# Strategic delays of delivery, market separation and demand discrimination

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Very preliminary version

#### Abstract

We show that an adequate choice of delays to delivet a dutable good allows a monopoly to teduce competition between his two tetailets on two different matkets. Instead of preventing each tetailet from selling on both matkets, the producet separates the matkets by directing the choices of consumets between the tetailets. The consumet whose willingness to pay is the lowest obtains the good later than the other, and both pay their highest valuations for the good: the producet perfectly discriminates the demand. The European cat matket where producets try to testrict competition between tetailets is an application of our findings.

JEL Classification : L22, L12, L40

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# 1 Introduction

For several years, the European car market has been under the attention of the European Commission: illegal market separation practices and others refusal to trade have been severely punished by the DG Competition. For example in 1998 Volkswagen AG has been fined for an infringement to the article &l(1) of the EC Treaty<sup>1</sup>: they were explicitly preventing German and Austrian consumers from buying in Italy, where prices were substantially lower, through agreements with Autogerma SA, their importer for Italy. More recently in 2000 Opel Nederland BV has been punished for the same kind of illegal practices, namely preventing sales to consumers from other Member States in the Nederlands<sup>2</sup>. The fines were respectively 90 million euros and 43 million euros. Finally in 2001, Volkswagen AG has been fined again in Germany (30,96 million euros) as well as DaimlerChrysler in Germany and Belgium (71,825 million euros).

Indeed according to the EC Treaty, article 81 paragraph 1, all the practices which may affect trade between Member States and which prevent, restrict or distort competition within the common market shall be prohibited as incompatible with the common market. Therefore all the practices that prevent retailers to sell the good to any consumers are illegal. In order to improve price transparency on this market and to increase parallel trade between the Member States, the European Commission regularly publishes price differences within Europe which aim to help consumers to buy on the market where the price is the lowest<sup>3</sup>. Moreover a new regulation for the European car market has been published, in order to improve the competition at each level of this industry, including after sale services <sup>4</sup>. Even if this regulation makes reference to almost all the illegal practices, some justified behavior whose effect would be to partly reduce competition between the retailers of the same brand may still persist. The scope of this paper is to look at some of this practices.

In this work we are looking to strategies of delayed delivery that may be used by a firm to separate different markets and consequently discriminate the demand. Our model describes the relations between a producer and his two retailers on two different but contiguous national markets, in presence of a transaction cost suffered by the buyer if he tries to buy the good abroad. The two buyers, one on each market, differ according to their willingness to pay for

<sup>&</sup>lt;sup>1</sup>Former article 85(1).

<sup>&</sup>lt;sup>3</sup>See EC decisions quoted in the references.

<sup>&</sup>lt;sup>3</sup>See the European Commission Website, http://europa.eu.int/comm/competition/.

<sup>\*</sup>See the Official Journal of the European Communities, 16.3.2002, C67/2 to C67/26.

one unit of the good: it is higher on the national market of the manufacturer than on the foreign market. Finally we assume that the manufacturer is able to extract the total profit earned by the retailers through fixed fees and a wholesale price, but cannot control their presence on each market. However he is able to commit to a delay in the delivery of the good on each market.

In this framework, we show that by delivering the durable good at different dates, the producer succeeds in restricting competition between his two retailers and conduct them to charge a higher price than in the situation where they are in competition at the same instant. In that case each consumer buys on his domestic market, and not on the foreign market. The consumer whose willingness to pay is the highest obtains the good at the beginning of the game, while on the foreign market a strictly positive delay in the delivery exists. Social welfare is then lower than without delays since a part of the surplus is destroyed by the delay on the foreign market, but the profit of the monopoly is higher.

This work is connected to the classical issues on intertemporal price discrimination for durable goods, as explored by Stockey [1979] and Bagnoli, Salant, and Swierzbinski [1989] among others. Stockey shows that price discrimination cannot occur if the production costs are not falling at a sufficient speed within time. Contrary to that work, the originality of our model is to consider the delay in the delivery as a strategic tool to restrict competition in a vertical restraint framework: exactly the same good is sold on both markets, but retailers can no longer be in frontal competition. In our case price discrimination occurs without diminishing production costs. Bagnoli et alii are concerned with the problem of a monopolist selling a durable good: they characterize the conditions under which the Coase conjecture (among other famous results) may be wrong.

More recently Anderson and Ginsburgh [1994] introduced a second-hand market in a model of a durable good sold by a monopolist to look at his best strategic behavior: even if the second-hand market may be used to achieve second-degree price discrimination, the result for the monopoly is not always to increase the gains. When the quality of the good sold may be chosen, the result may be to deteriorate quality to lower the value of the second-hand good, or to sold a good that will be as good as new in the end. Kühn [1998] showed that differences in quality and production costs between a durable and a non-durable good may be used to discriminate the demand. In our framework, the good produced by the manufacturer is homogenous and the discrimination comes from the screening of the market that can be done through the strategy of delayed delivery.

Price discrimination in the European car market has been the object of a more specific

paper by Kirman and Schueller [1990]. They show that differences in prices may be explained by differences between producers rather than differences in demands, but also differences in taxes. They consider a n firms competition model, where on each market there is a dominant producer except on one specific market where there is no national producer. In particular they show that in markets where the dominant producer has high costs, the prices of all the products are higher. Moreover in markets where the tax rate is high the pre-tax prices are lower. Finally in the market where there is no dominant producer prices are lower. Differently to their study, the rationale for demand discrimination in our framework is the difference between consumers's willingness to pay. It occurs because of the temporal separation of the markets: a producer even charging the same wholesale price for both retailers leads them to charge the highest price. Verboven [1996] among others estimated a model to explain price differences between new cars within the European Community: he shows that three factors are particularly interesting, namely the existence of a local market power (national producers in France, Germany, United Kingdom and Italy, benefit from a lower price elasticity than others), binding import quotas constraints (in France and Italy against Japanese cars), and collusion (which cannot be rejected in Germany and United Kingdom).

Our paper is organized as follows: in the second section we present the model. In the third section we analyze the benchmark case, where there are no delays in the delivery, to prove that discrimination cannot occur in that case. In the fourth section we solve the general model and determine the producer's choices in delays. Finally the fifth section discuss our results and the last section concludes.

## 2 The model

Let a producer of a manufactured good  $P_N$  be able to sell his homogenous, indivisible and durable good through two retailers,  $R_N$  on the domestic market and  $R_F$  on a foreign market. On each market there is only one consumer, respectively  $C_N$  and  $C_F$ , who behaves competitively. We assume without loss of generality that the marginal cost of production is equal to zero.

