The costs of dualism*

Jonathan Temple†
Department of Economics, University of Bristol
8 Woodland Road, Bristol BS8 1TN and CEPR

July 4, 2002

Abstract

This paper shows how to calibrate a two-sector general equilibrium model of production using a small number of parameter assumptions and readily available data. The framework is then used to analyze the costs of labor market dualism. The paper quantifies the effects of rural-urban wage differentials and urban unemployment on total output, wages and returns to capital, factor shares, and sectoral structure. One of the main findings is that labor market rigidities can have a major impact on the extent of industrialization.

JEL Categories: D61, O40

Keywords: dualism, productivity, wage differentials, minimum wages

*I am grateful to Bryan Graham, Stephen Redding, Kevin Roberts, Patrick Toche and seminar participants at the IIES in Stockholm, LSE, and the universities of Bristol, Kent, Leicester, North London, Oxford, Tilburg and Warwick for helpful comments and discussion. Any errors are my responsibility.

†Email jon.temple@bristol.ac.uk

1 Introduction

Models of dual economies have become a familiar part of development economics and the theory of international trade. Often, theoretical analysis is based upon a 2 x 2 model with two sectors and two factors, in which there is a differential in factor returns across sectors. A number of potential effects have been identified, together with policy measures that could eliminate dualism. Yet we know remarkably little about the likely magnitude of the various effects, or the gains to be expected from policy intervention.[1] In other words, previous research has largely failed to address one of the most important questions: how significant are the costs of dualism?

This paper seeks to answer that question, by quantifying the costs along a number of dimensions. The paper examines the effects of dualism on aggregate output, sectoral structure, wages and returns to capital, and the distribution of factor income. One of the most striking findings is that labor market rigidity can have a major impact on sectoral structure - in other words, on the extent of industrialization.

The paper also makes a methodological contribution. The paper shows how to calibrate a two sector general equilibrium model of production, in a way that is undemanding in terms of both data requirements and parameter assumptions. Among other things, the paper describes assumptions under which technology parameters and the intersectoral allocation of capital can be recovered from data on sectoral output and employment shares. As I will discuss below, these ideas have wider application, since general equilibrium models of production are relevant in a variety of fields.

Here the calibration technique is applied to labor market rigidity and its consequences. It is well known that the marginal product of labor is likely to differ across sectors in countries at an early stage of development. The paper considers the effects of one specific form of wage differential, namely that between a modern urban sector and a rural agricultural sector. This is often thought to be one of the most important of the possible distortions in developing country labor markets (Rosenzweig 1988, p. 751).

A natural starting point for the analysis of wage differentials is the framework of Harris and Todaro (1970). There are two key assumptions, to be discussed at greater length later in the paper. First, an exogenously fixed wage in the urban sector implies that the urban labor market fails to clear, leading to urban unemployment. Second, intersectoral migration takes place unless expected

[1] See for example Freeman (1992, 1993), who argues that not enough is known about the magnitude of the distortions associated with labor market imperfections.

wages are equal in the urban and rural sectors. Since it is expected wages which are equalized, rather than actual, there will be an intersectoral wage differential in equilibrium. The wage differential will be related to the extent of urban unemployment.

The paper calibrates a model of this general form, based on the version of the Harris-Todaro model introduced by Corden and Findlay (1975). Although the calibrated model is stylized, its simplicity has considerable advantages. The calibration exercise requires, somewhat remarkably, only data on agricultural output and employment shares, and assumptions about three parameters: the elasticity of substitution in production, the share of labor in national income, and the urban unemployment rate. As a result, the calibration is simple to carry out, the underlying assumptions are readily understood, and results can easily be communicated for a variety of cases. Perhaps more importantly, the simplicity of the model also allows the results to be understood and interpreted using existing trade theory, and casts some light on the quantitative importance of various effects identified by theorists. Hence the analysis provides a useful complement to more detailed computable general equilibrium (CGE) models, although admittedly at a significant cost in terms of realism.

The calibrated model is used to compare the outcomes under dualism with the first-best allocation, in which there is no unemployment and the marginal product of labor is equal across sectors. In line with most previous work, the results indicate that the impact of wage differentials on aggregate output is limited, especially if the elasticity of substitution in production is low. Furthermore, for the cases considered here, the movement from dualism to the first-best is associated with higher returns to capital but lower wages in both sectors, especially in non-agriculture.[2] Perhaps more importantly, dualism has important consequences for sectoral structure. Large shifts in the relative importance of the two sectors are possible in moving from the dualistic economy to the first-best. To put this in slightly more colourful terms, where we observe that a country has not industrialized, this paper shows that the nature of the urban labor market is a possible explanation.[3]

The general form of approach is related to a number of previous contributions. Johnson (1966) also studied the effects of factor market distortions in general equilibrium. He argued that the effects of differentials on aggregate output

[2]The result that wages are lower in both sectors in the first-best may seem surprising, but is in line with previous theoretical work. This point will be discussed later in the paper.

[3]In this respect, the paper supports the arguments of Agénor (1996) and Freeman (1992), that more research on the connections between labor market imperfections and economic growth could be fruitful.

are probably limited, based on visual inspection of production possibility frontiers drawn for the Cobb-Douglas case. Since then, the issue has been revisited under more general assumptions. Prominent examples include Dougherty and Selowsky (1973), de Melo (1977), and Williamson (1987, 1989). These papers tend to focus on the output effects of eliminating wage differentials for specific countries or historical cases. My work differs in considering a wider range of possible scenarios, and in giving more emphasis to the impact of dualism on wages and returns to capital, the distribution of factor income, and particularly sectoral structure, issues which have received relatively little attention in the literature thus far.

As noted earlier, the paper is also innovative in developing a calibration method that could easily be applied to a large number of countries, and offer insight into a range of other issues. Jones (1965) pointed out that simple general equilibrium models of production have not only been a workhorse of trade theory, but have also found applications in almost all branches of applied economics. The simple 2 x 2 model has long been left behind in the applied literature on international trade, but is still of interest in other areas, notably the study of economic growth and structural change. It would be a simple matter to extend the approach of this paper to analyze such questions, and to cast more light on various properties of the canonical 2 x 2 model.

The paper also casts indirect light on the effects of minimum wage legislation, when the coverage of such legislation is incomplete. In the analysis of this paper, the sectors are labelled 'agriculture' and 'modern'. The two sectors could be labelled more generally as 'uncovered' and 'covered' respectively, as in the work of Mincer (1976) and Fields (1997) on minimum wages. Under this alternative interpretation of the paper, the analysis quantifies the effects of a sector-specific minimum wage on output and wages, and reveals the consequences of eliminating such controls. Hence the analysis of the paper has relevance and interest beyond the specific application I emphasize, which is to labor markets in developing countries.

The paper has the following structure. Section 2 sets out the general model, defining both the first-best allocation and the dualistic one, and showing how the transition between the two may be understood using trade theory. Section 3 introduces the strategy for calibrating the model, starting with the relatively simple case where the sectoral production functions are Cobb-Douglas, and then turning to the more general case of CES production functions. Section 4 describes the data and assumptions. Section 5 reports the calibration results and uses them to compare the dual economy with the first-best allocation. Section

6 provides some further discussion, before section 7 concludes.

2 A model of dualism

This section describes a simple general equilibrium model of a dual economy, essentially the version of Harris and Todaro (1970) due to Corden and Findlay (1975). There are two sectors, rural agriculture and an urban 'modern' sector. The agricultural good is the numeraire. I make the standard simplifying assumption that both goods are traded on world markets, and the economy is too small to be able to influence world prices. Hence the relative price of manufactures, p, is exogenously fixed by world prices. Appropriate choice of units would allow this price to be normalized to one in the equations that follow, but I prefer to keep the role for the relative price explicit.

Aggregate output, capital and labor are denoted Y, K and L respectively. As is standard in general equilibrium models of production, the economy is closed to international flows of capital and labor, and the aggregate capital stock and labor supply are taken to be exogenously fixed.

The technologies in the two sectors are:

$$Y_a = A_a F(K_a, L_a) \quad (1)$$

$$Y_m = A_m G(K_m, L_m) \quad (2)$$

where Y_i, A_i, K_i and L_i are output, TFP, capital and labor in sector i (agriculture/modern) respectively. Returns to scale are constant in both sectors, and both factors are paid their marginal products.

Capital is fully employed, so that

$$K = K_a + K_m \quad (3)$$

I assume that capital is perfectly mobile between sectors, so that rental rates are equalized:

$$A_a F_K = p A_m G_K \quad (4)$$

where the subscript K denotes the derivative with respect to capital.

I assume that parameter values are such that specialization is incomplete. Then, the first-best equilibrium is described by equations (1)-(4) and the following four equations:

$$w_a = A_a F_L \quad (5)$$

$$
\begin{aligned}
w_m &= pA_mG_L && (6)\\
L &= L_a + L_m && (7)\\
w_m &= w_a && (8)
\end{aligned}
$$

where the subscript L denotes the derivative with respect to labor. These equations represent the equality of wages and marginal products, full employment of labor, and a long-run migration equilibrium in which any intersectoral wage differential is eliminated.

The paper will compare this first-best equilibrium with a dualistic one. The dualistic equilibrium is again described by equations (1)-(6) but differs in its specification of the labor market, which follows Harris and Todaro (1970). I assume that the urban wage is fixed above the market-clearing wage that would hold in the first-best equilibrium. This results in unemployment in the urban sector, so (7) is replaced by:

$$L = L_a + L_m + L_u \quad (9)$$

where L_u is urban unemployment.

The model is completed by specifying the migration equilibrium condition. I assume that the unemployed receive no income. Migration takes place between sectors unless the agricultural wage (w_a) is equal to the *expected* wage in the modern sector, which is a function of both the fixed modern sector wage (w_m) and the probability of finding employment at this wage. Assuming that jobs are allocated by a lottery among the urban population, this probability is $1-u$ where $u = L_u/(L_u+L_m)$ is the rate of urban unemployment.[4] Note the standard Harris-Todaro assumption that workers must be present in urban areas to have a chance of finding urban employment. If we interpret the model in terms of 'covered' and 'uncovered' sectors, the corresponding assumption is that workers cannot look for work in the covered sector while holding a job in the uncovered sector.

