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The advantage of transparent instruments of monetary policy*

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ABSTRACT

A classic question in international economics is whether it is better to use the exchange rate or the money growth rate as the instrument of monetary policy. A common argument is that the exchange rate has a natural advantage since exchange rates provide signals of policymakers' actions that are easier to monitor than those provided by money growth rates. We formalize this argument in a simple model in which the government chooses which instrument it will use to target inflation. In it, the exchange rate is more transparent than the money growth rate in that the exchange rate is easier for the public to monitor. We find that the greater transparency of the exchange rate regime makes it easier to provide the central bank with incentives to pursue good policies and hence gives this regime a natural advantage over the money regime.

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> "by the simple virtue of being a price rather than a quantity, the exchange rate provides a much clearer signal to the public of the government's intentions and actual actions than a money supply target. Thus, if the public's inflationary expectations are influenced to a large extent by the ability to easily track and continuously monitor the nominal anchor, the exchange rate has a natural advantage." Calvo and Vegh (1999 p. 1589)

> "True, the exchange rate has some special properties. In particular, it is easily observable, so the private sector can directly monitor any broken promises by the central bank. But we know of no convincing argument that turns these properties into an explanation for why it would be a more efficient method to achieve credibility to target the exchange rate rather than, say, the money growth rate." (Persson and Tabellini 1994. p17)

A classic question in international economics is whether it is better to use exchange rates or money growth rates as the instrument of monetary policy. A common argument, illustrated by the quote of Calvo and Vegh, is that the exchange rate has a natural advantage over the money growth rate as an instrument since the exchange rate provides a signal of policymakers' actions that is easier to monitor than that provided by money growth rates. Skeptics of this view, such as Persson and Tabellini, agree that the exchange rate is easier for the public to monitor than money growth. They argue, however, that no clear theoretical argument has been given that turns this property of exchange rates into a rationale for why the exchange rate has a natural advantage as an instrument. This paper provides such a theoretical argument.

We formalize this argument using a simple model of sustainable monetary policy similar to that in Kydland and Prescott (1977) and Barro and Gordon (1983). In it, each period, the central bank chooses one of two regimes for monetary policy: one in which the exchange rate is the instrument or one in which money growth is the instrument. Under the exchange-rate regime, the central bank picks an exchange rate with some foreign country, and realized domestic inflation varies with shocks to the inflation rate in the foreign country. Thus, by choosing an exchange rate the central bank sets the mean inflation rate and realized inflation varies with foreign inflation shocks. Under the money regime the central bank picks a money growth rate, thus setting the mean inflation rate, and realized inflation varies with domestic inflation shocks. Hence under both regimes the central banks sets the mean inflation rate and the realized inflation varies with exogenous shocks.

The key difference between the two regimes is in their transparency. The exchange-rate regime is transparent in that private agents can directly observe the exchange rate. The money regime is opaque in that private agents cannot directly observe money growth rates, but rather they only observe inflation which serves as a noisy signal of money growth. In all other respects, the two regimes are symmetric. Note that in both regimes the government is targeting inflation, it is just using different instruments to implement its target.

Exchange-rate regimes gain an advantage from their transparency only because this transparency helps mitigate credibility problems. To emphasize this point we first consider an environment in which the central bank can commit to its policies and hence has no credibility problems. In evaluating regimes we compare the best equilibrium of both regimes. Here, even though exchange rates are easier to monitor, exchange-rate regimes have no natural advantage: an exchange-rate regime is preferred to a money regime if and only if the volatility of foreign inflation shocks is smaller than that of domestic inflation shocks.

We then consider an environment in which the central bank cannot commit to its policies. Under either regime the central bank has credibility problems in that it has an incentive in the short run to surprise the public with higher than expected inflation. In equilibrium, this short-run incentive is balanced against the costs that arise when private agents adjust their expectations of future policies and hence their future actions when they perceive a deviation by the central bank.

In the environment without commitment, the exchange-rate regime has a natural advantage because of its transparency in the sense that when the volatilities of foreign and domestic shocks are equal the exchange-rate regime is strictly preferred. Under the exchange-rate regime private agents can detect any deviation by the central bank from its expected action with certainty and thus they can adjust their actions precisely when a deviation occurs. Under the money regime, private agents can only respond to inflation, which is a noisy signal of the central bank's action and thus their response to a deviation by the central bank must necessarily be less precise. It is this inability of private agents to precisely tailor their behavior in response to deviations by the central bank that makes it more difficult to deter the central bank from surprise inflation in the money regime. This inability gives the exchange-rate regime its natural advantage.

We also characterize the outcomes that occur in the best exchange-rate regime and the best money regime. In the environment with commitment the outcomes are symmetric in the two regimes. In both the government chooses a single level for either the exchange rate or the money growth rate and it implements this level in every period.

In the environment without commitment the outcomes under the two regimes are very different. In the best equilibrium under an exchange-rate regime, the central bank chooses a low rate of depreciation of the exchange rate designed to achieve a low average inflation rate. It maintains this low rate in every period, regardless of the realization of inflation. This policy is sustained by the fear that if the central bank were ever to deviate from this exchange rate path, private agents would treat this deviation as a signal that the government was going to implement the highest sustainable inflation rate in the next period and, hence, set the growth in wages to a correspondingly high level.

The equilibrium outcome under the best money regime looks very different. Under a money regime, agents cannot distinguish whether high realized inflation was the result of the central bank's choice of a high money growth rate or simply the result of a large domestic inflation shock. As a result of this lack of transparency, the outcome necessarily oscillates at random between two extreme phases, the first with low average inflation and the second with high average inflation. This random switching between phases of low and high average inflation along the equilibrium path in the best money regime is analogous to the outcomes obtained by Green and Porter (1984) and Abreu, Pearce, and Stacchetti (1986) in their analysis of equilibrium price wars among oligopolists.

In terms of the literature on monetary policy, our analysis is most related to the seminal contribution of Canzoneri (1985), who was the first to use the logic of Green and Porter (1984) to explain periodic bouts of high inflation. (See also Zarazaga 1993.) There is also some work in this literature on the issue of transparency in monetary policy. Cukierman and Meltzer (1986) and Faust and Svensson (1998, 1999) explore linear signalling outcomes in models with unobserved types.

In terms of international economics literature, the most related work is by Canavan and Tommasi (1997) and Herrendorf (1999), who use two-period signalling games to argue that governments can signal their preferences for low inflation by choosing an exchange rate rather than a money growth rate. For related work in a domestic context see Backus and Driffill (1985).

Here we have used a simple reduced-form model of money. Chang (1998), Phelan and Stacchetti (1999), and Albanesi, Chari and Christiano (2001) have used the recursive methods of Abreu, Pearce, and Stacchetti (1990) to analyze some general equilibrium macroeconomic models with perfect monitoring.

1. Policy instruments: exchange rates vs. money growth rates

Here we present a model of monetary policy in which, each period, the government selects either an exchange-rate regime, in which it uses the exchange rate as its policy instrument or a money regime, in which it uses the money growth rate as its policy instrument. The model extends the work of Kydland and Prescott (1977) and Barro and Gordon (1983).

Time is discrete and denoted $t = 0, 1, 2, \ldots$. There is a central bank referred to as a *government*, which dislikes unemployment and inflation, and a continuum of agents who each choose the rate of change of their nominal wage.

