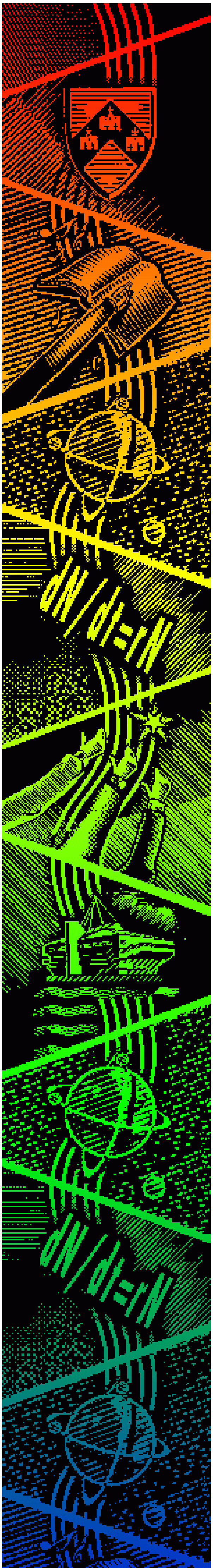

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Profitability of Horizontal Mergers with Price Interdependencies

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Abstract. We investigate how a downstream merger affects input prices and equilibrium profits when there are price interdependencies among firms. To do so, we develop a very simple model where different inputs, provided by monopolist suppliers, may be combined to produce differentiated products sold by oligopolist downstream units. We show that when the number of final products that may be produced is small, being an outsider is always better than participating in a downstream merger. When instead the number of final products is sufficiently large, some outsiders gain more than the participants but others lose. Thus if firms are uncertain about their rivals' willingness to merge, they might still have incentives to merge to eliminate the risk of being harmed by a merger between their competitors. We also show that if the products are not too differentiated no subsequent merger by the less benefitted/harmed firms will take place.

JEL classification: L13; L41

Keywords: downstream mergers, fixed-proportions technologies, preemption

1. Introduction

It is well known from the literature on horizontal integration that, when firms face identical and constant marginal costs of production, mergers are always more profitable for the outsiders, in both quantity and price competition settings, Salant et al., (1983), Deneckere and Davidson, (1985). In these contexts, a typical free-rider problem would emerge: even if mergers are profitable for the participants, each firm may find it more convenient to wait for other firms to merge so as to gain a higher payoff, Stigler, (1950). This however doesn't explain why firms usually oppose or try to prevent mergers involving their competitors. Lommerud et al., (2003), show how the introduction of endogenous input prices may provide a solution to the free-rider paradox. They consider a downstream merger in a model where three oligopolist downstream units are served by plant-specific input suppliers. They show that, when firms compete in quantities on the final market, this type of merger is always profitable for the participants and more profitable for a participant than for an outsider. The reason is that a merger induces the input suppliers

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of the participants to set lower prices for their inputs; since the inputs are complements, also the outsiders' suppliers decrease their prices but by less; this means that input prices are always lower for the participants and this obviously contributes positively to the profitability of a merger, thus eliminating any incentive to free-ride.

The complementarity between input prices is essential for these results to hold and it arises from the fact that each input supplier serves only one out of many oligopolist downstream units; often, however, we observe that firms purchase their inputs from different suppliers, who, in turn, serve many independent downstream units. The purpose of this work is precisely to investigate the incentives to merge in these types of settings. To do so, we develop a model in which different inputs, provided by monopolist suppliers, may be combined in fixed proportions to produce differentiated products sold by oligopolist firms.

In section 2, we show that when the number of final products that may be produced is small, being an outsider is always better than participating. This happens because a merger induces some of the suppliers of the participants to set lower prices for their inputs, thus producing a positive externality to some of the outsiders using the same inputs, whose price conditions improve more than for the participants.

When the number of final products is sufficiently large, however, the incentives to merge may drastically change. This is shown in section 3, where, following a merger, the outsiders combining some of the inputs used by the participants benefit from stronger cost reductions than the participants, but the outsiders using different inputs experience an increase in their production costs. If firms are uncertain about their rivals' willingness to merge, then they might still have incentives to merge to eliminate the risk of being harmed by a merger between their competitors.

Merging as a strategy to preempt rivals' mergers is also exemplified in Nilssen and Sorgard (1998), who analyse the interdependence of merging decisions over time when there are cost savings following a merger. Also, Fridolfsson and Stennek (2002) show that, although mergers are unprofitable, firms may still have incentives to merge to increase their market share. Brito, (2003), using a model of spatial competition, shows that even if some outsiders benefit from a merger more than the participants, there may still be incentives to merge to eliminate the risk of being the less benefitted outsiders.

In section 4, we investigate whether the initial merger might trigger subsequent mergers by the firms less benefitted or harmed and whether in this case the initial merger would still be profitable. To do that we limit our attention to the case where only horizontal integration in the downstream market is feasible. We find that if the products are not too differentiated then no harmed firm would have incentive to respond by merging. The initial merged units however might also consider the possibility of subsequent mergers by some of the initially benefitted outsiders. We find that there are always incentives for these units to merge with another benefitted outsider and that subsequent mergers by these firms always increase the profitability of the initial merger but only when the first merged units use different inputs.

In section 5 we conclude and make some suggestions for future research.

2. THE CASE OF THREE INPUT SUPPLIERS

We first consider the case in which the number of final goods that may be produced is so small that firms will always prefer to stay out of a merger. Suppose there are three different inputs, x_1, x_2 and x_3, that may be combined in fixed proportions to produce three different final goods, q_1, q_2 and q_3. More specifically, we assume:

$$q_1 = x_1 = x_2$$

$$q_2 = x_1 = x_3$$

$$q_3 = x_2 = x_3$$

where one unit for each of two different inputs is required to produce one unit of each output variety.

The inputs are supplied by three monopolist firms who serve three oligopolist downstream units, each producing a different variety of the final good, q.

Suppose that the degree of differentiation between any pair of varieties is the same and equal to $0 \leq b \leq 1$ and let w_j be the price per unit of input $j = 1, 2, 3$.

Thus the demand facing each individual firm i is

$$p_i = A - q_i - b \sum_{k=1, k \neq i}^{3} q_k$$

and the individual profits may be expressed as:

$$\pi_1 = (A - q_1 - b(q_2 + q_3) - w_1 - w_2)q_1$$

$$\pi_2 = (A - q_2 - b(q_1 + q_3) - w_1 - w_3)q_2$$

$$\pi_3 = (A - q_3 - b(q_1 + q_2) - w_2 - w_3)q_3$$

The Cournot solution for this market is:

$$q_1 = \frac{1}{2} \frac{2(w_1 - bw_3 + w_2) + A(b-2)}{b^2 - b - 2}$$

$$q_2 = \frac{1}{2} \frac{2(w_1 - bw_2 + w_3) + A(b-2)}{b^2 - b - 2}$$

$$q_3 = \frac{1}{2} \frac{2(w_2 - bw_1 + w_3) + A(b-2)}{b^2 - b - 2}$$

From which we can derive the input demands:

$$x_1 = q_1 + q_2 = \frac{2w_1 + A(b-2) + w_2(1-b) + w_3(1-b)}{(b+1)(b-2)}$$

$$x_2 = q_1 + q_3 = \frac{2w_2 + A(b-2) + w_1(1-b) + w_3(1-b)}{(b+1)(b-2)}$$

$$x_3 = q_2 + q_3 = \frac{2w_3 + A(b-2) + w_1(1-b) + w_2(1-b)}{(b+1)(b-2)}$$

Given the candidate equilibrium quantities each upstream unit chooses input prices simultaneously and non-cooperatively to maximise profits. These are given by:

$$\pi_j^U = x_j(w_j - C)$$

for each $j = 1, 2, 3$, and where C is the constant unit cost of production, common to all the upstream units. The unique solution to this problem is:

$$w_1^N = w_2^N = w_3^N = \frac{1}{2}\frac{A(b-2) - 2C}{b-3}$$

From which we can derive the equilibrium quantities and profits:

$$q_1^N = q_2^N = q_3^N = \frac{1}{2}\frac{A - 2C}{(b+1)(b-3)}$$

$$\pi_{D1}^N = \pi_{D2}^N = \pi_{D3}^N = \frac{1}{4}\frac{(A-2C)^2}{(b+1)^2(b-3)^2}$$

$$\pi_{U1}^N = \pi_{U2}^N = \pi_{U3}^N = \frac{1}{2}\frac{(A-2C)^2(2-b)}{(b+1)(b-3)^2}$$

Now consider a horizontal merger between firms 1 and 2 in the downstream market; in this case, the post-merger entity maximises:

$$\pi_{12} = (A - q_1 - b(q_2 + q_3) - w_1 - w_2)q_1 + (A - q_2 - b(q_1 + q_3) - w_1 - w_3)q_2$$

while the other firms continue to act independently.

