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Abstract

We consider the interaction between the risk of unemployment, random house prices, consumption and savings. A critical decision is that of refinancing house purchase, up to 100% mortgages are possible. There is also a fixed transaction cost of refinancing. In a CARA framework we derive the value function for a finite horizon, the policy of refinance and the consumption function. Either there is a maximum mortgage or a zero mortgage depending on interest rates, house prices and the transaction cost. The consumption function is linear in wealth and in the uncertainty caused by employment status and house prices of the future. Since there is either 100% or 0% equity withdrawal, consumption jumps when there is refinancing.

Keywords: precautionary savings, employment risk, mortgages, housing

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1 Introduction

In many European countries the housing market plays a special role. Together with pensions, housing is one of the major spending and financial decisions facing consumers. Some economies have a liquid rental market for housing and an illiquid retail market; others have a negligible rental market outside metropolitan areas but an active retail market (Chiuri and Jappelli, 2001). Flavin and Yamashita (2003) stress that with a thin rental market, housing decisions have to balance financial asset portfolio considerations with the need for housing services. This means that the typical life cycle portfolio composition sees systematic changes in the share of housing in wealth. Much applied work shows that house prices play a significant role in determining many economic variables at the aggregate level, such as consumption, savings and GDP growth (Muellbauer and Murphy, 1997). There is also a recent literature that looks at the collateral/buffer stock effect of housing investment on precautionary savings. Here the argument is that where the retail housing market is illiquid, housing equity serves as a buffer stock of wealth against low probability but very bad income shocks. Thus, housing is rarely traded but it allows a higher level of mean consumption since if the worst income events occur there is a buffer stock of wealth that can be realised (Pelizzon and Weber, 2003). This ignores the fact that most house owner/buyers hold mortgage debt against their housing (Campbell and Cocco, 2003) so that net housing wealth may be quite low. A further discussion is about irrationality of the mortgage market, there is inertia in the refinancing decision. For example, in the US, where most mortgages are at fixed nominal interest rates, there is active refinancing as interest rates change (Majumdar, 2004), but in Europe this is less common (Smith and Vass, 2004).

One of the worst income events is that of unemployment. Important questions here are whether individuals build up buffer stocks of bond or net housing wealth to allow adjustment to bad future employment shocks. Since one must always live somewhere, a buffer stock of housing wealth is only useful if it can be used as collateral against loans, which is predominantly executed by remortgaging. By contrast, with a differentiated housing stock, a consumer entering a period of unemployment can trade down in their housing; similarly, towards the end of life, when the remaining uncertainty about labour income is minimal¹.

Households face various risks in their employment and asset decisions. Labour income risk consists partly of shocks to the wage rate and partly of shocks to the availability of jobs. In financial assets there is nominal and real interest rate uncertainty and in the case of real assets such as housing, the risk of future house prices. Illiquid financial assets with a long term (like mortgages) also involve liquidity risk - to keep the house the individual must be able to keep up repayments (Fratantoni, 2001). In addition there are trends, so that a common assumption is of a hump-shaped mean life cycle real wage, perhaps with some trend growth, probably some small trend growth in real house prices and a small

¹It is possible that the consumer could get better equity withdrawal from trading down (see section 7).

positive trend in the real interest rate (Browning and Crossley, 2001). The importance of these different risks varies over the life cycle; typically for highly geared young households, with housing debt a high proportion of wealth and income, the liquidity risk is higher than for older households with, on average, more diversified wealth.

Another issue in the literature is related to the degree of substitutability of housing and financial wealth in determining consumption. Will two households with the same aggregate wealth, the same labour income prospects and the same preferences follow the same consumption function if one of them has a much higher proportion of housing wealth than the other? The empirical results here are mixed (Hoynes and McFadden, 1994; Bostic *et al.* 2004; Majumdar, 2004). The question is important since consumer spending fluctuations are often seen as an important determinant of business cycle fluctuations. The main transmission mechanism for converting changes in housing wealth into disposable resources is the mortgage; therefore, analysing how mortgage decisions are made when housing wealth changes is crucial.

Campbell and Cocco (2003) is a major study of the relative advantages of fixed and variable rate mortgages that is close to our concerns. They use a constant relative risk aversion utility function with an essentially time neutral consumer, which means that they rely on calibrated numerical simulation involving grid search over both their state and control variables to derive the optimum. This approach allows quite general assumptions about the stochastic processes that drive house prices, interest rates and labour income. However, they do not directly address our concerns that are about the optimal mortgage refinancing policy and the effects of the combination of house price uncertainty, imperfect capital markets and employment risk on the mortgage refinancing decision and savings.

We use a real model with time additive utility over a finite horizon with a positive rate of time preference. The felicity function exhibits prudence. We have employment and real house price risk and time varying wages and real interest rates (the latter are driven by stochastic processes but are foreseen by the individual). There are market imperfections in the mortgage market: a fixed transaction cost and the mortgage can never exceed the latest realised house price. The mortgage is an adjustable rate mortgage taken for the remaining life of the consumer but can be prepaid at any date, this means that we include the effects of the risk that mortgage repayments may exceed current income.

We find that in our framework there is a simple rule for refinancing: consumers refinance when the gain from the wedge between the mortgage rate and the savings interest rate (which depends on the current house price and mortgage, if any) is sufficiently high to cover the transaction cost. The refinancing may either involve reducing the current mortgage to zero or increasing it to its maximum level, permitted by the imperfect capital markets. In either case it is a bang-bang policy and the refinancing decision is driven just by financial efficiency. This is a simple strong result. In addition we analyse the effects of house price uncertainty and employment risk on consumption and savings. Generally employment risk raises precautionary savings; but future house price

uncertainty may either raise or reduce current consumption. On the one hand, the chance of high future house prices gives the prospect of possible high future collateralised borrowing possibilities; on the other hand, it gives the risk of ending up with a low housing asset value when meeting a spell of unemployment. Since the optimal refinancing has all or nothing features, when it is optimal to refinance, consumption will typically jump; it is like a windfall gain in wealth that can then be optimally consumed over the remaining future.

The financial instruments that we consider are very simple: a fixed term mortgage for up to the length of life and a one period bond. This means that, since the individual needs a house throughout life, the only way of avoiding large disposable wealth in the final period of life when the house is sold is to accumulate debts in the one period bond, which can then be paid off in the final period with the proceeds from the house sale. In the simulations we provide this occurs but in several cases consumption also jumps in the last years of life. In related papers it also occurs (Fratantoni, 2001; Campbell and Cocco, 2003, where utility/felicity rises at the end of life).

To see how often it pays to refinance and its quantitative effect on consumption we give some simulations mainly based on the typical hump-shaped pattern of real wage earnings of the employed and with random but trendless house prices. Here we find that the extent of refinancing critically depends on the transaction cost, but, apart from this, optimal savings have the usual pattern of the literature depending on the relative time preference of the consumer, except that the refinancing tends to serve as a lumpsum shock to cash-on hand in periods subsequent to those in which it occurs. Patient consumers will tend to produce a hump-shaped pattern of financial assets and growing consumption. Impatient consumers will tend to have falling consumption financed by debt in the first half of life, paying it off in middle age and later life. Typically the simulations do not exhibit hump-shaped consumption, this partly reflects the effect of the high value of wealth at the end of life caused by the house, perhaps partly the parameter values used.

An interesting special case arises when consumers know that they will retire before the end of the horizon. In this scenario individuals typically save more than when they can work all their life, but refinance in exactly the same way and for the same reasons. Interestingly in later life, when retired, consumers have growing consumption whether they are patient or impatient. An alternative simulation has wages and house prices with a positive trend increase, and still noise in house prices. Generally the consumer uses mortgage finance more heavily in this case, and also goes into debt in financial assets whatever the relation between the rate of time preference and interest rates. Foreseeing high future income on average makes it worth borrowing in the early part of life.

The plan of the paper is to give the assumptions in section 2, derive the overall value function explicitly in sections 3 and 4. In section 5 we analytically derive the consumption function. The calibrated simulations are in section 6. We then briefly discuss extensions and conclude.

2 The Model Assumptions

We take a finite time horizon T of discrete time periods t . The general form of the budget constraint without any mortgage refinancing is

$$A_{t+1} = (1 + r_t)A_t + w_t^s - \rho_t M_t - c_t$$

where ρ_t, r_t are the mortgage and interest rate respectively and are perfectly foreseen. M_t is the mortgage debt at the start of period t . w_t^s represents labour income and has two possible values: either the individual has a job and labour income is the wage w_t or there is no job and labour income is unemployment benefit of B_t . A_t are nonmortgage financial assets at the start of period t , which earn a one period interest rate of r_t .

The mortgage is a pure bond which lasts the horizon but which can be paid off/refinanced each period. House prices \tilde{p}_t are random and realised at the start of the period. There is a mortgage financing constraint saying $M_t \leq p_{t-1}$. In reality in the UK 100% mortgages are possible but usually only on relatively standard property. ²In every period before the final one, there is a chance α of being in employment. In the final period the individual is unemployed for sure.

Within a period $t < T$ the timing is that initially there is a portfolio (A_t, M_t) and p_t, ρ_t, r_t, w_t are all known at the period start. Interest income is paid and received at the start of the period. The mortgage is a debt of given face value with a variable one period interest rate and with maturity date of up to T . Each period consumers can refinance the mortgage if they wish, repaying the existing debt and taking out a new mortgage, again with a maturity date of T . The reason for doing this is to alter the debt position to take account of house price and relative real interest changes. If refinancing is undertaken, the consumer chooses a new mortgage size $M_{t+1} \leq p_t$ and has to pay a transaction cost of k . Then consumption c_t is chosen, next employment status is realised for period t and finally assets to carry forward into the next period A_{t+1} are determined within the budget constraint. This means that assets initially bear all the effects of shocks in employment, but this only affects the current period and is insignificant over the lifetime³. Allowing for refinancing the mortgage, the budget constraint in periods before the final one becomes

$$A_{t+1}^s = (1 + r_t) A_t + w_t^s + (1 - \rho_t) M_{t+1} - M_t - c_t$$

where w_t^s is either w_t or B_t . The carry forward of assets is random depending on the employment state at t - whether $w_t^s = w_t, B_t$. Note also that as viewed from earlier periods M_{t+1} is random, since it depends on the realisation of random house prices through the constraint $M_{t+1} \leq p_t$. Without refinance $M_{t+1} = M_t$

²In addition there is usually a current income multiplier so that the constraint might read $M_t \leq \min[p_t, \mu w_t]$. This could include the case in which, when unemployed, no refinancing is possible. See section 7.