Labor market equilibrium occurs when:

$$w_a = (1-u)w_m \quad (10)$$

This is the equilibrium condition associated with Harris and Todaro (1970). The model described by (1)-(6), (9) and (10) is the version of the Harris-Todaro model introduced by Corden and Findlay (1975). It combines the standard 2 x 2 model of trade theory with the Harris-Todaro labor market assumptions.

[4]The lottery assumption could be relaxed as in Moene (1988). The Harris-Todaro equilibrium condition will then be a good approximation provided that the rate of job turnover is high or the discount rate is low.

Note that one immediate implication of the Harris-Todaro condition is a wage differential across sectors: the ratio of marginal products w_m/w_a is equal to $1/(1-u)$. Hence output is lower than in the first-best for two reasons. First, because some of the labor force are unemployed, and second, because the marginal products of labor are not equalized for those who are employed.

With this in mind, I introduce a new device for analyzing the nature of the Harris-Todaro economy relative to the first-best allocation. The idea is to make use of theoretical work based on the case of exogenous wage differentials, as in the classic analyses of Jones (1971) and Magee (1973, 1976). These models differ from the Harris-Todaro approach in assuming full employment. A common form of such models can be described by equations (1)-(7) together with a fixed wage differential:

$$w_m = kw_a$$

How can we make use of existing results for such models? The movement from the Harris-Todaro economy to the first-best can be thought of as occurring in two stages, both of which can be understood individually using trade theory. The trick is to construct an artificial economy that forms an intermediate stage between the Harris-Todaro economy and the first-best. In the first stage, moving from the Harris-Todaro to the intermediate economy, we keep total employment fixed. All those currently employed are reallocated so that the marginal products of labor and capital are equalized across the two sectors. In the second stage, we then complete the move to the overall first-best by increasing total employment, by the number of unemployed in the original Harris-Todaro economy. This brings us to the first-best, with no unemployment and once again marginal products of labor that are equalized across the two sectors.

Analytically, the first stage is equivalent to the elimination of a wage differential in a model with an exogenous differential, given by $k = 1/(1-u)$, and in which employment is fixed. Hence standard results for the exogenous wage differential case, such as those of Jones (1971), can be applied to analyze the first stage. One result worth noting is that, at constant commodity prices, a higher premium paid to labor in the modern sector can sometimes be associated with an *increase* in that sector's relative output. Another result, more important to the analysis below, is that if labor's share of income is lower in the modern sector than in agriculture, a rise in the wage premium in the modern sector will be associated with higher wages in *both* sectors (Jones 1971, p. 442). This suggests that dualism may be associated with higher wages than in the first-best, a potential outcome to be confirmed below.

Now consider the second stage of the transition to the first-best, namely the elimination of unemployment, starting from the intermediate point where the returns to both factors are equal across sectors. The movement to the first-best is now just an increase in the labor force in an otherwise conventional 2 x 2 trade model with intersectoral factor mobility. In other words, it can be seen as a change in relative factor abundance which can be analyzed using the standard results of Heckscher-Ohlin trade theory. At constant commodity prices, the rise in employment will have no effect on factor prices, provided the economy remains incompletely specialized. The fall in the aggregate capital-labor ratio does give rise to a Rybczynski effect: with the capital stock fixed, it yields an increase in the output of the labor-intensive sector (here, agriculture) and a reduction in the output of the capital-intensive sector (non-agriculture).

I will adopt the analytical device of a two-stage transition in presenting the calibration results because, as seen above, it allows me to explain and interpret the overall findings using existing trade theory. By analyzing separately the effects of eliminating the wage differential holding employment constant, and then of increasing total employment, it is possible to gain a greater understanding of how the first-best outcomes are related to those under dualism.

Note that, from the analysis above, the introduction of an exogenously fixed wage in the urban sector can be associated with an *increase* in modern sector output. This paradoxical result for the 2 x 2 Harris-Todaro model was first pointed out by Corden and Findlay (1975, p. 66-67) using different reasoning, and has recently been emphasized by Allen (2001, p. 524). One contribution of the calibration exercise below, however, will be to show that this case is empirically unlikely.

Before progressing further, I briefly discuss some limitations of the approach adopted here. As with any stylized model, the Harris-Todaro framework is not without its critics. A clear drawback in this context is that the rigidity of the urban wage is assumed rather than modelled. Some recent contributions, notably Moene (1988) and MacLeod and Malcomson (1998), have analyzed models in which the urban wage is endogenously determined. Both these papers work with relatively simple representations of the production technologies in the two sectors, however. Combining their specifications for the labor market with more complex technologies would not be straightforward. Perhaps more importantly, the use of such a model would make it harder to relate the findings to existing trade theory, and harder to shed new light on some of the effects identified by trade theorists.

The model used here is stylized in other respects as well. Recent work on migration by Stark (1991) and others emphasizes that migration decisions are often collective and made in the interests of a household. Among the important real-world considerations I abstract from the potentially substantial share of rural household income gained in the non-agricultural sector, and the flow of resources between members of households divided between urban and rural areas, or more general forms of household income sharing.

Although these considerations could be integrated into the model, the present framework allows some interesting results to be derived in a way that is both simple and transparent. The simplicity of the framework allows a clear focus on the essential aspects of the present analysis, namely the existence of urban unemployment and a wage differential. Incidentally, although relating these two using the Harris-Todaro equilibrium condition simplifies some of the algebra, it is by no means crucial to the results. What the paper has to say about the aggregate costs of wage differentials is of interest even though wage gaps in the real world are rarely well described by the Harris-Todaro condition.

3 Calibrating the model

To take the Harris-Todaro model to the data, I assume that observed shares of agriculture in employment and output correspond to a Harris-Todaro equilibrium. The assumption that we observe the world in equilibrium is a strong one, but it is typical to most exercises in calibration or CGE modelling, and avoids imposing an arbitrary extent of disequilibrium.

Once the equilibrium assumption has been made, it is relatively straightforward to derive the nature of the first-best allocation, in which urban unemployment and marginal product differentials are eliminated. Recall that the main aim of the paper is to compare these two alternative equilibria, and therefore offer some insight into the costs of dualism.

The remainder of this section describes how to derive the first-best allocation, given the observed data. I start with the relatively simple Cobb-Douglas case to illustrate the basic ideas, before turning to the more complicated model with CES production functions. The technologies in the two sectors are:

$$\begin{aligned} Y_a &= A_a K_a^{\alpha} L_a^{1-\alpha} \\ Y_m &= A_m K_m^{\theta} L_m^{1-\theta} \end{aligned}$$

I will denote the agricultural employment share (L_a/L) by a and the output share (Y_a/Y) by s. Modern sector employment is given by:

$$L_m = (1-u)(1-a)L \tag{11}$$

Denote the share of labor in agricultural income by $\eta_a = w_a L_a / Y_a$ and in modern sector income by $\eta_m = w_m L_m / pY_m$. The share of labor in total national income is:

$$\eta = \frac{w_a L_a + w_m L_m}{Y}$$

Using (10) and (11), this expression can be simplified to $\eta = w_a L / Y$. Hence the share of labor in agricultural income can be written as:

$$\eta_a = \frac{w_a L_a}{Y_a} = \frac{w_a L}{Y} \frac{L_a}{L} \frac{Y}{Y_a} = \frac{\eta a}{s} \tag{12}$$

Given that the agricultural production function is Cobb-Douglas, the agricultural technology parameter α will be given by

$$1 - \alpha = \eta_a = \frac{\eta a}{s} \tag{13}$$

Similarly we can also derive an expression for the labor share in the modern sector, and hence the modern sector technology parameter θ:

$$1 - \theta = \eta_m = \eta \left(\frac{1-a}{1-s} \right) \tag{14}$$

Hence with two Cobb-Douglas production functions, constant returns to scale, and intersectoral factor mobility, we can infer the technology parameters using only an assumption about the aggregate labor share (η) and readily available data on the agricultural employment share (a) and output share (s).

Using the production functions, we can rewrite the long-run equilibrium condition (10) as:

$$(1-\alpha)\frac{Y_a}{aL} = (1-u)w_m = \frac{(1-u)p(1-\theta)Y_m}{(1-u)(1-a)L}$$

Hence we derive that

$$\frac{Y_a}{pY_m} = \frac{s}{1-s} = \left(\frac{1-\theta}{1-\alpha} \right) \frac{a}{1-a} \tag{15}$$

Using (15) combined with the production functions, it is possible to derive an equation which ties down the urban unemployment rate u in terms of a, α, θ, p, A_a, A_m, K_a and K_m. However, I will assume throughout that we do not know the last five variables, so that it is better to work instead with an assumed value for the urban unemployment rate, u. I will now show that using only data on a and s, and assumptions about u and η, it is possible to calculate the agricultural employment share in the first-best economy, denoted b.

In the first-best economy, workers will be paid the same in each sector. Thus we can derive an equation corresponding to (15), where Y_a' and Y_m' are sectoral outputs under the first-best allocation, and r is the agricultural share of output in the first-best economy:

$$\frac{Y_a'}{pY_m'} = \left(\frac{1-\theta}{1-\alpha}\right)\frac{b}{1-b} = \frac{r}{1-r} \tag{16}$$

In the Harris-Todaro economy, denote the proportion of capital used in agriculture by $x = K_a/K$, and the proportion in manufacturing by $(1-x)$. For the first-best economy, denote this proportion by z.

