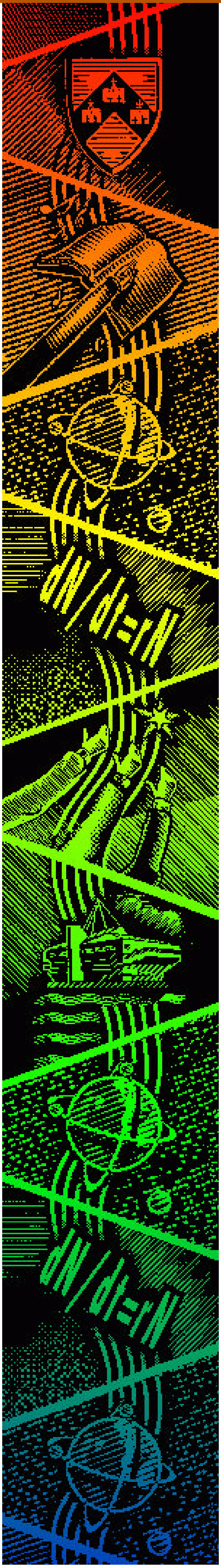

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Fiscal Policy in an Imperfectly Competitive Dynamic Small Open Economy

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Fiscal Policy in an Imperfectly Competitive Dynamic Small Open Economy*

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Abstract

In this paper we develop a general model of an imperfectly competitive small open economy. There is a traded and non-traded sector, whose outputs are combined in order to produce a single final good that can be either consumed or invested. We make general assumptions about preferences and technology, and analyse the impact of fiscal poclicy on the economy. We find that the fiscal mutiplier is between zero and one, and provide sufficient conditions for it to be increasing in the degree of imperfect competition. We also are able to compare the multiplier under free-entry and with a fixed number of firms, the speed of convergence to equilibrium and welfare. A simple graphical representation of the model is developed.

JEL: E20, E62, F4
Keywords: imperfect competition, open economy, fiscal policy.

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1 Introduction

There is now a well established literature which explores the effects of imperfect competition in output markets on fiscal policy in static closed economy models (Dixon (1987), Mankiw (1988), Startz (1989), Dixon and Lawler (1996)) and more recently in dynamic closed economy models (Rotemberg and Woodford (1995), Heijdra (1998)). Since the labour market is perfectly competitive, the key result is that the presence of imperfect competition in the product market leads to higher marginal profitability and hence the "profit multiplier", by which an initial increase in output generates a positive feed-back onto consumption via profits which is stronger with imperfect competition. As Startz (1989) argued, this effect will be absent when there is free-entry. Another way of understanding this is that there is a distortion of the consumption/leisure decision by increasing the cost of consumption relative to leisure. The household thus tends to choose relatively more leisure relative to consumption than under perfect competition. An increase in Government expenditure financed by a lump-sum tax will tend to lead to a larger reduction in leisure than crowding out of consumption, hence leading to a larger output multiplier.

This paper seeks to extend this analysis to a dynamic small open economy model. We develop the Walrasian framework of Turnovsky et al. (Sen and Turnovsky (1990,1991), Brock and Turnovsky (1994), Turnovsky (1991)[1]), but introduce monopolistic competition in the non-traded goods sector. We have also kept the traditional Ramsey assumption of a single final output which can be used for consumption, investment or government expenditure, with the traded and non-traded goods as intermediates. There are two inputs (capital and labour), the production and factor markets are Hecksher-Ohlin, with the traded sector being the more capital intensive[2]. This approach can be seen as combining the Hecksher-Ohlin two-sector model with the one-sector Ramsey model. The Ramsey household holds two assets, capital and an international bond and solves the standard intertemporal optimization problem giving rise to the dynamics of the economy.

Our setup differs in certain key respects from other papers. We allow

[1] Other papers that have looked at this issue in an essentially dynamic context include Gosh (1992), Mendoza (1995), Obstfeld (1982), 1989), Serven (1995), van Wincoop(1993) inter alia (see Obstfeld and Rogoff (1995b) for more references).

[2] This is a very common assumption, see Obstfeld (1989), Mendoza (1995) for a discussion.

for a general non-separable utility function over consumption and leisure (in many papers, either there is no disutility of work (e.g. Dornbusch (1983), Turnovsky (1991)), or it is additive (e.g. Sen and Turnovsky 1991)). Whilst it is standard in RBC models to have leisure in utility, it takes special functional forms (e.g. Backus et al. (1995)). We impose constant returns to scale on production in the two sectors rather than explicit functional forms. Adopting more general functional specifications enables us to understand which results are due to specific forms and which are truly general.

The results of the paper are as follows. For the case we are considering, the dynamics and equilibrium are very simple. Consumption and employment are constant, as are prices. Capital is accumulated by running a trade deficit and decumulating bonds, whilst switching production form the traded to the non-traded sector. We are able to construct a general analysis of the impact of fiscal policy. We find that the fiscal multiplier is positive but less than unity. This reflects the impact of taxation in increasing the labour supply and reducing consumption.

We are able to provide a simple graphical analysis of the steady-state effects of fiscal policy. We also consider the relationship between the multiplier and imperfect competition. We are able to show that

- whenever there is imperfect competition, the multiplier is larger when there is a fixed number of firms as opposed to the free-entry case.
- the multiplier is increasing in the degree of imperfect competition when preferences and technology are Cobb-Douglas.
- the speed of convergence to the steady state is slower when there is free entry, and in the case of Cobb-Douglas preferences and technology convergence is slower when there is more imperfect competition.
- the "Profit Effect" of imperfect competition without free-entry in the non-traded sector is vital for understanding the multiplier and resultant welfare effects.

The paper is organised as follows. Section 2 describes the disaggragated microlevel in the final output market, the two intermediate sectors and the factor markets, where the relationships are primarily *intratemporal*. In section 3 we analyse the aggregate level with a representative Ramsey consumer which captures the *intertemporal* relationships and the portfolio behaviour which yields the dynamic equilibrium of the economy. The steady state and dynamic properties of the equilibrium are described in section 4 and given graphical expression. The impact of imperfect competition on the multiplier and the speed of convergence is analysed in section 5.

2 Output and Factor Markets.

There are two intermediate sectors in the small non-monetary open economy: a perfectly competitive traded good sector producing a good at a given international price and an imperfectly competitive non-traded sector. There is a single final output in the economy, a good that can be consumed, invested or used by the government: this good is derived from combining the output of the two sectors according to a linear homogenous function. The combination can either be seen as occurring due to production (the traded and non-traded good are intermediate products) or preferences (a sub-utility function). For the formal structure of the paper, we will adopt the former physical interpretation.

2.1 Final Output

In the economy there are two types of good produced. N is the output of the non-traded (NT) good, itself a composite; T is domestic output of the traded-good, and m the (net) imports from abroad. The traded and the non-traded good are combined to produce a single non-traded[3] final output Y which can be consumed C, invested I or used as government expenditure G

$$Y = C + I + G$$

The traded good is taken as the numeraire and the price of the non-traded good is p_N , final output P.

Y is produced by a competitive industry with a constant returns to scale

[3] In the case where we view Y as a homothetic sub-utility, the assumption that Y is non-traded is obvious and inevitable. In the case of intermediate production it is less so. However, since the delivery of traded goods to the consumer involves many domestic inputs (delivery and retail services, assembly, packaging etc.), the assumpotion is not unreasonable in the case of Y being a final output.

production function, which we represent as a unit cost-function[4] $P(\ p_N)$. Price taking behaviour implies that

$$P = P(p_N) \tag{1}$$

By Shephard's Lemma we have the following conditional input demands for the traded good T^d and non-traded good N^d:

$$N^d = \alpha(p_N).\frac{PY}{p_N} \tag{2}$$

$$T^d = [1 - \alpha(p_N)]PY \tag{3}$$

where $\alpha(p_N)$ is the factor share of the NT good[5]. Along with the standard properties of a cost function, we also assume that $1 > \alpha(p_N) > 0$ for all $p_N > 0$, (this rules out corner solutions where only one good is used to produce Y. In fact we only need this property to hold near the equilibrium price.

2.2 The Traded Sector

The traded sector is perfectly competitive, and produces the traded good with a constant returns to scale production function which can be represented in factor-intensive form

$$T = F_T(L_T, K_T) = L_T f_T(k_T) \tag{4}$$

where L_T, K_T are employment and capital in the sector and k_T is the capital-labour ratio. The wage in terms of the numeraire is w, and the rental on capital r_k. Cost-minimisation implies the standard marginal productivity conditions

$$f_T'(k_T) = r_k \tag{5}$$

$$f_T - k_T f_T' = w \tag{6}$$

[4] The unit-cost function P is the homogenous to degree 1 in (p_N, p_T). We write function $P = P(p_N)$ since T is the numeraire and $p_T = 1$.

[5] The total input of the traded good comes both from domestic production and (net) imports, $T^d = T + m$.

2.3 The Non-traded Sector

The non-traded sector consists of two tiers: the higher-level is competitive and produces homogenous output (or composite good) N the lower tier is monopolistic[6], consisting of a large (and possibly variable) number n of firms j producing imperfectly differentiated products. The non-traded good is produced from the firms' output by the symmetric constant returns production function, which we represent by the unit cost-function $c(\mathbf{P})$,where $\mathbf{P}$ the n-vector of firms' prices p_j. Since the final NT output market is competitive, $p_N = c(\mathbf{P})$.We assume that there is no *Ethier effect*[7] so that cost is not affected by the number of firms n, and also for convenience assume that if all input prices are equal ($\mathbf{P} = p_N.\mathbf{1}$, where $\mathbf{1}$ is the unit n-vector), then $c(p_N.\mathbf{1}) = p_N$. By Shephard's Lemma the conditional demand for monopolist $j's$ output x_j is:

$$x_j = \frac{\partial c}{\partial p_j} N = \overline{\alpha}_j\ (\mathbf{P}) \frac{p_N N}{p_j} \tag{7}$$

where $\overline{\alpha}\ (\mathbf{P})$ is the homogeneous to degree zero factor-share for input j. From the demand for firm j (eq.7) we can derive the elasticity of demand $\varepsilon_j(\mathbf{P})$ which is homogeneous of degree zero in $\mathbf{P}$[8]. In particular, if all prices are the same, then the elasticity of demand is $\varepsilon^* = \varepsilon_j(\mathbf{1})$ for all j. The only restrictions we need to put on the technology for producing the non-traded good is that $\varepsilon^* > 1$ and the elasticity is non-decreasing[9]. We also assume that

[6] The assumption that the NT sector is imperfectly competitive is common, reflecting the view that the sector is protected from international competition. However, there is no reason in principle why we could not allow for the traded sector to be imperfectly competitive as well. We have opted for a perfectly competitive traded sector for modelling convenience only.