The timing of actions within each period is as follows. The government chooses the regime at the beginning of the period, namely, whether to use the exchange rate or money growth as its instrument of monetary policy. Under the exchange-rate regime, the government opens a trading desk at which it trades domestic and foreign currency. Under the money regime the government does not open this desk. The presence or absence of the trading desk is an observable indicator of the current monetary regime. Agents then choose their nominal wages. Finally, depending on the regime, the government chooses the level of either the exchange rate or money growth. The government is free to switch regimes at the beginning of the next period.

It is convenient to describe the economy for a given period t starting at the end of the period and working backward to the beginning. At the end the period, the government chooses the level of either the exchange rate or the money growth rate. The government takes as given the average rate of wage inflation x set by agents earlier in the period. Unemployment is equal to a constant U plus the gap between wage inflation and realized inflation π. The government's payoff for a given value of x and a realization of π is

$$(1) \qquad r(x,\pi) = -\frac{1}{2}\left[(U + x - \pi)^2 + \pi^2\right].$$

Realized inflation is a function of monetary policy as follows. Under the exchange-rate regime, the government chooses a rate of change in the exchange rate denoted $e_t = s_t - s_{t-1}$. For simplicity we refer to e_t as the *exchange rate*. The choice of the exchange rate e_t is observed. Inflation in the home country is given by

$$(2) \qquad \pi = e + \pi^*$$

where π^* is inflation in the foreign country, which is normal with mean 0 and variance $\sigma^2_{\pi^*}$. Thus, by choosing an exchange rate the government sets the mean inflation rate to be e, and the variance of domestic inflation is determined by shocks in the foreign country outside of its control. Foreign inflation π^* is observed only after the exchange rate is chosen. We let $g(\pi|e)$ denote the density of realized inflation at home given the choice of exchange rate e.

Under the money regime, the government chooses a money growth rate μ. Given μ, inflation π is given by

(3) $\pi = \mu + \varepsilon$

where ε are domestic shocks distributed normally with mean 0 and variance σ_π^2. Thus, by choosing the money growth rate the government sets the mean inflation rate to be μ, and the variance of domestic inflation is determined by domestic shocks outside of the government's control. We can think of the imperfect connection between money growth and inflation as arising from some combination of imperfect control over actual (as opposed to desired) money growth and a noisy relation between money growth and inflation. We let $f(\pi|\mu)$ denote the density of realized inflation given the choice of money growth rate μ. We call σ_π^2 the variance of domestic inflation shocks.

To model the idea that exchange rates are more easily monitored than money growth rates, we assume that under both regimes agents can see the exchange rate and the inflation rate but they cannot observe the money growth rate. Thus, under an exchange-rate regime agents directly see the actions of the government, namely, e, while under the money regime they do not.

Under both regimes (2) and (3) hold. In the exchange-rate regime e is the choice variable and money growth is endogenously determined, while in the money regime μ is the choice variable and e is endogenously determined. In these regimes the government's choice of either e or μ determines the mean inflation rate. The only difference in the regimes, besides observability of the instruments, is the variance of the resulting inflation. In this sense, in both regimes the government is targeting the mean rate of inflation.

The government's expected payoff under an exchange rate e is

$$S(x,e) = \int r(x,\pi)g(\pi|e)d\pi,$$

and under a money growth rate μ is

$$R(x,\mu) = \int r(x,\pi)f(\pi|\mu)d\pi.$$

With our functional forms these become

$$S(x,e) = -\frac{1}{2}\left[(U+x-e)^2 + e^2\right] - \sigma_{\pi^*}^2, \tag{4}$$

$$R(x,\mu) = -\frac{1}{2}\left[(U+x-\mu)^2 + \mu^2\right] - \sigma_{\pi}^2. \tag{5}$$

Notice that the government payoffs in the two regimes are symmetric with respect to the policy variables e and μ. In particular, the functions S and R differ only with respect to the uncontrollable variances $\sigma_{\pi^*}^2$ and σ_{π}^2 which are constants. For technical reasons we assume that the policies e and μ are bounded above and below by some arbitrarily large constants. These bounds ensure that the government payoffs are bounded.

In the middle of the period, each agent chooses the change in his wage rate $z_t = w_t - w_{t-1}$. We let x_t denote the average change in the wage rate in period t, which, for simplicity, we refer to as *average wages*. An agent's payoff for a given value of z and a realization of π is

$$r^A(z,\pi) = -\frac{1}{2}\left[(z-\pi)^2 + \pi^2\right]. \tag{6}$$

Each agent can choose z differently depending on whether the regime is an exchange-rate regime or a money regime. We denote these choices z_e and z_μ. An agent's expected payoff under an exchange-rate regime with exchange rate e is

$$S^A(z_e,e) = \int r^A(z_e,\pi)g(\pi|e)d\pi = -\frac{1}{2}\left[(z_e-e)^2 + e^2\right] - \sigma_{\pi^*}^2 \tag{7}$$

while this agent's expected payoff under a money regime with money growth rate μ is

$$R^A(z_\mu, \mu) = \int r^A(z_\mu, \pi) f(\pi|\mu) d\pi = -\frac{1}{2}\left[(z_\mu - \mu)^2 + \mu^2\right] - \sigma_\pi^2. \tag{8}$$

Notice that, under either regime, agents aim to choose wages equal to mean inflation, either e or μ depending on the regime.

Notice also that the objective function of agents differs from that of the government. In our simple reduced-form model this difference generates the conflict of interests between the government and the agents that leads to a time consistency problem. We think of this setup as a reduced-form way of capturing the tension that occurs in a general equilibrium model in which the government and the agents share the same objectives but there are distortions in the economy. (See Chari, Kehoe, and Prescott 1989 for a more complete discussion.)

The discounted payoff for the government is

$$(1-\beta)\sum_{t=0}^{\infty} \beta^t [(1-i_t) S(x_{et}, e_t) + i_t R(x_{\mu t}, \mu_t)] \tag{9}$$

where $1 \geq \beta > 0$ is the discount factor and i_t is an indicator variable that denotes the regime chosen in period t, where $i_t = 0$ if the exchange-rate regime is chosen and $i_t = 1$ if the money regime is chosen. Here x_{et} denotes the wages chosen in period t if the exchange-rate regime is chosen and $x_{\mu t}$ denotes the wages chosen in period t if the money regime is chosen. The discounted payoffs for the agents are written in a similar manner.

2. An environment with commitment

Here we suppose that the government can commit to a policy once-and-for-all in period 0. We show that with commitment the classic result holds: an exchange-rate regime is preferred if and only if the volatility of foreign inflation shocks is smaller than that of domestic inflation shocks. Thus, for this environment, the exchange rate has no natural advantage as an instrument even though it is more easily monitored.

In this environment, at the beginning of period 0, the government chooses the sequence $\{i_t, e_t, \mu_t\}_{t=0}^{\infty}$ indicating the regime it will follow and the exchange rate or money growth rate that it will implement under that regime in each period. After this, in each period t, agents choose wages z_{et} or $z_{\mu t}$ depending on the regime. Given (7) and (8), it is clearly optimal for agents to choose $z_{et} = e_t$ and $z_{\mu t} = \mu_t$, and hence average wages satisfy

(10) $\quad x_{et} = e_t$ and $x_{\mu t} = \mu_t$.

