The oil extraction puzzle: theory and evidence*

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ABSTRACT

This paper considers the relationship between the extraction rates and remaining reserves of a non-renewable resource. Under general conditions the derived extraction rule is firstly linear, and secondly exhibits a slope term common to all extractors regardless of pricing behaviour and costs whilst differences are captured by the intercept. Data from the world oil industry supports the hypothesis of linearity but the implied test was rejected in some cases. Latterly, it appears that either OPEC members are discounting at a higher rate than the competitive fringe or they are overstating their reserve levels.

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The ratios of extraction rates to estimated remaining reserves are widely reported for different countries, and indeed on occasion are mistakenly used to extrapolate time until exhaustion. Less attention is given to what might constitute the optimal relationship between extraction and remaining reserves. This paper solves an optimal extraction problem for a variety of alternative market structures and proposes a simple linear extraction-reserves relationship.

The theoretical work proposes that *regardless of the market structure*, the relationship between extraction and remaining reserves boils down to a surprisingly simple representation. At any point in time, optimal extraction (q_{it}) depends on a firm-specific constant (λ_{it}) and the product of some common slope parameter (μ_t) and own remaining reserves (R_{it}) – a straightforward linear function wherein differences in costs and market structure are all contained within the intercept term. Furthermore, the slope parameter is the same across all firms and extraction is independent of other firms' remaining reserves, hence can be written simply as $q_{it} = \lambda_{it} + \mu_t R_{it}$. Estimation of the model using data from the world oil industry gives reasonable support for the hypothesis of linearity and finds that the parameter estimates are robust. The test of the implied restriction is passed for the case of 1981 but rejected using data from 1991 and 2000. The oil extraction puzzle is that in 1991 and 2000 at least, the extraction-reserves relationship is different for OPEC members than for non-OPEC countries whether price setting or not. Some alternative resolutions of this puzzle are offered.

The paper is organised as follows. Section 1 describes the model foundations; section 2 formally derives the extraction paths within the context of a two-period model and section 3 derives solutions within the three-period context. The model is estimated and tested in section 4 and section 5 concludes.

1. Model Foundations

Following Eswaran and Lewis (1985), Hansen, Epple and Roberds (1985) and Hartwick and Sadorsky (1990) two firms, which can be thought of as OPEC and non-OPEC groups of countries extract from a fixed resource base over two, and later three periods. The framework is rich enough to permit analysis of competitive, monopolistic, and various intermediate market structures. The theoretical work confirms findings of the existing literature; in simulations prices start lowest (highest) and finish highest (lowest) under competition (monopoly). Intermediate market structures imply price and extraction paths somewhere between the two extremes. Simulations suggest that there is little difference in aggregate extraction (and therefore price) between the competitive and Stackelberg leadership cases, although the composition of output differs markedly. Analysis of the three-period case confirms the result of Eswaran and Lewis (1985) that within the linear-quadratic model, the Stackelberg leader model is dynamically consistent.

The model is based on that of Hansen, Epple and Roberds (1985), having removed the stochastic component of their specification. In the first instance there are two suppliers of a resource who allocate extraction over two time periods, t = 0,1. The problem is expressed as an unconstrained maximisation problem,

$$U_i = p_0 q_{i0} - \frac{\phi_i}{2} q_{i0}^2 + \left(\frac{1}{1+r}\right)\left\{ p_1 \left(R_i - q_{i0}\right) - \frac{\phi_i}{2}\left(R_i - q_{i0}\right)^2 \right\} \tag{1}$$

Where p_t is the resource price at time t, q_{it} is extraction by firm i at time t, r is the discount rate, and R_i are firm-i's reserves at the start of the initial period and are taken as given. Extraction costs are quadratic, and are allowed to vary across the firms according to the parameter, ϕ_i. Extraction costs are presupposed to be low enough so as to make the reserve constraint, $R_i = q_{i0} + q_{i1}$ bind. Maximisation is also subject to a conventional inverse linear demand function,

$$p_t = a - b\left[q_{1t} + q_{2t}\right] \tag{2}$$

where a and b are positive parameters.

A great deal of literature has focussed on the nature of extraction technology as represented by the cost function. This originates with Hotelling himself who analyses the case of increasing costs "as the mine goes deeper" and cumulative extraction increases (or the stock of remaining reserve decreases). Subsequent theoretical work includes that by Solow and Wan (1976), Heal (1976), Hanson (1980) and Farzin (1992). The quadratic form used here is relatively simplistic but has the advantage that it permits a simple and testable linear extraction rule. Alternative representations of the technology, discussed in more detail below, result in considerably more complicated extraction rules.

The method by which the firms attempt to maximise net present value depends on how they view their own and their competitor's ability to manipulate the price. Hansen, Epple and Roberds analyse three 'symmetric', and three 'non-symmetric' games. The symmetric games are as follows:

Competitive Equilibrium: Each supplier maximises its respective objective function treating prices as exogenous.

Symmetric Nash Game: Each supplier maximises its respective objective function taking into account its own influence on prices but treating the decisions of the other supplier as exogenous.

Collusive Game: The two suppliers collude and maximise the sum of the two objective functions, taking into account their influence on prices.

The non-symmetric games all imply that in some sense firm 2 (i.e. OPEC or its core) is a leader. These games are characterised as

Nash-Competitive (Myopic Leader) Game: The second player maximises its objective function, taking into account its influence on prices but treating the decisions of the first player as exogenous.

Dominant-Competitive (Stackelberg) Game: The second supplier maximises its objective function taking into account its influence both on prices and on the decisions of the first player.

Hansen, Epple and Roberds also analyse a variant of game E that they call the time consistent dominant-competitive game. However, Eswaran and Lewis (1985) demonstrate that within a linear-quadratic framework without uncertainty the open-loop solution to game E is dynamically consistent.[1] Our analysis is therefore restricted to the five cases.

2. The Two-Period Model

For the symmetric games, and indeed game D, Hansen, Epple and Roberds reformulate the maximisation problem of (1) as a problem faced by a social planner. In these cases the social planner maximises

$$U = \sum_{t=0}^{1} \beta^t \left\{ \mathbf{a}'[\mathbf{q_t}] - \frac{1}{2}[\mathbf{q_t}]'\mathbf{\Omega}[\mathbf{q_t}] - \frac{1}{2}[\mathbf{q_t}]'\boldsymbol{\theta}[\mathbf{q_t}] \right\} \qquad (3)$$

where $\beta = \dfrac{1}{1+r}$, $\mathbf{a} = \begin{bmatrix} a \\ a \end{bmatrix}$, $\mathbf{q_0} = \begin{bmatrix} q_{10} \\ q_{20} \end{bmatrix}$, $\mathbf{q_1} = \begin{bmatrix} R_1 - q_{10} \\ R_2 - q_{20} \end{bmatrix}$, $\boldsymbol{\theta} = \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix}$, and $\mathbf{\Omega}$ is a 2x2 matrix that captures the structure of the game. In the competitive case (A), $\mathbf{\Omega^a} = \begin{bmatrix} b & b \\ b & b \end{bmatrix}$, implying that both firms set marginal cost equal to price. In case C $\mathbf{\Omega^c} = \begin{bmatrix} 2b & 2b \\ 2b & 2b \end{bmatrix}$, implying that both firms equate marginal cost to marginal revenue. Hence, maximisation of (3) in the competitive case requires that

$$a - b(q_{10} + q_{20}) - \theta_i q_{i0} = \beta[a - b(R_1 - q_{10} + R_2 - q_{20}) - \theta_i(R_i - q_{i0})] \qquad (4)$$

for i =1,2. The present value of the difference between price and marginal cost is constant, which is the familiar condition under competition. On the other hand, maximisation under monopoly requires that

$$a - 2b(q_{10} + q_{20}) - \theta_i q_{i0} = \beta[a - 2b(R_1 - q_{10} + R_2 - q_{20}) - \theta_i(R_i - q_{i0})], \qquad (5)$$

and in this case, the present value of the difference between marginal revenue and marginal costs is constant across time.

