	**Vegetables**	**Others**	**Grain**
β							
	−.018	−.043	−.245	−.070	−.027	−.024	−.184
	(−8.331)	(−.529)	(−2.980)	(−1.098)	(−.362)	(−.309)	(−4.487)
σ^2_{ij}							
Tubers	4.640	2.244	1.101	.931	.835	.526	
Fruit		2.152	.815	.640	.560	.381	
Animal			.715	.326	.295	.191	
Fish				.577	.182	.149	
Vegetables					.360	.112	
Others						.196	
δ_{ij}							
Constant	−1.987	−1.239	−.127	−.747	−1.033	−.879	
	(−8.811)	(−8.958)	(−6.744)	(−8.856)	(−17.811)	(−20.578)	
Infants	.008	−.089	−.042	−.075	−.032	.002	
	(.090)	(−1.702)	(−.625)	(−2.414)	(−1.469)	(.106)	
Children	−.010	−.055	−.059	−.043	−.045	−.026	
	(−.183)	(−1.713)	(−1.280)	(−2.254)	(−3.352)	(−2.511)	
Adults	−.042	−.089	−.080	−.053	−.037	−.006	
	(−.726)	(−2.528)	(−1.796)	(−2.611)	(−2.559)	(−.579)	
$\widehat{\theta}$	.040	.064	.206	.120	.098	.133	.339

Note:
(a) Log-likelihood = 5347.0301. Number of parameters = 52.
(b) Log-likelihood under the null that demographic variables have no effect = 5053.409. Number of parameters = 34.
(c) T-statistics are in parentheses.
(d) Number of random draws = 100.
(e) $\widehat{\theta}_i = \frac{e^{\bar{z}\delta_i}}{\sum_{j=1}^{m} e^{\bar{z}\delta_j}}$.

fish, and grain. That means that each of the seven goods including grain needs not to be 'essential'. The coefficients on the demographic variables are mostly negative. The overall significance of the demographic variables can be tested with a likelihood ratio test statistic. Under the null hypothesis of no demographic variables, the log-likelihood function is 5053.409. The likelihood ratio statistic is 587.80 (with 18 degrees of freedom), which is very significant. We therefore reject the specification without demographic effects. For the adult and children variables, they have more significant coefficients on the various expenditure shares equations than those of the infants variable. The overall impacts of the demographic variables on expenditures of various goods can better be interpreted in terms of elasticities as follows.

TABLE 3.

LES (SMLE) – Demographic Effects

Incremental:	**Tubers**	**Fruit**	**Animal**	**Fish**	**Vegetables**	**Others**	**Grain**
Infant	5.12	−11.67	−7.21	−7.97	−0.81	3.20	4.03
Child	2.86	−4.90	−13.04	−2.32	−1.98	0.32	4.57
Adult	−1.19	−10.12	−19.68	−3.09	−0.36	3.27	5.42

TABLE 4.

LES (SMLE)– Price Elasticities

Quantities **Prices**	**Tubers**	**Fruit**	**Animal**	**Fish**	**Vegetables**	**Others**	**Grain**
Tubers	−1.7055	.0653	.3437	.1022	.0397	.0373	.2640
Fruit	.0404	−2.3130	.4726	.1405	.0545	.0512	.3629
Animal	.0846	.1880	−4.8148	.2942	.1142	.1073	.7602
Fish	.0248	.0551	.2901	−1.6324	.0335	.0315	.2228
Vegetables	.0175	.0389	.2048	.0609	−1.2175	.0222	.1573
Others	.0169	.0376	.1978	.0588	.0228	−1.1398	.1519
Grain	.0171	.0379	.1996	.0593	.0230	.0216	−1.2989

Table 3 presents estimates of the effects of incremental household members (by type) on consumption. The percentage changes in expenditures are evaluated in response to an incremental of infant, child or adult at the sample mean demographic variables. Let $\hat{\theta}_i = \frac{e^{\bar{z}\delta_i}}{\sum_{j=1}^m e^{\bar{z}\delta_j}}$ where $\bar{z}$ as the sample averages for the demographic variables. The expenditure shares corresponding to $\hat{\theta}$'s are $v_i x_i = v_i \beta_i + \hat{\theta}_i(1 - \sum_{j=1}^m v_j \beta_j)$. Let $\hat{\theta}_i^+$ be the corresponding θ_i with an incremental of a household member. The percentage change of an expenditure share in response to an incremental member will be $(\hat{\theta}_i^+ - \hat{\theta}_i)(1 - \sum_{j=1}^m v_j \beta_j)/v_i x_i$ in percentage. The results in Table 3 indicates that adding an infant to a household having mean demographic characteristics reduces the consumption of fruit, animal products, fish, and vegetables, at 11.67, 7.21, 7.97, and 0.81 percents, respectively, but increases the consumption of tubers, grain and other. Adding a child rather than an infant leads to an even greater reduction in animal product consumption (13.04 percent). Fruit and fish are also reduced but not as much as the reduction with an additional infant. With an additional adult, animal product will be reduced even further (19.68 percent) followed by fruit consumption (10.12 percent). With an additional infant or child, the consumption of tuber can increase. Grain and others have their consumption increase in response to an additional household member of any type. Grain,