Using the production functions, the following two equations must hold:

$$\frac{r}{1-r} = \frac{Y_a'}{pY_m'} = \frac{A_a z^\alpha K^\alpha b^{1-\alpha} L^{1-\alpha}}{pA_m(1-z)^\theta K^\theta (1-b)^{1-\theta} L^{1-\theta}} \tag{17}$$

$$\frac{s}{1-s} = \frac{Y_a}{pY_m} = \frac{A_a x^\alpha K^\alpha a^{1-\alpha} L^{1-\alpha}}{pA_m(1-x)^\theta K^\theta (1-a)^{1-\theta}(1-u)^{1-\theta} L^{1-\theta}} \tag{18}$$

By dividing (17) by (18), and then using (15) and (16), it is possible to derive the following equation implicitly defining b:

$$\left(\frac{z}{x}\right)^\alpha \left(\frac{1-x}{1-z}\right)^\theta \left(\frac{a}{b}\right)^\alpha \left(\frac{1-b}{1-a}\right)^\theta (1-u)^{1-\theta} = 1 \tag{19}$$

Assuming that rental rates are equalized, we have

$$\alpha\frac{Y_a}{K_a} = p\theta\frac{Y_m}{K_m}$$

Then we have in the Harris-Todaro economy, using (15),

$$\frac{K_a}{K_m} = \frac{x}{1-x} = \left(\frac{\alpha}{\theta}\right)\frac{Y_a}{pY_m} = \left(\frac{\alpha}{\theta}\right)\left(\frac{1-\theta}{1-\alpha}\right)\frac{a}{1-a} \tag{20}$$

and similarly in the first-best economy, using (16),

$$\frac{z}{1-z} = \left(\frac{\alpha}{\theta}\right)\left(\frac{1-\theta}{1-\alpha}\right)\frac{b}{1-b} \tag{21}$$

where b is the agricultural share of employment in the first-best economy. Solving (20) and (21) for x and z, and substituting in (19) we have

$$\left(\frac{1-a}{1-b}\right)^{\alpha-\theta} \left(\frac{1+\frac{\alpha}{\theta}\left(\frac{1-\theta}{1-\alpha}\right)\frac{a}{1-a}}{1+\frac{\alpha}{\theta}\left(\frac{1-\theta}{1-\alpha}\right)\frac{b}{1-b}}\right)^{\alpha-\theta} (1-u)^{1-\theta} = 1$$

Simplifying further,

$$\left(\frac{1-a\left(1-\frac{\alpha}{\theta}\left(\frac{1-\theta}{1-\alpha}\right)\right)}{1-b\left(1-\frac{\alpha}{\theta}\left(\frac{1-\theta}{1-\alpha}\right)\right)}\right)^{\alpha-\theta}(1-u)^{1-\theta}=1 \tag{22}$$

Rearranging gives

$$b=\frac{\theta(1-\alpha)}{\theta-\alpha}-\left(\frac{\theta(1-\alpha)}{\theta-\alpha}-a\right)(1-u)^{\frac{1-\theta}{\alpha-\theta}} \tag{23}$$

Hence equation (23) yields the agricultural share of employment in the first-best economy, assuming that an observed economy is in a Harris-Todaro equilibrium.

It would also be useful to know the ratio of output in the first-best economy to that in the Harris-Todaro economy, which I denote by Λ_Y. Given that commodity prices are exogenously fixed, the ratio is given by the ratio of nominal outputs:

$$\Lambda_Y=\frac{Y_a'+pY_m'}{Y_a+pY_m}$$

Using (15) and (16), the expression for Λ_Y can be written as

$$\Lambda=\frac{pY_m'\left(1+\frac{Y_a'}{pY_m'}\right)}{pY_m\left(1+\frac{Y_a}{pY_m}\right)}=\frac{Y_m'}{Y_m}\left(\frac{1+\frac{b}{1-b}\left(\frac{1-\theta}{1-\alpha}\right)}{1+\frac{a}{1-a}\left(\frac{1-\theta}{1-\alpha}\right)}\right) \tag{24}$$

Making use of the modern sector production function, and (20) and (21) we can derive:

$$\Lambda=\left(\frac{1+\frac{a}{1-a}\left(\frac{1-\theta}{1-\alpha}\right)\frac{\alpha}{\theta}}{1+\frac{b}{1-b}\left(\frac{1-\theta}{1-\alpha}\right)\frac{\alpha}{\theta}}\right)^{\theta}\left(\frac{1-b}{1-a}\right)^{1-\theta}(1-u)^{\theta-1}\left(\frac{1+\frac{b}{1-b}\left(\frac{1-\theta}{1-\alpha}\right)}{1+\frac{a}{1-a}\left(\frac{1-\theta}{1-\alpha}\right)}\right) \tag{25}$$

Overall, the key equations are (13), (14), (23) and (25). These equations allow us to calculate the agriculture share in the first-best economy, and the ratio of output in the first-best economy to that in the dual economy, using only information on the four variables u, a, s, and η.

Note that if $\theta > \alpha$ then $b < a$ from inspection of (22). In other words, agricultural employment is lower in the first-best economy than in the dual economy, if the modern sector is relatively capital intensive. This is consistent with Corden and Findlay (1975), who show that in general the outcome is determined by the 'manufacturing elasticity', the proportional change in labor input in manufacturing divided by the proportional change in marginal product. For the Cobb-Douglas case, this elasticity is greater than one. Corden and Findlay show that, as found here, agricultural employment will then be higher in the dual economy than in the first-best, provided that the modern sector is relatively capital intensive.

Although the Cobb-Douglas case is an easy one to handle, it is clear that the results are likely to be insufficiently general. Compared to the dual economy, output is higher in the first-best because extra labor is brought into employment, and because labor is reallocated between agriculture and non-agriculture. Clearly the elasticities of substitution between capital and labor in the two sectors will be key parameters governing this process. As Dougherty and Selowsky (1973) point out, the higher the elasticity of substitution, the larger the gain from labor reallocation, because the marginal product of labor changes more slowly as labor is reallocated. This suggests that it would be useful to experiment with different elasticities of substitution.

The simplest way to do this is to follow Kelley et al. (1972) and work with CES production functions. To keep the model tractable, I impose the restriction that the elasticity of substitution is the same in both sectors. Hence the technologies are now:

$$\begin{aligned} Y_a &= A_a \left[\delta_a K_a^{-\rho} + (1-\delta_a) L_a^{-\rho}\right]^{-\frac{1}{\rho}} \\ Y_m &= A_m \left[\delta_m K_m^{-\rho} + (1-\delta_m) L_m^{-\rho}\right]^{-\frac{1}{\rho}} \end{aligned}$$

where the elasticity of substitution $\sigma = 1/(1+\rho)$. As before, I use $b = L'_a/L$ to denote the proportion of employment in agriculture, $r = Y'_a/Y$ the share of agriculture in output and $z = K'_a/K$ the proportion of capital employed in agriculture, all in the first-best equilibrium. Once again Y'_m, K'_m and L'_m are the first-best levels of output, capital and labor in the modern sector respectively. In the first-best, the share of labor in national income is denoted ϕ, and the shares of labor income in agricultural and modern sector value added are denoted ϕ_a and ϕ_m respectively.

Appendix 1 shows how to derive two equations in terms of b, z and observable variables, which can then be solved for the first-best allocation of employment b and capital z. The first equation is:

$$\frac{b}{1-b} = \left(\frac{a}{1-a}\right)\left(\frac{1-x}{x}\right)(1-u)^{\frac{-\rho}{1+\rho}}\left(\frac{z}{1-z}\right) \tag{26}$$

The second equation is:

$$\frac{b}{1-b} = \left(\frac{z}{1-z}\right)\left(\frac{1-x}{x}\right)\left(\frac{s}{1-s}\right)\left(\frac{1+\left(\frac{1-\eta}{\eta}\right)\left(\frac{1-x}{1-a}\right)^{1+\rho}\left(\frac{1-b}{1-z}\right)^{\rho}(1-u)^{-\rho}}{1+\left(\frac{1-\eta}{\eta}\right)\left(\frac{x}{a}\right)^{1+\rho}\left(\frac{b}{z}\right)^{\rho}}\right) \tag{27}$$

Hence the calibration procedure is as follows. Three parameters have to be chosen: the aggregate labor share η, the elasticity of substitution $\sigma = 1/(1+\rho)$ and the urban unemployment rate u. We can then use data on agriculture's share of employment (a) and output (s) to solve for the share of agricultural capital in total capital, x, in the dualistic economy. Given a solution for x, we can then solve the two equations (26) and (27) numerically for b and z. Thus using only data or assumptions on a, s, η, u, and σ, it is possible to derive what agriculture's share of employment in the first-best economy (b) would be. It is then possible to calculate the ratio of output in the first-best economy to that in the Harris-Todaro economy, and other relevant outcomes, using the further equations derived in Appendix 1.

4 Data and assumptions

This section describes the data and assumptions that will be needed in the calibration exercise. I consider three different cases for the agricultural output and employment shares, based on data from three regional groupings of developing countries. The regions are sub-Saharan Africa, East Asia, and Latin America. Grouping countries by region is potentially useful given that technology parameters may vary with geographic location. Perhaps more importantly, the three groups differ in terms of their level of development and the extent of industrialization.

In calibrating the model, one important consideration is the choice of labor's income share (η). Evidence from developed countries tends to suggest that the labor share is roughly in the range 0.60 to 0.70. It is sometimes argued that the share may be lower in developing countries (see for instance Collins and Bosworth 1996). From a pragmatic point of view, a lower value for η is useful here because it makes the sectoral labor shares (η_a and η_m) more likely to be less than one, as is clear from equation (12). Accordingly, in what follows I set $\eta = 0.50$.

Next, I choose values for a and s for each of the three regional groups. Table 1 shows the median agricultural output and employment shares for 1960 and 1985 for these groups. The data are taken from the World Bank, and the samples exclude small countries, defined as those with a labor force lower than 250,000 in 1985.[5] Based on these data, I calibrate the model for a typical country in

[5] The data are taken from the *World Development Indicators* CD-Rom. Where necessary, the WDI data are supplemented by figures from the 1990 *Production Yearbook* of the Food and Agricultural Organization of the United Nations (FAO) and the 1987 *World Development Report*.

each regional grouping using the following values. The agricultural employment and output shares, a and s, are assumed equal to 0.80 and 0.50 respectively for sub-Saharan Africa, 0.65 and 0.35 for East Asia, and 0.50 and 0.30 for Latin America.

These choices depart slightly from those suggested by Table 1, in order to ensure that the implied labor shares (η_a and η_m) are less than one. Given that these departures are required, this is evidence that the model does not well approximate reality for all observed combinations of a and s. This is perhaps not wholly surprising, given the highly stylized nature of the model. Nevertheless, the results are still likely to be of some interest, not least because there is an obvious case for understanding the costs of dualism in a simple framework before moving to the analysis of more complex and realistic models.