[7] An "Ethier Effect" means that when there are more goods (n is bigger) the unit cost decreases (this is sometimes called the "love of variety" effect). See for example Heijdra (1998).

[8] It is often wrongly believed that this property holds only for a special class of homothetic functions (i.e CES, CD). The elasticity of demand is

$$\varepsilon_j(\mathbf{P}) = \frac{p_j}{c_j} c_{jj}$$

Since c is Homogeneous of degree 1 in $\mathbf{P}$, it follows that $\varepsilon_j(\mathbf{P})$ is Homogeneous of degree 0.

[9] We require that $\varepsilon^* > 1$ from the first order an interior optimum with strictly positive costs. ε_j being non-decreasing in p_j is sufficient to ensure that marginal revenue is decreasing and the second-order conditions are satisfied.

ε^* is unaffected by n (as is standard in the case of monopolistic competition).

Each firm has a technology

$$x_j = F_N(L_j, K_j) - F = L_j f_N(k_j) - F \tag{8}$$

where $\{L_j, K_j\}$ are labour and capital inputs of firm j, F is the fixed cost per firm, F_N is homogeneous of degree 1, k_j the capital-labour ratio and $f_N(k_j)$ the labour-intensive representation of N.

Monopolistic firm j chooses $\{p_j, L_j, K_j\}$ to maximise profits

$$\max\, p_j x_j - wL_j - r_k K_j$$

given the demand for its input from the producers of N (7) and technology (8). Under symmetry ($p_j = p_N, k_j = k_N$ etc.), this yields the standard price-markup (the Lerner-index)

$$\mu = \frac{p_N - MC}{p_N} = \frac{1}{\varepsilon^*} \tag{9}$$

where MC is marginal cost: by cost-minimisation $MC = \left(r_k / f_N^{'}(k_N)\right) = w/(f_N - k_N f_N^{'})$.

This price-markup equation (9) implies the marginal revenue products are equated to factor cost

$$p_N(1-\mu) f_N^{'}(k_N) = r_k \tag{10a}$$

$$p_N(1-\mu)\left[f_N(k_N) - k_N f_N^{'}(k_N)\right] = w \tag{10b}$$

These two equations can be combined with (6,5) to show that there is a misallocation of resources: the marginal product of each factor is larger in the NT sector than the T sector. This reflects the fact that the price of the non-traded good is higher than its marginal cost, so that equating marginal revenue products across sectors does not equate marginal value products.

In this paper, we shall be examining the behaviour of the NT sector in two cases (a) with a fixed number n of firms, (b) with free-entry (a zero profit condition).With a fixed number of firms the relationship between aggregate N and total factor inputs is

$$N = F_N(K_N, L_N) - nF = L_N f_N(k_N) - nF \tag{11}$$

Profits in the NT sector are given by

$$\Pi = p_N F(K_n, L_n) - (wL_N + r_K K_N) - np_N F$$

With free-entry, we have the zero-profit condition $\mu L_N f_N(k_N) = nF$ so that[10]

$$N = (1-\mu)L_N f_N(k_N) \tag{12}$$

Hence, with free-entry although there are fixed costs and increasing returns at the firm level, there are constant returns at the industry level.

For simplicity we shall be considering three combinations of parameters μ and F

- *Walrasian* $\mu = 0, F = 0$. In the Walrasian case, no equilibrium will exist if $F > 0$. When $F = 0$,the number of firms is indeterminate and has no effect.
- *Free-entry* $\mu > 0, F > 0$. When $\mu > 0$, no free entry equilibrium will exist unless $F > 0$. The actual level of F does not matter, as is apparent from (12): the level of nF is determined. For any given $F > 0$ the Walrasian case is the limit of the free entry case as $\mu \to 0$.
- *Fixed number of firms*: $\mu > 0, F \geq 0$. With a fixed number of firms, the fixed costs in the economy are constant (nF).

2.4 Factor market equilibrium

This occurs on two levels. First, we have the four marginal productivity equations (5),(6), (10a), (10b). For our purposes these can be seen as having

[10] Since N is Homogeneous of degree1

$$p_N F(K_N, L_N) = p_N(f_N - k_N f'_N)L_N + p_N f'_N K_N$$

But from (10a), (10b)

$$wL_N + r_K K_N = (1-\mu)p_N F(K_n, L_n)$$

Hence

$$\Pi = \mu p_N F(K_n, L_n) - np_N F$$

Zero-profits immediately implies (12).

four endogenous variables $\{k_N, k_T, w, r_K\}$, with predetermined variable(s) $\{p_N(1-\mu)\}$. Second, we have the aggregation market clearing conditions

$$\begin{aligned} K &= K_N + K_T && \text{(13a)} \\ L &= L_N + L_T && \text{(13b)} \end{aligned}$$

We can write the sectoral levels of output and factor-employment in the following way

$$L_N = \frac{Lk_T - K}{k_T - k_N} \; ; L_T = \frac{K - Lk_N}{k_T - k_N}$$

relating them to aggregate employment and the sectoral capital-labour ratios (themselves determined by $\{p_N, \mu\}$).Finally, outputs are given by

$$N = \frac{Lk_T - K}{k_T - k_N} f_N - nF; \; T = \frac{K - Lk_N}{k_T - k_N} f_T \tag{14}$$

In this section we have solved for the factor market equilibrium conditional on the price p_N. The equilibrium price will be determined when we solve for the dynamic equilibrium, which requires that we now look at the aggregate economy and the representative household.

3 The Household and Aggregate Dynamics.

In this section we turn to the aggregate level and the intertemporal structure of the economy. First we consider the household's intertemporal optimization. Second, we combine the household's behaviour with the micro-structure in order to derive the fundamental dynamic equations describing the economy. In the next section we will examine the steady state and linearised dynamics of the economy.

3.1 The Household's Optimization.

The economy is represented by a single household which owns all capital, supplies all labour, owns the net foreign (real) assets b, receives all of the profits from the domestic firm and pays (lumpsum) taxes which equal in each instant government expenditure G[11].

[11] The timing of taxation is irrelevant so long as the present values are equivalent since Ricardian equivalence holds.

The representative consumer has the following lifetime utility:

$$\int_0^{\infty} U(C, 1-L)e^{-\rho t}dt$$

where $U(C, 1-L)$ gives the flow of utility from current consumption C and leisure $l = 1 - L$: the household has one unit of leisure-endowment per instant and works for L of this. The discount rate ρ is assumed equal to the world interest rate r. For this paper, we assume that U is strictly concave in $(C, 1-L)$, with both consumption and leisure being normal goods.

The household's budget constraint is:

$$\dot{b} = rb + wL + r_K K + \Pi - P(C + I + G) \tag{15}$$

The current-value Hamiltonian for the household's optimization is:

$$H = U(C, 1-L) + \lambda[rb + wL + r_K K + \Pi - P(C + I + G)] + qI \tag{16}$$

For which the first order conditions are:

$$U_c = \lambda P \tag{17a}$$
$$U_L = -\lambda w \tag{17b}$$
$$-\lambda P = q \tag{17c}$$
$$\dot{\lambda} = 0 \tag{17d}$$
$$\dot{q} - rq = -\lambda r_K \tag{17e}$$

with the transversality conditions :

$$\lim_{t\to\infty} \lambda b e^{-\rho t} = \lim_{t\to\infty} qKe^{-\rho t} = 0 \tag{18}$$

and initial conditions $b(0) = b_0, K(0) = K_0$.

From (17d), λ is constant over time, $\lambda(t) = \lambda^*$. This implies that the marginal value of bonds to lifetime-utility is equalised over time, as is standard in such open economy models when the discount rate and world interest rate are the same.

Secondly the two equation (17a,17b) yield C and L conditional on (λ, P, w). Since U is strictly concave, we can write the resultant functions (known as Frisch demands[12]):

$$C = C(\lambda, P, w) \tag{19a}$$
$$L = L(\lambda, P, w) \tag{19b}$$

[12] See Cornes (1992,pp.163-165).

Where λ is the marginal lifetime utility of a foreign bond, with derivatives given in the appendix. The real wage $W = w/P$ defines the income-expansion path (IEP): λP the position on the path.

Thirdly, substituting (17c) into (17e) and differentiating with respect to time we obtain an *arbitrage equation* between capital and bonds

$$\dot{P} = rP - r_k \tag{20}$$

3.2 Market Clearing and Dynamic Behaviour.

We now take the optimality conditions of the household and combine them to obtain dynamic equations for the key variables p_N, K, b, λ which determine the state of the economy.

First, we can combine the arbitrage equation (20) with the factor market equations, noting that $P = P(p_N)$ and $r_K = r_K(P_N, \mu)$ to obtain a differential equation in p_N

$$\dot{p_N} = \frac{p_N\left[r.P(p_N) - r_k(p_N, \mu)\right]}{\alpha(p_N).P(p_N)} = \Gamma(p_N, \mu) \tag{21}$$

The dynamics of p_N are determined only by (p_N, μ): as such, (21) constitutes an autonomous subsystem within the model. In particular, it defines a non-trivial stationary point $p_N^* > 0$. Given our assumption that $\alpha \in (0, 1)$ for $p_N > 0$ then $\Gamma(p_N, \mu) = 0$ iff

$$r.P(p_N) - r_k(p_N, \mu) = 0 \tag{22}$$

A non-trivial stationary point must be unique, from the strict monotonicity of the LHS of (22) in p_N. (22) implicitly defines the strictly increasing function $p_N(\mu)$ and hence we can also define $P(\mu) = P(p_N(\mu))$ (strictly increasing) and $w(\mu)$ (strictly decreasing). An increase in the degree of monopoly in the NT sector thus increases the relative price of the NT output and the Consumption good, whilst reducing the numeraire wage in steady state.

Since we assume that $k_T > k_N$ we have

$$\frac{d\Gamma(p_N^*, \mu)}{dp_N} > 0$$

Equation (21) dictates how p_N must vary to ensure that the rates of return are equalised between capital and bonds. In the case where $k_T > k_N$, a deviation away from p_N^* will be reinforced (i.e. if p_N is above p_N^* it will need to be increasing).