³This is unimportant and arises from the definition of the period, but it simplifies the analysis to use this timing. You may think of shopping on Saturday and being paid the subsequent Friday.

of the literature works with isoelastic felicity, this generally requires approximation to get solutions and there is some evidence (Gourinchas and Parker, 2002) that the error involved can be substantial; on the other hand, since isoelastic preferences have unbounded marginal utility at zero consumption, it generally serves to keep cash on hand positive for sure and so almost acts like a liquidity constraint. With CARA, marginal utility is finite at zero consumption, so we may expect to see the consumer actively go into debt. However the lifetime budget constraint prevents him dying in debt.

3 Value Function

Based on Merton (1992), and Berloff and Simmons (2003), we conjecture that the value function at t is

$$V_t(A_t, M_t, p_t) = \alpha_t - \beta_t \exp[-b\delta_t((1+r_t)A_t - M_t)]$$

In the final period there is no issue of refinancing for $T+1$, the house is sold, any outstanding mortgage is redeemed (in which case the transaction cost has to be paid) and all remaining financial assets are consumed

$$\begin{aligned} c_T &= (1+r_t)A_T + p_T + w_T^s - M_T - k_T && \text{if } M_T > 0 \\ &= (1+r_t)A_T + p_T + w_T^s && \text{if } M_T = 0 \end{aligned}$$

We also assume that in the last period for sure the individual is unemployed so that $w_T^s = B_T$. This is without loss of generality since consumption is determined prior to knowledge of employment status and then in the last period it would have to be reined back to a level that will prove feasible if it turns out that the individual is unemployed in that period, as it is impossible to die in debt. We can also think of this as retirement in the final period. For the last period the value function has the above form. with

$$\begin{aligned} \alpha_T &= 1 \\ \beta_T &= \exp(-bB_T) \exp(-bp_T) && \text{if } M_T = 0 \\ \beta_T &= \exp(-bB_T) \exp(-b(p_T - k)) && \text{if } M_T > 0 \\ \delta_T &= 1 \end{aligned}$$

Moreover, since there is no employment risk in the last period, expectations only have to be taken over the house price:

$$\begin{aligned} E_{T-1}\beta_T &= \exp(-bB_T)E \exp(-b(p_T - k)) && \text{if } M_T > 0 \\ &= \exp(-bB_T)E \exp(-bp_T) && \text{if } M_T = 0 \end{aligned}$$

For earlier periods the form of the value function depends on whether it is optimal to undertake refinancing. We derive the value functions at t with and without refinancing and then compare them to determine the optimal refinancing decision.

With refinancing in any period before the final one c_t is determined to :

$$\max_{c_t} \{u(c_t) + \phi EV_{t+1}(A_{t+1}) | A_{t+1}^s = (1+r_t)A_t + (1-\rho_t)M_{t+1} - M_t - c_t + w_t^s - k\}$$

Define the expected utility term corresponding to next periods labour income as

$$W_t = [\alpha \exp(-b\delta_{t+1}(1+r_{t+1})w_t) + (1-\alpha) \exp(-b\delta_{t+1}(1+r_{t+1})B_t)]^{1/[1+\delta_{t+1}(1+r_{t+1})]}$$

and the "discounted future interest rate" as

$$\Delta_t = \frac{\delta_{t+1}(1+r_{t+1})}{1+\delta_{t+1}(1+r_{t+1})}$$

The appendix shows that conditional on the refinancing decision, the value function with refinancing is

$$\begin{aligned} V_t^R(A_t) &= 1 + \phi\alpha_{t+1} - [\phi(E\beta_{t+1})\delta_{t+1}(1+r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\ &\quad \cdot \exp(-b\Delta_t[(1+r_t)A_t - M_t])W_t \\ &\quad \cdot \exp(-b\Delta_t[\{(1-\rho_t)M_{t+1} - k\} - M_{t+1}/(1+r_{t+1})])/\Delta_t \end{aligned}$$

At the start of t , given that remortgaging takes place, the mortgage refinancing decision is to choose M_{t+1} to maximise $V_t^R(A_t)$ within the constraint $M_{t+1} \leq p_t$. Defining $\lambda_t = (1-\rho_t) - 1/(1+r_{t+1})$, this is equivalent to minimising

$$\exp[-b\Delta_t M_{t+1} \lambda_t]$$

The decision rule is then:

$$\begin{aligned} M_{t+1} &= p_t & \text{if } \lambda_t > 0 \\ M_{t+1} &= 0 & \text{if } \lambda_t < 0 \end{aligned}$$

The individual always chooses a corner solution for mortgage refinance - either zero or 100%. The choice is made comparing the interest and mortgage rates that set the relative cost of financing the housing debt via borrowing in bonds or in a mortgage. Integrating this into the value function

$$\begin{aligned} V_t^R(A_t) &= 1 + \phi\alpha_{t+1} - [\phi(E\beta_{t+1})\delta_{t+1}(1+r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\ &\quad \cdot \exp(-b\Delta_t[(1+r_t)A_t - M_t])W_t \cdot \exp(-b\Delta_t[\max[\lambda_t p_t, 0] - k])/\Delta_t \end{aligned}$$

Similar arguments show that without refinancing

$$\begin{aligned} V_t^{NR}(A_t) &= \max_{c_t} \{u(c_t) + \phi[\alpha_{t+1} - (E\beta_{t+1})(E \exp(-b\delta_{t+1}(1+r_{t+1})w_t^s)) \\ &\quad \cdot \exp[-b\delta_{t+1}((1+r_{t+1})\{(1+r_t)A_t - \rho_t M_t - c_t\} - M_t)]]\} \\ &= 1 + \phi\alpha_{t+1} - [\phi(E\beta_{t+1})\delta_{t+1}(1+r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\ &\quad \cdot \exp(-b\Delta_t[(1+r_t)A_t - M_t]) \cdot \exp(-b\Delta_t \lambda_t M_t)W_t/\Delta_t \end{aligned}$$

This then gives us a condition for refinancing to occur. At t the individual chooses to refinance the mortgage if $V_t^R(A_t) > V_t^{NR}(A_t)$, i.e. if:

$$\exp[-b\Delta_t(p_t \max((\lambda_t, 0) - k)] < \exp[-b\Delta_t M_t \lambda_t]$$

The refinancing condition is then:

$$p_t \max(\lambda_t, 0) - k - M_t \lambda_t > 0$$

Notice that this is independent of current financial assets, future employment or house price uncertainty: it is a matter of pure financial efficiency. Without restrictions on M_t , if the mortgage rate were above the savings rate the consumer could make unbounded wealth gains by borrowing infinitely in A_t and "investing" in M_t ; similarly, if $r > \rho$. The constraint $0 \leq M_t \leq p_{t-1}$ limits the size of these gains. Even though there are transaction costs, current mortgage decisions are not affected by the risk that in the future house prices may be high or low, nor by the risk of being unemployed. This is partly because decisions can be reversed next period, but it is also partly due to the CARA form of preferences with which the value function separates out current disposable wealth from future uncertainty. It follows that the effect of risk in either house prices or employment is all on consumption and savings.

If $\lambda_t > 0$ (the mortgage interest rate at t is relatively low with respect to r_{t+1}), in terms of the debt service costs it would pay to refinance to the highest extent possible by setting $M_{t+1} = p_t$ so long as the interest gain on the sum involved more than covers the transaction cost of refinancing, i.e.

$$p_t - M_t > \frac{k}{\lambda_t}$$

This occurs when the difference between the house price and the present mortgage is high and so is the interest differential between bonds and mortgages. This is the case of maximum equity withdrawal.

On the other hand, if $\lambda_t < 0$ (the mortgage interest rate at t is relatively high with respect to r_{t+1}), there would be a debt costs service advantage from replacing the mortgage by bond finance, which will be undertaken so long as the cost saving at least covers the transaction cost. Thus the consumer refinances the mortgage and chooses $M_{t+1} = 0$ when

$$\begin{aligned} k &< M_t \frac{1 - (1 - \rho_t)(1 + r_{t+1})}{1 + r_{t+1}} \\ &= -M_t \lambda_t \end{aligned}$$

i.e. when the cost of refinancing the mortgage is not too high.

We can summarise the possible remortgage actions at time t as

$\lambda_t > 0$	$M_t = 0$	$\lambda_t p_t > k$	$M_{t+1} = p_t$
$\lambda_t > 0$	$M_t = 0$	$\lambda_t p_t < k$	$M_{t+1} = 0$
$\lambda_t > 0$	$M_t = p_{t-s}$	$\lambda_t p_t > \lambda_t p_{t-s} + k$	$M_{t+1} = p_t$
$\lambda_t > 0$	$M_t = p_{t-s}$	$\lambda_t p_t < \lambda_t p_{t-s} + k$	$M_{t+1} = p_{t-1}$
$\lambda_t < 0$	$M_t = 0$		$M_{t+1} = 0$
$\lambda_t < 0$	$M_t = p_{t-s}$	$\lambda_t p_{t-s} + k < 0$	$M_{t+1} = 0$
$\lambda_t < 0$	$M_t = p_{t-s}$	$\lambda_t p_{t-s} + k > 0$	$M_{t+1} = p_{t-1}$

Notice that, if starting from a position with a zero mortgage there is a run of periods in each of which $\lambda_t < 0$, then there will be inactivity and zero outstanding mortgages in each of these periods. To move from this to a positive mortgage requires both that λ_t should switch sign and that, when it does, the expected financial gain from remortgaging outweighs the transaction cost. Similarly, the current mortgage at t may have been taken out at a level of p_{t-s} s periods ago, since when it has not been optimal to refinance.

