A Merton Model Approach to Assessing the Default Risk of UK Public Companies*

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Abstract

This paper shows how a Merton-model approach can be used to develop measures of the probability of failure of individual quoted UK companies. Probability estimates are then constructed for a group of failed companies and their properties as leading indicators of failure assessed. Probability estimates of failure for a control group of surviving companies are also constructed. These are used in Probit-regressions to evaluate the information content of the Merton-based estimates relative to information available in company accounts and in assessing Type I and Type II errors. We also look at power curves and accuracy ratios. The paper shows that there is much useful information in the Merton-style estimates.

JEL Classification: G12; G13

Keywords: Merton Models; Corporate Failure; Implied Default Probabilities

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1 Introduction

The quantitative modelling of credit risk initiated by Merton (1974) shows how the probability of company default can be inferred from the market valuation of companies. This paper employs a Merton-style approach to estimate default risk for public non-financial UK companies and assesses the reliability of these estimates using a range of different techniques.

The original Merton model is based on some simplifying assumptions about the structure of the typical firm's finances. The event of default is determined by the market value of the firm's assets in conjunction with the liability structure of the firm. When the value of the assets falls below a certain threshold (the default point), the firm is considered to be in default. A critical assumption is that the event of default can only take place at the maturity of the debt when the repayment is due. In order to implement the model in practical situations, this paper shows how this assumption can be modified to allow for default to occur at any point in time and not necessarily at maturity.

KMV Corporation[1] also uses the broad Merton approach to estimate the probability of firm failure in a number of different countries over a range of different forecast horizons. But the KMV approach does not rely solely on an analytical Merton model. Instead, it uses the Merton framework to estimate the "distance-to-default" of an individual company and then uses a proprietary data-base of US company histories to map this into an "expected default frequency" (EDF), estimated by the proportion of companies with a given distance to default that have failed in practice.[2] By contrast, the calculations reported here use only publicly available information on market prices and time series estimates of parameters to measure the probability of default.

A key concern is that the estimated probabilities of failure are both reliable and efficient. This paper assesses the reliability of the estimates by examining their success in predicting the failure or survival of both failed companies and survivors. The efficiency of the estimates is assessed by testing the extent to which the predictive power of the estimates could be improved by incorporating other information publicly available in company accounts. Models that combine a Merton approach with additional financial information are referred in the literature as "hybrid models". Sobehart and Keenan (2001) provide an excellent summary of this class of models.

The paper is organised as follows. Section 2 briefly reviews the literature on equity-based models of firm default. Section 3 shows how the original Merton model may be extended so that it can be implemented in practice. Section 4 outlines how the model may be tested. Section 5 describes the data on UK quoted companies and the sample

[1] Moody's has recently acquired KMV Corporation. The combined business of Moody's Risk Management Services and KMV is called Moody's KMV. Throughout this paper we use the terms KMV and Moody's to refer to the KMV Corporation and Moody's, respectively, before this acquisition took place.

[2] For a review of the KMV approach see Kealhofer and Kurbat (2002).

that is used in constructing estimates of failure. Section 6 sets out the results. Section 7 concludes.

2 Literature Review

There is a wide range of papers studying aggregate company defaults. Here we concentrate on those papers that adopt a structural or hybrid approach. The analysis by the KMV Corporation and Moody's are the most well known. See Nandi (1998) for a more extensive discussion of models for valuing financial instruments that are subject to default risk.

Crosbie and Bohn (2002) summarise KMV's default risk model. This is based on a modified version of the Black-Scholes-Merton framework that "assumes the firm's equity is a perpetual option with the default point acting as the absorbing barrier for the firm's asset value. When the asset value hits the default point, the firm is assumed to default." There are essentially three steps in the determination of the default probability. The first step is to estimate the asset value and volatility from the market value and volatility of equity and the book value of liabilities using their Merton approach. Second, the distance-to-default is calculated using the asset value and asset volatility. And finally, a default database of US incidents of default is used to derive an empirical distribution relating the distance-to-default to a default probability.

Sobehart, Stein, Mikityanskaya and Li (2000), from Moody's, construct a hybrid default risk model for US non-financial public firms. Moody's model provides a one-year estimated default probability using a variant of Merton's option theoretic model, Moody's rating (when available), company financial statement information, additional equity market information and macroeconomic variables.

As with the KMV model, the variant of the Merton model applied by Sobehart et al. (2000) is not used directly to calculate default probabilities but rather to calculate the market value and volatility of the firm's assets from equity prices. These inputs are used to derive the "distance to default", the number of standard deviations that the value of the firm's assets must drop in order to reach the default point. Moody's combine this information into a logistic regression to obtain some default probabilities that are further adjusted to correct for the fact that their in-sample data set had a slightly different proportion of defaulting-to-non-defaulting obligors from that observed in reality. According to the authors there appears to be a significant jump in performance as one moves from pure statistical models to those that include structural information. Interestingly, there is also a large gap between the pure structural model (Merton model) and Moody's hybrid model. The gap would represent the gain in accuracy derived from financial statement and rating data.