Second, we consider non-traded good market clearing condition. This can be obtained from the non-traded good market clearing condition, equating supply (11 or 12) with demand (3).With a fixed n we have

$$p_N.\left[L_N f_N(k_N) - nF\right] = \alpha(p_N)P(p_N)[C + G + \dot{K}]$$

the capital accumulation equation can be obtained by rearranging to yield

$$\begin{aligned} \dot{K} &= \frac{1}{P'}\left[\frac{Lk_T - K}{k_T - k_N} f_N - nF\right] - C - G \\ &\equiv \Phi^n(\lambda, K, p_N, G, n) \end{aligned} \tag{23}$$

where we note that

$$\Phi_K^n = -\frac{1}{P'}\frac{f_N}{k_T - k_N} < 0 \tag{24}$$

When there is free entry we have (using (12))

$$\begin{aligned} \dot{K} &= \frac{1}{P'}\left[\frac{Lk_T - K}{k_T - k_N}(1-\mu) f_N\right] - C - G \\ &= \Phi^e(\lambda, K, p_N, G) \end{aligned} \tag{25}$$

so that

$$\Phi_K^e = (1-\mu)\Phi_K^n$$

The accumulation of bonds can be derived by noting that imports m are the difference between domestic consumption (public and private) of the traded good (2) and domestic supply (14) yields[13]

$$\begin{aligned} \dot{b} &= rb + \frac{K - Lk_N}{k_T - k_N} f_T - \left[\frac{1-\alpha}{\alpha}\right] p_N \frac{Lk_T - K}{k_T - k_N} f_N \\ &\equiv \Psi(\lambda, K, p_N, b) \end{aligned} \tag{26}$$

Having derived the dynamic equations for the household's intertemporal optimization and the underlying microeconomic market clearing conditions, we will now examine the equilibrium solution.

[13] This uses the fact that

$$PY = \frac{1}{\alpha} p_N N$$

4 The Steady State and Linearised Dynamics

The dynamic equilibrium for the economy can be represented by the three differential equations for $\{K, b, p_N\}$ along with the transversality conditions This gives us four equations in four unknowns (λ, K, b, p_N). The system decomposes: we can solve (21) on its own to derive the time path of p_N; next we can solve the capital accumulation equation (25 or 23) for K conditional on λ given p_N and G; then (26) for b conditional upon λ given p_N and K; lastly the transversality condition (18) yields λ. There are two versions of the dynamic equation for capital (fixed n and free-entry): since the *dynamics* are not affected by the issue of entry, we will drop the superscript in this section indicating that both cases are covered. We will first consider the dynamics of the linearised system, treating the steady-state value λ^* as given (since it is constant, it does not influence the dynamics). We will then describe the full steady state.

4.1 Linearised Dynamics.

Turning first to the steady state, it is defined (for given λ^*) by :

$$0 = \Gamma(p_N^*, \mu) = \Phi(\lambda^*, K^*, p_N^*, G) = \Psi(\lambda^*, K^*, p_N^*, b^*)$$

The dynamics of the system follow Brock and Turnovsky (1994) closely, so the details will only be sketched here. Linearising around the steady state yields:

$$\begin{bmatrix} \dot{p}_N \\ \dot{K} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} \Gamma_p(p_N^*) & 0 & 0 \\ \Phi_p(p_N^*, K^*) & \Phi_K(p_N^*, K^*) & 0 \\ \Psi_p(p_N^*, K^*, b^*) & \Psi_K(p_N^*, K^*, b^*) & r \end{bmatrix} \begin{bmatrix} p_N - p_N^* \\ K - K^* \\ b - b^* \end{bmatrix} \tag{27}$$

From the point of view of the dynamics, b is a residual which is entirely determined via Ψ given λ^* and the current values of p_N and K. The sub-system in $\{K, p_N\}$ possesses two eigenvalues $\Gamma_p > 0$ and $\Phi_K < 0$ so that (27) is saddle-stable. Since the arbitrage equation is unstable, p_N is constant at its steady state value, $p_N(t) = p_N^*$ for all t. The solution to the linearised dynamic system is then:

$$p_N(t) = p_N^* \tag{28a}$$

$$K(t) = K^* + (K_0 - K^*)e^{\Phi_K t} \tag{28b}$$

$$b(t) = b^* + \Omega(K_0 - K^*)e^{\Phi_K t} \tag{28c}$$

We have solved for $b(t)$ following Brock and Turnovsky (1994), with

$$\Omega = \frac{\Psi_K}{\Phi_K - r} < 0 \tag{29}$$

Equation (29) indicates that there is a negative relationship between the accumulation of capital and bonds. This can be simplified:

Proposition 1 *(a)Free Entry and Walrasian*: $\Omega = -P$

$$\textit{(b)Fixed } n : \Omega = -P - \frac{\mu P_N}{\Phi_K^n - r} \cdot \frac{f_N}{k_T - k_N} < 0$$

All proofs are in the appendix. This Proposition is similar to Sen and Turnovsky (1995) in the case where there are no profits (Walrasian and free entry). Capital is accumulated by running down the stock of bonds: the household reallocates its total asset-stock from bonds to capital at the relative price P. Imperfect competition influences this since P is a function of μ (the NT component of capital is more expensive).

However, with a fixed n there is an additional *profit effect*: capital accumulation reduces the output[14] and hence profits in the NT sector. This means that whilst capital is below (above) the steady state level, the level of bonds is higher (lower) than it would have been without the profit effect. In fact, the additional term has a very precise interpretation: it is minus the marginal effect of $K(t)$ on the present value of profits. Whilst the dynamics of the economy reflect the effect of capital accumulation on the economy via Ω, the household does not take this into account, since the flow of profit income in all periods is taken as exogenous.

The adjustment path of the economy described is the following. Consumption and Employment are immediately set at their steady-state values, $C(t) = C^*, L(t) = L^*$ for all $t \in [0, \infty)$. The constancy of p_N (28a) implies that w and P are constant as well. The combination of a fixed λ and fixed prices means that there is perfect utility smoothing.

Capital is accumulated through the *sectoral reallocation* of labour and capital towards the non-traded sector (relative to the steady state), coupled with an increase in imports to boost the availability of the traded good (the level of sectoral outputs follow from $(L^*, K(t))$ and are given by (14)). Thus output Y is raised above its steady-state Y^* to allow for positive investment, but the cost is that the stock of bonds is reduced to finance the imports.

[14] Since the NT sector is labour intensive, an increase in capital reduces output in that sector.

4.2 The Steady State

In this section, we describe and depict the steady-state equilibrium. We already have the IEP which gives the relationship between consumption and leisure (which are both constant over time). In order to determine the level of Consumption and leisure (in effect, λ^*) we consider the life-time budget constraint ($LTBC$) of the representative household. Noting that since p_N is constant $r_K = rP$, and integrating (15) into present value terms using the transversality conditions (18) we have

$$\frac{P(C+G)}{r} = \frac{wL}{r} + [b_0 + PK_0] + \int_0^\infty \Pi(t)e^{-rt}dt \tag{30}$$

The steady-state values of consumption and leisure (and hence capital and bonds) will be determined by the intersection of (30) and the IEP.

4.2.1 The Walrasian and Free-entry cases.

In the Walrasian and free-entry cases there are no profits, so that we can define the $LTBC$ function $J^e(L, \mu)$

$$C + G = J^e(L, \mu) = \frac{1}{P(\mu)}\left[r\left(b_0 + P(\mu)K_0\right) + wL\right] \tag{31}$$

We can depict the steady state equilibrium graphically in consumption-leisure space. The relationship between consumption and leisure is represented by the income expansion path, as in Figure 1. A higher value of μ implies a lower real wage w/p and hence the IEP lies further to the right. We depict the J^e line for $\mu = 0$ (Walrasian) and $\mu > 0$. As μ increases, the intercept when $L = 0$ falls (since $b_0/P(\mu)$ falls) and the budget line rotates around to reflect the lower real wage. In effect, the productive inefficiency caused by free-entry with imperfect competition reduces the consumption possibilities for any given level of labour supply. When combined with the IEP we have the equilibrium. We depict the two equilibria for $\mu = 0$ (W) and $\mu > 0$ (IC). Clearly, from the geometry if we compare W and IC, we can see that whilst consumption is lower in IC, leisure may be higher or lower due to a conflict of income and substitution effects. Under the standard assumption that substitution effects dominate, leisure will be lower (as depicted).

The steady-state capital stock K^* corresponding to the steady-state level of employment and consumption is from (25) with entry and (23) without entry

$$K^* = \frac{1}{\Phi_K}(C^* + G) + k_T L^* \tag{32}$$

The corresponding steady-state bond holdings are derived from (28c) setting $t = 0$

$$b^* = b_0 - \Omega(K_0 - K^*) \tag{33}$$

Note that in the absence of profits (Walrasian and free-entry) the accumulation of capital has no effect on the household's $LTBC$ since the arbitrage condition implies that the returns on bonds and capital are always equal. Hence in this case, the only effect of imperfect competition is to alter the real wage and price level. Corresponding to $J^e(L, \mu)$ we have the implicit number of firms $n^e(L, \mu)$, where

$$n^e(L, \mu) = \frac{\mu \alpha P}{p_N F} J^e(L, \mu) \tag{34}$$

4.2.2 Fixed number of Firms

In the case of a fixed number of firms, we have to allow for profits in the $LTBC$. The the present value of profits will depend on both the steady state profit flow Π^* and the profits accumulated along the adjustment path (which can be expressed as a deviation form steady-state profits). Using the dynamic solution for $K(t)$ (28b) we have

$$\Pi(t) = \Pi^* + \frac{d\Pi}{dK}(K_0 - K^*)\exp(\Phi_K^n t) \tag{35}$$

Where Π^* is the steady-state profit

$$\Pi^* = \mu p_N \left[\frac{Lk_T - K^*}{k_T - k_N} f_N - n \frac{F}{\mu} \right]$$

$\frac{d\Pi}{dK} = -\frac{\mu P_N f_N}{k_T - k_N}$ is the marginal effect of an increase in K on the flow of profits (which is constant) and K^* is given by (32). Hence we have

$$\int_0^\infty \Pi(t) e^{-rt} dt = \frac{\Pi^*}{r} + \frac{d\Pi}{dK}(K_0 - K^*) \frac{-1}{\Phi_K^n - r}$$

so that the $LTBC$ becomes

$$\begin{aligned} C+G &= J(L,n,\mu) \\ &= \frac{1}{P}\left[r\left(b_0+PK_0\right)+wL+\Pi^*+\frac{d\Pi}{dK}(K_0-K^*)\frac{-r}{\Phi_K^n-r}\right] \end{aligned} \quad (36)$$

After some manipulation, we have

Proposition 2 *Consider the case of a fixed n. The steady-state relationship between consumption and employment $J(L,\mu)$ is given by*

$$J(L,n,\mu)=A+BL$$

where

$$\begin{aligned} (a) \quad B &> w/p \\ (b) \quad A &= A_0-A_1n \end{aligned}$$

with $A_0>r(\frac{b_0}{P}+K_0)$ and $A_1>0$.