Table 1
Agricultural employment and output shares in 1960 and 1985

		a	s
Sub-Saharan Africa	1960	0.86	0.47
	1985	0.75	0.34
East Asia	1960	0.63	0.29
	1985	0.41	0.18
Latin America and the Caribbean	1960	0.52	0.23
	1985	0.30	0.13

Notes

The variable 'a' is the share of agricultural employment in total employment and 's' is the agricultural output share.

Another key assumption concerns the urban unemployment rate. To gain some idea about appropriate magnitudes, I have calculated figures based on the ILO's *Yearbook of Labor Statistics, 1995.* Agricultural or rural areas are often not represented in these unemployment statistics, and this suggests that it is indeed valid to assume that all the reported unemployment is in the urban sector. This allows the derivation of an urban unemployment rate $u = U/(1-a)$ using data on the national unemployment rate (U) and the agricultural share of employment a.

The calculated urban unemployment rates (not reported) vary between zero and about 35%. The figures for Africa are very low, but are not based on labor force surveys, and are therefore likely to be inaccurate. This is particularly so given the ambiguity surrounding the concept of unemployment in developing countries. For instance, the World Bank (1995, p. 28) reports that in Ghana the measured rate of unemployment is 1.6%, but the underemployment rate is calculated to be about 24%. Knight (1998) writes that "on sensible definitions the open unemployment rate probably now exceeds a quarter in Zimbabwe and a third in South Africa" (p. 11). Elsewhere in the world, an urban unemployment rate between 20% and 30% is calculated for such countries as El Salvador, India, and Sri Lanka.

With all this in mind, I calibrate the model for an urban unemployment rate of 30%. Another way to judge this assumption is to look at the implied wage differential. An urban unemployment rate of 30% corresponds to a marginal product of labor around 40% higher in the modern sector than in agriculture, given that we have

$$\begin{aligned} w_m/w_a &= 1/(1-u) \\ &= 1/(1-0.3) \approx 1.4 \end{aligned}$$

It is important to emphasize that a marginal product differential of this magnitude is not implausible. Squire (1981, p. 102) cites evidence implying that the nominal wage gap for unskilled labor, unadjusted for differences in the cost of living between urban and rural areas, can easily be this large. For his sample of twenty-three developing countries the median differential is 34%. Squire comments, based on data in Clark (1957), that these figures are not out of line with wage gaps observed for seven developed countries in the second half of the nineteenth century. Similarly Williamson (1987) argues that rural-urban real wage gaps in England during the Industrial Revolution were of the order of 30%-50%. Figures from the World Bank (1995, p. 76) suggest that the wage gap may be even higher in many developing countries.

In practice, these observed rural-urban wage gaps may not represent equilibrium phenonomena of the kind envisaged by Todaro (1969) and Harris and Todaro (1970), as discussed by Hatton and Williamson (1991). Furthermore, observed wage gaps do not imply different returns to workers of potentially identical productivity, but could simply reflect differences in average skills across sectors. In these circumstances testing for the presence of differentials is not straightforward, and it can often be argued that measured differentials reflect unobserved characteristics (Magnac 1991). Interpreting the available evidence is also made more complicated by the role of urban amenities, and spatial variation in the cost of living. Overall, however, it is clear that assuming an urban unemployment rate of 30% implies equilibrium wage gaps that are not implausibly high.

5 Calibration results

In this section, I will use the results of section 3 to calibrate the model described in section 2. The basic idea is to combine an assumption about urban unemployment (u) with data on agricultural output and employment shares, under the assumption that the observed data correspond to a Harris-Todaro economy. I can then infer the nature of the first-best economy and compare it with the dual economy. The aim will be to examine the effects of dualism on aggregate output, sectoral output, sectoral structure, wages and rental rates, and the distribution of factor income.

As argued in section 2, greater understanding of the differences between Harris-Todaro and the first-best can be achieved by thinking of the movement between them as in two stages. In moving away from Harris-Todaro, we can first consider a movement to an intermediate economy in which the wage differential has been eliminated but total employment is held constant at the Harris-Todaro level. Secondly, we then complete the move to the first-best by eliminating unemployment. The results below will follow this decomposition, and it requires only a few simple changes to the algebra of section 3 to analyze the characteristics of the intermediate economy.

I first consider results for the Cobb-Douglas case, presented in Table 2.[6] The table shows the effects of eliminating a wage ratio of about 1.4, holding total employment constant. There is one column for each region. The top two rows show the sectoral labor shares implied by combining the agricultural output and employment shares with the assumption that the aggregate labor share $\eta = 0.5$. These labor shares are quite similar across regions, consistent with technology parameters that are similar across regions. The large differences in capital intensity across sectors will be discussed later in the paper.

The next six rows reveal the changes in sectoral structure in moving from a dual economy to an intermediate economy with the wage differential eliminated but employment held constant. The two equilibria can be compared in terms of agricultural employment shares (a and b), output shares (s and r) and sectoral capital allocations (x and z). The table also shows the ratio of modern sector employment in the intermediate economy to that in the dual economy.

The next section of the table reports the ratio of modern sector wages in the intermediate economy, to those in the dual economy; the corresponding ratio for agricultural wages; and the ratio of rental rates. The last row of this section reports the labor share in the intermediate economy. Finally, the fourth

[6] All calculations were carried out by computer programs written by the author, using the mathematical software Maple.

section of the table reports the ratio of modern sector output in the intermediate economy to that in the dualistic one; the corresponding ratio for agricultural output; and perhaps of especial interest, the change in total output.

Table 2
Eliminating the wage differential ($\sigma = 1$, $\eta = 0.5$, $u = 0.30$)

Variable	Region	Africa	East Asia	Latin America
	1. *Sectoral labor shares*			
η_m	Modern sector	0.20	0.27	0.36
η_a	Agricultural sector	0.80	0.93	0.83
	2. *Sectoral structure*			
a	Old employment share	0.80	0.65	0.50
b	New employment share	0.75	0.54	0.16
s	Old output share	0.50	0.35	0.30
r	New output share	0.42	0.25	0.08
x	Old capital allocation	0.20	0.05	0.10
z	New capital allocation	0.16	0.03	0.02
L'_m/L_m	New/old modern employment	1.26	1.32	1.67
	3. *Factor payments*			
w'_m/w_m	New modern wage/old	0.68	0.69	0.67
w'_a/w_a	New agricultural wage/old	0.98	0.99	0.96
r'/r	New rental rate/old	1.10	1.15	1.25
ϕ	New aggregate labor share	0.45	0.44	0.39
	4. *Output changes*			
Y'_m/Y_m	New modern output/old	1.16	1.17	1.36
Y'_a/Y_a	New agricultural output/old	0.86	0.73	0.27
Y'/Y	New total output/old	1.01	1.02	1.03

Notes

The output and employment shares are those of agriculture, as is the capital allocation. 'Old' corresponds to the Harris-Todaro economy, and 'New' to an intermediate economy with the same total employment but no wage differential.

Some of the most interesting results are those in section 3 of the table, which show how factor payments change when the wage differential is eliminated. It can be seen that wages are lower in both sectors in the absence of the differential. It may appear surprising that the wage falls in agriculture, but as noted earlier, this result has been derived in the trade theory literature on exogenous wage differentials with fixed commodity prices. Jones (1971, p. 442-443) shows that if an increased premium is paid to labor in the sector where labor receives the smaller distributive share (in the framework of this paper, if w_m/w_a increases and $\eta_m < \eta_a$) this must raise the wage rate, relative to the return to capital, in *both* sectors. Furthermore, what Jones refers to as the 'magnification effect' implies that the 'real' return to labour or capital moves in the same direction as the relative return. Hence in a model with exogenous wage differentials, the elimination of dualism will be associated with lower wages in both sectors. It should also be noted, however, that for the cases considered here the decline in the agricultural wage is not substantial. This finding nicely demonstrates how calibrating the 2 x 2 model can shed direct light on the quantitative implications of effects identified by trade theorists.

The final section of the table indicates the output gains associated with the elimination of the wage differential, holding total employment constant. It is clear that these gains are very small, even though I am considering a case where the modern sector wage is initially roughly 40% higher than that in agriculture. The output gains typically associated with the elimination of a wage differential may appear surprisingly low, but this result is fully consistent with a variety of previous work using other approaches, notably that of Johnson (1966).

Now I consider the difference made to these findings by the increase in employment, in the second stage of the transition from dualism to the first-best. Table 3 reports the overall first-best outcomes relative to the Harris-Todaro economy. Considering section 2 of the table, the most interesting result here is the large rise in modern sector employment in moving to the first-best. This is driven by two forces: the contraction of the agricultural sector, and the elimination of unemployment. In the African and East Asian cases, modern sector employment rises by more than two-thirds. In the Latin American case, it almost doubles.

The ratios of factor payments in section 3 of the table are unchanged compared with those in Table 2. This simply reflects a standard result in this form of 2 x 2 trade model, namely that factor prices are independent of factor endowments while the economy remains incompletely specialized.

The final section of the table indicates the output gains in moving from dualism to the first-best. One point to note is that, compared with the intermediate economy described in Table 2, modern sector output has fallen and agricultural sector output has risen. This reflects the Rybczynski effect at work, in the second stage of the transition from the dual economy to the first-best. The reduction in the capital-labor ratio, with employment increased and the capital stock unchanged, leads to a fall in the output of the capital-intensive good and a rise in the output of the labor-intensive good.

