Contracts and Inequity Aversion*

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Abstract

Inequity aversion is a special form of other regarding preferences and captures many features of reciprocal behavior, an apparently robust pattern in human nature. Using this concept we analyze the Moral Hazard problem and derive several results which differ from conventional contract theory. Our three key insights are: First, inequity aversion plays a crucial role in the design of optimal contracts. Second, there is a strong tendency towards linear sharing rules, giving a simple and plausible rationale for the prevalence of these schemes in the real world. Third, the Sufficient Statistics result no longer holds as optimal contracts may be "too" complete. Along with these key insights we derive a couple of further results, e.g. that the First Best contract for risk neutral agents is unique or that for risk neutral agents the First Best is not implementable if effort is not observable.

1 Introduction

> *It is when equals have or are assigned unequal shares, or people who are not equal, equal shares, that quarrels and complaints break out.*
>
> *Aristotle*

Inequity aversion is a special form of other regarding preferences. Capturing many features of reciprocal behavior, an apparently robust pattern in human nature, it was formulated by Ernst Fehr und Klaus Schmidt (1999) only recently[1]. The approach adheres rather closely to orthodox utility theory but is well capable of explaining a wide array of phenomena.

Inequity averse people do not only care about their own payoff, but also about how payoffs are distributed amongst their mates. Inequitable distributions cause them disutility. This disutility from inequitable distributions may outweigh the utility created by personal payoffs. So people would rather forego profits than accept inequitable distributions. This is a very stable pattern in many empirical and experimental studies. One example is the well known ultimatum game, where the "responders" usually turn down "unfair" offers, although these offers would still assign them an appreciable amount of money.[2]

Reciprocal patterns and notions of fairness seem to play an important role in human interactions and especially in labour markets, where people work together closely and earn their income.[3] However, so far nobody has investigated to what extent the incentive structure of labor contracts has to adopt to the prevalence of norms of reciprocity. In this paper we tackle the standard moral hazard problem and

[1]However, inequity aversion is not really a new concept. In the 1960's it was already discussed by sociologists. See e.g. Gouldner (1960), Goranson and Berkowitz (1966) or Berkowitz (1968). For a discussion of previous attempts to model concerns for equity in the economics literature refer to Austin (1977), Selten (1978), Young (1994) or Selten (1998).

[2]See Fehr, Gaechter and Kirchsteiger (1997) for reference on ultimatum game studies and Fehr and Schmidt (1999) for a survey of experimental and empirical studies

[3]See for example Akerlof(1982), Akerlof and Yellen (1990) and Fehr and Falk (1999).

introduce inequity averse agents.[4] From that several new results, shedding light on some well known puzzles in standard contract theory, are derived. First it is shown that the First Best contract for risk neutral agents is unique. The second result says that for risk neutral agents the First Best is not implementable if effort is not observable. The third point we make is that the Sufficient Statistics result does no longer hold. Finally it is proven that the optimal contract is linear in the First Best under risk neutrality and in the Second Best in the limit of strong inequity aversion. These results are partly in contradiction to orthodox theory.

In the real world we often observe linear contracts. Examples range from the persistence of sharecropping to the compensation schemes used by most start up businesses, where it is very common to pay the employees a rather low fixed wage but to give them in turn a share of the company's equity.

While standard economic theory predicts a much more complex and - from a practical point of view - generally undetermined structure to be the optimal solution to the principal agent problem (see e.g. Holmström (1979) or Mirrlees (1999)) there are only few attempts to explain the occurrence of linear contracts. Holmström and Milgrom (1987) consider a setting where the agent controls the drift rate of a Brownian motion. They show that the optimal contract is - for a rather specific setting - linear in overall outcome. However, the Holmström and Milgrom result depends very delicately on the assumptions they make on the stochastic process and on the form of the utility function.[5]

Innes (1990) assumes instead that the agent is risk neutral but wealth constrained. Then the optimal contract makes the agent the residual claimant if the outcome is such that it exceeds his wealth. In those regions the contract takes a linear form. This implies that the optimal contract has a slope of one, something which we rarely observe.

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[4]There are some other problems, too, that have been approached by using inequity aversion. There have been attempts to explain financial bubbles in an inequity aversion framework (Gebhardt (2001)). Also it has been shown that inequity aversion in an adverse selection set up can explain wage compression (von Siemens (2001)).