The key point to note is that steady-state profits increase with L, so that an increase in L yields an increased flow of profits in addition to wage income w/P. Hence $B>w/P$. The intercept term falls as the number of firms n increases.

In figure 2 we represent $J(L,n,\mu)$ and $J^e(L,n,\mu)$. Note that $J(L,n,\mu)$ is not the *perceived* budget constraint: it represents the *actual* trade-off between consumption and leisure, including the income derived from profits. The steady-state budget constraint corresponding to a point such as b on $J(L,n,\mu)$ will have the slope w/P as represented by the dotted line, since the profit income enters as a lump sum into the budget constraint. The value of profits (losses) is represented by the vertical distance above (below) the point where the free-entry budget line $J^e(L,\mu)$ intersects the $L=0$ line, Π_B. The key point to note is that the slope of the $J(L,n,\mu)$ line is greater than w/P since it includes additional profits earned from the NT sector as output increases (Proposition 2 part (a)). At point E where $J(L,n,\mu)=J^e(L,\mu)$, $n=n^e(L,\mu)$: at points to the left of E, $n<n^e(L,\mu)$ and profits are positive; to the right $n>n^e(L,\mu)$ leading to losses. If we alter the number of firms, there is a vertical shift in $J(L,n,\mu)$ (part (b) of the proposition).

5 Fiscal Policy

In this section, we look at the effect of fiscal policy on the economy. We will consider the effects of a permanent but unanticipated change in total government expenditure financed by a lump-sum tax.

Let us first turn to the steady state effects. We will be considering the output[15] and employment multipliers in particular. Since government expenditure does not affect the equilibrium prices, the full effect occurs only through the income effect (*resource withdrawal effect* in Turnovsky's terminology). Hence

Proposition 3 *In all cases (free-entry, fixed n, Walrasian) we have*

$$0 < \frac{dY}{dG} < 1, \quad 0 < \frac{dL}{dG}, \quad 0 < \frac{dK}{dG}.$$

This is depicted in consumption leisure space, Figure 3. In all three cases, there is a linear $LTBC$. An increase in G merely results in a vertical shift downwards in the $LTBC$, with no other change. This resource withdrawal effect is allocated by the household according to its consumption-leisure preferences as represented by the IEP, upward sloping since consumption and leisure are normal. In the limiting case where leisure has an infinite marginal utility (vertical IEP), then there would be 100% crowding out and zero multipliers for employment output and capital. Conversely, if labour has no marginal disutility (IEP Horizontal) then there is no crowding out.

We can now compare the multiplier under free-entry and for a fixed-n. Assuming that we start off from an initial position with zero-profits (a point like A in figure 3).

Proposition 4 *If either (a) for general preferences with $n = n^e(L, \mu)$, or (b) for any n with $U(C, 1-L)$ homothetic*

$$\left.\frac{dY}{dG}\right|_n > \left.\frac{dY}{dG}\right|_e \text{ and } \left.\frac{dL}{dG}\right|_e > \left.\frac{dL}{dG}\right|_n$$

[15] Y is related to national income (measured in terms of the numeraire) by the following identities: $GDP = PY - m$ and $GNP = GDP + rb$. In steady state, since $rb = m$, we have $GNP = PY$.

Note that we are not able to make general comparisons with the capital multipliers. In figure 4, we depict the new equilibria after the increase in G from the initial equilibrium at A. If there were no change in Leisure, then the new equilibrium would be at A'. However, the household chooses to allocate the increase in taxes by working harder, to be on its IEP. With free-entry, the new equilibrium us at C; with fixed-n the new equilibrium is at B (the dotted line passing through B is the households steady state budget constraint). Clearly, the reduction in consumption is smaller in the case of fixed-n than with free-entry. The reason is that with a fixed number of firms an increase in output is achieved more efficiently (since the level of fixed costs is constant), the efficiency being reflected in the additional profit income Π_B.

The above argument applies to general preferences, only relying on the upward slope of the IEP. In this case the initial starting position has to be the same $n = n^e(L, \mu)$ since this ensures that the slope is the same. In the case of homothetic preferences, the IEP is linear with constant slope, so that the proposition can be relaxed to any n.

Lastly, we can contrast the *impact* effect of a permanent increase with the steady state effects. On impact capital and bonds are at their initial value (unchanged by dG). Clearly, since price is unaffected by changes in G, and $\{\lambda, C, L\}$ jump to their steady state values immediately, we need only look at $\{Y, N, T, m\}$.

Proposition 5 *The impact effects of a permanent increase in government expenditure for all three cases (Walrasian, free-entry and fixed n) are*

$$\frac{dY(0)}{dG} > \frac{dY^*}{dG} > 0 \qquad \frac{dN(0)}{dG} > \frac{dN^*}{dG} > 0$$

$$\frac{dT^*}{dG} > 0 > \frac{dT(0)}{dG} \qquad \frac{d\dot{b}(0)}{dG} < 0$$

The impact response of a change in G is *larger* for aggregate output Y and N than the steady-state response. The output of the traded intermediate T actually falls initially: an increase in labour with a constant capital stock decreases the output of the capital intensive sector from (14). The time paths of variables are depicted in Figure 5, where the change in G occurs at time t'. We can see that in order to accumulate capital, resources are switched to the NT sector and away from the T. This results in an increase in imports that is financed by running down the stock of bonds.

5.1 Imperfect Competition and the Multiplier

Having considered the effect of fiscal policy for a given μ, we can now explore the effect of the markup on the multiplier. In general, we cannot say what the effect of μ is on the multiplier. However, we are able to make some general statements for specific functional forms. In particular, we consider the Cobb-Douglas (C-D) case

C-D Preferences $U(C, L) = \frac{1}{1-\sigma}\left(C^{\nu}(1-L)^{1-\nu}\right)^{1-\sigma}$

C-D Technology $f_N(k_N) = (k_N)^{\delta}; f_T(k_T) = (k_T)^{\beta}$

We are able to show that

Proposition 6 *Imperfect Competition and the multiplier in a Cobb-Douglas Economy*

(a) Fixed n and $\mu \geq 0$, C-D Preferences and technology:

$$\frac{d^2Y}{dGd\mu} > 0$$

(b) Fixed n and $\mu \geq 0$, C-D preferences, general technology

$$\left.\frac{dY}{dG}\right|_{\mu>0} > \left.\frac{dY}{dG}\right|_{\mu=0}$$

(c) Walrasian and free-entry cases, C-D technology and preferences

$$\frac{dY}{dG} = 1 - \nu$$

In the case of a fixed number of firms, we find that imperfect competition leads to less crowding out of consumption and a larger multiplier. However, in the Walrasian and free-entry cases the multiplier is unaffected by the degree of imperfect competition (this is not surprising given that there is no profit effect.

The mechanism behind the multiplier is the resource withdrawal effect, the wealth effect of the increase in taxation to finance expenditure, thus reducing consumption and increasing employment. However, with imperfect competition and a fixed number of firms the increase in output will lead to an increase in profits which will tend to offset the resource withdrawal effect and reduce the crowding out of consumption. Imperfect competition has a second effect of reducing efficiency, which tends to increase the reduction in consumption (the household is less able to offset the increase in taxation by working harder). As in Dixon and Lawler (1996), these two effects make the effect of μ on the multiplier ambiguous in general, but the two propositions above show that in the case of specific functional forms for preferences and technology, we can derive some clear-cut results.

Lastly, we can determine the welfare effects of fiscal policy. Since we are treating G as waste, the resource withdrawal effect reduces utility. In fact, we can derive the following simple characterization of the effect of a change in G on lifetime utility $\bar{U}$

$$\bar{U} = \int_0^{\infty} U(C, 1-L)\exp[-rt]dt$$

Proposition 7 *Consider the free-entry and Walrasian cases.*

$$\frac{d\bar{U}}{dG} = \frac{-U_c}{r} < 0$$

In these cases, the reduction in welfare is due solely to the taxation effect $(-U_c)$. However, the reduction in welfare is greater when there is imperfect competition. This is because the marginal utility of consumption is higher both because the consumption/leisure ratio is lower (the IEP is flatter) and the level of utility is lower.

When there is a fixed number of firms, there is an additional profit effect, which offsets the tax effect.

Proposition 8 *With $\mu > 0$ and fixed n*

$$\frac{d\bar{U}}{dG} = -\frac{U_c}{r} + \frac{U_c}{r}\left[\frac{1}{P}\frac{d\Pi^*}{dG}\frac{\Phi_K^n}{(\Phi_K^n - r)}\right] < 0$$

The term in square brackets represents the change in steady state profits as a result of the increase in G and also the profits cumulated along the path towards the steady state. There is a clear contrast between the welfare results and the multipliers. When there is imperfect competition, the output multiplier is larger for the case of fixed n and the reduction in welfare is smaller. However, if we compare the case of free entry ($\mu > 0$) with the Walrasian multiplier, we find that the larger multiplier is combined with a larger fall in welfare. It is clearly misleading to make simple comparisons of welfare changes based on simple output multipliers. In particular, the effect of profits is crucial in determining welfare.

5.2 Imperfect competition and the speed of convergence

From (28b) we can define the speed of convergence to steady-state as the appropriate $|\phi_K|$. To see why, note that from an initial position of $K(t)$, the time τ taken to reach half way to K^* is[16]

$$\tau = \frac{1}{\Phi_K} \log(\frac{1}{2})$$

A large value of $|\phi_K|$ hence indicates rapid convergence. We can now compare the speed of convergence with free entry and fixed n.

Proposition 9 *Speed of Convergence. Let $\mu > 0$.*

(a) Convergence is slower with free-entry, since we have

$$|\phi_K^e| = (1-\mu)\, |\phi_K^n|$$

(b) Cobb-Douglas preferences and technology. With a fixed number of firms the speed of convergence is decreasing in μ.

$$\frac{d\,|\phi_K^n|}{d\mu} < 0$$

The reason for result (a) is that capital accumulation is more costly when there is free entry. For a *given value* of μ, more resources are used to produce a single unit of output. Hence the rate of capital accumulation is reduced. With a fixed number of firms and CD preferences and technology the effect of

[16]This follows from

$$\frac{K_{t+\tau} - K^*}{K_t - K^*} = e^{\phi_K \tau}$$

an *increase* in μ is to reduce the speed of accumulation. The reason for this is that $\mu > 0$ implies that there is too little N relative to T: the accumulation of capital makes this misallocation worse by reducing N and increasing T. Also, the higher price P makes capital more costly.