The Nash solution to this problem is characterised by:

$w_2 = w_3 < w_1$

where

$$w_1 = \frac{(5A-2C)b^3 - 2(4C+5A)b^2 - 8(2A+C)b + 24(a+c)}{2b^3 - 7b^2 - 10b + 18}$$

$$w_2 = w_3 = \frac{Ab(b^2-5) - b^2(3C+2A) + 6(a+c)}{2b^3 - 7b^2 - 10b + 18}$$

Mergers are profitable for the participants if $\pi_{12} - 2\pi_D^N > 0$; as shown in Figure 1, this is the case only if the degree of product differentiation is not too low, for $b \leq 0.85$.

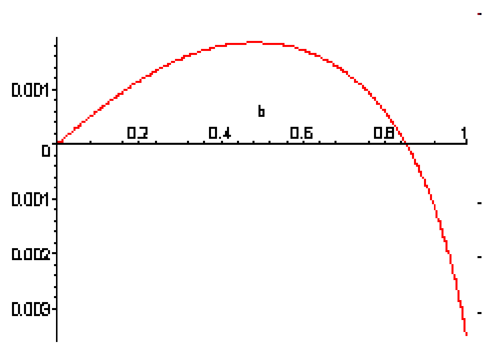

Figure 1: merger profitability for the participants

Mergers are instead always profitable for the downstream outsiders, $\pi_3 - \pi_D^N > 0$, and there is never a merger advantage, $\pi_{12} - 2\pi_3 < 0$.

The intuition behind these results may be traced by considering how a downstream merger affects the pricing behaviour of the input suppliers. Following a merger, the demand for input 1 becomes less price responsive; input 1 in fact is used in the production of both good 1 and 2 and thus the merged unit cannot shift production between its two plants to induce lower prices from its supplier; as a result input 1 price increases.

A merger instead increases the demand elasticity with respect to inputs 2 and 3; this effect is due to the fact that, for the case of the merged unit, input 2 is used only in the production of good 1 whereas input 3 is used only in the production of good 2; thus the merged unit can now shift production between the two goods to induce lower input prices from its suppliers; as a result the prices of input 2 and 3 decrease thus benefitting the firm left out of the merger, who now faces more favourable input price conditions. Whether the merged unit will earn or lose with respect to a pre-merger situation will depend on the relative strength of these two effects. Figure 1 shows that the merger profitability for the merged unit is first increasing, but at a decreasing rate, with the degree of product substitutability, till it reaches a maximum at $b = 0.53$ and then starts to decrease to become negative for sufficiently low degrees of product differentiation; for $0 \leq b \leq 0.53$, in fact, the input prices are strategic substitutes and, following a merger, the reduction in the prices of input 2 and 3 is relatively small; for $b > 0.53$, however, the prices of input 2 and 3 become strategic complements so that the incentives to set lower prices for input 2 are reinforced by the price reduction for input 3 and vice versa. As b gets sufficiently large, the reduction in the prices of input 2 and 3 is so strong that the input price conditions for the outsider become extremely favourable and the participants cannot any longer increase prices to profitably counteract the fall in demand.

The most interesting results however concern the profitability of the upstream units, which is always negative. In our model with firm specific input suppliers we found that, in the absence of externality effects, a merger was always unprofitable for the upstream units serving the merged entity; our new model seems to confirm this result, if we consider that now all the upstream units serve the merged entity. The profitability of the three suppliers however is affected in different ways. For the upstream units providing inputs 2 and 3, the main reason why a merger is unprofitable is the decrease of their input price not compensated by a sufficiently strong demand increase: lower input prices, in fact, induce higher input demand from the outsider, but due to the increase in the price of input 1, lower demand from the participants. The supplier of input 1, instead, following a horizontal merger, is induced to increase its input price, which causes a stronger reduction in its input demand.

3. The case of five input suppliers

In this section we will show that when many different inputs may be combined in fixed proportions to produce differentiated products then the participants may gain more than some of the outsiders; even if in this case there is never an absolute merger advantage, firms may still have incentives to participate for the fear of being the less benefitted outsiders.

We now consider the case in which there are five different inputs, x, that may be combined to produce ten differentiated products, q. More specifically we assume that one unit of input $i = 1..5$ may be combined with one unit of input $j = 1..5$, with $i \neq j$, to produce one unit of a differentiated product q_k, with $k = 1..10$. In this case each downstream unit always operates at a point where:

$$q_1 = x_1 = x_2$$

$$q_2 = x_1 = x_3$$

$$q_3 = x_1 = x_4$$

$$q_4 = x_1 = x_5$$

$$q_5 = x_2 = x_3$$

$$q_6 = x_2 = x_4$$

$$q_7 = x_2 = x_5$$

$$q_8 = x_3 = x_4$$

$$q_9 = x_3 = x_5$$

$$q_{10} = x_4 = x_5$$

The demand facing each individual downstream oligopolist is therefore:

$$p_i = A - q_i - b \sum_{k=1, k \neq i}^{10} q_k$$

and the input demands are:

$$x_1 = q_1 + q_2 + q_3 + q_4$$

$$x_2 = q_1 + q_5 + q_6 + q_7$$

$$x_3 = q_2 + q_5 + q_8 + q_9$$

$$x_4 = q_3 + q_6 + q_8 + q_{10}$$

$$x_5 = q_4 + q_7 + q_9 + q_{10}$$

The Nash solution to this problem is:

$$w_j^N = \frac{1}{3}\frac{5bC + 2(A+C) - bA}{b+2}$$

$$q_i^N = \frac{1}{3}\frac{(A-2C)(5b+2)}{(9b+2)(b+2)}$$

$$\pi_{Di}^N = \frac{1}{9}\frac{(A-2C)^2(5b+2)^2}{(9b+2)^2(b+2)^2}$$

$$\pi_{Uj}^N = \frac{4}{9}\frac{(A-2C)^2(2-b)(5b+2)}{(9b+2)(b+2)^2}$$

for each $i = 1..10$ and for all $j = 1..5$

We first consider a horizontal merger between two downstream oligopolists purchasing the inputs from different suppliers, for example, firms 1 and 10. In this context, the profits of the merged entity may be expressed as:

$$\pi_m = \left(A - q_1 - b\sum_{k=1,k\neq 1}^{10} q_k - w_1 - w_2\right) q_1 + \left(A - q_{10} - b\sum_{l=1,l\neq 10}^{10} q_l - w_4 - w_5\right) q_{10}$$

Notice that in this case all the input suppliers, apart from the supplier providing input 3, serve the merged units; thus the equilibrium input prices w_j are identical for $j = 1..5$ and $j \neq 3$:

$$w_j = \frac{(b^2-5b-2)(5b^3-47b^2+33b+14)C - (b-1)(b-2)(b^3-14b^2+45b+14)A}{(b+2)(3b^4-44b^3+168b^2-95b-42)}$$

$$w_3 = \frac{2(4b^5-35b^2+226b^3-132b-28-60b^4)C - (b-2)(b^4-20b^3+106b^2-64b-28)A}{2(b+2)(3b^4-44b^3+168b^2-95b-42)}$$

and $w_3 > w_j$

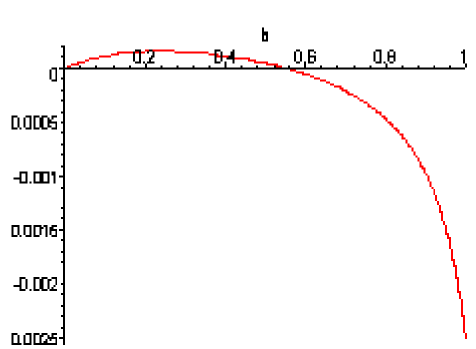

Figure 2: merger profitability of firms using input 3

Thus mergers are always more profitable for the outsiders not purchasing input 3, but less profitable for firms using input 3. Further, if the degree of product differentiation is sufficiently low, the merging process causes the profits of firms using input 3 to fall, Figure 2.