[5]See Hellwig and Schmidt (2000) for a detailed discussion.

There are a few studies that show that sharecropping contracts, perhaps the most widespread form of incentive contracts, are not only linear, but have predominantly a slope of 1/2. Bardhan and Rudra (1980), Bardhan (1984), Young (1996) and Young and Burke (2001) report empirical studies from India and Illinois where 60% to 90% of sharecropping contracts stipulate the equal split.

We use inequity aversion to explain these stylized facts. A linear and equitable sharing rule avoids the problem that in some situations one party is well-off, while the other earns little. For standard utility maximizers this is not a problem, as expected utility is all that counts. However, under inequity aversion an inequitable distribution of surplus reduces the agent's utility. So redistribution until an equitable outcome is achieved increases social benefit. If the surplus in different states of the world is not divided equitably the agent suffers from being better off or worse off respectively. Therefore inequity aversion adds a driving force towards linear and equitable sharing rules.

Using a model with convex costs of inequity aversion, we show that the First Best contract for risk neutral agents is, in contrast to standard contract theory, unique and linear with slope 1/2. This contract minimizes the principal's cost as it minimizes inequity, as every marginal unit is split equitably. But as soon as effort is no longer observable the First Best is, even for risk neutral agents, not implementable. This result is due to the fact that while the agent is risk neutral with respect to his monetary payoff we induce risk-averse-like preferences by introducing convex disutility of inequitable outcomes. In the Second Best the slope of the wage scheme is strictly above 1/2 as there is a need to give proper incentives. The strongest incentives are given if the firm is sold to the agent, i.e. the slope of the wage scheme is 1. But as this would result in very inequitable distributions of wealth, this is not optimal. It is cheaper for the principal to have lower incentives and lower exerted effort, but not to be forced to pay a high compensation to the agent in order to make up for the expected inequity. The stronger the concern for equitable distributions becomes the more the contract tends to the linear sharing rule. In the limit of infinite inequity aversion the concern for equity dominates any other aspect of the problem and the unique solution is a linear contract with slope 1/2.

Allowing for risk aversion introduces a countervailing force. While inequity aversion pushes towards equitable sharing, i.e. a slope of 1/2, risk aversion demands flat wage schemes, i.e. a slope of 0. Therefore, in the First Best without an incentive problem, the slope of the scheme is strictly below 1/2, but, again, tends more to the equitable split the more inequity averse the agent becomes. The intuition again is straightforward. In the First Best under risk neutrality the slope of the wage scheme was 1/2. Now the presence of risk aversion without a need for proper incentives calls for a less risky wage scheme.

Turning to the Second Best with risk averse agents leaves us with a shortage of clear-cut results. In this case there are three forces: inequity aversion, risk aversion and the need for incentives, working in opposing directions. While inequity aversion calls for a slope of 1/2, risk aversion pushes towards a slope of 0 and incentives are maximal at a slope of 1. Therefore the only stable effect is that the wage scheme is strictly increasing in profit. And again it holds true that the more inequity averse the agents become the stronger is the tendency towards the equitable sharing rule.

The last main result concerns the well known Sufficient Statistics result. This standard result says that a contract should condition on a specific variable if and only if it is a sufficient statistic for effort choice. This does not hold if agents are inequity averse. As agents have a concern for the distribution of payoffs the optimal contract conditions also on those parts of the profit which do not contain any information about the agent's effort choice.

The paper is structured as follows: In Section 2 the basic results for the case of risk neutral agents are derived. Section 3 discusses the general case of risk aversion, where we concentrate on the limit case of extreme inequity aversion. In Section 4 it is shown that the Sufficient Statistics result no longer holds. Section 5 concludes the paper.

2 Inequity Averse Agent

We model the interaction between a risk neutral profit maximizing principal and a utility maximizing agent who exhibits inequity aversion. To illustrate the problem, consider a landlord and a tenant.

The landlord, who is the principal in the model, is assumed throughout to be selfish and risk neutral while the tenant, i.e. the agent, is inequity averse[6]. The landlord hires the tenant to cultivate his fields. The crop x at the end of the season is distributed in an interval $[\underline{x}, \bar{x}]$ with density $f(x \mid e)$ which is determined by the effort e exerted by the tenant. As the principal is neither risk averse nor inequity averse he wants to maximize his expected net profit

$$EU_P = \int_{\underline{x}}^{\bar{x}} f(x \mid e)[(x - w(x)]dx$$

where $w(x)$ is the wage paid to the agent.

