A Hiring Subsidy for Long-Term Unemployed in a Search Model with PES and Random Search

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ABSTRACT: Our search model combines two search methods, the public employment service (PES) and random search. The separation rate is endogenous, the job matching process consists of three stages. In the first and the second respectively the short-term (STU) and the long-term unemployed (LTU) randomly search for a vacancy. In the last stage the PES matches registered jobseekers with registered vacancies. The LTU cause training costs and, during the training period, have a lower marginal product than the STU. The effects of the hiring subsidy and of profiling techniques to increase the effectiveness of the PES depend on the target group. For target groups, who have relatively low private search costs in comparison with their productivity, not only the hiring subsidy but also the job placement activities of the PES are counterproductive and reduce overall employment.

KEY-WORDS: Matching model, hiring subsidy, endogenous separation rate, active labour market policy, PES, search market

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1. INTRODUCTION

Millard/Mortensen (1997), Mortensen/Pissarides (1999, 2001) and Pissarides (2000, ch. 9) are search models which analyse the effects of active labour market policies (ALMP) on equilibrium unemployment. In these models, the labour market is characterised by a matching technology which represents the two-sided search process with its frictions – due to imperfect information, mobility costs and heterogeneities. Each new match of a job seeker with a vacancy is entitled to a hiring subsidy. The hiring subsidy increases both the number of newly created jobs and the amount of job destruction. Therefore, its overall effect on equilibrium unemployment is ambiguous. Millard/Mortensen (1997) and Mortensen/Pissarides (1999, 2001) thus estimate the net effects of the subsidy with the help of numerical simulations.

Our model differs from the above-mentioned in the following respects. First, two search methods are available, the public employment service (PES) and random search (Pissarides 1979). Second, ALMP is available only for the long-term unemployed (LTU). A hiring subsidy is paid to firms which register their vacancy and fill it with a LTU worker placed by the PES. The PES also has the option to subsidise matches established through random search. Third, the model's matching process consists of three subsequent stages. In the first and second stage respectively the short-term unemployed (STU) and the active job seekers among the LTU randomly search for a vacancy, in the third stage the PES matches registered vacancies with the registered unemployed. Fourth, the unemployed choose between a passive and an active search strategy. The active LTU combine both methods of search. The passive unemployed wait for a placement by the PES.

The model generates the following results. (1) Equilibrium unemployment depends negatively on both unemployment incidence and duration, and on the fraction of passive job seekers. (2) Moreover, the hiring subsidy increases job destruction and unemployment duration of passive job seekers, and reduces the proportion of active job seekers among the STU and LTU as well as the job-to-job transitions. As a consequence, it decreases overall employment. (3) It increases the fraction of the LTU, their average outside wage, and the expenses of the PES for passive and active measures. (4) Furthermore, the LTU must accept a wage penalty. (5) Intuition - embodied for example in the German "Job-Aqtiv-law" – recommends increasing the effectiveness of the public placement service in order to reduce equilibrium unemployment. This intuition is not confirmed by our model. (6) Strengthening the job seekers' bargaining power decreases equilibrium unemployment.

This paper is structured as follows: In Section 2, the equilibrium rate of unemployment is derived. Section 3 introduces the asset equations of filled jobs and employed workers. Section 4 deals with job creation. Section 5 covers the asset equations of the unemployed and wage negotiations. In Section 6, the equilibrium values of the filled jobs, the dispersions of the outside wages of the LTU and the job destruction condition are derived. Section 7 presents a numerical simulation and section 8 concludes. The Appendix contains all proofs and a graphical presentation of the simulation results.

2. Steady State and Hiring Subsidy

The time of the model is discrete. Job creation takes place at the beginning, job destruction at the end of a period, s. Fig. 1. The search process is two-sided. A continuum of vacancies searches for applicants, who can be of two different types: The first type are the short-term unemployed (STU) who have lost their job only at the end of the previous period. The second type has been without a job for one period or more and is either threatened by long-term unemployment or already belongs to the long-term unemployed (LTU).[1]

Methods and strategies of search. The model analyses the interactions of two search methods, the PES and random search, and two strategies of search, active search on the private search market and passive search through the job placement service of the PES. The search strategy of the vacancies is not specialised. Vacancies are simultaneously posted on the private search market where they randomly search for a worker and are registered with the PES. Unemployed workers choose between the active and the passive search strategy and the combinations of both strategies.

Workers who lose their job register with the PES to claim unemployment benefit and to avail of the job placement service. But registration often takes place only after days or weeks have passed since the beginning of the unemployment spell. Once the worker has notified the PES it reviews the right to the claim, registers and advises the unemployed worker and refers him to the placement service. The placement agency then looks for available jobs and either makes a job offer or not, depending on the number of registered vacancies and registered unemployed waiting for a placement. When the first offer arrives at the end of the *reaction time* the worker has often already found a job. How much time passes between the first day of an unemployment spell und the first job offer of the PES? Despite extensive research we have not found data on the distribution of the *reaction time* and have thus introduced the following assumption.

We assume that the *reaction time* lasts at least one period. Hence, the STU workers who decide on the active search strategy are certainly dependent on their own search efforts alone. Moreover, STU workers opting for the passive strategy and leaving all search activities up to the PES can expect a job offer in the second period of the unemployment spell at the earliest.

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[1] With respect to the differentiation between the STU and the LTU a model in discrete time is to be preferred to a model in continuous time. It is also easier to solve. However the assumption of a discrete time structure produces the following trade-off: On the one hand the reference period should not be too short so as not to vary too much from the usual time limitations between the STU and the LTU (North America: 6 months and over, Europe: 12 months and over). On the other hand the reference period should not be too long because, as we will assume, at least one model period elapses before an unemployed worker receives the first job offer from the PES.

Given the *reaction time* the PES can place only LTU workers – workers who are either threatened with long-term unemployment or who are already long-term unemployed.

Passive LTU workers leave the job search up to the PES. Active jobseekers among the LTU use both search methods simultaneously the PES and the private search market. Hence, in equilibrium their transition probability is higher than that of the LTU who have chosen to wait for a placement by the PES. But using the search market generates search costs so that, in equilibrium, only a part of the unemployed decide on the active search strategy.

Search process. The search process consists of three stages, s. Fig. 1. In the first, only the S_I active job seekers among the STU are searching. They possess the best information about current labour market conditions and, therefore, their applications are more targeted and arrive earlier than the placements of the PES or the applications of the active job seekers among the LTU.

In the second stage, advertised vacancies meet the *S* active job seekers among the LTU. In the last stage of the matching process the PES arranges matches between registered vacancies and the registered unemployed.

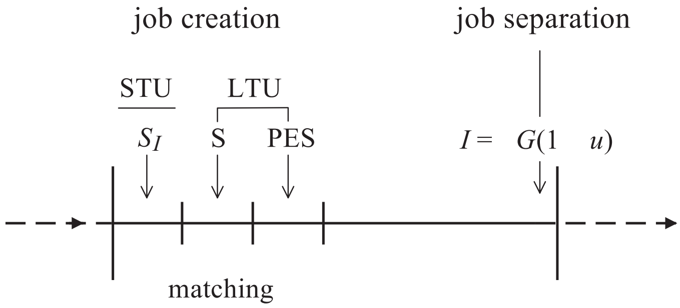

Fig. 1: The search process

Transition probabilities. The labour force is normalised to unity. Of the $1-u$ employed, $I = \lambda G(R)(1-u)$ lose their job at the end of a period, s. Fig. 1. $\lambda G(R)$ is the unemployment incidence where λ is the probability of an idiosyncratic productivity shock, $G(x)$ with support $0 \le \alpha \le x \le 1$ is the c.d.f. of the multiplicative shock *x* and *R* is the endogenous reservation productivity. yx, with $y > 0$, is the flow output of a job. If a match draws productivity *x* with $R \le x \le 1$, worker and firm decide to continue the job. If $x < R$, the match terminates, the job becomes vacant, and the worker unemployed.

Of the *I* workers who lose their job, S_I decide in favour of the active search strategy and immediately at the beginning of the next period start to search randomly for an unfilled vacancy, s. Fig. 1. The other $I - S_I \ge 0$ workers prefer the passive strategy, and, with the beginning of the subsequent period, they are threatened with long-term unemployment and belong to the group of LTU. The matching technology of the search market, which is specified below, generates the transition probability p_I that a given job seeker among the STU will find a vacancy. As the STU have a marginal product which is at least as high as that of the

LTU and do not cause training costs, each match of a STU worker with an advertised vacancy results in an employment contract. Therefore the measure of the STU, u_S, is given by

$$I - p_I S_I = u_S . \tag{1}$$

Of the pool u of LTU, $u - S \geq 0$ workers choose the passive search strategy and wait for a placement via the PES. Their unemployment duration hazard i.e. their transition probability into employment is $P(1-q_I)(1-q_S)F(T_P)$, where P denotes the probability of a contact with a vacancy found by the PES, q_I and q_S are the probabilities that the vacancy is already filled either by one of the S_I job seekers among the STU or by one of the S job seekers among the LTU.

Each match with a LTU worker generates match specific training costs $t \geq 0$, of which *ex ante* only the c.d.f. $F(t)$ with support $0 \leq t < \infty$ and the endogenous reservation costs T_P are known. The reservation costs T_P are the training costs up to which firms and the LTU are interested in signing a job contract. A match with a LTU worker with training costs $t > T_P$ is immediately broken up again; the job remains vacant and the job seeker unemployed. Then, $F(T_P)$ denotes the probability that the match partners face training costs $t \leq T_P$.