The speed of convergence to the steady state matters, in that it can be taken as an indicator of persistence. The more rapid convergence, the less the economy is influenced by the initial conditions. Imperfect competition tends to make convergence less rapid, and hence increases persistence. Furthermore, slower convergence will influence the accumulation of profits: slower convergence means that profits will deviate by more form the steady state value.

6 Conclusion

In this paper we have attempted to develop a general yet tractable framework with which to analyses the conduct of fiscal policy in the context of a small open economy with an imperfectly competitive non-traded sector. We have developed a simple and intuitive yet general framework which does not rely on special assumptions about preferences or technology and is able to yield some clear results. In addition we have developed a diagrammatic approach to the analysis of fiscal policy to complement the mathematical analysis.

There is clearly much more work to be done, in terms of exploring in more detail special cases within the general class of models allowed by our framework: this could include specific functional forms or even specific parameterizations of functional forms. However, we also hope that the analysis will provide a background for understanding the special cases that are often found in the existing literature.

7 *Bibliography*

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8 Appendix

Frisch demands

We compute the derivative of labour and consumption function $C(\lambda, w, P)$ and $L(\lambda, w, P)$.

$$\frac{\partial C}{\partial \lambda} = \frac{PU_{LL} + U_{CL}w}{U_{CC}U_{LL} - (U_{cl})^2} < 0, \qquad \frac{\partial L}{\partial \lambda} = -\frac{U_{CC}w + PU_{CL}}{U_{CC}U_{LL} - (U_{CL})^2} > 0,$$

$$\frac{\partial C}{\partial w} = \frac{\lambda U_{CL}}{U_{CC}U_{LL} - (U_{CL})^2}, \qquad \frac{\partial L}{\partial w} = -\frac{\lambda U_{CC}}{U_{CC}U_{LL} - (U_{CL})^2} > 0,$$

$$\frac{\partial C}{\partial P} = \frac{\lambda U_{LL}}{U_{CC}U_{LL} - (U_{CL})^2} < 0, \qquad \frac{\partial L}{\partial P} = -\frac{U_{CL}\lambda}{U_{CC}U_{LL} - (U_{CL})^2}.$$

Note that the concavity of the utility function implies that $U_{CC}U_{LL} - (U_{CL})^2 > 0$.

Proof of proposition 1

(a) Fixed n

When the number of firms is fixed, Ω is equal to

$$\Omega = \frac{\Psi_K}{\Phi_K - r}, \tag{37}$$

where

$$\Phi_K = \frac{p_N}{\alpha P}\frac{\partial N}{\partial K} < 0, \tag{38}$$

$$\Psi_K = \frac{\partial T}{\partial K} + p_N \frac{\partial N}{\partial K} - \left[\frac{p_N}{\alpha}\right]\frac{\partial N}{\partial K} > 0. \tag{39}$$

In order to simplify Ω, we rewrite equation (38)

$$\left[\frac{p_N}{\alpha}\right]\frac{\partial N}{\partial K} = P\Phi_K. \tag{40}$$

>From the budget constraint, the value of the sum of the increase in output in both sectors as a result of a higher capital stock is equal to the rental price of capital (since there are no profits)

$$\frac{\partial T}{\partial K} + p_N \frac{\partial N}{\partial K} = rP + \frac{d\Pi}{dK}. \tag{41}$$

Substituting (40), (41) into (39) yields

$$\Psi_K = rP + \frac{d\Pi}{dK} - P\Phi_K = P(r - \Phi_K) + \frac{d\Pi}{dK}.$$

Hence, (37) becomes

$$\Omega = \frac{(r - \Phi_K)\, P + \frac{d\Pi}{dK}}{\Phi_K - r} = -P + \frac{\mu P_N}{\Phi_K - r} \frac{f_N}{k_T - k_N} < 0. \tag{42}$$

(b) Free entry and Walrasian,

In the Walrasian case, the proof is as in (a) except that $\mu = 0$ and equation (41) is altered to take account of the fact that there is no profit income

$$\frac{\partial T}{\partial K} + p_n \frac{\partial N}{\partial K} = rP.$$

Hence,

$$\Psi_K = \frac{\partial T}{\partial K} - P\Phi_K + p_n \frac{\partial N}{\partial K} = P(r - \Phi_K).$$

Then, Ω becomes (as required)

$$\Omega = \frac{(r - \Phi_K)\, P}{\Phi_K - r} = -P < 0.$$

In the case of free entry, the proof follows the same steps than in the Walrasian case but, in this case, the derivative of the nontraded output with respect to the capital stock is equal to

$$\frac{\partial N}{\partial K} = \frac{(1 - \mu)\, f_N}{k_N - k_T}. \tag{43}$$

Proof of proposition 2

We compute the level of consumption at the steady state as function of employment and public expenditure. The consumer budget constraint at the steady state is equal to:

$$C + G = \frac{r\,(b^* + PK^*) + wL + \Pi}{P}. \tag{44}$$

We use that in the level of bonds on the steady state is equal to

$$b^{*}=b_{0}-\Omega\left(K_{0}-K^{*}\right), \tag{45}$$

In the case of free entry and perfect competition there are not profits $\Pi = 0$,and since $\Omega = -P$, we have that

$$C+G=\frac{rb_{0}}{P}+rK_{0}+\frac{w}{P}L. \tag{46}$$

In the case of imperfect competition with no entry, we compute the level of profits and capital stock at the steady state

$$K^{*}=\frac{1}{\Phi_{K}}\left(C+G\right)+k_{T}L. \tag{47}$$

$$\Pi=\mu\alpha\left(p_{N}\right)P\left(C+G\right). \tag{48}$$

If we substitute this two equation into equation (44) and we use that in this case Ω is defined in equation (42), it yields

$$C+G=\left(\frac{\Phi_{K}\alpha\mu f_{T}}{P\left(\Phi_{K}\left(1-\mu\alpha\right)-r\right)}+\frac{w}{P}\right)L+rK_{0}+\left[\frac{\Phi_{K}-r}{\Phi_{K}\left(1-\mu\alpha\left(p_{N}\right)\right)-r}\right]\frac{rb_{0}}{P},$$

where $\dfrac{\Phi_{K}\alpha\mu f_{T}}{P\left(\Phi_{K}\left(1-\mu\alpha\right)-r\right)}>0$ and $\left[\frac{\Phi_{K}-r}{\Phi_{K}(1-\mu\alpha(p_{N}))-r}\right]>1.$

Proof of proposition 3

>From the market clearing condition and the budget constraints in the steady state, we can compute the derivative of the marginal utility of wealth λ^{*} and the capital stock K^{*}.

The derivatives of the marginal utility of wealth with respect to public expenditure are equal to

Case	$\dfrac{\partial\lambda^*}{\partial G}$
Walrasian, Free-entry.	$\dfrac{P}{l_\lambda w - PC_\lambda} > 0.$
Fixed n.	$\dfrac{P - \dfrac{p_N\mu}{\Phi_K - r}\dfrac{\partial N}{\partial K}}{l_\lambda w - PC_\lambda + (l_\lambda r k_T + C_\lambda)\dfrac{p_N\mu}{\Phi_K - r}\dfrac{\partial N}{\partial K}} > 0.$

In all three cases we have

$$\frac{\partial C^*}{\partial G} = C_\lambda \frac{\partial\lambda^*}{\partial G} < 0 \qquad \frac{\partial L^*}{\partial G} = L_\lambda \frac{\partial\lambda^*}{\partial G} > 0 \ .$$

The derivative of capital stock with respect to public expenditure in the case of imperfect competition with non entry is equal to

$$\left.\frac{\partial K^*}{\partial G}\right|_{\mu>0,n} = \frac{l_\lambda P\left(\frac{\alpha}{p_N}k_N f_T + (1-\alpha)\,k_T f_N\right)}{f_N\left[l_\lambda w - PC_\lambda + (l_\lambda r k_T + C_\lambda)\dfrac{p_n\mu\frac{\partial N}{\partial K}}{\Phi_K - r}\right]} > 0.$$

If we fix the mark up equal zero $\mu = 0$, we obtain the derive of capital stock respect to public expenditure in the case of perfect competition

$$\left.\frac{\partial K^*}{\partial G}\right|_{\mu=0} = \frac{l_\lambda P\left(\frac{\alpha}{p_N}k_N f_T + (1-\alpha)\,k_T f_N\right)}{f_N\left[l_\lambda w - PC_\lambda\right]} > 0.$$

Finally, the derivative of the capital stock in the case of free entry is equal to

$$\left.\frac{\partial K^*}{\partial G}\right|_{\mu>0,e} = \frac{l_\lambda P\left(\frac{\alpha}{p_N}k_N f_T + (1-\alpha)\,k_T\,(1-\mu)\,f_N\right)}{(1-\mu)\,f_N\left[l_\lambda w - PC_\lambda\right]}. > 0.$$

Proof of proposition 4

(a) If we start from long-run equilibrium, then all derivatives are evaluated from the same initial position. In order to prove that the fiscal multiplier is higher in the case of imperfect competition with non-entry than the case of entry, we need to prove that

$$\left.\frac{\partial\lambda^*}{\partial G}\right|_n = \frac{P - \dfrac{p_n\mu\dfrac{\partial N}{\partial K}}{\Phi_K - r}}{l_\lambda w - PC_\lambda + (l_\lambda r k_T + C_\lambda)\dfrac{p_n\mu\frac{\partial N}{\partial K}}{\Phi_K - r}} < \frac{P}{l_\lambda w - PC_\lambda} = \left.\frac{\partial\lambda^*}{\partial G}\right|_e,$$

this inequality is satisfied if

$$(l_\lambda w - PC_\lambda)\left[P - \frac{p_n\mu\frac{\partial N}{\partial K}}{\Phi_K - r}\right] < P\left(l_\lambda w - PC_\lambda + (l_\lambda r k_T + C_\lambda)\frac{p_n\mu\frac{\partial N}{\partial K}}{\Phi_K - r}\right),$$

this expression is equal to

$$(l_\lambda w - PC_\lambda)P - (l_\lambda w - PC_\lambda)\frac{p_n\mu\frac{\partial N}{\partial K}}{\Phi_K - r} < P(l_\lambda w - PC_\lambda) + P(l_\lambda r k_T + C_\lambda)\frac{p_n\mu\frac{\partial N}{\partial K}}{\Phi_K - r}, \tag{49}$$

If we subtract in both sides $(l_\lambda w - PC_\lambda)P$, we obtain

$$-(l_\lambda w - PC_\lambda)\frac{p_n\mu\frac{\partial N}{\partial K}}{\Phi_K - r} < P(l_\lambda r k_T + C_\lambda)\frac{p_n\mu\frac{\partial N}{\partial K}}{\Phi_K - r},$$

since $\dfrac{p_n\mu\dfrac{\partial N}{\partial K}}{\Phi_K - r} > 0$,we have that

$$-w < Prk_T,$$

which is always satisfied.