Kealhofer and Kurbat (2002) (KMV) try to replicate Moody's empirical results (Sobehart et al. (2000)) on the Merton approach. They obtain contrary results. The Merton approach outperforms Moody's ratings and various accounting ratios in predicting default. Kealhofer and Kurbat (2002) explain this discrepancy by the fact that their implementation of the Merton model is more accurate than Moody's approach. This greater accuracy, according to the authors, come from the special approaches developed to estimate asset volatility.

3 Implementation of the Merton Model

The basic insight of the Merton (1974) model is that the pay-offs to the shareholders of a firm are very similar to the pay-offs had they purchased a call option on the value of the firm with a strike price given by the amount of debt outstanding. As such, the option pricing techniques of Black and Scholes (1973) may be used to estimate the value of the option and the underlying probability of default.

The Merton model and the variation of the Merton model adopted in this paper assume a simple capital structure for the firm: debt plus equity. We denote the notional amount of debt by B, with $(T - t)$ being the time to maturity, the value of the firm is A_t, and $F(A, T, t)$ is the value of the debt at time t. The equity value at t is denoted by $f(A, t)$. Then, we can write the value of the firm A_t as:

$$A_t = F(A, T, t) + f(A, t) \tag{1}$$

The original Merton model assumes that the firm promises to pay B to the bondholders at maturity T. If this payment is not met, that is, if the value of the firm is less than B, the bondholders take over the company and the shareholders receive nothing. Furthermore, the firm is assumed not to issue any new senior claims nor pay cash dividends nor repurchase shares prior to the maturity of the debt. Under these assumptions the shareholders are effectively long a European call option on the value of the firm.

This paper relaxes the assumption that default (or insolvency) can only occur at the maturity of the debt. The model developed here assumes that insolvency occurs the first time that assets falls short of the redemption value of debt. In other words, insolvency occurs the first time that the value of the firm falls below the default point.

To model this we use the concept of a barrier option.[3] Barrier options are options where the payoff depends on whether the underlying asset price reaches a certain level during a certain period of time. A down-and-out call option is one type of knock-out option. It is a regular call option that ceases to exist if the asset price reaches a certain level, the barrier. The barrier level is below the initial asset price. The owner of a knock-out option receives a prespecifed, nonnegative, cash payment (the rebate) if the underlying asset price breaches the barrier prior to expiration.

[3]Other equity-based models of credit risk that use the concept of barrier options are Black and Cox (1976), Longstaff and Schwartz (1995) and Briys and de Varenne (1997).

To derive the probability of default using a barrier option we suppose that the value of the firm's underlying assets follows the following stochastic process:

$$dA = \mu_A A dt + \sigma_A A dz \tag{2}$$

where $dz = \varepsilon\sqrt{dt}$ and $\varepsilon \sim N[0,1]$

and we assume a non-stochastic process for the liabilities:

$$dL = \mu_L L dt \tag{3}$$

Let us denote the asset-liability ratio by k:

$$k = A/L \tag{4}$$

Default occurs when k falls below the default trigger or default point called $\underline{k}$ at any time within a given period. To estimate this probability of default we need to model how k changes over time.[4] Differentiate (4) to obtain

$$dk = (\mu_A - \mu_L)kdt + \sigma_A kdz$$

and define:

$$\mu_A - \mu_L = \mu_k$$

$$\sigma_A = \sigma_k$$

The values for μ_k and σ_k are needed to calculate the probabilities of default. Maximum likelihood techniques are used to obtain estimates of those two parameters.[5]

4 Testing the Model

To test the performance of the Merton approach adopted in this paper, we calculate the probabilities of default (PDs) implied by our model for a sample of UK non-financial firms that includes a number of defaulters.[6] We then do three types of tests: (1) we evaluate our model against the actual default experience; (2) we compare our model with other default models; and (3) we use measures of statistical power based on power curves and accuracy ratios.

For the first type of test, we compare the PD profiles for a subsample of defaulters with the timing of actual defaults to assess the accuracy of the model in predicting those failures. We also calculate the Type I and II errors. Ideally we want both type of errors

[4] Note that the value of the firm's assets, A, is unobservable and hence so is the k ratio. What we can observe is the equity-liability ratio (y). Nickell and Perraudin (2002), page six, derive a mapping between the equity-liability ratio and the value of the firm's assets-liability ratio that this paper borrows.