The agent's utility function exhibits risk neutrality in wealth and inequity aversion. To decide whether an allocation is fair or unfair the agent compares her payoff $w(x)$ and the principal's net payoff $[x - w(x)]$[7]. Therefore the agent's utility is given by

$$\begin{aligned} U_A \quad & = w(x) - c(e) - \alpha G\left[[x - w(x)] - w(x)\right] \\ \text{with} \quad \xi G'(\xi) > 0 \quad & , \; G''(\xi) > 0 \\ G(0) = 0 \quad & , \; G'(0) = 0 \; , \; G''(0) = 0 \end{aligned}$$

The function $G(\cdot)$ captures the fact that the agent suffers from inequitable outcomes. Whenever the share of the principal, $x - w(x)$ and her own share, $w(x)$, differ, this causes disutility, which is assumed to be convex[8]. α is the weight the agent puts on

[6] One could extend the model and allow for an inequity averse principal, too. Section 5 contains a discussion of this topic.

[7] All our results hold for a richer model, too, where the agent compares her net payoff $[w(x) - c(e)]$ to the principal's net payoff $[x - w(x)]$. But in order to keep the exposition as simple as possible we present only this version.

[8] Dropping the assumption of convexity does no harm to our main results, but for simplicity - and as we think it is realistic - we keep it throughout the paper.

achieving equitable outcomes. One could think of this weight embedded in G, but as limit results are derived in Section 3 it is written explicitely. As effort can be easily redefined there are no restrictions imposed on $c(e)$ apart from $c'(e) > 0$.

The way inequity aversion is modelled is very similar to Fehr and Schmidt (1999). However we differ in one important aspect. The Fehr and Schmidt function for the two player case has the following form:

$$U_i(x_i) = x_i - \alpha_i \max\{x_j - x_i, 0\} - \beta_i \max\{x_i - x_j, 0\}$$

While utility in our model is also additively separable in income, effort and inequitable outcomes the way to describe the disutility caused by inequitable outcomes is altered. We assume that the disutility from inequality is convex. There are two reasons for this. The first reason is technical. Disutility from inequity is another deadweight loss, next to costs from bearing risk, and usually convex cost functions are assumed in order to guarantee interior solutions. The second reason is economical: Convexity implies an aversion towards lotteries over different levels of inequity, while concavity would imply affinity towards such lotteries. Assuming risk averse agents later on in this paper one should not have such features of risk loving built in the model.

In the first best case where one can write contracts on effort levels e the principal faces the following optimization problem

$$\max_{e,w(x)} \; EU_P = \int_{\underline{x}}^{\bar{x}} f(x \mid e)[(x - w(x)]dx$$

$$\text{s.}t.(PC) \; EU_A = \int_{\underline{x}}^{\bar{x}} f(x \mid e)\{w(x) - \alpha G[x - 2w(x)]\}dx - c(e) \geq \bar{U}$$

where PC denotes the agent's participation constraint.

In the standard case without inequity aversion this problem has multiple solutions. The principal chooses the optimal effort level while the wage scheme can take any given form as both agent and principal only care about expected wage. This is different in the case with an inequity averse agent. Proposition 1 below shows that the optimal contract is unique and linear.[9]

[9]The exact structure is given by $w(x) = \frac{\lambda-1}{4\alpha\lambda} + 1/2\ x$, i.e. a constant plus an equitable share of the profit. λ denotes the Lagrange parameter.

Proposition 1

If both agent and principal are risk neutral and the agent exhibits inequity aversion, the First Best contract is unique and linear with slope $1/2$.[10]

The intuition for this result is obvious. The principal wants to hold the agent down to her outside option and to extract as much rent as possible. Due to the convexity of G the costs from inequitable outcomes are minimized by holding the extent of inequity in every possible state of the world constant. But this implies that principal and agent have to share the surplus equally in every state of the world. Therefore the optimal contract is given by a fixed payment not depending on the realized profit level while every additional unit of surplus is divided equally between the two.