The transition probability for the S active job seekers among the LTU is $(1-q_I)[p_S F(T_S) + (1-p_S)P(1-q_S)F(T_P)]$. First, the located vacancy must still be free. The probability to locate an unfilled vacancy through random search is $p_S(1-q_I)$. Even if random search is not successful, the active job seekers among the LTU still have the chance to be placed by the PES. The probability to be placed by the PES after random search has failed is $P(1-q_I)(1-q_S)$. Second, the job seeker must draw training costs that are below the reservation costs of the respective search method. If active labour market policy discriminates between the two methods of search and subsidizes for example only placements by the PES then reservation costs depend on the search method. T_S is the reservation cost of the search market and T_P the reservation cost of the matching process organised by the PES.

Summarising the flows into employment which result from the above transition probabilities and taking into account that u_S denotes the inflow into the pool of long-term unemployed u yields the steady state condition:

$$u_S = P(1-q_I)(1-q_S)F(T_P)u + p_S(1-q_I)[F(T_S) - P(1-q_S)F(T_P)]S . \tag{2}$$

The LTU prefer the active search strategy only if the transition probability from combining the two methods of search is higher than that of the passive search strategy alone. Given the above transition probabilities this necessary condition for active job search is fulfilled iff $F(T_S) > P(1-q_S)F(T_P)$.

Matching function. The function $m(x, v)$ represents the matching technology of the search market, where m is the number of contacts per period for a given measure of job seekers x and advertised vacancies v. The matching function has constant returns to scale, is strictly concave and monotone in both arguments. Immediately at the beginning of a period, $m(S_I, v)$ of the v

advertised vacancies are filled by the STU who are actively searching. For a given vacancy posted at the beginning of a period, the probability of a match with a STU worker is $q(\theta_I) \equiv m(1/\theta_I, 1) = m(S_I, v)/v$, with $\theta_I = v/S_I$ denoting the tightness of the labour market in the first stage of the matching process. The transition probability of a given active job seeker among the STU is $p(\theta_I) = \theta_I q(\theta_I)$. For convenience we write $q_I = q(\theta_I)$ and $p_I = p(\theta_I)$.

The S active job seekers among the LTU workers face the same v advertised vacancies. $m(S,v)$ represents the measure of contacts, and $q(\theta_S) \equiv m(1/\theta_S, 1) = m(S,v)/v$ is the contact probability of a given vacancy with one of the active job seekers among the LTU – with $\theta_S = v/S$ denoting the tightness of the labour market in the second stage of the matching process. The contact probability of a given job seeker is $p(\theta_S) = \theta_S q(\theta_S)$, and we write $q_S = q(\theta_S)$ and $p_S = p(\theta_S)$.

As all vacancies are advertised as well as registered, v is also an argument in the matching function $M(u,v)$ of the PES, which has the same properties as $m(x,v)$. M is the measure of contacts per period which the PES brings about with v registered vacancies and a stock of u registered long-term unemployed. For a given vacancy, therefore, $Q(\Theta) \equiv M(1/\Theta, 1) = M(u,v)/v$ is the contact probability with a registered LTU worker via the PES – with $\Theta = v/u$ denoting the tightness between both registers of the PES. Thus, for the registered unemployed, the probability of a contact with a registered vacancy is $P(\Theta) = \Theta Q(\Theta)$.

Unemployment duration. The duration of an unemployment spell depends on the search strategy chosen by an unemployed. If the unemployed leaves the job search up to the PES, then he will be out of work for at least one period and taking into account the above transition probabilities the average length of time required for a job search will be $D_P = 1 + d_P$, where $d_P = 1/P(1-q_I)(1-q_S)F(T_P)$. d_P is the duration of job search of a long-term unemployed who prefers the passive search strategy. The duration of unemployment of an actively searching LTU worker is $d_S = 1/(1-q_I)[p_S F(T_S) + (1-p_S)P(1-q_S)F(T_P)]$. An unemployed who combines the passive search strategy in the first period of his unemployment spell with the active search strategy in all subsequent periods faces a duration of unemployment equal to $D_S = 1 + d_S$. While an unemployed worker who opts for the active search strategy from the beginning on faces an expected duration of job search of $(1-p_I)D_S$ periods.

Inserting equation (1) into equation (2), using $I = \lambda G(R)(1-u)$ and taking into account the above definitions of the tightness in the three labour market segments, we obtain the following equation for equilibrium unemployment in the steady state

$$u = \frac{\lambda G(R)}{\lambda G(R) + \sigma_I p_I + \sigma_S / d_S + (1-\sigma_S)/d_P}, \tag{3}$$

where $\sigma_S = S/u \leq 1$ is the share of active LTU job seekers among the unemployed and $\sigma_I = S_I/u$ is the ratio of active job seekers among the STU to the pool of unemployed u.

Contrary to the share σ_S the ratio σ_I is not bounded from above. The unemployment rate (3) *cet. par*. increases if the job destruction rate λG increases, the durations of job search of the active or the passive search strategy increase, the ratio of job-to-job transitions to the number of unemployed, $\sigma_I p_I$, or the share of active job seekers among the LTU worker decrease. The impact of σ_S on u is due to the fact that $d_S < d_P$ is a necessary condition for the decision of the LTU workers to search actively for a job. But if $d_S < d_P$, then u increases with a decreasing share σ_S of active job seekers among the LTU workers.

Hiring subsidy. The PES is fully integrated (OECD 1996) and provides the following services. First, it pays unemployment benefits. Second, it matches registered vacancies with registered job seekers, and third it pursues active labour market policies (ALMP). In this last function, the PES pays a hiring subsidy to firms that enter into an employment contract with a LTU worker. The hiring subsidy is paid when the match partners sign the contract and incur the training costs $t \geq 0$. The subsidy therefore compensates for the training costs. Training expenditures can be monitored by the PES without costs. Since the support of the distribution of training costs is not bounded from above, the PES establishes an upper bound H on the hiring subsidy so that the training costs of all matches with $t \leq H$ are fully subsidized, whereas matches with $t - H > 0$ have to finance the balance out of the match rent.

3. FILLED JOBS

Each match combines a vacant job with a job seeker. The partners of a new contact first determine the match specific training costs t. If t exceeds the reservation costs, the agents separate immediately. Otherwise, they negotiate the conditions of the employment contract and start production thereafter.

An employment contract $[w_i, w(x), R]$ has three components. The first is the outside wage w_i which is paid to the worker throughout the initial period, the training period. It depends on his status i as a job seeker - on whether he is a STU worker with $i = I$ or a LTU worker who has opted either for the passive, $i = P$, or the active search strategy (which combines the two methods of search), $i = S$. The second component of the contract is the match specific inside wage represented by the wage function $w : [R,1] \rightarrow \Re$. At the end of a period the productivity x, which is valid during the subsequent period, is revealed to the match. If $x \in [R,1]$, the match continues and the worker earns the bargained inside-wage $w(x)$. Therefore, after each shock to the match specific productivity, worker and firm renegotiate the conditions of the employment contract, especially the wage[2]. The third component defines the reservation threshold R at which the job will be destroyed.

Continuation periods. After the training period all jobs have the same productivity y. Shocks hit a match with probability $\lambda \geq 0$, are match specific, and manifest in the multiplica-

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[2] Mortensen/Pissarides (1999) and Pissarides (2000) present a discussion of objections against the plausibility of this assumption and the two-tier wage structure which results from the possibility of renegotiation.

tive productivity component *x*, which is a random variable with c.d.f. $G(x)$ defined on $x \in [\alpha, 1]$. Within each period only one shock can occur. Furthermore, shocks are iid.

Let $\Pi(x)$ be the present value of a filled job after the manifestation of a shock $x \in [\alpha, 1]$. Worker and firm, considering their reservation utility, are both interested in continuing the match as long as $\Pi(x) \geq 0$ and agree on job destruction as soon as $\Pi(x) < 0$. Since $\Pi(x)$ is a continuously monotonically increasing function of *x*, as will be shown below, a reservation threshold *R* exists, for which

$$\Pi(R) = 0 . \tag{4}$$

Only jobs with $x \geq R$ will be continued. We assume that the firm markets the output *yx* at the end of the period at the same time as it pays the bargained wage $w(x)$. Then the steady state equation for the present value $\Pi(x)$ of a filled job is

$$\Pi(x) = \rho\left\{yx - w(x) + \lambda \int_R^1 \Pi(h)\, dG(h) + (1-\lambda)\Pi(x)\right\} . \tag{5}$$

Flow and stock variables are discounted at the rate ρ, where $0 < \rho = 1/1 + r < 1$ with the interest rate $r > 0$. With probability λ the job is hit by a shock and changes into a state *h*. If $R \leq h \leq 1$ the match is continued and the continuation asset value becomes $\Pi(h)$. With probability $1-\lambda$ the match specific productivity does not change.

A worker who is employed at the match specific productivity *yx* gets the wage $w(x)$, and his human capital has the present value $W(x)$. The asset pricing equation for $W(x)$ is

$$W(x) = \rho\left\{w(x) + \lambda\left[\int_R^1 W(h)\, dG(h) + G(R)U_I\right] + (1-\lambda)W(x)\right\} . \tag{6}$$

With probability λ a shock occurs and the match draws the productivity *h*. If $h \geq R$, the value of the worker is $W(h)$ and the match continues. If, on the other hand, $h < R$, which occurs with probability $G(R)$, the job is destroyed, the worker becomes a short-term unemployed and the value of his human capital is U_I .