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[1] In the two-period case, a commitment to extraction in period one entails an irreversible commitment to extraction in the second period, given sufficiently low extraction costs. Dynamic consistency of the open-loop solution to game E in the three-period case is also proven below.

In the Symmetric Nash game (B), the game structure matrix, $\mathbf{\Omega}$, is defined as $\mathbf{\Omega^b} = \begin{bmatrix} 2b & b \\ b & 2b \end{bmatrix}$. This structure is discussed in Sargent (1979); both players regard the others output as given and beyond its own influence. Firms do however consider the effect upon price of changing their own output. The first order conditions in this case are that

$$a - b(2q_{i0} + q_{j0}) - \theta_i q_{i0} = \beta\left[a - b\left(2(R_i - q_{i0}) + (R_j - q_{j0})\right) - \theta_i (R_i - q_{i0})\right], \qquad (6)$$

for i, j = 1, 2; $i \neq j$. Both firms take the other's output as given, and therefore set the present value of the difference between marginal revenue and marginal costs to be equal across time.

The simplest non-symmetric game structure is that of the Nash-competitive game (game D). Player 2 behaves as in game B and (myopically) takes player 1's output as given, but player 1 now acts competitively. The game structure matrix is defined as $\mathbf{\Omega}^d = \begin{bmatrix} b & b \\ b & 2b \end{bmatrix}$ and the first order conditions now require that

$$a - b(q_{10} + q_{20}) - \theta_1 q_{10} = \beta\left[a - b(R_1 - q_{10} + R_2 - q_{20}) - \theta_1 (R_1 - q_{10})\right], \qquad (7a)$$

and $$a - b(q_{10} + 2q_{20}) - \theta_2 q_{20} = \beta\left[a - b\left(R_1 - q_{10} + 2(R_2 - q_{20})\right) - \theta_2 (R_2 - q_{20})\right]. \qquad (7b)$$

For firm one the present value of the difference between price and marginal cost is constant, and for firm 2, the present value of the difference between marginal revenue minus marginal cost is constant (and again, taking firm 1 output as given).

In each of the first four cases there are two equations, and two unknowns – first period extraction of both firms. For example rearranging (4) for q_{i0} gives two equations,

$$q_{i0} = \frac{ar}{(2+r)(b+\phi_i)} - \left[\frac{b}{b+\phi_i}\right] q_{j0} + \frac{R_i}{2+r} + \left[\frac{b}{(2+r)(b+\phi_i)}\right] R_j \qquad (8)$$

for i = 1,2; j =1,2; $i \neq j$.

Combining these two equations gives

$$q_{i0} = \frac{\phi_j ar}{(2+r)(b\phi_i + b\phi_j + \phi_i\phi_j)} + \frac{R_i}{2+r} \tag{9}$$

for both firms. It can readily be seen that in this context, firm *j*'s remaining reserves do not determine firm *i*'s extraction profile. The same procedure in the other three market structures yield the extraction rules detailed in table 1.

The dominant-firm (Stackelberg) game (E) requires a different solution method, because player two now takes into account the effect that her actions have on price *and* the output decision of the follower, player 1. In game E, player 1, the (competitive) follower, reacts to player 2, the (price fixing) leader. The follower thus adheres to the condition in (4) and the reaction function that this implies is therefore given by equation (8) where i = 1. However, the difference here is that the leader (firm 2) knows that the follower will adhere to this rule. Given this, then the leader incorporates the followers reaction function directly into her own objective function, and then optimises, choosing her own and implicitly the followers' extraction profile. The leader therefore maximises

$$U_2 = \sum_{t=0}^{1} \beta^t \left\{ p_t\left(q_{1t}^*, q_{2t}\right) q_{2t} - \frac{\phi_2}{2} q_{2t}^2 \right\} \tag{10}$$

subject to the demand function, the reserve constraint and the reaction function of the follower $\left(q_{1t}^*\right)$. This enables a rule for leader-extraction in the first period, and consequently an equation for follower extraction. These equations are also reported in table 1.

Table 1

First period extraction rules under different market structures in the 2-period model

Game structure	Extraction Rules	
Competition (A)	$q_{i0} = \frac{\phi_j ar}{(2+r)(b\phi_i + b\phi_j + \phi_i\phi_j)} + \frac{R_i}{2+r}$	(11A)
Symmetric Nash (B)	$q_{i0} = \frac{(b+\phi_j)ar}{(2+r)(3b^2 + 2b\phi_i + 2b\phi_j + \phi_i\phi_j)} + \frac{R_i}{2+r}$	(11B)
Monopoly (C)	$q_{i0} = \frac{\phi_j ar}{(2+r)(2b\phi_i + 2b\phi_j + \phi_i\phi_j)} + \frac{R_i}{2+r}$	(11C)
Nash-Competitive (D)	$q_{10} = \frac{(b+\phi_2)ar}{(2+r)[b^2 + 2b\phi_1 + b\phi_2 + \phi_1\phi_2]} + \frac{R_1}{2+r}$	(11D1)
	$q_{20} = \frac{\phi_1 ar}{(2+r)[b^2 + 2b\phi_1 + b\phi_2 + \phi_1\phi_2]} + \frac{R_2}{2+r}$	(11D2)
Stackelberg (E)	$q_{10} = \frac{[b\phi_1 + b\phi_2 + \phi_1\phi_2]ar}{(2+r)(b+\phi_1)[2b\phi_1 + b\phi_2 + \phi_1\phi_2]} + \frac{R_1}{2+r}$	(11E1)
	$q_{20} = \frac{\phi_1 ar}{(2+r)[2b\phi_1 + b\phi_2 + \phi_1\phi_2]} + \frac{R_2}{2+r}$	(11E2)

Table 1 shows that in all cases first period extraction (and therefore second period extraction) depends upon the parameters of the demand function, the cost parameters of both firms, the discount rate and own initial reserve stocks. A notable outcome is that in all cases firm *i*'s extraction is independent of firm *j*'s remaining reserves. This is not to say that firm *i*'s extraction plan is independent of the extraction path of firm *j*; the intercept term varies according to the rule by which firm j is operating. Another key result is that the 'slope' coefficient, by which today's (first period) extraction is related to own initial remaining reserves is equal to 1/(2+*r*) across all industries. The distinctions between the extraction profiles in the different industrial structures are contained entirely in the intercept terms.

Employing a more complicated technology would readily result in a more complicated extraction function. A typical representation used in the literature has marginal costs inversely related to remaining reserves, for example, $\partial C_i / \partial q_i = \phi_i q_i / R_i$. However, using the above procedure for the competitive case, even this relatively simple formulation leads to an optimal extraction rule for firm i of

$$q_{i0} = \frac{R_i}{1+r} + \frac{\phi_j \left[r(1+a) - b(R_i + R_j)\right] R_i}{(1+r)\left\{\phi_i \phi_j (1+r) + b(2+r)\left[\phi_i R_j + \phi_j R_i\right]\right\}}$$

and other market structures yield comparable results. Such extraction rules differ from the simple linear rules in a number of key respects. Firstly, firm i's extraction plan depends on the reserve levels of both firms and is highly non-linear. Secondly, if these more complicated technologies are significantly more meaningful than the model derived here then we should expect the intercept and slope estimates to be unstable across countries that follow the same price setting behaviour. The hypotheses of non-linearity and instability of parameter estimates are tested below.