[5] The equations used to calculate these probabilities of default are derived in Tudela and Young (2002).

[6] We describe the composition of the sample in the section below.

to be small, but clearly there is a trade-off between the two.

For the second type of test, we compare our model with other approaches. To compare the performance of our Merton approach with the information content of company account data only, we estimate a probit model. The dependent variable is a dummy that takes on the value of unity if the company went bankrupt, and zero otherwise, and regressors are company account indicators. To select the company account variables included in the probit estimations we follow Geroski and Gregg (1997), one of the most comprehensive empirical studies of the determinants of company default in the UK. To compare the accuracy of both models we calculate Type I and II errors.

The power of the PDs in explaining company default is assessed formally against other models by testing for their significance when added to the estimated probit model above. If the coefficient of the PD variable is significantly different from zero, we can conclude that the Merton approach here implemented adds value to the company account variables.

For the third type of test, following Kocagil, Escott, Glormann, Malzkom and Scott (2002), we use power curves and accuracy ratios to assess the statistical power of the models. Both testing tools evaluate the accuracy of a model in ranking defaulters and non-defaulters using the estimated probabilities of default. To plot a power curve, for a given model we rank the companies in our sample by risk score (PD) from the riskiest to the safest (horizontal axis). For a given percentage of this sample we calculate the number of defaulters included in that percentage as a proportion of the total number of defaulters in our sample (vertical axis). Thus for a sample in which one per cent of companies default, a perfect model would include all the defaulters within the riskiest percentile. By contrast, in a random model the first percentile would tend to include only one per cent of the defaulters and its power curve would be represented by a 45 degree line. The better the model at ranking companies the more bowed towards the upper-left corner its power curve would be. The power curve is sample dependent in that its shape is dependent on the proportion of companies in the sample that default.

The accuracy ratio, even if less visual, gives a single statistic that summarises the information content of the power curve. The accuracy ratio has values that rank from zero per cent (random model) to 100 per cent (perfect model) and it is defined as the ratio of the area between the power curves of the actual and random models to the area between the power curves of the perfect and random models.

5 Sample and Data Description

The model presented in Section 3 is estimated for a sample of UK non-financial quoted[7] companies. Specifically, we collect 7,459 financial statements from 1990 to 2001, 65 of

[7] Clearly, we cannot directly apply the model to private companies given the data requirements: we need equity market capitalisation series.

which correspond to firm defaults.[8] The sample of failed companies was constructed collecting news from FT.com about companies that went into receivership. The sample constructed in this manner was checked against the "deaduk" dataset in Thompson Financial Datastream and the "Companies House" website. The default date was selected as being the last day in which an equity price movement was observed. This may be not the exact date of default but it is a good approximation given the discrepancy and/or inaccuracy observed in the different sources consulted and the difficulty of defining a default date.

Table 1 disaggregates the number of failures and non-failures by year for the sample we use in the estimations (and for the sample we initially gathered for illustration purposes). It is immediately apparent that defaults are concentrated in 1990–92 i.e. the recession years.

All our data are downloaded from Thompson Financial Datastream. To estimate the PDs we use market capitalisation and liability data. Total liabilities are defined as total assets minus total share capital and reserves and short-term liabilities are defined as current liabilities. Following the findings of Crosbie and Bohn (2002) that the asset value at which a firm defaults generally lies somewhere between total liabilities and current liabilities, the liability measure that we use is 50 per cent of long-term liabilities (total liabilities minus short-term liabilities) plus all short-term liabilities.

The PDs are estimated on a weekly basis using a five-year rolling window. Moreover, we do not include in the maximisation procedure those observations when a dividend pay-out was made. This is to avoid any uninformative jump in equity prices. The dividend payments dates are also obtained from Datastream. The equity data (market capitalisation) are weekly data,[9] but the liability data are annual. In order to generate the necessary weekly liability data we use cubic spline interpolation routines. The PDs are calculated for different time horizons, from one year to five years, but here we concentrate on one-year and two-year PDs.

In order to estimate the competing probit models we need company account data, in particular, profit margins, the ratio of debt to total assets, the ratio of cash to liabilities, the number of employees and sales growth. Profit margins are defined as EBITDA relative to sales and we further construct three binary (0,1) dummy variables[10] for negative profit margins, profit margins between zero and three per cent, and profit margins between three and six per cent. The debt to assets ratio is defined as gross debt (borrowing of maturity less than a year plus capital loans with maturity greater than one year) relative to total assets. The cash to liabilities ratio is the 'total cash and equivalent' variable from Datastream relative to liabilities as defined above.

[8] Initially, we identify 76 firm defaults but due to the lack of some company account data needed for our econometric specifications we use 65 to present comparable results across estimations.