4 Overall Value Function

The overall value function is the larger of $V_t^R(A_t, M_t)$ and $V_t^{NR}(A_t, M_t)$, which can be written as

$$\begin{aligned}
V_t(A_t, M_t, p_t) &= 1 + \phi\alpha_{t+1} - [\phi(E\beta_{t+1})\delta_{t+1}(1+r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\
&\quad \cdot \exp(-b\Delta_t[(1+r_t)A_t - M_t]) \\
&\quad \cdot \exp(-b\Delta_t \max\{\lambda_t M_t, \max[\lambda_t p_t, 0] - k\})W_t/\Delta_t \\
&= \alpha_t - \beta_t \exp[-b\delta_t((1+r_t)A_t - M_t)]
\end{aligned}$$

This generates recurrence relations for the unknown functions

$$\begin{aligned}
\alpha_t &= 1 + \phi\alpha_{t+1} \\
\delta_t &= \Delta_t = \frac{\delta_{t+1}(1+r_{t+1})}{1 + \delta_{t+1}(1+r_{t+1})} \\
\beta_t &= [\phi(E\beta_{t+1})\delta_{t+1}(1+r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\
&\quad \cdot \exp(-b\frac{\delta_{t+1}(1+r_{t+1})}{1 + \delta_{t+1}(1+r_{t+1})} \max\{\lambda_t M_t, \max[\lambda_t p_t, 0] - k\})W_t/\delta_t
\end{aligned}$$

or

$$\beta_t = \left[\phi(E\beta_{t+1}) \frac{\delta_t}{1 - \delta_t} \right]^{1 - \delta_t} \exp(-b\delta_t \max\{\lambda_t M_t, \max[\lambda_t p_t, 0] - k\})W_t/\delta_t \quad (1)$$

Solving these equations for α_t, δ_t

$$\begin{aligned} a_{T-t} &= \sum_{s=0}^t \phi^s \\ \delta_{T-t} &= \frac{\prod_{s=0}^{t-1} (1 + r_{T-s})}{1 + \sum_{\tau=0}^{t-1} \prod_{s=0}^{\tau} (1 + r_{T-s})} \end{aligned} \quad (2)$$

For example

$$\delta_{T-3} = \frac{(1 + r_T)(1 + r_{T-1})(1 + r_{T-2})}{1 + (1 + r_T) + (1 + r_T)(1 + r_{T-1}) + (1 + r_T)(1 + r_{T-1})(1 + r_{T-2})}$$

The recurrence relation in β , which captures the effect of future house prices and employment uncertainty, requires some careful analysis.

4.1 $E\beta_{t+1}$: The Effects of Future House Price Uncertainty

The effects of future house price uncertainty at time t work through the expression $E\beta_{t+1}$. Since we have assumed that any trend in p_t is not stochastic $E_{t+1}\beta_{t+2}$ is not a function of p_{t+1} so at time t the random term in β_{t+1} is

$$\begin{aligned} F_{t+1} &= \exp(-b\delta_{t+1} \max[\lambda_{t+1}M_{t+1}, p_{t+1} \max\{\lambda_{t+1}, 0\} - k]) \\ &= \exp(-b\delta_{t+1}G_{t+1}) \end{aligned}$$

where

$$\begin{aligned} G_{t+1} &= \lambda_{t+1}M_{t+1} \text{ if } \lambda_{t+1} < 0 \text{ and } \lambda_{t+1}M_{t+1} > -k \\ &= -k \text{ if } \lambda_{t+1} < 0 \text{ and } \lambda_{t+1}M_{t+1} < -k \\ &= \lambda_{t+1}M_{t+1} \text{ if } \lambda_{t+1} > 0 \text{ and } p_{t+1}\lambda_{t+1} - k < \lambda_{t+1}M_{t+1} \\ &= p_{t+1}\lambda_{t+1} - k \text{ if } \lambda_{t+1} > 0 \text{ and } p_{t+1}\lambda_{t+1} - k > \lambda_{t+1}M_{t+1} \end{aligned} \quad (3)$$

$$(4)$$

We want to compute EF_{t+1} over p_{t+1} .

If $\lambda_{t+1} < 0$, either the consumer does not wish to refinance, or refinances to carry forward a zero mortgage. In either event G_{t+1} and the conditions are not random, so $EF_{t+1} = F_{t+1}$.

If $\lambda_{t+1} > 0$ it is more complex. The third case (3) holds when house prices are such that the consumer potentially wishes to refinance to the maximum permissible extent but the savings from doing so will not cover the transaction cost of refinance. This occurs when

$$p_{t+1} < M_{t+1} + \frac{k}{\lambda_{t+1}}$$

and in the fourth case (4) which holds for p_{t+1} above this, the gains from taking out a new maximum mortgage do cover the transaction cost..

Hence when $\lambda_{t+1} > 0$, if we define the probability that the maximum remortgage will not cover the transaction cost by $\gamma_t = \Pr(p_{t+1} < M_{t+1} + \frac{k}{\lambda_{t+1}})$:

$$EF_{t+1} = \exp(-b\delta_{t+1}\lambda_{t+1}M_{t+1})\gamma_t + (E \exp(-b\delta_{t+1}[p_{t+1}\lambda_{t+1}-k])|p_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+1}})(1-\gamma_t)$$

Suppose that the support of the distribution of p_{t+1} is $[\underline{p}_{t+1}, \bar{p}_{t+1}]$. If

$$\begin{aligned} M_{t+1} + \frac{k}{\lambda_{t+1}} < \underline{p}_{t+1} & \quad \text{then} \quad \gamma_t = 0 \\ M_{t+1} + \frac{k}{\lambda_{t+1}} > \bar{p}_{t+1} & \quad \text{then} \quad \gamma_t = 1 \end{aligned}$$

That is we have two boundary cases where either house prices are always so low that a maximum remortgage will not cover the transaction cost, or where they are so high that with certainty the maximum remortgage is profitable. Since

$$E\beta_{t+1} = \left[\phi(E\beta_{t+2}) \frac{\delta_{t+1}}{1-\delta_{t+1}} \right]^{1-\delta_{t+1}} \cdot EF_{t+1}W_{t+1}/\delta_{t+1}$$

this then gives us the relations in table 2.

Thus, there are only effects of immediate future house price uncertainty on the current value function if $\lambda_{t+1} > 0$. In fact, extending this argument, the distribution of house prices at any future data τ only affects the current value function for those periods in which $\lambda_\tau > 0$. Since there is no intertemporal stochastic dependence in house prices:

$$\begin{aligned} E_t\beta_{t+1} &= E_t \left\{ (E_{t+1}\beta_{t+2})^{1-\delta_{t+1}} \left(\frac{\phi\delta_{t+1}}{1-\delta_{t+1}} \right)^{1-\delta_{t+1}} F_{t+1}W_{t+1}/\delta_{t+1} \right\} \\ &= (E_{t+1}\beta_{t+2})^{1-\delta_{t+1}} \left(\frac{\phi}{1-\delta_{t+1}} \right)^{1-\delta_{t+1}} \delta_{t+1}^{-\delta_{t+1}} (E_t F_{t+1})W_{t+1} \end{aligned}$$

and so we can solve this equation recursively to get

$$\begin{aligned} E_t\beta_{t+1} &= (E_{T-1}\beta_T)^{(1-\delta_{t+1})(1-\delta_{t+2})\dots(1-\delta_{T-1})} \\ &\quad \left(\frac{\phi}{1-\delta_{t+1}} \right)^{1-\delta_{t+1}} \left(\frac{\phi}{1-\delta_{t+2}} \right)^{(1-\delta_{t+1})(1-\delta_{t+2})} \dots \left(\frac{\phi}{1-\delta_{T-1}} \right)^{(1-\delta_{t+1})\dots(1-\delta_{T-1})} \\ &\quad E_t F_{t+1} (E_{t+1} F_{t+2})^{(1-\delta_{t+1})} \dots (E_{T-2} F_{T-1})^{(1-\delta_{t+1})\dots(1-\delta_{T-2})} \\ &\quad W_{t+1} (W_{t+2})^{(1-\delta_{t+1})} \dots (W_{T-1})^{(1-\delta_{t+1})\dots(1-\delta_{T-2})} \\ &\quad \delta_{t+1}^{-\delta_{t+1}} \delta_{t+2}^{-\delta_{t+2}(1-\delta_{t+1})} \dots \delta_{T-1}^{-\delta_{T-1}(1-\delta_{t+1})\dots(1-\delta_{T-2})} \end{aligned}$$

The future EF s only include elements of the distribution of house prices for cases in which their corresponding future λ is positive. Using this together with

		$E\beta_{t+1}$
$\lambda_{t+1} < 0$	$\lambda_{t+1}M_{t+1} > -k$	$\left[\phi(E\beta_{t+2}) \frac{\delta_{t+1}}{1-\delta_{t+1}} \right]^{1-\delta_{t+1}} \exp(-b\delta_{t+1}\lambda_{t+1}M_{t+1})W_{t+1}/\delta_{t+1}$
$\lambda_{t+1} < 0$	$\lambda_{t+1}M_{t+1} < -k$	$\left[\phi(E\beta_{t+2}) \frac{\delta_{t+1}}{1-\delta_{t+1}} \right]^{1-\delta_{t+1}} (\exp(b\delta_{t+1}k))W_{t+1}/\delta_{t+1}$
$\lambda_{t+1} > 0$	$M_{t+1} + \frac{k}{\lambda_{t+1}} < \underline{p}_{t+1}$	$\left[\phi(E\beta_{t+2}) \frac{\delta_{t+1}}{1-\delta_{t+1}} \right]^{1-\delta_{t+1}} (E \exp(-b\delta_{t+1}[\underline{p}_{t+1}\lambda_{t+1} - k]))W_{t+1}/\delta_{t+1}$
$\lambda_{t+1} > 0$	$M_{t+1} + \frac{k}{\lambda_{t+1}} > \bar{p}_{t+1}$	$\left[\phi(E\beta_{t+2}) \frac{\delta_{t+1}}{1-\delta_{t+1}} \right]^{1-\delta_{t+1}} (E \exp(-b\delta_{t+1}\lambda_{t+1}M_{t+1}))W_{t+1}/\delta_{t+1}$
$\lambda_{t+1} > 0$	$\underline{p}_{t+1} < M_{t+1} + \frac{k}{\lambda_{t+1}} < \bar{p}_{t+1}$	$\exp(-b\delta_{t+1}\lambda_{t+1}M_{t+1})\gamma_t +$
		$(E \exp(-b\delta_{t+1}[\underline{p}_{t+1}\lambda_{t+1} - k]))\underline{p}_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+1}})(1 - \gamma_t)$