The formulation permits a complete description of the extraction profile and it is of interest to compare the extraction profiles within the different industrial structures. In proposition one the relative sizes of aggregate extraction in the first period across the different industrial structures are summarized.

Proposition 1: Denoting Q_{0G} for aggregate first period extraction with game structure G = A, B, C, D, E then $Q_{0A} > Q_{0E} > Q_{0D} > Q_{0B} > Q_{0C}$. The proof of this statement is straightforward and contained in the appendix.

The composition of output depends on the specific parameterisation. As a base case of the model, the demand specification is posited as $p_t = 50 - 0.023(q_{1t} + q_{2t})$. This ensures that marginal revenue is positive over the relevant extraction range and that interior solutions are obtained for all market structures. The extraction cost parameter is the same for both firms; $\phi_1 = \phi_2 = 0.006$, ensuring that marginal cost does not exceed marginal revenue for output ranges relevant to the example. The interest rate is set at 50% (imagine the two periods correspond to ten yearly intervals). Initial reserves, R_1 and R_2 are set at 500 and 1500 to reflect an advantageous leader resource endowment. The results of this base case are presented in table 2.

Table 2

Extraction profiles, prices and profits in the 2-period model base case

Game Structure	p_0	p_1	q_{01}	q_{11}	Π_1	Q_{02}	q_{12}	Π_2
Competitive	22.8	31.2	392.3	107.7	10685	792.3	707.7	29885
Symmetric Nash	25.5	28.5	333.3	166.7	11270	733.3	766.7	30470
Monopoly	26.9	27.1	302.0	198.0	11350	702.0	798.0	30550
Nash Competitive	23.4	30.6	496.2	3.8	10939	661.3	838.7	29863
Stackelberg	23.0	31.0	439.1	60.9	10786	733.3	766.7	29930

Table 2 demonstrates the result that prices start lowest, and finish highest under competition. Conversely prices start highest and finish lowest under monopoly. Profits (Π_i) are highest for both firms under monopoly, but the full cooperation implied by monopoly is unlikely given the incentives to increase first period extraction unilaterally. When firm 1 acts competitively and firm 2 engages in price manipulation but ignores the effects of its output upon firm 2's output (market structure D) firm 1 benefits from firm 2's output reduction. However, because of the fact that firm 2 ignores its own impact upon firm 1's extraction decision firm 2 is actually worse off than even the competitive case. When firm 2 acts as a Stackelberg leader (thereby recognising the impact of its own extraction pattern upon the smaller firm) initial output is increased somewhat and firm 2 profits are higher relative to case D. The price path in the Stackelberg case is relatively unchanged from the competitive solution, although both firms benefit from this price-leadership.

It is unlikely that both firms will have the same value of the cost function parameter. Given that firm 2 extracts a larger amount in each period due to its larger reserves, then the same cost parameter implies that marginal costs are higher for firm 2 than for firm 1, which is somewhat incongruous. To examine the more realistic case, where marginal costs of firm 2 are less than those for firm 1 the base case is amended, setting ϕ_2 = 0.00003 – implying that marginal costs are approximately equal to £2 at an output level of 750. The results of this simulation are presented in table 3.

Table 3

Extraction profiles, prices and profits, 2-period model, leader cost advantage

Game Structure	p_0	p_1	q_{01}	q_{11}	Π_1	q_{02}	q_{12}	Π_2
Competitive	21.6	32.4	202.2	297.8	10500	1032.1	467.9	32391
Symmetric Nash	25.2	28.8	323.5	176.5	11161	755.5	744.5	33314
Monopoly	26.6	27.4	201.1	298.9	10509	816.2	683.8	34188
Nash Competitive	23.3	30.7	485.8	14.2	10908	674.5	825.6	32599
Stackelberg	22.6	31.4	373.0	127.0	10650	816.7	683.3	32760

When firm 2 has a cost advantage as well as greater initial reserves, table 3 shows that the price path and therefore the *aggregate* extraction path is similar to the base case. However, the composition of output in each period is changed quite markedly depending on the market structure. Firm 1 first period extraction is reduced in all cases, as would be expected given firm 2's new cost advantage. This reduction is most dramatic in the competitive instance, and relatively small in the two non-symmetric market structures.

These results imply that there is relatively little difference in aggregate output between the competitive and Stackelberg outcomes, even when the leader has a considerable cost advantage and is in possession of 75% of global reserves. The difference in first-period aggregate output under these two market structures is given by $\frac{b\phi_1^3 ar}{(2+r)(b+\phi_1)[b\phi_1+b\phi_2+\phi_1\phi_2][2b\phi_1+b\phi_2+\phi_1\phi_2]}$; hence the output difference depends on all the model parameters. Nonetheless for all plausible parameterisations this difference is relatively quite small.

The two-period model enables an examination of the price and output paths under different market structures. Optimal extraction plans do not depend on other firms' reserve stocks, and furthermore the direct relationship between extraction and own reserves is remarkably simple. We now turn to the three period case to see if this result holds, and furthermore to examine the possibility of dynamic inconsistency.

3. The Three-Period Model

Analysis of a three-period model enables us to test the robustness of the two-period results. It also enables examination of the problem of dynamic inconsistency. Recall that in the two period model there is no problem of pre-commitment, because the leader is constrained to exhaust in period 2 and there is clearly no benefit from changing that plan once period 2 arrives. However, in the three-period context, there may exist incentives to the leader to change the extraction plan once the first period is over. Newbery (1981) discusses such a context where fringe costs are lower than monopoly costs; having 'encouraged' the fringe to extract rapidly, the leader may be able to hike the price given the absence of any competitive constraints. The approach used here enables a fuller analysis of this issue as the extraction paths are explicitly derived.

The solution method[2] is as before, under the maintained assumptions of the model there are two first order conditions applying to both firms. For example in the competitive case,

$$a-b(q_{10}+q_{20})-\phi_i q_{i0} = \frac{a-b(q_{11}+q_{21})-\phi_i q_{i1}}{1+r}$$

and
$$\frac{a-b(q_{11}+q_{21})-\phi_i q_{i1}}{1+r} = a-b(R_1-q_{10}-q_{11}+R_2-q_{20}-q_{21})-\phi_i(R_i-q_{i0}-q_{i1}).$$

Solution of these four equations yields four unknowns (q_{10}, q_{11}, q_{20} and q_{21}). The extraction profiles are presented in table 4. In the case of the Stackelberg equilibrium, the leader takes the competitive responses in periods 0 and 1 (and therefore implicitly the final period) as given and optimises by choosing period 0 and 1 extraction rates. This is an open-loop equilibrium, and therefore potentially dynamically inconsistent.

Table 4 confirms the main findings of the two-period model. Optimal extraction is linear in own reserves and the slope coefficient is equal across firms regardless of whether or how the firm is price setting and the cost structure. Extraction profiles are also independent of competitors' remaining reserves. The extraction profiles in the base case[3] and the leader cost-advantage case are presented in tables 5 and 6.

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[2] The derivations are available as a Maple 6 file.
[3] Initial reserves are respectively set equal to 1000 and 2000, and the discount rate set equal to 0.3 so as to ensure an internal solution. These modifications do not alter the substance of the analysis.