If we compare the input demand elasticities before and after a merger, we find that a merger makes the input demands for the participants more price responsive; this induces their input suppliers to set lower prices. The intuition is that now the participants use different inputs and therefore they can shift their production between the two composite goods to induce lower input prices from their suppliers. Thus the prices of all the inputs used by the participants decrease and this produces a positive externality to all the outsiders using the same inputs; the cost of production for these firms decreases.

This implies that, following a merger, the participants and the outsiders using the same inputs will face the same input price conditions and, as in the case of exogenous input prices, the outsiders will take a free-ride earning higher profits.

The outsiders using the other inputs, instead, will face less favourable price conditions; following a merger, in fact, we can show that the demands for these inputs get less price responsive; from the pre-merger and post-merger input 3 demands we can derive:

$$\frac{\delta x_3^N}{\delta w_3} = \frac{4(2+5b)}{(9b+2)(b-2)}$$

$$\frac{\delta x_3^M}{\delta w_3} = 4\frac{b^2-2-5b}{(b^2-9b-2)(b-2)}$$

where the difference $\left|\frac{\delta x_3^N}{\delta w_3}\right| - \left|\frac{\delta x_3^M}{\delta w_3}\right|$ is positive and increasing with the degree of substitutability. Thus, following a merger, the input suppliers not serving the merged units will set higher prices for their inputs and the incentives to do so will be stronger the lower is the degree of product differentiation.

The intuition is that, following a merger, all the outsiders using at least one of the inputs used by the participants will benefit from a reduction in their production costs; this will induce them to increase their levels of production and therefore their demand of the other inputs, whose price therefore increases. When the degree of product differentiation is sufficiently great the incentives for the suppliers not serving the participants to set higher prices are lower, so that also the profitability of the outsiders purchasing from them will improve.

As in the case of only 3 suppliers, all the upstream units serving the merged entity are worse off, Figure 3; this seems to confirm that horizontal downstream mergers are always unprofitable for the suppliers of the participants; and the main explanation is that now the participants can shift their production between the two composite goods thus inducing stronger price competition between their suppliers, who are therefore forced to set lower prices.

The profitability of the upstream unit producing input 3 instead improves, if the degree of substitutability is not too high, Figure 4.

We now consider a merger between two downstream units purchasing one of their inputs from the same supplier, for example a merger between firms 1 and 2. In this case the merged units maximise:

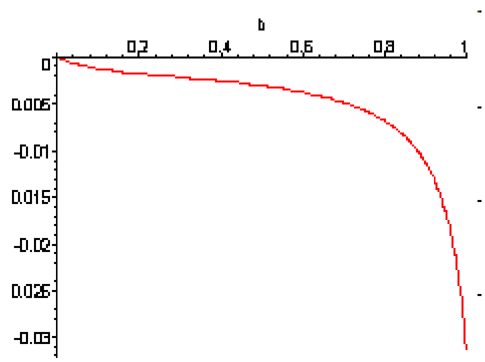

Figure 3: merger profitability of the suppliers serving the merged units

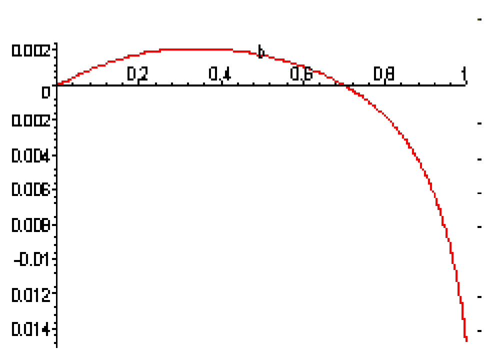

Figure 4: merger profitability of the supplier of input 3

$$\pi_m = \left(A - q_1 - b \sum_{k=1,k\neq 1}^{10} q_k - w_1 - w_2 \right) q_1 + \left(A - q_2 - b \sum_{l=1,l\neq 2}^{10} q_l - w_1 - w_3 \right) q_2$$

and the Nash solution to this problem is characterised by:

$$w_4 = w_5 > w_1 > w_2 = w_3$$

As in the case of only three inputs, the prices of inputs 2 and 3 fall, while the price of input 1 increases. The other input prices increase.

Again the intuition behind these results may be found analysing the input demand elasticities before and after a merger. It may be shown that input 2 and 3 input demands get more price responsive, thus leading their suppliers to set lower prices; this happens because the merged units may shift production between the two composite goods thus inducing fiercer price competition between their input suppliers. Input 1 demand, instead, becomes less price responsive and this induces its input supplier to set a higher price: since input 1 is used in identical proportion in the production of both the composite goods, there is no possibility of substitution between the two composite goods for the merged unit and this increases the bargaining power of its supplier.

The prices of the inputs not used by the participants are affected in two opposite ways: the decrease in the prices of inputs 2 and 3 produces a positive externality to all the outsiders using at least one of these inputs, who therefore increase their demand of the other inputs; the increase in the price of input 1 however will produce a negative externality to all the outsiders using this input, who will therefore decrease their demand of the other inputs; since however there are more outsiders using inputs 2 and 3 than outsiders using input 1, and thus more outsiders benefitting from improved price conditions, the net effect will be an increase in the demand of the other inputs, whose price will therefore increase.

We can now analyse the implications in terms of firms' profitability of these price changes. The less benefitted firms are those using the inputs not used by the participants; the prices of these inputs, as we have seen, increase more than any other input's, thus

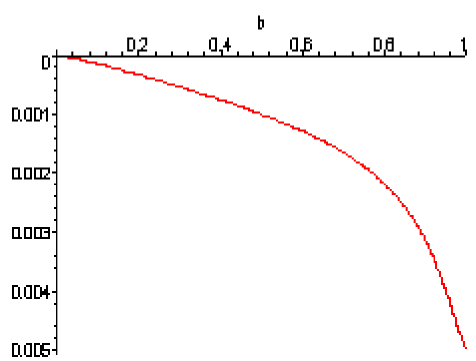

Figure 5: profitability of the less benefitted outsiders

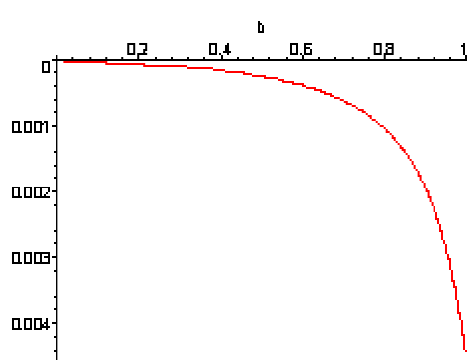

Figure 6: merger profitability for the firms using the participants' most expensive input

reducing the input demands of the downstream units using them. The profitability for these firms is always negative, as shown in Figure 5.

For the firms combining input 1 (the participants' input, whose price increases) with any other inputs not used by the participants, price conditions, following a merger, worsen though less than for the previous firms; the implications in terms of profitability are however negative also in this case, Figure 6.

The most benefitted firm uses the same inputs as the participants whose prices, following a merger, decrease: profitability for this firm is always positive, Figure 7.

The second more benefitted firms are those combining one of the least expensive inputs with one of the most expensive inputs; also for these firms profitability is always positive,

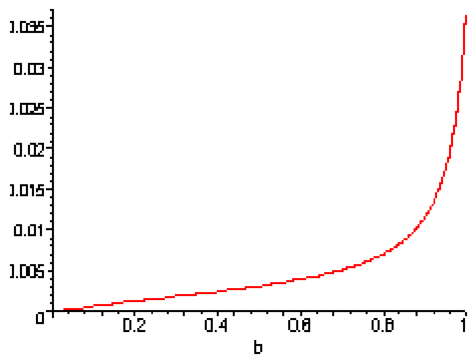

Figure 7: merger profitability of the most benefitted outsider

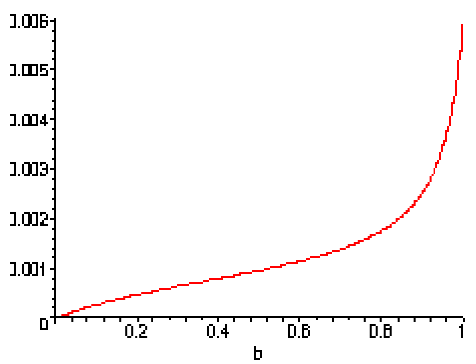

Figure 8: merger profitability for the firms combining the less expensive inputs with the more expensive ones

Figure 8.