Training period. Firms choose the initial productivity when they set up the match and negotiate the outside wage. If the job seeker is a STU worker, the initial productivity is set at $x = 1$. Moreover, the STU do not generate training costs and therefore the initial value Π_I of a job filled by a STU worker is

$$\Pi_I = \rho\left\{y - w_I + \lambda \int_R^1 \Pi(h)\, dG(h) + (1-\lambda)\Pi(1)\right\}, \tag{7}$$

with w_I denoting the negotiated outside wage. If the match is not hit by a shock, the worker's productivity remains at $x = 1$ in the continuation period as well, and the filled job has the value $\Pi(1)$. The human capital of a worker who starts production as a STU worker is

$$W_I = \rho\left\{w_I + \lambda\left[\int_R^1 W(h)dG(h) + G(R)U_I\right] + (1-\lambda)W(1)\right\}, \tag{8}$$

where $W(1)$ denotes the value of the worker in the continuation period if no shock occurs.

The LTU find a vacancy either through random search or via the PES. When wage negotiations between a vacancy and a LTU worker start, jobs filled by the STU are already productive. Moreover, the LTU need a training period and, therefore, we assume that their initial productivity yz, with $z \le 1$, is lower than the initial productivity of a STU worker. The allocation of the training costs and the hiring subsidy is subject to negotiation, but the outside wage $w_i(t)$ and the initial value of the job $\Pi_i(t)$ depend only on t if t exceeds the subsidy bound H, where $i = P$ if the LTU worker has opted for the passive and $i = S$ if he has chosen the active search strategy.

For the sake of brevity, we present the asset equations only for the case where the training costs exceed the subsidy H. The indicator variable $\tau \in \{0,1\}$ takes on the value of one if the PES also subsidises the matches formed by random search, while $\tau = 0$ if H is paid only to matches arranged by the PES. Considering the status of the job seeker $i = P, S$, the present value of a job filled with a LTU worker is given by

$$\Pi_i(t) = \rho\left\{yz - w_i(t) + \lambda\int_R^1 \Pi(h)dG(h) + (1-\lambda)\Pi(1)\right\}, \tag{9}$$

where in (9) and also in (10) $H \le t \le T_P$ for $i = P$ and $\tau H \le t \le T_S$ for $i = S$.

Taking into account the negotiated outside wage $w_i(t)$ the present value of the worker's human capital during the training period is

$$W_i(t) = \rho\left\{w_i(t) + \lambda\left[\int_R^1 W(h)dG(h) + G(R)U_I\right] + (1-\lambda)W(1)\right\}. \tag{10}$$

4. JOB CREATION AND RESERVATION COSTS

All vacancies are advertised and registered, no vacancy specialises in one of the two search methods[3]. Entrance into the labour market is free for all vacancies, but open only at the beginning of a period. The flow of vacancies therefore persists until the present value of a vacancy is driven to zero, $V = 0$. Considering this infinitely elastic supply of vacancies, the *job creation* condition is $0 = -k + q_I \Pi_I + (1 - q_I)V_I$, where k denotes the flow costs for advertising and for registration with the PES. If there is no contact with a STU worker in the first stage of

[3] Specialisation may occur due to the heterogeneity of the job seekers or the jobs or because of increasing search costs. We assume that the search cost function of a vacancy with respect to the two search methods is sub-additive, so that, considering the asset value of a vacancy, it is advantageous for firms to offer vacancies through both channels.

the matching process - an event which has the probability $1-q_I$ - the vacancy takes on the value of its outside option $V_I > 0$.

There are three reasons for the existence of an outside option with the value V_I. First, vacancies are not specialised. Second, the matching process consists of three stages. A vacancy that is not filled during the first has the option to meet a LTU worker who is actively searching for a job or placed by the PES in the second or third stage of the matching process respectively. The value of this option is V_I**.** Third, the supply of vacancies is perfectly inelastic in the last two stages of the matching process.

The above *job creation* condition can also be interpreted as follows. Due to search costs, each successful match generates a positive rent, which is distributed between worker and firm through the wage. $\Pi_I - V_I$ is the firm's share of the rent of a match with a STU worker, Π_I is the value of the filled job, and V_I is the value of the outside option of the vacancy. The price which the firm pays for participating in the matching process is *k*, the *implicit* price for the first stage is $k - V_I$. Thus, the *job creation* condition states that the flow of vacancies into the labour market lasts until the implicit search cost a firm has to incur to take part in the first stage of the matching process equals its share of the match rent:

$$\frac{k - V_I}{q_I} = \Pi_I - V_I . \tag{11}$$

The option value V_I of a vacancy in the first stage of the matching process, when the search costs *k* are sunk, is

$$V_I = q_S V_S + [1 - q_S F(T_S)] Q V_P , \tag{12}$$

where q_S denotes the probability that the vacancy will be filled by a LTU worker who is actively seeking**.** V_S is the conditional expected value of a job which has contact with such a worker. If the vacancy does not meet an active LTU worker or if the training costs of the applicant exceed T_S – a composite event with the probability $1 - q_S F(T_S)$ – then the vacancy still has the third option to meet a LTU worker placed by the PES. The probability of a match with a LTU worker placed by the PES is *Q*, and the conditional option value of the job is V_P[4].

Reservation costs. The hiring subsidy of the PES compensates for the training costs up to the bound *H*. Matches with the LTU placed by the PES and with training costs *t* higher than *H* must finance the balance $t - H \geq 0$ out of the match rent. The allocation of the balance is part of the contract negotiations, and the value of the filled job, $\Pi_P(t)$, therefore depends on *t*. As will be shown, $\Pi_P(t)$ is a monotonically *increasing* function of *t*, while the net value of the job, $\Pi_P(t) + H - t$, is a contraction and fulfils the reservation property with respect to *t*. Hence, reservation costs T_P exist, with

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[4] Appendix AI contains the asset pricing equations for V_S and V_P.

$$T_P = \Pi_P(T_P) + H\,. \tag{13}$$

Match partners whose training costs are lower than T_P sign an employment contract while with $t > T_P$ they separate immediately.

A vacancy filled by a LTU worker who is actively searching has the value $\Pi_S(t)$ if the match draws training costs *t*, with $t - \tau H \geq 0$. In view of the third stage of the matching process, QV_P is the value of the outside option of the firm. Therefore, the job will only be filled if its net value is at least as high as the value of the option to meet a LTU worker placed by the PES, $\Pi_S(t) + \tau H - t \geq QV_P$. Since the net value of the job has the reservation property, reservation costs T_S also exist for the method of random search

$$T_S = \Pi_S(T_S) + \tau H - QV_P\,. \tag{14}$$

5. The Value of Unemployment and Wage Negotiations

A worker who has lost his job must decide between the active and the passive search strategy. Given the exogenous unemployment benefit *b* and the real return from leisure *l* the worker chooses the strategy which maximises the present-discounted value of his human capital U_I

$$U_I = \max\left\{\rho(b + l + U),\quad -c_I + p_I W_I + (1 - p_I)\rho(b + l + U)\right\}, \tag{15}$$

The choice set of the Bellman equation (15) contains two alternatives. First, if the worker prefers the passive strategy, he receives the unemployment benefit *b* and the imputed income from leisure activities *l*, while his human capital takes on the value *U*.[5] In the subsequent period he has to decide again whether to wait for a placement via the PES or to search and apply for a vacancy on the market. In the first case, the value of his human capital is U_P, in the second, it is U_S. The worker will opt for the search strategy that maximises the present value of his human capital so that

$$U = \max\left\{U_P, U_S\right\}. \tag{16}$$

Second, if the STU worker chooses to search randomly, he incurs search costs $c_I > 0$. With probability p_I, he will locate a vacancy, and his value is W_I. With probability $1 - p_I$ his search fails, he receives the unemployment benefit *b* and the real income from leisure *l*, and his human capital takes on the value *U*.

The present value of the human capital of a LTU worker who waits for a placement via the PES is

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5 For simplicity we assume that *b* is exogenous. The endogenisation of *b* (Mortensen / Pissarides 2001) lowers the incentive to search and thus strengthens the comparative static effects of *H*, which are shown in Section 7.

$$U_P = P\{(1-q_I)(1-q_S)\Big[F(H)W_P + \int_H^{T_P} W_P(t)dF(t) + [1-F(T_P)]\rho(b+l+U)\Big] + [q_I + (1-q_I)q_S]\rho(b+l+U)\} + (1-P)\rho(b+l+U) \tag{17}$$

If the worker is matched and if the vacancy for which he applies is not yet filled – the probability for this composite event is $P(1-q_I)(1-q_S)$ – the value put on the worker by the market is W_P provided that the subsidy compensates fully for the training costs, that is if $t-H \leq 0$. Otherwise, if the training costs exceed H but are lower than the reservation costs T_P, the integral in (17) denotes the expected value of the employed worker. If the training costs exceed T_P, vacancy and applicant separate, and the present value of the worker is $\rho(b+l+U)$ as in the cases where the vacancy is already filled or the LTU is not offered a vacancy by the PES.

If the LTU worker decides on the active search strategy, he will incur search costs $c_S > 0$. Considering the contact probability p_S generated by the search market, his present-discounted value is U_S with

$$U_S = -c_S + p_S\Big\{(1-q_I)\Big[F(\tau H)W_S + \int_{\tau H}^{T_S} W_S(t)dF(t) + [1-F(T_S)]U_P\Big] + q_I U_P\Big\} + (1-p_S)U_P \tag{18}$$

If the job search fails – either because the LTU worker is confronted with a vacancy already filled or because he incurs training costs that exceed T_S or because he does not find a vacancy – his value is equal to the value of the passive strategy U_P because placement via the PES is the final option which concludes the matching process.

Wage negotiations. Job search takes time and causes search costs. Therefore, each match appropriates a monopoly rent which is distributed between the match partners through the wage. The distribution rules are obtained according to the generalised Nash solution to a bargaining problem, with $\beta \in (0,1)$ denoting the bargaining strength of the job seeker.