Table 2

the expression for β_t ⁶ and the fact that

$$\begin{aligned}\frac{1}{1 + \delta_{t+1}(1 + r_{t+1})} &= 1 - \delta_t \\ \beta_t &= \left(\frac{\phi}{1 - \delta_t}\right)^{1 - \delta_t} \delta_t^{-\delta_t} W_t (E_t \beta_{t+1})^{1 - \delta_t} F_t\end{aligned}$$

we can write:

$$\begin{aligned}\beta_t &= (E_{T-1} \beta_T)^{(1 - \delta_t)(1 - \delta_{t+1}) \dots (1 - \delta_{T-1})} \\ &\quad \left(\frac{\phi}{1 - \delta_t}\right)^{1 - \delta_t} \left(\frac{\phi}{1 - \delta_{t+1}}\right)^{(1 - \delta_t)(1 - \delta_{t+1})} \dots \left(\frac{\phi}{1 - \delta_{T-1}}\right)^{(1 - \delta_t)(1 - \delta_{t+1}) \dots (1 - \delta_{T-1})} \\ &\quad F_t (E_t F_{t+1})^{1 - \delta_t} (E_{t+1} F_{t+2})^{(1 - \delta_t)(1 - \delta_{t+1})} \dots (E_{T-2} F_{T-1})^{(1 - \delta_t)(1 - \delta_{t+1}) \dots (1 - \delta_{T-1})} \\ &\quad W_t (W_{t+1})^{1 - \delta_t} \dots (W_{T-1})^{(1 - \delta_t)(1 - \delta_{t+1}) \dots (1 - \delta_{T-1})} \\ &\quad \delta_t^{-\delta_t} \delta_{t+1}^{-\delta_{t+1}(1 - \delta_t)} \dots \delta_{T-1}^{-\delta_{T-1}(1 - \delta_t)(1 - \delta_{t+1}) \dots (1 - \delta_{T-1})}\end{aligned}$$

Note that there is an effect on β_t of the time to go to the horizon, the longer the remaining future, the higher the number of terms in the product for β since there are more future nodes. Therefore, in earlier periods β tends to be higher which reflects the effect of the greater amount of uncertainty remaining. Conversely towards the end of life, there is little remaining uncertainty and so on these grounds less of a need for precautionary savings.

Combining elements of Table 2 with (2),(??) and

$$V_t(A_t, M_t, p_t) = \alpha_t - \beta_t \exp[-b\delta_t((1 + r_t)A_t - M_t)]$$

gives the form of the overall value function. At t the initial mortgage M_t and house price p_t determine the mortgage of $t + 1$. Together with λ_{t+1} and house prices at $t + 1$, the form of $E\beta_{t+1}$ and of the value function at t is determined. However, these also depend on $E\beta_{t+2}$, which itself depends *inter alia* on the mortgage at $t + 2$. Hence a combination of the history and elements of the whole future determine the current value function.

5 Consumption

As usual with CARA preferences consumption with or without refinancing is basically linear in disposable wealth, but there are some complications.

1. With refinancing ($p_t \max(\lambda_t, 0) - k - M_t \lambda_t > 0$) and from:

$$\begin{aligned}\exp(c_t^R) &= \left(\phi (E\beta_{t+1}) \delta_{t+1} (1 + r_{t+1})\right)^{-\frac{1}{b(1 + \delta_{t+1}(1 + r_{t+1}))}} \exp[\delta_t((1 + r_t)A_t - M_t - k)] \\ &\quad \cdot W_t^{-\frac{1}{b(1 + \delta_{t+1}(1 + r_{t+1}))}} \exp[\delta_t p_t \max(\lambda_t, 0)]\end{aligned}$$

⁶ If the probability of unemployment was either history dependent or uncertain, then so long as it is independent of house prices there is little impact on the expression for β : terms in W would become $E_t W_{t+1}$.

we have:

$$c_t^R = -\frac{1}{b(1 + \delta_{t+1}(1 + r_{t+1}))} \ln(\phi(E\beta_{t+1})\delta_{t+1}(1 + r_{t+1})) + \delta_t((1 + r_t)A_t - M_t) \\ - \frac{1}{b(1 + \delta_{t+1}(1 + r_{t+1}))} \ln(W_t) + \delta_t(p_t \max(\lambda_t, 0) - k)$$

2. Without refinancing ($p_t \max(\lambda_t, 0) - k - M_t \lambda_t < 0$) we have:

$$c_t^{NR} = -\frac{1}{b(1 + \delta_{t+1}(1 + r_{t+1}))} \ln(\phi(E\beta_{t+1})\delta_{t+1}(1 + r_{t+1})) + \delta_t((1 + r_t)A_t - M_t) \\ - \frac{1}{b(1 + \delta_{t+1}(1 + r_{t+1}))} \ln(W_t) + \delta_t \lambda_t M_t$$

The main features of consumption are that:

(i) Both with and without refinancing, the effect of future period employment risk and house price uncertainty is to shift the intercept of the consumption function by an amount that depends on the degree of risk aversion. Since it is possible that $\phi(E\beta_{t+1})\delta_{t+1}(1 + r_{t+1}) < 1$, the future uncertainty in house prices may actually increase rather than reduce consumption. We know that for sure $\delta_{t+1}(1 + r_{t+1}) < 1$ so long as $r_{t+1} < 0.5$ and $\phi < 1$ but the relation of $\phi\delta_{t+1}(1 + r_{t+1})$ to $E\beta_{t+1}$ is unclear so that the total effect is ambiguous. Uncertain future house prices give the opportunity of high prices and so the chance of high future equity withdrawal, reducing the need for current buffer stock savings against future employment uncertainty. If future house prices are certain then the expression for $E\beta_{t+1}$ changes when $\lambda_{t+1} > 0$. We can gauge the effect of this by comparing $E\beta_{t+1}$ with its value when house prices are constant at their mean Ep_{t+1} :

Case 1

If $\lambda_{t+1} > 0$ and $p_{t+1}\lambda_{t+1} - k > M_{t+1}\lambda_{t+1}$, then $Ep_{t+1}\lambda_{t+1} - k > M_{t+1}\lambda_{t+1}$. Hence, with certain house prices:

$$EF_{t+1}^c = F_{t+1}^c = \exp(-b\delta_{t+1}(\lambda_{t+1}Ep_{t+1} - k))$$

as opposed to

$$EF_{t+1} = E \exp(-b\delta_{t+1}[p_{t+1}\lambda_{t+1} - k])$$

>From Jensen's inequality it follows that $EF_{t+1} = E \exp(-x) < \exp(-Ex) = EF_{t+1}^c$ since $\exp(-x)$ is concave.

$$c^C - c^U = \frac{1}{b(1 + \delta_{t+1}(1 + r_{t+1}))} \ln\left(\frac{EF_{t+1}}{EF_{t+1}^c}\right) < 0$$

where c^C is consumption when future house prices are certain and c^U is consumption under uncertainty. Here future house prices are for sure going to

allow a higher mortgage and an interest cost saving, so effectively the house price uncertainty raises future wealth unambiguously.

Case 2

If $\lambda_{t+1} > 0$ and $Ep_{t+1}\lambda_{t+1} - k > M_{t+1}\lambda_{t+1}$ then

$$EF_{t+1}^c = F_{t+1}^c = \exp(-b\delta_{t+1}(\lambda_{t+1}Ep_{t+1} - k))$$

as opposed to

$$EF_{t+1} = \exp(-b\delta_{t+1}\lambda_{t+1}M_{t+1})\gamma_t + (E \exp(-b\delta_{t+1}[p_{t+1}\lambda_{t+1} - k])|p_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+1}})(1 - \gamma_t)$$

Again from the Jensen's inequality it follows that $E \exp(-x) < \exp(-Ex)$ since $\exp(-x)$ is concave. And since $Ep_{t+1}\lambda_{t+1} - k > M_{t+1}\lambda_{t+1}$ we know that $\exp(-b\delta_{t+1}\lambda_{t+1}M_{t+1}) > \exp(-b\delta_{t+1}[\lambda_{t+1}Ep_{t+1} - k])$. So the overall comparison is ambiguous: there is a risk that future house prices are low, constraining the refinancing possibilities.

Case 3

On the other hand if $\lambda_{t+1} > 0$ and $Ep_{t+1}\lambda_{t+1} - k < M_{t+1}\lambda_{t+1}$ then

$$EF_{t+1}^c = F_{t+1}^c = \exp(-b\delta_{t+1}\lambda_{t+1}M_{t+1}) < \exp(-b\delta_{t+1}(\lambda_{t+1}Ep_{t+1} - k))$$

Hence:

$$\frac{EF_{t+1}}{EF_{t+1}^c} = \gamma_t + \frac{(E \exp(-b\delta_{t+1}[p_{t+1}\lambda_{t+1} - k])|p_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+1}})(1 - \gamma_t)}{\exp(-b\delta_{t+1}\lambda_{t+1}M_{t+1})}$$

Since

$$\frac{(E \exp(-b\delta_{t+1}[p_{t+1}\lambda_{t+1} - k])|p_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+1}})}{\exp(-b\delta_{t+1}\lambda_{t+1}M_{t+1})} < 1$$

then

$$\frac{EF_{t+1}}{EF_{t+1}^c} < 1$$

and

$$c^C - c^U = \frac{1}{b(1 + \delta_{t+1}(1 + r_{t+1}))} \ln \left(\frac{EF_{t+1}}{EF_{t+1}^c} \right) < 0$$

In other words, when with certain future house prices no refinancing is undertaken ($Ep_{t+1}\lambda_{t+1} - k < M_{t+1}\lambda_{t+1}$), precautionary savings are negative.