A merger is always profitable for the participants, as shown in Figure 9, but less profitable than for the outsiders using at least one of the least expensive inputs.

Finally we consider the implications of these price changes on the profitability of the suppliers. We start by considering the profitability of the suppliers of the merged units: as expected mergers are always unprofitable for the upstream units serving the participants, Figures 10 and 11.

For the suppliers not serving the participants, instead, a merger is always profitable if the degree of substitutability is not too low, Figure 12; as b increases, in fact the incentives

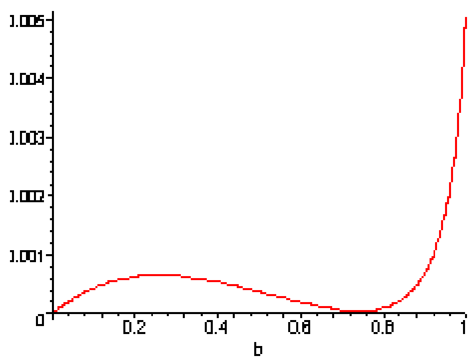

Figure 9: merger profitability for the participants

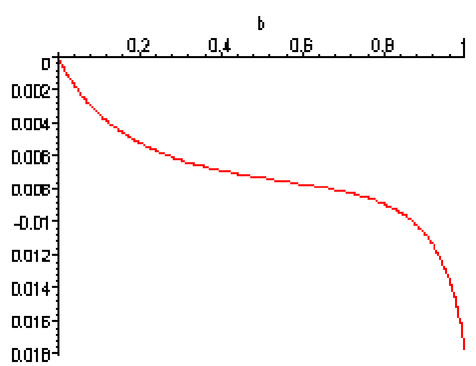

Figure 10: merger profitability of the supplier of the more expensive input provided to the participants

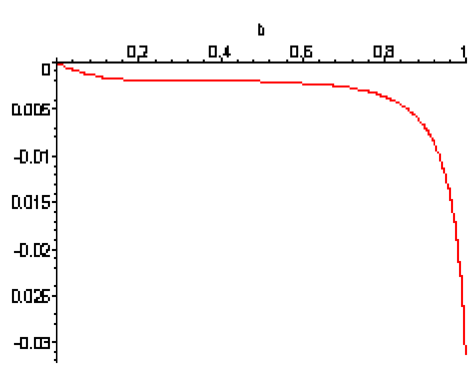

Figure 11: merger profitability of the input supplier of the less expensive input provided to the participants

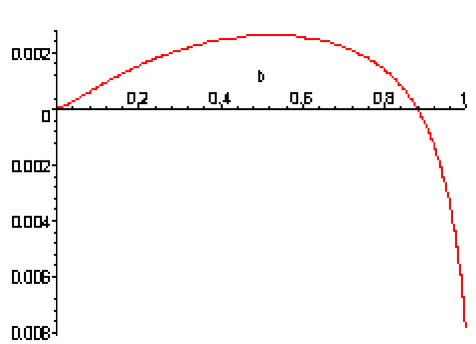

Figure 12: merger profitability of the input suppliers not serving the participants

for the input suppliers to set higher prices tend to increase thus reducing the input demand from their purchasers.

4. Subsequent mergers

One question is whether this type of merger might encourage subsequent mergers by the firms less benefitted or harmed and whether in this case the first merged units would still have incentives to merge. To investigate this issue we limit our attention to the case in which only horizontal integration in the downstream market is feasible. This assumption is not implausible if we consider that the upstream market is much more concentrated than the downstream market and that vertical integration could be used by the downstream units to price discriminate against their rivals.

We further assume that only two firm mergers are allowed. For the case of five input suppliers we have seen that, following a downstream merger between two units using different inputs, the outsiders using the other input, input 3, will face less favourable cost conditions. Thus these outsiders, units 2, 5, 8 and 9, might have incentives to respond by merging to recover the lost profits. More specifically, suppose that unit 2 merges with unit 5 and that unit 8 joins in a merger with unit 9. The profits of the post-merger units in this case are given by:

$$\pi_m = \left(A - q_1 - b \sum_{k=1, k\neq 1}^{10} q_k - w_1 - w_2 \right) q_1 + \left(A - q_{10} - b \sum_{l=1, l\neq 10}^{10} q_l - w_4 - w_5 \right) q_{10}$$

$$\pi_{2,5} = \left(A - q_2 - b \sum_{k=1, k\neq 2}^{10} q_k - w_1 - w_3 \right) q_2 + \left(A - q_5 - b \sum_{l=1, l\neq 5}^{10} q_l - w_2 - w_3 \right) q_5$$

$$\pi_{8,9} = \left(A - q_8 - b \sum_{k=1, k\neq 8}^{10} q_k - w_3 - w_4 \right) q_8 + \left(A - q_9 - b \sum_{l=1, l\neq 9}^{10} q_l - w_3 - w_5 \right) q_9$$

where π_m denotes the profits of the initial merged units.

The Nash solution to this problem is:

$$w_i = \frac{(b^2 - 5b - 2)(3b^4 - 40b^3 + 117b^2 - 68b - 28)C + (b-1)(b-2)(5b^3 - 45b^2 + 88b + 28)A}{3b^6 - 45b^5 + 191b^4 - 107b^3 - 574b^2 + 460b + 168}$$

for $i = 1, 2, 4, 5$

and

$$w_3 = \frac{\left(3b^6 - 537b^3 - 53b^5 + 297b^4 + 54b^2 + 260b + 56\right) C + (b-2)(4b^2 - 17b + 14)(b^2 - 7b - 2)A}{3b^6 - 45b^5 + 191b^4 - 107b^3 - 574b^2 + 460b + 168}$$

where $w_i < w_3$

Thus subsequent mergers between the less benefitted units lead to the same pattern of equilibrium input prices and therefore the relative merging gains do not change: mergers are always profitable for the originally merged units and for the outsiders, but more for the outsiders; they may instead be unprofitable for the new merged units, if the degree of product differentiation is sufficiently low, for $b > 0.48$, Figure 13. In this case however the less benefitted units would have no incentive to merge in the first place; this is shown

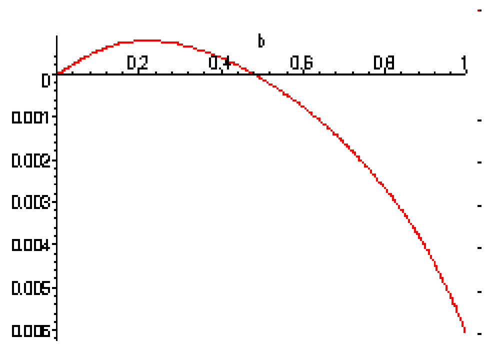

Figure 13: merger profitability for the new merged units

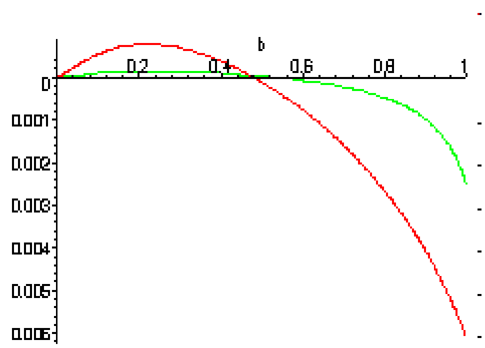

Figure 14: merger incentives for the new merged units

in Figure 14 where for $b > 0.45$, the pre-merger profitability, though negative, is always greater than the post-merger profitability.

The intuition behind this result may be traced by considering that the most expensive input, input 3, is common to all the less benefitted units and thus a merger between these units cannot induce lower prices for this input.