If a STU worker meets a vacancy, the outside wage w_I for the initial period of the match is derived from the sharing rule

$$W_I - U_I = \frac{\beta}{1-\beta}(\Pi_I - V_I), \tag{19}$$

with V_I denoting the reservation value of the vacancy, which follows from the firm's option to fill the vacant job with a LTU worker.

If the vacancy meets a LTU worker, the sharing rule depends on the design of the ALMP, whether the PES compensates fully for the training costs, or whether the agents have to negotiate the allocation of the balance $t-H \geq 0$. For wage negotiations with a LTU worker who is randomly searching the sharing rule is

$$W_S(t) - U_S = \begin{cases} \frac{\beta}{1-\beta}[\Pi_S - QV_P], \text{ for } 0 \le t \le \tau H \\ \\ \frac{\beta}{1-\beta}[(\Pi_S(t) + \tau H - t) - QV_P], \text{ for } \tau H \le t \le T_S \end{cases} \tag{20}$$

where $W_S(t) - U_S$ is the job seeker's share of the rent, and QV_P is the reservation value of the vacancy given the third stage of the matching process.

For wage negotiations with a LTU worker placed by the PES the following sharing rule is implemented:

$$W_P(t) - U_P = \begin{cases} \frac{\beta}{1-\beta}\Pi_P, \text{ for } 0 \le t \le H \\ \\ \frac{\beta}{1-\beta}[\Pi_P(t) + H - t], \text{ for } H \le t \le T_P \end{cases} \tag{21}$$

where $\Pi_P(t) + H - t$ is the firm's share of the rent if $H \le t \le T_P$.

Taking into account the idiosyncratic shock $x \in [R,1]$, the value of a STU worker, U_I, and the fact that in equilibrium the asset price of a vacancy at the initial stage of the search process is $V = 0$, the sharing rule used for the negotiations with an insider is

$$W(x) - U_I = \frac{\beta}{1-\beta}\Pi(x). \tag{22}$$

Considering the asset pricing equations (5) – (10) and the sharing rules (19) – (22), we obtain

LEMMA 1 [BARGAINED WAGES]. *Given the reservation income rU_I of a STU worker and the asset values U_S and U_P of the LTU worker who prefer the active or the passive search strategy respectively, the agents negotiate the following inside and outside wages.*

(i) *The bargained inside wage at a match specific productivity $x \in [R,1]$ is*

$$w(x) = rU_I + \beta(yx - rU_I). \tag{23}$$

(ii) *A STU worker who makes a job-to-job transition and produces, in the initial period, with productivity $x = 1$ receives the outside wage*

$$w_I = w(1) - \beta V_I \rho^{-1}, \tag{24}$$

where $w(1)$ is the inside wage (23) for $x = 1$, and $\rho^{-1} = 1 + r$.

(iii) *If the PES subsidises the training costs, a LTU worker with human capital U_P placed by the PES receives the outside wage w_P with*

$$w_P = w(1) - \beta(1-z)y + (1-\beta)(U_P - U_I)\rho^{-1}\text{, for } t \leq H\text{,} \tag{25}$$

where yz, with $z \leq 1$*, is the flow output in the training period.*

If $H \leq t \leq T_P$*, the outside wage* $w_P(t)$ *in the training period is*

$$w_P(t) = w_P - \beta(t-H)\rho^{-1}. \tag{26}$$

(iv) *If* $t \leq \tau H$ *a LTU worker with human capital* U_S *who finds a job through random search receives the outside wage* w_S*:*

$$w_S = w(1) - \beta(1-z)y + (1-\beta)(U_S - U_I)\rho^{-1} - \beta Q V_P \rho^{-1}\text{, for } t \leq \tau H. \tag{27}$$

If the training costs exceed τH *the bargained wage is*

$$w_S(t) = w_S - \beta(t-\tau H)\rho^{-1}\text{, for } \tau H \leq t \leq T_S. \tag{28}$$

As equation (23) shows, the inside wage equals the reservation income of the worker plus a share of the current match rent that depends on his bargaining strength β. As (19) shows, the value of the outside option V_I diminishes the rent of a match with a STU worker, and, as a consequence, reduces the share of the current rent (24) a STU worker can appropriate in the contract negotiation. The time of the model is discrete. While the reservation value of the vacancy refers to the beginning of the period, wages are paid at the end; V_I, therefore, is discounted in (24) to the end of the period.

The lower the productivity $z \leq 1$ of a LTU worker during the training period, the lower the bargained outside wages, as equations (25) and (27) show. Moreover, training costs higher than *H* are partially passed on to the worker, so that the outside wages (26) and (28) respectively fall monotonically with *t*.

Finally, the outside wages (25) and (27) depend on the balance of the present values of a LTU and a STU worker, $U_i - U_I$, $i = P, S$, and hence on the search strategies the unemployed prefer. To determine the signs and the magnitudes of the rents $U_i - U_I$, $i = P, S$, we first have to explain which search strategies the LTU and the STU workers use in equilibrium.

Choice of the search strategy. If $U_I > \rho(b + L + U)$, then all STU workers immediately search for a new job. The number of active job seekers S_I among the STU rises, the tightness θ_I of the search market in the first stage of the matching process diminishes, and the transition rate p_I falls. The adjustment comes to an end either because the gains from private job search are driven to zero, as $U_I = \rho(b + L + U)$, or because the total inflow searches randomly for a job, so that $S_I = I$. In the following, we look at the first case and assume that in equilibrium the gains from search vanish so that $U_I = \rho(b + L + U)$ and $S_I \leq I$.

The LTU choose the active search strategy if $U_S > U_P$. The number of active job seekers *S* increases, the tightness θ_S of the labour market in the second stage of the matching process decreases, and the contact probability p_S diminishes until either all workers in the unemployment pool u search actively for a job, so that $S = u$, or the gains from private job search vanish, so that $U_S = U_P$ and $S \le u$. In the following, we investigate the second case and assume that, in equilibrium, $U_S = U_P$ and $S \le u$.

With $U_S = U_P$, the LTU are indifferent to the strategies of search, and from the wage equations (25) and (27) it follows for the outside wage of a random searcher among the LTU: $w_S = w_P - \beta Q V_P \rho^{-1}$, for $t \le \tau H$. Moreover, with $U_S = U_P$, it suffices to determine the sign and the magnitude of the rent $U_P - U_I$. If in equilibrium the STU are indifferent to the active and the passive search strategy, then the differential rent $U_P - U_I$ can be derived from the asset equation (17), the sharing rule (21), and equation (A1) for the option value V_P of a vacancy that, in view of the third stage of the matching process, expects to meet a LTU worker placed by the PES (s. App. III):

$$U_P - U_I = \frac{\beta}{1-\beta} \frac{P(1-q_I)(1-q_S)V_P}{[1 - P(1-q_I)(1-q_S)F(T_P)]}. \tag{29}$$

If the STU are indifferent to both search strategies then the differential rent (29) is strictly positive. The reason for this is the *reaction time* of the PES: the PES is available to the LTU whereas it is not to the STU who have just lost their jobs; the STU must wait at least one period - after the PES has reviewed their claims, has registered and referred them to the job placement service - until the first job offer arrives. During this time, which we assume lasts one period or more, the STU have to rely on their own search efforts.

The differential rent (29) increases together with the probability P for a contact via the PES, the reservation costs T_P, the probability $(1-q_I)(1-q_S)$ of finding a job that is not yet filled by one of the active job seekers, and with the option value V_P.

6. THE VALUE OF A FILLED JOB, WAGE DISPERSIONS AND JOB DESTRUCTION

With the wage equations from Lemma 1, the asset equations from section 3, and the condition of the reservation productivity (4), we can now derive the value of a filled job.

LEMMA 2 [FILLED JOBS]. (i) *The continuation value of a filled job producing with the idiosyncratic productivity* $x \in [R,1]$ *is*

$$\Pi(x) = (1-\beta)y\frac{x-R}{\lambda + r}. \tag{30}$$

(ii) *Taking into account the reservation value* V_I*, a job filled by a STU worker has the present value*

$$\Pi_I = \Pi(1) + \beta V_I, \tag{31}$$

where $\Pi(1)$ *is the continuation value (30) for the match productivity* $x = 1$.

(iii) *A job filled by a LTU worker who is placed and whose training costs are subsidised by the PES has the value*

$$\Pi_P = \Pi(1) - \rho(1-\beta)(1-z)y - (1-\beta)(U_P - U_I), \text{ for } t \le H. \tag{32}$$

A job filled by a subsidised LTU worker whose training costs exceed H has the present value

$$\Pi_P(t) = \Pi_P + \beta(t-H), \textit{for } H \le t \le T_P. \tag{33}$$

(iv) *Since the LTU workers are indifferent between the two search strategies, taking into account the reservation value QV_P, a job filled by a worker who is actively searching has the asset price*

$$\Pi_S = \Pi_P + \beta QV_P, \text{ for } t \le \tau H. \tag{34}$$

For training costs t with $\tau H \le t \le T_S$ we finally obtain

$$\Pi_S(t) = \Pi_S + \beta(t - \tau H). \tag{35}$$

From the value equations for the filled jobs, we can derive the reservation costs T_P and T_S.