Note that in contrast to the standard principal agent literature, which focuses on the incentive constraint as the reason for contingent contracts, here it is the participation constraint, which causes a contract to depend on the outcome in a linear form.

A straightforward generalization of Proposition 1 yields:

Corollary

For m *risk neutral principals and* n *risk neutral and inequity averse agents the slope of the First Best incentive scheme is* $1/(m+n)$.

Still keeping our agent risk neutral we now turn to the second best where the contract cannot condition upon e. This gives an additional constraint in the principal's optimization problem

$$\text{(IC) } e \in \arg\max_{e} \ EU_A = \int_{\underline{x}}^{\bar{x}} f(x \mid e)\{w(x) - \alpha G[(x - 2w(x)]\}dx - c(e)$$

where IC denotes the agent's incentive constraint. As usual in the literature we use the first order approach to derive our results.

In standard principal agent models the First Best is still implementable even if effort is not observable as long as the agent is risk neutral. The principal just sells

[10]The proofs for this and all other propositions can be found in the appendix of this paper.

the firm to the agent. Now the agent has full marginal incentives and - because of risk neutrality - no disutility from bearing risk.

This no longer works under the presence of inequity aversion, as selling the firm results in almost all cases in very inequitable allocations.

Proposition 2

In the Second Best case where both agent and principal are risk neutral and the agent is inequity averse, the First Best is not implementable.

To see this recall that the First Best contract is unique and has slope 1/2. As one wants to give full incentives, i.e. slope 1, in the Second Best the First Best outcome cannot be achieved. The intuition is quite straightforward. The convexity of the disutility from inequitable outcomes induces risk-averse-like preferences and therefore the First Best is no longer feasible .

In general, only the following statement on the optimal wage scheme can be made:

Lemma 1

In the Second Best case where both agent and principal are risk neutral and the agent exhibits inequity aversion, the slope of the optimal contract is strictly greater than 1/2 *if the Monotone Likelihood Ratio Property (MLRP)*[11] *holds.*

The reason for this result is as follows. In the First Best case, without any need to give proper incentives, the slope already was 1/2. Now, as the need for incentives is present and a higher value of x is, due to the MLRP, a signal for a higher effort level, one should observe higher powered incentive schemes. The reason why one in general will not observe a slope of one is that one has to balance off the need to provide incentives and the increasing disutility from inequity.

A close look at the slope of $w(x)$, $w'(x) = \frac{1}{2} + \frac{1}{4\alpha} \frac{\mu \frac{\partial}{\partial x}\left[\frac{f_e(x|e)}{f(x|e)}\right]}{\left[\lambda + \mu \frac{f_e(x|e)}{f(x|e)}\right]^2 G''(\cdot)}$, shows that if the agent's concern for inequity, i.e. α, increases, the contract converges to the equal sharing rule as the relative weight on the first term increases compared to the

[11] The MLRP says that the ratio $\frac{f_e(x|e)}{f(x|e)}$ has to be strictly increasing in x.

weight of the second term, $\frac{1}{4\alpha}$. If the agent is extremely inequity averse the contract is virtually linear.

3 Risk Averse Agent

Introducing risk aversion alters the agent's utility function to

$$EU_A = \int_{\underline{x}}^{\bar{x}} f(x \mid e)u_A[w(x)] - \alpha G[x - 2w(x)]dx - c(e)$$

with $u'_A[w(x)] > 0$ and $u''_A[w(x)] < 0$. I.e. the agent's utility is concave in his monetary payoff. This implies that he dislikes gambles over his wealth. We still assume that utility is still additively separable in a term depending on monetary payoffs, $u_A[w(x)]$, and the part that captures his aversion towards inequitable outcomes, $G(\cdot)$.

For the First Best case the principal's optimization problem has following form

$$\max_{w(x)} \ EU_P = \int_{\underline{x}}^{\bar{x}} f(x \mid e)[x - w(x)]dx$$

$$\text{s.t. (PC) } EU_A = \int_{\underline{x}}^{\bar{x}} f(x \mid e)\{u_A[w(x)] - \alpha G[x - 2w(x)]\}dx - c(e) \geq \bar{U}$$

Now there is a tradeoff between the agent's risk aversion, which pushes towards a fixed wage payment, and her inequity aversion, which pushes towards the equal split. We formulate this in Lemma 2.