The first merged units however might also consider whether there are incentives for the initially more benefitted outsiders to merge with one of the less benefitted firms. We find that these incentives exist only if the two firms use different inputs and if the products are sufficiently differentiated. However the more benefitted units have always stronger incentives to merge with another benefitted unit using different inputs; more specifically, both units 3 and 7 and units 4 and 6 could earn more from a merger between them rather than from a merger with one of the less benefitted units using different inputs. In this case the post-merger profits are given by:

$$\pi_m = \left(A - q_1 - b\sum_{k=1,k\neq 1}^{10} q_k - w_1 - w_2\right) q_1 + \left(A - q_{10} - b\sum_{l=1,l\neq 10}^{10} q_l - w_4 - w_5\right) q_{10}$$

$$\pi_{3,7} = \left(A - q_3 - b\sum_{k=1,k\neq 3}^{10} q_k - w_1 - w_4\right) q_3 + \left(A - q_7 - b\sum_{l=1,l\neq 7}^{10} q_l - w_2 - w_5\right) q_7$$

$$\pi_{4,6} = \left(A - q_4 - b\sum_{k=1,k\neq 4}^{10} q_k - w_1 - w_5\right) q_4 + \left(A - q_6 - b\sum_{l=1,l\neq 6}^{10} q_l - w_2 - w_4\right) q_6$$

while the less benefitted firms continue to behave independently. The Nash equilibrium of this game is characterised by:

$$w_i = \frac{1}{3}\frac{(3b+1)(12b^3 - 47b^2 + 33b + 14)C - (b-1)(b-2)(9b^2 - 24b - 7)A}{6b^4 - 9b^3 - 38b^2 + 43b + 14}$$

for $i = 1, 2, 4, 5$

$$w_3 = \frac{1}{6}\frac{(54b^4 + 138b - 180b^3 + 32b^2 + 28)C + (-9b^4 - 130b^2 + 63b^3 + 60b + 28)A}{6b^4 - 9b^3 - 38b^2 + 43b + 14}$$

where $w_3 > w_i$

and mergers are always profitable for the merged units, but may be unprofitable for the outsiders if the products are close substitutes, Figure15. Further, since these mergers induce lower input prices for the participants, the profitability of the initial merger improves more than the profitability of the initially more benefitted outsiders. This is shown in Figure 16 where the first merged units earn exactly the same profits as the new

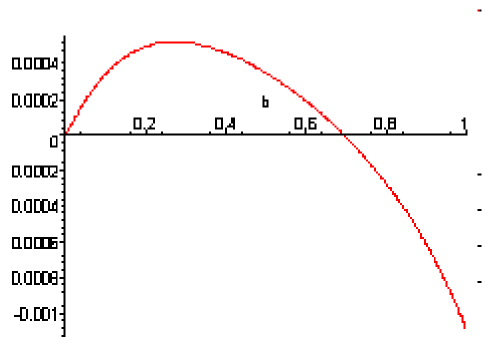

Figure 15: merger profitability for the outsiders

merged units. When however the products are sufficiently differentiated also the outsiders have incentives to merge. In this case the non-cooperative Nash equilibrium yields the following equilibrium input prices:

$$w_i = \frac{(3b+1)(24b+7-7b^2)C + (27b+7)(1-b)A}{11b^2+21-21b^3+85b} \tag{1}$$

for $i = 1, 2, 4, 5$

$$w_3 = \frac{(41b+45b^2+7-21b^3)C + (22b-17b^2+7)A}{11b^2+21-21b^3+85b} \tag{2}$$

with $w_3 > w_i$

and we get the same pattern of equilibrium input prices which leaves unchanged the relative merging gains. Mergers may still be unprofitable for the units using the most expensive input, input 3, if the products are not sufficiently differentiated, Figure 17. Since however these mergers further reduce the other input prices, the profitability of the other units increases, as shown in Figure 18.

Thus an initial merger between firms using different inputs is always profitable for the participants because, by inducing lower input prices from its suppliers, it triggers subsequent mergers that further decrease the monopoly power of the upstream units serving the original merged units.

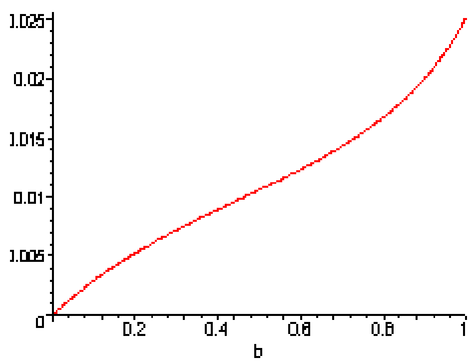

Figure 16: merger profitability for the participants

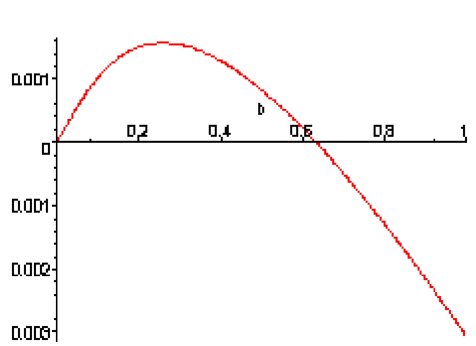

Figure 17: Merger profitability for the integrated units using a common input

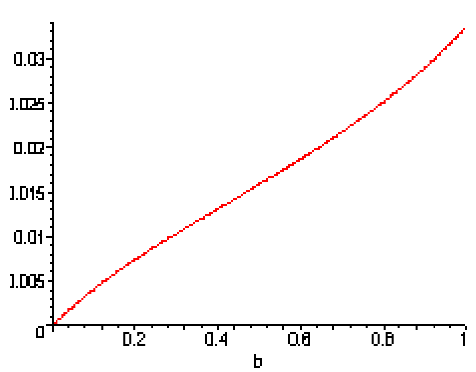

Figure 18: merger profitability for the integrated units using different input

We now consider the case in which the initial merger is between two units using a common input. As in the previous scenario, we assume that only one type of integration is feasible and that only two firm mergers are allowed by the antitrust authority. Are there incentives for the harmed downstream units to respond by merging? And if this is the case would be mergers still profitable for the first merged units? Before investigating this issue, we must notice that now only three units are harmed by the merging process: unit 10, which uses the two most expensive inputs (inputs 4 and 5), and units 3 and 4 that combine the most expensive input used by the participants, input 1, with respectively one of the most expensive inputs, inputs 4 and 5. Since only two firm mergers are allowed, one firm will necessarily be left out. We first consider the case in which units 3 and 4 agree to join in a merger. In this case the post merger profits are:

$$\pi_m = \left(A - q_1 - b \sum_{k=1, k\neq 1}^{10} q_k - w_1 - w_2 \right) q_1 + \left(A - q_2 - b \sum_{l=1, l\neq 2}^{10} q_l - w_1 - w_3 \right) q_2$$

$$\pi_{3,4} = \left(A - q_3 - b \sum_{k=1, k\neq 3}^{10} q_k - w_1 - w_4 \right) q_3 + \left(A - q_4 - b \sum_{l=1, l\neq 4}^{10} q_l - w_1 - w_5 \right) q_4$$

where π_m denotes the profits of the initially merged units. We notice that input 1 is used by both the merged units in the production of both their products. This means that the participants cannot shift production between their two plants to induce fiercer price competition between their suppliers; as a result of a merger, we should therefore expect a higher price for input 1. Each of the other inputs is instead used only in the production of one good; in this case the participants may force their suppliers to set lower input prices. Since input 1 is used only by the participants, a merger of this type should benefit all the outsiders thus eliminating any incentive for subsequent mergers. But there is still the question whether this merger is also profitable for the originally merged units. If this were not the case then the initial merger might not be carried through in the first place.

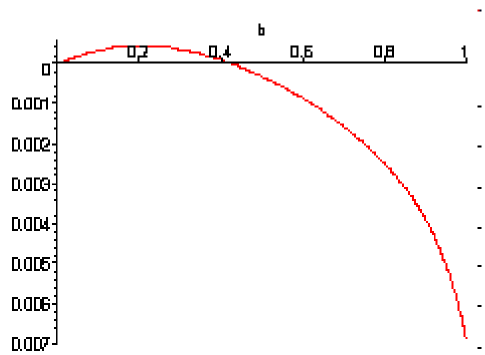

Figure 19: merger profitability for the merged units

Solving for the non-cooperative Nash equilibrium, we find that the solution is characterised by:

$$w_i = \frac{(5b+2)(b^4-31b^3+117b^2-68b-28)C+(b-1)(b-2)(17b^2-92b-28)A}{-168-476b+458b^2+237b^3-119b^4+5b^5}$$

for $i = 2,3,4,5$

$$w_1 = \frac{\left(-12b^5+338b^4-1118b^3+252b^2+536b+112\right)C+(b-2)(b^4-48b^3+226b^2-132b-56)A}{-168-476b+458b^2+237b^3-119b^4+5b^5}$$

where $w_1 > w_i$

Thus following a merger, all the outsiders face lower input prices and as a result earn higher profits. The profitability of the participants however may be negative if the products are not sufficiently differentiated, Figure 19. If however the products are not enough differentiated the new merged units would have no incentive to respond by merging. This is shown in Figure 20, where for $b > 0.47$, profits for these units are always higher if they don't merge.