[9] Actually, it is daily data but we use Wednesdays only to avoid any day-of-the-week effect.

[10] Following Geroski and Gregg (1997).

Apart from the company account data, we have also included some year dummies and/or a macroeconomic indicator[11] (GDP) in our probit estimates to account for the general economic situation. The macroeconomic data are obtained from the electronic version of the International Financial Statistics published by the International Monetary Fund.

6 Results

6.1 Implied Probabilities of Default

As an initial way of measuring the accuracy of our Merton approach, we first compare the PDs of defaulting and non-defaulting companies. For defaulting companies, we calculate the 1-year ahead PD in each month of the twelve months prior to default and take the simple average of these PDs as a measure of the default probability. This is what we call *1-year PD annual average*. For non-defaulting companies, we take a simple average of the 1-year ahead PDs in each month of the preceding calendar year. We investigate the sensitivity of our results to these definitions below.

The usefulness of the estimated default probabilities generated by the model can be assessed by examining the Type I and II errors for different failure thresholds (see results in Table 2). The lower the failure threshold, the smaller the Type I error (i.e. the proportion of companies classified as survivors that failed), but at the expense of a greater Type II error. As shown in Table 2, for the entire sample, choosing a failure threshold of five per cent, we fail to classify as defaulters 4.6 per cent of companies that went bankrupt. At this level, the Type I error is zero for eight of the years considered. The corresponding Type II error for the whole sample is 19.9 per cent of non-defaulting companies. If we increase the failure threshold to a PD greater or equal than ten per cent, then 9.2 per cent of our population of defaulters had not been classified as defaulters. But in this case, the Type II error is lower at 15 per cent.

We perform a test for the equality of 1-year PD means between the defaulters group and the non-defaulters group. The 1-year PD average for the defaulter group is 47.33, for the non-defaulter group it is 5.44. The null hypothesis is that the difference of the two means equals zero. Under the alternative of this difference being different from zero, we reject the null at the one per cent level of significance.

To check further the accuracy of the model we calculate the Type I and II errors for the 2-year PD annual average (defined as the average of the 2-year PDs, —the probability of default in two years time from now— from the 12^{th} month before the default month to the 24^{th} before the default month) and for the 1-year PD for the twelfth month before the default month. This last measure is very strict in the sense that it gathers information for one month only, whereas the other measures compile the information content of twelve months.

[11]Several macroeconomic variables were tested and finally we decided on GDP (see Section 6).

As expected, for the same thresholds, the Type I errors are bigger for the latter measures. For a five per cent threshold the Type I error for the 2-year PD annual average is 6.1 per cent, and for the 1-year PD as defined above is 24.6 per cent. To check if the latter is a spurious result, we calculate the Type I error for a five per cent cut-off for the 1-year PD for the eleventh month before the default month, the tenth, and so on until the seventh month before the default month. The Type I error for these measures are 16.9, 18.5, 16.9, 13.8 and 10.8 per cent, respectively. These figures are more in line with the results obtained for the 1-year and 2-year PD annual averages, indicating that the figure of 24.6 per cent is spurious. One, therefore, should always look at PDs for more than one month and relative to recent history.

By way of illustration and to assess the model's ability to reflect credit risk at the individual firm, Figure 1 represents 1-year and 2-year PDs (monthly averages) for those companies that failed in 1992.[12] The black line is the 1-year PD, that is, the probability of default in one year time in a given month. The grey curve is the 2-year PD, the probability of default in two years time in a given month. The dashed(dotted) vertical line cuts the time axis exactly one(two) year(s) before the failure date.

To correctly classify defaults, 1-year PDs should be above the chosen threshold to define failure when crossing the dashed vertical line. Similarly, 2-year PDs should be greater than the threshold when crossing the dotted vertical line. All the PD curves show rising profiles before the companies went bankrupt, and are very high in the months before the failure. From twelve to six months before failure the 1-year PD is always greater than 50.8 per cent, whereas from 24 to twelve months before failure the average PD is 29.1 per cent.

6.2 Adding other Company Account Information

To compare the performance of our Merton approach with the information content of company account data only, we estimate a probit model using company account data as regressors. Here the dependent variable is a dummy that takes on the value of unity if the company went bankrupt, and zero otherwise. Given the concentration of defaulters in the recession period, we also include in this probit model a macroeconomic indicator, GDP, as an additional regressor. We test the power of PDs to explain company failure by adding them to the probit model. If the coefficient of the PD variable is significantly different from zero after controlling for company account data, we can conclude that the Merton approach adds value to the company account variables.