Lemma 3 [Reservation Costs]. (i) *The reservation costs T_P which are applied to the LTU who are placed by the PES follow from (33) together with $\Pi_P(T_P) + H - T_P = 0$:*

$$T_P = \frac{\Pi_P}{1-\beta} + H. \tag{36}$$

From the asset pricing equations (33) - (35) and $T_S = \Pi_S(T_S) + \tau H - QV_P$ we can derive the reservation costs for the method of random search

$$T_S = T_P - (1-\tau)H - QV_P. \tag{37}$$

(ii) *As a consequence of the fact that $T_P - T_S = (1-\tau)H + QV_P > 0$, the percentage of LTU who cannot be placed via the search market, is always higher than the percentage of LTU who cannot be placed via the PES: $1 - F(T_S) > 1 - F(T_P)$.*[6]

The dispersions of the outside wages of the LTU during the training period depend on the method of search and the distribution of the training costs.

Lemma 4 [Wage Dispersions]. (i) *The dispersions of the outside wages of the LTU are defined on the ranges $[w_P(T_P), w_P]$ and $[w_S(T_S), w_S]$, where $w_i(T_i)$ is the lowest and w_i is the highest wage of the respective wage dispersion, $i = P, S$. From Lemma 1 and Lemma 3,*

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[6] With Lemma 2 and Lemma 3 the option values of a vacancy V_P and V_S are only functions of the subsidy limit *H*, the reservation costs T_P and T_S, the tightness Θ, and the design $\tau \in \{0,1\}$ of the ALMP (s. App. AI).

taking into account that in equilibrium $U_S = U_P$, *it follows that* $w_P(T_P) = w_S(T_S)$ *and* $w_P - w_S = \beta Q V_P \rho^{-1} > 0$.

(ii) *The average wages of the normalized dispersions are given by* $\overline{w}_P = [F(H)w_P + \int_H^{T_P} w_P(t)\,dF(t)] / F(T_P)$ *and* $\overline{w}_S = [F(\tau H)w_S + \int_{\tau H}^{T_S} w_S(t)\,dF(t)] / F(T_S)$. *If the training costs are exponentially distributed, then* $\overline{w}_P > \overline{w}_S$.

The *job destruction* rule can be derived by evaluating the asset equation (5) at the reservation threshold $x = R$. Taking into account the wage equation (23) we obtain:

$$0 = R - \frac{rU_I}{y} + \frac{\lambda}{\lambda + r}\int_R^1 (h - R)\,dG(h). \tag{38}$$

In order to close the model, we still have to determine the reservation income of a STU worker, rU_I, and the transition probabilities of the method of random search.

In equilibrium the STU and the LTU, by assumption, are indifferent to the active and the passive search strategies so that $U_I = \rho(b + l + U)$ and $U_P = U_S = U$. With these conditions, the reservation income of a STU worker is equal to the sum of the unemployment benefit, the real return of leisure and the differential rent $U_P - U_I$:

$$rU_I = b + l + (U_P - U_I). \tag{39}$$

Taking account of the fact that job seekers in equilibrium are indifferent between the two search strategies, we finally obtain the transition probabilities generated by the search market, $p(\theta_I)$ and $p(\theta_S)$, as follows.

Lemma 5 [Random Search]. (i) *From the Bellman equation (15) and* $U_I = \rho(b + l + U)$ *it follows that, in equilibrium, the expected search costs of a STU worker who is randomly searching are equal to his share of the match rent,* $c_I / p_I = W_I - U_I$. *From this, together with the sharing rule (19) and the asset equation (31), we obtain*

$$\frac{c_I}{p(\theta_I)} = \frac{\beta}{1-\beta}[\Pi(1) - (1-\beta)V_I]. \tag{40}$$

(ii) *Using the assumption* $U_P = U_S = U$ *and the asset equation (18), it follows that, in equilibrium, the expected search costs of a LTU worker who is randomly searching equal his expected share in the match rent:* $c_S/(1 - q_I)p_S = F(\tau H)(W_S - U_S) + \int_{\tau H}^{T_S}(W_S(t) - U_S)dF(t)$. *From this equilibrium condition we obtain with respect to the sharing rule (20) and the option value (A2)*

$$\frac{c_S}{[1 - q(\theta_I)]p(\theta_S)} = \frac{\beta}{1-\beta}[V_S - QV_P]. \tag{41}$$

The equilibrium of the search model consists of solutions $[\Pi(1), \Pi_P, \Theta, \theta_I, \theta_S, R, T_P, T_S, u]$ to the equations (A5) – (A12) in Appendix II and the equilibrium unemployment (3). The comparative static effects of the hiring subsidy are indeterminate as a consequence of the multiplicity of the channels through which ALMP work. Which effect dominates is an empirical question. We therefore have carried out a series of numerical experiments.

7. SIMULATION

Parameters and matching functions. The choice of the baseline parameter values, Table A1 in Appendix II, is made with respect to the design of the experiments of Mortensen/Pissarides (1999, 2001) and the restraint that in equilibrium the number of active job seekers, S_I and *S,* have to be "interior solutions" to the model**.**

The bargaining power of the workers is $\beta = 0.50$, the marginal product of a job at full productivity is $y = 100$. During their training period, the LTU produce a marginal product of $yz = 60$; the value of leisure is $l = 30$, UI benefits are $b = 30$; the real interest rate *r* is 2 %; the probability of a productivity shock λ is 10 %; the search costs are $c_I = 40$ and $c_S = 25$, and the recruiting costs of a vacancy amount to $k = 30$.

The distribution function $G(x)$ of the productivity shocks is assumed to be uniform on $[\alpha, 1]$, with the lower support $\alpha = 0.65$. Training costs $t \geq 0$ are exponentially distributed with mean $1/\delta = 15$.

The matching functions of the PES and the search market are of the Cobb Douglas type (Petrongolo/Pissarides 2001). For a given vacancy the probabilities of a contact with a job seeker are

$$\text{PES:} \qquad Q(\Theta) = ef * (1/\Theta)^{1-\Phi} \tag{42}$$

$$\text{Search market:} \qquad q(\theta) = d * (1/\theta)^{1-\phi}. \tag{43}$$

The values of the "total factor productivities" of the basic scenario are $ef = d = 0.30$; for the elasticities of the job matches *M* and *m* with respect to vacancies we use $\Phi = 4/5$ and $\phi = 1/5$ respectively. Thus, among the arguments of the matching technology of the PES, the vacancies dominate, while on the search market the active job seekers are the dominating input factor.

Indicators. The time period which corresponds to the duration of a model period is, as we assume, the yearly quarter. The following indicators are used to evaluate the simulations: (1) Quarterly unemployment rate *u* in percent; (2) quarterly unemployment incidence λG in percent; (3) unemployment duration of active and passive LTU job seekers d_S and d_P respectively in quarters; (4) ratio of the STU making job-to-job transitions, $\sigma_I p_I = p_I S_I / u * 100$; (5) fraction of active job seekers among the LTU, $\sigma_S = S/u * 100$; (6) fraction of active job seekers among the inflowing STU, $S_I / I * 100$; (7) the outside wage w_I negotiated by the

STU making job-to-job transitions; (8) the indicator for the outside wages of the LTU, which equals the mean of the distribution of outside wages, s. Lemma 4, as a percentage of the outside wage of the STU, $wIP = \overline{w}_P / w_I * 100$. $100 - wIP$ denotes the average wage penalty which a LTU worker placed by the PES must accept due to his low productivity and the training costs. (9) fraction of the LTU, $LTU = (1 - u_S / u) * 100$; (10) placement rate of the PES, *PES* (see App. II for a definition); (11) UI benefits as a percentage of the net product, $PLMP = u\,b / np * 100$. The net product is $np = (1-u) y \int_R^1 x\,dG(x) / [G(1) - G(R)]$, where the term $(1-u)y$, which denotes the net product for $x = 1$, is weighted with the conditional expected value of the productivity parameter $x \geq R$. (12) Expenses of the PES for active labour market policies in percent of the net product, *ALMP* (s. App. II).

The results of the simulation with the upper bound H for the hiring subsidy are shown graphically in the Appendices IV – VI. We distinguish between an ALMP design which supports only placements by the PES (regime $\tau = 0$) and a policy which gives equal support to both search methods (regime $\tau = 1$). Appendix IV shows the results for both regimes ($\tau = 0$ and $\tau = 1$) with the baseline parameter values from Tab. A1. Appendix V describes the results for $\tau = 0$ at different values for the workers' bargaining strength ($\beta = 0.45$ and $\beta = 0.55$). Appendix VI depicts the results for $\tau = 0$ at varying values for the matching productivity of the PES ($ef = 0.25$ and $ef = 0.35$). The App. V und VI show clearly that the results 1 – 4 are also stable with shocks which affect central model parameters. Moreover, App. V und VI show additional results which are based on the effects of the policy parameter *ef* and the structural parameter of the wage bargaining system β.

Throughout the following paragraphs we compare the results of our numerical experiments with the corresponding data of the OECD (2001 a, b).

Result 1. *The figures, App. IV – VI, show that consistent with Mortensen/Pissarides (1999, 2001) the hiring subsidy H increases the equilibrium rate of unemployment u.*

For example in the regime $\tau = 0$, where only PES placements are subsidised u increases from 7,4 % ($H = 0$) to 8,4 % ($H = 30$).

In comparison: in the year 2000 the rate of unemployment in the OECD was in total 6.4 % and in the EU 8.3 %.

In the standard model of Mortensen/Pissarides the hiring subsidy lowers the costs of job creation, so that on the one hand job creation is stimulated and the duration of unemployment falls. On the other hand the unemployment incidence increases. Because of the increasing tightness the opportunity costs of a filled job rise and the match partners separate faster. The second effect outweighs the first so that overall employment decreases.

In our model four factors have an affect on equilibrium unemployment, s. equation (3). First the incidence, second the length of the unemployment spells of active and passive long-term unemployed, third the ratio of job-to-job transitions and forth the share of passive job seekers among the LTU workers.