Consider now the case in which unit 10 merges with unit 3. In this context, the profits of the post-merger units are:

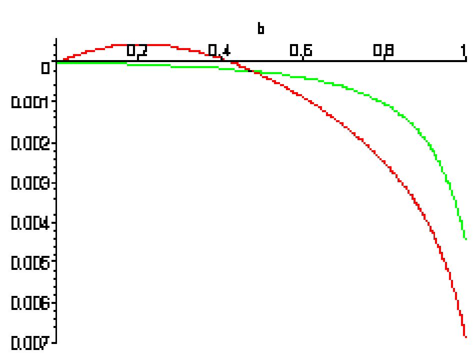

Figure 20: merger incentives for the new merged units

$$\pi_m = \left(A - q_1 - b \sum_{k=1,k\neq 1}^{10} q_k - w_1 - w_2 \right) q_1 + \left(A - q_2 - b \sum_{l=1,l\neq 2}^{10} q_l - w_1 - w_3 \right) q_2$$

$$\pi_{3,10} = \left(A - q_3 - b \sum_{k=1,k\neq 3}^{10} q_k - w_1 - w_4 \right) q_3 + \left(A - q_{10} - b \sum_{l=1,l\neq 10}^{10} q_l - w_4 - w_5 \right) q_{10}$$

This scenario seems more favourable for the originally merged units. A merger between these units in fact will induce a higher price for input 1, but the merger between units 3 and 10 will have an opposite effect. In this case the Nash equilibrium is characterised by:

$w_1 < w_5 < w_2 = w_3 < w_4$

and mergers are always profitable for the originally merged units, but more profitable for some of the outsiders. The most benefitted outsiders use the less expensive inputs, input 1 and input 5, and they always benefit from a merger. The outsiders using the most expensive inputs, however, might be harmed if products are close substitutes, Figure 21. The profitability of the new merged units instead may be positive only if the products are sufficiently differentiated, Figure 22.

The analysis is further complicated by the fact that each originally harmed outsider might find it more convenient to wait for the other harmed units to merge. If all the harmed units have the same reasoning then a possible outcome could be a hold up situation

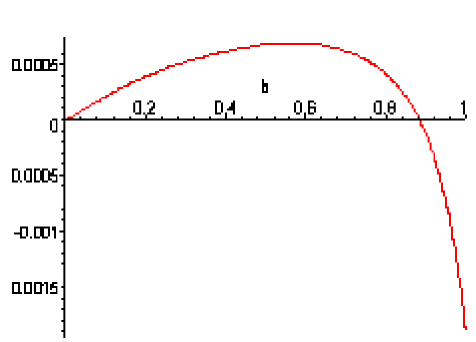

Figure 21: merger profitability of the outsiders using the most expensive inputs

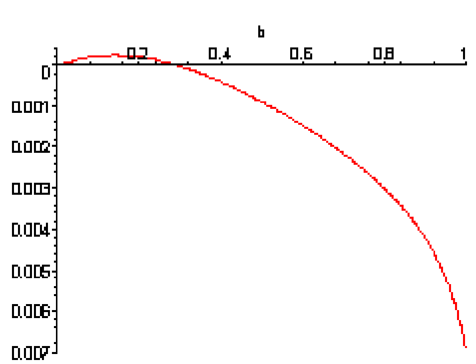

Figure 22: merger profitability of the new merged units

in which no further merger follows to the initial one. Since however a merger with the most harmed outsider is always less profitable, another possible outcome could be a merger between the less harmed units, which is the worst possible scenario for the originally merged units. Anticipating all this, would be the original merger still carried through? If the products are not too differentiated, then the original harmed units would have no incentives to merge and the initial merger would not trigger subsequent mergers by the harmed units.

The initial merged units however might also consider the possibility of subsequent mergers between other units. For instance, units 6, 7, 8, 9, though benefitting from the initial merger, could improve their profitability by merging with the unit using different inputs; more specifically, if there were no merger restrictions, unit 6 could profitably merge with unit 9 and unit 7 with unit 8. But if these two mergers took place, then unit 5 and unit 10 could improve their profitability by merging in turn. The post-merger profits in this case are given by:

$$\pi_m = \left(A - q_1 - b\sum_{k=1,k\neq 1}^{10} q_k - w_1 - w_2\right) q_1 + \left(A - q_2 - b\sum_{l=1,l\neq 2}^{10} q_l - w_1 - w_3\right) q_2$$

$$\pi_{5,10} = \left(A - q_5 - b\sum_{k=1,k\neq 5}^{10} q_k - w_2 - w_3\right) q_5 + \left(A - q_{10} - b\sum_{l=1,l\neq 10}^{10} q_l - w_4 - w_5\right) q_{10}$$

$$\pi_{6,9} = \left(A - q_6 - b\sum_{k=1,k\neq 6}^{10} q_k - w_2 - w_4\right) q_6 + \left(A - q_9 - b\sum_{l=1,l\neq 9}^{10} q_l - w_3 - w_5\right) q_9$$

$$\pi_{7,8} = \left(A - q_7 - b\sum_{k=1,k\neq 7}^{10} q_k - w_2 - w_5\right) q_7 + \left(A - q_8 - b\sum_{l=1,l\neq 8}^{10} q_l - w_3 - w_4\right) q_8$$

$$\pi_3 = \left(A - q_3 - b\sum_{k=1,k\neq 3}^{10} q_k - w_1 - w_4\right) q_3$$

$$\pi_4 = \left(A - q_4 - b \sum_{k=1, k \neq 4}^{10} q_k - w_1 - w_5 \right) q_4$$

The Nash solution to this game is characterised by the following equilibrium input prices:

$w_2 = w_3 < w_4 = w_5 < w_1$

and mergers are always profitable for the new merged units and the outsiders but may be unprofitable for the first participants if the products are not sufficiently differentiated, Figures $23 - 25$.

The intuition behind this result is very simple: in contrast to the first merged units, each of the new merged entities uses different inputs and thus may shift production between its two goods to induce lower prices from its suppliers and enjoy better cost conditions.

A comparison of Figures 9 and 25 suggests however that the initial merger would be still carried through, if the products are sufficiently differentiated. In this case however also the last two outsiders would have incentives to merge and the non-cooperative Nash equilibrium would yield the same input prices as in (1) and (2) but with $w_1 > w_i$ and $i = 2, 3, 4, 5$. As we have already shown, in this case mergers are always profitable for the units using different inputs, but may be unprofitable for the firms using a common input (in this case, input 1), if the products are not enough differentiated, Figures $17 - 18$.

Anticipating the reactions of the non-merging firms, the first merged units would choose to merge, whenever possible, with a firm using different inputs; in this case, in fact, the cost reduction induced by a merger is stronger and subsequent mergers have a positive effect on the profitability of the initial merger.

5. CONCLUSIONS

We have analysed the profitability of downstream mergers when a limited number of inputs, provided by monopolist suppliers, may be combined to produce differentiated products sold by oligopolist downstream firms. We have shown that when the number

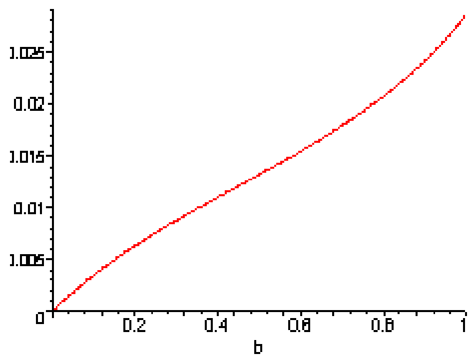

Figure 23: merger profitability for the new merged units

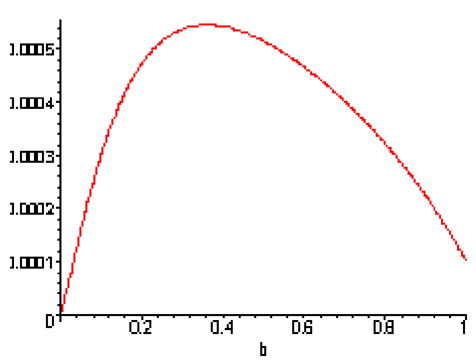

Figure 24: Merger profitability for the outsiders

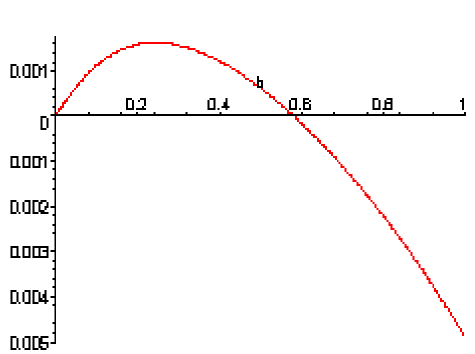

Figure 25: merger profitability for the initial merged units

of products that may be produced is very small, it is always better to be an outsider rather than participate in a merger. This happens because, following a merger, some input suppliers of the merged units are induced to set lower prices for their inputs, thus producing a positive externality to some of the outsiders using the same inputs; cost conditions for these outsiders improve more than for the participants and this obviously contributes positively to their profitability.