In Table 3 we collect the results from these models. We use different measures of PDs for robustness tests. When using 1-year PDs the company account data is lagged one year, that is, it corresponds to the year before the default year —columns (1), (2) and (5). If we include 2-year PDs in the probit estimation, we lag the company account data 2 years

[12]We choose this year because it is the year with the highest number of defaults (see Table 1).

—columns (3) and (4). We always use the GDP of the year of default.

In column (1) of Table 3 we use the 1-year PD annual average. The results show that the PD variable is significant and that only one company account variable, the debt to assets ratio, is significant and at a lower level.[13] The number of employees (included to account for size) is marginally significant. The variable accounting for the macroeconomic environment is also significant.[14] The conclusion is that the PD variable contains information over and above that included in publicly available company accounts.

In column (2) we re-estimate the model of column (1) excluding the PD variable. Interestingly, the profitability measures are now significant. Having negative profit margins instead of profit margins greater than six per cent significantly increase the likelihood of failure. Profit margins between zero and three per cent (instead of profit margins greater than six per cent) also increases the probability of failure. The coefficient for this last measure is, as expected, smaller than the coefficient for negative profit margins. The coefficient of profit margins between three and six per cent is smaller than the two previous coefficients but it is not significant. If we compare these three coefficients with the ones in column (1) we clearly see the effect of omitting the PD variable. In column (1) these coefficients were not significant and did not have the correct signs or the expected increasing-in-value pattern.

Moreover, the exclusion of the PD variable increases the significance level of the debt to assets ratio. The size and the macroeconomic variables are still significant. It is interesting to note that the constant is not significant in the model of column (1), but becomes significant once we exclude the PD variable, signalling the better fit of the model in column (1).

In the final rows of Table 3 we report the average log-likelihood and the pseudo-R^2 to compare models. We include two measures of pseudo-R^2 based on McFadden (1974) and Cragg and Uhler (1970).[15] Comparing the values for the average log-likelihood we see that this is bigger for the model of column (1), that is the model that includes PDs as regressor. Moreover, the pseudo-R^2 of the model of column (1) is more than twice[16] the

[13] The fact that the debt to asset ratio is significant even controlling for PDs, which use a similar ratio in their calculation, reveals a highly non-linear relationship between likelihood of default and the debt to asset ratio.

[14] We have also included yearly dummy variables instead of the macroeconomic variable with 2001 as the reference year. The dummies for the years 1990–92 and 1995 were significant. If we include the yearly dummies plus GDP, the yearly dummies are no longer significant. We also tried GDP growth, GDP deviation from its long-run trend, Industrial Production Index and its deviation from trend. All these variables were significant, but when included with the yearly dummies some of them were still significant. For this reason we report the results for the model that includes GDP (GDP=100 for 1995). Different measures of interest rates and prices were also included but failed to be significantly different from zero.

[15] For a discussion of these measures we refer the reader to Maddala (1983), pages 37–41.

[16] Strictly speaking one cannot compare R^2's across models with different number of regressors since the higher the number of regressors the higher the R^2. Notwithstanding, we have excluded one variable

pseudo-R^2 for the model that excludes the PDs. Both statistics indicate, therefore, the superiority of the first model.

We run the model of column (1) by alternatively eliminating one defaulter at a time. The aim of this exercise was to check if the results were driven by a possible outlier. Since the results did not change substantially we can discard this possibility.

We also estimate the models of columns (1) and (2) in Table 3 for the years 1990–93[17] one year at a time. The results remain broadly the same.

Columns (3) and (4) use information on PDs and company account data two years prior to the year when the default occurred. We do this as a robustness check and to evaluate the statistical power of 2-year PDs. The results are very similar to the ones of columns (1) and (2).

The model of column (5) is as model (1) but with a different PD variable. Here we only take the information of the 1-year PD of the 12^{th} month prior to the default month. Even if the coefficient for the PD measure is still significant, the coefficients for the accounting variables (that collect information for twelve months instead of one month only as the PD) are significant: negative profit margins and profit margins between zero and three per cent. Please note that the 1-year PD twelve months before failure is a measure twelve months prior to the default month, whereas the accounting variables are simply those of the year before failure. For negative profit margins not being significant we have to include information on 1-year PD from twelve to five months before failure.

6.3 Power Curves and Accuracy Ratios

Figure 2 plots the power curve for some of the models estimated in this paper. The hybrid model is the model of column (1) in Table 3. Company account data is the model of Table 3, column (2). The other three curves correspond to different PD measures as stated in the graph. The power curves of Figure 2 have been constructed for the same proportion of defaulters in each model, which means that we can compare each curve with the other. But it is not possible to compare power curves produced by other models that use different data sets (the same applies for the AR index).