It should be clear that here the optimal policies and allocations solve the Ramsey problem of choosing $\{i_t, e_t, \mu_t, x_{et}, x_{\mu t}\}_{t=0}^{\infty}$ to maximize (9) subject to (10). This problem reduces to a sequence of static problems of choosing e and μ to solve $\max_e S(e, e)$ and $\max_\mu R(\mu, \mu)$ and then choosing the regime that leads to the higher value. Since the government payoffs are symmetric with respect to the policy variables, the optimal exchange rate and money growth rate are identical (both 0) and the government simply picks the regime with the lower variance of inflation. We denote this maximum payoff as v^R and refer to it as the *Ramsey payoff*. We summarize this result as follows.

Proposition 1. (No natural advantage with commitment) Under commitment the exchange-rate regime is preferred to the money regime if and only if $\sigma_{\pi^*}^2 \leq \sigma_{\pi}^2$.

3. An environment without commitment

Here we suppose that the government cannot commit. Instead, in each period it chooses the regime and then, after agents set wages, it chooses the level of its policy instrument. We show that without commitment the exchange-rate regime has a natural advantage because of its transparency.

In this environment both the government and private agents choose their actions as functions of the observed history of aggregate variables: the choice of regime, the exchange rate and inflation. In period t this history is given by $h_t = (i_0, e_0, \pi_0; \ldots; i_{t-1}, e_{t-1}, \pi_{t-1})$. A strategy for government is a sequence of functions $\sigma^G = \{i_t(h_t), e_t(h_t), \mu_t(h_t)\}$ which map histories into the choice of regime i_t and corresponding money growth rates μ_t or exchange rates e_t. A strategy for agents is a sequence of functions $\sigma^A = \{z_{et}(h_t), z_{\mu t}(h_t)\}_{t=0}^{\infty}$ which map histories into actions z_t, where $z_{et}(h_t)$ is only relevant if $i_t(h_t) = 0$ and $z_{\mu t}(h_t)$ is only relevant if $i_t(h_t) = 1$. We also define a sequence of functions $\sigma^X = \{x_{et}(h_t), x_{\mu t}(h_t)\}_{t=0}^{\infty}$ which record the average wages chosen by agents after each history. Let $\sigma = (\sigma^G, \sigma^A, \sigma^X)$ denote the strategies of the government and agents and the average wages. Notice that in the histories we need not record the past averages of the actions of agents since a deviation by any one agent cannot affect this average. (See, for example, Chari and Kehoe 1990 for details.)

A *perfect equilibrium* in this environment is a collection of strategies σ such that i) after every history h_t, the private agents' strategy σ^A is optimal given the government's strategy σ^G and the average of other private agents' wages σ^X, and ii) after every history h_t, the government's strategy σ^G is optimal given the average of private agents' strategies σ^X.

Let V denote the set of equilibrium payoffs. In what follows it will prove convenient to allow public randomization to guarantee that this set V is convex and thus equal to an interval $[v^w, v^b]$ where v^w is the lowest or the *worst equilibrium payoff* and v^b is the highest or the *best equilibrium payoff.* This public randomization is accomplished by adding to the model a random variable θ_t that the agents and the government observe at the beginning of each period. We modify the histories h_t to include the realizations of this variable from period 0 through period t.

It should be clear given our functional forms (7) and (8), that, given a government strategy σ^G, after any history h_t, it is optimal for private agents to choose wages $z_{et}(h_t) = e_t(h_t)$ and $z_{\mu t}(h_t) = \mu_t(h_t)$. Thus, in any perfect equilibrium, average wages must satisfy

$$x_{et}(h_t) = e_t(h_t) \text{ and } x_{\mu t}(h_t) = \mu_t(h_t).$$

That is, wage inflation must equal expected inflation.

We formulate the incentive constraint of the government recursively by drawing on Abreu, Pearce and Stacchetti (1986, 1990). Their basic idea is as follows. In a repeated game a strategy is a prescription for current actions and all future actions. When evaluating the government's current payoffs and current incentive constraints, however, we need not specify the whole sequence of future actions following every possible current action; rather, we need specify only how the government's payoff from next period on, namely, its *continuation value*, will vary as its current action varies. In a perfect equilibrium these continuation values are also equilibrium payoffs for the repeated game starting from next period on. This simple observation forms the basis for a recursive approach for describing the incentive compatibility constraints for the government and to finding the set of equilibrium payoffs.

Consider a period in which the government has chosen an exchange-rate regime and agents have chosen wages x_e. We can formulate the current incentive constraint on the government's choice of exchange rate e recursively as follows. What matters to the government in choosing the exchange rate is how its current period payoff and its continuation value vary with its action e. Its current period payoff is $(1-\beta)S(x_e, e)$, where the agents' choice of x_e is taken as given. Since agents observe the government's choice of exchange rate e, their future choices of wages, and thus the future payoffs of the government, can vary with e directly. Rather than describe the entire sequence of future actions taken by private agents and the government, contingent on the government's current choice of e, we simply describe the government's continuation payoff from those actions as some function $w(e)$. Since, in a perfect equilibrium, the strategies that private agents and the government follow from next period on must also be perfect equilibrium strategies of the repeated game starting from that period, the government's continuation values $w(e)$ must lie in the set V of perfect equilibrium payoffs for the government. Given any such continuation value function $w(e) \in V$, we say that an *exchange rate* e *is incentive compatible in the current period* if

$$(11) \quad (1-\beta)S(x_e, e) + \beta w(e) \geq (1-\beta)S(x_e, e') + \beta w(e')$$

for all e'. This constraint simply requires that the government get a higher discounted sum of current and future payoffs from choosing e than it does from choosing any other e'. It is a standard result that such a recursive incentive constraint is necessary and sufficient for full incentive compatibility.

Consider next a period in which the government has chosen a money regime and agents have chosen wages x_μ. We can formulate the current incentive constraint on the government's choice of money growth rate μ recursively as well. This constraint is different from the constraint (11) above because, here, agents do not observe the government's action, here the money growth rate μ, but rather inflation $\pi = \mu + \varepsilon$, which is a noisy signal of μ. Hence the government's continuation value cannot vary with μ directly, but rather it can vary only with π. Thus we write the continuation value function for the government in this case as $w(\pi)$.

These continuation values $w(\pi)$ must also lie in the set V of perfect equilibrium payoffs of the government.

Given any such continuation value function $w(\pi) \in V$, we say that a *money growth rate μ is incentive compatible in the current period* if

$$(12) \quad (1-\beta)R(x_\mu, \mu) + \beta \int w(\pi) f(\pi|\mu) d\pi \geq (1-\beta)R(x_\mu, \mu') + \beta \int w(\pi) f(\pi|\mu') d\pi$$

for any possible μ'. This constraint simply requires that the government get a higher payoff from this period on from choosing μ than it does from choosing any other μ'. Notice that here the government's continuation payoffs vary with μ only to the extent that changes in the money growth rate μ shift the distribution of inflation π.

Notice that the set of equilibrium payoffs V is independent of which regime is used in the current period. This is because we have assumed that the government can switch regimes at the beginning of any period and, hence, the game from next period on is independent of the regime used in the current period. Also note that the set V in which the government's continuation values $w(e)$ must lie is unknown. We can solve for this set recursively. To show that an exchange-rate regime has a natural advantage, however, we do not need to solve for V. Instead, we treat this set $V = [v^w, v^b]$ as a parameter and show that an exchange-rate regime has a natural advantage from transparency for any nondegenerate equilibrium set V.