Lemma 2

In the First Best case where the agent is risk averse and has convex disutility from inequity, the slope of the optimal contract is strictly below 1/2.

The intuition again is straightforward. As in the First Best case under risk neutrality the slope of the wage scheme was 1/2, now the presence of risk aversion without a need for proper incentives calls for a less risky wage scheme.

Again, looking at $w'(x) = \frac{1}{2} + \frac{u''_A[w(x)]}{\alpha 8G''(\cdot) - 2u''_A[w(x)]}$ shows that for an increasing α, i.e. more inequity averse agents, the contract converges to the equitable sharing rule, as the latter part becomes smaller and smaller as α increases.

Now analyzing the second best case one has to add another constraint:

$$\text{(IC) } e \in \arg\max_{e} \; EU_A = \int_{\underline{x}}^{\bar{x}} f(x \mid e)\{u_A[w(x)] - \alpha G[(x - 2w(x)]\}dx - c(e)$$

Out of the above stated optimization problem one gets, by using the first order approach, the following first order conditions.

$$\frac{\partial u_A[w(x)]}{\partial w(x)} = \frac{1}{\lambda + \mu \frac{f_e(x|e)}{f(x|e)}} - 2\alpha G'(\cdot)$$

From this we derive Lemma 3, which replicates a standard result from conventional contract theory.

Lemma 3

The optimal Second Best contract for a risk and inequity averse agent is strictly increasing in x if the MLRP holds.

This result is not surprising. The result in standard contract theory is that the optimal contract is increasing in x if the MLRP holds. Now inequity aversion is added which is an additional force towards profit oriented payment schemes.[12]

For the general case, i.e. $\alpha \neq 0$, the slope of the wage scheme is given by

$$w'(x) = \frac{1}{2} + \frac{\frac{1}{2}u''_A[w(x)] + \left[\frac{\mu \frac{\partial}{\partial x}\left[\frac{f_e(x|e)}{f(x|e)}\right]}{\left[\lambda + \mu \frac{f_e(x|e)}{f(x|e)}\right]^2}\right]}{\alpha 4G''(\cdot) - u''_A[w(x)]}$$

Now consider the case of extreme inequity aversion, i.e. $\alpha \to \infty$. This would describe a person suffering so much from inequity that she could not be compensated for. She would be willing to incur enormous costs in order to achieve an equitable allocation.[13]

[12]Note, that, assuming $\alpha = 0$, i.e. a non inequity averse agent, the First Order Condition boils down to the well known expression $\frac{1}{\frac{\partial u_A[w(x)]}{\partial w(x)}} = \lambda + \mu \frac{f_e(x|e)}{f(x|e)}$ where the curvature of the incentive scheme is determined by the likelihood ratio.

[13]One might think of Heinrich von Kleist's novel *Michael Kohlhaas*, where the Saxon horse dealer Kohlhaas triggered off a war when he was betrayed by the authorities, i.e. an inequitable outcome was imposed on him.

Proposition 3

If the agent is infinitely inequity averse the optimal Second Best contract is linear with slope 1/2.

There is no longer any scope for the principal to successfully propose a scheme that deviates from the equitable split as any deviation would harm the agent infinitely. One can see this fairly easy in the condition for the slope of the optimal contract, as the latter term converges to zero if α becomes very large.

4 The Sufficient Statistics Result

In this section it is shown that one of the basic concepts of contract theory no longer holds if agents are inequity averse. The agent's concern for equitable outcomes calls for a profit contingent contract even in the case where profit is not a sufficient statistic[14] for the effort choice. This fact is captured in Proposition 4.

Proposition 4

If the agent is inequity averse the sufficient statistics result does not apply.

To prove this consider the following setup: The firms' profit Π can be separated into two parts x and y, i.e. $\Pi = x+y$. While the distribution of x depends on the effort e exerted by the agent, y is purely randomly distributed. In the appendix it is shown that contrary to the well known sufficient statistics result, the optimal contract when the agent exhibits inequity aversion conditions on y, although this variable contains no information concerning the effort choice.

The intuition should be clear. Profit serves not only as a signal whether or not the agent exerted enough effort, but also is important for the agent's utility as she has a concern for equitable distributions. As the agent compares her payoff to the firm's profit, y is taken into account when equitability is judged. Therefore it has to be taken into account when the contract is written.