Observing the different curves we see that the hybrid model outperforms all other models. The 1-year PD annual average is almost identical to the hybrid model at small proportions of sample excluded. The model that uses only company account information is clearly

from our model (1) to check if the pseudo-R^2 was still of the same order of magnitude. Even excluding the GDP variable that has been proved to be highly significant the MacFadden pseudo-R^2 is 0.2785 (and higher if we exclude one of the non-significant variables). This exercise was undertaken for the rest of the models presented in Table 3 and the same result applies. Therefore we are confident in the comparison of pseudo-R^2's across specifications.

[17]The number of defaulters is too small for the individual years from 1994 to 2001 to obtain reliable results.

inferior to hybrid models or Merton models.

We also calculate the accuracy ratios for the models of Figure 2. Sobehart and Keenan (2001) report the accuracy ratios for the KMV's implementation of the Merton model (using 1-year probabilities of default) and for a hybrid model as described in Sobehart et al. (2000). These ratios are 69.0 per cent and 72.7 per cent, respectively. We can use these figures as an approximate benchmark to evaluate the accuracy ratios calculated for our models. The closest models to compare with those figures are the ones for the 1-year PD annual average and the hybrid model, that is, 76.7 per cent and 77.09 per cent, respectively.

Comparing the accuracy ratios for the different models, a reduced form model of the type of Geroski and Gregg (1997) (accuracy ratio of 42 per cent) is easily outperformed by a Merton approach (ratios between 53 to 77 per cent depending on the specific PD measured considered), reflecting the information incorporated into market prices.[18] The jump in performance from a pure structural (Merton-based) approach to a hybrid model (with a ratio of 77 per cent) is not as acute. One can always argue that this gap may be enhanced by the inclusion of more accounting variables. But what is important here is the existence of some information that is not captured by the Merton approach this paper uses.

7 Conclusions

This paper describes the derivation of default probabilities from an extended version of the Merton model and applies this to a number of UK non-financial quoted companies over the period 1990–2001.

The probability of default derived from our Merton model implementation provides a strong signal of failure one year in advance of its occurrence. The mean value of the 1-year PD annual average measure for our entire sample is 47.3 per cent for those companies that went bankrupt, and 5.4 per cent for those that did not default. A more restrictive probability of default measure shows a similar pattern. The mean value of the 1-year PD for twelve months before the default date is 32.0 per cent for defaulters and 5.2 per cent for non-defaulters.

Calculation of Type I and II errors suggests that PDs are successful in discriminating between failing and non-failing firms. Using a threshold of ten per cent, that is, classifying defaults as those firms with a 1-year PD greater or equal to ten per cent, the Type I error is relatively modest at 9.2 per cent (with a Type II error of 15.0 per cent). For a 2-year PD measure the Type I and II errors for the same threshold are 12.3 and 29.9 per

[18] We use the terms reduced form and statistical models for those estimated models which are not based on a theoretical model. The term structural model is applied to the estimated models which are derived from a theoretical model, e.g. the Merton model.

cent, respectively.

If we compare our model with a reduced form model of the type of Geroski and Gregg (1997), we can state that the structural model (Merton approach) clearly outperforms the reduced form model. This is independent of the specific PD measure, including the comparison of 2-year PDs and a statistical model that uses one-year lagged accounting ratios. But it also shows that the hybrid models, i.e. those combining company account information and the PDs derived from a Merton model, outperform pure structural models, if only marginally.

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Tables and Figures

Table 1: Distribution of Defaults over Time

Year	Whole Sample		Estimation Sample	
	Non-Defaults	Defaults	Non-Defaults	Defaults
1990	412	13	410	9
1991	447	15	443	10
1992	474	13	471	13
1993	484	8	482	7
1994	498	3	495	3
1995	510	6	508	6
1996	554	3	552	3
1997	597	5	595	5
1998	667	4	664	3
1999	917	0	907	0
2000	996	2	816	2
2001	1078	4	1051	4
Total	7634	76	7394	65

Table 2: Type I & II Errors: Merton Model, 1-year PDs annual average[1]