(b) In the case of homothetic preferences, l_λ, C_λ are the same along the whole IEP (they depend only on w/P). Hence, the derivation given in part (a) is valid irrespectively of the initial position.

Proof of proposition 5

We consider the effects of the fiscal policy in the short run and the long run equilibrium. We use the linearised system to compute the short run variation in the different variables (again, Φ_K takes the free-entry or fixed-n values as appropriate

$$\begin{aligned}\frac{dI(0)}{dG} &= -\frac{dK^*}{dG}\Phi_K > 0, \\ \frac{dC^*}{dG} &= \frac{dC(0)}{dG} = C_\lambda \frac{d\lambda^*}{dG} < 0, \\ \frac{dL^*}{dG} &= \frac{dL(0)}{dG} = l_\lambda \frac{d\lambda^*}{dG} > 0.\end{aligned}$$

Hence, the inequality for the long-run and impact effect on Y

$$\frac{dY^*}{dG} = 1 + \frac{dC^*}{dG} = \frac{dY(0)}{dG} - \frac{dI(0)}{dG}.$$

We can compare the variation of output by sector

$$\frac{dN(0)}{dG} = \frac{k_T f_T}{k_T - k_N}\frac{dL^*}{dG} > \frac{k_T f_N}{k_T - k_N}\frac{dL^*}{dG} - \frac{f_N}{k_T - k_N}\frac{\partial K^*}{\partial G} = \frac{dN^*}{dG}$$

In the short run, the increase in non-traded output is purely due to the increase in labour supply: in the long-run there is an additional offsetting effect as capital accumulates which reduces N. However, the net long-run effect is positive: from the NT market clearing condition.

$$\frac{dN^*}{dG} = \frac{\alpha P}{p_N}\frac{dY}{dG} > 0$$

The Traded sector output falls in the short-run due to the increase in labour supply, but increases over time as capital accumulates

$$\frac{dT(0)}{dG} = \frac{-k_N f_T}{k_T - k_n}\frac{dL^*}{dG} < 0.$$

The long-run effect of G on T can be found from the trade-balance condition $\dot{b} = 0$ (26) which can be written as

$$0 = rb + T^* - (1-\alpha)PY,$$

so that

$$\frac{dT}{dG} = -r\frac{db}{dG} + (1-\alpha)P\frac{dY}{dG} > 0.$$

The evolution of the current account depends on the evolution of investment. The evolution of bonds is equal to:

$$\begin{aligned}
\frac{db(t)}{dG} &= \Omega\frac{dK(t)}{dG} = \Omega\frac{dK^*}{dG}\left(1-\exp\left(\Phi_K t\right)\right) < 0, \\
\frac{d\dot{b}(t)}{dG} &= -\Omega\frac{dK^*}{dG}\Phi_K \exp\left(\Phi_K t\right) < 0.
\end{aligned}$$

Proof of Proposition 6

(a) With Cobb-Douglas preferences and technology we have for fixed n

$$\frac{\partial C^*}{\partial G} = \frac{-\upsilon\left(\left(1-\beta\right)+\alpha\left(\beta-\delta\right)-\alpha\mu\left(1-\delta\right)\right)}{\left(\left(1-\beta\right)+\alpha\left(\beta-\delta\right)+\mu\alpha\left(\delta-\upsilon\right)\right)}. \tag{50}$$

From which $\dfrac{\partial C^*}{\partial G}$ is increasing in μ.

(b) With Cobb-Douglas preferences and a general technology the derivative of consumption with respect to public expenditure in the case of perfect competition and free entry are equal

$$\frac{\partial C^*}{\partial G} = -\upsilon. \tag{51}$$

In the case of fixed n we have

$$\frac{\partial C^*}{\partial G} = \frac{-\upsilon\left(1-\frac{1}{P}\frac{p_N\mu}{\Phi_K - r}\frac{\partial N}{\partial K}\right)}{\left(1-\frac{1}{P}\frac{p_N\mu}{\Phi_K - r}\frac{\partial N}{\partial K}\right)+\frac{1}{P}\left(\frac{\left(1-\upsilon\right)f_T}{w}\right)\frac{p_N\mu}{\Phi_K - r}\frac{\partial N}{\partial K}}. \tag{52}$$

Now we compare the case of monopolistic competition with respect to the case of perfect competition

$$\frac{-\upsilon\left(1-\frac{1}{P}\frac{p_N\mu}{\Phi_K - r}\frac{\partial N}{\partial K}\right)}{\left(1-\frac{1}{P}\frac{p_N\mu}{\Phi_K - r}\frac{\partial N}{\partial K}\right)+\frac{1}{P}\left(\frac{\left(1-\upsilon\right)f_T}{w}\right)\frac{p_N\mu}{\Phi_K - r}\frac{\partial N}{\partial K}} > -\upsilon, \tag{53}$$

This inequality is always satisfied since

$$\upsilon\frac{1}{P}\frac{\left(1-\upsilon\right)f_T}{w}\frac{p_N\mu}{\Phi_K - r}\frac{\partial N}{\partial K} > 0 \tag{54}$$

(c) In the Walrasian and free-entry cases the effect of G on C^* is given by (50) setting $\mu = 0$.

Proof of proposition 7

$$\begin{aligned}
\bar{U} &= \frac{U(C, 1-L)}{r}, \\
\frac{dU}{dG} &= U_c \frac{dC}{dG} - U_L \frac{dL}{dC} = U_c \left[\frac{dC}{dG} - \frac{w}{P} \frac{dL}{dG} \right]. \qquad (55)
\end{aligned}$$

We now evaluate $\frac{dL}{dG}$. Differentiating (15) with respect to time and rearranging we have (noting that $\{L, C\}$ are constant over time and there are no profits):

$$\frac{dL}{dG} = \frac{1}{w} \frac{d\,\dot{b}\,(t)}{dG} - r \frac{db(t)}{dG} \frac{1}{w} - r \frac{P}{w} \frac{K(t)}{dG} + \frac{P}{w} \frac{dC}{dG} + \frac{P}{w} \frac{dI(t)}{dG} + \frac{P}{w}.$$

Substituting this in (55) yields

$$\frac{dU}{dG} = U_c \left(-\frac{1}{P} \frac{d\,\dot{b}\,(t)}{dG} + \frac{1}{P} r \frac{db(t)}{dG} + r \frac{dK(t)}{dG} - \frac{dI(t)}{dG} - 1 \right).$$

Noting that

$$\begin{aligned}
\frac{db(t)}{dG} &= \Omega \frac{dK(t)}{dG}, \\
\frac{d\,\dot{b}\,(t)}{dG} &= \Omega \frac{dI(t)}{dG},
\end{aligned}$$

where $\Omega = -P$ yields

$$\begin{aligned}
\frac{dU}{dG} &= -U_c \\
\frac{d\bar{U}}{dG} &= -\frac{U_c}{r}
\end{aligned}$$

the desired result.

Proof of proposition 8

In the case of imperfect competition with profits the proof here follows the proof of proposition 8, except that it allows for profits. Differentiating (15) with respect to public expenditure and re-arranging yields

$$\frac{dC}{dG}=\frac{1}{P}\left(-\frac{d\,\dot{b}\,(t)}{dG}+r\frac{db(t)}{dG}+w\frac{dL}{dG}+rP\frac{K(t)}{dG}+\frac{d\Pi(t)}{dG}-P\frac{dI}{dG}-P\right),$$

we substitute the above expression into (55), thus the variation in total welfare is

$$\frac{dU}{dG}=U_c\left(-\frac{1}{P}\frac{d\,\dot{b}\,(t)}{dG}+\frac{1}{P}r\frac{db(t)}{dG}+r\frac{K(t)}{dG}+\frac{1}{P}\frac{d\Pi(t)}{dG}-\frac{\dot{K}(t)}{dG}-1\right),$$

since bonds and capital accumulation are related through Ω

$$\frac{dU}{dG}=U_c\left(-\frac{1}{P}\Omega\frac{\dot{K}(t)}{dG}+\frac{1}{P}r\Omega\frac{K(t)}{dG}+r\frac{K(t)}{dG}+\frac{1}{P}\frac{d\Pi(t)}{dG}-\frac{\dot{K}(t)}{dG}-1\right),$$

if we rearrange terms

$$\frac{dU}{dG}=\frac{U_c}{P}\left(\left(\frac{1}{\Phi_K-r}\frac{p_N\mu f_N}{k_N-k_T}\right)\left(r\frac{dK(t)}{dG}-\frac{d\dot{K}(t)}{dG}\right)+\frac{d\Pi(t)}{dG}-P\right),$$

we use that

$$\frac{dK(t)}{dG}=\frac{dK^*}{dG}(1-e^{\Phi_K t}), \tag{56}$$

$$\frac{d\dot{K}(t)}{dG}=-\frac{dK^*}{dG}\Phi_K e^{\Phi_K t}, \tag{57}$$

so that

$$\frac{dU}{dG}=\frac{U_c}{P}\left(\left(\frac{1}{\Phi_K-r}\right)\frac{d\Pi^*}{dG}\left(r(1-e^{\Phi_K t})+\Phi_K e^{\Phi_K t}\right)+\frac{d\Pi(t)}{dG}-P\right) \tag{58}$$

now, we compute the profits along the time

$$\begin{aligned}
\Pi(t) &= \mu P_N \left[\frac{Lk_T - K(t)}{k_T - k_N} f_N - nF\right] \\
&= \mu P_N \left[\frac{Lk_T - (K^* + (K_0 - K^*).\exp(\Phi_K t))}{k_T - k_N} f_N - nF\right] \\
&= \Pi^* + \frac{d\Pi}{dK}(K_0 - K^*)\exp(\Phi_K t).
\end{aligned}$$

where $\frac{d\Pi}{dK} = -\frac{\mu P_N f_N}{k_T - k_N}$ is constant. Now

$$\frac{d\Pi(t)}{dG} = \frac{d\Pi^*}{dG} - \frac{d\Pi}{dK}\frac{dK^*}{dG}\exp(\Phi_K t) = \frac{d\Pi^*}{dG}\left[1 - \exp(\Phi_K t)\right]. \tag{59}$$