The main causes for the positive correlation between ALMP and the unemployment rate in our model are the following. First, firms and workers only enjoy the benefits of the hiring subsidy if they are matched by the PES (regime $\tau = 0$). The hiring subsidy therefore increases the opportunity costs of a start-up in the first and second stage of the search process. The consequences are that the fraction of active job seekers among the LTU, σ_S, and the ratio of job-to-job transitions, $\sigma_I p_I$, fall or that the fractions of those STU and LTU who prefer to wait for a placement by the PES increase. Second, the hiring subsidy reduces duration of unemployment but only the duration of the active job seekers, d_S, while the average spell length d_P of an unemployed worker who decides on the passive search strategy increases. The reason for this is that the growing number of passive job seekers is concentrated in the third stage of the matching process. As a result the tightness between the registers of the PES decreases, the *reaction time* $1+1/P(\Theta)$ rises and the probability of a successful match falls. It is not surprising that the duration of the unemployment spell of the active job seeker falls because, on the one hand, the supply of vacancies rises due to the hiring subsidy and, on the other hand, the number of active job seekers falls.

ALMP thus increases not only the job destruction rate but also the duration of the unemployment spell of passive job seekers as well as their fraction of all unemployed.

The development of the fraction of the LTU who are randomly searching depends on the design of the ALMP. In the regime $\tau = 0$, where only PES placements are subsidised, the fraction of active job seekers among the LTU decreases. As a result the fraction of passive job seekers rises in $\tau = 0$ not only among the STU but also among the LTU.

Result 2. *Although the hiring subsidy raises the fraction of active job seekers among the LTU, the symmetrical labour market policy ($\tau = 1$) lowers overall employment. The reasons are: First the symmetrical labour market policy increases the duration of job search of the active job seekers. Second it leads to a crowding-out of active job seekers among the STU and reduces the job-to-job transitions even below the level reached in the regime $\tau = 0$. Third, as in the regime $\tau = 0$ d_P increases. Nevertheless, due to the growing number of active job seekers among the LTU, the equilibrium rate of unemployment does not increase as much as it does in the regime $\tau = 0$.*

Result 3. *Active labour market policy has the following additional consequences (in $\tau = 0$): (1) The fraction of job seekers threatened with long-term unemployment or being long-term unemployed (LTU) – in the model those are LTU workers who are unemployed for longer than 3 months (1 quarter) – increases from 72.0 % ($H = 0$) to 73.6 % ($H = 30$). (2) The costs for PLMP increase from 2.7 % ($H = 0$) to 3.1 % ($H = 30$) of the net product, while the costs for ALMP ($H = 30$) reach the value of 0.3 % of the net product.*

In comparison: in the year 2000 the incidence of job seekers threatened with long-term unemployment or being long-term unemployed (3 months and over) was 65.3 % of total unemployment in the OECD and 75.6 % in the EU. Moreover, in 1999 the average OECD member incurred costs for UI benefits of 1.0 % of the GDP as well as costs for "subsidies to regular employment in the private sector" of 0.1 % of the GDP.

> **Result 4.** *Without ALMP ($H = 0$) the LTU placed by the PES must accept on average a 5.6 % wage penalty compared to a STU worker when making a job-to-job transition. ALMP ($H = 30$) turn this penalty into an advantage of 3.5 % for the LTU under the regime $\tau = 0$.*

In comparison: based on the first seven rounds of the British Household Panel Survey, Arulampalam (2001) estimates that, after an unemployment spell, a worker must accept a wage penalty of 5.7 % compared to making a job-to-job transition[7].

The App. V und VI show clearly that the results 1 – 4 are also stable with shocks, which affect central model parameters. In addition the graphs depict two further interesting effects.

> **Result 5.** *The more effective the matching service of the PES – measured by the total factor productivity ef of the PES matching function under the regime $\tau = 0$ and without ALMP ($H = 0$) – the higher equilibrium unemployment is.*

The reasons are: first the more effective job placement service of the PES raises the opportunity costs of the filled jobs and therefore the incidence. While a job with $ef = 0.30$ has a mean durability of $1/\lambda G(R)*100 = 36$ quarters or 9.0 years, the durability falls to 8.2 years for $ef = 0.35$. Second, the fraction of active job seekers among the STU and thus the ratio of the STU making job-to-job transitions decreases with increasing *ef*. Third, even though the higher productivity of the PES lowers the unemployment duration of both search strategies – for the passive strategy the duration falls from 5 to 4.8 quarters, for the active from 1.8 to 1.6 quarters – the first two negative effects outweigh the positive third effect.

Why does the duration of the unemployment spells decrease? The fact that d_P falls is obviously due to the higher productivity of the PES. The decrease of d_S results from the reduction in the number of the active job seekers among the LTU. This improves the chances of the remaining searchers who stick to their search strategy.

Of course the fraction of PES matches, and thus the success which the PES will claim, grows with the effectiveness of its placement service.

> **Result 6**. *The increase in the bargaining power of the workers as shown in App. V from $\beta = 0.50$ to $\beta = 0.55$ lowers the durability of a job from 9.0 to 8.7 years and increases the duration of the unemployment spells – the duration rises by about 18 days: d_P increases from 5.0 to 5.2 quarters and d_S from 1.8 to 1.9 quarters.*
>
> *Although the durability of a job falls and the length of an unemployment spell rises overall employment increases.*

Why? First, the higher bargaining power of the workers increases the share of the match rent appropriated by the applicants who are actively searching and decreases the profit share of the firms. Second, firms react to the smaller profit share with a lower supply of vacancies,

[7] This wage penalty increases to 14 % in the fourth year after the unemployment spell and then decreases again (Arulampalam 2001).

the transition rates of the unemployed decrease and, therefore, durations increase. Third, because of the increasing gains from active job search the fraction of active job seekers among the STU and the LTU as well as the proportion of job-to-job transitions grow. Because the duration of the unemployment spell of the active job seekers is less than the unemployment duration of the passive unemployed, $d_P - d_S = 3.3$ quarters ($H = 0$ and $\beta = 0.55$), equilibrium unemployment decreases, even though the incidence and the duration increase.

8. SUMMARY

This paper presents a search model in discrete time. Job seekers can choose between two methods of search, matching by the PES, where firms register their vacancies, and random search on the search market, where firms advertise vacancies. The matching process includes three stages. In the first only the active job seekers among the STU search randomly for a vacancy. The STU have lost their job at the end of the previous period and, therefore, of all the unemployed possess the best information about current labour market conditions. Their applications are more targeted and reach the firms earlier than the applications of all the other unemployed. In the second stage the active job seekers among the LTU apply for jobs, and finally, in the third stage, also those LTU who are sent by the PES. Firms prefer applications from the STU, not only because they arrive first, but also because unlike the LTU they immediately work with full productivity and do not generate training costs. The PES subsidises the training costs with a hiring subsidy. Two regimes are compared. Under one regime only the matches created by the PES are subsidised, under the other the subsidy is paid for each match with a LTU worker, irrespective of the method of search. Under both regimes the unemployment rate increases with an increasing hiring subsidy. The reasons are the increasing job destruction rate, the decreasing fraction of active job seekers among the STU and of job-to-job transitions, and the *increasing* duration of unemployment of the passive job seekers.

In contrast to the standard search model, the ratio of active job seekers is endogenous in our model. Therefore, an increase in the bargaining strength of the job seekers has three consequences. First, as in the standard model, the share of the match rent appropriated by the workers increases, while the profit share decreases. As a consequence the job destruction rate increases and the supply of vacancies is reduced. Second, a lower supply of vacancies reduces the transition rates into employment and durations grow. Third, the fact that unemployment does not increase but decreases is a consequence of the growing rate of active job seekers and of the fact that active in comparison with passive job seekers have a much shorter unemployment duration. The reason that the unemployed switch to the active search strategy and that the rate of passive job seekers falls is, naturally, the higher share of the match rent and the higher wages.

Of course, the PES can increase its placement success by improving the effectiveness of its matching service. Nevertheless, the job destruction rate will increase and the fraction of active job seekers among the STU will decrease so that the improved effectiveness of the PES will lead to an increase in equilibrium unemployment, although the unemployment duration for both groups of jobseekers, the passive and the active, is reduced.

The economic policy consequences of the model are clear: the effects of ALMP and profiling techniques to increase the effectiveness of the state placement service depend on the target group. For unemployed with low private search costs compared to their productivity, not only the policy instruments of ALMP but also the actual job placement activities of the PES are counterproductive. On the other hand the instruments of ALMP and the placement service of the PES have a stimulating effect in job creation for target groups with such high private search costs that in equilibrium without policy it is not worthwhile for these groups to actively search for a job. But policy makers have to take into account that despite their stimulating effects these instruments of ALMP reduce aggregate employment.

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APPENDIX I

Option values V_P and V_S. 1. When firms decide whether to post a vacancy they know the c.d.f. of the training costs $F(t)$, the reservation costs T_i and the conditions for PES subsidies $\tau \in \{1,0\}$. Before the training costs are revealed the asset value of a vacancy expecting a contact with a LTU worker placed by the PES is

$$V_P = \int_0^H \Pi_P \, dF(t) + \int_H^{T_P} [\Pi_P(t) + H - t] dF(t). \tag{A1}$$

If the training costs of the LTU are fully subsidised, the job has the value Π_P. The second term in (A1) denotes the expected value of the job if the training costs are higher than the subsidy bound *H* but below the reservation costs T_P. Finally, if the training costs exceed T_P, the match partners separate immediately. Analogously, before training costs are known the conditional option value of a vacancy that meets an active job seeker among the LTU is

$$V_S = \int_0^{\tau H} \Pi_S \, dF(t) + \int_{\tau H}^{T_S} [\Pi_S(t) + \tau H - t] dF(t) + \int_{T_S}^{\infty} Q V_P \, dF(t), \tag{A2}$$

where $\tau = 1$ if the PES also subsidises the matches formed by random search, otherwise $\tau = 0$. If the match specific training costs of the job seeker exceed T_S, the agents separate and, in view of the third stage of the matching process, the vacancy takes on the value of the outside option QV_P.