To market the good each retailer chooses a retail price  $p_x$  for x = N, F, given the unique wholesale price w charged by the producer  $P_N$  for any unit sold by the retailer, given the fixed fees  $T_N$  and  $T_F$  paid by each retailer to the producer, and given the dates at which the producer is able to deliver the good on each market,  $\tau_N$  and  $\tau_F$ . Let  $\mathcal{P}_N$  be the support of the pure strategies for retailer  $R_N$ , and  $\mathcal{P}_F$  for retailer  $R_F$ . We may define a mixed strategy as a probability distribution  $\sigma_x$  on a support  $\operatorname{Supp}(\sigma_x) \subseteq \mathcal{P}_x$  for x = N, F. As usual a mixed strategy  $\sigma_x$  may be any type of distribution on  $\operatorname{Supp}(\sigma_x)$ .

Each consumer chooses the retailer he wants to buy from, depending on the pairs  $(p_x, \tau_x)$ observed on each market. Let  $v_x$  be the constant instantaneous flow of benefit generated by the consumption of the durable good for x = N, F. For the sake of tractability we assume that  $v_N > v_F = 1$ . Moreover the good does not depreciate, and consumers are infinitely living and able to consume if they have bought the good. Finally we assume that buying on one's foreign market (i.e.  $C_N$  buying from  $R_F$  or  $C_F$  buying from  $R_N$ ) induces an additional transaction cost  $\epsilon \geq 0$  for the consumer. Let  $\delta$  be the discount factor identical for all agents. If he buys the good from retailer  $R_y$ , the net benefit for consumer  $C_x$  for x, y = N, F is then,

$$u_x(p_\mathbf{y},\tau_\mathbf{y}) = \sum_{t=\tau_\mathbf{y}}^{+\infty} \delta^t v_x - p_\mathbf{y} - \epsilon \cdot \mathbb{I}_{x\neq\mathbf{y}}$$

where we assume that the good is paid on order,  $\mathbb{I}_{x 
eq y}$  being equal to one when x 
eq y and 0 else.

Some simplifications give immediately that

$$u_N(p_N, \tau_N) = \frac{\delta^{\tau_N}}{1-\delta}v_N - p_N$$
 if  $C_N$  buys the good from  $R_N$ ,  
 $u_N(p_F, \tau_F) = \frac{\delta^{\tau_F}}{1-\delta}v_N - p_F - \epsilon$  if  $C_N$  buys the good from  $R_F$ ,  
 $u_F(p_N, \tau_N) = \frac{\delta^{\tau_N}}{1-\delta} - p_N - \epsilon$  if  $C_F$  buys the good from  $R_N$ ,  
 $u_F(p_F, \tau_F) = \frac{\delta^{\tau_F}}{1-\delta} - p_F$  if he buys the good from  $R_F$ .

When choosing their supplier, each consumer will simply compare the net benefits in both cases. We assume that when a consumer is indifferent between both retailers, he chooses to buy the good in his own country. Therefore each individual demand is addressed entirely to  $R_F$  or  $R_N$ , depending on the prices and the delays. Let  $d_x(p_N, \tau_N, p_F, \tau_F)$  be the individual demand for x = N, F, we obtain

$$d_x(p_N, \tau_N, p_F, \tau_F) = \begin{cases} 1 & \text{to} \quad R_x & \text{if} \quad u_x(p_x, \tau_x) \ge u_x(p_y, \tau_y), \quad u_x(p_x, \tau_x) \ge 0\\ 1 & \text{to} \quad R_y & \text{if} \quad u_x(p_x, \tau_x) < u_x(p_y, \tau_y), \quad u_x(p_y, \tau_y) \ge 0 \\ 0 & \text{if} \quad \max\left\{u_x(p_x, \tau_x), u_x(p_y, \tau_y)\right\} < 0 \end{cases}$$
(1)

The payoffs of the retailers are therefore

$$\Pi_{N}((p_{N}, p_{F})|(\tau_{N}, \tau_{F})) = (p_{N} - w) \times D^{N}((p_{N}, \tau_{N}), (p_{F}, \tau_{F})) - T_{N}$$
(2)

$$\Pi_{F}((p_{N}, p_{F})|(\tau_{N}, \tau_{F})) = (p_{F} - w) \times D^{F}((p_{N}, \tau_{N}), (p_{F}, \tau_{F})) - T_{F}$$
(3)

where  $D^{\mathbf{y}}$  is the aggregated demand addressed to the retailer  $R^{\mathbf{y}}$  for a given pair of delivery dates  $(\tau_N, \tau_F)$  and a pair of prices  $(p_N, p_F)$ , y = N, F.

A Nash equilibrium in mixed strategy in the retail price competition subgame is defined as usual as a pair  $(\sigma_N^*, \sigma_F^*)$  such that

$$\mathbb{E}_{\sigma_N^*,\sigma_F^*}\Pi_N(\sigma_N^*,\sigma_F^*) \ge \mathbb{E}_{\sigma_F^*}\Pi_N(p_N,\sigma_F^*) \quad \forall p_N \in \mathcal{P}_N$$

and

$$\mathbb{E}_{\sigma_N^*,\sigma_F^*}\Pi_N(\sigma_N^*,\sigma_F^*) \ge \mathbb{E}_{\sigma_N^*}\Pi_F(\sigma_N^*,p_F) \quad \forall p_F \in \mathcal{P}_F$$

Once the subgame results are taken into account, the payoff of producer  $P_N$  is

$$\Pi_F(w,\tau_N,\tau_F) = w \times (D^N(\tau_N,\tau_F) + D^F(\tau_N,\tau_F)) + T_N + T_F$$
(4)

where  $D^{\mathbf{y}}(\tau_N, \tau_F)$  denotes the quantity asked to retailer  $R_{\mathbf{y}}$  in equilibrium for a given pair of delivery dates  $(\tau_N, \tau_F), \mathbf{y} = N, F$ .

The game tree depicted in figure 1 summarizes our model.

# 3 The benchmark case : no delays in delivery

To show that introducing different delays in deliveries is a major strategic concern for producer  $P_N$ , let us first consider the situation where there are no delays at all. In that case, if the difference between the gross consumers benefits is sufficiently large compared to the transaction cost  $\epsilon$ , both retailers are in direct competition, meaning that the profits of each of them are low. They are not able to extract the entire consumers surplus in that case. We may demonstrate that point by first looking to the demands addressed to each retailer and then derive the equilibrium in the subgame where  $\tau_N = \tau_F = 0$ .