We introduce the above expression into equation (58)

$$\frac{dU}{dG} = \frac{U_c}{P}\left(\left(\frac{1}{\Phi_K - r}\right)\frac{d\Pi^*}{dG}\left(r(1 - e^{\Phi_K t}) + \Phi_K e^{\Phi_K t}\right) + \frac{d\Pi^*}{dG}\left[1 - e^{\Phi_K t}\right] - P\right),$$

if we simplify this expression, we obtain

$$\frac{dU}{dG} = \frac{U_c}{P}\left(\frac{d\Pi^*}{dG}\left(\left(\frac{\Phi_K}{\Phi_K - r}\right)\right) - P\right) < 0,$$

therefore

$$\frac{d\bar{U}}{dG} = \frac{U_c}{r}\left(\frac{1}{P}\frac{d\Pi^*}{dG}\left(\left(\frac{\Phi_K}{\Phi_K - r}\right)\right) - 1\right) < 0.$$

Proof of proposition 9

With C-D preferences and technology we have

$$\begin{aligned}
\Phi_K^n &= -\frac{(1-\beta)\, r}{\alpha\,(1-\mu)\,(\beta - \delta)}, \\
\frac{\partial \Phi_K^n}{\partial \mu} &= \frac{(1-\beta)\, r}{\alpha\,(\beta-\delta)}\frac{1}{((1-\mu))^2} > 0.
\end{aligned}$$