[14]See Holmström (1979) for a formal exposition.

A recent empirical study by Bertrand and Mullainathan (2001) has pointed out the relevance of such "too" complete contracts. They find that CEO pay is as sensitive to purely random variations of realized profit as to variations of profit that are due to the CEO's effort.

5 Conclusion

Introducing inequity aversion yields results that deviate from those known from the standard principal agent literature. If the principal can condition on the effort level, the optimal contract for the case of a risk neutral agent is unique and implies an equitable sharing rule. Also the unique Second Best contract in the limit of strong inequity aversion specifies an equitable sharing rule. Therefore we have a strong tendency towards linearity in the incentive schemes. In contrast to the standard principal agent literature, which focuses on the incentive constraint as the reason for contingent contracts, here it is the participation constraint, too, which causes a contract to depend on the outcome in a linear form.

Furthermore we show that even for risk neutral agents the First Best is in general not implementable as soon as effort is no longer observable. It is also shown that under inequity aversion the sufficient statistics result no longer holds, as the optimal contract conditions on the realized profit level even if profit is not a sufficient statistic for the effort choice.

There are several ways in which the model could be extended. One might explore the extent to which introducing inequity aversion on the principal's side will alter the results. This may be an additional driving force towards linear contracts, specifying an equitable sharing rule, as now both parties have a preference for equitable distributions. Therefore this should even strengthen our results. A logical next step is an application of this approach to team production problems and multi agent problems as inequity aversion should be even more important when agents have to interact with peers.

Summing up, assuming inequity averse preferences on the agent's side sheds some light on topics in contract theory which have not yet been satisfactorily resolved. Our basic claim seems rather robust. Introducing inequity aversion in the utility function adds a driving force towards linear contracts. This provides a very simple, plausible and experimentally well supported explanation for the predominance of linear wage schemes in real labor markets.

6 Appendix

6.1 Proof of Proposition 1

The First Order Condition of the First Best problem yields after rearranging

$$\begin{aligned} G'(\cdot) &= \frac{1-\lambda}{2\alpha\lambda} = constant \\ &\Longrightarrow x - 2w(x) = constant \equiv \gamma \\ &\Longrightarrow w(x) = \gamma + \frac{1}{2}x \end{aligned}$$

Given that $G'(\cdot)$ is constant for all x it follows by the strict convexity of G that the level of inequity over all realisations of x is constant, i.e. an equitable sharing rule is optimal.

q.e.d.

6.2 Proof of Proposition 2

First one has to note that here - in contrast to standard principal agent models - the optimal First Best contract is unique. The Lagrangian of the First Best Problem has the form

$$L = E[U_P(x - w(x))|\, e] - \lambda[\bar{U}_A - E[U_A \big|\, e]]$$

The derivative of the Lagrangian with respect to effort yields

$$\frac{\partial L}{\partial e} = \frac{\partial\, E[U_P(x - w(x))|\, e]}{\partial e} + \lambda \frac{\partial\, E[U_A|\, e]}{\partial e} = 0$$

The second expression is the derivative of the agent's incentive constraint and therefore has to be zero in optimum in the Second Best case. If we plug in the First Best wage scheme, which according to Proposition 1 has the form $w^*(x) = \gamma + 1/2x$, the term $\frac{\partial\, E[U_P(x-w(x))|e]}{\partial e}$ changes to $1/2\frac{\partial E[x|e]}{\partial e}$, which has to be zero in order to guarantee the First Best solution if the Incentive Constraint holds in the Second Best. But, as we assumed $c(e) > 0$, it can not be an equilibrium if $\left.\frac{\partial E[x|e]}{\partial e}\right|_{e=e^{FB}}$ is equal to zero, as we could reduce the effort, and hence $c(e)$ without reducing the expected value of x. Therefore $\frac{\partial E[U_A|e]}{\partial e} \neq 0$ must hold, which means that the First Best effort level is not implementable in the Second Best.

q.e.d.