Assume that

$$\frac{v_N - 1}{1 - \delta} \ge \epsilon \tag{5}$$

and let  $D^N(p_N, p_F)$  and  $D^F(p_N, p_F)$  be the aggregated demands addressed to each retailer, N and F, then

- If  $p_N > \frac{v_N}{1-\delta}$  and  $p_F > \frac{v_N}{1-\delta} \epsilon$ , then the demand addressed to each retailer is equal to 0,  $D^N(p_N, p_F) = 0$  and  $D^F(p_N, p_F) = 0$
- If  $p_N > p_F + \epsilon$ ,  $p_F \leq \frac{v_N}{1-\delta} \epsilon$  and  $p_F > \frac{1}{1-\delta}$ , then  $R_F$  capture the demand of  $C_N$  only and  $R_N$  does not sell,

$$D^N(p_N, p_F) = 0$$
 and  $D^F(p_N, p_F) = 1$  from  $C_N$ 

- If  $p_N > p_F + \epsilon$  and  $p_F \leq \frac{1}{1-\delta}$ , then  $R_F$  captures the demand of both consumers,  $D^N(p_N, p_F) = 0$  and  $D^F(p_N, p_F) = 2$  from  $C_N$  and  $C_F$
- If  $p_N \leq \frac{v_N}{1-\delta}$ ,  $p_N \leq p_F + \epsilon$ ,  $p_N \geq \frac{1}{1-\delta} \epsilon$ , and  $p_F > \frac{1}{1-\delta}$ , then  $R_N$  sells to his domestic consumer and  $R_F$  does not sell,

 $D^N(p_N, p_F) = 1$  from  $C_N$  and  $D^F(p_N, p_F) = 0$ 

- If  $p_N \ge p_F - \epsilon$ ,  $p_N \le p_F + \epsilon$ , and  $p_F \le \frac{1}{1-\delta}$ , then each retailer sells to his domestic consumer,

 $D^N(p_N,p_F)=1 \mbox{ from } C_N \mbox{ and } D^F(p_N,p_F)=1 \mbox{ from } C_F$ 

- If  $p_N < p_F - \epsilon$  and  $p_N \leq rac{1}{1-\delta} - \epsilon$ , then  $R_N$  captures the demand of both consumers,

$$D^{N}(p_{N}, p_{F}) = 2$$
 from  $C_{N}$  and  $C_{F}$  and  $D^{F}(p_{N}, p_{F}) = 0$ 

We may represent in the plan  $(p_N, p_F)$  the demands obtained by each retailer for a given pair of prices.

#### [Insert figure 2 here ]

Let (in this section)  $\Pi_N(p_N, p_F)$  and  $\Pi_F(p_N, p_F)$  be the payoffs of the retailers when the good may de delivered at date 0 by the producer. We may establish the following lemma in order to characterize the equilibrium in  $(p_N, p_F)$  given a wholesale price w.

**Lemma 1** If the difference between the consumers gross benefits is large enough, i.e. if (5) holds, there exists a mixed strategy equilibrium to the price competition between the retailers. The equilibrium expected payoffs for any wholesale price are given by

- $(E\Pi_N^*, E\Pi_F^*) = (0, 0)$  if  $w > \frac{v_N}{1-\delta}$
- $(\mathbb{E}\Pi_N^*, \mathbb{E}\Pi_F^*) = (\frac{v_N}{1-\delta} w, 0)$  if  $w \in ]\frac{v_N}{1-\delta} \epsilon$ :  $\frac{v_N}{1-\delta}]$

- $(\mathbb{E}\Pi_N^*, \mathbb{E}\Pi_F^*) = (\epsilon, 0) \text{ if } w \in ]\frac{1}{1-\delta} : \frac{v_N}{1-\delta} \epsilon]$
- $(\mathbb{E}\Pi_N^*, \mathbb{E}\Pi_F^*) = (\epsilon, \frac{1}{1-\delta} w) \text{ if } w \in ]\frac{1}{1-\delta} \epsilon: \frac{1}{1-\delta}]$
- $(E\Pi_N^*, E\Pi_F^*) = (\epsilon, \epsilon)$  if  $w \in [0; \frac{1}{1-\delta} \epsilon]$

*Proof* : Proving the existence of a mixed strategy equilibrium in every price subgame may be done by using theorems 5 and 5a given in Dasgupta-Maskin [1986]. Adapted to our framework their results may be basically summarized in the following manner :

(Adapted from Dasgupta-Maskin [1986], theorems 5 and 5a) Let  $\mathcal{P}_N$  and  $\mathcal{P}_F$  be two closed intervals, supports of the pure strategies of retailers  $R_N$  and  $R_F$ , then if

- Payoffs  $\Pi_N(p_N, p_F)$  and  $\Pi_F(p_N, p_F)$  are continuous except on a subset of a set of prices  $\mathcal{P}(N) = \{(p_N, p_F) \in \mathcal{P}_N \times \mathcal{P}_F / p_N = f(p_F)\}$  for retailer  $R_N$  and  $\mathcal{P}(F) = \{(p_N, p_F) \in \mathcal{P}_N \times \mathcal{P}_F / p_N = g(p_F)\}$  for retailer  $R_F$ , where f and g are one-to-one functions,
- The sum of payoffs  $\Pi_N(p_N, p_F) + \Pi_F(p_N, p_F)$  is upper hemi-continuous, i.e. is such that for any sequence  $\{p^n\} \in \mathcal{P}_N \times \mathcal{P}_F$  such that  $\{p^n\}$  converges to  $p \in \mathcal{P}_N \times \mathcal{P}_F$ ,  $\limsup_{n \to +\infty} \Pi_N(p^n) + \Pi_F(p^n) \leq \Pi_N(p) + \Pi_F(p)$
- $\Pi_x(p_x, p_y)$  is bounded and weakly lower hemi-continuous in  $p_x$ , i.e. is such that  $\forall \overline{p}_x \in \mathcal{P}(x), \exists \lambda \in [0; 1]$  such that  $\forall p_y, \overline{p}_x = g(p_y)$ ,

$$\begin{split} \lambda \liminf_{p_x \to \overline{p_x}} \Pi_x(p_x, p_y) + (1 - \lambda) \liminf_{p_x \to \overline{p_x}} \Pi_x(p_x, p_y) \geq \Pi_x(\overline{p}_x, p_y) \text{ for } x \neq y, \ x, y = N, F, \end{split}$$

and if moreover for some x = N, F there exists  $\widehat{p}_x \in P_x$  such that  $\forall \overline{p}_y \in P_y$ ,

$$\begin{array}{ll} -\lim_{p_x\to\,\widehat{p}_x} & \Pi_x(p_x,p_y) \text{ exists and is equal to } \Pi_x(\widehat{p}_x,\overline{p}_y), \\ & p_y\to\overline{p}_y \end{array}$$

-  $\lim_{p_x \to \hat{p}_x^+} \Pi_x(p_x, \bar{p}_y)$  exists, is less or equal to  $\Pi_x(\hat{p}_x, \bar{p}_y)$ , and is continuous in  $\bar{p}_y$ 

Then the game has a mixed strategy equilibrium.