2. With respect to the asset equations (32) – (35) and Lemma 3 the option values (A1) and (A2) of a vacancy transform to

$$V_P(T_P, H) = (1-\beta)[F(H)(T_P - H) + \int_H^{T_P} (T_P - t) dF(t)] \quad \text{and} \tag{A3}$$

$$V_S(\Theta, T_P, T_S, H, \tau) = (1-\beta)\left[F(\tau H)(T_S - \tau H) + \int_{\tau H}^{T_S} (T_S - t) dF(t)\right] + Q(\Theta) V_P(T_P, H) \tag{A4}$$

APPENDIX II

The model equations in implicit form are:

$$J^1(\Pi(1), \Pi_P, \Theta, \theta_I, \theta_S, R, T_P, T_S; H) \equiv \Pi(1) - (1-\beta) y \frac{1-R}{\lambda + r} = 0 \tag{A5}$$

$$J^2(\cdot) \equiv \Pi(1) - \rho(1-\beta)(1-z)y - \Pi_P - \frac{\beta P(1-q_I)(1-q_S)}{1 - P(1-q_I)(1-q_S)F(T_P)} V_P(T_P, H) = 0 \tag{A6}$$

$$J^3(\cdot) \equiv \Pi_P - (1-\beta)(T_P - H) = 0 \tag{A7}$$

$$J^4(\cdot) \equiv \frac{k - V_I(\Theta, T_P, T_S, \theta_S, H, \tau)}{q_I(\theta_I)} - [\Pi(1) - (1-\beta) V_I(\Theta, T_P, T_S, \theta_S, H, \tau)] = 0 \tag{A8}$$

$$J^5(\cdot) \equiv R - \frac{b}{y} - \frac{\Pi(1) - \rho(1-\beta)(1-z)y - \Pi_P}{(1-\beta)y} + \frac{\lambda}{\lambda + r} \int_R^1 (h - R) dG(h) = 0 \tag{A9}$$

$$J^6(\cdot) \equiv T_P - T_S - (1-\tau)H - Q(\Theta) V_P(T_P, H) = 0 \tag{A10}$$

$$J^7(\cdot) \equiv \frac{c_I}{p(\theta_I)} - \frac{\beta}{1-\beta}[\Pi(1) - (1-\beta) V_I(\Theta, T_P, T_S, \theta_S, H, \tau)] = 0 \tag{A11}$$

$$J^8(\cdot) \equiv \frac{c_S}{(1-q_I)p(\theta_S)} - \frac{\beta}{1-\beta}[V_S(\Theta, T_P, T_S, H, \tau) - Q(\Theta) V_P(T_P, H)] = 0\,. \tag{A12}$$

Tab. A1: The baseline parameter of the model

β	r	λ	y	z	l	b	k	c_I	c_S	$1/\delta$	α	ef	d	Φ	ϕ
0.50	0.02	0.10	100	0.60	30	30	30	40	25	15	0.65	0.30	0.30	4/5	1/5

The indicators *PES* and *ALMP* are defined as follows:

$$PES = \frac{P(1-q_I)(1-q_S)F(T_P)(u-p_S S)}{u_S} * 100 \tag{A13}$$

$$ALMP = \frac{ALP + ALS}{np}, \text{ where} \tag{A14}$$

$$ALP = P(1-q_I)(1-q_S)F(T_P)(u-p_S S)\left[\int_0^H t\,dF(t) + \int_H^{T_P} H\,dF(t)\right]\Big/F(T_P)$$

$$ALS = [u_S - P(1-q_I)(1-q_S)F(T_P)(u-p_S S)]\left[\int_0^{\tau H} t\,dF(t) + \int_{\tau H}^{T_S} \tau H\,dF(t)\right]\Big/F(T_S).$$

Appendix III

Proof of Lemma 1. (i) Write the sharing rule used for the negotiations with an insider (22) as

(P1) $$(1-\beta)U_I = (1-\beta)W(x) - \beta\Pi(x).$$

Substitute $\Pi(x)$ and $W(x)$ with the asset pricing equations (5) and (6) out of (P1) and the inside-wage (23) follows.

(ii) From the sharing rule used for the negotiations with the STU (19) follows

(P2) $$(1-\beta)U_I - \beta V_I = (1-\beta)W_I - \beta\Pi_I.$$

Now the outside-wage (24) for STU workers follows from (P2), the asset pricing equations (7) and (8) and (P1).

(iii) Write the sharing rule (21) as

(P3) $$(1-\beta)U_P - \beta(t-H) = (1-\beta)W_P(t) - \beta\Pi_P(t).$$

Substitute the values of the filled job and the employed worker with (9) and (10) out of (P3), and take account of (P1) and (23) to get the wage equation

(P4) $$w_P(t) = w(1) - (1-z)\beta y + (1-\beta)(U_P - U_I)\rho^{-1} - \beta(t-H)\rho^{-1}.$$

The wage equations (25) and (26) follow from (P4). Notice that the last term on the RHS of (P4) is equal to zero for $t \le H$.

(iv) Like in (iii) the wage equations (27) and (28) follow from the asset pricing equations (9) and (10), (P1) and the sharing rule (20), which we can write as

(P5) $$(1-\beta)U_S - \beta(t - \iota H) - \beta Q V_P = (1-\beta)W_S(t) - \beta \Pi_S(t).$$

Proof of equation (29). Rearrange the asset pricing equation (17), and take account of the equilibrium condition $U_I = \rho(b+U)$ to get

(P6) $$\begin{aligned}(U_P - U_I)[1 - P(1-q_I)(1-q_S)F(T_P)] = \\ P(1-q_I)(1-q_S)\left[F(H)(W_P - U_P) + \int_H^{T_P}(W_P(t) - U_P)dF(t)\right]\end{aligned}$$

Substitute the sharing rule (21) into the worker's share of the match rent on the RHS of (P6) and take account of the asset equation of the outside option (A1), to find the equation of the differential rent (29).

Proof of Lemma 2. (i) Equations (4) and (5) imply $0 = yR - w(R) + \lambda \int_R^1 \Pi(h)dG(h)$ and $(\lambda + r)\Pi(x) = yx - w(x) + \lambda \int_R^1 \Pi(h)dG(h)$. From these two equations together with the wage equation (23) the statement follows.

(ii) Insert the wage equation (24) into the asset equation (7) and take account of equation (5) to derive the asset pricing equation (31).

(iii) The asset pricing equations (32) and (33) follow from substituting the wage equation (P4) into (9) and rearranging terms with respect to the asset equation (5).

(iv) Similar to the above argument we can derive the asset pricing equations (34) and (35) from (9) by taking into account the wage equations (27) and (28).

Proof of Lemma 3. (i) Write the asset equation (33) as $\Pi_P(T_P) + H - T_P = \Pi_P - (1-\beta)(T_P - H)$, and take account of the condition of the reservations costs, $\Pi_P(T_P) + H - T_P = 0$.

Write (34) and (35) as $\Pi_S(T_S) + H - T_S = \Pi_P + \beta Q V_P - (1-\beta)(T_S - \iota H)$, take account of $\Pi_S(T_S) + H - T_S = QV_P$ to derive $T_S = \Pi_P/(1-\beta) + \tau H - QV_P$. Substitute (36) into the last equation and the statement follows.

Proof of Lemma 4. (i) Substitute $w_S = w_P - \beta Q V_P \rho^{-1}$ into the wage equation (28) to get $w_S(T_S) = w_P - \beta(T_S - \tau H + QV_P)\rho^{-1}$. By Lemma 3 $T_S - \tau H + QV_P = T_P - H$. Therefore we can conclude taking into account the wage equation (26): $w_S(T_S) = w_P - \beta(T_P - H)\rho^{-1} = w_P(T_P)$.

(ii) First we define the auxiliary functions $z(x)$ und $K(x,\tau)$, $x \in [0, T_P - H)$, as

(P7) $$z(x) = T_P - H - x$$

(P8) $$K(x,\tau) = x + \frac{\int_{\tau H}^{z(x)+\tau H}(t - \tau H)f(t)dt}{F(z(x)+\tau H)}.$$

$K(x,\tau)$ is continuously differentiable on $[0,T_P-H)$, if the p.d.f. of the training costs $f(t)$, $t>0$, is differentiable.

Inserting the wage equations (26) and (28) into the expectations of the wage distributions, $\overline{w}_P$ and $\overline{w}_S$, and taking account of $w_S = w_P - \beta QV_P\rho^{-1}$ and (37), we can write $\overline{w}_P$ and $\overline{w}_S$ with respect to (P8) as

(P9) $$\overline{w}_P = w_P - \beta K(0,1)\rho^{-1}$$

(P10) $$\overline{w}_S = w_P - \beta K(QV_P,\tau)\rho^{-1}.$$

Now, (P9) and (P10) imply: $\overline{w}_P > \overline{w}_S \quad\Leftrightarrow\quad K(0,1) < K(QV_P,\tau)$, where $\tau\in\{0,1\}$.