When instead the number of final products is sufficiently large, the incentives to merge may drastically change: even if some outsiders benefit more from a merger others lose and this may create incentives for firms to merge to eliminate the risk of being harmed by a merger between competitors. We have considered two different scenarios: a merger between firms using different inputs and a merger between firms using a common input. We have shown that in the first scenario, a merger induces the input suppliers of the participants to set lower prices for their inputs. Thus following a merger the participants and the outsiders using the same inputs will face identical cost conditions and, as in the case of exogenous input prices, the outsiders will take a free-ride and earn higher profits. The outsiders using the other inputs, instead, will face less favourable price conditions and, if the products are not too differentiated, earn lower profits. We therefore investigated whether in this case there might be incentives for the harmed units to respond by merging. Our analysis shows that these incentives exist only when the less benefitted units earn positive profits and therefore when the products are sufficiently differentiated. In this case, however, a merger between the less benefitted units leads to the same pattern of equilibrium input prices and as a consequence doesn't affect the relative merging gains. Thus even if a merger is more profitable for some of the outsiders, firms may still have incentives to merge to preempt a merger between their competitors even if this merger may trigger subsequent mergers from the less benefitted firms. Further, if the products are not too differentiated, no harmed unit will have incentives to merge.

For the case in which the initial merged units use a common input, we have shown that a merger induces the input suppliers of the participants to set a higher price for the common input but lower prices for the other inputs. This in turn produces a positive externality to the outsiders using at least one of the less expensive inputs also used by the participants, but a negative externality to all the other firms. Also in this case we therefore investigated the incentives for the harmed units to respond by merging. We showed that each harmed unit might benefit more from a merger between the other harmed units and that for each participant a merger with the most harmed outsider is always less profitable. Thus we could figure out two possible equilibrium outcomes: a hold up situation in which no harmed unit would merge or alternatively a merger between the less harmed units. In the first case the initial merged units would always have incentives to merge, but in the second scenario they would be better off staying out of the merger and no initial merger would be carried through. If however the products are not too differentiated, then the initially harmed units would have no incentives to respond by merging and no subsequent merger would follow. Thus if we limit our attention to the reactions of the less benefitted or harmed units, the initial merger will always be carried through unless the products are sufficiently differentiated.

The first merged units however might also consider the possibility of subsequent mergers by other units. We have seen that in both scenarios there is always an incentive for some of the initially benefitted units to merge with another benefitted unit using different inputs; this type of merger increases the profitability of the initial merger when the first merged units use different inputs but may be unfavourable for the initial participants when they use a common input. Thus mergers with firms using different inputs should be always preferred to mergers with firms using a common input, because, by reducing the monopoly power of the suppliers, they induce greater cost savings for the participants.

In our analysis we considered only a limited number of inputs, but we believe that the model may be easily extended to the case of many different inputs. Our results assumed fixed-proportion technologies. Thus our analysis could be extended to allow for variable-proportions technologies or substitutability between different inputs. Finally, in our equilibrium analysis we assumed that only one type of integration was feasible. It might be interesting to investigate which equilibrium structures would emerge if firms were allowed to form/break links with other units.

A. Appendix: equilibrium outcomes

A.1. The case of three input suppliers. Consider a horizontal merger between firms 1 and 2 in the downstream market; in this case, the post-merger entity maximises:

$$\pi_{12} = (A - q_1 - b(q_2 + q_3) - w_1 - w_2)q_1 + (A - q_2 - b(q_1 + q_3) - w_1 - w_3)q_2$$

while the other firms continue to act independently.

The Cournot solution for the downstream market is:

$$q_1 = \frac{1}{4}\frac{w_2(2b - b^2 - 4) + w3(6b - 3b^2) + A(2b^2 - 6b + 4) + 4w_1(b - 1)}{b^3 - 3b^2 + 2}$$

$$q_2 = \frac{1}{4}\frac{w_3(2b - b^2 - 4) + w_2(6b - 3b^2) + A(2b^2 - 6b + 4) + 4w_1(b - 1)}{b^3 - 3b^2 + 2}$$

$$q_3 = \frac{w_3(2 + b) - 2bw_1 + w_2(b + 2) - 2A}{b^2 - 2b - 2}$$

Total input demands for the upstream units are:

$$x_1 = q_1 + q_2$$

$$x_2 = q_1 + q_3$$

$$x_3 = q_2 + q_3$$

and, given the candidate equilibrium quantities, the solution to the upstream firms' problem yields the equilibrium input prices:

$$w_1 = \frac{(5A-2C)b^3 - 2(4C+5A)b^2 - 8(2A+C)b + 24(a+c)}{2b^3 - 7b^2 - 10b + 18}$$

$$w_2 = w_3 = \frac{Ab(b^2-5) - b^2(3C+2A) + 6(a+c)}{2b^3 - 7b^2 - 10b + 18}$$

and hence the equilibrium quantity and profit levels:

$$\pi_{12} = \frac{1}{8}\frac{(b+1)(5b^3 - 10b^2 - 16b + 24)^2(A-2C)^2}{(b^2-2b-2)^2(2b^3-7b^2-10b+18)^2}$$

$$\pi_3 = \frac{1}{16}\frac{(24 - 2b^3 + b^4 - 8b^2)^2(A-2C)^2}{(b^2-2b-2)^2(2b^3-7b^2-10b+18)^2}$$

$$\pi_1^U = -\frac{1}{8}\frac{(5b^3 - 10b^2 - 16b + 24)^2(A-2C)^2}{(b^2-2b-2)(2b^3-7b^2-10b+18)^2}$$

$$\pi_2^U = \pi_3^U = \frac{1}{4}\frac{(1-b)(b-3)^2(b+2)^2(b^2+4b-8)(A-2C)^2}{(b^2-2b-2)(2b^3-7b^2-10b+18)^2}$$

Mergers are profitable for the participants if:

$$\pi_{12} - 2\pi_D^N = \frac{b(576 + 1568b - 4144b^2 - 2851b^6 + 1106b^7 - 1099b^5)(A-2C)^2}{8(b^2-2b-2)^2(2b^3-7b^2-10b+18)^2(b+1)^2(b-3)^2} +$$

$$+\frac{(6832b^4 - 2124b^3 - 191b^9 + 25b^{10} + 266b^8)(A-2C)^2}{8(b^2-2b-2)^2(2b^3-7b^2-10b+18)^2(b+1)^2(b-3)^2} > 0$$

Mergers are instead always profitable for the downstream outsiders:

$$\pi_3 - \pi_D^N = \frac{b(b^5 - 8b^4 + 15b^3 + 22b^2 - 56b - 16)(b^6 - 29b^4 + 22b^3 + 152b^2 - 80b - 144)(A-2C)^2}{16(b^2-2b-2)^2(2b^3-7b^2-10b+18)^2(b+1)^2(b-3)^2} > 0$$

and there is never a merger advantage:

$$\pi_{12} - 2\pi_3 = \frac{b(29b^6 - 63b^5 - 192b^4 + 388b^3 + 432b^2 - 608b - 192 - b^7)(A - 2C)^2}{(b^2 - 2b - 2)^2(2b^3 - 7b^2 - 10b + 18)^2} < 0$$

A.2. The case of five input suppliers.