First we check that these conditions are verified in our model, in order to apply directly the tools developed by Dasgupta and Maskin without writing the proof. Then we will turn to the characterization of the mixed strategy equilibria in this game. Clearly the supports  $\mathcal{P}_N$  and  $\mathcal{P}_F$  are closed, since they may be restricted to any closed and bounded interval containing  $[0: \max\{\frac{v_N}{1-\delta}, \frac{1}{1-\delta} - \epsilon\}]$  for  $R_N$  and  $[0: \max\{\frac{v_N}{1-\delta} - \epsilon, \frac{1}{1-\delta}\}]$  for  $R_F$ . Furthermore no retailer will charge a negative retail price.

Taking as given the fees paid by each retailer to the producer, their payoff functions are

$$\Pi_N(p_N, p_F) = \begin{vmatrix} 0 & \text{if } p_N > \frac{v_N}{1-\delta}, p_F > \frac{v_N}{1-\delta} \\ 0 & \text{if } p_N > p_F + \epsilon, p_F \le \frac{v_N}{1-\delta} - \epsilon, p_F > \frac{1}{1-\delta} \\ 0 & \text{if } p_N > p_F + \epsilon, p_F \le \frac{1}{1-\delta} \\ p_N - w & \text{if } p_N \le \frac{v_N}{1-\delta}, p_N \le p_F + \epsilon, p_N \ge \frac{1}{1-\delta} - \epsilon, p_F > \frac{1}{1-\delta} \\ p_N - w & \text{if } p_N \ge p_F - \epsilon, p_N \le p_F + \epsilon, p_F \le \frac{1}{1-\delta} \\ 2 \times (p_N - w) & \text{if } p_N \le p_F - \epsilon, p_N \le \frac{1}{1-\delta} - \epsilon \end{vmatrix}$$

and

$$\Pi_F(p_N, p_F) = \begin{vmatrix} 0 & \text{if} \quad p_N > \frac{v_N}{1-\delta}, p_F > \frac{v_N}{1-\delta} \\ p_F - w & \text{if} \quad p_N > p_F + \epsilon, p_F \le \frac{v_N}{1-\delta} - \epsilon, p_F > \frac{1}{1-\delta} \\ 2 \times (p_F - w) & \text{if} \quad p_N > p_F + \epsilon, p_F \le \frac{1}{1-\delta} \\ 0 & \text{if} \quad p_N \le \frac{v_N}{1-\delta}, p_N \le p_F + \epsilon, p_N \ge \frac{1}{1-\delta} - \epsilon, p_F > \frac{1}{1-\delta} \\ p_F - w & \text{if} \quad p_N \ge p_F - \epsilon, p_N \le p_F + \epsilon, p_F \le \frac{1}{1-\delta} \\ 0 & \text{if} \quad p_N \le p_F - \epsilon, p_N \le \frac{1}{1-\delta} - \epsilon \end{vmatrix}$$

Checking that all the conditions are verified may be done by choosing 8 different points (1 on each line in the neighborhood of which the payoffs are discontinuous), and examining the limits of the payoffs when prices converge to these points. Our formulation satisfies these conditions and therefore a mixed strategy equilibrium exists.

Let us turn to the characterization of the equilibrium of the game. Obviously if  $P_N$  charges a wholesale price higher than the maximum willingness to pay  $w > \frac{v_N}{1-\delta}$  no retailer will sell on the market. Setting their retail prices at w insures them that nobody will buy the good, inducing a 0 profit.

If  $P_N$  charges  $w \in ]\frac{v_N}{1-\delta} - \epsilon : \frac{v_N}{1-\delta}]$ ,  $R_F$  cannot do a positive profit and obtain the demand of  $C_N$ . Then  $R_F$  does not want to sell and is indifferent between all the prices strictly higher than  $\frac{v_N}{1-\delta} - epsilon$ . Then  $R_N$  is able to extract all the surplus on his domestic market, by charging  $p_N^* = \frac{v_N}{1-\delta}$ . In equilibrium  $C_N$  buys the good from  $R_N$  and  $C_F$  does not buy at all.

If  $P_N$  charges  $w \in \left[\frac{1}{1-\delta}: \frac{v_N}{1-\delta} - \epsilon\right]$ , each retailer may realize a positive profit:  $R_F$  may compete to obtain the demand of  $C_N$ , but is unable to serve his own consumer  $C_F$ . The equilibrium prices are then such that  $p_N^* = w + \epsilon$  and  $p_F^* = w$ , where consumer  $C_N$  buys the good from  $R_N$ . Indeed as long as he may realize a positive profit,  $R_F$  will compete in price to obtain the demand of  $C_N$ : the decreasing auction stops when  $R_F$  offers w, and  $R_N$  the first price such that he keeps  $C_N$ , i.e.  $w + \epsilon$ .

If  $P_N$  charges  $w \in \left]\frac{1}{1-\delta} - \epsilon : \frac{1}{1-\delta}\right]$ , the retail price competition subgame does not possess a pure strategy Nash equilibrium. To check this point, let us consider the natural candidate  $(p_N^*, p_F^*) = (w + \epsilon, w)$ . Since  $R_F$  is not able to capture the demand of  $C_N$  and has to sell only the good to his customer  $C_F$ , he is better off by increasing his price up to  $p_F = \frac{1}{1-\delta}$ : his profit is strictly higher and he does not loose  $C_F$  by doing this, since  $R_N$  can never find profitable to capture  $C_F$  for this wholesale price. But  $R_N$  is then better off by increasing his price up to  $p_N^* = \frac{1}{1-\delta} + \epsilon$ , since he still keeps the demand of  $C_N$  but earns a higher revenue for the single unity he sells. May  $(p_N^*, p_F^*) = (\frac{1}{1-\delta} + \epsilon, \frac{1}{1-\delta})$  be an equilibrium ? Neither, since  $R_F$  will always lower his price to try to obtain all the demand from  $C_N$  and  $C_F$ . This argument my be applied to any pair of prices, and therefore the game admits only (at least one) mixed strategy equilibrium. We will determine it by restricting first the support of the strategies in equilibrium, and then determining the equilibrium itself.