Table 4

Extraction rules under different market structures in the 3-period model

Game structure	Extraction Rules	
(A)	$q_{i0} = \dfrac{\phi_j ar(3+r)}{(b\phi_i + b\phi_j + \phi_i\phi_j)(3+3r+r^2)} + \dfrac{R_i}{(3+3r+r^2)}$	(12A1)
	$q_{i1} = \dfrac{\phi_j ar^2}{(3+3r+r^2)(b\phi_i + b\phi_j + \phi_i\phi_j)} + \dfrac{(1+r)R_i}{(3+3r+r^2)}$	(12A2)
(B)	$q_{i0} = \dfrac{ar(3+r)(\phi_j + b)}{(3b^2 + 2b\phi_i + 2b\phi_j + \phi_i\phi_j)(3+3r+r^2)} + \dfrac{R_i}{(3+3r+r^2)}$	(12B1)
	$q_{i1} = \dfrac{ar^2(\phi_j + b)}{(3b^2 + 2b\phi_i + 2b\phi_j + \phi_i\phi_j)(3+3r+r^2)} + \dfrac{(1+r)R_i}{(3+3r+r^2)}$	(12B2)
(C)	$q_{i0} = \dfrac{\phi_j ar(3+r)}{(2b\phi_i + 2b\phi_j + \phi_i\phi_j)(3+3r+r^2)} + \dfrac{R_i}{(3+3r+r^2)}$	(12C1)
	$q_{i1} = \dfrac{\phi_j ar^2}{(3+3r+r^2)(2b\phi_i + 2b\phi_j + \phi_i\phi_j)} + \dfrac{(1+r)R_i}{(3+3r+r^2)}$	(12C2)
(D)	$q_{10} = \dfrac{ar(3+r)(\phi_2 + b)}{(b^2 + 2b\phi_1 + b\phi_2 + \phi_1\phi_2)(3+3r+r^2)} + \dfrac{R_1}{(3+3r+r^2)}$	(12D11)
	$q_{20} = \dfrac{\phi_1 ar(3+r)}{(3+3r+r^2)(b^2 + 2b\phi_1 + b\phi_2 + \phi_1\phi_2)} + \dfrac{R_2}{(3+3r+r^2)}$	(12D12)
	$q_{11} = \dfrac{ar^2(\phi_2 + b)}{(b^2 + 2b\phi_1 + b\phi_2 + \phi_1\phi_2)(3+3r+r^2)} + \dfrac{(1+r)R_1}{(3+3r+r^2)}$	(12D21)
	$q_{21} = \dfrac{\phi_1 ar^2}{(3+3r+r^2)(b^2 + 2b\phi_1 + b\phi_2 + \phi_1\phi_2)} + \dfrac{(1+r)R_2}{(3+3r+r^2)}$	(12D22)
(E)	$q_{10} = \dfrac{ar(3+r)(b\phi_1 + b\phi_2 + \phi_1\phi_2)}{(b+\phi_1)(2b\phi_1 + b\phi_2 + \phi_1\phi_2)(3+3r+r^2)} + \dfrac{R_1}{(3+3r+r^2)}$	(12E11)
	$q_{20} = \dfrac{\phi_1 ar(3+r)}{(3+3r+r^2)(2b\phi_1 + b\phi_2 + \phi_1\phi_2)} + \dfrac{R_2}{(3+3r+r^2)}$	(12E12)
	$q_{11} = \dfrac{ar^2(b\phi_1 + b\phi_2 + \phi_1\phi)}{(b+\phi_1)(2b\phi_1 + b\phi_2 + \phi_1\phi_2)(3+3r+r^2)} + \dfrac{(1+r)R_1}{(3+3r+r^2)}$	(12E21)
	$q_{21} = \dfrac{\phi_1 ar^2}{(3+3r+r^2)(2b\phi_1 + b\phi_2 + \phi_1\phi_2)} + \dfrac{(1+r)R_2}{(3+3r+r^2)}$	(12E22)

Once again prices starts off lowest (highest) and finish highest (lowest) in the competitive (monopoly) case. The intermediate market structures lead to price paths that concur with the above discussion. As before there is relatively very little to choose between the competitive and Stackelberg aggregate outcomes, although once again the composition of aggregate output is altered significantly in these two cases.

Table 5

Extraction profiles, prices and profits in the 3-period model base case

Game Structure	p_0	p_1	p_2	q_{10}	q_{11}	q_{12}	Π_1	q_{20}	q_{21}	q_{22}	Π_2
A	21.7	26.5	32.8	489.2	347.5	163.3	19841	739.8	673.3	586.9	37886
B	25.1	26.8	29.1	416.0	340.9	243.1	20766	666.7	666.7	666.7	38811
C	26.9	27.0	27.1	377.2	337.3	285.5	20892	627.8	663.1	709.0	38937
D	22.5	26.6	31.9	618.1	359.2	22.7	20242	577.3	658.5	764.2	37851
E	22.1	26.6	32.4	547.2	352.8	100.0	20000	666.7	666.7	666.7	37957

Table 6

Extraction profiles, prices and profits, 3-period model, leader cost advantage

Game Structure	p_0	p_1	p_2	q_{10}	q_{11}	q_{12}	Π_1	q_{20}	q_{21}	Q_{22}	Π_2
A	20.3	26.4	34.3	253.3	326.1	420.6	19549	1037	700.4	262.4	40594
B	24.7	26.8	29.5	403.9	339.7	256.4	20593	694.2	669.2	636.6	42052
C	26.5	27.0	27.5	252	325.9	422.1	19563	769.4	676	554.6	43432
D	22.4	26.6	32.0	605.2	358.1	36.8	20194	593.6	660	746.4	40922
E	21.6	26.5	32.9	465.2	345.3	189.5	19785	770.1	676.1	553.8	41177

We now turn our attention to the question of whether or not the Stackelberg equilibrium is dynamically consistent.

Proposition 2: The Stackelberg equilibrium is dynamically consistent. In particular optimal extraction in the second period detailed at the outset of the three period case is the same as optimal extraction were the leader to re-optimise at the start of the second period.

Proposition 2 is a special case of Proposition 1 of Eswaran and Lewis (1985). In order to prove proposition 2, it is required that equation (12E22) is equal to equation (11E2) above, accounting for the change in reserves held at the start of the optimisation period, hence from equation (11E2)

$$q_{21} = \frac{\phi_1 ar}{(2+r)[2b\phi_1 + b\phi_2 + \phi_1\phi_2]} + \frac{R_2^* - q_{20}}{2+r}. \qquad (13)$$

In equation (13) R_2^* are firm 2's initial reserves at the outset of period 0, thus $R_2^* - q_{20}$ are remaining reserves at the outset of period 1. Substituting in for q_{20} from equation (12E12) yields

$$q_{21} = \frac{\phi_1 ar}{(2+r)[2b\phi_1 + b\phi_2 + \phi_1\phi_2]} + \frac{1}{2+r}\left\{\left(\frac{2+3r+r^2}{3+3r+r^2}\right)R_2^* - \frac{\phi_1 ar(3+r)}{(3+3r+r^2)[2b\phi_1 + b\phi_2 + \phi_1\phi_2]}\right\}$$

which can be simplified to give

$$q_{21} = \frac{\phi_1 ar^2}{(3+3r+r^2)[2b\phi_1 + b\phi_2 + \phi_1\phi_2]} + \left(\frac{1+r}{3+3r+r^2}\right)R_2^*$$

which is equation (13E22).