A merger between two units using different inputs. The profits of the merged entity in this case may be expressed as:

$$\pi_m = \left(A - q_1 - b \sum_{k=1, k\neq 1}^{10} q_k - w_1 - w_2 \right) q_1 + \left(A - q_{10} - b \sum_{l=1, l\neq 10}^{10} q_l - w_4 - w_5 \right) q_{10}$$

And all the input suppliers, apart from the supplier providing input 3, serve the merged units; thus the equilibrium input prices w_j are identical for $j = 1..5$ and $j \neq 3$:

$$w_j = \frac{(b^2 - 5b - 2)(5b^3 - 47b^2 + 33b + 14)C - (b - 1)(b - 2)(b^3 - 14b^2 + 45b + 14)A}{(b + 2)(3b^4 - 44b^3 + 168b^2 - 95b - 42)}$$

$$w_3 = \frac{2(4b^5 - 35b^2 + 226b^3 - 132b - 28 - 60b^4)C - (b - 2)(b^4 - 20b^3 + 106b^2 - 64b - 28)A}{2(b + 2)(3b^4 - 44b^3 + 168b^2 - 95b - 42)}$$

$$q_m = \frac{(b - 2)(3b^5 - 52b^4 - 136b + 234b^3 - 61b^2 - 28)(A - 2C)}{(b^2 - 9b - 2)(b + 2)(3b^4 - 44b^3 + 168b^2 - 95b - 42)}$$

$$q_i = \frac{1}{2} \frac{(b^2 - 5b - 2)(b^4 - 20b^3 + 106b^2 - 64b - 28)(A - 2C)}{(b^2 - 9b - 2)(b + 2)(3b^4 - 44b^3 + 168b^2 - 95b - 42)}$$

$$q_k = -\frac{1}{2} \frac{(3b^5 - 52b^4 - 136b + 234b^3 - 61b^2 - 28)(A - 2C)}{(b^2 - 9b - 2)(b + 2)(3b^4 - 44b^3 + 168b^2 - 95b - 42)}$$

$$\pi_m = \frac{1}{2} \frac{(b + 1)(b - 2)^2(3b^5 - 52b^4 - 136b + 234b^3 - 61b^2 - 28)^2(A - 2C)^2}{(b^2 - 9b - 2)^2(b + 2)^2(3b^4 - 44b^3 + 168b^2 - 95b - 42)^2}$$

$$\pi_{Di} = \frac{1}{4}\frac{(b^2-5b-2)^2(b^4-20b^3+106b^2-64b-28)^2(A-2C)^2}{(b^2-9b-2)^2(b+2)^2(3b^4-44b^3+168b^2-95b-42)^2}$$

$$\pi_{Dk} = \frac{1}{4}\frac{(3b^5-52b^4-136b+234b^3-61b^2-28)^2(A-2C)^2}{(b^2-9b-2)^2(b+2)^2(3b^4-44b^3+168b^2-95b-42)^2}$$

$$\pi_{Uj} = \frac{1}{2}\frac{(4b^3-39b^2+24b+16)(b^3-14b^2+45b+14)^2(b-2)(1-b)(A-2C)^2}{(b^2-9b-2)(b+2)^2(3b^4-44b^3+168b^2-95b-42)^2}$$

$$\pi_{U3} = \frac{1}{2}\frac{(b^4-20b^3+106b^2-64b-28)^2(b^2-5b-2)(2-b)(A-2C)^2}{(b^2-9b-2)(b+2)^2(3b^4-44b^3+168b^2-95b-42)^2}$$

where firms $i = 2, 5, 8, 9$ use input 3 whereas firms $k = 3, 4, 6, 7$ don't.

Mergers are always more profitable for the outsiders not purchasing input 3:

$$\pi_m - 2\pi_{Dk} = \frac{1}{2}\frac{(b-3)b^2(3b^5-52b^4+234b^3-61b^2-136b-28)^2(A-2C)^2}{(b^2-9b-2)^2(b+2)^2(3b^4-44b^3+168b^2-95b-42)^2} < 0$$

but less profitable for firms using input 3:

$$\pi_m - 2\pi_{Di} = \frac{b(-64744b^4+4512b-248468b^7+117389b^6+11056b^2-18484b^3+448)}{2(b^2-9b-2)^2(b+2)^2(3b^4-44b^3+168b^2-95b-42)^2}+$$

$$+\frac{b(89401b^5-38023b^9+144450b^8+5094b^{10}+9b^{12}-340b^{11})}{2(b^2-9b-2)^2(b+2)^2(3b^4-44b^3+168b^2-95b-42)^2} > 0$$

A merger between two units using a common input. In this case the merged units maximise:

$$\pi_m = \left(A - q_1 - b\sum_{k=1,k\neq 1}^{10} q_k - w_1 - w_2\right) q_1 + \left(A - q_2 - b\sum_{l=1,l\neq 2}^{10} q_l - w_1 - w_3\right) q_2$$

and the Nash solution to this problem is characterised by:

$w_4 = w_5 > w_1 > w_2 = w_3$

where

$$
\begin{aligned}
w_1 \quad = \quad & \frac{a(2-b)(76b^6-1587b^5+10491b^4-21806b^3+3456b^2+8512b+1568)}{(-119168b^3+391b^7-6292b^6+26799b^5-4606b^4+46584b^2+55664b+9408)} + \\
& -\frac{(-543b^7+9770b^6-54129b^5+90182b^4+25032b^3-49784b^2-24752b-3136)c}{(-119168b^3+391b^7-6292b^6+26799b^5-4606b^4+46584b^2+55664b+9408)}
\end{aligned}
$$

$$
\begin{aligned}
w_2 \quad = \quad & w_3 = \frac{a(1-b)(b-2)(93b^5-1703b^4+9368b^3-12860b^2-9968b-1568)}{(-119168b^3+391b^7-6292b^6+26799b^5-4606b^4+46584b^2+55664b+9408)} + \\
& -\frac{(-577b^7+10256b^6-56125b^5+93346b^4+24472b^3-51816b^2-25200b-3136)c}{(-119168b^3+391b^7-6292b^6+26799b^5-4606b^4+46584b^2+55664b+9408)}
\end{aligned}
$$

$$
\begin{aligned}
w_4 \quad = \quad & w_5 = \frac{a(2-b)(46b^6-1137b^5+8763b^4-20794b^3+3912b^2+8512b+1568)}{(-119168b^3+391b^7-6292b^6+26799b^5-4606b^4+46584b^2+55664b+9408)} + \\
& -\frac{(-47960b^2-24752b+8750b^6+28168b^3-483b^7-48873b^5-3136+81246b^4)}{(-119168b^3+391b^7-6292b^6+26799b^5-4606b^4+46584b^2+55664b+9408)}
\end{aligned}
$$

Hence the equilibrium quantities and profits for the downstream units are:

$$
q_i = \frac{(b-2)(286b^7-5995b^6+40143b^5-82824b^4-19976b^3+51024b^2+24976b+3136)(a-2c)}{2(b^2-9b-2)(46584b^2+55664b+9408-119168b^3-4606b^4+26799b^5+391b^7-6292b^6)}
$$

$$
q_j = \frac{(47b^8-1368b^7+14140b^6-61031b^5+90094b^4+31912b^3-47976b^2-24752b-3136)(a-2c)}{(b^2-9b-2)(46584b^2+55664b+9408-119168b^3-4606b^4+26799b^5+391b^7-6292b^6)}
$$

$$
q_k = \frac{(30b^8-1006b^7+11713b^6-55807b^5+88020b^4+26104b^3-50112b^2-24976b-3136)(a-2c)}{(b^2-9b-2)(46584b^2+55664b+9408-119168b^3-4606b^4+26799b^5+391b^7-6292b^6)}
$$

$$
q_5 = \frac{-(-76b^7+34919b^5-14168b^3+53160b^2-80750b^4+17b^8+25200b+3136-3568b^6)(a-2c)}{(b^2-9b-2)(46584b^2+55664b+9408-119168b^3-4606b^4+26799b^5+391b^7-6292b^6)}
$$

$$q_{10} = \frac{(77b^8 - 2088b^7 + 19858b^6 - 76695b^5 + 95290b^4 + 38040b^3 - 47064b^2 - 24752b - 3136)(a - 2c)}{(b^2 - 9b - 2)(46584b^2 + 55664b + 9408 - 119168b^3 - 4606b^4 + 26799b^5 + 391b^7 - 6292b^6)}$$

$$\pi_m = 2(b+1)q_i^2,\ \pi_{Dj} = q_j^2,\ \pi_{Dk} = q_k^2,\ \pi_{D5} = q_5^2,\ \pi_{D10} = q_{10}^2$$

for $i = 1, 2,\ j = 3, 4,\ k = 6, 7, 8, 9.$

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