The supports of the Nash equilibrium in mixed strategy  $\operatorname{Supp}(\sigma_N)$  and  $\operatorname{Supp}(\sigma_F)$  are necessarily such that  $\operatorname{Supp}(\sigma_N) \subseteq [w + \epsilon; \frac{1}{1-\delta} + \epsilon]$  and  $\operatorname{Supp}(\sigma_F) \subseteq [w; \frac{1}{1-\delta}]$ . There is no point in using prices such that the expected profit is equal to zero or is negative whatever the choice of the opponent. Consequently retailer  $R_N$  will never charge less than  $w + \epsilon$  since  $R_F$  is unable to capture the demand of  $C_N$  in that case. Therefore  $R_F$  never charges less than w. Moreover  $R_N$ never charges more than  $\frac{1}{1-\delta} + \epsilon$  since  $R_F$  will always to undercut  $R_N$  leaving him a 0 profit.

Without loss of generality assume that the supports are given by two intervals  $[p_N^{\ell}; p_N^h]$  for retailer  $R_N$  and  $[p_F^{\ell}; p_F^h]$  for retailer  $R_F$ . In equilibrium the mower and upper bounds for each firm are such that

- (i)  $(p_N^h, p_F^h)$  is such that  $p_N^h \le p_F^h + \epsilon$ ,  $p_N^h \ge w + \epsilon$ , and  $p_F^h \le \frac{1}{1-\delta}$ ,
- (ii)  $(p_N^\ell, p_F^\ell)$  is such that  $p_N^\ell \leq p_F^\ell + \epsilon, \ p_N^\ell \geq w + \epsilon$ , and  $p_F^\ell \leq \frac{1}{1-\delta}$ ,
- (iii)  $(p_N^{\ell}, p_F^{h})$  is such that  $p_N^{\ell} \le p_F^{h} + \epsilon$ ,  $p_N^{\ell} \ge w + \epsilon$ , and  $p_F^{h} \le \frac{1}{1-\delta}$ ,
- $(\text{iv}) \ (p_N^h, p_F^\ell) \text{ is such that } p_N^h \ge p_F^\ell + \epsilon, \ p_N^h \le \frac{1}{1-\delta} + \epsilon, \text{ and } p_F^\ell \le \frac{1}{1-\delta}.$

We start by proving (i). Clearly  $(p_N^h, p_F^h)$  cannot be such that  $R_N$  does not sell: for any equilibrium choice  $p_F^h$ ,  $R_N$  will reduce his price in order to keep his demand and increase his

expected payoff. Therefore (i) must be true in equilibrium, causing (iii). Is it possible for (iv) not to be true ? No since it would mean that all the prices on which both players are randomizing are such that each retailer sells the good to his consumer. In that case, since the payoffs are linear in the price charged, each firm prefers to play in pure strategy, meaning that  $p_x^\ell = p_x^h$  for all x = N, F, but this is not possible since we have seen that no pure strategy equilibrium exists. Then (iv) is true. Finally (ii) has also to be true :  $p_F^\ell$  cannot be such that  $(p_N^\ell, p_F^\ell)$  belongs to the region where  $R_F$  sells 2 units, since increasing the price increases the expected payoff. Once  $p_F^\ell$  is such that  $(p_N^\ell, p_F^\ell)$  belongs to the region where both retailers sell exactly 1 unit,  $p_F^\ell$  has to be such that  $R_F$  cannot increase his expected payoff by raising  $p_F^\ell$ , i.e.  $R_F$  has to be indifferent between the extra gain obtained by a slight increase in price and the extra cost resulting from the fact that the region where  $R_F$  sells only one unit has a higher probability and the region where  $R_F$  sells 2 units a lower probability.

Once the supports are defined, we may characterize the expected payoffs of each retailer in equilibrium. First we show that a mixed strategy equilibrium with randomization on two prices exists. The pair of strategies  $(\sigma_F^*, \sigma_N^*)$  is an equilibrium if each player is indifferent between the two pure strategies constituting his mixed strategy. Let  $\eta_N^h$  the probability attached to the high price  $p_N^h$  for player  $R_N$ , and  $\eta_F^h$  for player  $R_F$ . These probabilities have to satisfy in equilibrium

$$\mathbb{E}_{\sigma_F^*} \Pi_N(p_N^\ell, \sigma_F^*) = \mathbb{E}_{\sigma_F^*} \Pi_N(p_N^h, \sigma_F^*)$$

which gives

$$\eta_F^h(p_N^\ell - w) + (1 - \eta_F^h)(p_N^\ell - w) = \eta_F^h(p_N^h - w) \Leftrightarrow \eta_F^h = \frac{(p_N^\ell - w)}{(p_N^h - w)} \ge 1$$

and

$$\mathbb{E}_{\sigma_N^*} \Pi_N(\sigma_N^*, p_N^\ell) = \mathbb{E}_{\sigma_N^*} \Pi_F(\sigma_N^*, p_F^h)$$

which gives

$$\eta_N^h 2(p_F^\ell - w) + (1 - \eta_N^h)(p_F^\ell - w) = \eta_N^h(p_F^h - w) + (1 - \eta_N^h)(p_F^h - w) \Leftrightarrow \eta_N^h = \frac{(p_F^h - w)}{(p_F^\ell - w)} - 1 < 1$$

and has to satisfy  $p_F^h \ge 2p_F^\ell - w$ .

The expected payoffs of the retailers in equilibrium are therefore straightforward to obtain

$$\mathbf{E}_{\sigma_N^*,\sigma_F^*} \Pi_N(\sigma_N^*,\sigma_F^*) = p_N^\ell - w \quad \text{and} \quad \mathbf{E}_{\sigma_N^*,\sigma_F^*} \Pi_F(\sigma_N^*,\sigma_F^*) = p_F^h - w$$

It remains to determine the values of  $p_N^{\ell}$  and  $p_F^{h}$  in equilibrium. First of all remark that  $p_F^{h} = \frac{1}{1-\delta}$  in equilibrium. The reason is as follows: whatever the price of  $R_N$  (low or high),

the equilibrium will always be such that each retailer will sell the good to his own customer. In that case the high price strategy for  $R_F$  is always to sell at  $p_F^h = \frac{1}{1-\delta}$  which generates the highest profit. Finally the low price strategy in equilibrium for  $R_N$  is  $p_N^\ell = w + \epsilon$ : no higher low price for  $R_N$  may be part of an equilibrium strategy since in that case  $R_F$  could always have randomize on low prices giving him the entire demand from both consumers. Then  $R_N$  would have been better off doing a profit equal to  $\epsilon$  than earning a zero profit, and then he has an incentive to charge a lower price, the lowest in the support of his mixed strategy, namely  $p_N^\ell = w + \epsilon$ .