We compare the exchange-rate regime to the money regime as follows. We first compute the highest payoff that can be achieved if the exchange-rate regime is used in the current period, and both the level of e and x_e and the continuation value function $w(e)$ are chosen to maximize the payoff of the government subject to the incentive constraints. This payoff is the highest perfect equilibrium payoff for the government given that it uses an exchange-rate regime in the current period and is free to switch regimes in each future period. We then compute the corresponding highest payoff for the government given that it uses a money regime in the current period and is free to switch regimes in each future period. We compare these payoffs to characterize when the exchange-rate regime is preferred to the money regime.

Given a set $V = [v^w, v^b]$ of perfect equilibrium payoffs, *the best payoff for the government under an exchange-rate regime* is the solution to the following problem: choose current actions x_e and e and continuation value function $w(e) \in V$ to maximize

$$(1-\beta)S(x_e, e) + \beta w(e)$$

subject to the incentive constraints $x_e = e$ and (11). Notice that the left-side of the incentive constraint (11) is the payoff to be maximized, so it is clearly optimal to set $w(e) = v^b$. To relax the incentive constraint (11) as much as possible it is clearly optimal to set $w(e') = v^w$ so as to minimize the right-side of this constraint . Using this argument and substituting out $x_e = e$ we can write this problem as

$$\text{(13)} \quad \max_e \ (1-\beta)S(e, e) + \beta v^b$$

subject to

$$\text{(14)} \quad (1-\beta)(S(e, e') - S(e, e)) \leq \beta(v^b - v^w)$$

for all e'.

Given a set $V = [v^w, v^b]$ of perfect equilibrium payoffs, *the best payoff for the government under a money regime* is the solution to the following problem: choose current actions x_μ and μ and continuation value function $w(\pi) \in V$ to maximize

$$\text{(15)} \quad (1-\beta)R(x_\mu, \mu) + \beta \int w(\pi) f(\pi|\mu) d\pi$$

subject to the incentive constraints $x_\mu = \mu$ and (12). Substituting $x_\mu = \mu$ and rearranging the incentive constraint we can write this problem as

$$(16) \quad \max_{\mu}(1-\beta)R(\mu,\mu) + \beta \int w(\pi) f(\pi|\mu) d\pi$$

subject to

$$(17) \quad (1-\beta)(R(\mu,\mu') - R(\mu,\mu) \leq \beta \int w(\pi)(f(\pi|\mu) - f(\pi,\mu'))d\pi.$$

We begin with a preliminary result that we use in establishing Proposition 2.

Lemma 1. (V nondegenerate) If the variance of foreign inflation shocks is less than or equal to that of domestic inflation shocks, the set $V = [v^w, v^b]$ has $v^b > v^w$ and v^b is greater than payoff from the static Nash outcome repeated in every period.

We then have the following.

Proposition 2. (A natural advantage without commitment) When there is no commitment, the exchange-rate regime is preferred to the money regime even if the variances of foreign and domestic inflation shocks are the same.

Proof. When $\sigma^2_{\pi^*} = \sigma^2_{\pi}$, the current period payoffs are the same in that $S(\mu,\mu') = R(\mu,\mu')$. Clearly, the exchange-rate regime is weakly preferred to the money regime. To show that the exchange-rate regime is strictly preferred we proceed as follows. The continuation value for the government under the money regime is lower than it is under the exchange-rate regime since $w(\pi) \leq v^b$ implies

$$(18) \quad \int w(\pi) f(\pi|\mu) d\pi \leq v^b.$$

Suppose first that $w(\pi)$ is such that (18) is an equality. Then $w(\pi) = v^b$ (almost everywhere), the government's continuation payoff is independent of its current action and the only incentive compatible actions under a money regime are the static Nash actions. From Lemma 1 we know that the government can achieve a payoff that is strictly higher than that of static Nash with an exchange-rate regime. Hence, if (18) is an equality, an exchange-rate regime is strictly preferred to a money regime.

Next, suppose that $w(\pi)$ is such that (18) is a strict inequality. Note that the incentive constraint under a money regime is tighter than it is under an exchange-rate regime since

$$\int w(\pi)(f(\pi|\mu) - f(\pi, \mu'))d\pi < v^b - v^w.$$

As a result, here also, the best payoff the government can achieve under an exchange-rate regime is strictly higher than the best payoff it can achieve under a money regime. *Q.E.D.*

We illustrate the results of Propositions 1 and 2 in Figure 1. In the figure we show how the optimal regime varies with the variances of domestic and foreign inflation shocks. When there is commitment, the exchange-rate regime is preferred if and only if the variance of foreign inflation shocks, $\sigma^2_{\pi^*}$, is lower than the variance of domestic shocks, σ^2_{π}. This is the region labelled A in the figure. When there is no commitment, the exchange-rate regime is preferred even if the variances of the shocks are equal. Thus, the region for which the exchange-rate regime is preferred expands to include the region labelled B as well as A.

4. Alternative models of transparency

In modelling the idea that exchange rates are easier to monitor than money growth rates we have made the simple but extreme assumptions that inflation is the only signal of the money growth rate and that money growth rates are never observed. Here we show that we can relax these assumptions and still obtain our main result.

Suppose first that, in addition to inflation, there is a direct noisy signal of money growth. Specifically, we add a direct signal of money growth η to the model. Let $f(\pi, \eta|\mu)$ be the density of inflation π and the noisy signal η given the money growth μ. Here the government's continuation value can vary only with π and η and can be written as $w(\pi, \eta)$. The government's incentive constraint becomes

$$(19)\quad (1-\beta)R(x_{\mu}, \mu)+\beta \int\int w(\pi, \eta) f(\pi, \eta|\mu) d\pi d\eta \geq (1-\beta)R(x_{\mu}, \mu')+\beta \int\int w(\pi) f(\pi, \eta|\mu') d\pi d\eta$$

for any possible μ'. It should be clear that it is easy to prove the analogue of Proposition 2. in this environment.

Suppose next that inflation is the only signal but that the money growth rate is perfectly observable with a lag. Specifically, we modify our model by letting money growth become known with a one-period lag. Here, the history for the private agents is

$$h_t = (i_0, e_0, \pi_0; i_1, e_1, \pi_1, \mu_0; \ldots; i_{t-1}, e_{t-1}, \pi_{t-1}, \mu_{t-2})$$

Thus, the money growth μ_{t-1} only is observed after private agents set their wages in period t. The history for the government is

$$H_t = (i_0, e_0, \pi_0, \mu_0; i_1, e_1, \pi_1, \mu_0; \ldots; i_{t-1}, e_{t-1}, \pi_{t-1}, \mu_{t-1}).$$

The strategies are defined as functions of these histories in the standard way. Let $V = \left[v^w, v^b\right]$ be the set of equilibrium payoffs for the government of the game starting in period 0. We assume

$$(20)\quad v^b < v^R$$

so that the best equilibrium payoff to the game with no commitment is strictly less than the Ramsey payoff.

The intuition of why exchange rates have a natural advantage in this environment is clear. Under the money regime any deviation in period t is not directly observed in that period. Thus, in period $t+1$ the private agents can react only to a noisy signal of that action. Of course, in period $t+2$ agents have observed the government's action in period t and agents at that time can precisely react to any deviation in period t. This lag in the ability to precisely react leads to a tighter incentive constraint under the money regime and thus gives the exchange rate regime its advantage.