Sample	Type Error	Threshold 5%	10%	15%	20%	30%
Whole Sample	I	4.61	9.23	13.85	20.00	36.92
	II	19.95	14.97	11.79	9.43	6.32
1990	I	0.00	22.22	22.22	33.33	33.33
	II	20.24	14.15	10.73	8.05	5.37
1991	I	0.00	0.00	10.00	20.00	30.00
	II	31.83	26.86	22.12	18.51	13.54
1992	I	0.00	0.00	0.00	7.69	23.08
	II	25.90	19.11	15.29	12.31	14.29
1993	I	0.00	0.00	0.00	0.00	14.29
	II	30.50	23.44	19.50	16.18	11.83
1994	I	0.00	0.00	0.00	0.00	100.00
	II	17.17	11.92	9.09	6.87	4.44
1995	I	16.67	16.67	33.33	33.33	50.00
	II	13.98	10.04	7.09	5.12	3.54
1996	I	0.00	0.00	0.00	33.33	33.33
	II	14.13	10.51	9.06	7.25	5.80
1997	I	20.00	20.00	20.00	20.00	60.00
	II	14.45	11.43	8.57	6.55	3.70
1998	I	33.33	66.67	66.67	66.67	100.00
	II	15.21	10.54	7.98	6.32	4.07
1999	I					
	II	19.07	14.55	11.47	9.26	6.06
2000	I	0.00	0.00	0.00	0.00	0.00
	II	19.73	15.20	11.89	9.31	5.64
2001	I	0.00	0.00	25.00	25.00	25.00
	II	21.60	15.70	12.18	9.99	6.47

[1] *1-year PD annual average* is the average of the 1-year PD, —probability of default in one year time from now— for the 12 months preceding the default month.

Table 3: Using Company Account Data[1]

Variable	(1)	(2)	(3)	(4)	(5)
Constant	0.43	1.36**	0.89	1.53**	0.93
	(0.64)	(2.17)	(1.29)	(2.28)	(1.46)
1-year PD annual average[2]	0.02***				
	(11.03)				
2-year PD annual average[3]			0.01***		
			(5.90)		
1-year PD 12 months before failure[4]					0.01***
					(5.90)
Profitability< 0%	0.17	0.68***	0.25	0.51***	0.49***
	(1.11)	(5.00)	(1.63)	(3.51)	(3.40)
0% <Profitability< 3%	0.17	0.42***	−0.09	0.11	0.30*
	(0.97)	(2.81)	(−0.47)	(0.59)	(1.91)
3% <Profitability< 6%	−0.01	0.14	−0.04	0.07	0.07
	(−0.03)	(0.92)	(−0.23)	(0.48)	(0.45)
Debt over assets	0.31**	0.48***	0.25**	0.33***	0.39***
	(2.52)	(4.61)	(2.37)	(3.52)	(2.99)
Cash over liabilities	0.01	−0.12	−0.04	−0.18	−0.07
	(0.13)	(−1.18)	(−0.37)	(−1.38)	(−0.71)
log number of employees	−0.6*	−0.05*	−0.04	−0.04	−0.05*
	(−1.75)	(−1.66)	(−1.11)	(−1.45)	(−1.76)
Sales growth	−0.11	−0.06	−0.16*	−0.26***	0.00
	(−0.91)	(−0.44)	(−1.73)	(−2.98)	(2.02)
GDP	−0.03***	−0.04***	−0.04***	−0.04***	−0.03***
	(−4.60)	(−6.18)	(−5.10)	(−5.81)	(−5.45)
Avg. Log-likelihood	−0.034	−0.042	−0.040	−0.040	−0.041
McFadden Pseudo-R^2	0.3105	0.1501	0.1787	0.1296	0.1878
Cragg & Uhler Pseudo-R^2	0.2999	0.1438	0.1717	0.1246	0.1801

[1]Company Account Data is for the year before the default year for models using 1-year PDs, columns (1), (2) and (5) and 2 years before the default year for models using 2-year PDs, columns (3) and (4). In this table we present the estimated coefficients and the z-statistics in parenthesis. ***,** and * mean that the coefficient is significant at the 1%, 5% and 10% level, respectively.

[2] *1-year PD annual average* is the average of the 1-year PD, —probability of default in one year time from now— for the 12 months preceding the default month.

[3] *2-year PD annual average* is the average of the 2-year PD, —probability of default in 2 years time from now— from the 12^{th} month before the default month to the 24^{th} before the default month.

[4] *1-year PD 12 months before failure* is the 1-year PD for the 12^{th} month before the default month.