6.3 Proof of Lemma 1

Differentiating the Lagrangian of the Second Best problem with respect to $w(x)$ yields after rearranging this condition for $G'(\cdot)$.

$$G'(\cdot) \quad = \quad \frac{1}{2\alpha}\left[\frac{1}{\lambda + \mu\frac{f_e(x|e)}{f(x|e)}} - 1\right]$$

Differentiating this equation with rerspect to x gives us the following condition for $w'(\cdot)$

$$w'(x) \quad = \quad \frac{1}{2} + \frac{1}{\alpha}\frac{\mu\frac{\partial}{\partial x}\left[\frac{f_e(x|e)}{f(x|e)}\right]}{4\left[\lambda + \mu\frac{f_e(x|e)}{f(x|e)}\right]^2 G''(\cdot)}$$

where $\frac{\partial}{\partial x}\left[\frac{f_e(x|e)}{f(x|e)}\right]$ denotes the derivative of the Likelihood Ratio with respect to x, which has to be positive if the MLRP holds. Therefore the slope of the incentive scheme has to be strictly above 1/2, as the second term is strictly positive.

q.e.d.

6.4 Proof of Lemma 2

The first derivative of the Lagrangian with respect to $w(x)$ gives after rearranging

$$u'_A[w(x)] \quad = \quad -2\alpha G'(\cdot) + 1/\lambda$$

Differentiating the whole expression with respect to x yields

$$w'(x) = \frac{1}{2} + \frac{u''_A[w(x)]}{\alpha 8G''(\cdot) - 2u''_A[w(x)]}$$

As the latter part is stricly smaller than zero the slope is strictly below 1/2. q.e.d.

6.5 Proof of Lemma 3

Differentiating the First Order Condition with respect to x gives us the following condition for $w'(x)$

$$\left[\overbrace{-u_A''}^{+}+\overbrace{4\alpha G''}^{+}\right]w'(x) \quad = \quad \left[\overbrace{2\alpha G''}^{+}+\frac{\overbrace{\mu\frac{\partial}{\partial x}\left[\frac{f_e(x\mid e)}{f(x\mid e)}\right]}^{+}}{\underbrace{\left[\lambda+\mu\frac{f_e(x\mid e)}{f(x\mid e)}\right]^2}_{+}}\right]$$

From this expression one can see fairly easy that $w'(x)$ has to be positive in order to guarantee this equation to hold.

q.e.d.

6.6 Proof of Proposition 4

Suppose the firms' profit Π can be separated into two parts x and y, i.e. $\Pi = x + y$. While the distribution $f(x \mid e)$ of x depends on the effort e exerted by the agent, y is purely randomly distributed and its density is given by $g(y)$. To show that the sufficient statistics result does not apply when the agent exhibits inequity aversion consider the principal's optimisation problem

$$\max\ EU_P = \int_{\underline{x}}^{\bar{x}} f(x\mid e)x dx + \int_{\underline{y}}^{\bar{y}} g(y)y dy - \int_{\underline{x}}^{\bar{x}}\int_{\underline{y}}^{\bar{y}} w(x,y)f(x\mid e)g(y)dxdy$$

$$\text{s.}t.(PC) \qquad \bar{U} \leq \int_{\underline{x}}^{\bar{x}}\int_{\underline{y}}^{\bar{y}} \{u[w(x,y)] - \alpha G[x+y-2w(x,y)]\}f(x\mid e)g(y)dxdy - c(e)$$

$$\text{s.}t.(IC) \qquad e \in \arg\max_e \int_{\underline{x}}^{\bar{x}}\int_{\underline{y}}^{\bar{y}} \{u[w(x,y)] - \alpha G[x+y-2w(x,y)]\}f(x\mid e)g(y)dxdy - c(e)$$

where $g(y)$ is the density function for y, the random part of the profit. By using the first order approach one gets

$$(IC')\ 0 = \int_{\underline{x}}^{\bar{x}}\int_{\underline{y}}^{\bar{y}} f_e(x\mid e)g(y)[u[w(x,y) - \alpha G[x+y-2w(x,y)]]dxdy - c_e(e)$$

The first order condition for the principal's optimization problem has the following form

$$0 = -1 + \lambda \left[u'[w(x,y)] + 2\alpha G'[\cdot]\right] + \mu \frac{f_e(x,y \mid e)}{f(x,y \mid e)} \left[u'[w(x,y)] + 2\alpha G'[\cdot]\right]$$

An application of the implicit function theorem yields

$$\frac{\partial w}{\partial y} = \frac{\alpha G''[\cdot]}{4\alpha G''[\cdot] - u''[w(x,y)]} \neq 0 \qquad \forall\, \alpha \neq 0$$

As w depends on y, which does not contain any information about the agent's effort choice the sufficient statistics result does not apply .[15]

q.e.d.