The technical difficulty in proving this result is that this game does not lend itself to recursive analysis as easily as our original game. In our original game both the private agents and the governments have the same information at the beginning of each period, hence the game starting from any period t looks identical to the game starting in period 0. In the game with information lags, in the game starting from period t the government has private information, namely its actions in period $t-1$ that the private agents do not have. In the game starting in period 0, however, the government has no private information. Hence, when there are lags the game starting at time t does not look identical to the game starting in period 0.

We can prove the analog of Proposition 2 in this environment, however, without using recursive analysis. We prove this in three steps. The first step is to show that in any period the government's continuation payoff can never be lower than the payoff to the worst equilibrium payoff v^w starting from time 0. This follows because each period the government always has the option of not conditioning its action on what it has done in the past. So, therefore, the government cannot be held to payoffs lower than it can attain at date 0 when there is no past on which to condition. Thus, the punishment following a money regime, when the government has private information, can never be worse than that following an exchange rate regime, when it does not.

The second step is the following lemma.

Lemma 2. If the variance of foreign and domestic inflation shocks is the same, then, for any equilibrium strategy profile σ that starts in the first period with a money regime, there exists an alternative equilibrium strategy profile $\tilde{\sigma}$ that starts in the first period with an exchange rate regime and which delivers the same payoff for the government.

Proof. The current period payoff functions $R(x_\mu, \mu)$ and $S(x_e, e)$ are identical and the distribution of realized inflation π is also identical whenever $e = \mu$ and $x_e = x_\mu$. The alternative equilibrium strategy profile $\tilde{\sigma}$ is constructed by having the government choose an exchange rate regime in the first period, private agents setting wages $x_e = x_\mu$, government choosing $e_0 = \mu_0$, and, in subsequent periods having agents' and the government's actions vary with realized inflation π_0 in exactly the same way that these actions varied with π_0 under the original strategy profile σ. The more subtle part is that if the government deviates under a money regime this deviation can be detected only statistically, while if it deviates under an exchange rate regime this deviation is detected for sure. Following such a deviation with $e_0 \neq \mu_0$, under $\tilde{\sigma}$ instead of using whatever punishment would have happened in the money regime we instead use the worst possible punishment, namely a strategy profile that delivers the payoff v^w for the government. From Lemma 1 above, we know that this punishment for deviations $e_0 = \mu_0$ is (weakly) more severe than any punishment for deviations specified in σ, and hence, $\tilde{\sigma}$ must also be incentive compatible.

The third step finishes the proof as follows.

Proposition 3. If v^b is strictly less than the Ramsey payoff and the variance of the domestic and foreign inflation shocks are equal, then an exchange rate regime is strictly preferred to a money regime.

Proof: Let σ be an equilibrium strategy profile in which the government chooses a money regime in period 0. By Lemma 2, we can construct an alternative equilibrium in which the government chooses an exchange rate regime and attains the same value. We now show that by choosing an exchange rate regime we can relax the period 0 incentive constraint for the government and thus attain a strictly higher payoff. This completes the proof since, under the assumption that it is infeasible to attain the Ramsey payoff, the incentive constraint strictly binds in any money regime.

Given any value of realized inflation in period 0 and any money growth in period 0, the government's continuation payoff from period 1on lies between v^w and v^b. Since private agents' actions in period 1 cannot be contingent on μ_0 but rather must depend on realized inflation, the cost to the government of a deviation period 0 in terms of the change in the expected continuation value must be strictly less than $v^b - v^w$. In contrast, under an exchange rate regime, there is an equilibrium in which the cost to the government after a deviation is equal to $v^b - v^w$. Hence, the incentive constraint when an exchange rate regime is chosen in period 0 is strictly looser than the incentive constraint when a money regime is chosen. Thus, the best equilibrium payoff is strictly higher under an exchange rate regime than it is under a money regime. *Q.E.D.*

Here we have assumed that the best equilibrium has a strictly lower payoff than the Ramsey payoff. Clearly, if β is high enough, then the Ramsey equilibrium can be achieved under both regimes and the best exchange rate regime is tied with the best money regime.

5. The best equilibrium without commitment

So far we have compared the best payoffs the government can achieve under exchange rate and money regimes. Here we characterize the outcomes that give rise to these best payoffs.

When the exchange-rate regime is the preferred regime the equilibrium is simple. The government chooses the exchange-rate regime in each period and sets the exchange rate equal to e^b which denotes the best exchange rate policy. If there are any deviations, agents and the governments revert to the actions that implement the worst equilibrium payoff v^w. These actions may correspond either to an exchange rate regime or a money regime depending on the variances of the shocks. In equilibrium, of course, there are no deviations and, hence, the exchange rate is set to e^b in every period. This result follows immediately from (13).

The equilibrium outcome under the best money regime looks very different. Under this regime the government starts off setting the money growth equal to some low growth rate μ^b and continues to do so as long as low inflation is realized, that is, as long as the domestic inflation shocks ε are small enough so that $\mu^b + \varepsilon \leq \pi^b$. When a sufficiently large domestic inflation shock occurs, agents and the government revert to the actions that implement the worst equilibrium payoff v^w. We prove this result in Proposition 4.

It turns out that when the variances are such that a money regime implements the best payoff, the worst payoff is also implemented by a money regime. In this regime the government starts off setting the money growth equal to some high growth rate μ^w and continues to do so as long as high inflation is realized. When a sufficiently small domestic inflation shock occurs, agents and the government revert to the actions that implement the best equilibrium payoff. We prove this result in Proposition 6.

These results about the nature of the best and worst money regimes are reminiscent of those concerning equilibrium price wars in models of oligopoly discussed by Green and Porter (1984) and Abreu, Pearce, and Stacchetti (1986).

We begin by characterizing the best money regime. To do so we need to solve the problem (15) and proceed as follows. We begin by replacing the incentive constraint (12) by the first-order condition associated with maximizing the left-side of this incentive constraint with respect to μ and evaluating it at the proposed government policy. The resulting constraint is

$$(21) \quad (1-\beta)R_\mu(x_\mu,\mu) + \beta \int w(\pi) f_\mu(\pi|\mu)\, d\pi = 0$$

where $R_\mu(x,\mu) = \partial R(x,\mu)/\partial\mu$ and $f_\mu(\pi|\mu) = \partial f(\pi|\mu)/\partial\mu$. This first-order condition is necessary and sufficient to ensure that (12) holds when the function defined by the left-side of (12) is concave in μ. In Proposition 4, we simply assume that this approach is valid and characterize the resulting $w(\pi)$. In Proposition 5 we show that, given the resulting form of $w(\pi)$, the left-side of (12) is concave in μ when the variance of domestic inflation shocks is sufficiently large.

Under the assumption that our first-order condition approach is valid, in the problem (15) we can replace the government's incentive constraint (12) with the constraint (21). In any solution to this problem the continuation values necessarily have a bang-bang form

$$(22) \quad w^b(\pi) = \left\{ \begin{array}{c} v^b \text{ if } \pi \leq \pi^b \\ v^w \text{ if } \pi > \pi^b \end{array} \right\}$$

That is, there is a cutoff inflation level π^b such that the optimal continuation value function $w^b(\pi)$ is set to the best payoff v^b if realized inflation is less than π^b and to the worst payoff

v^w if realized inflation is greater than π^b.