Table 3
Overall results ($\sigma = 1$, $\eta = 0.5$, $u = 0.30$)

Variable	Region	Africa	East Asia	Latin America
	1. *Sectoral labor shares*			
η_m	Modern sector	0.20	0.27	0.36
η_a	Agricultural sector	0.80	0.93	0.83
	2. *Sectoral structure*			
a	Old employment share	0.80	0.65	0.50
b	New employment share	0.77	0.59	0.31
s	Old output share	0.50	0.35	0.30
r	New output share	0.45	0.30	0.16
x	Old capital allocation	0.20	0.05	0.10
z	New capital allocation	0.17	0.04	0.05
L'_m/L_m	New/old modern employment	1.67	1.67	1.98
	3. *Factor payments*			
w'_m/w_m	New modern wage/old	0.68	0.69	0.67
w'_a/w_a	New agricultural wage/old	0.98	0.99	0.96
r'/r	New rental rate/old	1.10	1.15	1.25
ϕ	New aggregate labor share	0.47	0.46	0.43
	4. *Output changes*			
Y'_m/Y_m	New modern output/old	1.14	1.16	1.32
Y'_a/Y_a	New agricultural output/old	0.94	0.90	0.59
Y'/Y	New total output/old	1.04	1.07	1.10

Notes

The output and employment shares are those of agriculture, as is the capital allocation. 'Old' corresponds to the Harris-Todaro economy and 'New' to the first-best.

One of the most important findings is that eliminating dualism, even in the initial presence of substantial urban unemployment, raises total output per head by around 10% at most. A comparison of Tables 2 and 3 shows that most of this output gain is driven by reduced unemployment rather than by the elimination of the wage differential. Overall, the results suggest that the effects of dualism are felt more keenly in sectoral structure than in aggregate output. Section 6 will discuss the implications of this result.

Some of the properties noted here follow from the 'other things equal' nature of the comparison and, in particular, the decision to compare equilibria while holding the capital stock fixed. Although this is the most natural starting point, it does give rise to some counter-intuitive results. For example, with a fixed capital stock, the elimination of dualism is associated with a fall in labor productivity of the employed in both sectors, reflecting the fall in the aggregate capital-labor ratio.[7] This suggests that it would be useful to repeat these experiments allowing the capital stock to be determined endogenously. However, complications quickly arise, and these will be discussed in detail in section 6.

The two main findings, insignificant output gains but potentially major sectoral shifts, are borne out by more general experiments based on CES production functions. For brevity, I report only the case where agriculture accounts for 65% of employment and 35% of output, which corresponds roughly to East Asia for the early 1960s. As before, the urban unemployment rate is assumed to be 30%. I consider three values for the elasticity of substitution: 0.5, 1.5 and 2.

The results are presented in Table 4. There are two main points to note. First, higher elasticities of substitution are associated with a larger change in output, as one might expect given that diminishing returns will set in more slowly. This pattern can be seen in section 4 of the table. The second main point to note is that the extent of structural change is very sensititive to the elasticity of substitution. With an elasticity of 0.5, there is only a small change in employment shares and total modern sector employment. With an elasticity of 2, the changes are dramatic.

The framework developed here can also be used to analyze the paradox noted by Corden and Findlay (1975), namely that modern sector output may be greater in the dual economy than in the first-best, despite the rigidity in the urban labor market. Corden and Findlay showed that a sufficient condition for this would be fixed coefficients in production in both sectors, combined with the assumption that the modern sector is relatively capital intensive. They also noted that the paradoxical result would still hold for a limited degree of technical substitution. I have carried out experiments for the cases used above, based on a 'typical' country for each of the three regional groupings. These experiments suggest that σ has to be very low before the Corden-Findlay paradox emerges. In each case, σ must be around 0.15 or below for the dual economy to be associated with greater modern sector output than in the first-best. Many economists would be unhappy with assuming an elasticity this low, at least at this level of aggregation, which suggests that the Corden-Findlay paradox is perhaps best seen as a theoretical curiosity.

[7] To see this, note that with Cobb-Douglas production functions, average products are a multiple of marginal products (and hence wages). If wages fall in both sectors then so must the average products of employed labor. Aggregate output can still rise, because more people are employed, and aggregate total factor productivity has risen.

Table 4
Results for CES production functions ($\eta = 0.5$, $u = 0.30$, $a = 0.65$, $s = 0.35$)

Variable	Region	$\sigma = 0.5$	$\sigma = 1.5$	$\sigma = 2$
	1. *Sectoral labor shares*			
η_m	Modern sector	0.27	0.27	0.27
η_a	Agricultural sector	0.93	0.93	0.93
	2. *Sectoral structure*			
a	Old employment share	0.65	0.65	0.65
b	New employment share	0.69	0.45	0.24
s	Old output share	0.35	0.35	0.35
r	New output share	0.35	0.22	0.11
x	Old capital allocation	0.05	0.05	0.05
z	New capital allocation	0.05	0.03	0.01
L'_m/L_m	New/old modern employment	1.28	2.24	3.12
	3. *Factor payments*			
w'_m/w_m	New modern wage/old	0.69	0.69	0.69
w'_a/w_a	New agricultural wage/old	0.99	0.99	0.99
r'/r	New rental rate/old	1.13	1.17	1.20
ϕ	New aggregate labor share	0.47	0.46	0.45
	4. *Output changes*			
Y'_m/Y_m	New modern output/old	1.06	1.29	1.49
Y'_a/Y_a	New agricultural output/old	1.05	0.68	0.35
Y'/Y	New total output/old	1.06	1.08	1.09

Notes

The output and employment shares are those of agriculture, as is the capital allocation. 'Old' corresponds to the Harris-Todaro economy and 'New' to the first-best.

6 Further discussion

This section provides some further discussion of the results, and their possible generality. I start with the results concerning overall output gains, and then discuss the more noticeable effects on sectoral structure. I also consider the implications for policy, and in particular, whether or not dualism should be a major concern.

We have seen that the output losses associated with wage differentials are small. The framework above may even overstate the extent of marginal product differentials. It is easy to show that if the unemployed receive at least some income, perhaps in the informal sector, then the intersectoral disparity between marginal products in the long-run migration equilibrium will be smaller for a given urban unemployment rate.

Are there any ways in which dualism could be associated with greater output losses? The comparison of the Harris-Todaro and first-best economies in this paper holds the capital stock fixed, and relaxing this assumption would tend to be associated with larger output effects. The most obvious way to make the capital stock endogenous would be to open the economy to capital flows, but this is not straighforward. With the urban wage and goods prices assumed to be fixed, and the other assumptions retained, then specifying an exogenous rental rate is unlikely to be consistent with a diversified equilibrium. One solution to this would be to introduce a specific factor in non-agriculture, but I leave this to further work.[8]

It is also worth noting that greater output effects might be found if the technology parameters took other values. Johnson (1966, p. 697) drew attention to this issue for Cobb-Douglas production functions, based on plotting production possibility frontiers for various cases. He found that the output losses associated with a wage differential are greater when the exponents in the two sectoral production functions are similar across sectors.

In the calibration experiments undertaken here, the technology parameters are some distance apart. One way to retain the same basic framework, but consider alternative values for parameters, is to assume that a certain fraction of agricultural output is not measured in the national accounts, perhaps because it is produced for non-marketed domestic consumption. If a fraction ϵ of agricultural output is unmeasured, we can adjust the observed agricultural output shares as follows:

$$s^* = \frac{s}{1 - \epsilon + \epsilon s}$$

where s^* is the 'true' output share adjusted for mismeasurement. If half of agricultural output is unmeasured ($\epsilon = 0.5$) then the output share becomes 0.67 in the African case, 0.52 in the East Asian case, and 0.46 in the Latin American case. In turn this implies that the technology parameters are closer together. Calibrating the model for the Cobb-Douglas case, using these new values for the technology parameters, results in larger output effects, as suggested by the informal geometric analysis in Johnson (1966). Nevertheless, the output gains remain a relatively small fraction of GDP, with the possible exception of the Latin American case.

Various other changes to the simple Harris-Todaro framework could also be associated with higher output costs of dualism. For example, it could be

[8] Another solution would be to follow Yabuuchi (1993) in assuming that capital is sector-specific, and that only the agricultural sector is open to international capital flows.

argued that workers receive their average product in agriculture rather than their marginal product, through household income sharing for example. Given diminishing returns in agriculture then, for a given urban unemployment rate, the differentials in marginal products across sectors will be greater.

Alternatively, workers may be risk averse. It is sometimes argued that observed urban-rural wage gaps are larger than those implied by the simplest version of the Todaro (1969) framework combined with observed urban unemployment rates (for example, Rosenzweig 1988). If workers are risk averse, they will require a greater wage premium for urban employment to compensate them for the risk of being unemployed. This will be associated with a larger wage differential, and hence the costs of dualism will typically be greater.

Overall, though, the main consequences of dualism appear to be for sectoral structure rather than for output, as we saw in section 5. When the elasticity of substitution in production is relatively high, the movement from the Harris-Todaro economy to the first-best is associated with large shifts in sectoral structure, partly due to the elimination of the wage differential and the contraction of agriculture, and partly due to the elimination of unemployment.

How general is this kind of result likely to be? The model adopted here perhaps overstates the extent of structural change that is likely to be associated with the ending of dualism. One reason is that capital may be sector-specific in developing countries (Robinson 1989) and this will tend to limit the extent of sectoral shifts. I have experimented with the sector-specific case for Cobb-Douglas production functions, using results derived in Appendix 2. These experiments tend to suggest that sectoral shifts are indeed moderated under sector-specific capital, although only for the Latin American case are the differences marked.[9] The total output gains are very similar in magnitude.

Another extension to the work above would be to incorporate a role for non-traded goods. Relative price changes would then probably limit the extent of structural differences between the dualistic economy and the first-best. The simplest way to study this issue is to assume that the proportions of income spent on each good are fixed, within an economy that is closed to international trade. This involves some extensions to the algebra of section 3, and in particular the introduction of a price index. In carrying out this extension I have found, not surprisingly, that the introduction of constant budget shares implies that employment shares are very similar across the dualistic and first-best economies. Perhaps more interestingly, the magnitude of the output losses appears to be

[9]This can be inferred from Tables 3 and 4. Note that z, the share of capital allocated to agriculture in the first-best, is often close to x, agriculture's share of capital in the Harris-Todaro economy. Only in the Latin American case is there much difference.

robust to this alternative assumption about price determination. In the case of fixed budget shares, the output costs of dualism are very similar to those in the case of fixed prices, although slightly smaller.