TABLE 5.

LES (SMLE)– Total Expenditure (Income) Elasticities

Tubers	Fruit	Animal	Fish	Vegetables	Others	Grain
1.2796	1.7592	3.6845	1.0799	0.7625	0.7364	0.7431

which has the largest average expenditure, is consumed in greater amount with incremental household members in age order.

Tables 4 and 5 provide, respectively, the matrix of price elasticities and the vector of income elasticities. The own price elasticities for the LES system are

$$\frac{\partial \ln x_i}{\partial \ln v_i} = -1 + \frac{(1-\hat{\theta}_i)\beta_i}{x_i} = -1 + \frac{(1-\hat{\theta}_i)v_i\beta_i}{v_i x_i}$$

for $i = 1, \cdots, m$. The cross price elasticities for the system are

$$\frac{\partial \ln x_i}{\partial \ln v_j} = -\hat{\theta}_i \frac{v_j \beta_j}{v_i x_i}, \quad i \neq j,\ j = 1, \cdots, m.$$

The income elasticities are

$$\frac{\partial \ln x_i}{\partial \ln M} = \frac{\hat{\theta}_i}{v_i x_i}, \quad i = 1, \ldots, m.$$

The reported elasticities in Tables 4 and 5 are evaluated for a representative household having sample mean demographic characteristics and mean shares. The price and expenditure elasticities are highest (in absolute value) for animal products and lowest for grain, vegetables and others. All these seven goods are price elastic. Animal products are commonly found to be highly income elastic in developing countries. Among these seven goods, grain, vegetables and others are ‘necessities’ as the expenditure elasticities are less than one. Animal products, fish, fruit and tubers are all ‘luxury’ goods. All the elasticities seem sensible.

To illustrate the difference on the specification with general correlated disturbances and that with a restricted setting. Tables 6-9 report MLE of the model with an error component structure for the disturbances. These results can be compared with those of Tables 2-5. While the estimated coefficients and the implied demographic effects and elasticities in Tables 6-9 are overall confirmable, there are important exceptions. As the error component specification has fewer unknown parameters than those of the general covariance specification, some parameter estimates in Table 6 tend to be more significant than those of Table 2. The estimated β for grains

TABLE 6.

LES – ML Parameter Estimates (Error Component Model)

	Tubers	**Fruit**	**Animal**	**Fish**	**Vegetables**	**Others**	**Grain**
β	−.0267	−.0474	−.2343	−.0536	−.0327	−.0299	.0122
	(−8.8708)	(−11.8979)	(−11.9465)	(−12.2722)	(−11.0840)	(−9.4441)	(3.3862)
δ_{ij}							
Constant	−1.9602	−1.3167	−.1338	−.6223	−.6226	−.4649	
	(−13.4296)	(−12.3139)	(−1.2209)	(−7.1550)	(−9.2591)	(−7.3691)	
Infants	−.0407	−.0361	−.0623	−.0890	−.0395	−.0057	
	(−.9398)	(−1.0944)	(−2.2052)	(−2.9782)	(−1.7160)	(−.2600)	
Children	−.0953	−.1167	−.1159	−.0904	−.0911	−.0694	
	(−3.6856)	(−5.6813)	(−6.5021)	(−5.1844)	(−6.4036)	(−5.1035)	
Adults	−.1104	−.1252	−.1128	−.0731	−.0722	−.0448	
	(−3.7798)	(−5.7829)	(−6.3366)	(−3.6181)	(−4.6829)	(−2.9864)	
σ	1.0286	.7309	.5546	.6556	.4391	.3702	.6421
$\widehat{\theta}$	.0287	.0506	.1683	.1189	.1237	.1673	.3425

Note:
(a) T-statistics are in parentheses.
(b) $\widehat{\theta}_i = \frac{e^{\bar{z}\delta_i}}{\sum_{j=1}^{m} e^{\bar{z}\delta_j}}$.