Part of the rationale for why the optimal continuation value takes the form (22) is intuitive. Since higher money growth rates make higher inflation more likely, if we are to discourage the government from choosing a high money growth rate, the continuation value must specify a low level when the realized inflation is high. Slightly less intuitive is that the best continuation value function must assign only the best and the worst possible equilibrium payoffs. Mechanically, this occurs because both the payoffs and the incentive constraint are linear in the continuation values. We demonstrate this formally in Proposition 3.

Proposition 4. Under the assumption that the first-order condition approach is valid, the optimal continuation value function has the form of (22).

Proof. Letting λ be the multiplier on the government's incentive constraint (21), the term in the Lagrangian that involves $w(\pi)$ is

$$\beta \int w(\pi)[1+\lambda \frac{f_\mu(\pi|\mu)}{f(\pi|\mu)}]f(\pi|\mu)d\pi.$$

Notice that this term is linear in each value of $w(\pi)$, so that it is optimal to set

$$w^b(\pi) = \left\{ \begin{array}{c} v^b \text{ if } (1+\lambda \frac{f_\mu(\pi|\mu)}{f(\pi|\mu)}) > 0 \\ v^w \text{ if } (1+\lambda \frac{f_\mu(\pi|\mu)}{f(\pi|\mu)}) < 0 \end{array} \right\}$$

These first-order conditions imply the optimal continuation values are always extreme, that is, either v^b or v^w. The only issue is for what values of π are the payoffs v^b and v^w assigned. To determine these values we start by observing that with our assumption of normality $f_\mu(\pi|\mu) = f(\pi|\mu)(\pi-\mu)/\sigma_\pi$ so that our densities satisfy the monotone likelihood ratio property in that

$$\frac{f_\mu(\pi|\mu)}{f(\pi|\mu)} = (\pi-\mu)/\sigma_\pi$$

is increasing in π. Thus, $w^b(\pi)$ is increasing in π if $\lambda > 0$ and decreasing in π if $\lambda < 0$. We will show $\lambda < 0$ so that it is decreasing in π.

Clearly, if we are to induce a low current money growth rate the continuation value must specify a low level when the realized inflation is high, since higher money growth rates make higher inflation more likely. We demonstrate this formally as follows. First, note that at the optimum $R_\mu(x^b, \mu^b) \geq 0$. This follows since the optimum must weakly improve upon static Nash and thus must have a money growth rate less than or equal to the static Nash level. That is, $x^b = \mu^b \leq U$. Since $R_\mu(x, \mu) = U + x - 2\mu$, then $\mu^b \leq B(x^b)$, and $R_\mu(x^b, \mu^b) \geq 0$. Next, since $R_\mu(x^b, \mu^b) \geq 0$ the incentive constraint (21) implies that

$$(23) \quad \int w^b(\pi) f_\mu(\pi|\mu)\, d\pi \leq 0.$$

Since inflation is normally distributed with mean μ, increasing μ increases the distribution of inflation in the sense of first-order stochastic dominance. Thus, increasing μ increases $\int w^b(\pi) f(\pi|\mu) d\pi$ when $w^b(\pi)$ is increasing and decreases this integral when $w^b(\pi)$ is decreasing. Thus, to satisfy (23), $w^b(\pi)$ must be decreasing. Q.E.D.

In the Appendix we prove the following proposition, justifying our use of the first-order approach. We let ϕ and Φ denote the density and cumulative distribution functions of a standard normal, respectively.

Proposition 5. (First-order approach valid) Given that $w^b(\pi)$ has the bang-bang form (22) and is decreasing, if $\sigma_\pi^2 > \frac{\beta}{1-\beta}(v^b - v^w)\phi(1)/2$, then the incentive constraint (12) is satisfied if and only if the first-order condition (21) holds.

As we have noted above, when a sufficiently large domestic inflation shock occurs, agents and the government revert to the actions that implement the worst equilibrium payoff

v^w. Along the equilibrium path such a shock must eventually occur, so the actions that implement the worst equilibrium payoff are eventually observed. To complete our characterization of the best money regime we must also characterize the worst equilibrium outcome.

The worst equilibrium payoff v^w can occur under either an exchange-rate regime or a money regime depending on the variances of domestic and foreign inflation shocks. This worst equilibrium payoff is the larger of two payoffs: the worst payoff under an exchange-rate regime v_e^w and the worst payoff under a money regime v_μ^w. That is, $v^w = \max\{v_e^w, v_\mu^w\}$. The worst equilibrium payoff is the larger of these two payoffs because, at the beginning of each period, the government can choose which regime it prefers. In the remainder of this section we characterize the worst payoffs v_e^w and v_μ^w under the two regimes and the parameter values for which $v^w = v_e^w$ and $v^w = v_\mu^w$.

Given a set $V = [v^w, v^b]$ of perfect equilibrium payoffs, the worst payoff for the government under an exchange-rate regime v_e^w is the solution to the following problem: choose current actions x_e and e and continuation value function $w(e) \in V$ to minimize

$$(1-\beta)S(x_e, e) + \beta w(e)$$

subject to the incentive constraints $x_e = e$ and (11). To relax the constraint (11) as much as possible, it is optimal to set $w(e') = v^w$ for all $e' \neq e$ so as to minimize the right-side of this incentive constraint. Since the continuation value following a deviation is independent of the deviation, the best deviation against e is simply the static best response $B(e)$. Using this argument and substituting out $x_e = e$ we can write this problem as

$$(24) \quad \min_{e,w} \ (1-\beta)S(e,e) + \beta w$$

subject to $w \in [v^w, v^b]$ and

(25) $(1-\beta)S(e,e) + \beta w \geq (1-\beta)S(e, B(e)) + \beta v^w$

for all e'.

Notice two points. First, the left-side of the constraint (25) is the objective function that we are trying to minimize. Second, the left-side of the constraint can be made arbitrarily small by increasing e. Hence, this constraint must bind.

Since the incentive constraint binds, we can find the solution by minimizing the right-side of the incentive constraint subject to (25) written as an equality. This problem is

(26) $\min_e (1-\beta)S(e, B(e)) + \beta v^w$

subject to $v^w \leq w \leq v^b$ and

(27) $\beta w = (1-\beta)[S(e, B(e) - S(e,e)] + \beta v^w$

Clearly, since $S(e, B(e))$ is decreasing in e, the solution involves finding the w that allows for the largest choice of e. Given our functional forms, $S(e, B(e)) - S(e,e)$ is increasing in e for e greater than the static Nash level, U. Hence, the solution involves setting $w = v^w$ and choosing e to be the largest solution to (27) with $w = v^w$. (This result is reminiscent of a result in Abreu 1986.)

In the worst money regime, continuation values $w^w(\pi)$ are assigned to give the government the incentive to choose a higher money growth rate than it would choose in the static Nash outcome. This entails giving the government high continuation values in the event that high inflation is realized and giving it low continuation values in the event that low inflation is realized. Thus, if the worst money growth rate is realized as part of the path of equilibrium play, the government chooses a high money growth rate and keeps choosing this high rate unless a sufficiently high level of inflation is realized. If such a sufficiently high level is realized, the path of play reverts to the best equilibrium path of play, whether that be an exchange-rate regime or a money regime. In this sense, in the worst money regime, extremely high inflation must be realized before inflation can fall. This result is proved in the next proposition.