I now turn to the potential implications of the paper's findings. It is often thought that intersectoral mobility, and labor market flexibility in general, plays a role in successful development and growth. For example, in the course of a study of the Korean labor market, Kim and Topel (1995) suggest that the "implied mobility of the labor force [in Korea] may be a boon to development and structural change". As a general proposition this may be true, but the results of this paper suggest that the case for intersectoral mobility will sometimes have to go beyond static efficiency considerations. It is certainly true that in the absence of mobility, growth will usually be associated with a widening urban-rural wage gap. Yet it turns out that the output loss associated with such a gap is typically small, unless the labor market is so poorly integrated that the wage differential becomes very large.

One argument that could be made is that the net present value of even a small output loss may be very large. The relevance of this argument to policy depends on the nature of the costs involved in eliminating dualism. If dualism can be eliminated by a one-off policy change, the point is a strong one, but many of the proposals for moving to the first-best are based on policies that are likely to involve substantial recurrent costs. Theoretical analysis of dual economies has often focused on wage subsidies as a solution, but in the likely absence of lump sum taxes, raising the revenue for such subsidies will involve deadweight losses for the duration of the subsidy scheme.[10]

Overall, the case for eliminating dualism might have to be based on two other considerations: the consequences of dualism for inequality, and interaction with other imperfections or distortions. In the dual economy, the rural-urban wage gap and urban unemployment both contribute to income inequality. The results above show that there are also other distributional effects at work. The elimination of dualism is associated with a movement in factor shares that usually works in favour of capital.[11]

The most important costs of dualism may arise if wage differentials interact with other imperfections and distortions to generate much more significant welfare losses. The example Williamson (1987, 1989) emphasizes is the interaction with capital market failure. Say that the modern sector must finance most of its

[10] On wage subsidies in the Harris-Todaro model, see Basu (1980), Bhagwati and Srinivasan (1974), Corden and Findlay (1975) and Ray (1998, p. 382-388).

[11] In the cross-country data, dualism is associated with greater inequality. See Bourguignon and Morrisson (1998).

investment from its own profits. As a result, anything which constrains the size of the modern sector will tend to be associated with a lower capital stock and lower labor productivity. More generally, Fishlow and David (1961) established that the joint impact of imperfections in the capital and labor market may be deadweight losses that are rather more significant than those arising from imperfections in one market alone. Finally, however, note that when a differential in rental rates is present in the 2 x 2 Harris-Todaro model, a reduction in the minimum wage has an ambiguous effect on aggregate output (Khan and Naqvi 1983).[12]

Perhaps the most important reason for concern about dualism is that there could be significant externalities in the modern sector. These could be associated with, for example, the beneficial effects of learning-by-doing or agglomeration. As we saw above, the elimination of dualism can be associated with dramatic changes in sectoral structure, especially if the elasticity of substitution in production is high. In the presence of sector-specific externalities, these changes in sectoral structure could have significant consequences for welfare, and the presence of dualism would then be a major policy concern. Some of the relevant issues are discussed in Graham and Temple (2001), who calibrate a model of a dual economy with a sector-specific externality that gives rise to multiple equilibria.

7 Conclusion

This paper has sought to quantify the costs of dualism. The paper first shows how to calibrate a two-sector general equilibrium model of production using readily available data on sectoral output and employment shares. Using results introduced here, it is straightforward to recover technology parameters and the intersectoral capital allocation from the available data. This approach may have wider application, especially to the study of structural change and growth.

The particular calibration exercise I pursue is based upon the extension of the Harris-Todaro model by Corden and Findlay (1975), and provides some insight into the consequences of dualism for aggregate output, factor returns, factor shares and sectoral structure. There are three main findings. First, the elimination of dualism is not associated with particularly large output gains, even when one starts from a position of urban unemployment. Second, the movement to the first-best is associated with slightly lower wages in agriculture.

[12] A reduction in the rental differential will always raise aggregate output in the 2 x 2 model. This is not the case, however, in a model with a role for land in the agricultural production function. See Chao and Yu (1992).

Rental rates rise, and the distribution of factor income shifts against labor.

The third and perhaps most interesting finding is that the elimination of dualism can give rise to large changes in sectoral structure. The novel implication is that a failure to industrialize could have its origin in the labor market. An interesting avenue for future research would be to investigate the generality of this claim, perhaps based on computable general equilibrium models that include a role for non-traded goods, and a more detailed specification of labor market rigidities and imperfections.

The paper also casts light on the effects of minimum wage legislation or other forms of wage floor, when coverage is incomplete. As noted earlier, we can interpret the agricultural sector as one uncovered by minimum wages, and the urban sector as covered. The paper analyzes a case where the minimum wage generates a 30% unemployment rate in the covered sector. Despite unemployment of this extent, the effects of such legislation on aggregate output and wages in the uncovered sector are generally found to be small for the cases considered here. The minimum wage does succeed in redistributing income towards labor, however.

8 Appendix 1

This appendix describes the calibration technique for the version of the model with CES production functions in both sectors. Once again the labor shares in the two sectors in the dual economy, η_a and η_m, can be recovered using equations (12) and (14). We can then use these equations to recover the share of agricultural capital in total capital, $x = K_a/K$. Start by noting that under intersectoral capital mobility, where r is the rental rate:

$$\begin{aligned} 1 - \eta_a &= \frac{rK_a}{Y_a} = \frac{rK}{Y}\frac{K_a}{K}\frac{Y}{Y_a} \\ &= (1-\eta)\frac{x}{s} \end{aligned} \tag{28}$$

If we substitute in for η_a using (12), and rearrange, we find that:

$$x = \frac{s - \eta a}{1 - \eta} \tag{29}$$

which defines x in terms of two observable variables (a and s) and the aggregate labor share η. The rest of the solution procedure aims to establish two simultaneous equations in two unknowns, the first-best agricultural employment share $b = L'_a/L$ and the first-best share of capital allocated to agriculture, $z = K'_a/K$.

The distribution parameters δ_a and δ_m are assumed to be unobserved, and must be recovered from the data. Using the equation for the labor share in agriculture ($\eta_a = w_a L_a / Y_a$) and assuming that labor in agriculture is paid its marginal product, the following equation can be derived:

$$\delta_a = \frac{1 - \eta_a}{1 - \eta_a + \eta_a \left(\frac{K_a}{L_a}\right)^{-\rho}} \tag{30}$$

A corresponding equation holds for δ_m:

$$\delta_m = \frac{1 - \eta_m}{1 - \eta_m + \eta_m \left(\frac{K_m}{L_m}\right)^{-\rho}} \tag{31}$$

(An equivalent equation was derived by Klump and Preissler, 2000, p. 45).

In the dual economy, equalization of rental rates implies:

$$\begin{aligned} \delta_a A_a^{-\rho} \left(\frac{Y_a}{K_a}\right)^{1+\rho} &= p \delta_m A_m^{-\rho} \left(\frac{Y_m}{K_m}\right)^{1+\rho} \\ \left(\frac{x}{1-x}\right)^{1+\rho} &= \frac{\delta_a}{\delta_m} \left(\frac{pA_m}{A_a}\right)^{\rho} \left(\frac{s}{1-s}\right)^{1+\rho} \end{aligned} \tag{32}$$

With CES production functions, the Harris-Todaro condition (10) can be written as:

$$(1-\delta_a) A_a^{-\rho} \left(\frac{Y_a}{L_a}\right)^{1+\rho} = (1-u)p(1-\delta_m) A_m^{-\rho} \left(\frac{Y_m}{L_m}\right)^{1+\rho} \tag{33}$$

$$(1-u)^{-\rho} \left(\frac{a}{1-a}\right)^{1+\rho} = \left(\frac{1-\delta_a}{1-\delta_m}\right) \left(\frac{pA_m}{A_a}\right)^{\rho} \left(\frac{s}{1-s}\right)^{1+\rho} \tag{34}$$

Now turn to the equations for the first-best economy, where $b = L'_a/L$ is the proportion of employment in agriculture, $r = Y'_a/Y$ is the share of agriculture in output and $z = K'_a/K$ is the proportion of capital employed in agriculture. Y'_m, K'_m and L'_m are output, capital and labor in manufacturing respectively. Let the share of labor in national income in the first-best be ϕ, and the shares of labor income in agricultural and manufacturing income be ϕ_a and ϕ_m respectively. Intersectoral capital mobility in the first-best economy yields an equation similar to (32):

$$\left(\frac{z}{1-z}\right)^{1+\rho} = \frac{\delta_a}{\delta_m} \left(\frac{pA_m}{A_a}\right)^{\rho} \left(\frac{r}{1-r}\right)^{1+\rho} \tag{35}$$

We can now start to derive two simultaneous equations in terms of b, z and observable variables, which can then be solved for the first-best allocation of employment b and capital z. The simplest way to derive the first simultaneous equation is to divide the expressions for the marginal products in the Harris-Todaro economy by those in the first-best economy. In the case of labor's marginal products, the result is:

$$\left(\frac{Y_a}{Y_a'}\right)^{1+\rho} = (1-u)^{-\rho}\left(\frac{a}{b}\right)^{1+\rho}\left(\frac{1-b}{1-a}\right)^{1+\rho}\left(\frac{Y_m}{Y_m'}\right)^{1+\rho}$$

and similarly for capital:

$$\left(\frac{Y_a}{Y_a'}\right)^{1+\rho} = \left(\frac{x}{z}\right)^{1+\rho}\left(\frac{1-z}{1-x}\right)^{1+\rho}\left(\frac{Y_m}{Y_m'}\right)^{1+\rho}$$

Combining these two expressions yields the first key equation:

$$\frac{b}{1-b} = \left(\frac{a}{1-a}\right)\left(\frac{1-x}{x}\right)(1-u)^{\frac{-\rho}{1+\rho}}\left(\frac{z}{1-z}\right) \tag{36}$$

To derive the second simultaneous equation, note that as before, we have two equations for the income shares, similar to (12) and (14):

$$\phi_a = \frac{\phi b}{r} \tag{37}$$

$$\phi_m = \phi\left(\frac{1-b}{1-r}\right) \tag{38}$$

The equations corresponding to (30) and (31) are the same in form, with the appropriate change of symbols:

$$\delta_a = \frac{1-\phi_a}{1-\phi_a+\phi_a\left(\frac{K_a'}{L_a'}\right)^{-\rho}} \tag{39}$$

$$\delta_m = \frac{1-\phi_m}{1-\phi_m+\phi_m\left(\frac{K_m'}{L_m'}\right)^{-\rho}} \tag{40}$$