Summing up, in each of these cases housing is acting like an intertemporal buffer stock in wealth with effects on the current level of savings. Knowing that in the future there will be a redeemable asset (though of uncertain value), the consumer can afford to borrow today. In addition the composition of wealth between net housing wealth ($p_t - M_t$) and A_t has an impact on consumption. The marginal propensity to consume out of wealth is $\delta_t(1 + r_t)$ but out of a reduction of the current mortgage is $\delta_t(1 - \lambda_t)$.

In addition there is an effect on consumption of the remaining length of the horizon. We have seen that $E\beta_t$ tends to fall through time. This serves to yield consumption growth through time *ceteris paribus*. As time passes there is less remaining uncertainty and so less of a need for precautionary savings.

(ii) Consumption is linear in net wealth $(1+r_t)A_t - M_t$ but is nonlinear in expected current period labour income. The latter is essentially an artifact of the assumed within period timing where consumption has to be chosen before the employment status of the current period is realised. In fact this labour income risk reduces consumption: suppose that income were certain at the level $\bar{w}_t = \alpha w_t + (1-\alpha)B_t$. Then since the exponential is a convex function and $\delta > 0$

$$\alpha \exp(-b\delta_{t+1}(1+r_{t+1})w_t) + (1-\alpha) \exp(-b\delta_{t+1}(1+r_{t+1})B_t) < \exp(-b\delta_{t+1}(1+r_{t+1})\bar{w}_t)$$

and then since $\ln(\cdot)$ is an increasing function, consumption is depressed by the labour income uncertainty. This argument also applies to the future labour income risk terms in $E\beta_{t+1}, W_{t+s}$. If labour income of some future period were certain at its mean level this would increase the term in W_{t+s} which *ceteris paribus* would raise $E\beta_{t+1}$ and tend to raise current consumption.

(iii) When it is optimal to refinance, the effect of refinancing on current consumption is unambiguously non-negative. This is current equity withdrawal. However, with no refinancing the current mortgage state may increase, reduce or leave consumption unchanged. It will reduce consumption when $\lambda_t < 0$ but the cost saving from reducing the current mortgage M_t to zero does not cover the transaction cost of doing so.

(iv) The current mortgage interest rate only has effects via λ_t . When $\lambda_t < 0$ an increase in the mortgage rate may cause a switch from no refinance to reducing the current positive mortgage to zero, which will generate a step jump in consumption. If the mortgage is already zero, an increase in the current mortgage rate will have a zero effect on consumption. When $\lambda_t > 0$ and $p_t > M_t$ there is scope for increasing the mortgage so long as the interest gain covers the transaction cost. A fall in the current mortgage rate increases the chance of refinancing and so may cause a switch from no refinancing of the current mortgage (which may be zero) to refinancing to the maximum extent possible which causes a jump in consumption. Only a part of the current wealth change arising from remortgaging is consumed in the current period (the coefficient δ) and part of the wealth change is used to smooth future consumption.

(v) The current savings interest rate has obvious income effects on consumption. First, there is a consumption increasing effect through raising capital income when the consumer has positive financial assets, but a decreasing effect through raising the debt service cost when assets are negative. Future savings rates and especially the savings rate of the next period have much more complex effects: directly through altering the slope of the consumption function in most variables, indirectly through varying λ_t and through affecting the discounting terms in $E\beta_{t+1}$. If at t the foreseen r_{t+s} increases ($s > 0$) all the terms in δ_τ for $t \leq \tau < s$ are increased. In particular if $s = 1$ then δ_{t+1} is unaffected and so

the term $1/(1 + \delta_{t+1}(1 + r_{t+1}))$ falls and so the marginal propensity to consume out of labour income expected for this period falls.

(vi) As the degree of risk aversion rises, the absolute value of the intercept of the consumption function falls in any period and so between any two periods there is less variability in consumption. In particular there are smaller jumps in consumption when refinancing occurs.

6 Calibrated Simulations

We study the optimal consumption and mortgage choices of an individual with a 40 period horizon. In every period the chance of being in employment is $\alpha = 0.90$, in which case annual earnings are w_t .

6.1 Case 1: Hump Shaped Earnings and Trendless House Prices

In this case we assume a hump-shaped profile of wages:

$$w_t = 0.5 \cdot \exp(-((t - 20)/20)^2)$$

so that they peak at $t = 20$ and start and end at about 0.2.

If the individual is jobless, she gets unemployment benefits, which are 50% of the wage in the first period and then slightly grow at a constant rate $g = 0.015$.

The interest rate is held constant ($r = 0.025$) and the mortgage rate is defined by the process:

$$\rho_t = 1.1 \cdot r + \varepsilon_t \quad \text{where } \varepsilon_t \sim U(-.27, .5)$$

This is without loss of generality, since in the mortgage refinancing decision what really matters is the fluctuation of ρ_t around r (and not the fact that r is time-varying). We assume also that there is no trend in house prices:

$$p_t \sim U(5B_1, 10B_1)$$

This is based on the rule of thumb that generally the house price income ratio is around 3 and so the house price/unemployment benefit ratio is around 6 or 7. We assume that in the first period the consumer takes out a mortgage equal to the house price.

In figures 2 to 5 we plot the realized wage (which is either w_t or B_t depending on the employment status), house prices, λ_t and the mortgage function. In the benchmark case the cost of refinancing the mortgage is $k = 0.1B_1$, the discount factor is $\phi = 1/1.15$ and the coefficient of risk aversion is $b = 1.63$ ⁷

⁷The justification for this value is given in Berloff & Simmons (2003).

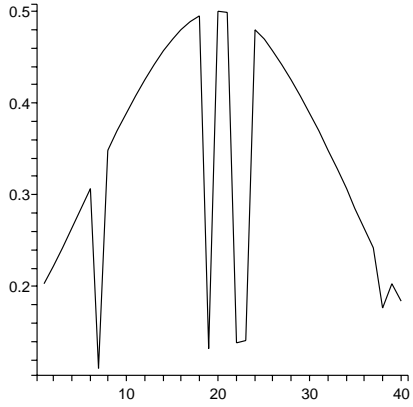


Fig 2: Labour income

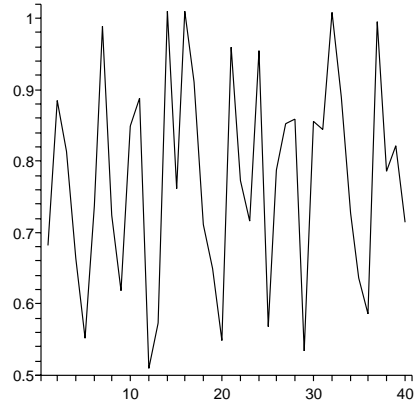


Fig 3: House prices

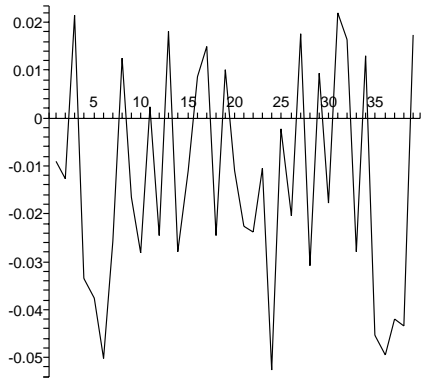


Fig 4: $\lambda_t = 1 - \rho_t - 1/(1 + r_{t+1})$

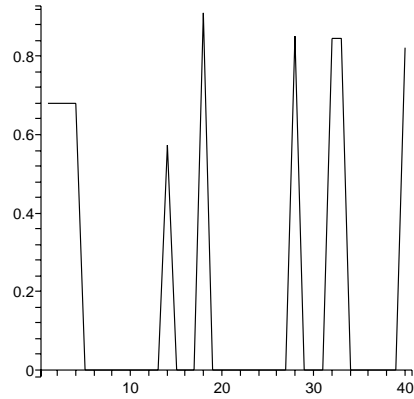


Fig 5: Mortgage refinancing

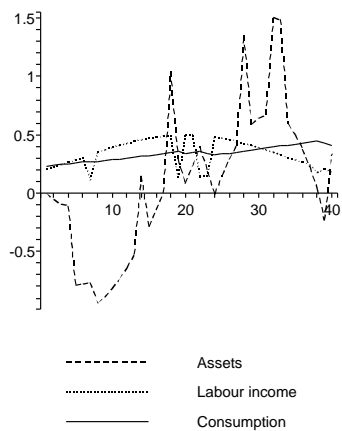


Fig 6: Patient consumer

The shape of the life-cycle profile of consumption and assets is consistent with the general result (Deaton, 1991) that patient consumers are natural lenders, not borrowers. In the presence of uncertainty, they save early in life and decumulate later; initial consumption is low, but grows rapidly during the life-cycle. In our simulations, the individual experiences four spells of unemployment, one of which lasts for two periods. In response to these unexpected changes in income, consumption jumps discontinuously. The fall in last period consumption with these particular realisations depends on the fact that the realised house price at T is relatively low.

Since refinancing the mortgage involves fixed transaction costs, consumers do not make smooth adjustments. The higher the transaction costs, the less frequent the refinancing. We now consider two alternative values of the refinancing cost: in the first case $k = 0.4B_1$, in the second $k = 0.05B_1$. As Figure (7) shows, the mortgage choice is very sensitive to the level of k . A fixed transaction cost equal to the 40% of the initial annual benefit is sufficient to prevent the consumer from any refinancing.