As before, under the assumption that the first-order condition approach is valid we can write the problem of finding the worst payoff under a money regime as

$$(28) \qquad \min_{\mu,x,w(\pi)} (1-\beta)R(x,\mu) + \beta \int w(\pi) f(\pi,\mu)\, d\pi$$

subject to the constraints $x = \mu$ and (21).

Proposition 6. Under the assumption that the first-order approach is valid, the optimal continuation value function for the worst equilibrium in the money regime has the form

$$(29) \qquad w^w(\pi) = \left\{ \begin{array}{c} v^w \text{ if } \pi \leq \pi^w \\ \\ v^b \text{ if } \pi > \pi^w \end{array} \right\}$$

for some cutoff inflation rate π^w.

Proof. The proof is similar to that of Proposition 3. Specifically, the first-order condition of the problem (28) with respect to $w(\pi)$ implies that $w^w(\pi)$ has a bang-bang form around some cutoff π^w. To show that $w^w(\pi)$ must be increasing, note that at the optimum $R_\mu(x^w, \mu^w) \leq 0$ so that current period payoff for the government is decreased when the government deviates to a higher money growth rate. Accordingly, the incentive constraint (21) implies that

$$\int w^w(\pi) f_\mu(\pi|\mu)\, d\pi \geq 0$$

which gives the result that $w^w(\pi)$ is increasing. *Q.E.D.*

We use an argument similar to that in Proposition 2 to characterize the regions of the parameter space. When the variance of domestic and foreign inflation shocks is the same, the worst payoff under an exchange-rate regime is lower than that under a money regime, that is, $v_e^w < v_\mu^w$. This is because, in this case, the current period payoff functions R and S are the same and, under an exchange-rate regime, the incentive constraint is looser than it is under a money regime. Hence, when these variances are the same, the worst equilibrium payoff $v^w = \max\{v_e^w, v_\mu^w\}$ is equal to that under a money regime. Clearly, increasing the variance of foreign inflation shocks above that of the domestic shocks reduces v_e^w and leaves v_μ^w unchanged. Hence, $v^w = v_\mu^w$ when the variance of foreign inflation shocks exceeds that of domestic inflation shocks.

We combine this result with that in Proposition 2 to characterize equilibrium outcomes in Figure 2. If the variance of foreign shocks is sufficiently high relative to that of domestic shocks, as in Region C, then the government follows a money regime in both the best and the worst equilibrium. If the variance of foreign shocks is sufficiently low relative to that of domestic shocks, as in Region E, then the government follows an exchange-rate regime in both the best and the worst equilibrium. When the variances of the two shocks are similar, as in Region D, then the government uses an exchange-rate regime in the best equilibrium and a money regime in the worst equilibrium.

In Regions D and E the observed outcome is a constant e in every period. The observed outcome in Region C is more interesting. In this region a money regime is used in both the best and worst equilibrium. In Figure 3 we illustrate a typical path of money growth and inflation observed in the best equilibrium in periods 0 to 10. In period 0 agents choose low wages $x_\mu = \mu^b$, the government chooses low money growth μ^b and realized inflation is this low money growth plus the domestic inflation shock $\pi_0 = \mu^b + \varepsilon_0$. Since realized inflation π_0 is less than the critical value π^b, in period 1 agents again choose wages $x_\mu = \mu^b$, the government chooses low money growth μ^b and realized inflation is $\pi_1 = \mu^b + \varepsilon_1$. The outcomes continue in this fashion, with agents choosing low wages and the government choosing low money growth until the domestic inflation shock is large enough so that realized inflation exceeds the critical value π^b. In the figure this occurs in period 4. In period 5, agents choose high wages $x_\mu = \mu^w$, the government chooses high money growth rate μ^w and realized inflation is $\pi_5 = \mu^w + \varepsilon_{5.}$ This pattern continues until the domestic inflation shock is high enough so that realized inflation exceeds the high critical value π^w. In the figure this occurs in period 7. In period 8, the outcome reverts back to the pattern of agents choosing low wages and the government choosing low money growth. After that, the outcome cycles stochastically between these two phases, depending on the realizations of the domestic inflation shocks.

6. Conclusion

Here we have considered the advantage of transparency in a model in which the exchange rate is observable and the money growth rate is only observable with noise, at least contemporaneously. In the best equilibrium of the exchange-rate regime the rate of depreciation of the exchange rate is constant. This occurs because our simple model abstracts from all shocks that would lead the optimal mean inflation rate to vary over time. As such our model does not provide a rationale for fixed exchange rates, rather it provides a rationale for using exchange rates as policy instruments rather than money growth rates.

This paper shows that a certain price, namely, the exchange rate, has an advantage over a certain quantity, namely, the money growth rate, as an instrument for monetary policy. This basic idea that prices have an advantage over quantities as instruments of monetary policy might also be applied to a comparison of interest rates and any other quantity instrument that is more difficult to monitor.

Appendix

Proof of Lemma 1. In any period, given some wages x_e, the government's static best response to x_e is to choose e to maximize $S(x_e, e)$. This best response is given by $B(x_e) = (U + x_e)/2$. Likewise, the static best response to x_μ is $B(x_\mu) = (U + x_\mu)/2$, and the static Nash outcomes are $e = x_e = U$ and $\mu = x_\mu = U$. Repeating the static Nash outcomes in every period, regardless of the history, is a perfect equilibrium that leads to a payoff for the government of

$$v^N = \max[S(U, U), R(U, U)].$$

Thus, $v^N \in V$.

We now construct a higher equilibrium payoff using the following trigger strategies. Let $\hat{e}$ be some exchange rate that is strictly lower than the static Nash exchange rate U, and let $\hat{v} = S(\hat{e}, \hat{e})$ be the government's payoff when $x_e = \hat{e}$ and this $\hat{e}$ is played in every period. The trigger strategies specify the following. Begin with the government choosing an exchange-rate regime, agents setting $x_e = \hat{e}$, and the government choosing $\hat{e}$. Continue with these actions in every period unless the government deviates from $\hat{e}$. Following any such deviation both the government and the private agents revert to the static Nash outcome. These strategies constitute an equilibrium if the government has no incentive to deviate in that

$$(30) \quad (1 - \beta)(S(\hat{e}, B(\hat{e})) - S(\hat{e}, \hat{e})) \leq \beta(\hat{v} - v^N)$$

holds. It is easy to show with our functional forms that (30) is satisfied for $\hat{e} = U - \varepsilon$ for some sufficiently small ε. Thus, $\hat{v}$ and v^N are equilibrium payoffs that satisfy $v^b \geq \hat{v} > v^N \geq v^w$. *Q.E.D.*

Proof of Proposition 4. Here we show that the solution to the problem with incentive constraint (12) is satisfied if and only if the first-order condition (21) holds when $\sigma_\pi^2 > \frac{\beta}{1-\beta}(\bar{w} - \underline{w})\frac{\phi(1)}{2}$. Using (22), the constraint (12) can be written

$$(31) \quad \mu \in \arg\max_{\mu}(1-\beta)R(x,\mu) + \beta\left[\bar{w}\Phi(\frac{\pi^h - \mu}{\sigma_\pi}) + \underline{w}(1 - \Phi(\frac{\pi^h - \mu}{\sigma_\pi}))\right].$$

Since $F(\pi^h, \mu) = \Phi((\pi^h - \mu)/\sigma_\pi)$, we can write the first and second order conditions of the maximization problem (31) as

$$(32) \quad (1-\beta)R_\mu(x^h,\mu) - \beta\frac{(\bar{w} - \underline{w})}{\sigma_\pi}\phi(\frac{\pi^h - \mu}{\sigma_\pi}) = 0$$

and for all μ

$$(33) \quad (1-\beta)R_{\mu\mu}(x^h,\mu) - \beta(\bar{w} - \underline{w})\frac{(\pi^h - \mu)}{\sigma_\pi^2}\phi(\frac{\pi^h - \mu}{\sigma_\pi}) \leq 0$$

which can be written

$$(34) \quad -2(1-\beta) - \beta\frac{(\bar{w} - \underline{w})}{\sigma_\pi^2}\phi(z)z \leq 0$$

for all $z \in [-\infty, \infty]$. The expression $\phi(z)z$ in (34) is minimized at $z = -1$. Since $\phi(-1) = \phi(1)$, the inequality $\sigma_\pi^2 > \frac{\beta}{1-\beta}(\bar{w} - \underline{w})\frac{\phi(1)}{2}$ guarantees that the second order condition holds globally, and thus (21) is both necessary and sufficient for (12). Q.E.D.