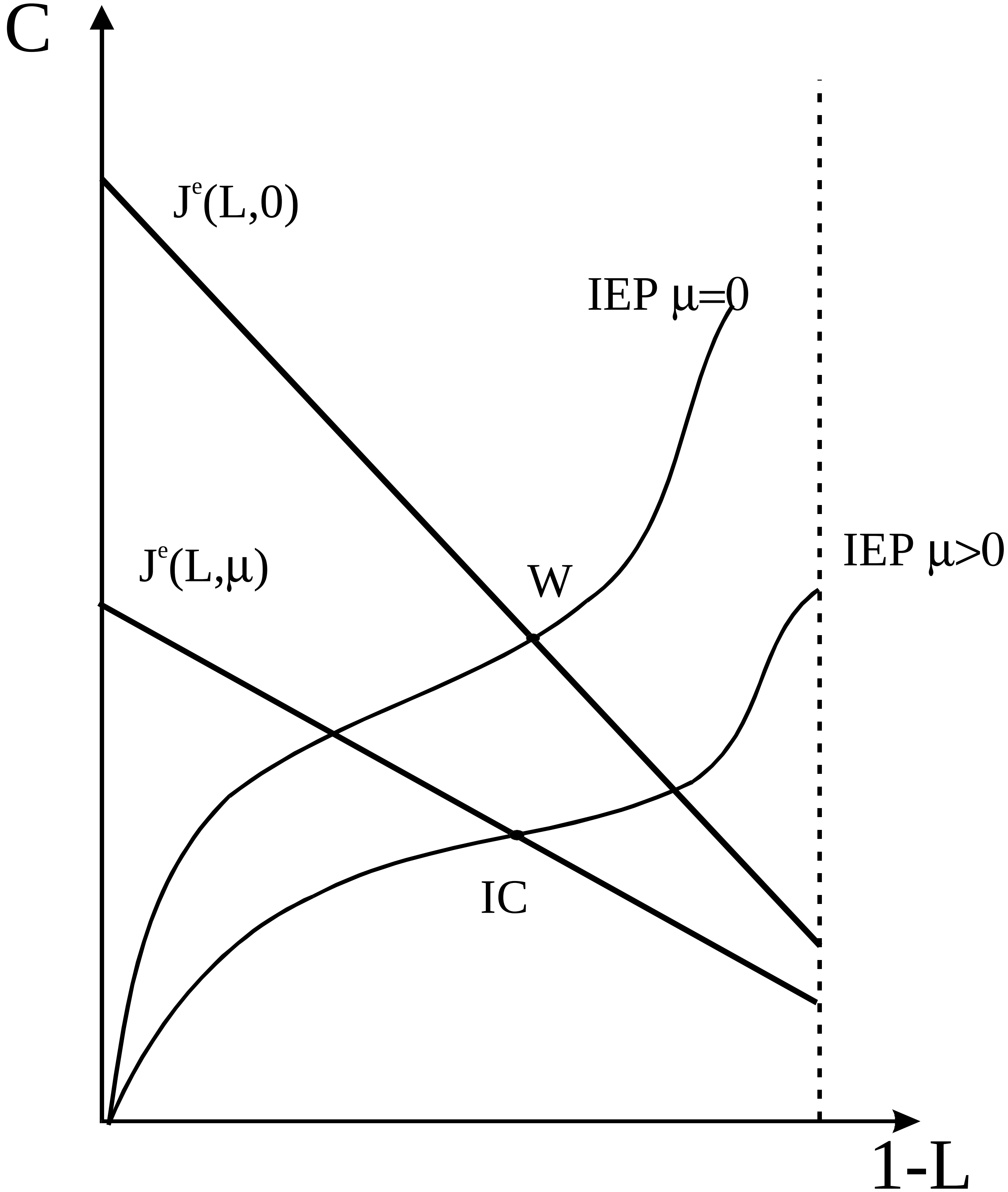

Figure 1: Equilibrium For Free-entry And Walrasian Cases

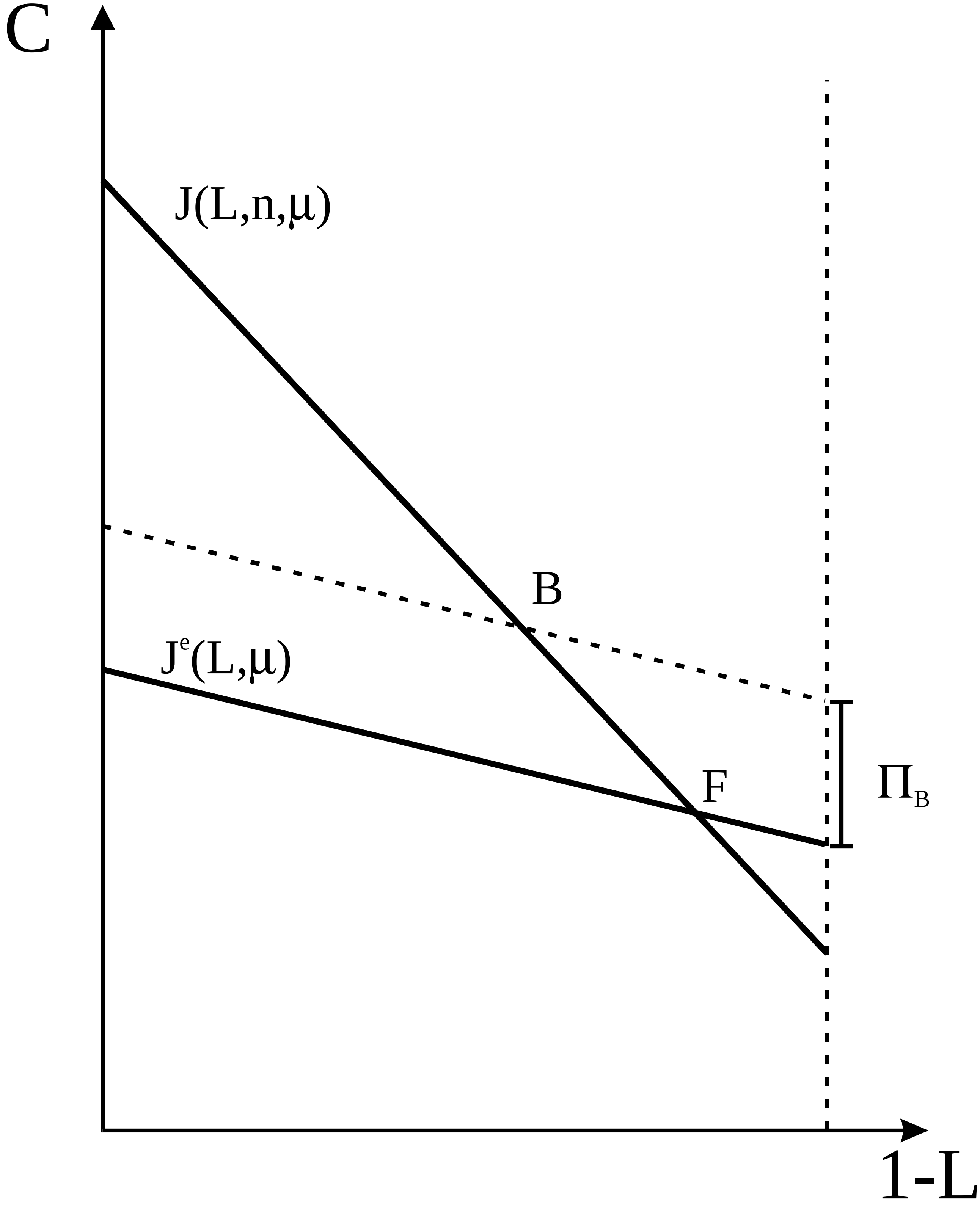

Figure 2: Free entry and Fixed n
Compared with $\mu>0$

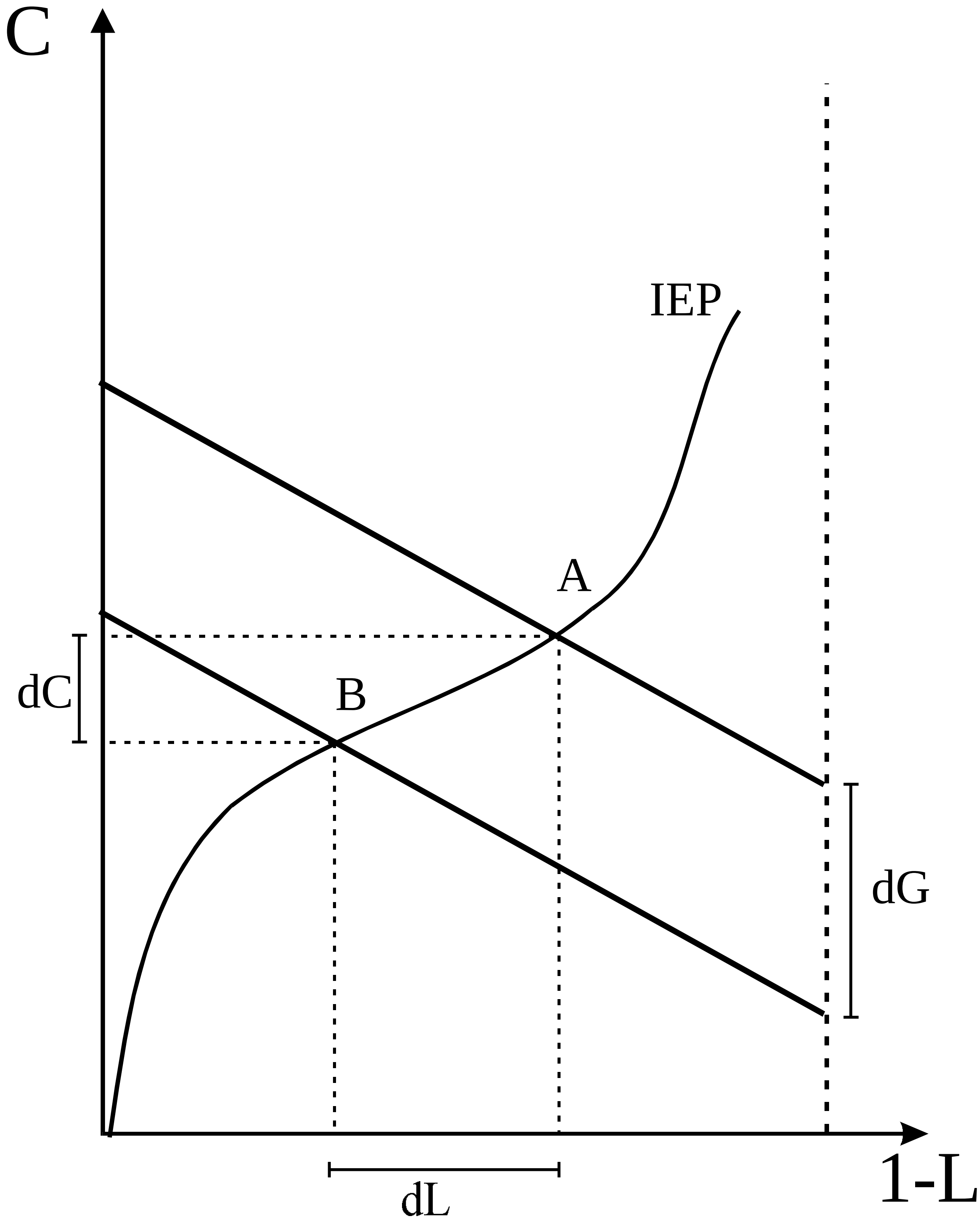

Figure 3: The Multiplier and Resource Withdrawal Effect.

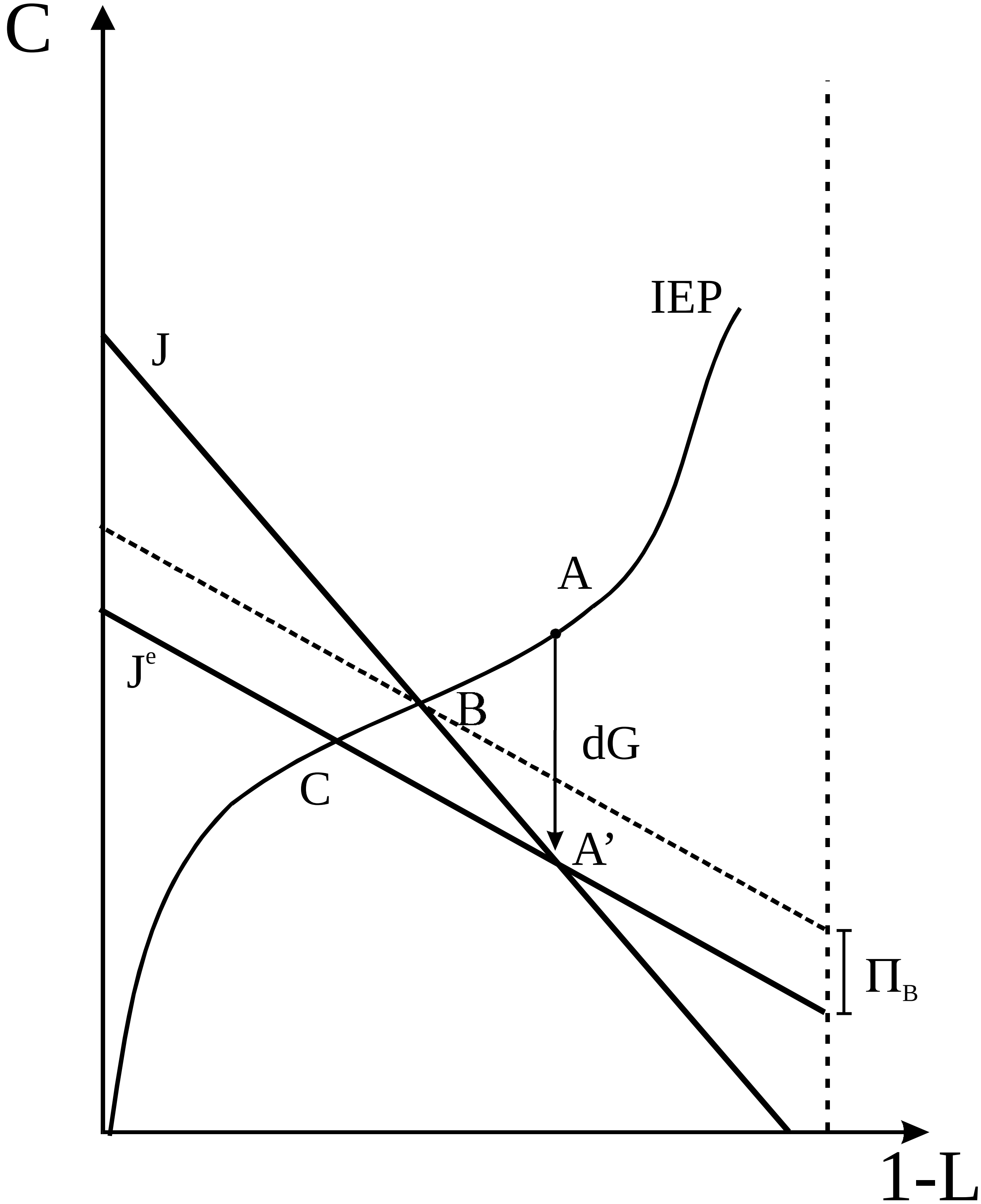

Figure 4: The Multiplier under Free Entry and with a Fixed Number of Firms

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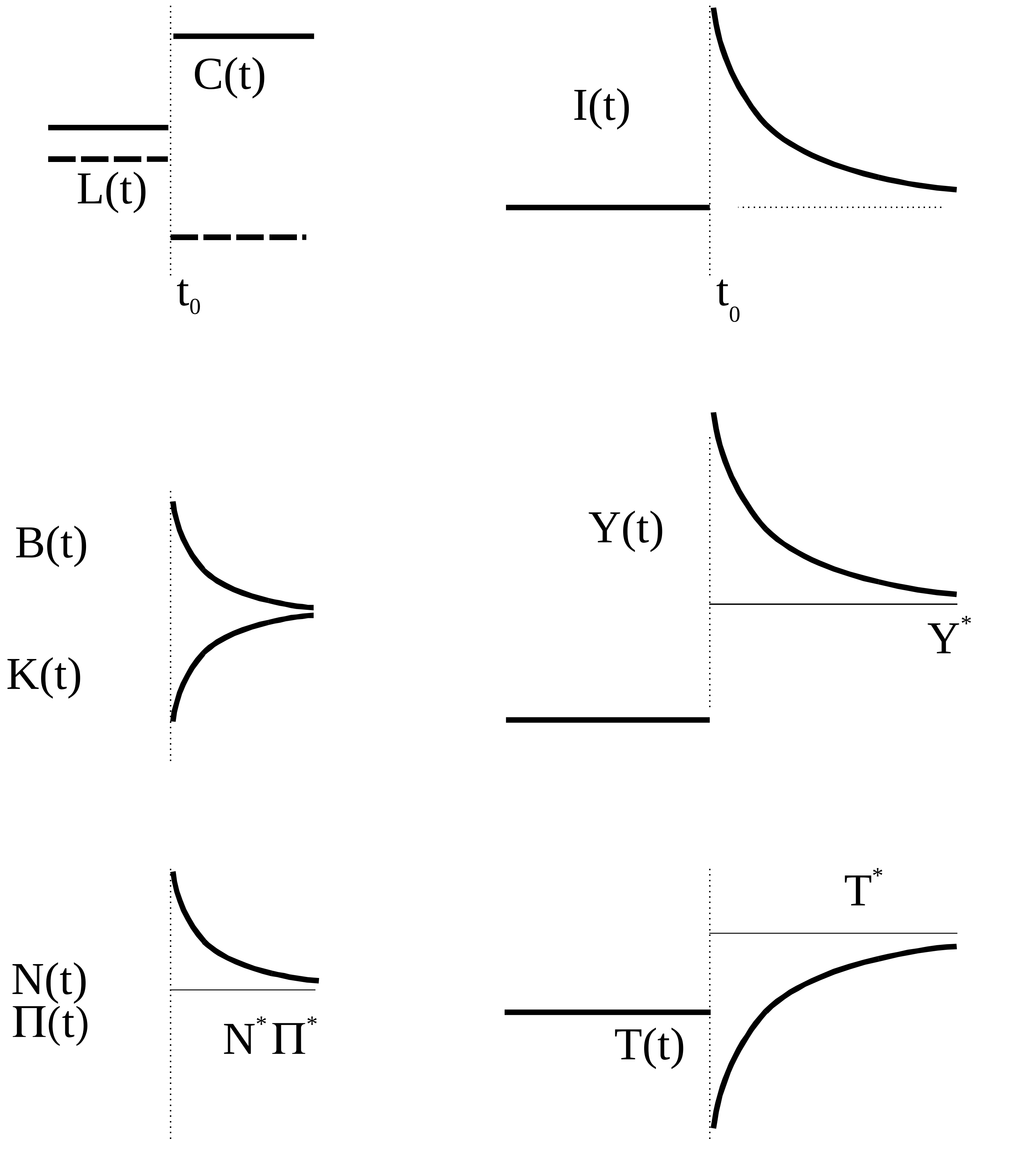

Fig.5: Dynamic Responses to a Permanent increase in G at time t_0