Assume that

(P11) $$K(x,\tau) > K(0,\tau), \quad \text{and}$$

(P12) $$K(0,0) \geq K(0,1),$$

Assume $\tau=1$, then the statement follows from (P11). For $\tau=0$ the inequalities (P11), (P12) and $QV_P>0$ imply that $K(QV_P,0) > K(0,0) \geq K(0,1)$. Using the inequalities again the statement follows.

If the training costs are exponentially distributed, $x\in[0,T_P-H)$, and $H\geq 0$, then the inequalities (P11) and (P12) hold.

1. Let $f(t)=\delta e^{-\delta t}$, $\delta>0$, then

(P13) $$K(x,\tau) = x + \frac{e^{-\delta\tau H}\left[1-(1+\delta z(x))e^{-\delta z(x)}\right]}{\delta\left(1-e^{-\delta\left(1-e^{-\delta(z(x)+\tau H)}\right)}\right)}$$

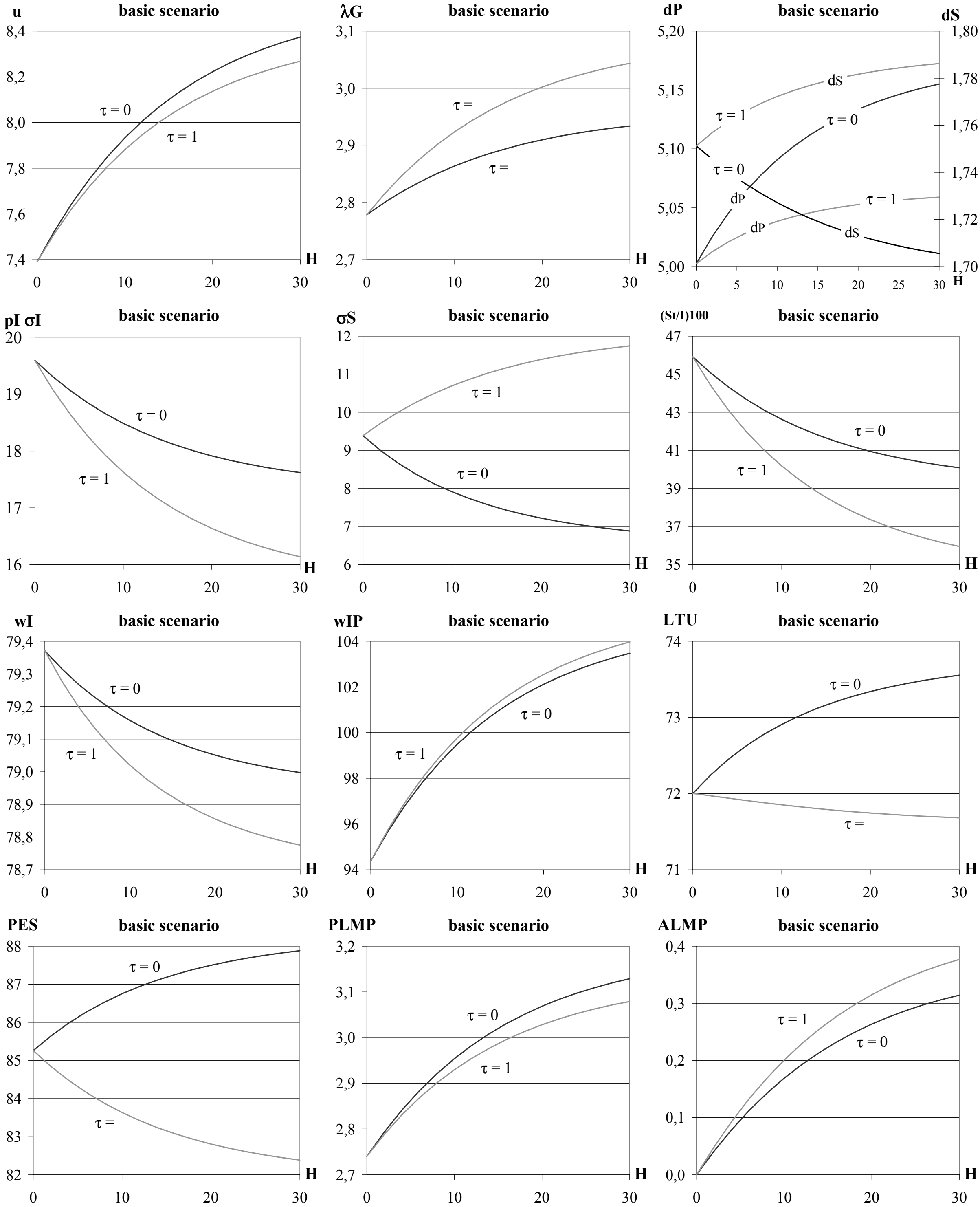
2. From (P13) and $H\geq 0$ we can conclude that (P12) holds.

3. The function $K(x,\tau)$ is continuously differentiable on $[0,T_P-H)$. To prove that $K(x,\tau)$ is strictly monotonically increasing with respect to x, we compute the partial derivative of $K(x,\tau)$:

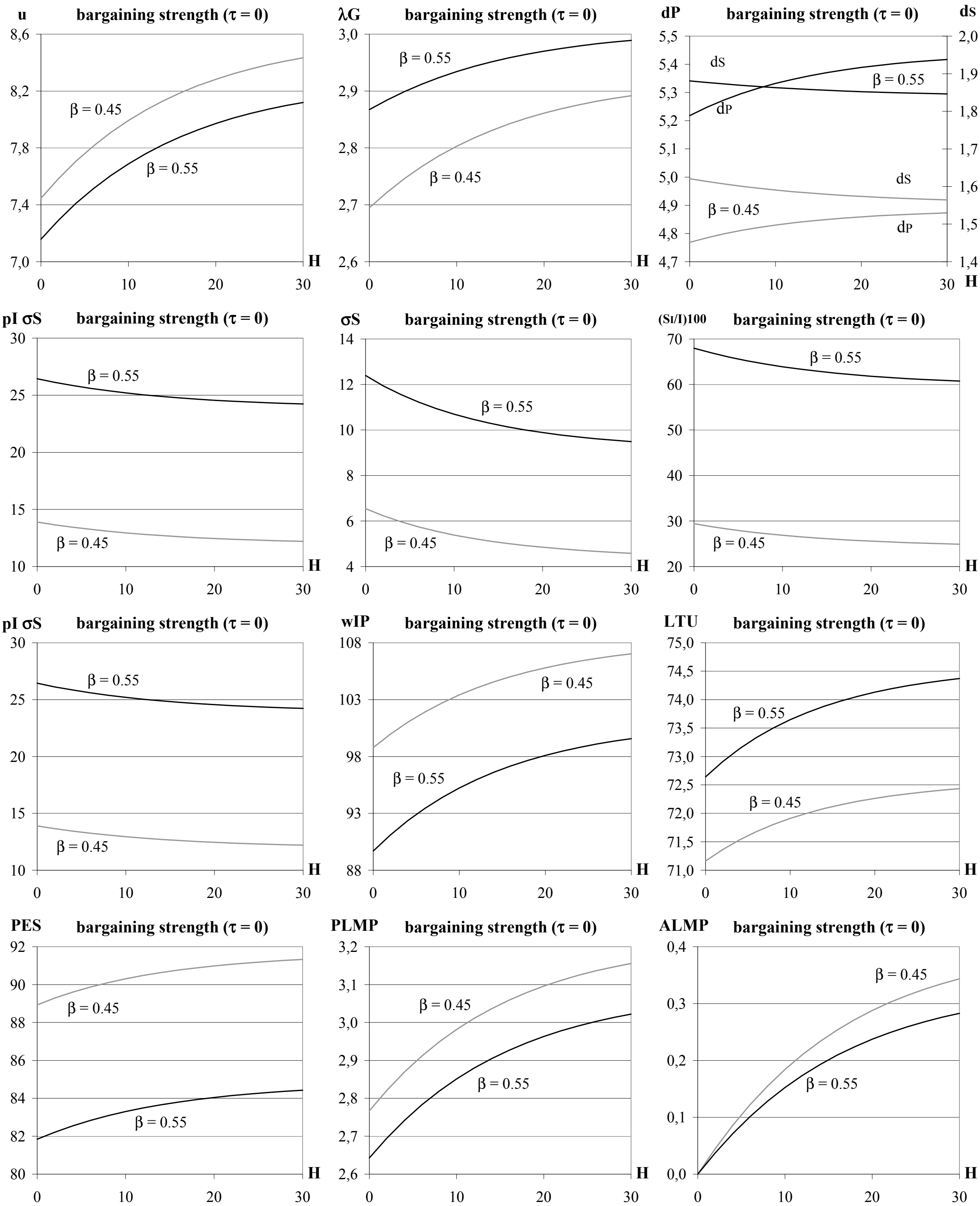
(P14) $$\frac{\partial K(x,\tau)}{\partial x} = 1 - \frac{e^{-\delta(z+\tau H)}\left[\delta z - e^{-\delta\tau H}\left(1-e^{-\delta z}\right)\right]}{\left(1-e^{-\delta(z+\tau H)}\right)^2}.$$

For $z>0$ it is true that $\partial K(x,\tau)/\partial x > 0 \quad\Leftrightarrow\quad 1 > e^{-\delta(z+\tau H)}\left[(2+\delta z)-e^{-\delta\tau H}\right]$. As the inequality on the RHS of the equivalence holds for $\tau H\geq 0$ the statement follows.

APPENDIX IV BASIC SCENARIO ($\tau = 0$, $\tau = 1$)



APPENDIX V BARGAINING STRENGTH (τ= 0)



APPENDIX VI EFFICIENCY ($\tau = 0$)