The prices part of the equilibrium strategies have also to satisfy

$$p_F^\ell \geq \frac{w}{2} + \frac{1}{2(1-\delta)}, \quad p_N^h > p_F^\ell + \epsilon, \quad p_N^h < \frac{1}{1-\delta} + \epsilon$$

in order for  $\eta_F^h$  to be well defined, and in order to satisfy (i)-(iv). Indeed  $p_N^h = \frac{1}{1-\delta} + \epsilon$  can never be part of an equilibrium strategy:  $R_F$  by slightly decreasing his high price strategy will be able to capture both consumers leaving a 0 profit to  $R_N$ . The same argument applies to check that  $p_N^h > p_F^\ell + \epsilon$ : in equilibrium  $R_N$  has to loose the market when  $R_F$  plays his low price strategy. Therefore when  $P_N$  charges  $w \in \left[\frac{1}{1-\delta} - \epsilon; \frac{1}{1-\delta}\right]$ , there exist several mixed strategy Nash equilibrium, such that the expected payoffs of the retailers are

$$\mathbb{E}\Pi_N(\sigma_N^*,\sigma_F^*)=\epsilon \quad ext{and} \quad \mathbb{E}\Pi_F(\sigma_N^*,\sigma_F^*)=rac{1}{1-\delta}-w$$

If  $P_N$  charges a wholesale price  $w \in ]0: \frac{1}{1-\delta} - \epsilon]$ , the retail price competition subgame does not possess a pure strategy Nash equilibrium similarly to the previous case.  $(p_N, p_F) = (w, w)$ cannot be an equilibrium since every retailer has an incentive to increase his price up to  $w + \epsilon$ .  $(p_N, p_F) = (w + \epsilon, w)$  and  $(p_N, p_F) = (w, w + \epsilon)$  cannot be equilibrium pairs since again the low price retailer has an incentive to increase his price. Finally price pairs such that both retailers are charging a high price and each of them serves his own consumer cannot be equilibrium, since each retailer has always an incentive to increase his price. Border price pairs such that  $p_x = p_y + \epsilon$  for  $x \neq y$  cannot be equilibrium neither since a retailer has always an incentive to undercut his rival and take all the market. Therefore the subgame does not admit a pure strategy Nash equilibrium.

Again we may characterize the main properties of the mixed strategy Nash equilibrium. As before the supports are  $\operatorname{Supp}(\sigma_N) = [w + \epsilon; \frac{1}{1-\delta} + \epsilon]$  and  $\operatorname{Supp}(\sigma_F) = [w + \epsilon; \frac{1}{1-\delta}]$ . There is no point in using prices such that the expected profit is equal to zero or is negative whatever the choice of the opponent. Consequently retailer  $R_N$  will never charge less than  $w + \epsilon$  since  $R_F$  is unable to capture the demand of  $C_N$  in that case, and symmetrically for  $p_F \geq w + \epsilon$ .

We may then establish the following proposition.

**Proposition 1** In absence of delays in the delivery, producer  $P_N$  is unable to price discriminate the demand.

*Proof*: This result is the immediate consequence of the lemma 1. Whatever the wholesale price he is choosing, the producer is unable to restrict the competition between the retailers since the differential in willingness to pay is high enough to start the price war. Depending on the values of the parameters the equilibrium will be to choose w such that every consumer buys the good or on the contrary only one consumer,  $C_N$ , buys.

Now let us turn to the general case where the producer  $P_N$  introduces delays in the delivery to the retailers.

## 4 Strategic delivery

When the producer  $P_N$  is able to impose delays in the delivery to his retailers, the nature of the competition in the subgame changes. We may see this easily by characterizing the demands addressed to the retailers for a given pair of delays  $(\tau_N, \tau_F)$ . The conditions in (1) may be re-expressed as follows.

Consumer  $C_N$  buys from retailer  $R_N$  if his net benefit is higher than the benefit obtained by trading with  $R_F$ 

$$u_N(p_N,\tau_N) \ge u_N(p_F,\tau_F) \quad \Leftrightarrow \quad p_N \le p_F + \epsilon + \frac{\delta^{\tau_N} - \delta^{\tau_F}}{1 - \delta} v_N \tag{6}$$

Buying to  $R_N$  gives  $C_N$  a positive benefit if

$$u_N(p_N, \tau_N) \ge 0 \quad \Leftrightarrow \quad p_N \le \frac{\delta^{\tau_N}}{1-\delta} v_N$$
(7)

and buying to  $R_F$  gives  $C_N$  a positive benefit if

$$u_N(p_F, \tau_F) \ge 0 \quad \Leftrightarrow \quad p_F + \epsilon \le \frac{\delta^{\tau_F}}{1 - \delta} v_N$$
(8)

Reciprocally consumer  $C_F$  will buy from retailer  $R_F$  if the net benefit he obtains from consuming is positive and higher than the benefit of buying to retailer  $R_N$ ,

$$u_F(p_F, \tau_F) \ge u_F(p_N, \tau_N) \quad \Leftrightarrow \quad p_N \ge p_F - \epsilon + \frac{\delta^{\tau_N} - \delta^{\tau_F}}{1 - \delta} \tag{9}$$

The net benefit is then positive if

$$u_F(p_N, \tau_N) \ge 0 \quad \Leftrightarrow \quad p_N + \epsilon \le \frac{\delta^{\tau_N}}{1 - \delta}$$
 (10)

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$$u_F(p_F, \tau_F) \ge 0 \quad \Leftrightarrow \quad p_F \le \frac{\delta^{\tau_F}}{1-\delta}$$
 (11)

When aggregating the individual demands there are three cases depending on the delays in delivery on each market.