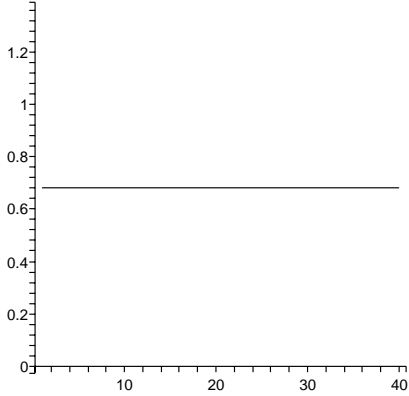


Fig 7: Mortgage size ($k = 0.40B_1$)

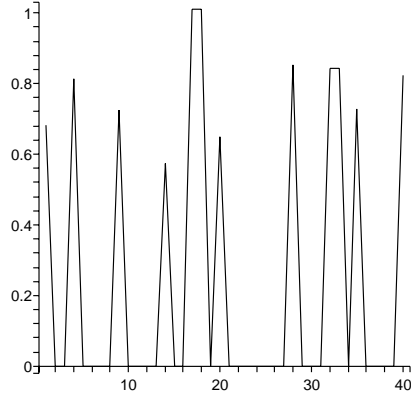


Fig 8: Mortgage size ($k = 0.05B_1$)

A crucial assumption in our simulation concerns the discount factor. Life-cycle models usually assume $\phi = 1/(1+r)$ or in the case of interest rate uncertainty $E(1+r) = 1/\phi$; in this case there is no trend in consumption and investors are more likely to borrow when income is known to be growing than when it is falling (Fig 9).

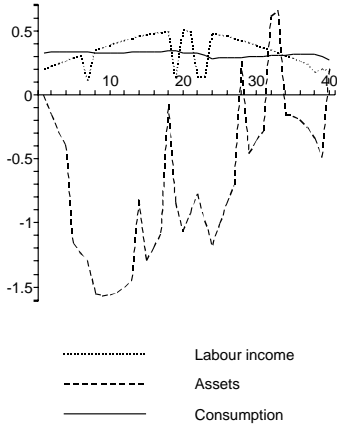


Fig 9: Time neutral consumer

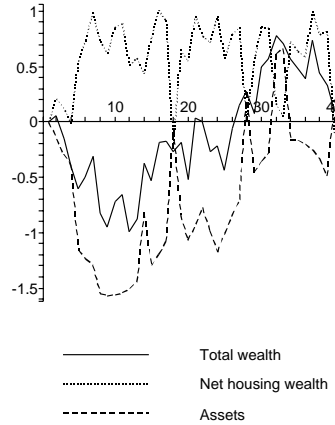


Fig 10: Composition of wealth

For a time neutral consumer the life cycle composition of wealth is shown in Fig 10

Here we define total wealth as the sum of financial assets and net housing wealth at the start of the period ($A_t + p_t - M_t$). Fluctuations in housing wealth are caused by either refinancing or shocks in house prices. Switching total debt between the financial asset and the mortgage is apparent in Fig 11, but there are periods in which financial assets and net housing wealth move in the same

direction. Net housing wealth is largely trendless, and the lifecycle effect of hump shaped wages is captured mainly in financial assets. Interestingly total wealth rises over the second half of life until the last few periods even though the consumer is in debt in financial assets for these late periods. The perfect financial capital market is allowing the consumer to borrow against the value of the house at the end of the horizon.

Finally, for impatient individuals ($\phi = 1/1.04$) optimal consumption starts at a high level and decreases rapidly during the life-cycle; to carry out their consumption plans, these investors borrow large amount of money early in life. This reflects the lack of borrowing restrictions in our model, whose only constraint is that terminal assets must be nonnegative.

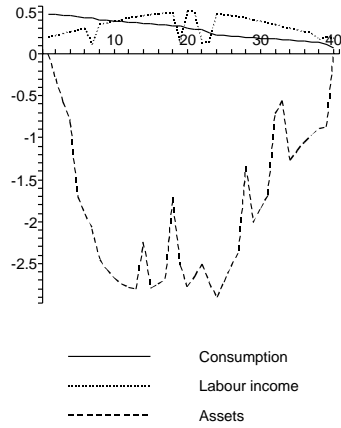


Fig 11: Impatient consumer

6.1.1 Case 1.1: Foreseen Compulsory Retirement

In this case we assume that the individual must retire at $t = 30$ after which his non-capital income is just the benefit. The wage peaks at $t = 15$ and starts at $w_1 \simeq 0.2$. The other assumptions are maintained.

$$w_t = 0.5 \cdot \exp(-((t - 15)/15)^2) \quad t = 1..30$$

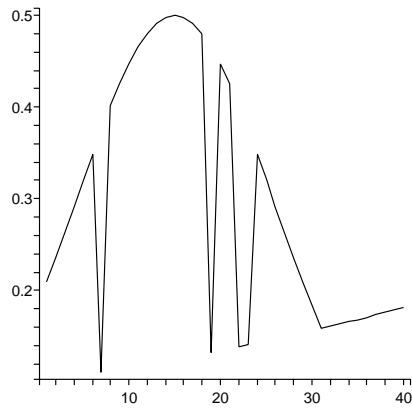


Fig 12: Wages

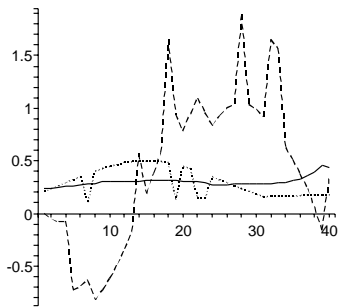


Fig 13: Time neutral consumer

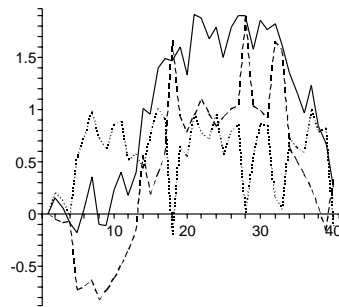


Fig 14: Composition of wealth

For the time neutral consumer the composition of wealth is shown in Fig 14. As with humpshaped wages, net housing wealth is broadly trendless and the biggest jumps in financial assets are matched by simultaneous reverse big jumps in net housing wealth as the consumer switches the source of finance for housing debt. There is a strong life cycle effect in financial assets: initial borrowing followed by accumulation of financial assets which are used during retirement to finance consumption.

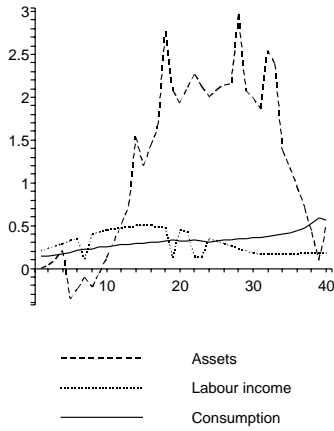


Fig 15: Patient consumer

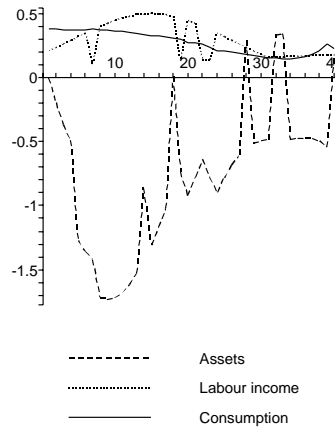


Fig 16: Impatient consumer

The lack of labour income in later life makes both the patient and the time neutral consumer ($\phi = 1/(1+r)$) save sufficiently to keep financial assets nearly always positive. The growth rate of consumption for these consumers is generally positive. The impatient consumer saves more but still has falling consumption (Fig 16). The compulsory retirement has no effect on the refinancing decision so these consumers refinance in exactly the same way as consumers who can keep working until $t = 40$.

6.2 Case 2: Growing Wages and House Prices

In this case wages grow from an initial level of unity at a smooth rate of 5% and house prices at time t are uniformly distributed on a time varying interval $[3w_t, 6w_t]$.

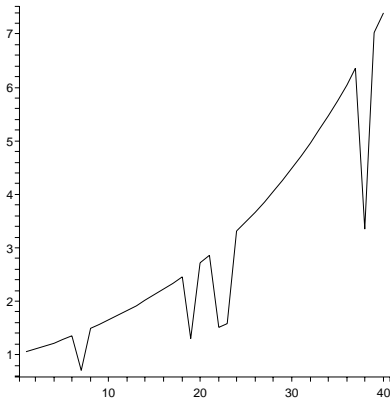


Fig 17: Growing wages

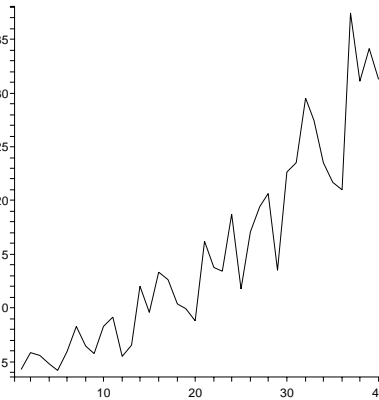


Fig 18: House prices

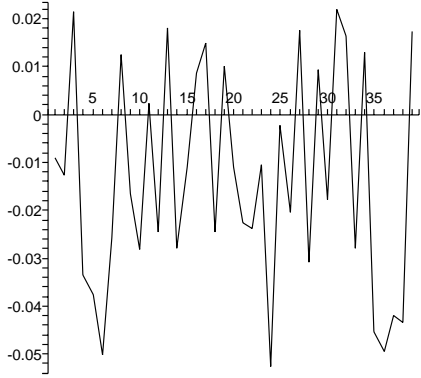


Fig 19: $\lambda_t = 1 - \rho_t - 1/(1 + r_{t+1})$

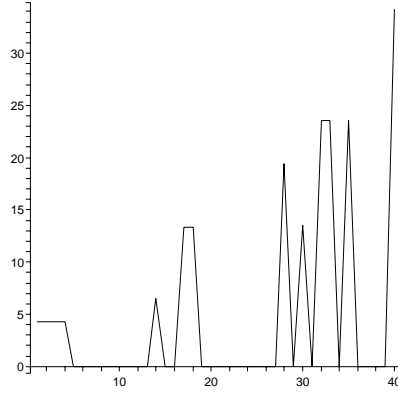


Fig 20: Mortgage refinancing

The growth in real house prices and wages has an effect on the time profile of consumption that dominates time preference effects, consequently we give only the details of the simulation for the time neutral consumer.