Now we make use of the fact that the distribution parameters δ_a and δ_m are aspects of the production technologies, and so are the same for the Harris-Todaro and the first-best economies. Equating (30) to (39), and (31) to (40), we can derive equations for the labor shares in the first-best:

$$\phi_a = \frac{1}{1+\left(\frac{1-\eta}{\eta}\right)\left(\frac{x}{a}\right)^{1+\rho}\left(\frac{b}{z}\right)^{\rho}} \tag{41}$$

$$\phi_m = \frac{1}{1+\left(\frac{1-\eta}{\eta}\right)\left(\frac{1-x}{1-a}\right)^{1+\rho}\left(\frac{1-b}{1-z}\right)^{\rho}(1-u)^{-\rho}} \tag{42}$$

Using these equations together with (37) and (38), we have:

$$\frac{\phi_a}{\phi_m} = \left(\frac{b}{1-b}\right)\left(\frac{1-r}{r}\right) = \frac{1+\left(\frac{1-\eta}{\eta}\right)\left(\frac{1-x}{1-a}\right)^{1+\rho}\left(\frac{1-b}{1-z}\right)^{\rho}(1-u)^{-\rho}}{1+\left(\frac{1-\eta}{\eta}\right)\left(\frac{x}{a}\right)^{1+\rho}\left(\frac{b}{z}\right)^{\rho}} \tag{43}$$

Now we need to eliminate the unobservable variable r. If we combine equations (32) and (35), we find that r, z, x and s are related in the following way:

$$\frac{r}{1-r} = \left(\frac{z}{1-z}\right)\left(\frac{1-x}{x}\right)\left(\frac{s}{1-s}\right) \tag{44}$$

Hence equation (43) can be rewritten as:

$$\frac{b}{1-b} = \left(\frac{z}{1-z}\right)\left(\frac{1-x}{x}\right)\left(\frac{s}{1-s}\right)\left(\frac{1+\left(\frac{1-\eta}{\eta}\right)\left(\frac{1-x}{1-a}\right)^{1+\rho}\left(\frac{1-b}{1-z}\right)^{\rho}(1-u)^{-\rho}}{1+\left(\frac{1-\eta}{\eta}\right)\left(\frac{x}{a}\right)^{1+\rho}\left(\frac{b}{z}\right)^{\rho}}\right) \tag{45}$$

Equations (36) and (45) can be solved numerically to obtain b and z.

The remaining results are computed as follows. To calculate the output ratio across the two economies, first note that the equality of wages with marginal products for CES production functions implies the following modern sector labor share for the Harris-Todaro economy:

$$\eta_m = (1-\delta_m)\, A_m^{-\rho}\left(\frac{Y_m}{L_m}\right)^{\rho} \tag{46}$$

and for the first-best:

$$\phi_m = (1-\delta_m)\, A_m^{-\rho}\left(\frac{Y'_m}{L'_m}\right)^{\rho} \tag{47}$$

From (24) in the text, the output ratio across the two economies can be written as:

$$\Lambda = \frac{Y'_m}{Y_m}\left(\frac{1+\frac{Y'_a}{pY'_m}}{1+\frac{Y_a}{pY_m}}\right) = \frac{Y'_m}{Y_m}\left(\frac{1+\frac{r}{1-r}}{1+\frac{s}{1-s}}\right)$$

Using (44), (46), and (47) this can be rewritten as:

$$\begin{aligned}\Lambda &= \left(\frac{\phi_m}{\eta_m}\right)^{\frac{1}{\rho}}\left(\frac{L'_m}{L_m}\right)\left(\frac{1+\left(\frac{z}{1-z}\right)\left(\frac{1-x}{x}\right)\left(\frac{s}{1-s}\right)}{1+\frac{s}{1-s}}\right)\\ &= \left(\frac{\phi_m}{\eta_m}\right)^{\frac{1}{\rho}}\left(\frac{1-b}{(1-u)(1-a)}\right)\left(\frac{1+\left(\frac{z}{1-z}\right)\left(\frac{1-x}{x}\right)\left(\frac{s}{1-s}\right)}{1+\frac{s}{1-s}}\right)\end{aligned}$$

The only additional information this requires is the solution for (42) which can be calculated using the solutions for b and z. Finally, note that we can also obtain the ratio of modern sector wages in the two economies using

$$\frac{w'_m}{w_m} = \left(\frac{Y'_m/L'_m}{Y_m/L_m}\right)^{1+\rho} = \left(\frac{\phi_m}{\eta_m}\right)^{\frac{1+\rho}{\rho}}$$

where the second equality follows from equations (46) and (47). Corresponding expressions can easily be derived for the ratios of agricultural wages and rental rates across the two economies.

9 Appendix 2

This appendix describes simple modifications to the Cobb-Douglas results of section 3 that apply when capital is sector-specific, rather than mobile between sectors. We can consider this case by setting $z = x$. Then the first-best agricultural employment share b_{SS} is implicitly defined by

$$\left(\frac{a}{b_{SS}}\right)^{\alpha}\left(\frac{1-b_{SS}}{1-a}\right)^{\theta}(1-u)^{1-\theta} = 1 \tag{48}$$

Given a, u, α and θ this equation can be solved for b_{SS} numerically. We can also find the effects of dualism on aggregate output when capital is sector-specific. Using equation (24) in the main text and substituting in the Cobb-Douglas production functions for Y_m and Y'_m, and cancelling terms, gives

$$\Lambda_{SS} = \left(\frac{1-b_{SS}}{1-a}\right)^{1-\theta}(1-u)^{\theta-1}\left(\frac{1+\frac{b_{SS}}{1-b_{SS}}\left(\frac{1-\theta}{1-\alpha}\right)}{1+\frac{a}{1-a}\left(\frac{1-\theta}{1-\alpha}\right)}\right) \tag{49}$$

Other results follow in a straightforward way.

References

[1] Agénor, P.-R. (1996). The labor market and economic adjustment. *IMF Staff Papers*, 43, 261-335.

[2] Allen, J. (2001). The state of the art in modeling migration in the LDCs: a comment. *Journal of Regional Science*, 41(3), 521-528.

[3] Basu, K. (1980). Optimal policies in dual economies. *Quarterly Journal of Economics*, 95, 187-196.

[4] Bhagwati, J. N. and Srinivasan, T. N. (1974). On reanalyzing the Harris-Todaro model: policy rankings in the case of sector-specific sticky wages. *American Economic Review*, 64, 502-508.

[5] Bourguignon, F. and Morrisson, C. (1998). Inequality and development: the role of dualism. *Journal of Development Economics*, 57, 233-257.

[6] Chao, C. and Yu, E. S. H. (1992). Capital markets, urban unemployment and land. *Journal of Development Economics*, 38, 407-413.

[7] Clark, C. (1957). *The conditions of economic progress* (3rd edn.) London: Macmillan.

[8] Corden, W. M. and Findlay, R. (1975). Urban unemployment, intersectoral capital mobility and development policy. *Economica*, 42, 59-78.

[9] de Melo, Jaime A. P. (1977). Distortions in the factor market: some general equilibrium estimates. *Review of Economics and Statistics*, 59(4), 398-405.

[10] Dougherty, C. and Selowsky, M. (1973). Measuring the effects of the misallocation of labor. *Review of Economics and Statistics*, 5, 386-390.

[11] Fields, Gary S. (1997). Wage floors and unemployment: a two-sector analysis. *Labour Economics*, 4, 85-91.

[12] Fishlow, A. and David, P. A. (1961). Optimal resource allocation in an imperfect market setting. *Journal of Political Economy*, 69, 529-546.

[13] Freeman, R. B. (1992). Labor market institutions and policies: help or hindrance to economic development? *Proceedings of the World Bank Annual Conference on Development Economics*, 117-144.

[14] Freeman, R. B. (1993). Labor markets and institutions in economic development. *American Economic Review*, 83, 403-408.

[15] Graham, B. S. and Temple, J. R. W. (2001). Rich nations, poor nations: how much can multiple equilibria explain? *CEPR discussion paper no. 3046.*

[16] Harris, J. R. and Todaro, M. (1970). Migration, unemployment and development: a two-sector analysis. *American Economic Review*, 60, 126-142.

[17] Hatton, T. J. and Williamson, J. G. (1991). Integrated and segmented labor markets: thinking in two sectors. *Journal of Economic History*, 51, 413-425.

[18] Johnson, Harry G. (1966). Factor market distortions and the shape of the transformation curve. *Econometrica*, 34(3), 686-698.

[19] Jones, Ronald W. (1965). The structure of simple general equilibrium models. *Journal of Political Economy*, 73(6), 557-572.

[20] Jones, Ronald W. (1971). Distortions in factor markets and the general equilibrium model of production. *Journal of Political Economy*, 79(3), 437-459.

[21] Kelley, A. C., Williamson, J. G. and Cheetham, R. J. (1972). *Dualistic economic development: theory and history*. Chicago: University of Chicago Press.

[22] Khan, M. A. and Naqvi, S. N. H. (1983). Capital markets and urban unemployment. *Journal of International Economics*, 15, 367-385.

[23] Kim, D.-I. and Topel, R. H. (1995). Labor markets and economic growth: lessons from Korea's industrialization, 1970-1990. In Freeman, R. B. and Katz, L. F. (eds.) *Differences and changes in wage structures*. Chicago: NBER and University of Chicago Press, 227-264.

[24] Klump, R. and Preissler, H. (2000). CES production functions and economic growth. *Scandinavian Journal of Economics*, 102(1), 41-56.

[25] Knight, J. (1998). The labor market and its reform in southern Africa and China: similarities and contrasts. Manuscript, Oxford University.

[26] MacLeod, W. Bentley and Malcomson, James M. (1998). Motivation and markets. *American Economic Review*, 88(3), 388-411.

[27] Magee, Stephen P. (1973). Factor market distortions, production and trade: a survey. *Oxford Economic Papers*, 25, 1-43.