If the delays are such that

$$\frac{\delta^{\tau_F}}{1-\delta}v_N - \epsilon \geq \frac{\delta^{\tau_F}}{1-\delta} \quad \text{and} \quad \frac{\delta^{\tau_N} - \delta^{\tau_F}}{1-\delta}v_N + \epsilon > \frac{\delta^{\tau_N} - \delta^{\tau_F}}{1-\delta}$$

then the demands addressed to each firm are

We may summarize this graphically

If the delays are such that

$$\frac{\delta^{\tau_F}}{1-\delta}v_N - \epsilon < \frac{\delta^{\tau_F}}{1-\delta} \quad \text{and} \quad \frac{\delta^{\tau_N} - \delta^{\tau_F}}{1-\delta}v_N + \epsilon > \frac{\delta^{\tau_N} - \delta^{\tau_F}}{1-\delta}$$

then the demands addressed to each firm are

We may summarize this graphically

If the delays are such that

$$\frac{\delta^{\tau_F}}{1-\delta}v_N - \epsilon \geq \frac{\delta^{\tau_F}}{1-\delta} \quad \text{and} \quad \frac{\delta^{\tau_N} - \delta^{\tau_F}}{1-\delta}v_N + \epsilon \leq \frac{\delta^{\tau_N} - \delta^{\tau_F}}{1-\delta}$$

then the demands addressed to each firm are

We may summarize this graphically

Instead of characterizing all the equilibria in all the subgames, let us first look at the situation in which there is no delay for  $R_N$  and a positive delay for  $R_F$ . Start in a situation where without delays on both markets, retailers are always in frontal price competition, i.e.

 $\frac{v_N-1}{1-\delta} > \epsilon$ . In that case increasing the delay changes the demand: as long as  $\tau_F$  is low and for  $\tau_N = 0$ , the parameters are such that

$$\delta^{\tau_F} \frac{v_N - 1}{1 - \delta} > \epsilon$$

and

$$(1-\delta^{\tau_F})\frac{v_N-1}{1-\delta}>0>-2\epsilon$$

therefore the demands are given by figure 3.

When  $\tau_F$  is high enough, since  $\delta < 1$ , the demands are given by figure 4, since parameters are such that

$$\delta^{\tau_F} \frac{v_N - 1}{1 - \delta} < \epsilon$$

and

$$(1-\delta^{\tau_F})\frac{v_N-1}{1-\delta}>0>-2\epsilon$$

Characterizing the equilibrium may be done as for lemma 1, with again the need to look at mixed strategy equilibrium when w is low. However a subgame equilibrium particularly interesting is the following. When  $P_N$  charges a wholesale price such that  $w \in [\delta^{\tau_F} \frac{v_N}{1-\delta} - \epsilon; \delta^{\tau_F} \frac{1}{1-\delta}]$ , the equilibrium prices are  $(p_N^*, p_F^*) = (\frac{v_N}{1-\delta}, \delta^{\tau_F} \frac{1}{1-\delta})$ . Indeed there is no point in deviating for any retailer in that case: no increase or decrease in prices can improve their profits. The following proposition is straightforward.

**Proposition 2** If the producer  $P_N$  may choose the delays for delivering the good when he decides the wholesale prices, he is able to discriminate the demand by charging a unique wholesale price higher than his marginal cost of production.

*Proof* : The intuitive argument is the following. Start in the situation where the difference between the gross benefits from consuming is large enough, i.e.  $\frac{v_N-1}{1-\delta} > \epsilon$ . In that case the demands are given as in figure 2. We know that the price competition subgame ends in a situation where the producer either looses a market or looses a large amount of profit. What happens if  $\tau_N = 0$  and  $\tau_F$  increases ? The demands are for a while given as in figure 3, but when  $\tau_F$  is high enough, the demands are given as in figure 4. The condition to be in "figure 4 configuration" when  $\tau_N = 0$  is

$$\frac{\delta^{\tau_F}}{1-\delta}v_N - \epsilon < \frac{\delta^{\tau_F}}{1-\delta} \quad \text{and} \quad \frac{1-\delta^{\tau_F}}{1-\delta}v_N + \epsilon > \frac{1-\delta^{\tau_F}}{1-\delta} - \epsilon$$

The second condition is always true. The first condition resumes to

$$\delta^{\tau_F} \frac{v_N - 1}{1 - \delta} < \epsilon \quad \Leftrightarrow \quad \delta^{\tau_F} < \frac{\epsilon(1 - \delta)}{v_N - 1}$$

which gives

$$\tau_F > \ln(\frac{\epsilon(1-\delta)}{v_N - 1}) / \ln \delta > 0$$

Choosing a wholesale price  $w \in \left[\frac{\delta^{r_F}}{1-\delta}v_N - \epsilon; \frac{\delta^{r_F}}{1-\delta}\right]$  induces a pure strategy equilibrium given by

$$(p_N^*,p_F^*)=(\frac{1}{1-\delta}v_N,\frac{\delta^{\tau_F}}{1-\delta})$$

and this equilibrium will be more profitable for  $P_N$ . Clearly the consumers are doing a 0 benefit in that case, and the producer will be able to obtain all the profit in the game through the fees for example, or through the wholesale prices in some cases if he may discriminate ion a "sensible and coherent manner" with respect to competition policy (w has to be lower for  $R_F$ than  $R_N$ ).

Since the utility of  $C_F$  diminishes with  $\tau_F$ ,  $P_N$  chooses the first delay such that he may separate the markets,  $\tau_F$  solution of

$$\frac{\delta^{\tau_F}}{1-\delta}v_N-\epsilon=\frac{\delta^{\tau_F}}{1-\delta}$$

and he charges  $w = \delta^{ au_F} rac{1}{1-\delta} . \|$ 

**Proposition 3** By price discriminating through the delays, a producer obtains the entire surplus of the economy but the economic welfare is lower than without delays: one consumer,  $C_F$ , gets a lower utility from buying the good later.

*Proof* : Assume that the production cost is equal to 0. Then the profit of  $P_N$  is equal to

$$\Pi = \frac{v_N}{1-\delta} + \frac{\delta^{\tau_F}}{1-\delta}$$

The economic welfare without delays is equal to

$$EW = \frac{v_N}{1-\delta} + \frac{1}{1-\delta}$$

which is clearly higher. ||

Now let us discuss our model and results.

# 5 Discussion

Introducing the possibility for consumers to resell the good on a second hand market does not change the results: since  $C_F$  obtains the good later than  $C_N$ , there is no possibility for  $C_F$  to compete with  $R_N$  through the second-hand market to supply  $C_N$ . However if the good depreciates, perhaps a good deal could be for  $C_N$  to resell on the second-hand market to  $C_F$ and buy a new unit from  $R_F$ , depending on the speed at which the good depreciates and on the individual willingness to pay. What are the equilibrium delays in that case ?

In order to discriminate efficiently (i.e. without letting the revenues decrease too much because of time), the producer  $P_N$  has to charge a price higher than his marginal cost. If there are more consumers, for example if they are distributed on a line with a linear or quadratic transportation cost, this could induce a double-marginalization in the subgame. Therefore firms will sell less: the discrimination strategy through delivery delays will have an additional cost, since less consumers will be able to buy the good at the monopoly price charged by the retailers. To fight against this effect the producer will have to increase the delay, in order to reduce the the difference between the wholesale price and his marginal cost. There will be a trade-off between discrimination and coverage of the market.