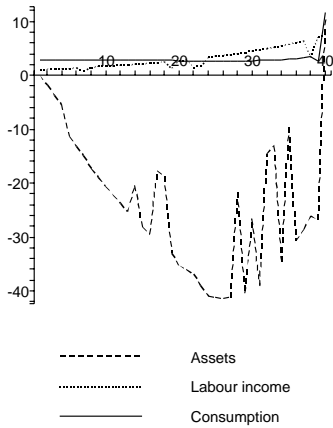


Fig 21: Time neutral consumer

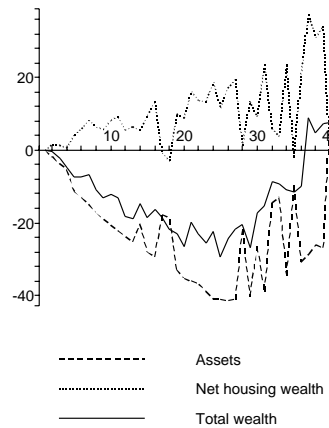


Fig 22: Composition of wealth

For the time neutral consumer in this case the composition and evolution of wealth are shown in Fig 22.

The growth in real house prices causes heteroscedasticity in net housing wealth since the consumer is switching between mortgages of zero and of a maximum size that is growing over time. Since both wages and house prices are growing the consumer anticipates that on average he will have high cash on hand in later life, and reacts by consistently borrowing in financial assets until about midlife when debts are repaid out of high labour incomes.

The individual refinances more frequently than with trendless wages, presumably because of the induced trend and heteroscedasticity in house prices: at

later dates house prices are more variable, and high house price realisations allow a relatively large mortgage on refinancing, which can then anticipate future house price and earnings growth. The individual borrows also in the financial asset, but financial debts show quite high variability as the proceeds of a remortgage are partly used to pay off some of them. The rapid jump in consumption at the end of the plan is due to a high realisation of the final house price, which not only covers repayment of financial debt and the mortgage, but also allows a final spree.

7 Extensions

Our approach suggests some obvious areas for future research and has various special assumptions whose force we try to evaluate here.

First we have assumed no liquidity constraints in the financial asset: so long as the lifetime budget constraint is respected consumers can borrow as much as they wish. This is important in making the key determinant of the refinancing decision the relative interest rate advantages of borrowing against the house or against future wealth (including future labour earnings and the future value of the house). We can think of this as using the house value as collateral for borrowing in financial assets - since the lifetime budget constraint is satisfied, there is no default risk and, if during life, financial assets become negative, these debts are just rolled forward to the next period. An alternative would be to impose the constraint $A_t \geq 0$ in which case the refinancing decision becomes much less transparent - it has to take account of the fact that remortgaging now influences the chance with which next period the consumer may end up being liquidity constrained e.g. if they lose employment.

Second generally the amount that can be borrowed on a mortgage is limited not only by the house value but also by the current income level. The rationale for this seems to be on debt service cost grounds. In the UK usually this multiplier limits the mortgage to no more than three or four times income. Our simulations nearly all respect this constraint so the simulated results will still represent optimal behaviour even with this income limit. Figure 23 shows the income/mortgage ratio in the first simulated case as against the ratios of respectively 3.0 and 4.0

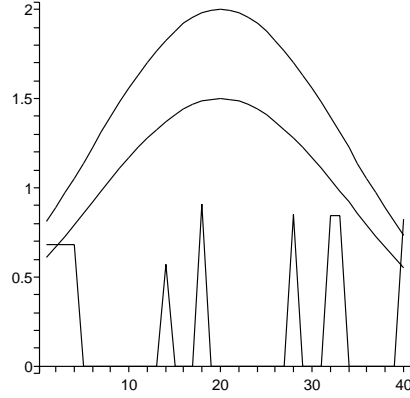


Fig. 23: Mortgage-income ratio

More generally we could quite easily incorporate the constraint into the analytical framework, it raises nothing new conceptually but makes the algebra more complicated. Given that the individual wishes to refinance in period t , the new mortgage decision would be⁸

$$\begin{aligned} M_{t+1} &= \min[p_t, \mu w_{t-1}] \quad \text{if } \lambda_t > 0 \\ &= 0 \quad \text{if } \lambda_t < 0 \end{aligned}$$

Again refinancing will be optimal if there is a financial advantage:

$$\min[p_t, \mu w_{t-1}] \max[\lambda_t, 0] - k_t - M_t \lambda_t > 0$$

Putting these together, the term in mortgage activity in the overall value function would become

$$\min\{\exp(-b\Delta_t \lambda_t M_t), \exp(-b\Delta_t(1+r_{t+1})[\max[\lambda_t \min[p_t, \mu w_{t-1}], 0] - k])\} W_t / \Delta_t$$

Again all the effects of uncertainty are captured in β whose recurrence relation (1) becomes

$$\beta_t = \left[\phi(E\beta_{t+1}) \frac{\delta_t}{1 - \delta_t} \right]^{1 - \delta_t} \exp(-b\delta_t \max\{\lambda_t M_t, \max[\lambda_t \min[p_t, \mu w_{t-1}], 0] - k\}) W_t / \delta_t$$

Then following the methods of section 4.1, the time path of β can be deduced.

An obvious extension would be to allow for more than one type of house, e.g. a large expensive house with price p_t and a small cheaper house with price

⁸Since labour income for t is unknown at the time of refinancing, the income constraint works on past labour income.

π_t , each of these prices are uncertain but for sure always cheaper $\pi_t < p_t$. Then consumers could trade down from large to small houses and vice versa. We might then expect to see systematic trading down close to retirement. In terms of housing decisions at t there are three choices: retain the existing house and mortgage; retain the existing house but refinance; change house and refinance. It makes sense to add a second transaction cost k_t^h which is incurred when changing house (in addition to the refinancing transaction cost). The effect is to add a third branch to the value function and the overall value function is then the maximum over the three branches. If we keep the other assumptions maintained (especially no liquidity constraints), the refinancing decision will have the same form, once any house purchase/sale has been decided. Again all uncertainty will be channelled through β , the value function will have a similar structure and the recurrence relation for β , (1) will become

$$\beta_t = [\phi(E\beta_{t+1})\delta_{t+1}(1+r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \cdot \exp(-b\Delta_t \max\{p_t - \pi_t - k_t^h + (\pi_t \max((\lambda_t, 0) - k), \lambda_t M_t, \max[\lambda_t p_t, 0] - k)\} W_t / \Delta_t)$$

Furthermore, in the compulsory retirement case we could make the date of retirement uncertain. This is similar to dropping the perfect foresight assumption on wages and has significant effects (see Berloffia and Simmons, 2003, and below).

The main special assumptions that we have made are:

- CARA preferences depending only on consumption and independent of housing or leisure or socio-demographics.

A problem with CARA is that optimally consumption may turn out to be negative since marginal utility is finite at zero consumption. That never happens in our simulations. We could make utility vary with housing and leisure-in the context of a model with a single indivisible house type, the former adds little, but the latter would be interesting and, although most of the structure of the value function, the refinancing decision and consumption will remain unchanged, there will be some additional preference effects (see Berloffia and Simmons, 2003).

- Perfect foresight of unemployment benefit, wage and interest rates.

This is an important simplification with potentially large implications. If interest rates are uncertain then the consumer has a real portfolio choice, not just a choice driven by choosing the asset with the highest return. We might then expect to get some diversification of the portfolio depending on the covariance between the interest rates. In addition the covariance between interest rates and house prices will play a role.

Uncertainty in the real wage when employed can readily be incorporated so long as it is uncorrelated with house prices and with the chance of having a job. The value function, the refinancing decision and consumption will have a similar functional structure where the expected labour income term

W_t becomes

$$W_t = \alpha E \exp(-b\delta_{t+1} (1 + r_{t+1}) w_t^s) + (1 - \alpha) \exp(-b\delta_{t+1} (1 + r_{t+1}) B_t)$$

If there is correlation between house prices and wages it is more complex. Another assumption we have made is that the real house price is stationary, we could relax this, allowing for a random walk with drift and get similar results.

- A chance of being jobless that is constant (and foreseen) through time.

It would be easy to allow the chance of unemployment to be time dependent - essentially it is just a notational change, the effect will be absorbed into W_t , so all the results will carry through.

- Omission of the impact of the tax system.

The treatment of interest income and payments, capital gains on housing and of implicit user services of owned housing differs between tax systems, so it is important to interpret these variables as post-tax.

- A single one period financial asset with a perfect capital market.

The biggest omission here is the role of voluntary or involuntary contributions to pension schemes. Pension wealth can be defined by either the value of accumulated contributions to date or by the estimated pension income that will accrue at maturity. With the former approach and using the Family Resources Survey, Warren *et al.* (2001) find that individual median wealth was about £63k which decomposed into median wealths of pensions £26k, financial wealth £1k and housing wealth £24k. The pension wealth divided into about 39% in state pensions, 53% in occupational pensions and only 8% in "discretionary" pensions. This asset structure accords with that found by others where liquid or risky financial assets are an insignificant proportion of individual wealth. Warren *et al.* also find significant age, household composition and cohort effects, although over the life cycle the change in the share of financial wealth is not very sizeable-the wealthiest group (couples of above retirement age) had median wealth of £165k and the highest level of financial wealth but even here the median value for financial wealth was only £6k. Pension wealth is clearly important and serves both to remove some effects of uncertain date of death and to act as a buffer against asset shocks, e.g. falling real house prices late in life. Using the projected benefits approach and a cohort approach, Attanasio and Rohwedder (2003) find substantial age and cohort differences in the level of pension wealth.

A further factor is that in reality there are wedges between the saving rate and the borrowing rate in bond type finance. Including this will affect the refinancing decision.