Whereas the slope terms are identical regardless of market structure and costs the intercepts will only be equal in very special circumstances.[4] If the market operates under competitive conditions and the cartel has a cost advantage over the fringe then we would expect the cartel intercept to exceed the fringe intercept. If OPEC (or some subset) attempts to fix prices along the lines of the Stackelberg model and their cost advantage were sufficiently small then we would expect to find the converse. From equations (12E11) and (12E12) this requires $\phi_1 < \frac{b\phi_1 + b\phi_2 + \phi_1\phi_2}{b + \phi_1}$ and therefore that $\frac{\phi_1}{\phi_2}(\phi_1 - \phi_2) < b$ although there is no reason why either might hold. The cartel intercept may thus be above or below the fringe intercept.

[4] For example Eswaran and Lewis (1985) consider the case of perfect competition and identical cost parameters.

4. Empirical Analysis

The theory has some surprisingly strong and testable implications. Very simply, we just have

$$q_{it} = \alpha_t + \beta_t R_{it} + \gamma_t OPEC_t \qquad (14)$$

where β_t is the common slope parameter and α_t is the intercept parameter of the fringe extractors and $OPEC_t$ is a dummy variable set equal to one if the country is a member of OPEC. The unrestricted econometric model is written as

$$q_{it} = \alpha_{Ft} + \beta_{Ft} R_{it} + \eta_{Ft} R_{it}^2 \qquad (15)$$

$$q_{it} = \alpha_{Ct} + \beta_{Ct} R_{it} + \eta_{Ct} R_{it}^2 \qquad (16)$$

where α_{Ft} and α_{Ct} respectively denote the fringe and cartel intercept terms and β_{Ft} and β_{Ct}, are the respective slope terms. Under the null of a linear extraction rule with common slopes then the quadratic parameters, respectively η_{Ft} and η_{Ct} are equal to zero and $\beta_{Ft} = \beta_{Ct}$.

The data employed come from the BP Statistical Review of World Energy, 2001. This provides data on 'proved reserves' as of the end of 1980, 1990 and 1999, and average daily extraction rates[5] in millions of barrels for every major oil extracting country in the world. This data is previewed in figure 1.

The approach taken here is a cross sectional analysis for each of these three time periods due to the time varying nature of the parameters. The full sample of countries is divided into the OPEC members[6] and the 'competitive fringe'. Estimation[7] of (15) and (16) over the relevant sub-samples and estimation of the restricted model (14) using ordinary least squares over the full data set yielded the results in table 7.

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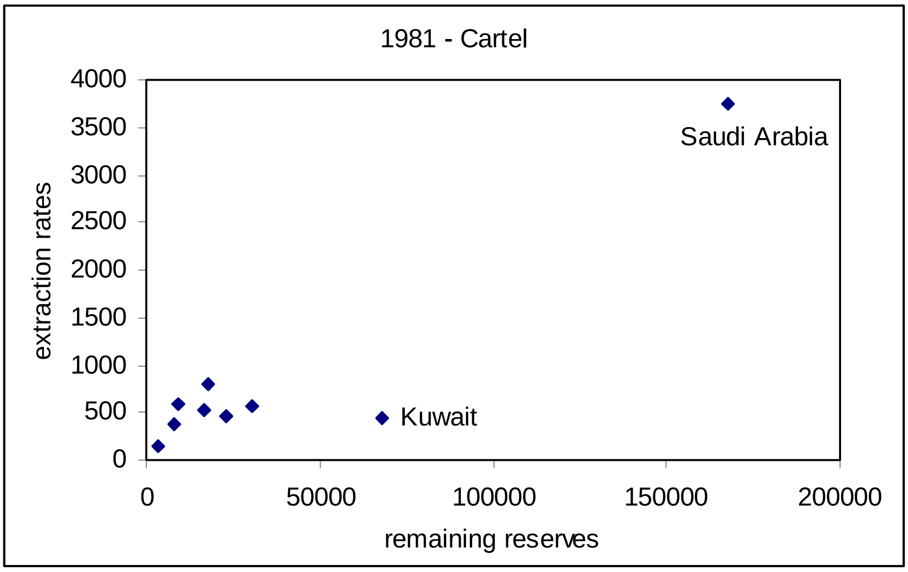
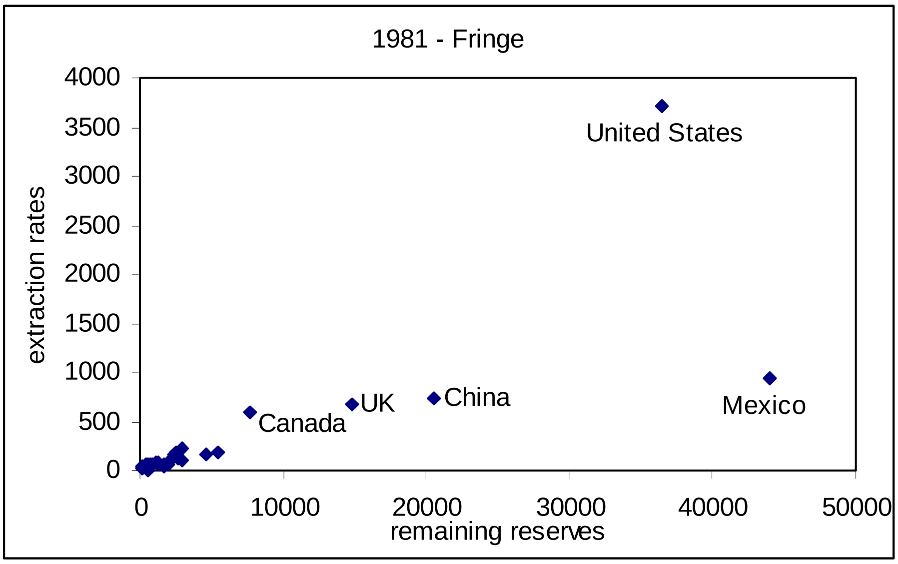
[5] Multiplied by 365 to give a figure for annual extraction rates.
[6] Algeria, Indonesia, Iran, Iraq, Kuwait, Libya, Nigeria, Qatar, Saudi Arabia, Venezuela and the United Arab Emirates.
[7] For the 1981 observation Iran and Iraq are omitted from the sample and in the 1991 case Iraq and Kuwait are omitted.