[28] Magee, Stephen P. (1976). *International trade and distortions in factor markets*. New York: Marcel Dekker.

[29] Magnac, Th. (1991). Segmented or competitive labor markets. *Econometrica*, 59(1), 165-187.

[30] Mincer, Jacob (1976). Unemployment effects of minimum wages. *Journal of Political Economy*, 84(4), S87-S104.

[31] Moene, Karl Ove (1988). A reformulation of the Harris-Todaro mechanism with endogenous wages. *Economics Letters*, 27(4), 387-390.

[32] Ray, D. (1998). *Development Economics*. Princeton University Press, Princeton, NJ.

[33] Robinson, S. (1989). Multisectoral models. In H. Chenery and T. N. Srinivasan (eds.) *Handbook of Development Economics, Volume II*. North Holland, Amsterdam.

[34] Rosenzweig, M. R. (1988). Labor markets in low-income countries. In H. Chenery and T. N. Srinivasan (eds.) *Handbook of Development Economics, Volume I*. Amsterdam: North Holland.

[35] Squire, L. (1981). *Employment policy in developing countries*. New York: Oxford University Press.

[36] Stark, O. (1991). *The migration of labor*. Oxford: Basil Blackwell.

[37] Todaro, M. P. (1969). A model of labor migration and urban unemployment in less developed countries. *American Economic Review*, 59, 138-148.

[38] Williamson, J. G. (1987). Did English factor markets fail during the Industrial Revolution? *Oxford Economic Papers*, 39, 641-678.

[39] Williamson, J. G. (1989). The constraints on industrialisation: some lessons from the First Industrial Revolution. In J. G. Williamson and V. R. Panchamukhi (eds.) *The balance between industry and agriculture in economic development*, vol. 2. Macmillan: London.

[40] World Bank (1995). *World Development Report*. New York: Oxford University Press.

[41] Yabuuchi, S. (1993). Urban unemployment, international capital mobility and development policy. *Journal of Development Economics*, 41, 399-403.

10 Notes for referees

The derivations reported in the text are based on some tedious algebra. For convenience, the notes below fill in some of the missing steps.

Since the process of checking the algebra is an arduous one, it may be worth stating that I am reasonably confident that the expressions given in the text are correct, for no fewer than four reasons beyond the usual revisits to the algebra. Several of the points below can be confirmed by referring to the attached printout of the Maple computer program I have written for the Cobb-Douglas case; the more complex programs for the CES case are available immediately on request.

1. Whenever the model is calibrated, I use the results together with the production functions to calculate the marginal products of capital and labour in both the Harris-Todaro and the first-best economies: I then check that the marginal products are related in the appropriate way (for instance, they should be equal in the first-best). In the computer program, this corresponds to checking that wdiff=0, wdiff2=0 and rdiff=0. This version of the software does not check the equality of rental rates under dualism, but that is easily done (and has been).

2. When the model is calibrated I also check that the sum of the values of the marginal products times the input quantities is equal to nominal output (i.e. I check the value added identity) where total nominal output is found by summing the nominal outputs of the two sectors. Various other consistency checks have been implemented in developing the software - some of these can be seen in the computer printout. Note that the program provided checks that various components sum to zero; in earlier and longer versions of the software, I have checked that the relevant individual components sum to zero (for example, x-x2 should equal zero, outfac2-outfac=0 etc.).

3. The results from the calibration exercise are in agreement throughout with the relevant trade theory, as discussed at greater length in the main text.

4. The calibration results found for the Cobb-Douglas case (elasticity of substitution equal to one) are almost exactly identical to those found for the CES case with an elasticity of substitution of 1.0001. Since the algebra for the two cases is derived almost completely independently, and both derivations give the same answers for this value of the elasticity of subsitution, this tends to reinforce my belief that these lengthy derivations are correct.

10.1 Derivation of (23)

Start from equation (22):

$$
\begin{aligned}
1 &= \left(\frac{1-a\left(1-\frac{\alpha}{\theta}\left(\frac{1-\theta}{1-\alpha}\right)\right)}{1-b_M\left(1-\frac{\alpha}{\theta}\left(\frac{1-\theta}{1-\alpha}\right)\right)}\right)^{\alpha-\theta}(1-u)^{1-\theta} \\
b_M &= \frac{1-\left[1-a\left(1-\frac{\alpha}{\theta}\left(\frac{1-\theta}{1-\alpha}\right)\right)\right](1-u)^{\frac{1-\theta}{\alpha-\theta}}}{1-\frac{\alpha}{\theta}\left(\frac{1-\theta}{1-\alpha}\right)}
\end{aligned}
$$

Note that the denominator is equal to:

$$\frac{\theta-\alpha}{\theta(1-\alpha)}$$

Using this we can derive

$$
\begin{aligned}
b_M &= \frac{\theta(1-\alpha)}{\theta-\alpha}-\frac{\theta(1-\alpha)}{\theta-\alpha}\left[1-a\left(1-\frac{\alpha}{\theta}\left(\frac{1-\theta}{1-\alpha}\right)\right)\right](1-u)^{\frac{1-\theta}{\alpha-\theta}} \\
&= \frac{\theta(1-\alpha)}{\theta-\alpha}-\left[\frac{\theta(1-\alpha)}{\theta-\alpha}-\frac{\theta(1-\alpha)}{\theta-\alpha}a\left(1-\frac{\alpha}{\theta}\left(\frac{1-\theta}{1-\alpha}\right)\right)\right](1-u)^{\frac{1-\theta}{\alpha-\theta}} \\
&= \frac{\theta(1-\alpha)}{\theta-\alpha}-\left[\frac{\theta(1-\alpha)}{\theta-\alpha}-\frac{\theta(1-\alpha)}{\theta-\alpha}a\left(\frac{\theta\,(1-\alpha)}{\theta(1-\alpha)}-\frac{\alpha}{\theta}\left(\frac{1-\theta}{1-\alpha}\right)\right)\right](1-u)^{\frac{1-\theta}{\alpha-\theta}} \\
&= \frac{\theta(1-\alpha)}{\theta-\alpha}-\left[\frac{\theta(1-\alpha)}{\theta-\alpha}-a\right](1-u)^{\frac{1-\theta}{\alpha-\theta}}
\end{aligned}
$$

which is equation (23) in the text.

10.2 Derivation of (30)

Agricultural labor is paid its marginal product, so with a CES function this implies:

$$w_a=(1-\delta_a)\,A_a^{-\rho}\left(\frac{Y_a}{L_a}\right)^{1+\rho}$$

See for instance Chung (1994, p. 111). Then agriculture's share in agricultural income is:

$$
\begin{aligned}
\eta_a &= \frac{w_aL_a}{Y_a}=(1-\delta_a)\,A_a^{-\rho}\left(\frac{Y_a}{L_a}\right)^{1+\rho}\frac{L_a}{Y_a} \\
&= (1-\delta_a)\,A_a^{-\rho}\left(\frac{Y_a}{L_a}\right)^{\rho} \\
&= (1-\delta_a)\,A_a^{-\rho}A_a^{\rho}\frac{[\delta_aK_a^{-\rho}+(1-\delta_a)L_a^{-\rho}]^{-1}}{L_a^{\rho}} \\
&= \frac{(1-\delta_a)}{\left(\delta_aK_a^{-\rho}+(1-\delta_a)L_a^{-\rho}\right)L_a^{\rho}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(1-\delta_a)}{\delta_a\left(\frac{K_a}{L_a}\right)^{-\rho}+(1-\delta_a)} \\
\eta_a\delta_a\left(\frac{K_a}{L_a}\right)^{-\rho}+\eta_a(1-\delta_a) &= (1-\delta_a) \\
\delta_a\left(1-\eta_a+\eta_a\left(\frac{K_a}{L_a}\right)^{-\rho}\right) &= 1-\eta_a \\
\delta_a &= \frac{1-\eta_a}{\left(1-\eta_a+\eta_a\left(\frac{K_a}{L_a}\right)^{-\rho}\right)}
\end{aligned}
$$

10.3 Derivation of labor shares ready for (43):

First of all, note the equations in the text (30) and (31) imply that:

$$
\frac{\delta_a}{1-\delta_a} = \frac{1-\eta_a}{\eta_a\left(\frac{x}{a}\right)^{-\rho}\left(\frac{K}{L}\right)^{-\rho}} \tag{50}
$$

and

$$
\frac{\delta_m}{1-\delta_m} = \frac{1-\eta_m}{\eta_m\left(\frac{1-x}{(1-u)(1-a)}\right)^{-\rho}\left(\frac{K}{L}\right)^{-\rho}}
$$

From (39) we have:

$$
\begin{aligned}
\delta_a &= \frac{1-\phi_a}{1-\phi_a+\phi_a\left(\frac{K'_a}{L'_a}\right)^{-\rho}} \\
\delta_a\left[1-\phi_a+\phi_a\left(\frac{K'_a}{L'_a}\right)^{-\rho}\right] &= 1-\phi_a \\
\phi_a\left[1-\delta_a+\delta_a\left(\frac{K'_a}{L'_a}\right)^{-\rho}\right] &= 1-\delta_a \\
\phi_a &= \frac{1-\delta_a}{\left[1-\delta_a+\delta_a\left(\frac{K'_a}{L'_a}\right)^{-\rho}\right]} \\
\phi_a &= \frac{1}{\left[1+\frac{\delta_a}{1-\delta_a}\left(\frac{K'_a}{L'_a}\right)^{-\rho}\right]}
\end{aligned}
$$

Substituting in using (50) above and using the definitions $z = K'_a/K$ and $b = L'_a/L$:

$$
\phi_a = \frac{1}{\left[1+\left(\frac{1-\eta_a}{\eta_a\left(\frac{x}{a}\right)^{-\rho}\left(\frac{K}{L}\right)^{-\rho}}\right)\left(\frac{z}{b}\right)^{-\rho}\left(\frac{K}{L}\right)^{-\rho}\right]}
$$

Simplifying and using the expressions in the text for η_a and $1-\eta_a$ given by equations (12) and (28) ultimately yields the equation in the text. Similar calculations give the equation in the text for ϕ_m.

References

[1] Chung (1994). *Utility and production functions.*