TABLE 7.

LES – Demographic Effects (Error Component Model)

Incremental:	**Tubers**	**Fruit**	**Animal**	**Fish**	**Vegetables**	**Others**	**Grain**
Infant	−1.33	−1.19	−12.57	−8.00	−1.25	2.87	2.89
Child	−4.03	−9.79	−20.90	−4.05	−3.73	−1.04	6.06
Adult	−6.66	−12.59	−22.73	−2.80	−2.41	.94	5.24

TABLE 8.

LES – Price Elasticities (Error Component Model)

Quantities							
Prices	**Tubers**	**Fruit**	**Animal**	**Fish**	**Vegetables**	**Others**	**Grain**
Tubers	−1.1802	.0266	.1482	.0389	.0286	.0326	−.0125
Fruit	.0080	−1.7570	.2246	.0590	.0434	.0494	−.0190
Animal	.0174	.0873	−3.4022	.1277	.0940	.1069	−.0411
Fish	.0062	.0310	.1727	−1.3365	.0334	.0380	−.0146
Vegetables	.0055	.0279	.1553	.0408	−1.2130	.0341	−.0131
Others	.0053	.0266	.1495	.0393	.0289	−1.1637	−.0127
Grain	.0043	.0217	.1212	.0318	.0234	.0266	−.9603

TABLE 9.

LES – Total Expenditure (Income) Elasticities (Error Component Model)

Tubers	Fruit	Animal	Fish	Vegetables	Others	Grain
.9177	1.3913	3.099	1.070	.9623	.9263	.7509

turns out to be slightly positive and indicates that grains can be an 'essential' good. The demographic variables have now all negative coefficients. An increased household member of any type has a sharper reduction in the consumption of animal products. The consumption of tubers will be reduced by an additional infant or child instead of an increasing amount in the general covariance model. Again, grain is consumed in greater amount with incremental household members of any type. For price and expenditure elasticities, they are still highest for animal products and lowest for grain. Grain becomes price inelastic and its price increase has now lead to demand reduction of all goods. Tubers may become a 'necessity' in addition to grain, others and vegetables.

As the covariance matrix of the error component disturbances imposes equal correlations, it is nested into the general covariance model. Therefore, the error component structure can be tested by a likelihood ratio statistic. The log-likelihood function for the error component model is found to be 3125.34 while the log-likelihood function for the general model is 5347.03. The implied likelihood ratio statistic is 4444.87; and the data reject the restricted error covariance specification.[3]

6. CONCLUSION

We apply a new simulation method that solves the multiple integrals that arise in the ML estimation of consumer demand systems with binding non-negativity constraints. Our study shows that the econometric implementation of the SML approach can effectively avoid high-dimensional integration. We demonstrate the feasibility of the SML approach for the LES with non-negativity constraints, and we present the results of a seven-goods demand system.[4] Direct simulation methods as in the simulation methods of moments and simulated pseudo-likelihood methods (e.g., Laroque and Salanie, 1993 in a different context) that require the simulation of demand

[3]We experimented with the model by treating demands as continuous dependent variables, thus neglecting their discrete nature. Using the MLE of Table 6 as initial values, parameter estimates changed sharply after only a few iterations. This suggests that explicitly treating the binding non-negativity constraints importantly affects the estimates.

[4]The model and the estimation methods can be easily modified to handle consumer demand for goods with quantity rationing.

quantities subject to non-negativity constraints for each consumer in the sample would be computationally expensive. The SML approach avoids solving for simulated demand quantities since the likelihood function is conditional on observed demand quantities. In the SML approach, only the likelihood function needs to be simulated.

In principle, the SML approach can be applied to the estimation of more general consumer demand systems. However, it remains a difficult problem for the estimation of a flexible demand system such as the one derived from the quadratic utility function or the translog demand system, which do not satisfy model monotonicity coherency conditions on its whole parameter space. Monotoncity coherency conditions for such systems impose very complex inequality constraints on their parameters.[5] Effective procedures which impose such inequality constraints remain to be found. For the latter, Bayesian methodologies may become attractive. These will be left for future research.

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[5]Alternative flexible cost and profit and consumer demand functions with global concavity or convexity properties have recently been introduced in Diewert and Wales (1987, 1988). The usefulness of such system for empirical studies remains to be seen.

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