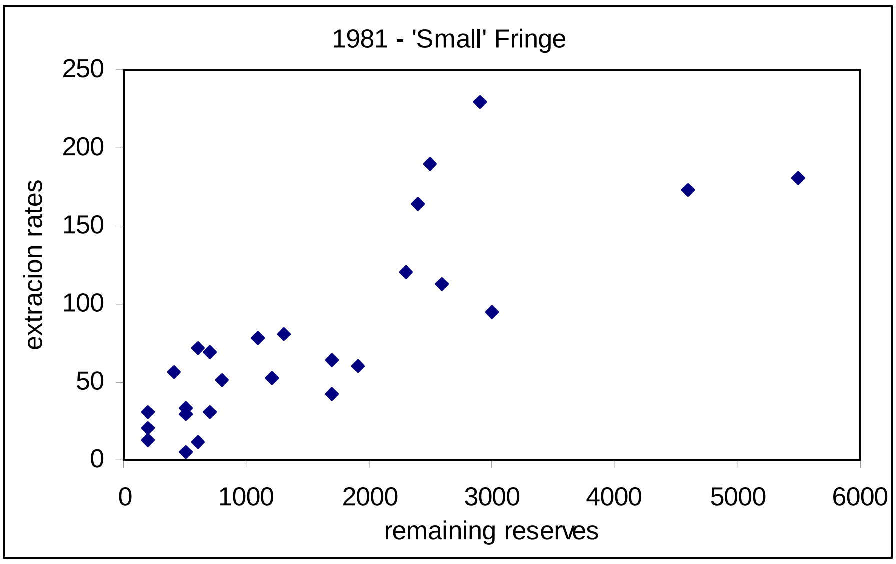


Fig. 1a. *Extraction Rates and Remaining Reserves in 1981*

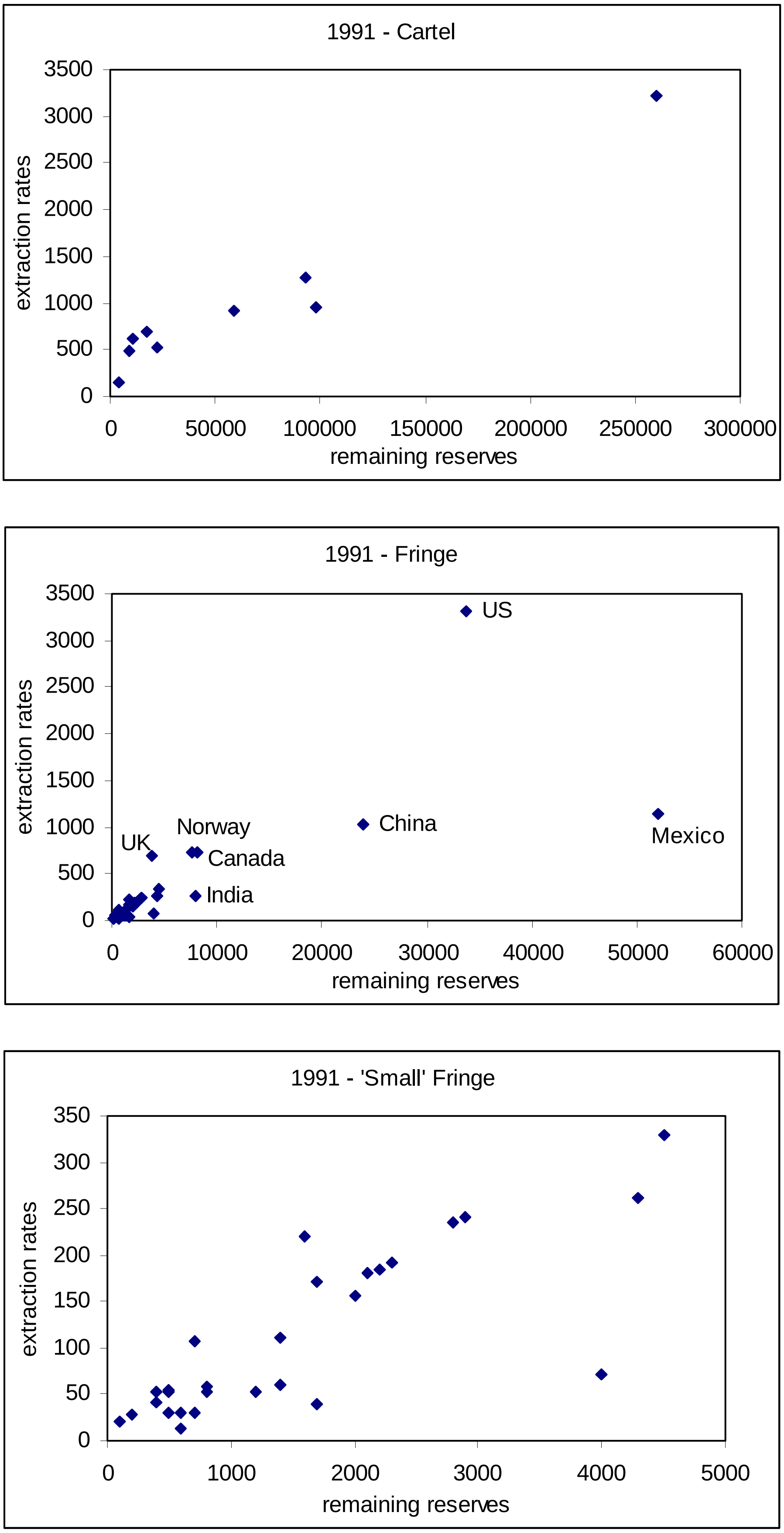


Fig. 1b. *Extraction Rates and Remaining Reserves in 1991*

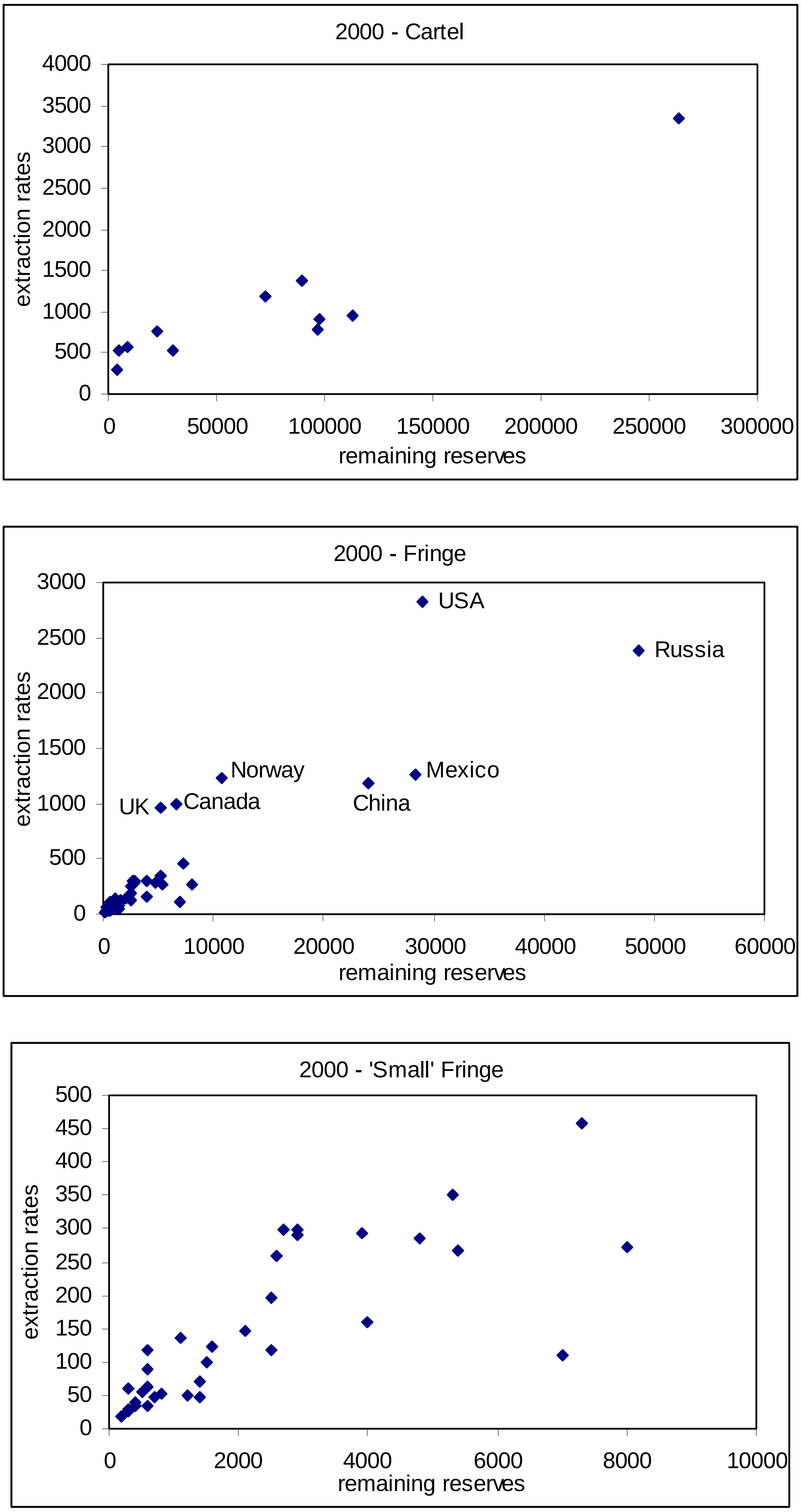


Fig. 1c. *Extraction Rates and Remaining Reserves in 2000*

Table 7

OPEC and fringe estimated oil extraction equations, 1981, 1991 and 2000

Sample	Estimation results[8]	N[9]	$\bar{R}^2$ [10]	BP[11] {p-value}
1981 Restricted; ($F_{1,36}$ = 15.37)[12]	$q_i = 156301 - 213921\,OPEC + 0.0236\,R_{i,-1}$ (1.68) (–0.97) (7.34)	40	0.61	1.39 {0.239}
1981 Unrestricted fringe	$q_i = 8680 + 0.051\,R_{i,-1}$ (0.10) (6.80)	31	0.60	22.08 {0.000}
1981 Unrestricted cartel	$q_i = 99741 + 0.020\,R_{i,-1}$ (0.533) (6.50)	9	0.84	0.858 {0.35}
1991 Restricted; ($F_{1,40}$ = 22.69)	$q_i = 247234 - 84343\,OPEC + 0.013\,R_{i,-1}$ (3.09) (–0.40) (6.55)	44	0.56	0.213 {0.644}
1991 Unrestricted fringe	$q_i = 92991 + 0.043\,R_{i,-1}$ (1.26) (6.72)	35	0.56	19.87 {0.000}
1991 Unrestricted cartel	$q_i = 298404 + 0.011\,R_{i,-1}$ (3.36) (12.09)	9	0.95	0.04 {0.85}
2000 Restricted; ($F_{1,49}$ = 97.91)	$q_i = 306394 - 239854\,OPEC + 0.0131\,R_{i,-1}$ (4.07) (–1.14) (6.71)	53	0.53	0.43 {0.511}
2000 Unrestricted fringe	$q_i = 66278 + 0.057\,R_{i,-1}$ (1.37) (12.89)	42	0.80	12.106 {0.000}
2000 Unrestricted cartel	$q_i = 279302 + 0.010\,R_{i,-1}$ (2.03) (7.62)	11	0.85	3.59 {0.058}

[8] Estimation by ordinary least squares, t-ratios are reported in parentheses.
[9] The sample size.
[10] The 'R-bar' statistic for overall goodness of fit.
[11] A Breusch-Pagan (1979) test for heteroscedasticity. Under the null of homoscedasticity the statistic NR^2 from the regression $\varepsilon_i^2 = \lambda + \mu R_{i,-1}$ follows a chi-squared distribution with one degree of freedom. The significance level is reported in {}.
[12] The F test of the model restriction, with one degree of freedom in the numerator and n degrees of freedom in the denominator where n is the number of degrees of freedom relating to the unrestricted regressions.

Each regression exhibits a reasonable amount of explanatory power, and the estimated coefficients exhibit their expected signs in all cases. Visual inspection of fig. 1 lends support for the hypothesis of linearity; indeed in all cases the quadratic terms in reserves were found to be insignificant and are therefore omitted. However, there is consistent evidence that the slope coefficient differs substantially between OPEC and the competitive producers, contrary to the theory outlined above. The F-test of the null hypothesis of common slopes is rejected at the 1% level in all three cases. However, the Breusch-Pagan test indicates considerable heteroscedasticity within the fringe, which could reflect parameter heterogeneity within the fringe sample. Visual inspection of fig. 1 also suggests a problem of outliers in some instances.