Figure 1: Implied Probabilities of Default

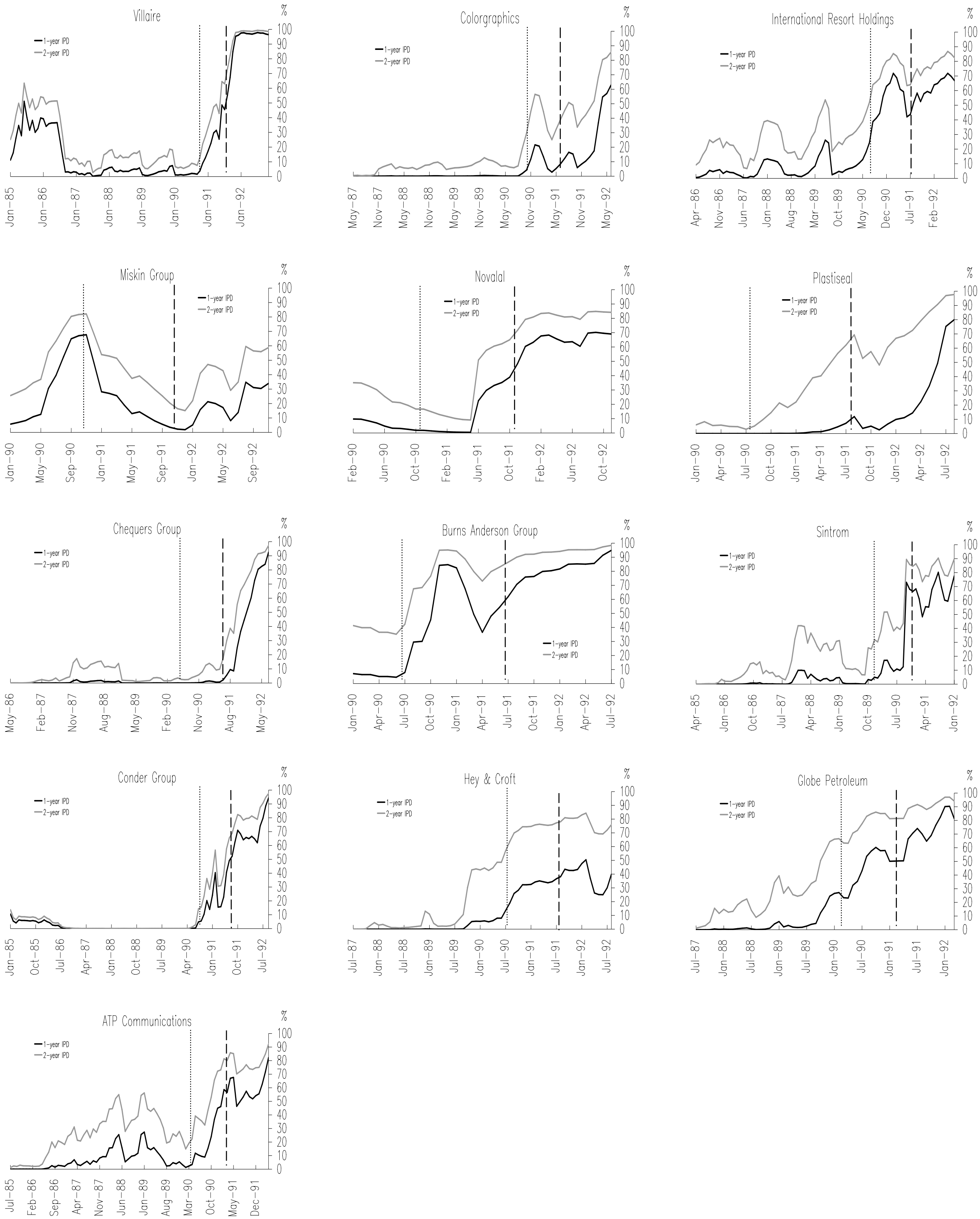
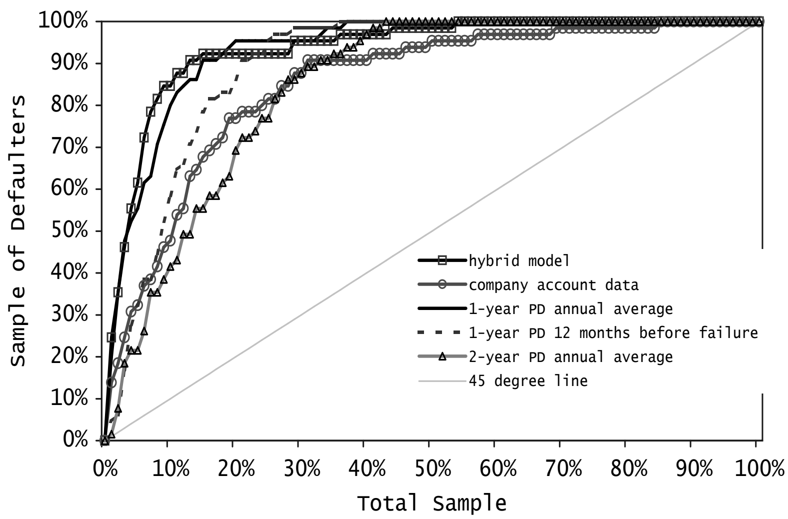


Figure 2: Power Curve



Note: To plot the power curve, for a given model we rank the companies in our sample by risk score (PD) from the riskiest to the safest (horizontal axis). For a given percentage of this sample we calculate the number of defaulters included in that percentage as a proportion of the total number of defaulters in our sample (vertical axis).