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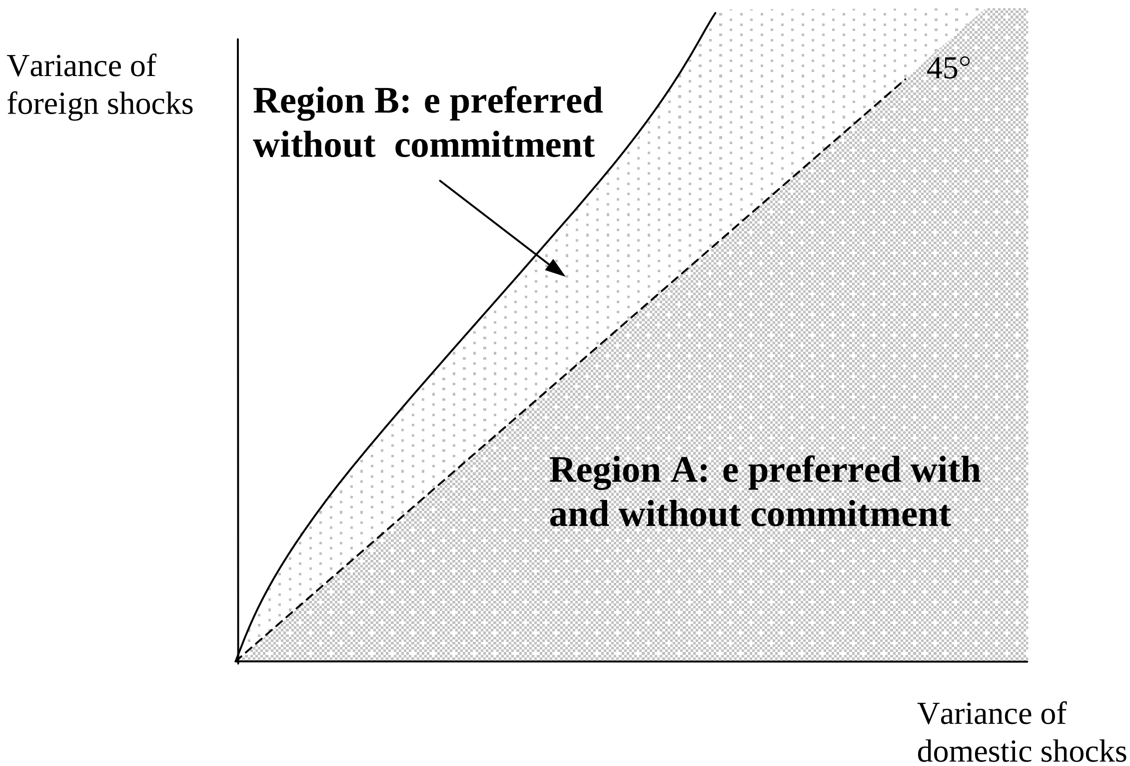
Figure 1:
Parameter regions for which exchange rate regime preferred to money regime with and without commitment*



*With commitment, exchange rate regimes are preferred in region A, where the variance of domestic inflation shocks is larger than the variance of foreign inflation shocks. With no commitment, exchange rate regimes have an additional advantage. They are preferred in both region A and in region B.

Figure 2:
Regimes in best and worst equilibrium

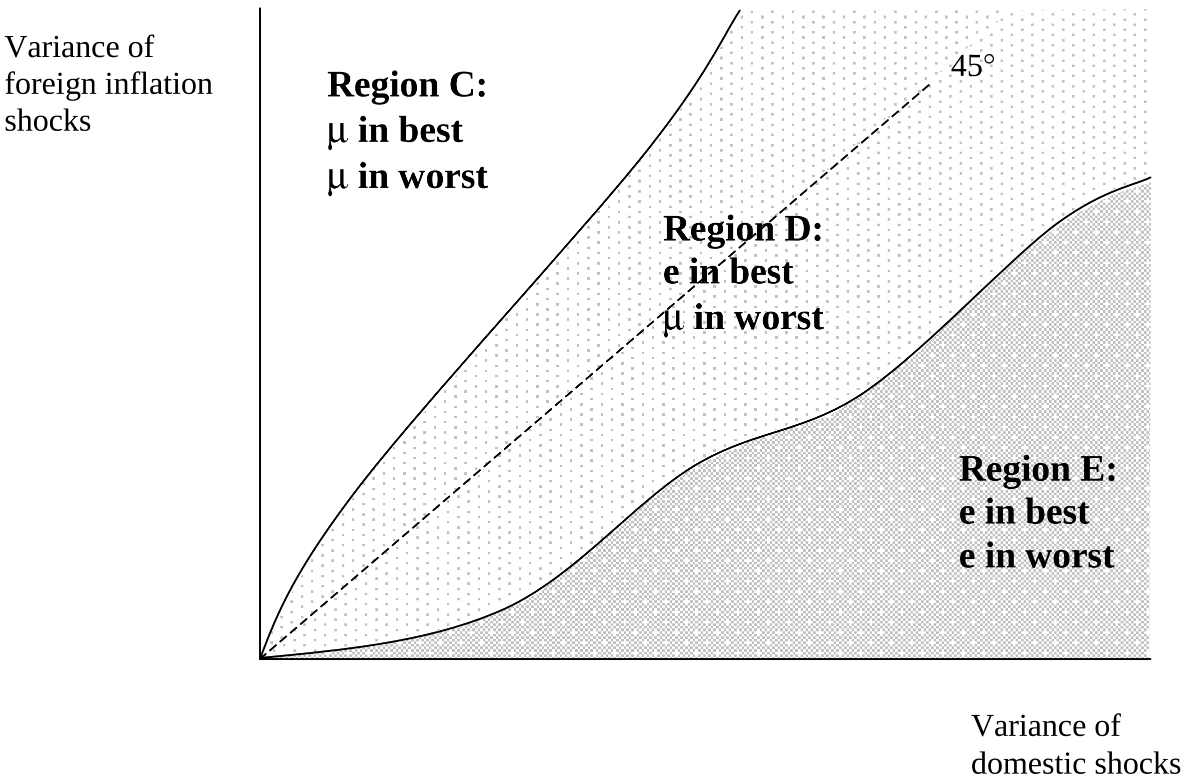


Figure 3: Outcomes when money regime followed in best and worst equilibrium