Examination of the residuals reveals that the USA is an extreme outlier in all cases. In particular, extraction rates are in excess of the regression-predicted amount by over three standard errors in every case.[13] Omission of the USA[14] improved estimation in the following way: The slope coefficient for the 1981 and 1991 fringe samples was noticeably reduced in size, but increased in significance. Furthermore, the F-statistic implied by the model restriction becomes insignificantly different from zero for the 1981 sample, although there is still persistent evidence of heteroscedasticity. The model restriction was again firmly rejected in the other two samples.

In order to deal with the heteroscedasticity the weighted least squares procedure described by Pindyck and Rubinfeld (1991, p.130) is employed. This method presupposes that the source of the heteroscedasticity is that implied by the Breusch-Pagan test (i.e. the errors are correlated with remaining reserves – see footnote 11). Estimation, omitting the US and correcting for heteroscedasticity yielded the results in table 8.

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[13] We do not conjecture why the US chooses to 'over-extract' in this way.
[14] The full results are available on request.

Table 8

WLS estimation results, omitting the USA and correcting for heteroscedasticity

Sample	Parameter (t-stat) α	β	γ	N	$\overline{R}^2$	BP {p-value}	F-test {p-value}
1981 Restricted	14802 (4.34)	0.039 (6.48)	1754 (0.02)	39	0.32	0.59 {0.44}	1.95 {0.17}
1981 Unrestricted cartel	118937 (1.85)	0.023 (3.20)		9	0.23	0.63 {0.43}	
1981 Unrestricted fringe	13284 (3.47)	0.044 (5.97)		30	0.28	1.11 {0.29}	
1981 Unrestricted small fringe	13268 (3.12)	0.043 (5.01)		26	0.26	4.66 {0.03}	
1991 Restricted	15321 (4.44)	0.058 (8.06)	-152311 (-1.04)	43	0.33	0.13 {0.71}	6.95 {0.012}
1991 Unrestricted cartel	154363 (2.04)	0.019 (2.73)		9	0.28	2.27 {0.13}	
1991 Unrestricted fringe	13268 (3.69)	0.066 (8.25)		34	0.28	0.02 {0.88}	
1991 Unrestricted small fringe	13874 (4.27)	0.063 (7.90)		28	0.39	0.502 {0.478}	
2000 Restricted	18727 (3.41)	0.058 (7.44)	60091 (0.48)	52	0.16	0.066 {0.80}	13.99 {0.000}
2000 Unrestricted cartel	338429 (7.18)	0.01 (2.25)		11	0.84	3.27 {0.07}	
2000 Unrestricted fringe	12441 (2.20)	0.073 (8.33)		41	0.09	0.193 {0.66}	
2000 Unrestricted small fringe	15512 (2.92)	0.065 (7.30)		35	0.18	1.47 {0.22}	

Application of weighted least squares (WLS) eliminates the previously identified heteroscedasticity. The estimated coefficients maintain their signs and significance, although overall explanatory power is reduced following omission of the USA. Similarly, the OPEC dummy variable remains insignificant although given that the cartel intercept might be higher or lower depending on the cost parameters and the demand function slope this is not necessarily a criticism of the model in itself. Recall that if the market were characterised by competition and the cartel had a large enough cost advantage, then we would expect to find that the OPEC parameter might be positive. Given that it is widely acknowledged that such a cost advantage does exist for OPEC members, then the absence of a positive parameter might indicate that OPEC has been attempting to exercise market power, perhaps along the lines of the Stackelberg model, especially in the early 1980s.