The second remark concerns the introduction of competition between producers, with new retailers on each market. In that case increasing the delay may be dangerous since the retailer may loose consumers on the market where the delay is long. However this effect depends drastically on the differentiation between the two producers: if the goods they are producing are close substitutes (i.e. if the retailers are close on the Hotelling line), clearly increasing the delays will hurt a producer with a third effect, an increased competition made by his competitor if his retailer benefits from short delays. However if the goods are differentiated enough (i.e. if the retailers are far one from the other on the Hotelling line), increasing the delay will not decrease the demand addressed to the retailer by a large amount, and then a incentive to discriminate through this method will reappear.

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# 6 Appendix: graphics

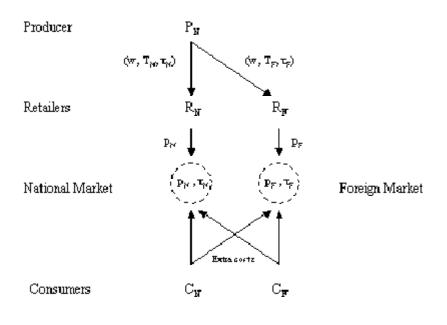


Figure 1: The game tree

Figure 1: The Game Tree

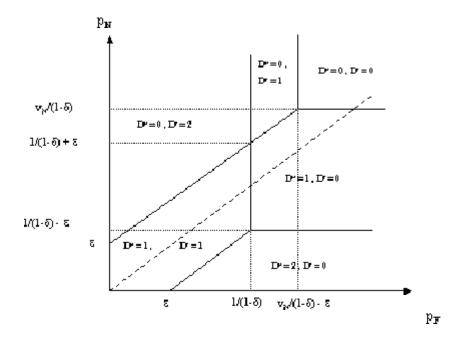
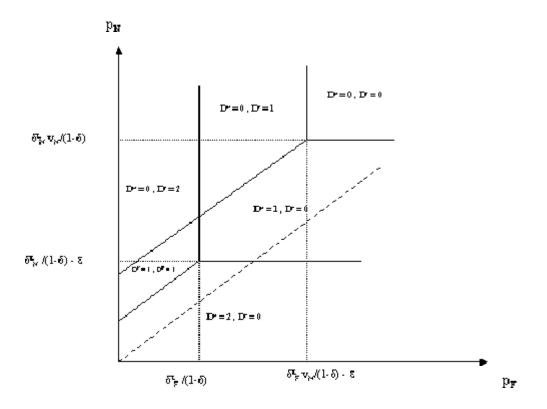


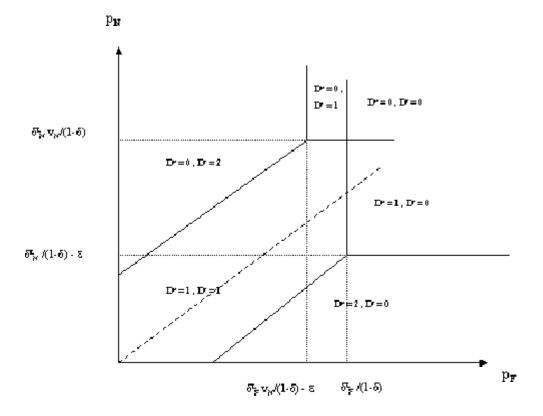
Figure 2: Demands when  $(v_{N'}1)(1-\delta) \geq \varepsilon$  for  $t_{M} = t_{F} = 0$ 

Figure 2: Demands when  $\frac{v_N-1}{1-\delta} > \epsilon$  for  $\tau_N = \tau_F = 0$ .



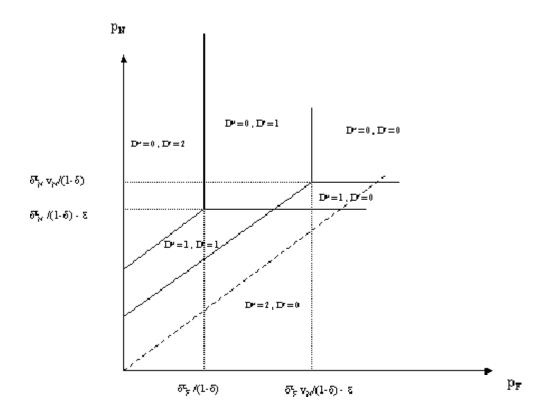
 $Figure \ 3: Demands \ when \ \delta_F \ v_N / (1 \cdot \delta) \cdot \epsilon \geq \ \delta_F / (1 \cdot \delta) \ \text{and} \ (\delta_N^* - \ \delta_F) \ v_N / (1 \cdot \delta) + \epsilon \geq (\delta_N^* + \ \delta_F) / (1 \cdot \delta) \cdot \epsilon$ 

Figure 3: Demands when  $\delta^{\tau_F} \frac{v_N-1}{1-\delta} \ge \epsilon$  and  $(\delta^{\tau_N} - \delta^{\tau_F}) \frac{v_N-1}{1-\delta} > -2\epsilon$ .



 $Figure \ 4: Demands \ when \ \delta_F \ v_{\mu}/(1 \cdot \delta) + \epsilon \leq \delta_F \ /(1 \cdot \delta) \ \text{and} \ (\delta_W^* - \delta_F^*) \ v_{\mu}/(1 \cdot \delta) + \epsilon \geq (\delta_W^* + \delta_F^*) \ /(1 \cdot \delta) + \epsilon$ 

Figure 4: Demands when  $\delta^{\tau_F} \frac{v_N-1}{1-\delta} < \epsilon$  and  $(\delta^{\tau_N} - \delta^{\tau_F}) \frac{v_N-1}{1-\delta} > -2\epsilon$ .



 $Figure \ 5: Demands \ when \ \delta_F \ v_N / (1 \cdot \delta) + \epsilon \geq \ \delta_F / (1 \cdot \delta) \ \text{and} \ (\delta_N^* - \ \delta_F) \ v_N / (1 \cdot \delta) + \epsilon \leq (\delta_N^* - \ \delta_F) / (1 \cdot \delta) + \epsilon$ 

Figure 5: Demands when  $\delta^{\tau_F} \frac{v_N-1}{1-\delta} \ge \epsilon$  and  $(\delta^{\tau_N} - \delta^{\tau_F}) \frac{v_N-1}{1-\delta} \le -2\epsilon$ .