8 Conclusions

This paper uses a framework which allows us to analytically solve for the value function and the optimal lifecycle policies for consumption, saving in financial assets and mortgage debt when a finitely lived, risk averse individual faces employment risk and uncertainty of house prices. This gives the advantage of being able to derive general propositions as opposed to specific simulation results without resorting to approximations which may have substantial inaccuracy. The financial asset market is perfect and up to 100% variable rate mortgages are allowed. Using CARA preferences and the assumption that house prices do not have stochastic trends while interest rates are certain and wages are foreseen facilitate explicit solution. However, the form of value function and optimal policy that we find is generally robust to relaxation of these special assumptions, similar results would follow if we had more constraints on available mortgages, uncertain wages, preferences depending not only on consumption but also on housing services, more than one type of house. One main result in all these cases is that there is a single "sufficient statistic" through which the effects of uncertainty on the value function and the optimal policies are channelled. This is due to the CARA form of preferences.

In terms of detail, we find that consumption is linear in wealth with an intercept that depends on future employment and house price risk, and a slope that depends on risk aversion and interest rates. Depending on the interest rate differential and the mean of future house prices, house price uncertainty may raise or reduce consumption in a period. Housing wealth and mortgage finance impact on consumption so that in periods when it is optimal to refinance consumption jumps corresponding to equity withdrawal. Therefore, sometimes consumption tracks cash on hand and is not fully smoothed. In other periods there is an ambiguous effect of mortgage debt on consumption. We perform some simulations which illustrate these properties. Broadly, when real wages are trendless, the trend in consumption reflects the degree of patience of the consumer so that although we mainly use hump shaped wages, we do not find hump shaped consumption. However if real wage growth is expected then even if the consumer is patient, it is optimal to borrow in early periods against later high labour income. In one set of simulations we explore the effects of a period of retirement on consumption and financial decisions. Generally there is accumulation of the financial asset during working life to finance retirement.

The effects of housing wealth on consumption and saving/borrowing decisions primarily work through the mortgage. Consequently, the analysis of the refinancing of mortgages is important to understand how housing wealth can act as a buffer stock against bad shocks, e.g. in employment. We find that without liquidity constraints and with foreseeable interest rates, the refinancing decision is driven by financial efficiency considerations. The individual will refinance to the maximum extent possible in those periods in which the financial gains from doing so cover the transaction cost. Hence we should expect to see individuals with either zero mortgages or 100% mortgages. The financial gains from refinance are used partly to finance present and partly future consumption.

Since the optimal mortgage is always at a limiting value of zero or the maximum permissible, any smoothing of consumption is achieved through varying the financial asset/debt position. Consequently, life-cycle effects or trends in labour income are translated into an optimal consumption path via nonlinear trend type variation in financial assets. This means that during life the composition of wealth varies. On the other hand, housing does act as a buffer stock in the sense that knowing that in the last period the house will have for sure a value high in relation to labour income and an even higher mean value, the consumer can borrow earlier in life in financial assets.

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A Appendix

A.1 Value Function

At T all available wealth is consumed and no mortgage interest is paid since no new mortgage debt is contracted so

$$0 = (1 + r_t) A_T + p_T - M_T - c_T + B_T - k$$

In the last period the value function is:

$$\begin{aligned} V_T &= 1 - \exp(-b((1 + r_T) A_T - M_T)) \exp(-bB_T) \exp(-bp_T) \quad \text{if } M_T = 0 \\ &= 1 - \exp(-b((1 + r_T) A_T - M_T)) \exp(-bB_T) \exp(-b(p_T - k)) \quad \text{if } M_T > 0 \end{aligned}$$

This result is obtained simply substituting the budget constraint at T into the instantaneous CARA utility function.

So at T

$$\begin{aligned} \alpha_T &= 1 \\ \beta_T &= \exp(-bB_T) \exp(-bp_T) \quad \text{if } M_T = 0 \\ \beta_T &= \exp(-bB_T) \exp(-b(p_T - k)) \quad \text{if } M_T > 0 \\ \delta_T &= 1 \end{aligned}$$

In any period before the final one, with refinancing the Bellman's equation says that c_t is determined to :

$$\max_{c_t} \{u(c_t) + \phi EV_{t+1}(A_{t+1}) | A_{t+1} = (1 + r_t) A_t + (1 - \rho_t) M_{t+1} - M_t - c_t + w_t^s - k\}$$

That is equivalent to:

$$\begin{aligned} & \max_{c_t} \{u(c_t) + \phi E(\alpha_{t+1} - \beta_{t+1} \exp[-b\delta_{t+1} ((1 + r_{t+1}) A_{t+1} - M_{t+1})]) \\ & \quad \cdot | A_{t+1} = (1 + r_t) A_t + (1 - \rho_t) M_{t+1} - M_t - c_t + w_t^s - k\} \end{aligned}$$

or

$$\begin{aligned} & \max_{c_t} \{1 - \exp(-bc_t) + \phi \{ \alpha_{t+1} - E(\beta_{t+1} \exp[-b\delta_{t+1} (1 + r_{t+1}) \{(1 + r_t) A_t \\ & \quad + (1 - \rho_t) M_{t+1} - M_t - c_t + w_t^s - k\} - M_{t+1}]) \} \} \end{aligned}$$

Since we are assuming that the risk of unemployment is foreseen and is independent of the uncertain house prices:

$$\begin{aligned} & E[\beta_{t+1} \exp(-b\delta_{t+1} (1 + r_{t+1}) w_t^s)] \\ = & E\beta_{t+1} [\alpha \exp(-b\delta_{t+1} (1 + r_{t+1}) w_t) + (1 - \alpha) \exp(-b\delta_{t+1} (1 + r_{t+1}) B_t)] \end{aligned}$$

The first order condition gives:

$$\begin{aligned} \exp(-bc_t) &= [\phi(E\beta_{t+1})\delta_{t+1} (1 + r_{t+1})] \cdot \exp[b\delta_{t+1} (1 + r_{t+1}) c_t] \\ & \quad \cdot \exp[-b\delta_{t+1} ((1 + r_{t+1}) [(1 + r_t) A_t + (1 - \rho_t) M_{t+1} - M_t - k] - M_{t+1})] \\ & \quad \cdot [\alpha \exp(-b\delta_{t+1} (1 + r_{t+1}) w_t) + (1 - \alpha) \exp(-b\delta_{t+1} (1 + r_{t+1}) B_t)] \\ = & [\phi(E\beta_{t+1})\delta_{t+1} (1 + r_{t+1})] \cdot \exp(-b\delta_{t+1} (1 + r_{t+1}) [(1 + r_t) A_t - M_t]) \\ & \quad \cdot \exp[b\delta_{t+1} (1 + r_{t+1}) c_t] \cdot \exp(-b\delta_{t+1} [(1 + r_{t+1}) \{(1 - \rho_t) M_{t+1} - k\} - M_{t+1}]) \\ & \quad \cdot [\alpha \exp(-b\delta_{t+1} (1 + r_{t+1}) w_t) + (1 - \alpha) \exp(-b\delta_{t+1} (1 + r_{t+1}) B_t)] \end{aligned}$$

Hence:

$$\begin{aligned} \exp(-bc_t) &= [\phi(E\beta_{t+1})\delta_{t+1} (1 + r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\ & \quad \cdot \exp(-b \frac{\delta_{t+1} (1 + r_{t+1})}{1 + \delta_{t+1} (1 + r_{t+1})} [(1 + r_t) A_t - M_t]) \\ & \quad \cdot [\alpha \exp(-b\delta_{t+1} (1 + r_{t+1}) w_t) + (1 - \alpha) \exp(-b\delta_{t+1} (1 + r_{t+1}) B_t)]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\ & \quad \cdot \exp(-b \frac{\delta_{t+1}}{1 + \delta_{t+1} (1 + r_{t+1})} [(1 + r_{t+1}) \{(1 - \rho_t) M_{t+1} - k\} - M_{t+1}]) \end{aligned}$$

and

$$\begin{aligned}
\exp [b\delta_{t+1} (1 + r_{t+1}) c_t] &= [\phi(E\beta_{t+1})\delta_{t+1} (1 + r_{t+1})]^{-\delta_{t+1}(1+r_{t+1})/[1+\delta_{t+1}(1+r_{t+1})]} \\
&\cdot \exp\left(b\frac{(\delta_{t+1} (1 + r_{t+1}))^2}{1 + \delta_{t+1} (1 + r_{t+1})} [(1 + r_t) A_t - M_t]\right) \\
&\cdot \exp\left(b\frac{\delta_{t+1}^2 (1 + r_{t+1})}{1 + \delta_{t+1} (1 + r_{t+1})} [(1 + r_{t+1}) \{(1 - \rho_t) M_{t+1} - k\} - M_{t+1}]\right) \\
&\cdot [\alpha \exp(-b\delta_{t+1} (1 + r_{t+1}) w_t) + (1 - \alpha) \exp(-b\delta_{t+1} (1 + r_{t+1}) B_t)]^{-\frac{\delta_{t+1}(1+r_{t+1})}{1+\delta_{t+1}(1+r_{t+1})}}
\end{aligned}$$

So conditional on the refinancing decision, taking expectations over the employment status at t the value function with refinancing is

$$\begin{aligned}
V_t^R(A_t) &= 1 + \phi\alpha_{t+1} - [\phi(E\beta_{t+1})\delta_{t+1} (1 + r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\
&\cdot \exp\left(-b\frac{\delta_{t+1} (1 + r_{t+1})}{1 + \delta_{t+1} (1 + r_{t+1})} [(1 + r_t) A_t - M_t]\right) \\
&\cdot [\alpha \exp(-b\delta_{t+1} (1 + r_{t+1}) w_t) + (1 - \alpha) \exp(-b\delta_{t+1} (1 + r_{t+1}) B_t)]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\
&\cdot \exp\left(-b\frac{\delta_{t+1} (1 + r_{t+1})}{1 + \delta_{t+1} (1 + r_{t+1})} [\{(1 - \rho_t) M_{t+1} - k\} - M_{t+1}/(1 + r_{t+1})]\right) \\
&\cdot \left[\frac{1 + \delta_{t+1} (1 + r_{t+1})}{\delta_{t+1} (1 + r_{t+1})}\right]
\end{aligned}$$