To test the robustness of these parameter estimates the regression was performed on a reduced sample of fringe countries that excludes the larger actors indicated in fig. 1. It is noteworthy that the size and significance of the slope and intercept coefficients are virtually unchanged for the smaller sample. This supports the conjecture of the economic model, that the reserve level itself does not determine the slope relationship, i.e. that the slope term is stable. Thus, although the relationship seems to get noisier as reserve levels increase, the underlying linear relationship seems to hold across countries with different declared levels of remaining reserves whilst belonging to the same competitive type. The strong findings of linearity and the robustness of parameter estimates go some way against the argument that the model is mis-specified.

The model restriction test is passed in 1981, but fails (at the 5% level) in the later two samples. The finding in the 1991 and 2000 samples is that the slope coefficient is significantly lower for the cartel members than the fringe.

One explanation for this lies in the argument that discount rates may differ under different market structures. Examination of the structural model suggests that the slope term is inverse in the discount rate, thus a smaller slope term implies a higher discount rate within the cartel membership. Adelman (1986) argues that OPEC members are more likely to discount at a higher rate primarily due to increased risk exposure from amplified output fluctuations, a non-diversified 'national portfolio' and political risk. Others have countered that given imperfect capital markets OPEC members may discount at lower rates due to the opportunity cost of capital being lower. The findings reported here support Adelman's position, but it should be noted that this is not the only explanation of our findings.

Alternatively, mis-measurement may explain the results. Either the fringe possesses more reserves than they are saying, or OPEC members' true reserves are less than declared. It is unclear why the fringe might systematically understate its own remaining reserves. Laherrere (2001) focuses on the latter possibility, arguing that reported remaining reserves depend upon the image that the author wishes to give (Laherrere, 2001, p9). The award of quota within OPEC, and intra-cartel bargaining strength may thus depend on declared remaining reserves. (Indeed, reported total OPEC reserves increased by 80% over the 1980s as opposed to fringe reserve increases of 10%). Furthermore, if such an overstatement of remaining reserves explains the oil extraction puzzle, then table 8 suggests that the degree of overstatement has increased from 1981 through 1991 to 2001.

5. Conclusions

This paper analyses a duopoly model of non-renewable resource extraction. Optimal extraction plans are derived within two and three period models for five alternative market structures subject to the constraint of market demand, a resource constraint and quadratic extraction technology. An important finding is that regardless of the market structure, the relationship between extraction and remaining reserves reduces to a simple representation – a straightforward linear function wherein differences in costs and market structure are all contained within the intercept term.

The theoretical work lends itself to straightforward econometric estimation. The hypotheses of linearity and parameter stability were widely supported. The key restriction implied by the model is that the slope term of the linear extraction relationship is the same regardless of the cost parameters and market structure. Applying cross-sectional data to the model suggested that the linear relationship with common slopes could not be rejected for the case of 1981 but failed using the 1991 and 2000 data. This puzzle may be explained either by OPEC members discounting at a higher rate or overstatement of their reserves.

Appendix

The proof of proposition 1 requires proof of four statements:

1. $Q_{0A} > Q_{0E}$. Using equations (11A), (11E1) and (11E2) this requires that

$$\frac{(\phi_1+\phi_2)ar}{(2+r)(b\phi_i+b\phi_j+\phi_i\phi_j)} > \frac{[b\phi_1+b\phi_2+\phi_1\phi_2+(b+\phi_1)\phi_1]ar}{(2+r)(b+\phi_1)[2b\phi_1+b\phi_2+\phi_1\phi_2]}$$

hence $\quad (\phi_1+\phi_2)(b+\phi_1)[2b\phi_1+b\phi_2+\phi_1\phi_2] > [2b\phi_1+b\phi_2+\phi_1\phi_2+\phi_1^2](b\phi_1+b\phi_2+\phi_1\phi_2)$,

$$(\phi_1+\phi_2)(b+\phi_1)[2b\phi_1+b\phi_2+\phi_1\phi_2] > [(b+\phi_1)(\phi_1+\phi_2)+b\phi_1](b\phi_1+b\phi_2+\phi_1\phi_2),$$

and $\quad (\phi_1+\phi_2)(b+\phi_1)b\phi_1 > b\phi_1(b\phi_1+b\phi_2+\phi_1\phi_2)$.

2. $Q_{0E} > Q_{0D}$. Using equations (11D1), (11D2), (11E1) and (11E2) then

$$\frac{[b\phi_1+b\phi_2+\phi_1\phi_2+(b+\phi_1)\phi_1]ar}{(2+r)(b+\phi_1)[2b\phi_1+b\phi_2+\phi_1\phi_2]} > \frac{(b+\phi_1+\phi_2)ar}{(2+r)[b^2+2b\phi_1+b\phi_2+\phi_1\phi_2]},$$

hence

$$[b\phi_1+b\phi_2+\phi_1\phi_2+(b+\phi_1)\phi_1][b^2+2b\phi_1+b\phi_2+\phi_1\phi_2] > (b+\phi_1+\phi_2)(b+\phi_1)[2b\phi_1+b\phi_2+\phi_1\phi_2],$$

$$[2b\phi_1+b\phi_2+\phi_1\phi_2+\phi_1^2][b^2+2b\phi_1+b\phi_2+\phi_1\phi_2] > (b^2+2b\phi_1+b\phi_2+\phi_1^2+\phi_1\phi_2)[2b\phi_1+b\phi_2+\phi_1\phi_2],$$

and $\quad \phi_1^2[b^2+2b\phi_1+b\phi_2+\phi_1\phi_2] > \phi_1^2[2b\phi_1+b\phi_2+\phi_1\phi_2]$.

3. $Q_{0D} > Q_{0B}$. Using equations (11B), (11D1) and (11D2)

$$\frac{(b+\phi_1+\phi_2)ar}{(2+r)(b^2+2b\phi_1+b\phi_2+\phi_1\phi_2)} > \frac{(2b+\phi_1+\phi_2)ar}{(2+r)(3b^2+2b\phi_1+2b\phi_2+\phi_1\phi_2)},$$

hence $\quad (b+\phi_1+\phi_2)(3b^2+2b\phi_1+2b\phi_2+\phi_1\phi_2) > (2b+\phi_1+\phi_2)(b^2+2b\phi_1+b\phi_2+\phi_1\phi_2)$,

$$(b+\phi_1+\phi_2)(2b^2+b\phi_2) > b(b^2+2b\phi_1+b\phi_2+\phi_1\phi_2),$$

$$(b+\phi_1+\phi_2)(2b+\phi_2) > b^2+2b\phi_1+b\phi_2+\phi_1\phi_2,$$

and $\quad (b+\phi_1+\phi_2)(2b+\phi_2) > (2b+\phi_2)(b+\phi_1)-b^2$.

4. $Q_{0B} > Q_{0C}$. Using equations (11A) and (11B) this implies

$$\frac{(2b+\phi_1+\phi_2)ar}{(2+r)(3b^2+2b\phi_1+2b\phi_2+\phi_1\phi_2)} > \frac{(\phi_1+\phi_2)ar}{(2+r)(2b\phi_1+2b\phi_2+\phi_1\phi_2)},$$

hence $$(2b+\phi_1+\phi_2)(2b\phi_1+2b\phi_2+\phi_1\phi_2) > (\phi_1+\phi_2)(3b^2+2b\phi_1+2b\phi_2+\phi_1\phi_2),$$

$$2b(2b\phi_1+2b\phi_2+\phi_1\phi_2) > 3b^2(\phi_1+\phi_2),$$

and $$4b^2(\phi_1+\phi_2+\phi_1\phi_2) > 3b^2(\phi_1+\phi_2).$$

Proposition 1 therefore must hold given ϕ_1, ϕ_2, $b > 0$.

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