[15]Not surprisingly, for $\alpha = 0$, i.e. a purely selfish agent, the sufficient statistics result applies again, as there $w_y(y) = 0$ holds.

References

(1) G.A. Akerlof: *Labor contracts as a partial gift exchange*, Quarterly Journal of Economics, Vol. 97 (1982), S. 543-69

(2) G.A. Akerlof and J.L. Yellen: *Fairness and unemployment*, American Economic Review, Vol. 78 (1990), 44-49

(3) W. Austin: *Equity Theory and social comparison processes*,in Social Comparison Processes, edited by J.M. Suls und R.L. Miller, Washington D.C., Hemisphere Publishing Corporation 1977

(4) P. Bardhan: *Land, Labor, and Rural Poverty*, New York: Columbia University Press, 1984

(5) P. Bardhan and A. Rudra: *Terms and Conditions of Sharecropping Contracts: An Analysis of Village Survey Data in India*, Journal of Development Studies , (1980), 287-302

(6) L. Berkowitz: *Responsibility, Reciprocity and social distance in help-giving: An experimental investigation of english social class differences*, Journal of Experimental Social Psychology, Vol. 4 (1968), 46-63

(7) M. Bertrand and S. Mullainathan: *Are CEOs Rewarded for Luck? The Ones without Principals are*, Quarterly Journal of Economics, Vol. 116(3) (2001), 901-932

(8) E. Fehr and A. Falk: *Wage rigidity in a competitive incomplete contract market*, Journal of Political Economy, Vol. 107 (1999), 106-34

(9) E. Fehr / S. Gächter / G. Kirchsteiger: *Reciprocity as a contract enforcement device: Experimental evidence*, Econometrica, Band 65 (1997), 833-60

(10) E. Fehr and K.M. Schmidt: *A theory of fairness, competition and cooperation*, Quarterly Journal of Economics, Vol. 114(3) (1999), 817-868

(11) G. Gebhardt: *Inequity Aversion and Asset Prices*, mimeo University of Munich, (2001)

(12) R.E. Goranson and L. Berkowitz: *Reciprocity and responsibility reactions to prior help*, Journal of Personality and Social Psychology, Vol. 3(2) (1966), 227-32

(13) A.W. Gouldner: *The norm of reciprocity: A preliminary statement*, American Sociological Review, Vol. 25 (1960), 161-78

(14) M. Hellwig and K.M. Schmidt: *Discrete-Time Approximations of the Holmström-Milgrom Brownian Motion Model of Intertemporal Incentive Provision*, mimeo, University of Munich, (2000)

(15) B. Holmström: *Moral Hazard and Observability*, Bell Journal of Economics, Vol. 10 (1979), 74-91

(16) B. Holmström and P. Milgrom: *Agregation and Linearity in the Provision of Intertemporal Incentives*, Econometrica, Vol. 55 (1987), 303-328

(17) R. Innes: *Limited Liability and Incentive Contracting with Ex-Ante Action Choices*, Journal of Economic Theory, Vol. 52 (1990), 45-67

(18) J. Mirlees: *The Theory of Moral Hazard and Unobservable Behaviour, Part I*, Review of Economic Studies, Vol. 66 (1999), 3-22

(19) R. Selten: *The equity principle in economic behavior*, Decision Theory and Social Ethics, H.W. Göttinger und W. Leinfellner (Hrsg.), (1978), 289-301

(20) R. Selten: *Features of experimentally observed bounded rationality*, European Economic Review, Vol. 42 (1998), 413-436

(21) F. v. Siemens: *Adverse Selection and Inequity Aversion*, mimeo University of Munich, (2001)

(22) H.P. Young: *Equity: In Theory and Practice*, Princeton University Press, Princeton NJ, (1994)

(23) H.P. Young: *The Economics of Convention*, Journal of Economic Perspectives, Vol. 10 (1996), 105-122

(24) H.P. Young and M.A. Burke: *Competition and Custom in Economic Contracts: A Case Study of Illinois Agriculture*, American Economic Review, Vol. 91 (2001), 559-573